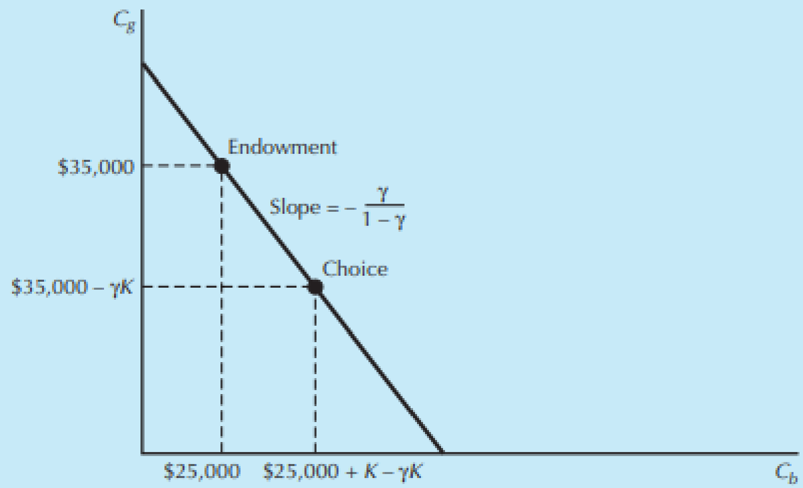
Assignment 1.

12.1:

Plot that illustrates the budget line associated with the purchase of insurance:



Insurance. The budget line associated with the purchase of insurance. The insurance premium γ allows us to give up some consumption in the good outcome (C_g) in order to have more consumption in the bad outcome (C_b).

The concept you're describing is a classic application of microeconomic theory to the decision-making process of purchasing insurance. In this context, the "good state" refers to the scenario where the loss doesn't occur, and the "bad state" refers to the scenario where the loss does occur.

The Budget Line

The budget line represents all the combinations of consumption in the good and bad states that a person can afford given their endowment and the price of insurance. The slope of this budget line is given by $-\frac{\gamma}{1-\gamma}$, where γ is the cost of insurance per dollar of coverage.

Indifference Curves

Indifference curves represent combinations of consumption in the good and bad states that provide the same level of utility to the consumer. A convex shape of the indifference curve implies that the consumer prefers a balanced or "smoothed" consumption profile across states of the world.

Consumption Points to the Left of the Endowment

Consumption points to the left of the endowment point represent scenarios where the consumer has decided to "self-insure" to some extent. In other words, they are willing to bear more risk in the bad state in exchange for higher consumption in the good state.

Here's how it works:

- **In the Good State**: The consumer would have more than their initial endowment of \$35,000 because they did not spend as much on insurance premiums (γK).
- **In the Bad State**: The consumer would have less than their initial endowment of \$25,000 because they did not purchase enough insurance to cover the full loss.

By moving to a point to the left of the endowment, the consumer is essentially "betting" that the bad state will not occur, and thus they can enjoy higher consumption in the good state at the risk of lower consumption in the bad state.

Optimal Choice

The optimal amount of insurance to purchase is determined by the tangency between the budget line and the highest possible indifference curve. At this point, the marginal rate of substitution between consumption in the good and bad states is equal to the slope of the budget line, or the "price" of trading off consumption between states.

The **Budget Line** represents all the combinations of consumption in the good and bad states that a person can afford given their endowment and the price of insurance. The slope of the budget line is given by $-\frac{\gamma}{1-\gamma}$.

The **Marginal Rate of Substitution (MRS)** is the rate at which a consumer is willing to trade off consumption in the good state for consumption in the bad state while keeping utility constant. It is represented by the slope of the indifference curve at a given point.

Optimal Choice

The optimal amount of insurance to purchase is determined by the point where the budget line is tangent to the highest possible indifference curve. At this point, the MRS is equal to the slope of the budget line. In other words, the rate at which you are willing to trade off consumption between the good and bad states (MRS) should be equal to the "price" at which you can actually make that trade (slope of the budget line).

So, while the budget line and the MRS are not the same, they become equal at the point of optimal choice. This is known as the **tangency condition** in economics.

The consumer's attitude toward risk—whether risk-loving, risk-neutral, or risk-averse—will significantly influence their decision to buy or sell insurance. Let's break down how each type of consumer would react:

1. Risk-Averse Consumer

- **Behavior**: Prefers certainty to uncertainty and is willing to pay a premium to avoid risk.
- **Insurance Decision**: Likely to buy insurance to minimize the impact of the bad state, even if the premium is slightly higher than the actuarially fair price.
- **Graphical Representation**: Would choose a point where the budget line is tangent to a high-level indifference curve, maximizing utility given the constraint.

2. Risk-Neutral Consumer

- **Behavior**: Indifferent between a certain and an uncertain prospect if they offer the same expected value.
- **Insurance Decision**: Would buy insurance only if it's priced at the actuarially fair rate, meaning the premium equals the expected loss.
- **Graphical Representation**: Would choose a point where the budget line intersects an indifference curve, but the curve's shape wouldn't matter as much.

3. Risk-Loving Consumer

- **Behavior**: Prefers uncertainty to certainty and is willing to pay a premium for a gamble.
- **Insurance Decision**: Unlikely to buy insurance and might even sell insurance to others, taking on more risk for the chance of higher returns.
- **Graphical Representation**: Would choose a point where the budget line intersects a lower-level indifference curve, indicating a willingness to accept more risk for potentially higher rewards.

In summary, the shape of the indifference curves and their interaction with the budget line will differ based on the consumer's risk preferences. A risk-averse consumer will have steep, convex indifference curves, a risk-neutral consumer will have linear indifference curves, and a risk-loving consumer will have concave indifference curves.

12.2

The Expected Utility Theory posits that if a utility function can be written as a weighted sum of utilities for different states of the world, then that utility function satisfies the expected utility property. In mathematical terms, a utility function $u(c_1, c_2, \pi_1, \pi_2)$ has the expected utility property if it can be written as:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 u(c_1) + \pi_2 u(c_2)$$

Let's examine each of the given utility functions:

(a) $u(c_1, c_2, \pi_1, \pi_2) = a(\pi_1 c_1 + \pi_2 c_2)$

This utility function can be rewritten as $a\pi_1 c_1 + a\pi_2 c_2$, which is a weighted sum of utilities for c_1 and c_2. Therefore, it satisfies the expected utility property.

(b) $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2^2$

This utility function cannot be written as a weighted sum of utilities for c_1 and c_2 because of the c_2^2 term. Therefore, it does not satisfy the expected utility property.

(c) $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + \pi_2 \ln c_2 + 17$

This utility function can be written as a weighted sum of utilities for c_1 and c_2, with an additional constant term 17. The constant term does not affect the expected utility property, so this function satisfies it.

In summary:

- Function (a) satisfies the expected utility property.
- Function (b) does not satisfy the expected utility property.
- Function (c) satisfies the expected utility property.

12.3:

To determine which option a risk-averse individual would choose, we can use the concept of expected utility. A risk-averse individual aims to maximize their expected utility, not just their expected monetary gain.

Option 1: The Gamble

The gamble pays \$1000 with a probability of 25% and \$100 with a probability of 75%. The expected monetary value of the gamble is:

$$\text{Expected Value (Gamble)} = (0.25 \times 1000) + (0.75 \times 100) = 250 + 75 = 325$$

Option 2: The Payment of \$325

The payment of \$325 is a certain outcome, so its expected value is simply \$325.

Expected Utility

For a risk-averse individual, the utility of a certain outcome is generally higher than the expected utility of a risky gamble with the same expected value. In this case, both the gamble and the certain payment have the same expected value of \$325. However, the risk-averse individual would prefer the certainty of receiving \$325 to the gamble.

Conclusion

A risk-averse individual would choose the certain payment of \$325 over the gamble, even though both options have the same expected monetary value.

12.4 :

If the certain payment is reduced to \$320, the decision becomes more nuanced and would depend on the specific utility function of the individual. However, we can still make some general observations.

Option 1: The Gamble

The expected monetary value of the gamble remains the same at \$325.

Option 2: The Payment of \$320

The certain payment is now \$320, which is slightly less than the expected value of the gamble.

Expected Utility

For a risk-averse individual, the utility function is concave, meaning that they derive less additional utility from each extra dollar earned. In this scenario, the expected utility of the gamble would be calculated as follows (assuming a generic utility function $u(x)$):

$$\text{Expected Utility (Gamble)} = 0.25 \times u(1000) + 0.75 \times u(100)$$

The utility of the certain payment of \$320 would simply be $u(320)$.

Decision Criteria

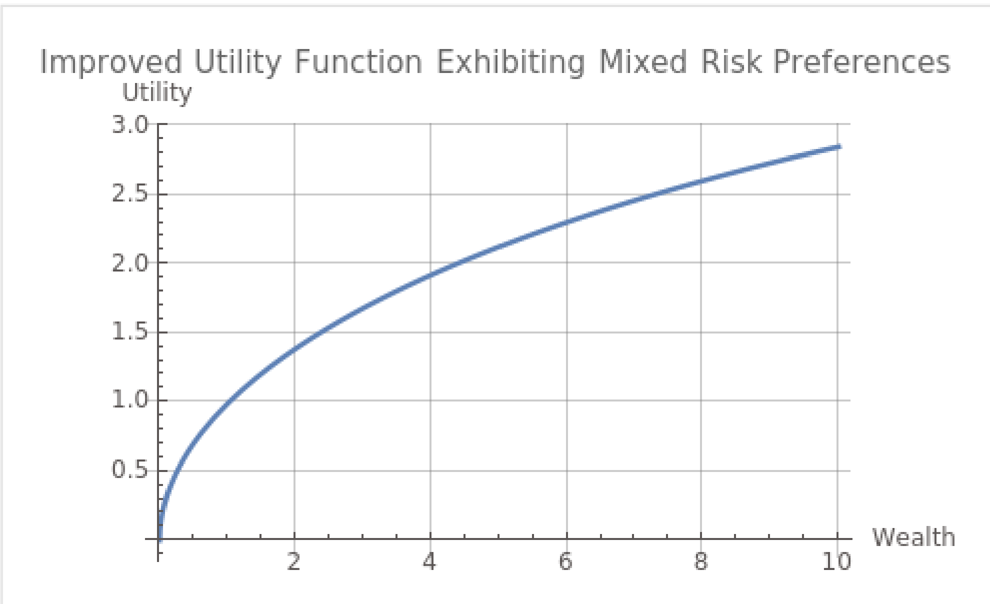
- If $u(320) > 0.25 \times u(1000) + 0.75 \times u(100)$, the individual would choose the certain payment of $320.
- If $u(320) < 0.25 \times u(1000) + 0.75 \times u(100)$, the individual might opt for the gamble.

For a risk-averse individual with a concave utility function, it's possible that the certainty of $320 could be more appealing than the gamble with an expected value of $325, especially if the utility derived from the lower but certain amount outweighs the expected utility of the gamble.

12.5

- **Risk-loving behavior for small gambles**: The function is convex for smaller values of wealth (x).
- **Risk-averse behavior for larger gambles**: The function becomes concave for larger values of wealth (x).

The utility function used here is $U(x) = x^{0.5} - 0.01x^{1.5}$.



- The x-axis represents "Wealth."
- The y-axis represents "Utility."

In this improved version, the utility function remains non-negative and shows a transition from risk-loving to risk-averse behavior as wealth increases. Does this better address your concerns?

Key Points:

1. **Utility Function (Blue Curve)**: The blue curve represents a utility function $u(x) = x^3 - 0.1x^4$. This function is designed to exhibit risk-loving behavior for small gambles and risk-averse behavior for larger gambles.
2. **Linear Utility (Orange Line)**: The orange line represents a linear utility function $u(x) = x$, which is the utility function of a risk-neutral individual.
3. **Wealth vs. Utility**: The x-axis represents wealth, and the y-axis represents utility.

Behavior Explanation:

- **Risk-Loving for Small Gambles**: For small values of x, the utility function is concave upwards, indicating risk-loving behavior.
- **Risk-Averse for Larger Gambles**: For larger values of x, the utility function becomes concave downwards, indicating risk-averse behavior.

Here is the graph of the utility function $U(x) = x^{0.5} - 0.01x^{1.5}$ along with tangent lines at different points:

- The **blue curve** represents the utility function.
- The **orange line** is the tangent at $x = 1$.
- The **green line** is the tangent at $x = 4$.
- The **red line** is the tangent at $x = 8$.

Interpretation:

1. **Risk-Loving at $x = 1$**: The tangent line (orange) lies below the utility curve, indicating that the curve is convex at this point. This represents risk-loving behavior.
2. **Risk-Neutral around $x = 4$**: The tangent line (green) is almost parallel to the utility curve, suggesting that the curve is neither clearly convex nor concave at this point. This represents risk-neutral behavior.
3. **Risk-Averse at $x = 8$**: The tangent line (red) lies above the utility curve, indicating that the curve is concave at this point. This represents risk-averse behavior.

12.6

Self-insuring for flood damage versus fire damage presents different challenges for a neighborhood group, and there are several reasons why self-insuring for flood damage might be more difficult:

1. Scale of Damage:

- **Flood**: Floods often affect a large area and many properties simultaneously.
- **Fire**: Fires are generally more localized and may affect only one or a few properties at a time.

2. **Predictability:**

- **Flood**: Floods can be seasonal or result from extreme weather events, making them somewhat predictable but still hard to prepare for due to their scale.
- **Fire**: Fires can often be prevented through good practices and quick response, making them somewhat more predictable and manageable.

3. **Cost of Mitigation:**

- **Flood**: Flood defenses like dams, levees, and drainage systems are expensive and require long-term planning and construction.
- **Fire**: Fire prevention measures like smoke detectors, fire extinguishers, and fire-resistant materials are relatively inexpensive and easy to implement.

4. **Correlated Risks:**

- **Flood**: If one house in a neighborhood is at risk of flooding, it's likely that many others are too, making the risk highly correlated.
- **Fire**: The risk of fire is generally less correlated among neighbors unless there's a widespread natural disaster like a wildfire.

5. **Insurance Pooling:**

- **Flood**: Due to the correlated and large-scale nature of flood risks, pooling resources to self-insure is more challenging.
- **Fire**: The risks are less correlated and generally smaller in scale, making it easier to pool resources for self-insurance.

6. **Government Support:**

- **Flood**: Government flood insurance programs may exist, but they often have limitations and may not cover all types of flood damage.
- **Fire**: Government support in the form of fire departments is generally more readily available and effective.

7. **Frequency:**

- **Flood**: Some areas may be prone to frequent flooding, making self-insurance unsustainable in the long run.
- **Fire**: Fires are generally less frequent, especially if proper safety measures are in place.

For these reasons, a neighborhood group might find it more challenging to self-insure for flood damage compared to fire damage.

Assignment 2

The utility function $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$ can be simplified to $\pi v(c_1) + (1 - \pi) v(c_2)$ given that $\pi_1 = \pi$ and $\pi_2 = 1 - \pi$.

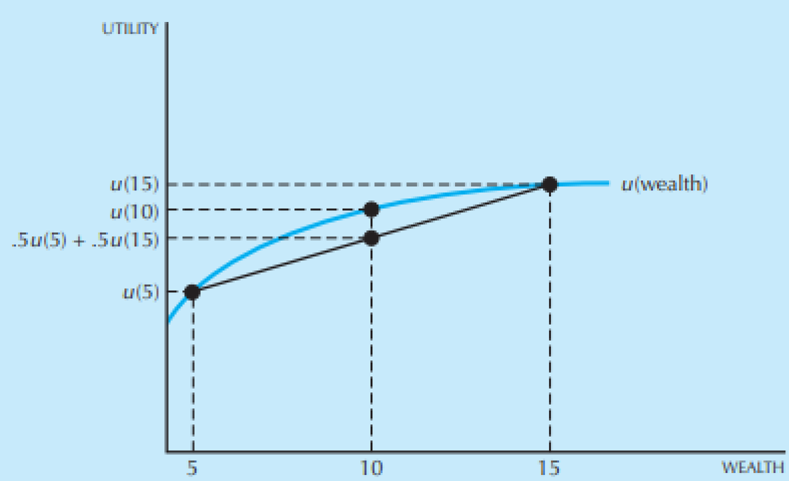
Geometric Argument:

1. **Indifference Curves**: In a 2D space where the x-axis represents consumption in state 1 (c_1) and the y-axis represents consumption in state 2 (c_2), the indifference curves represent combinations of c_1 and c_2 that yield the same utility.
2. **Shape of Indifference Curves**:
    - **Concave v (Risk-Averse)**: If v is concave, the indifference curves will also be concave. This means the consumer is willing to give up more consumption in state 2 to gain a little in state 1, indicating risk aversion.
    - **Convex v (Risk-Loving)**: If v is convex, the indifference curves will be convex. This means the consumer is willing to take on more risk for the chance of higher returns, indicating risk-loving behavior.
    - **Linear v (Risk-Neutral)**: If v is linear, the indifference curves will be linear. This means the consumer is indifferent between risk and certainty as long as the expected value remains the same, indicating risk neutrality.
3. **Budget Line**: The budget line represents all possible combinations of c_1 and c_2 that the consumer can afford. A risk-averse consumer will choose a point where the budget line is tangent to a concave indifference curve, a risk-loving consumer will choose a point where the budget line is tangent to a convex indifference curve, and a risk-neutral consumer will choose a point where the budget line intersects a linear indifference curve.
4. **Tangency and Optimization**: The point of tangency between the budget line and the indifference curve represents the optimal choice for the consumer. The shape of the indifference curve at this point reflects the consumer's attitude toward risk.

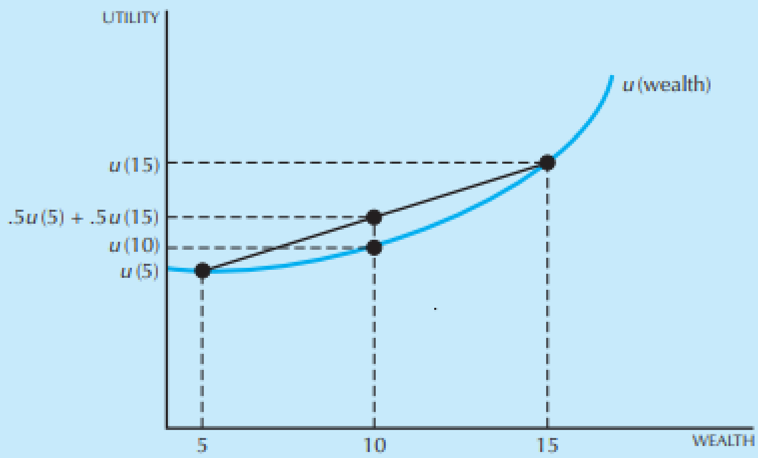
Summary:

- **Risk-Averse**: Concave v → Concave Indifference Curves → Prefers Certainty
- **Risk-Loving**: Convex v → Convex Indifference Curves → Prefers Risk
- **Risk-Neutral**: Linear v → Linear Indifference Curves → Indifferent to Risk vs. Certainty

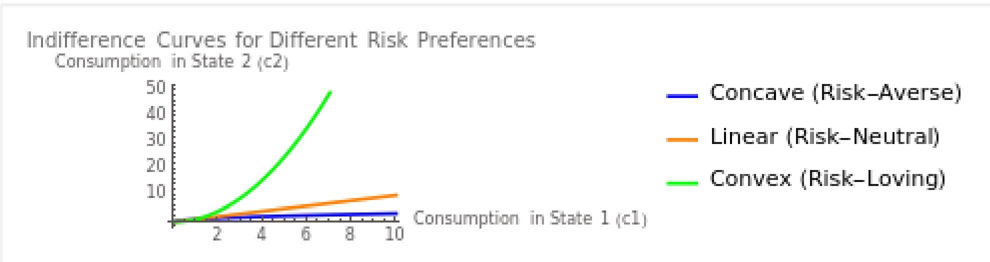
Plot that shows the shapes of indifference curves for different risk preferences:



Risk aversion. For a risk-averse consumer the utility of the expected value of wealth, $u(10)$, is greater than the expected utility of wealth, $.5u(5) + .5u(15)$.



Risk loving. For a risk-loving consumer the expected utility of wealth, $.5u(5) + .5u(15)$, is greater than the utility of the expected value of wealth, $u(10)$.



Key Points:

1. **Concave (Risk-Averse) - Blue Curve**: The blue curve represents a concave utility function, which is typical for a risk-averse consumer. The consumer would prefer more certainty and is willing to give up some consumption in one state to gain a little in another.
2. **Linear (Risk-Neutral) - Orange Line**: The orange line represents a linear utility function, typical for a risk-neutral consumer. The consumer is indifferent between risk and certainty as long as the expected value remains the same.
3. **Convex (Risk-Loving) - Green Curve**: The green curve represents a convex utility function, which is typical for a risk-loving consumer. The consumer prefers taking risks for the chance of higher returns.
4. **Axes**: The x-axis represents consumption in state 1 (c_1), and the y-axis represents consumption in state 2 (c_2).

The shape of these curves reflects the consumer's attitude toward risk.

Assignment 3.

The consumer's problem is to maximize utility subject to a budget constraint. The utility function is given by:

$$\pi v(c_1) + (1 - \pi)v(c_2)$$

The budget constraint is derived from the consumption equations $c_1 = y_1 + (1 - \gamma)K$ and $c_2 = y_2 - \gamma K$. We can rewrite these equations in terms of c_1 and c_2:

$$c_1 = y_1 + (1-\gamma)K$$

$$c_2 = y_2 - \gamma K$$

We can eliminate K from these equations to get a single constraint involving c_1 and c_2:

$$K = \frac{y_2 - c_2}{\gamma} = \frac{c_1 - y_1}{1-\gamma}$$

$$(y_2 - c_2)\gamma = (c_1 - y_1)(1-\gamma)$$

$$y_2\gamma - c_2\gamma = c_1 - y_1 - c_1\gamma + y_1\gamma$$

$$c_1 + y_1\gamma - y_2\gamma = c_2 - c_1\gamma$$

$$c_1 + c_2\gamma - c_1\gamma = y_2\gamma - y_1\gamma$$

$$c_1 + (1 - \frac{\gamma}{\gamma})c_2 = y_2\gamma - y_1\gamma + y_1$$

$$c_1 + (1 - \frac{\gamma}{\gamma})c_2 = y_1 + (y_2 - y_1)\gamma$$

$$c_1 + (1 - \frac{\gamma}{\gamma})c_2 = y$$

where $y = y_1 + (y_2 - y_1)\gamma$.

So the consumer's maximization problem can be written as:

$$\max_{c_1, c_2} \quad \pi v(c_1) + (1-\pi)v(c_2)$$

subject to

$$c_1 + (1 - \frac{\gamma}{\gamma})c_2 = y$$

This confirms that the consumer's maximization problem can indeed be written in the form provided.

b)

We have two options on how to proceed:

It appears that the equations for the Lagrange method are not straightforward to solve symbolically. This is common for optimization problems involving utility functions, as the solutions often depend on the specific functional form of $v(c)$.

Substitution Method: Summary

For the substitution method, we found that the optimal c_2 satisfies:

$$v'(c_2) = \frac{\pi}{1-\pi} v'(y - c_2)$$

Lagrange Method: Summary

For the Lagrange method, the first-order conditions are:

$$\pi v'(c_1) = \lambda$$

$$(1-\pi)v'(c_2) = \lambda$$

$$y = c_1 + c_2$$

These equations would give us the optimal c_1 and c_2 if we could solve them, but they depend on the specific form of $v(c)$.

Economic Interpretation

1. **Substitution Method**: The equation $v'(c_2) = \frac{\pi}{1-\pi} v'(y - c_2)$ tells us how the marginal utility of consumption in state 2 ($v'(c_2)$) relates to the marginal utility of consumption in state 1 ($v'(y - c_2)$). The consumer trades off consumption between the two states based on the probability π of state 1 occurring.
2. **Lagrange Method**: The Lagrange method would give us the marginal utility of income (λ) that equates the marginal utilities across states, subject to the budget constraint. This is a more general approach that doesn't require making one good a function of the other.

Using the Lagrange:

Certainly! Let's start by setting up the consumer's maximization problem using the Lagrangian method. The consumer aims to maximize the utility function:

$$U = \pi v(c_1) + (1-\pi)v(c_2)$$

subject to the budget constraint:

$$c_1 = y_1 + (1-\gamma)K$$

$$c_2 = y_2 - \gamma K$$

The Lagrangian function $\mathcal{L}$ is:

$$\mathcal{L} = \pi v(c_1) + (1-\pi)v(c_2) + \lambda[y_1 + (1-\gamma)K - c_1] + \mu[y_2 - \gamma K - c_2]$$

First-Order Conditions

The first-order conditions (FOCs) are the partial derivatives of $\mathcal{L}$ with respect to c_1, c_2, K, λ, and μ, set equal to zero:

1. $\frac{\partial \mathcal{L}}{\partial c_1} = \pi v'(c_1) - \lambda = 0$
2. $\frac{\partial \mathcal{L}}{\partial c_2} = (1-\pi)v'(c_2) - \mu = 0$
3. $\frac{\partial \mathcal{L}}{\partial K} = \lambda(1-\gamma) - \mu\gamma = 0$
4. $\frac{\partial \mathcal{L}}{\partial \lambda} = y_1 + (1-\gamma)K - c_1 = 0$
5. $\frac{\partial \mathcal{L}}{\partial \mu} = y_2 - \gamma K - c_2 = 0$

Solving the FOCs

1. From FOC 1 and 2, we get:
$$\lambda = \pi v'(c_1)$$
$$\mu = (1-\pi)v'(c_2)$$
2. From FOC 3, we get:
$$\lambda(1-\gamma) = \mu\gamma$$
3. Substituting λ and μ from step 1 into step 2, we get:
$$\pi v'(c_1)(1-\gamma) = (1-\pi)v'(c_2)\gamma$$
$$\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)} = \frac{\gamma}{1-\gamma}$$

Economic Interpretation

The condition $\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)} = \frac{\gamma}{1-\gamma}$ represents the optimal trade-off between consumption in the two states. It shows that the marginal rate of substitution between c_1 and c_2 should be equal to the ratio $\frac{\gamma}{1-\gamma}$, which is the cost of transferring one unit of consumption from state 2 to state 1 via insurance.

In simpler terms, the consumer will choose an insurance coverage K such that the marginal utility of consumption in state 1, weighted by the probability π, is equal to the marginal utility of consumption in state 2, weighted by $1-\pi$, adjusted for the cost of insurance $\frac{\gamma}{1-\gamma}$.

c)

The utility function is $U = \pi v(c_1) + (1-\pi)v(c_2)$.

For $v(c) = \log(c)$, the first and second derivatives are:

1. $v'(c) = \frac{1}{c}$
2. $v''(c) = -\frac{1}{c^2}$

The Marginal Rate of Substitution (MRS) can be defined as:

$$\frac{dc_2}{dc_1} = -\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)}$$

Taking the derivative of $\frac{dc_2}{dc_1}$ with respect to c_1 gives:

$$\frac{d^2c_2}{dc_1^2} = \frac{-\pi v''(c_1)}{(1-\pi)v'(c_2)} + \frac{\pi v'(c_1)v''(c_2)}{(1-\pi)v'(c_2)^2}$$

Substituting the values for $v'(c)$ and $v''(c)$, we get:

$$\frac{d^2c_2}{dc_1^2} = \frac{\pi}{c_1^2(1-\pi)c_2} - \frac{\pi}{c_1(1-\pi)c_2^2}$$

Since $c_1 < c_2$ and $\pi < 1$, $\frac{d^2c_2}{dc_1^2} > 0$, confirming that the indifference curves are convex.

Economic Interpretation

The convexity of the indifference curves implies that the consumer is willing to make trade-offs between c_1 and c_2, but the rate at which they are willing to do so changes. Specifically, the consumer is more willing to give up c_2 for c_1 when they have less of c_1 and more of c_2.

d)

The consumer aims to maximize the utility function:

$$U = \pi v(c_1) + (1-\pi)v(c_2)$$

subject to the budget constraints:

$$c_1 = y_1 + (1-\gamma)K$$

$$c_2 = y_2 - \gamma K$$

The first-order condition for optimization, which equates the MRS to the price ratio, is:

$$\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)} = \frac{\gamma}{1-\gamma}$$

Case 1: $\gamma = \pi$

When $\gamma = \pi$, the first-order condition simplifies to:

$$\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)} = \frac{\pi}{1-\pi}$$

This equation simplifies to $v'(c_1) = v'(c_2)$, meaning that the marginal utility of consumption is the same in both states. In this case, the consumer will choose full insurance to equalize consumption across states, as there is no cost or benefit to shifting consumption from one state to the other.

Case 2: $\gamma > \pi$

When $\gamma > \pi$, the first-order condition becomes:

$$\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)} < \frac{\gamma}{1-\gamma}$$

In this case, the cost of transferring consumption from state 2 to state 1 ($\frac{\gamma}{1-\gamma}$) is greater than the consumer's willingness to pay for it ($\frac{\pi v'(c_1)}{(1-\pi)v'(c_2)}$). Therefore, the consumer will choose less than full insurance to balance the marginal utilities across states while considering the higher cost of insurance.

Economic Interpretation

1. **Full Insurance ($\gamma = \pi$)**: The consumer equalizes the marginal utility of consumption across states, effectively eliminating all risk.
2. **Partial Insurance ($\gamma > \pi$)**: The consumer is less willing to pay the higher premium and will therefore choose a level of insurance that balances the marginal utilities across states, leading to less than full insurance.

e)

The decision to buy insurance is influenced by several factors, including the consumer's risk aversion, the cost of insurance (γ), and the perceived risk (π). Here's a breakdown of the conditions under which insurance will or will not be bought:

Conditions for Buying Insurance:

1. **Fair Premium ($\gamma = \pi$)**: When the premium rate is equal to the probability of the adverse event (flood), a risk-averse consumer will opt for full insurance to eliminate all risk.
2. **High Risk Aversion**: A consumer with a highly concave utility function (indicating strong risk aversion) is more likely to buy insurance, even if $\gamma > \pi$.
3. **High Perceived Risk (π)**: If the consumer perceives a high risk of the adverse event occurring, they are more likely to buy insurance.
4. **Marginal Rate of Substitution**: If $|MRS_{12}(y_1, y_2)| > \frac{\gamma}{1-\gamma}$, the consumer will buy insurance. This condition suggests that the consumer values consumption in the state of the adverse event (c_1) more than in the good state (c_2).

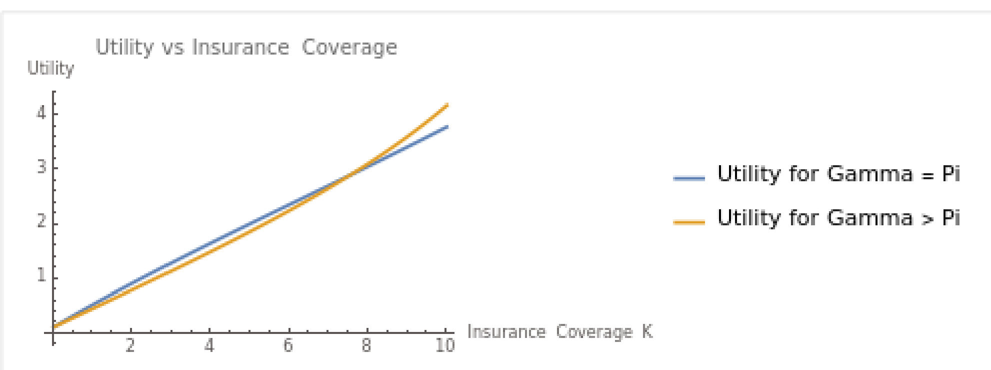
Conditions for Not Buying Insurance:

1. **Unfair Premium ($\gamma > \pi$)**: When the premium rate is higher than the perceived risk, a rational consumer may opt for partial or no insurance, depending on their level of risk aversion.
2. **Low Risk Aversion**: A consumer with a less concave or linear utility function may opt not to buy insurance, as they are less concerned about the risk.
3. **Low Perceived Risk (π)**: If the consumer perceives a low risk of the adverse event occurring, they may decide that insurance is not worth the cost.
4. **Marginal Rate of Substitution**: If $|MRS_{12}(y_1, y_2)| \leq \frac{\gamma}{1-\gamma}$, the consumer may not buy insurance. This suggests that the consumer is less concerned about consumption in the state of the adverse event compared to the good state.

Summary:

- **Will Buy Insurance**: When $\gamma = \pi$ or $|MRS_{12}(y_1, y_2)| > \frac{\gamma}{1-\gamma}$.
- **May Not Buy Insurance**: When $\gamma > \pi$ and $|MRS_{12}(y_1, y_2)| \leq \frac{\gamma}{1-\gamma}$.

Graphical Analysis: Utility vs Insurance Coverage with Utility Function



The utility function is increasing in both c_1 and c_2, and the surface is concave, reflecting the consumer's risk-averse nature.

Marginal Rate of Substitution (MRS)

The MRS between c_1 and c_2 for the utility function $\pi \log(c_1) + (1 - \pi) \log(c_2)$ is:

$$|MRS_{12}| = \left| \frac{c_2 \pi}{c_1(1 - \pi)} \right|$$

Condition for Buying Insurance

You mentioned that $|MRS_{12}(y_1, y_2)| > \frac{\gamma}{1-\gamma}$ is a condition for buying insurance. Let's verify this:

$$|MRS_{12}| = \left| \frac{y_2 \pi}{y_1(1 - \pi)} \right| > \frac{\gamma}{1 - \gamma}$$

This inequality suggests that the consumer would buy insurance if the marginal rate of substitution between y_1 and y_2 is greater than the ratio $\frac{\gamma}{1-\gamma}$.

Interpretation

1. **MRS and Insurance**: The MRS captures how much of c_2 the consumer is willing to give up for an additional unit of c_1. If this rate is greater than $\frac{\gamma}{1-\gamma}$, the consumer finds it beneficial to buy insurance, as the marginal utility of c_1 is relatively higher.
2. **Risk Aversion**: The condition $|MRS_{12}(y_1, y_2)| > \frac{\gamma}{1-\gamma}$ can be seen as a measure of the consumer's risk aversion. A higher MRS indicates a greater willingness to trade off c_2 for c_1, which is consistent with a risk-averse consumer wanting to buy insurance.