Seminar 1 and 2

September 9, 2022

1 From the Seminar 1:

1.1 Exercise 2D:

Verify that the indirect utility function is homogeneous of degree zero in prices and income.

Basically, we want to show that if the prices and income both increase with the same proportion, there is no decrease or increase in utility.

$$\begin{aligned} v\left(t_p, t_y\right) &= \frac{t \cdot y^2}{\left(tp_1\right)\left(tp_2\right) \cdot \frac{1}{4}} \\ &= \frac{t \cdot y^2 \cdot t^{-1}}{p_1 \cdot p_2} \cdot \frac{1}{4} = \frac{y^2}{p_1 p_2} \cdot \frac{1}{y} = V(p, y) \end{aligned}$$

1.2 *Exercise 2F:*

Verify that the expenditure function is strictly increasing in utility, increasing in prices, homogeneous of degree 1, and concave in prices and satisfies Shepard's lemma.

Increasing in utility:

$$e = 2\sqrt{p_1 p_2 u}$$

$$\frac{\partial e}{\partial u} = \frac{p_1 p_2}{\sqrt{p_1 p_2 u}} > 0$$

Increasing in prices:

$$\frac{\partial e}{\partial p_1} = \frac{p_2 u}{\sqrt{p_1 p_2 u}} > 0$$

Concave in prices:

$$e''\left(P_1\right) = -\frac{P_2^2 u^2}{2\left(P_1 p_2 u\right)^{\frac{3}{2}}} < 0$$

Note for your exam! Do not even think about higher dimensions unless you are told otherwise, in this course or in other ones. The higher dimensions are not a part of this course, and you can therefore just calculate the second derivative. Please do not make things more complicated than they are. :)

Homogeneous of degree 1:

We want to show the following:

$$e(u, tp) = t^1 e(p, u), \forall t > 0$$

$$\begin{aligned} e\left(u_1, p_p\right) &= 2\sqrt{\left(t_{p_1}\right)\left(t_{p_2}\right)u} \\ &= 2 \cdot \sqrt{u} \cdot \sqrt{t\left(p_1 p_2\right)} \\ &= 2 \cdot \sqrt{utp_1 p_2} \\ &= 2 \cdot \sqrt{t^1 up_1 p_2} = t^1 e\left(p, u\right) \end{aligned}$$

Verify Shephards lemma:

$e(p, u)$ is differentiable

$$\frac{\partial e(p, u)}{\partial p_i} = x_i^h(p, u) i = 1, 2, \ldots, n$$

Ok, we already know that the expenditure function is differentiable. We are now simply going to verify that the Hicksian demands are returned when we make the partial derivatives on the expenditure function. Remember: The Hicksian demands are whats called the compensated demands. Given a set utility, when the price changes a little, how will your demand change to achieve the same level of utility?

See? When we are looking at Hicksian demands, we are in the indifference curves and the map of them. We are keeping utility constant! We can never observe Hicksian demands!

Now, back to the assignment :)

$$\begin{aligned} \frac{\partial e}{\partial p_1} &= \frac{\partial}{\partial p_1} 2\sqrt{p_1 p_2 u} > 2 \frac{\partial}{\partial p_1}\left(\sqrt{p_1 p_2 u}\right) \\ &= \frac{1}{2\sqrt{p_1 p_2 u}} \cdot \frac{\partial}{\partial p_1}\left(p_1 p_2 u\right. \\ &= \frac{\partial \cdot 1}{2 \cdot \sqrt{p_1 p_2 u}} \cdot p_2 u = \frac{\sqrt{p_2}}{p_1} \cdot \sqrt{u} \end{aligned}$$

Note:

The following expressions, just in case you are wondering, are equivalent.

$$\sqrt{u}\frac{\sqrt{p_2}}{\sqrt{p_1}} = \frac{p_2 u}{\sqrt{p_1 p_2 u}}$$

1.3 Exercise 2G:

Demonstrate that the following relations hold:
This exercise takes us trough relations between what are seemingly somewhat

different concepts. It will demonstrate to us how «easy» we can move from indirect utility to the expenditure function.

We can divide this into two theorems.

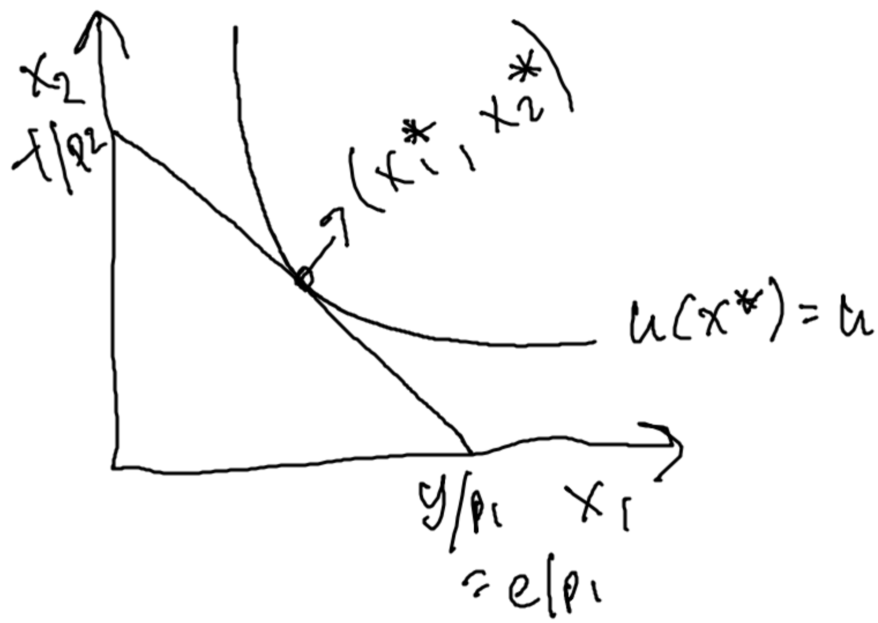
Theorem 1: Let v(p,y) and e(p,u) be the indirect utility and expenditure function for some consumer who's utility function is continuous and strictly increasing. (Rational),. Then for all p >0 and y >0,

1. Relation i)

2. Relation ii)

The power of that theorem is that is allows to derive any of the consumers respective function with only one optimization outcome.

Figure: Maximized utility and minimized expenditure:



Intuition: If x* solves the utility maximisation problem, let u = u(x*), then x* solves the expenditure minimization problem, and vice versa.

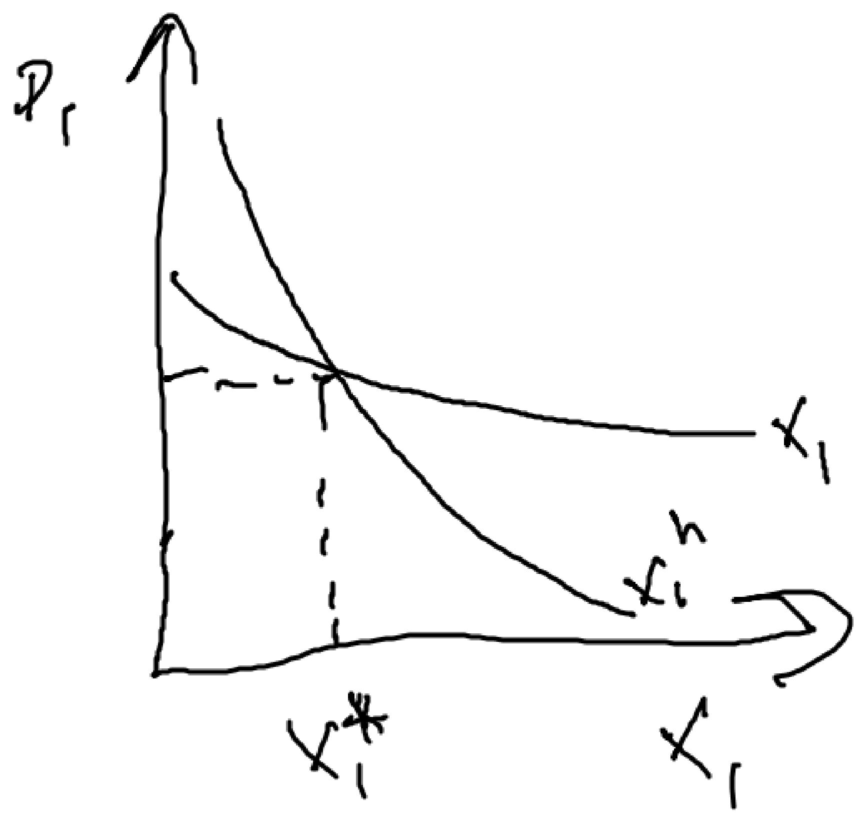
Moreover, we can derive a direct relationship between the quantities demanded.

Theorem 2: If a consumers preferences can be represented by a utility function that is continuous and strictly increasing, and strictly quasi-concave (in the positive realm (normal quadrant of the graph)), we have the following relationship between the Hicksian and the Marshallian demand functions for positive prices and levels of income.

Relation iii) and iv).

That theorem says that the marshallian demand d at prices p and income y is equal to the Hicksian demand at prices p and the utility level u is the same as the marshallian demand evaluated at those prices.

Figure: Relation between marshallian demand and Hicksian



$$x_1^* = x_1^h(p, u) = x_1^h\left(p_1 v(p, y)\right) = x_1(p, y)$$

2 Seminar 2c on uncertainty

An agent with initial wealth

$$W_0$$

Faces a gamble, g, with equal chances to win or loose. Winning is noted as h.

$0 < h < w_0$

$u(w) = \ln(w)$

A) explain that this gamble may be written as:

$g \equiv \left((1/2) \cdot (w_0 + h), (1/2) \cdot (w_0 - h)\right)$

And that,

$$E(g) = w_0$$

Solution:
Well, we may use several approaches here. By looking at the first equation, this clearly states what the gamble may result in either winnings or a loss of money, with the equal chance of both outcomes.

The second equation, that the expected result of the outcome, is that you end up with your winnings. Why enter into this gamble, you may ask?

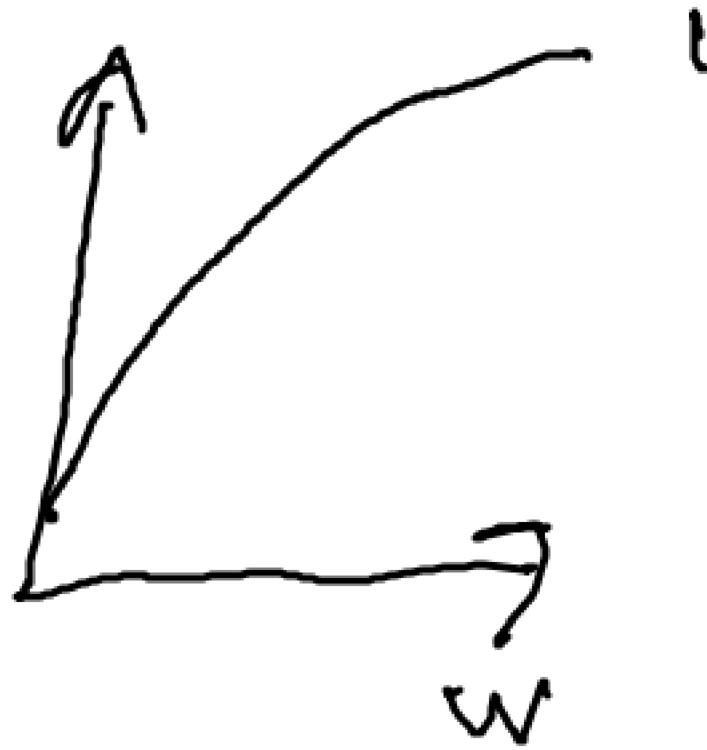
B)
From definition 2.4 we know that an agent is risk averse at g if

$$u(E(g)) > u(g)$$

So, lets test and see if we can demonstrate what we are told!

$$\begin{aligned} E(u(g)) &= \frac{1}{2}\ln(w_0 + h) + \frac{1}{2}\ln(w_0 - h) \\ &= \ln\sqrt{(w_0^2 - h^2)} < \ln w_0 \end{aligned}$$

Note, you could also argue here, as the examples does, that since this is strictly



quasi-concave, remember the figure from the g

The agent is therefore also risk-averse. We will also demonstrate later in the assignment other ways to show the risk aversion, with the Arrow-Pratt measure.

C)

From definition 2.5: The certainty equivalent of any simple gamble g over wealth levels is an amount of wealth, CE, offered with certainty, such that u(g) = u(CE).
The risk premium is an amount of wealth, P, such that P = E(g) - CE.

$$\ln(CE) = \frac{1}{2}\ln(w_0 + h) + \frac{1}{2}\ln(w_0 - h) = \ln\left(w^2 - h^2\right)^{\frac{1}{2}}$$

do ln on both sides

$$CE = \left(w_0^2 - h^2\right)^{\frac{1}{2}}$$

The P is given directly from the solutions above. Clearly this is positive for a risk averse person.

D)

The Arrow-Pratt measure of absolute risk aversion:

$$R_a(w) \equiv \frac{u''(w)}{u'(w)}$$

The Arrow-Pratt measure of relative risk aversion:

$$R_r(w) \equiv R_a(w)w$$

So:

$$u(w) = \ln(w)$$
$$u'(w) = \frac{1}{w}$$
$$u''(w) = -\frac{1}{w^2}$$

$R_a(w) = -\frac{\frac{1}{x^2}}{\frac{1}{x}} = \frac{\frac{1}{w^2}}{\frac{1}{w}}$ Remember:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{c \cdot b} \Rightarrow \frac{1 \cdot w}{w^2 \cdot 1} = \frac{w}{w^2} = \frac{1}{w}$$

As this is positive, we can see that the individual is risk averse, but the aversion is decreasing with the wealth/income.

The relative measure:

$$R_r(w) = \frac{1}{w} \cdot w = 1$$

As this is constant, the agent has constant relative risk aversion.

In combination, the two measures indicates that as the individual gets wealthier, it decreases its risk aversion, and it decreases at a constant rate.