Seminar 11 - Fall Economincs 3220/4220

This solution set is written and made by Jarle Kvile. It should not be taken as the full solution, and should not be thought to be sufficient to get any type of grade on an exam or a test of any kind.

1

a

Clearly a scenario where they have private valuations and don't care about others. This is private value action.

b

If the book considers all the scenarios to be equally likely, and the reservation price influences whether the auction will occur, then yes, it could be suggesting that the reservation price should be set in a way that takes into account all possible bidder valuation configurations.

Let's break down the book's argument with a zero reservation price:

1. In the case of (8,8), the highest bid will be $8.
2. In the case of (8,10) and (10,8), the highest bid will be $9 because the bidder with the $10 valuation will only need to outbid the $8 valuation by the minimum increment.
3. In the case of (10,10), the highest bid will be $10.

If all scenarios are equally likely, the expected profit is the average of the highest bids across these scenarios:

- Expected profit = (Probability of (8,8) * Highest bid in (8,8)) + (Probability of (8,10) * Highest bid in (8,10)) + (Probability of (10,8) * Highest bid in (10,8)) + (Probability of (10,10) * Highest bid in (10,10))
- Expected profit = (0.25 * $8) + (0.25 * $9) + (0.25 * $9) + (0.25 * $10)
- Expected profit = $2 + $2.25 + $2.25 + $2.5
- Expected profit = $9

This is how the book arrives at an expected profit of $9 with a zero reservation price.

When the book considers a reservation price of $10:

- The auction will only occur when at least one bidder values the item at $10. This happens in two of the four scenarios: (10,8) and (10,10). Therefore, with a reservation price of $10, the item will sell half of the time.

If the book's expected profit with a reservation price of $10 is $7.50, it implies that:

- Expected profit = (Probability of sale * Sale price)
- $7.50 = Probability of sale * $10

To get a probability of sale that would result in an expected profit of $7.50:

- Probability of sale = $7.50 / $10
- Probability of sale = 0.75

This suggests that the book is assuming there is a 75% chance of the item selling when the reservation price is $10

c)

To guarantee that the bidders with the two highest values get the books in a sealed-bid auction, you can use a variant of the sealed-bid auction called the "second-price sealed-bid auction," also known as a Vickrey auction, but with a slight modification to accommodate two identical items instead of one.

Here's how the modified second-price sealed-bid auction would work for two items:

1. Each of the three students submits a bid in a sealed envelope, without knowing the others' bids.
2. All bids are opened simultaneously, and the two highest bidders win the items.
3. Instead of paying their own bids, both winners pay a price equal to the third-highest bid (the highest losing bid).

This auction format has several desirable properties:

- **Truthful Bidding**: It encourages each bidder to bid their true value for the books because they know they will only need to pay the amount of the third-highest bid if they win. There's no advantage to bidding higher or lower than their actual valuation since the payment is determined by the next highest loser's bid.
- **Efficiency**: The auction is efficient because the two students who value the books the most end up winning them.
- **Simple and Transparent**: The rules are straightforward, and the outcome is easy to determine once all bids are revealed.

This method guarantees that the students with the highest and second-highest valuations will get the books and pay a price equal to the third-highest valuation. If you have identical items and wish to sell them in a single auction to multiple highest bidders, this adaptation of the Vickrey auction is one of the simplest and most effective methods.

d)

It was efficient in the sense that it awarded the license to the firm that valued it most highly. But it took a year for this to happen, which is inefficient. A Vickrey auction or an English auction would have achieved the same result more quickly

e)

The auction of a jar filled with pennies can represent elements of both private-value and common-value auction types depending on specific factors:

1. **Private-Value Auction**: If each bidder has a unique estimation of the value of the jar based on their own subjective assessment and private information (like a particular use for the pennies or an aesthetic appreciation for the jar), then it leans towards a private-value auction. Here, the value of the jar to each bidder is independent of the others' valuations.
2. **Common-Value Auction**: If the value of the jar is essentially the same to all bidders because it is determined by the objective content (the number of pennies in the jar), but they don't know that exact number, it is more like a common-value auction. In this scenario, all the bidders are effectively estimating the same underlying value (the market value of the pennies), but they might have different information or abilities to estimate that value.

For the jar of pennies, the situation could be described as a "common value" auction because, despite each bidder's private estimate, the actual value of the jar is determined by the total amount of money in pennies inside it — a fact that is the same for all bidders.

As for the winning bidder making a profit, this depends on several factors, including the accuracy of the bids about the jar's true value and the competition in the auction. In common-value auctions, there's a well-known phenomenon called the "winner's curse," which suggests that the winner of an auction for a common-value item often overpays due to competition and the pressure to win. This happens because the winner may tend to bid more than the item's actual value, having been influenced by their overestimation or by the aggressive bidding of others who also overestimate the value.

If the bidders are rational and aware of the winner's curse, they may adjust their bids accordingly, trying to avoid winning the auction at a cost that exceeds the jar's actual value. However, if the bidders fail to account for the winner's curse, the winning bidder might pay too much and therefore would not make a profit. In classroom settings, where bidders might not be as experienced or may get caught up in the excitement of the auction, the winner's curse is a very likely outcome.

2

1)

V : valuation $= \frac{3}{5}$
b: bid
$\hat{v}$: opponents valuation $\hat{V} \sim$ Uniform over $(0,1)$
$\hat{b}$: opponents bid

$$\hat{b}(\hat{v}) = \hat{v}^2$$

Your payoff:

$$\begin{array}{ll} v - b = \frac{3}{5} - b & \text{if } b \geq \hat{b} \\ 0 & \text{if } b < b \end{array}$$

Expected payoff
Payout when losing - prob of losing + Payout when winding - prob of winning

$$\begin{aligned} &= 0 + \left(\frac{3}{5} - b\right) \cdot p(\hat{b} < b) \\ &\quad + \left(\frac{3}{5} - b\right) \cdot \rho\left(\hat{v}^2 \leq b\right) \\ &\quad + \left(\frac{3}{5} - b\right) \cdot \rho(\hat{v} < \sqrt{b}) \end{aligned}$$

Since $\hat{V}$ is uniformly distributed over o to 1, we know that $\rho(\hat{v} < \sqrt{b}) = \sqrt{b}$

So

$$\left(\frac{3}{5} - b\right) \cdot \rho(\hat{v} < \sqrt{b}) = \left(\frac{3}{5} - b\right) \times \sqrt{b}$$

$$E(u) = \frac{3}{5} b^{\frac{-1}{2}} - b^{\frac{3}{2}}$$

$$
\begin{aligned}
\frac{dE(u)}{db} &= \frac{1}{2}\frac{3}{5}b^{-\frac{1}{2}} \cdot \frac{3}{2}\sqrt{6} = 0 \\
&= \frac{3}{10}b^{-\frac{1}{2}} = \frac{3}{2}b^{\frac{1}{2}} \\
&= \frac{3}{10} \cdot \frac{2}{3} = 6 \\
&= 0,2 \\
&= \frac{1}{5} = b
\end{aligned}
$$

In conclusion, with the strategy $b_n = \frac{v_n}{3}$, your optimal bid would indeed be slightly above 0.2, such as $b = 0.2 + \epsilon$, where ϵ is an infinitesimally small amount, ensuring you win against any opponent bid based on a valuation up to 0.6. This is the maximum bid you would want to make to ensure a non-negative payoff. If your bid is $b = 0.2$ (exactly) as per the suggested strategy, you maximize your expected payoff while avoiding the risk of overbidding.

b)

In a second-price (or Vickrey) auction, each bidder submits a bid in a sealed envelope, and the highest bidder wins, but the price paid is the second-highest bid. The dominant strategy in this auction format is for each bidder to bid their true valuation of the item. This is because the winner pays the second-highest bid, not their own, so there is no advantage to bidding either above or below one's true valuation.

To show that bidding one's true valuation v_i is weakly preferred to bidding any $x < v_i$:

Let's consider a bidder with valuation v_i, and let's assume there are two cases:

1. The bidder bids v_i and wins the auction. This means their bid was at least as high as the highest other bid, say b_{other}. They pay b_{other}, which is less than or equal to their valuation v_i, so their utility is $u = v_i - b_{\text{other}}$, which is non-negative.
2. The bidder bids x where $x < v_i$ and still wins the auction. This would only happen if $b_{\text{other}} < x < v_i$. They pay b_{other}, so their utility is still $u = v_i - b_{\text{other}}$, which is the same as if they had bid their true valuation.
3. The bidder bids x and loses the auction to someone who bid b_{other} where $x < b_{\text{other}} \leq v_i$. They would have won if they had bid v_i, and since b_{other} is less than or equal to v_i, they would have derived positive utility from winning (since $v_i > b_{\text{other}}$).

Therefore, in the first two cases, the bidder gets the same utility whether they bid x or v_i, but in the third case, bidding below their valuation causes them to lose an auction that they would have preferred to win (since their valuation is higher than the winning bid). Bidding v_i ensures that they never lose an auction to a bid that is less than their valuation, guaranteeing they do not miss out on a positive utility outcome.

So, in a second-price auction, bidding less than your valuation could cause you to lose the item and miss out on positive utility while bidding your true valuation does not have this risk. That's why bidding v_i is weakly preferred to bidding any $x < v_i$; it gives the bidder the same or better utility in every case.

c)

1)

In a second-price sealed-bid auction, each bidder submits one bid without knowing the others' bids. The highest bidder wins the item, but the price paid is the second-highest bid.

To determine the winner and the price paid in this auction for the desk, we look at the valuations each person has for the desk:

- Ann: $45
- Bill: $53
- Colin: $92
- Dave: $61
- Ellen: $26
- Frank: $78
- Gale: $82
- Hal: $70
- Irwin: $65
- Jim: $56

The winner will be the person with the highest valuation since in a second-price auction, it's a dominant strategy for each bidder to bid their true valuation.

Colin has the highest valuation at $92, so Colin will win the auction.

The price Colin will pay is equal to the second-highest bid. The second-highest valuation is Gale's at $82.

Therefore, Colin will win the desk and pay $82 for it.

2

In a first-price sealed-bid auction, the highest bidder wins the item and pays the amount they bid. When valuations are common knowledge among bidders, the strategy in a first-price auction becomes more about outbidding the second-highest valuation by the minimum bid increment rather than trying to estimate the others' valuations.

In this case, since the bidders know each other's valuations and everyone is bidding rationally and strategically, the person with the highest valuation (Colin, with $92) knows that he only needs to bid slightly more than the second-highest valuation to win the desk. Since the valuations are common knowledge and all the bidders are rational, they all understand that bidding above their own valuation would result in negative utility (paying more than what the item is worth to them), and bidding below the highest valuation they know about is futile if they want to win.

Given that the next highest valuation after Colin is Gale at $82, Colin should bid just slightly above $82. If we assume the minimum bid increment is $1, Colin should bid $83.

So, in the Nash equilibrium for this game, Colin will win the desk by bidding $83. No other bidder has an incentive to bid higher because they know they cannot derive positive utility by doing so (it would exceed their valuation), and Colin has no incentive to bid higher than necessary to outbid Gale.

3

Since the seller knows the valuation, the seller should simply announce the highest price. Colin wins again and pays $92.

d)

As in the text, without the reservation price the privce should be 300.

The example provided in the question illustrates how setting a reserve price in an auction can potentially increase the expected revenue for the seller compared to holding an auction without a reserve price. Here's a step-by-step explanation of the example:

1. **Expected Revenue Without Reserve Price**:
   The text states that the expected revenue without a reserve price is $300. This is based on the bidding strategies of the players and the distribution of their valuations.
2. **Setting a Reserve Price**:
   The auctioneer decides to set a reserve price r. For simplicity, let's consider a sealed-bid first-price auction. Players will only bid if their valuation v_i is greater than the reserve price r. The probability that a player's valuation is less than r is $\frac{r}{900}$, because valuations are uniformly distributed between 0 and 900.
3. **Probability of Both Valuations Below r**:
   The probability that both players have valuations less than r is $\left(\frac{r}{900}\right)^2$.
4. **Setting r at 450**:
   The auctioneer sets $r = 450$. The probability that at least one player has a valuation above 450 is the complement of the probability that both valuations are below 450. So, it's:
   $$1 - \left(\frac{450}{900}\right)^2 = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$
5. **Expected Revenue With Reserve Price**:
   If at least one player's valuation is above the reserve price, the auctioneer gets at least r in revenue. The expected revenue when $r = 450$ is therefore:
   $$450 \times \frac{3}{4} = 337.5$$

This expected revenue of $337.5 is higher than the $300 expected revenue without a reserve price.

What this shows is that by setting a reserve price at the midpoint of the distribution, the auctioneer can increase the expected revenue because it's more likely that at least one player will have a valuation above the reserve price and the item will sell for at least that reserve price. This example makes clear assumptions about the behavior of the bidders and their valuations, which can vary in more complex models or real-world scenarios.

e)

Given Player 1's bid b_1, Player 2's bid b_2, and their respective valuations v_1 and v_2, each uniformly distributed on [0, 1], the expected payoff for Player 1 when he bids b is:

$$\text{Expected Payoff}_1 = v_1 \Pr(\hat{b} \leq b) - b$$

Here, $\hat{b}$ is the bid of Player 2 (the opponent), and $\Pr(\hat{b} \leq b)$ is the probability that Player 2's bid is less than or equal to Player 1's bid b. If the bid function of the opponent is $b(v)$, then we have:

$$\Pr(\hat{b} \leq b) = \Pr(b(\hat{v}) \leq b) = \Pr(\hat{v} \leq b^{-1}(b)) = b^{-1}(b)$$

The expected payoff can therefore be written as $v_1 b^{-1}(b) - b$. The first-order condition for maximum expected payoff implies:

$$v_1 b^{-1\prime}(b) = 1$$

This implies:

$$b'(v_1) = 1$$

Now, integrating this with respect to v_1, we obtain the bidding function $b(v)$:

$$b(v) = \frac{1}{2}v^2 + A$$

Since the bid function must start from zero (i.e., $b(0) = 0$), it implies that $A = 0$. Thus, we have:

$$b(v) = \frac{1}{2}v^2$$

This is the symmetric Nash equilibrium bidding function for both players. Each player bids half the square of his valuation, and that is their best response to the bidding strategy of the other player.