Seminar 3

This solution set is written and made by Jarle Kvile. It should not be taken as the full solution, and should not be thought to be sufficient to get any type of grade on an exam or a test of any kind.

Game Theory Overview

Introduction

We choose static game with complete information as our benchmark because it's the simplest game. More complex games need stricter conditions to locate the equilibria, i.e., refinement (rule out unrealistic NE).

- **Be sure to know which type the game is before answering any question!**

Complete Information

Players have symmetric information (no private information) and know clearly which "type" their opponents are.

Static (Watson Part II)

Players act simultaneously and independently.

- Belief, Pure/Mixed strategy
- Rationality, Dominance & Nash Equilibrium (NE)

Dynamic (Watson Part III)

One move after another.

- Empty threat: not every NE are realistic, refinement needed.
- Subgame Perfect Nash Equilibrium (SPNE)
- Imperfect information: examine every subgame by hand (Watson pp. 190)
- Perfect information: backward-induction method
- Repeated game & trigger strategy

Incomplete Information (Watson Part IV)

At least one player does not know which type his/her opponents are.

- Exogenous move: nature decides player's type.
- Players without private information can only "guess" (assign probability/belief to) the type of his/her opponent.

Static (Watson pp.327 - 377)

- Bayesian Nash Equilibrium (BNE)
- Bayesian normal form

Dynamic (Watson pp.378 - 406)

More information can be obtained from the opponent's behavior.
⇒ Probability (belief) can be adjusted dynamically (updated).

- Perfect Bayesian (Nash) Equilibrium (PBNE)
- Screening: player **without** private information moves first (e.g., Insurance scheme to screen risky clients; Contract to rule out low-capacity workers).
- Signaling: player **with** private information moves first (High-risk clients pretend to be of low-risk).
- Pooling & Separating equilibrium.

Additional Notes

Many students treated a dynamic game as static last year...

Narrative like "player 2 acts before/after player 1" is definitely dynamic. You must refine the NE properly!

- Correctly drawn extensive form for complex dynamic games may gain some points :)
- Do more exercises on screening and signaling games until you can solve it by yourself. It may give you an extra 20 points (could be the difference between C+ and A-) in the exam.

Assignment 1: Payoffs

A)

Problem Statement

Consider a two-player game represented in normal form. The players are labeled as Player 1 and Player 2. The strategies available to Player 1 are U, M, D and those available to Player 2 are L, C, R.

The payoff matrix is as follows:

		Player 2		
		L	C	R
Player 1	U	$(10,0)$	$(0,10)$	$(3,3)$
	M	$(2,10)$	$(10,2)$	$(6,4)$
	D	$(3,3)$	$(4,6)$	$(6,6)$

A mixed strategy for Player 1 is denoted as $s_1 \in \Delta\{U, M, D\}$, where $s_1(U)$ is the probability that Player 1 plays strategy U, and so forth. Similarly, for Player 2, a mixed strategy is $s_2 \in \Delta\{L, C, R\}$.

For simplicity, we write s_1 as $(s_1(U), s_1(M), s_1(D))$, and similarly for Player 2.

Questions

1. $u_1(U, C)$
2. $u_2(M, R)$
3. $u_2(D, C)$
4. $u_1(s_1, C)$ for $s_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$
5. $u_1(s_1, R)$ for $s_1 = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$
6. $u_1(s_1, L)$ for $s_1 = (0, 1, 0)$
7. $u_2(s_1, R)$ for $s_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$
8. $u_2(s_1, s_2)$ for $s_1 = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and $s_2 = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$

Now, let's proceed to solve each of these questions.

To evaluate the payoffs in a game with mixed strategies, we can use the concept of expected utility. Let's assume that player 1 has a mixed strategy $s_1 = (s_1(U), s_1(M), s_1(D))$ and player 2 has a mixed strategy $s_2 = (s_2(L), s_2(C), s_2(R))$.

The expected payoff for player 1, given these mixed strategies, can be calculated as follows:

$$\text{Expected Payoff for Player 1} = \sum_{i \in \{U,M,D\}} \sum_{j \in \{L,C,R\}} s_1(i) \times s_2(j) \times \text{Payoff}_1(i, j)$$

Similarly, the expected payoff for player 2 can be calculated as:

$$\text{Expected Payoff for Player 2} = \sum_{i \in \{U,M,D\}} \sum_{j \in \{L,C,R\}} s_1(i) \times s_2(j) \times \text{Payoff}_2(i, j)$$

Where $\text{Payoff}_1(i, j)$ and $\text{Payoff}_2(i, j)$ are the payoffs for player 1 and player 2, respectively, when player 1 chooses strategy i and player 2 chooses strategy j.

Full Solutions

1. $u_1(U, C)$

To find $u_1(U, C)$, we look at the payoff for Player 1 when Player 1 chooses strategy U and Player 2 chooses strategy C. From the payoff matrix, this is $(0, 10)$. The payoff for Player 1 is 0.

4. $u_1(s_1, C)$ for $s_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$

To find $u_1(s_1, C)$, we use the expected utility formula for Player 1:

$$\text{Expected Payoff for Player 1} = \sum_{i \in \{U,M,D\}} s_1(i) \times \text{Payoff}_1(i, C)$$

Using $s_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$ and the payoffs from the matrix:

$$u_1(s_1, C) = \frac{1}{3} \times 0 + \frac{2}{3} \times 10 + 0 \times 4$$

$$u_1(s_1, C) = 0 + \frac{20}{3} + 0$$

$$u_1(s_1, C) = \frac{20}{3}$$

5. $u_1(s_1, R)$ for $s_1 = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$

To find $u_1(s_1, R)$, we use the expected utility formula for Player 1:

$$\text{Expected Payoff for Player 1} = \sum_{i \in \{U,M,D\}} s_1(i) \times \text{Payoff}_1(i, R)$$

Using $s_1 = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ and the payoffs from the matrix:

$$u_1(s_1, R) = \frac{1}{4} \times 3 + \frac{1}{2} \times 6 + \frac{1}{4} \times 6$$

$$u_1(s_1, R) = \frac{3}{4} + 3 + \frac{3}{2}$$

$$u_1(s_1, R) = \frac{3}{4} + \frac{12}{4} + \frac{6}{4}$$

$$u_1(s_1, R) = \frac{21}{4}$$

8. $u_2(s_1, s_2)$ **for** $s_1 = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ **and** $s_2 = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$

To find $u_2(s_1, s_2)$, we use the expected utility formula for Player 2:

$$\text{Expected Payoff for Player 2} = \sum_{i \in \{U,M,D\}} \sum_{j \in \{L,C,R\}} s_1(i) \times s_2(j) \times \text{Payoff}_2(i, j)$$

Using $s_1 = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and $s_2 = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$:

$$u_2(s_1, s_2) = \frac{1}{2} \times \frac{1}{4} \times 0 + \frac{1}{2} \times \frac{1}{4} \times 10 + \frac{1}{2} \times \frac{1}{2} \times 3 + \frac{1}{2} \times \frac{1}{4} \times 10 + \frac{1}{2} \times \frac{1}{4} \times 2 + \frac{1}{2} \times \frac{1}{2} \times 4$$

$$u_2(s_1, s_2) = 0 + \frac{5}{4} + \frac{3}{4} + \frac{5}{4} + \frac{1}{4} + \frac{2}{4}$$

$$u_2(s_1, s_2) = \frac{5}{4} + \frac{3}{4} + \frac{5}{4} + \frac{1}{4} + \frac{2}{4}$$

$$u_2(s_1, s_2) = \frac{16}{4}$$

$$u_2(s_1, s_2) = \frac{9}{2}$$

Quick Solutions

2. $u_2(M, R)$

The payoff for Player 2 when Player 1 chooses M and Player 2 chooses R is 4.

3. $u_2(D, C)$

The payoff for Player 2 when Player 1 chooses D and Player 2 chooses C is 6.

6. $u_1(s_1, L)$ **for** $s_1 = (0, 1, 0)$

The expected payoff for Player 1 is 2.

7. $u_2(s_1, R)$ **for** $s_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$

The expected payoff for Player 2 is $\frac{11}{3}$.

B)

Cournot Duopoly Game

Consider a Cournot duopoly game where two firms, labeled as Firm 1 and Firm 2, simultaneously and independently decide on the quantities to produce in a market. The quantity selected by Firm i is denoted as q_i and must be greater than or equal to zero ($q_i \geq 0$) for $i = 1, 2$.

Market Price

The market price p is determined by the equation:

$$p = 100 - 2q_1 - 2q_2$$

Costs and Payoffs

Each firm produces at a cost of 20 per unit. The payoff for each firm is defined as its profit.

Player 1's Belief

Firm 1 believes that Firm 2 is equally likely to select each of the quantities $q_2 = 6, 11, 13$.

Question

What is Firm 1's expected payoff if it chooses to produce a quantity $q_1 = 14$?

Economic Intuition

In the Cournot duopoly model, firms compete by choosing quantities to maximize their profits. The market price depends on the total quantity produced, and each firm's profit depends on both its own and its competitor's production levels. In this scenario, Firm 1 is trying to anticipate Firm 2's behavior to make an optimal decision. The belief that Firm 2 is equally likely to produce 6, 11, or 13 units adds a probabilistic element to Firm 1's decision-making process.

Full Solution

Step 1: Calculate Profit Function for Firm 1

The profit π_1 for Firm 1 is given by:

$$\pi_1 = p \times q_1 - 20 \times q_1$$

Substituting the equation for p:

$$\pi_1 = (100 - 2q_1 - 2q_2) \times q_1 - 20 \times q_1$$

$$\pi_1 = 100q_1 - 2q_1^2 - 2q_1q_2 - 20q_1$$

$$\pi_1 = 80q_1 - 2q_1^2 - 2q_1q_2$$

Step 2: Calculate Expected Payoff for $q_1 = 14$

Firm 1 believes that Firm 2 is equally likely to choose $q_2 = 6, 11, 13$. Therefore, the expected payoff $E[\pi_1]$ is:

$$E[\pi_1] = \frac{1}{3}(\pi_1(q_1 = 14, q_2 = 6) + \pi_1(q_1 = 14, q_2 = 11) + \pi_1(q_1 = 14, q_2 = 13))$$

Substituting into the profit function:

$$E[\pi_1] = \frac{1}{3}\left(80 \times 14 - 2 \times 14^2 - 2 \times 14 \times 6 + 80 \times 14 - 2 \times 14^2 - 2 \times 14 \times 11 + 80 \times 14 - 2 \times 14^2 - 2 \times 14 \times 13\right)$$

$$E[\pi_1] = \frac{1}{3}(1120 - 392 - 168 + 1120 - 392 - 308 + 1120 - 392 - 364)$$

$$E[\pi_1] = \frac{1}{3}(560 + 420 + 364)$$

$$E[\pi_1] = \frac{1}{3} \times 1344$$

$$E[\pi_1] = 448$$

Conclusion

If Firm 1 chooses to produce 14 units, its expected payoff, given its belief about Firm 2's behavior, is 448.

Assignment 2 : Dominance

A)

Problem Statement

Dominance in Normal Form Games

Consider the following normal form games involving two players, labeled as Player 1 and Player 2. In each game, the players simultaneously and independently choose from a set of strategies. The payoffs for each player are given in the form $(\textbf{Payoff for Player 1}, \textbf{Payoff for Player 2})$.

Games

Game a)

$$\begin{array}{cccc} & & \textbf{Player 2} & \\ & & L & R \\ \textbf{Player 1} & A & (3,3) & (2,0) \\ & B & (4,1) & (8,-1) \end{array}$$

Game b)

$$\begin{array}{ccccc} & & \textbf{Player 2} & & \\ & & L & C & R \\ \textbf{Player 1} & U & (5,9) & (0,1) & (4,3) \\ & M & (3,2) & (0,9) & (1,1) \\ & D & (2,8) & (0,1) & (8,4) \end{array}$$

Game c)

$$\begin{array}{cccccc} & & \textbf{Player 2} & & & \\ & & W & X & Y & Z \\ \textbf{Player 1} & U & (3,6) & (4,10) & (5,0) & (0,8) \\ & M & (2,6) & (3,3) & (4,10) & (1,1) \\ & D & (1,5) & (2,9) & (3,0) & (4,6) \end{array}$$

Game d)

$$\begin{array}{cccc} & & \textbf{Player 2} & \\ & & L & R \\ \textbf{Player 1} & U & (1,1) & (0,0) \\ & D & (0,0) & (5,5) \end{array}$$

Questions

Identify the dominated strategies for Player 1 and Player 2 in each game.

Solution

Game a)

- **Player 1**: Strategy D dominates U because $4 > 3$ and $8 > 2$.
- **Player 2**: Strategy L dominates R because $3 > 0$ and $1 > -1$.

Game b)

- **Player 1**: Strategy D does not dominate M because while $8 > 4$, $0 = 0$ and $2 < 8$.
- **Player 2**: Strategy L dominates R because $9 > 3$, $2 > 1$, and $8 > 4$.

Game c)

			Player 2		
		W	X	Y	Z
Player 1	U	$(3,6)$	$(4,10)$	$(5,0)$	$(0,8)$
	M	$(2,6)$	$(3,3)$	$(4,10)$	$(1,1)$
	D	$(1,5)$	$(2,9)$	$(3,0)$	$(4,6)$

Step 1: Check for Pure Strategy Dominance

1. **Player 1**: No pure strategy dominates another.
2. **Player 2**: Strategy X dominates Z because $10 > 8$ and $3 > 1$.

Step 2: Check for Mixed Strategy Dominance for Player 1

We are interested in a mixed strategy $s_1 = (p, 0, 1-p)$ where U is played with probability p and D is played with probability $1-p$. This mixed strategy should dominate M.

To find the range of p for which s_1 dominates M, we need to satisfy the following inequalities:

1. $3p + 1(1-p) > 2$
2. $4p + 2(1-p) > 3$
3. $5p + 3(1-p) > 4$
4. $0p + 4(1-p) > 1$

Step 3: Solve the Inequalities

1. $3p + 1 - p > 2 \longrightarrow 2p > 1 \longrightarrow p > \frac{1}{2}$
2. $4p + 2 - 2p > 3 \longrightarrow 2p > 1 \longrightarrow p > \frac{1}{2}$
3. $5p + 3 - 3p > 4 \longrightarrow 2p > 1 \longrightarrow p > \frac{1}{2}$
4. $4 - 4p > 1 \longrightarrow -4p > -3 \longrightarrow p < \frac{3}{4}$

Step 4: Identify the Range for p

Combining all inequalities, we find that $\frac{1}{2} < p < \frac{3}{4}$.

Conclusion

In Game c:

- For Player 1, a mixed strategy $s_1 = (p, 0, 1-p)$ where $\frac{1}{2} < p < \frac{3}{4}$ dominates M.
- For Player 2, strategy X dominates Z.

Let's re-examine the mixed strategy $s_1 = \left(\frac{2}{3}, 0, \frac{1}{3}\right)$ for Player 1 to see if it dominates strategy M.

Step 1: Calculate the Expected Payoff for Player 1 Using s_1

We want to find the expected payoff for Player 1 using the mixed strategy $s_1 = \left(\frac{2}{3}, 0, \frac{1}{3}\right)$ against each of Player 2's strategies W, X, Y, Z.

1. Against W: $\frac{2}{3} \times 3 + \frac{1}{3} \times 1 = 2 + \frac{1}{3} = \frac{7}{3}$
2. Against X: $\frac{2}{3} \times 4 + \frac{1}{3} \times 2 = \frac{8}{3} + \frac{2}{3} = \frac{10}{3}$
3. Against Y: $\frac{2}{3} \times 5 + \frac{1}{3} \times 3 = \frac{10}{3} + 1 = \frac{13}{3}$
4. Against Z: $\frac{2}{3} \times 0 + \frac{1}{3} \times 4 = 0 + \frac{4}{3} = \frac{4}{3}$

Step 2: Compare the Expected Payoffs to Strategy M

1. Against W: $\frac{7}{3} > 2$
2. Against X: $\frac{10}{3} > 3$
3. Against Y: $\frac{13}{3} > 4$
4. Against Z: $\frac{4}{3} > 1$

Conclusion

The mixed strategy $s_1 = \left(\frac{2}{3}, 0, \frac{1}{3}\right)$ for Player 1 dominates strategy M because the expected payoff for Player 1 using s_1 is greater than the payoff from using M against all of Player 2's strategies.

Game d)

- **Player 1**: No dominated strategies.
- **Player 2**: No dominated strategies.

Quick recap here:

The process of going from a general mixed strategy $s_1 = (p, 0, 1-p)$ where $\frac{1}{2} < p < \frac{3}{4}$ to a specific mixed strategy $s_1 = \left(\frac{2}{3}, 0, \frac{1}{3}\right)$ involves a few steps.

Steps to Validate a Specific Mixed Strategy

1. **Identify the General Mixed Strategy**: First, you identify the general form of the mixed strategy that could potentially dominate another strategy. In this case, it was $s_1 = (p, 0, 1-p)$ where $\frac{1}{2} < p < \frac{3}{4}$.
2. **Select a Specific p**: Next, you choose a specific value of p that satisfies the inequality $\frac{1}{2} < p < \frac{3}{4}$. In your example, $p = \frac{2}{3}$ was chosen.
3. **Calculate the Expected Payoffs**: For this specific p, calculate the expected payoffs against each of the opponent's strategies. This involves taking the weighted average of the payoffs for each of the pure strategies included in the mixed strategy.
4. **Compare to the Strategy Being Potentially Dominated**: Compare these expected payoffs to the payoffs from the strategy that is potentially dominated (in this case, M). If the expected payoffs are all greater, then the specific mixed strategy dominates the strategy in question.
5. **Validate the General Form**: Finally, it's good to note that the specific p you chose is just one example. The general form $s_1 = (p, 0, 1-p)$ where $\frac{1}{2} < p < \frac{3}{4}$ suggests that there are other p values that would also result in a dominating mixed strategy.

So, in summary, you start with a general form based on inequalities, pick a specific value that satisfies those inequalities, and then validate that this specific value indeed results in a dominating strategy by calculating and comparing expected payoffs.

Assignment 3: Best response

A)

Problem Statement

Best Response Sets in a Normal Form Game

Consider the following normal form game involving two players, labeled as Player 1 and Player 2. In this game, the players simultaneously and independently choose from a set of strategies. The payoffs for each player are given in the form $(\text{Payoff for Player 1}, \text{Payoff for Player 2})$.

		Player 2		
		L	C	R
Player 1	U	$(10,0)$	$(0,10)$	$(3,3)$
	M	$(2,10)$	$(10,2)$	$(6,4)$
	D	$(3,3)$	$(4,6)$	$(6,6)$

Questions

1. Find $BR1(u2)$ for $u2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.
2. Find $BR2(u1)$ for $u1 = \left(0, \frac{1}{3}, \frac{2}{3}\right)$.
3. Find $BR1(u2)$ for $u2 = \left(\frac{5}{9}, \frac{4}{9}, 0\right)$.
4. Find $BR2(u1)$ for $u1 = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$.

Solution

(a) $BR1(u2)$ for $u2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

To find the best response set for Player 1 given Player 2's mixed strategy $u2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, we need to calculate the expected payoff for each of Player 1's strategies U, M, D.

1. **Expected Payoff for** U: $\frac{1}{3} \times 10 + \frac{1}{3} \times 0 + \frac{1}{3} \times 3 = \frac{10+0+3}{3} = \frac{13}{3}$
2. **Expected Payoff for** M: $\frac{1}{3} \times 2 + \frac{1}{3} \times 10 + \frac{1}{3} \times 6 = \frac{2+10+6}{3} = \frac{18}{3} = 6$
3. **Expected Payoff for** D: $\frac{1}{3} \times 3 + \frac{1}{3} \times 4 + \frac{1}{3} \times 6 = \frac{3+4+6}{3} = \frac{13}{3}$

The best response set for Player 1 is $BR1(u2) = \{M\}$

(b) $BR2(u1)$ for $u1 = \left(0, \frac{1}{3}, \frac{2}{3}\right)$

Expected payoffs for Player 2's strategies L, C, R:

1. **Expected Payoff for** L: $0 \times 0 + \frac{1}{3} \times 10 + \frac{2}{3} \times 3 = 0 + \frac{10}{3} + 2 = \frac{16}{3}$
2. **Expected Payoff for** C: $0 \times 10 + \frac{1}{3} \times 2 + \frac{2}{3} \times 6 = 0 + \frac{2}{3} + 4 = \frac{14}{3}$
3. **Expected Payoff for** R: $0 \times 3 + \frac{1}{3} \times 4 + \frac{2}{3} \times 6 = 0 + \frac{4}{3} + 4 = \frac{16}{3}$

Best response for Player 2: $BR2(u1) = \{L, R\}$

(c) $BR1(u2)$ for $u2 = \left(\frac{5}{9}, \frac{4}{9}, 0\right)$

Expected payoffs for Player 1's strategies U, M, D:

1. **Expected Payoff for** U: $\frac{5}{9} \times 10 + \frac{4}{9} \times 0 = \frac{50}{9}$
2. **Expected Payoff for** M: $\frac{5}{9} \times 2 + \frac{4}{9} \times 10 = \frac{10}{9} + \frac{40}{9} = \frac{50}{9}$
3. **Expected Payoff for** D: $\frac{5}{9} \times 3 + \frac{4}{9} \times 4 = \frac{15}{9} + \frac{16}{9} = \frac{31}{9}$

Best response for Player 1: $BR1(u2) = \{U, M\}$

(d) $BR2(u1)$ **for** $u1 = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$

Expected payoffs for Player 2's strategies L, C, R:

1. **Expected Payoff for** L: $\frac{1}{3} \times 0 + \frac{1}{6} \times 10 + \frac{1}{2} \times 3 = 0 + \frac{5}{3} + \frac{3}{2} = \frac{5}{3} + \frac{9}{6} = \frac{15}{9} + \frac{18}{9} = \frac{33}{9}$
2. **Expected Payoff for** C: $\frac{1}{3} \times 10 + \frac{1}{6} \times 2 + \frac{1}{2} \times 6 = \frac{10}{3} + \frac{1}{3} + 3 = \frac{11}{3} + \frac{18}{6} = \frac{33}{9} + \frac{18}{9} = \frac{51}{9}$
3. **Expected Payoff for** R: $\frac{1}{3} \times 3 + \frac{1}{6} \times 4 + \frac{1}{2} \times 6 = 1 + \frac{2}{3} + 3 = \frac{3}{3} + \frac{2}{3} + \frac{9}{3} = \frac{14}{3}$

Best response for Player 2: $BR2(u1) = \{C\}$

B)

Problem Statement

Cournot Duopoly Game with Best Response Analysis

Consider a version of the Cournot duopoly game where firms 1 and 2 simultaneously and independently select quantities to produce in a market. The quantity selected by firm i is denoted q_i and must be greater than or equal to zero ($q_i \geq 0$), for $i = 1, 2$. The market price p is given by:

$$p = 100 - 2q_1 - 2q_2$$

Each firm produces at a cost of 20 per unit. The payoff for each firm is defined as its profit.

Questions

1. Is it ever a best response for player 1 to choose $q_1 = 25$?
2. Suppose that player 1 believes that player 2 is equally likely to select each of the quantities 6, 11, and 13. What is player 1's best response?

You are correct, and I apologize for the oversight. The profit function for player 1 is indeed not constant with respect to q_1. Let's correct the analysis.

Solution

1. Is it ever a best response for player 1 to choose $q_1 = 25$?

Player 1's profit function is:

$$\text{Profit}_1 = (100 - 2q_1 - 2q_2)q_1 - 20q_1$$

$$\text{Profit}_1 = 100q_1 - 2q_1^2 - 2q_1q_2 - 20q_1$$

$$\text{Profit}_1 = 80q_1 - 2q_1^2 - 2q_1q_2$$

To maximize this profit, we take the first derivative with respect to q_1 and set it equal to zero:

$$\frac{d(\text{Profit}_1)}{dq_1} = 80 - 4q_1 - 2q_2 = 0$$

Solving for q_1, we get:

$$BR1(q_2) = 40 - \frac{q_2}{2}$$

Since $q_2 \geq 0$, the maximum value for $BR1(q_2)$ is 40 when $q_2 = 0$. Therefore, $q_1 = 25$ is never a best response for any q_2.

2. What is player 1's best response if player 2 is equally likely to select 6, 11, or 13?

The first-order condition (FOC) gives us the best response function $BR1(q_2) = 40 - \frac{q_2}{2}$, but we can't directly plug in the expected values of q_2 into this function to get the expected best response for q_1.

To find the expected best response for q_1 given that q_2 is equally likely to be 6, 11, or 13, we should calculate the expected profit for q_1 and then maximize it.

The profit function for player 1 is:

$$\text{Profit}_1 = (100 - 2q_1 - 2q_2)q_1 - 20q_1$$

$$\text{Profit}_1 = 80q_1 - 2q_1^2 - 2q_1q_2$$

Given that q_2 is equally likely to be 6, 11, or 13, the expected profit for player 1 is:

$$E[\text{Profit}_1] = \frac{1}{3} \times (80q_1 - 2q_1^2 - 2q_1 \times 6) + \frac{1}{3} \times (80q_1 - 2q_1^2 - 2q_1 \times 11) + \frac{1}{3} \times (80q_1 - 2q_1^2 - 2q_1 \times 13)$$

$$E[\text{Profit}_1] = \frac{1}{3} \times (80q_1 - 2q_1^2 - 12q_1) + \frac{1}{3} \times (80q_1 - 2q_1^2 - 22q_1) + \frac{1}{3} \times (80q_1 - 2q_1^2 - 26q_1)$$

$$E[\text{Profit}_1] = \frac{1}{3} \times (68q_1 - 2q_1^2) + \frac{1}{3} \times (58q_1 - 2q_1^2) + \frac{1}{3} \times (54q_1 - 2q_1^2)$$

$$E[\text{Profit}_1] = \frac{1}{3} \times (180q_1 - 6q_1^2)$$

$$E[\text{Profit}_1] = 60q_1 - 2q_1^2$$

To find the best response for q_1, we take the derivative of $E[\text{Profit}_1]$ with respect to q_1 and set it equal to zero:

$$\frac{d(E[\text{Profit}_1])}{dq_1} = 60 - 4q_1 = 0$$

$$q_1 = 15$$

So, given the specified beliefs, player 1's best response is $q_1 = 15$.

C)

Problem Statement

Rock-Paper-Scissors Game in Normal Form

In the Rock-Paper-Scissors game, two players simultaneously choose one of three options: Rock, Paper, or Scissors. The payoffs are as follows:

	Rock (Player 2)	Paper (Player 2)	Scissors (Player 2)
Rock (Player 1)	(0, 0)	(-1, 1)	(1, -1)
Paper (Player 1)	(1, -1)	(0, 0)	(-1, 1)
Scissors (Player 1)	(-1, 1)	(1, -1)	(0, 0)

Questions

Determine the best response sets for Player 1 given the following mixed strategies for Player 2:

1. $u_2 = (1, 0, 0)$
2. $u_2 = \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$
3. $u_2 = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$
4. $u_2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

Solution

a) $BR1(u_2)$ for $u_2 = (1, 0, 0)$

Player 2 is playing Rock with probability 1. The best response for Player 1 is to play Paper, which beats Rock.

$BR1(u_2) = \text{Paper}$

b) $BR1(u_2)$ for $u_2 = \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$

The expected payoff for Player 1 for each of their strategies is:

- Rock: $0 \times \frac{1}{6} + (-1) \times \frac{1}{3} + 1 \times \frac{1}{2} = 0 - \frac{1}{3} + \frac{1}{2} = \frac{1}{6}$
- Paper: $1 \times \frac{1}{6} + 0 \times \frac{1}{3} + (-1) \times \frac{1}{2} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$
- Scissors: $-1 \times \frac{1}{6} + 1 \times \frac{1}{3} + 0 \times \frac{1}{2} = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$

The best response for Player 1 is either Rock or Scissors.

$BR1(u_2) = \{\text{Rock, Scissors}\}$

c) $BR1(u_2)$ for $u_2 = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

The expected payoff for Player 1 for each of their strategies is:

- Rock: $0 \times \frac{1}{2} + (-1) \times \frac{1}{4} + 1 \times \frac{1}{4} = 0$
- Paper: $1 \times \frac{1}{2} + 0 \times \frac{1}{4} + (-1) \times \frac{1}{4} = \frac{1}{4}$
- Scissors: $-1 \times \frac{1}{2} + 1 \times \frac{1}{4} + 0 \times \frac{1}{4} = -\frac{1}{4}$

The best response for Player 1 is Paper.

$BR1(u_2) = \text{Paper}$

d) $BR1(u_2)$ for $u_2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

The expected payoff for Player 1 for each of their strategies is:

- Rock: $0 \times \frac{1}{3} + (-1) \times \frac{1}{3} + 1 \times \frac{1}{3} = 0$
- Paper: $1 \times \frac{1}{3} + 0 \times \frac{1}{3} + (-1) \times \frac{1}{3} = 0$
- Scissors: $-1 \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3} = 0$

Player 1 is indifferent among all strategies when Player 2 plays a completely mixed strategy.

$BR1(u_2) = \{\text{Rock, Paper, Scissors}\}$

Assignment 4: Rationalizability

A)

Game a)

	X	**Y**
U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

Game b)

	X	**Y**	**Z**
U	(2,0)	(1,1)	(4,2)
M	(3,4)	(1,2)	(2,3)
D	(1,3)	(0,2)	(3,0)

Game c)

	X	**Y**	**Z**
U	(6,3)	(5,1)	(0,2)
M	(0,1)	(4,6)	(6,0)
D	(2,1)	(3,5)	(2,8)

Game d)

	X	**Y**	**Z**
U	(8,6)	(0,1)	(8,2)
M	(1,0)	(2,6)	(5,1)
D	(0,8)	(1,0)	(4,4)

Game e)

	X	**Y**
A	(2,2)	(0,0)
B	(0,0)	(3,3)

Game f)

	X	**Y**
A	(8,10)	(4,1)
B	(6,4)	(8,5)

Game g)

	X	**Y**
U	(3,10)	(4,1)
D	(6,4)	(8,5)

Rationalizable Strategies

Game a)

Since no strategies are strictly dominated for either player, all strategies are rationalizable.

- Rationalizable strategies for Player 1: {U, M, D}
- Rationalizable strategies for Player 2: {X, Y}

Game b)

For Player 1:

- U dominates D ($2 > 1$, $1 = 1$, $4 > 3$)

So, D is eliminated.

For Player 2:

- Z dominates Y ($2 > 1$, $3 > 2$, $0 = 0$)

So, Y is eliminated.

- Rationalizable strategies for Player 1: {U, M}
- Rationalizable strategies for Player 2: {X, Z}

Game c)

For Player 1:

- X dominates Y ($6 > 5$, $0 = 0$, $2 > 1$)

So, Y is eliminated.

For Player 2:

- Y dominates Z ($1 > 0$, $6 > 5$, $8 > 2$)
- X dominates Y ($3 > 1$, $4 > 2$, $5 > 0$)

So, Z is eliminated.

- Rationalizable strategies for Player 1: {U}
- Rationalizable strategies for Player 2: {X}

Game d)

- If Player 1 plays U, the expected payoff from playing Z is 2, but the expected payoff from the mixed strategy $[X, Y] = \left[\frac{1}{2}, \frac{1}{2}, 0\right]$ is $\frac{1}{2} \times 6 + \frac{1}{2} \times 1 = 3.5$.
- If Player 1 plays M, the expected payoff from playing Z is 1, while the expected payoff from the mixed strategy is $\frac{1}{2} \times 0 + \frac{1}{2} \times 6 = 3$.
- If Player 1 plays D, the expected payoff from playing Z is 4, while the expected payoff from the mixed strategy is $\frac{1}{2} \times 8 + \frac{1}{2} \times 0 = 4$.

In each case, the mixed strategy of $[X, Y]$ either performs better or equally as well as Z, making Z not rationalizable.

After eliminating Z, D becomes strictly dominated by M for Player 1:

- For X, M gives 1 while D gives 0.
- For Y, M gives 6 while D gives 0.

So, D is also not rationalizable.

The final rationalizable strategies are U, M for Player 1 and X, Y for Player 2.

Game e)

For Player 1, there are no dominated strategies.

For Player 2, there are no dominated strategies.

- Rationalizable strategies for Player 1: {A, B}
- Rationalizable strategies for Player 2: {X, Y}

Game f)

1. For Player 1 (Rows):
   - A strictly dominates B when Player 2 plays X ($8 > 6$).
   - B strictly dominates A when Player 2 plays Y ($8 > 4$).
2. For Player 2 (Columns):
   - X strictly dominates Y when Player 1 plays A ($10 > 1$).
   - Y strictly dominates X when Player 1 plays B ($5 > 4$).

Since neither strategy for each player is strictly dominated across all strategies of the opponent, all strategies are rationalizable:

- For Player 1, the rationalizable strategies are A, B.
- For Player 2, the rationalizable strategies are X, Y.

Game g)

Original Strategies

- Player 1 (Row player) has initial strategies: U, D
- Player 2 (Column player) has initial strategies: X, Y

	X	Y
U	$(3, 10)$	$(4, 1)$
D	$(6, 4)$	$(8, 5)$

Step-by-Step Analysis

1. **First Iteration:**
   - For Player 1 (Rows):
     - Strategy D strictly dominates U when Player 2 plays X ($6 > 3$).
     - Strategy D strictly dominates U when Player 2 plays Y ($8 > 4$).
   - For Player 2 (Columns):
     - Strategy X strictly dominates Y when Player 1 plays U ($10 > 1$).
     - Strategy Y strictly dominates X when Player 1 plays D ($5 > 4$).
2. **Second Iteration:**
   - For Player 1, the strategy U is strictly dominated by D across all strategies for Player 2. Therefore, U is eliminated, leaving D as the only rationalizable strategy.
   - For Player 2, strategy X is better when Player 1 plays U, and strategy Y is better when Player 1 plays D. Since U is eliminated for Player 1, Y becomes the only rationalizable strategy for Player 2.
3. **Conclusion:**
   - For Player 1, the only rationalizable strategy is D.
   - For Player 2, the only rationalizable strategy is Y.

B)

The concept of a *rationalizable strategy* is closely related to the idea of a *best response*, but the two are not identical. Let's break down the situation:

Statement 1: s_1 is a rationalizable strategy for player 1.

This means that there exists some belief that player 1 might have about the actions of player 2, under which s_1 is a best response.

Statement 2: s_2 is a best response to s_1.

This means that, given that player 1 is playing s_1, s_2 is the strategy that maximizes player 2's payoff.

Question: Is s_2 a rationalizable strategy for player 2?

To determine whether s_2 is rationalizable, we need to know whether there is some belief that player 2 could have about the actions of player 1, under which s_2 is a best response.

In this specific situation, we already know s_2 is a best response to s_1, and s_1 is a rationalizable strategy for player 1. Since s_1 is rationalizable, it means that s_1 could reasonably be played by player 1. Therefore, it's reasonable for player 2 to believe that player 1 might play s_1. Under this belief, s_2 is a best response.

Conclusion:

Yes, s_2 would be a rationalizable strategy for player 2 in this case, because it is a best response to a strategy (s_1) that could feasibly be played by player 1.

Assignment 5: Nash equilibrium

A)

Game a):

	X	**Y**
U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

- Player 1 (Row) has three strategies: U, M, and D.
- Player 2 (Column) has two strategies: X and Y.

There are no pure Nash equilibria in this game.

Game b):

	X	**Y**	**Z**
U	(2,0)	(1,1)	(4,2)
M	(3,4)	(1,2)	(2,3)
D	(1,3)	(0,2)	(3,0)

- Player 1 (Row) has three strategies: U, M, and D.
- Player 2 (Column) has three strategies: X, Y, and Z.

	X	**Y**	**Z**
U	~~(2, 0)~~	~~(1, 1)~~	4, 2
M	(3, 4)	~~(1, 2)~~	~~(2, 3)~~
D	~~(1, 3)~~	~~(0, 2)~~	~~(3, 0)~~

Nash equilibria:

- (U, Z): Player 1 plays U, and Player 2 plays Z.
- (M, X): Player 1 plays M, and Player 2 plays X.

Game c):

	X	**Y**	**Z**
U	(6,3)	(5,1)	(0,2)
M	(0,1)	(4,6)	(6,0)
D	(2,1)	(3,5)	(2,8)

- Player 1 (Row) has three strategies: U, M, and D.
- Player 2 (Column) has three strategies: X, Y, and Z.

	X	**Y**	**Z**
U	(6, 3)	~~(5, 1)~~	~~(0, 2)~~
M	~~(0, 1)~~	~~(4, 6)~~	~~(6, 0)~~
D	~~(2, 1)~~	~~(3, 5)~~	~~(2, 8)~~

Nash equilibrium:

- (U, X): Player 1 plays U, and Player 2 plays X.

Game d):

	X	**Y**	**Z**
U	(8,6)	(0,1)	(8,2)
M	(1,0)	(2,6)	(5,1)
D	(0,8)	(1,0)	(4,4)

- Player 1 (Row) has three strategies: U, M, and D.
- Player 2 (Column) has three strategies: X, Y, and Z.

	X	**Y**	**Z**
U	(8, 6)	~~(0, 1)~~	~~(8, 2)~~
M	~~(1, 0)~~	(2, 6)	~~(5, 1)~~
D	~~(0, 8)~~	~~(1, 0)~~	~~(4, 4)~~

Nash equilibria:

- (U, X): Player 1 plays U, and Player 2 plays X.
- (M, Y): Player 1 plays M, and Player 2 plays Y.

Game e):

	X	**Y**
A	(2,2)	(0,0)
B	(0,0)	(3,3)

- Player 1 (Row) has two strategies: A and B.
- Player 2 (Column) has two strategies: X and Y.

	X	**Y**
A	(2, 2)	~~(0, 0)~~
B	~~(0, 0)~~	(3, 3)

Nash equilibria:

- (A, X): Player 1 plays A, and Player 2 plays X.
- (B, Y): Player 1 plays B, and Player 2 plays Y.

Game f):

	X	**Y**
A	(8,10)	(4,1)
B	(6,4)	(8,5)

- Player 1 (Row) has two strategies: A and B.
- Player 2 (Column) has two strategies: X and Y.

	X	**Y**
A	(8, 10)	~~(4, 1)~~
B	~~(6, 4)~~	(8, 5)

Nash equilibria:

- (A, X): Player 1 plays A, and Player 2 plays X.
- (B, Y): Player 1 plays B, and Player 2 plays Y.

Game g):

	X	**Y**
U	(3,10)	(4,1)
D	(6,4)	(8,5)

- Player 1 (Row) has two strategies: U and D.
- Player 2 (Column) has two strategies: X and Y.

	X	**Y**
U	~~(3, 10)~~	~~(4, 1)~~
D	~~(6, 4)~~	(8, 5)

Nash equilibrium:

- (D, Y): Player 1 plays D, and Player 2 plays Y.

B)

Problem Statement

In a two-player normal-form game, the strategy sets for Player 1 and Player 2 are $S_1 = [0, 1]$ and $S_2 = [0, 1]$, respectively. The utility functions for the players are:

- $u_1(s_1, s_2) = 3s_1 - 2s_1s_2 - 2s_1^2$
- $u_2(s_1, s_2) = s_2 + 2s_1s_2 - 2s_2^2$

We are asked to find the Nash equilibrium of this game.

What Does "Interior" Mean?

The term "interior" in this context means that the Nash equilibrium occurs within the strategy sets S_1 and S_2 rather than at the boundaries. This implies that we can use calculus to find the equilibrium by taking derivatives and setting them equal to zero.

Solution

To find the Nash equilibrium, we need to find the strategies (s_1^*, s_2^*) that maximize each player's utility given the other player's strategy.

Step 1: Take Partial Derivatives

For Player 1, we take the partial derivative of u_1 with respect to s_1:

$$\frac{\partial u_1}{\partial s_1} = 3 - 2s_2 - 4s_1$$

For Player 2, we take the partial derivative of u_2 with respect to s_2:

$$\frac{\partial u_2}{\partial s_2} = 1 + 2s_1 - 4s_2$$

Step 2: Best Response Functions

Player 1's best-response function $BR1(s_2)$ is derived from the first-order condition $3 - 2s2 - 4s1 = 0$, which gives us:

$$BR1(s_2) = \frac{3}{4} - \frac{s2}{2}$$

Player 2's best-response function $BR2(s_1)$ is derived from the first-order condition $1 + 2s_1 - 4s_2 = 0$, which gives us:

$$BR2(s_1) = \frac{1}{4} + \frac{s1}{2}$$

Step 3: Solve for Nash Equilibrium

To find the Nash equilibrium, we need to solve the system of equations:

$$s_1 = \frac{3}{4} - \frac{s2}{2}$$

$$s_2 = \frac{1}{4} + \frac{s1}{2}$$

Substituting $BR1(s_2)$ into $BR2(s_1)$:

$$s_2 = \frac{1}{4} + \frac{1}{2}\left(\frac{3}{4} - \frac{s2}{2}\right)$$

$$s_2 = \frac{1}{4} + \frac{3}{8} - \frac{s2}{4}$$

$$s_2 = \frac{1}{2}$$

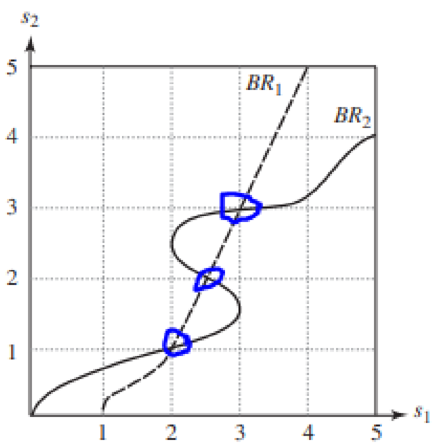
Substituting s_2 back into one of the original equations to find s_1:

$$s_1 = \frac{3}{4} - \frac{1}{4}$$

$$s_1 = \frac{1}{2}$$

So, the Nash equilibrium in this game is $(s_1^*, s_2^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

C)



Problem Statement

In a two-player game, the strategy sets for Player 1 and Player 2 are $S_1 = [0, 5]$ and $S_2 = [0, 5]$, respectively. The best-response functions $BR1(s2)$ and $BR2(s1)$ are given graphically. The curves intersect at points $(2, 1)$, $\left(\frac{5}{2}, 2\right)$, and $(3, 3)$.

Questions

(a) Does this game have any Nash equilibria? If so, what are they?
(b) What is the set of rationalizable strategy profiles for this game?

Solution

(a) Nash Equilibria

The Nash equilibria are points where each player's strategy is the best response to the other player's strategy. These points are determined by the intersections of the best-response functions $BR1(s2)$ and $BR2(s1)$. According to the graphical information, the curves intersect at points $(2, 1)$, $\left(\frac{5}{2}, 2\right)$, and $(3, 3)$.

Therefore, this game has three Nash equilibria: $(2, 1)$, $\left(\frac{5}{2}, 2\right)$, and $(3, 3)$.

(b) Rationalizable Strategy Profiles

The set of rationalizable strategy profiles is generally determined by the iterated elimination of strictly dominated strategies. In this case, the graphical information suggests that the rationalizable strategies for Player 1 are in the interval $[2, 3]$, and for Player 2 are in the interval $[1, 3]$.

Thus, the set of rationalizable strategy profiles is $R = [2, 3] \times [1, 3]$.

Note:

The key here is how the best-response function behaves around the point in question, not just at the point itself. A strategy is rationalizable if it could be a best response to some belief a player might have about their opponent's strategy.

In the case of the point $\left(\frac{5}{2}, 2\right)$:

- If $BR2$ is decreasing in s_1 at this point, then Player 2 would prefer to deviate if they believe Player 1 might deviate slightly. Specifically, if Player 1 were to play a strategy slightly less than $\frac{5}{2}$, then Player 2's best response would be to play a strategy slightly less than 2. Likewise, if Player 1 were to play a strategy slightly more than $\frac{5}{2}$, Player 2's best response would be to play a strategy slightly more than 2.

This implies that 2 is not a "stable" best response for Player 2 around this point, making it not rationalizable even though it forms part of a Nash equilibrium. A Nash equilibrium is a snapshot where both players' strategies are best responses to each other, but it doesn't necessarily capture the dynamics around that point.

The "direction" of the intersection (from the left or from the right) matters because it informs us how the best-response function behaves around that point. If $BR2$ is decreasing in s_1 at $\frac{5}{2}$, then Player 2's best response is "unstable" around this point, implying that the strategy is not rationalizable.

In contrast, for the points $(2, 1)$ and $(3, 3)$, if $BR2$ is increasing in s_1 at these points, then these strategies are stable best responses for slight deviations in Player 1's strategy, making them both Nash equilibria and rationalizable.

Assignment 6: Mixed-strategy Nash equilibrium

A)

Problem Statement

In a version of the lobbying game, two firms, X and Y, have the option to Lobby (L) or Not Lobby (N). The payoffs are as follows:

	$Y: L$	$Y: N$
$X: L$	$(-5, -5)$	$(x - 15, 0)$
$X: N$	$(0, 15)$	$(10, 10)$

Here, $x > 25$.

Question

Identify the pure-strategy Nash equilibria of this game, if any.

Solution

A pure-strategy Nash equilibrium occurs when each player's strategy is a best response to the other player's strategy. Let's examine each cell of the payoff matrix to see if either player would want to deviate, assuming the other player's strategy is fixed.

1. $(X : L, Y : L)$ **with payoffs** $(-5, -5)$
   - X would prefer to switch to N to get a payoff of 0 instead of -5.
   - Y would prefer to switch to N to get a payoff of 10 instead of -5.
2. $(X : L, Y : N)$ **with payoffs** $(x - 15, 0)$
   - X would not want to switch to N, as the payoff would be 10, and $x - 15 > 10$ (given $x > 25$).
   - Y would not want to switch to L, as the payoff would be -5.
3. $(X : N, Y : L)$ **with payoffs** $(0, 15)$

- X would not want to switch to L, as the payoff would be -5.
- Y would not want to switch to N, as the payoff would be 10.

4. $(X : N, Y : N)$ **with payoffs** $(10, 10)$
   - X would prefer to switch to L to get a payoff of $x - 15$ (given $x > 25$).
   - Y would prefer to switch to L to get a payoff of 15 instead of 10.

Both of these strategy profiles meet the criteria for a Nash equilibrium: neither player has an incentive to deviate unilaterally from their current strategy. Therefore, (N, L) and (L, N) are indeed pure-strategy Nash equilibria for this game.

b)

To find the mixed-strategy Nash equilibrium of the game, we need to consider the probabilities with which each player will play their respective strategies. Let's denote:

- p as the probability that Player X chooses to Lobby (L), and $1 - p$ as the probability that Player X chooses Not to Lobby (N).
- q as the probability that Player Y chooses to Lobby (L), and $1 - q$ as the probability that Player Y chooses Not to Lobby (N).

Mixed-Strategy Nash Equilibrium

For Player X to be indifferent between playing L and N, the expected payoffs must be equal:

$$-5q + (x - 15)(1 - q) = 10 - 10q$$

$$-5q + x - 15 - xq + 15q = 10$$

$$10q = 25 - x$$

$$q = \frac{25 - x}{10}$$

$$q = \frac{25 - x}{20 - x}$$

For Player Y to be indifferent between playing L and N, the expected payoffs must be equal:

$$-5p + 15 - 15p = 10 - 10p$$

$$-5p + 15 - 15p = 10 - 10p$$

$$-20p = -5$$

$$p = \frac{1}{4} \times 2$$

$$p = \frac{1}{2}$$

Mixed-Strategy Nash Equilibrium

The corrected mixed-strategy Nash equilibrium is:

- Player X plays L with probability $p = \frac{1}{2}$ and N with probability $1 - p = \frac{1}{2}$.
- Player Y plays L with probability $q = \frac{25-x}{20-x}$ and N with probability $1 - q = 1 - \frac{25-x}{20-x}$.

c)

The probability that the government makes a decision that favors firm X (i.e., the probability that the outcome is (L, N)) is the product of the probabilities that X chooses L and Y chooses N:

$$\text{Probability of } (L, N) = \frac{1}{2} \times \frac{20 - x - 25 + x}{20 - x}$$

$$= \frac{1}{2} \times \frac{-5}{20 - x}$$

$$= \frac{-5}{2(20 - x)}$$

$$= \frac{5}{2(x - 20)}$$

Here, $\frac{5}{2(x-20)}$ is the probability that the government makes a decision that favors firm X, given that $x > 25$.

d)

The derivative of the probability $\frac{5}{2(x-20)}$ with respect to x is $-\frac{5}{2(x-20)^2}$. Since the derivative is negative, it shows that the probability decreases as x increases.

From an economic standpoint, as x becomes larger, (L, N) is a 'better' outcome refers to the benefit or utility x that Firm X gains from lobbying. In this context, a larger x means a larger benefit for Firm X if they are successful in lobbying ((L, N) occurs).

So, as x increases, the "stakes" are higher for Firm X, making the outcome (L, N) more desirable for them. However, the probability of this outcome occurring decreases, as indicated by the negative derivative. This could be seen as a risk-reward trade-off: the higher the potential reward (x), the lower the probability of achieving it.

B)

Problem Statement

In a social dilemma, a pedestrian is hit by a car and is in immediate need of medical attention. There are n bystanders near the accident scene. Each bystander has two choices:

1. Call for help (dial 911), incurring a personal cost c.
2. Do not call for help.

The utility for each bystander who calls for help is $v - c$. If a bystander does not call but at least one other person does, the utility is v. If no one calls for help, the utility for each bystander is zero.

The utility obtained if someone calls for help is v, and the cost of calling for help is c. It is given that $v > c$.

Objectives:

1. Identify the Nash equilibria in this game.
2. Discuss the social implications of the equilibria.

To find the symmetric Nash equilibrium in this n-player game, let's consider the following:

Let q be the probability that any given bystander decides to call for help. Since we're looking for a symmetric equilibrium, each player will use the same mixed strategy q.

Payoffs:

1. **If a bystander calls for help**, their expected payoff is $v - c$.
2. **If a bystander doesn't call for help but someone else does**, their expected payoff is v.
3. **If no one calls for help**, the payoff is 0.

Expected Payoff for Not Calling:

If a bystander chooses not to call, the probability that at least one of the remaining $n - 1$ bystanders calls is $1 - (1 - q)^{n-1}$. Therefore, the expected utility for not calling is:

$$E[\text{Not Calling}] = v \times (1 - (1 - q)^{n-1})$$

Expected Payoff for Calling:

The expected utility for calling is simply $v - c$.

Equilibrium Condition:

Step 3: Solve for q

To find the equilibrium probability q, we'll solve the equation:

$$v - c = v - v \times (1 - q)^{n-1}$$

Simplifying, we get:

$$c = v \times (1 - q)^{n-1}$$

Dividing by v:

$$\frac{c}{v} = (1 - q)^{n-1}$$

Taking the $(n - 1)$-th root:

$$(1 - q) = \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$$

Finally, solving for q:

$$q = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$$

This q is the probability that a bystander will call for help in the mixed-strategy Nash equilibrium.

This gives us the symmetric Nash equilibrium strategy q for this n-player game, under the assumption that $v > c$.

b)

To compute the probability that at least one person calls for help in equilibrium, we need to consider the opposite scenario: the probability that no one calls for help. Then we can subtract this from 1 to get the probability that at least one person calls.

Probability that No One Calls:

The probability that a single person does not call for help is $1 - q$. Therefore, the probability that none of the n people call for help is $(1 - q)^n$.

Probability that At Least One Person Calls:

The probability that at least one person calls for help is $1 - (1 - q)^n$.

Substituting $q = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$, we get:

$$1 - \left(1 - \left(1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}\right)\right)^n$$

Simplifying, we find:

$$1 - \left(\left(\frac{c}{v}\right)^{\frac{1}{n-1}}\right)^n$$

$$1 - \left(\frac{c}{v}\right)^{\frac{n}{n-1}}$$

Dependence on n:

As n increases, the exponent $\frac{n}{n-1}$ approaches 1, making the term $\left(\frac{c}{v}\right)^{\frac{n}{n-1}}$ closer to $\frac{c}{v}$. This means that as the number of bystanders increases, the probability that at least one person calls for help approaches $1 - \frac{c}{v}$, which is greater than zero given $v > c$.

Perverse or Intuitive?

This result is somewhat intuitive in that the probability of someone calling for help increases with the number of bystanders. However, it's also somewhat perverse because it highlights the inefficiency of individual decision-making in this scenario. Even though the probability of someone calling increases with more bystanders, it's not guaranteed to be 1, leaving room for potential tragedies.

Economic effects

The symmetric Nash equilibrium strategy $q = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}$ has several interesting social implications:

1. The Bystander Effect:

As n (the number of bystanders) increases, the term $\frac{1}{n-1}$ approaches zero, making q closer to 1. This reflects the "bystander effect," where individuals are less likely to take action when more people are present, assuming that someone else will step in.

2. Cost and Value:

The equilibrium strategy q is influenced by both c (the cost of calling) and v (the value of someone calling). If the cost c is low compared to the value v, then q will be closer to 1, making it more likely that someone will call for help.

3. Social Welfare:

From a societal perspective, the ideal situation is for at least one person to call for help (v utility for everyone). However, the Nash equilibrium suggests that this may not always happen, especially if the cost c is high or if there are many bystanders (n is large).

4. Coordination Failure:

The game captures a classic problem of coordination failure in large groups. Even though everyone would be better off if at least one person calls, the individual incentives may not align with this collective optimum.

5. Policy Implications:

Understanding this equilibrium can help in designing policies or awareness campaigns. For example, reducing the perceived cost c (e.g., through awareness that calling 911 is anonymous and won't involve you in legal issues) could increase the likelihood of someone calling for help.