This solution set is written and made by Jarle Kvile. It should not be taken as the full solution, and should not be thought to be sufficient to get any type of grade on an exam or a test of any kind.

1: Baysesian games

A: Watson 24.1

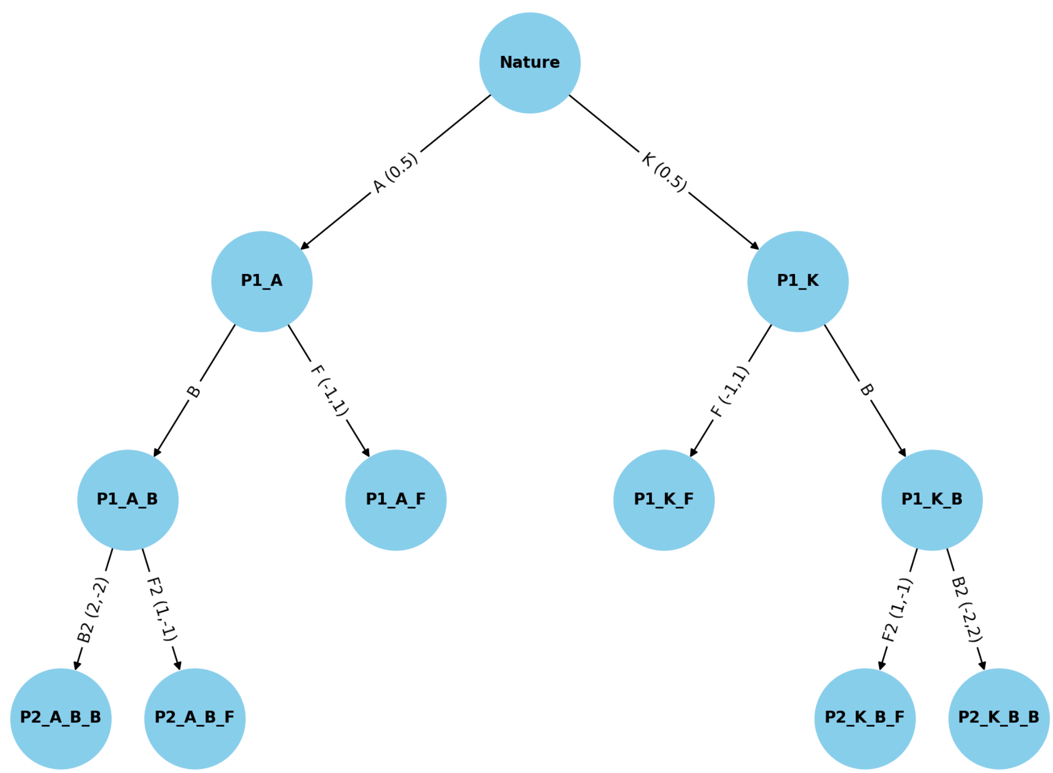
Alright, let's break this down step by step.

Extensive Form:

The extensive form of a game provides a visual representation of the sequence of actions players can take, starting from the beginning of the game to its conclusion. We'll represent the game as a tree, with nodes representing decision points and branches representing actions.

1. Nature chooses a card for Player 1: Ace (A) or King (K) with equal probabilities.
2. Player 1 observes his card and chooses to Bid (B) or Fold (F).
    - If Player 1 folds, the game ends with payoffs (-1, 1).
3. If Player 1 bids, Player 2 chooses to Bid (B2) or Fold (F2) without observing Player 1's card.
    - If Player 2 folds, the game ends with payoffs (1, -1).
    - If Player 2 bids and Player 1 has an Ace, the game ends with payoffs (2, -2).
    - If Player 2 bids and Player 1 has a King, the game ends with payoffs (-2, 2).

Extensive Form Representation of the Poker Game



- The initial node represents Nature's choice of dealing an Ace (A) or King (K) to Player 1 with equal probabilities (0.5 each).
- The subsequent nodes represent Player 1's decision to Bid (B) or Fold (F) based on the card they have.
- If Player 1 decides to bid, the next set of nodes represents Player 2's decision to Bid (B2) or Fold (F2).
- The terminal nodes (leaf nodes) indicate the payoffs for each player based on their decisions and the card dealt to Player 1.

Next, we'll represent this game in the Bayesian normal form.

Bayesian Normal Form:

The Bayesian normal form of a game represents the strategies and expected payoffs of each player in a matrix form. In this game, the strategies of each player depend on the information they have and the decisions they can make based on that information.

Let's denote Player 1's strategies as:

- The strategies for Player 1 are represented by the rows:
  - Bb: Bid when having an Ace
  - Bf: Bid when having a King
  - Fb: Fold when having an Ace
  - Ff: Fold when having a King
- The strategies for Player 2 are represented by the columns:
  - B: Bid after Player 1 bids
  - F: Fold after Player 1 bids

We'll create a matrix where each entry represents the expected payoff for each combination of strategies. The expected payoff is calculated based on the probabilities given by Nature's move and the payoffs specified in the problem description.

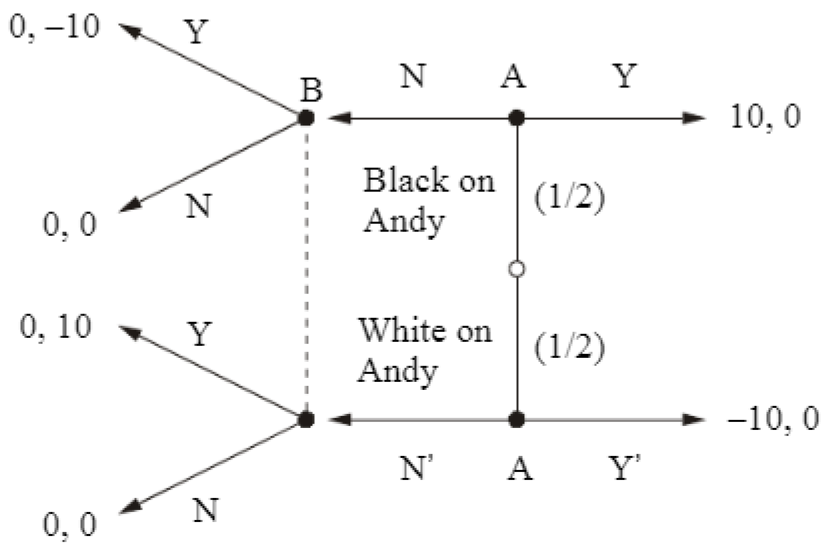
Here are the Bayesian normal form matrix for the poker game:

	B	F
Bb	$0, 0$	$1, -1$
Bf	$1/2, 1/2$	$0, 0$
Fb	$-3/2, 3/2$	$0, 0$
Ff	$-1, 1$	$-1, 1$

No NASH.

B: Watson 24.2

Extensive Form:



Normal form:

Let's analyze the game using the provided payoff matrix.

A chooses from {YY, YN, NY, NN} and B chooses from {Y, N}. We'll check for strictly dominated strategies first:

1. For A choosing between YY and YN against B's Y:
   - If B chooses Y, A gets 0 from YY and 5 from YN.
     Thus, YN dominates YY when B chooses Y.
2. For A choosing between YY and YN against B's N:
   - If B chooses N, A gets 0 from YY and 5 from YN.
     Again, YN dominates YY when B chooses N.

Since YN dominates YY for every choice of B, YY is a strictly dominated strategy for A. We can eliminate that row. The table then looks like:

A	Y	N
YY	$0, 0$	$0, 0$
YN	$5, 5$	$5, 0$
NY	$-5, -5$	$-5, 0$
NN	$0, 0$	$0, 0$

Next, for A choosing between YN and NY against B's Y:

- If B chooses Y, A gets 5 from YN and -5 from NY.
  Thus, YN dominates NY when B chooses Y.

For A choosing between YN and NY against B's N:

- If B chooses N, A gets 5 from YN and -5 from NY.
  Again, YN dominates NY when B chooses N.

Again, since YN dominates NY for every choice of B, NY is a strictly dominated strategy for A. We can eliminate that row too:

A	Y	N
YN	$5,5$	$5,0$
NN	$0,0$	$0,0$

For B:

1. When A chooses YN:
    - If A chooses YN, B gets 5 from Y and 0 from N.
      Thus, Y strictly dominates N for B when A chooses YN.
2. When A chooses NN:
    - If A chooses NN, B gets 0 from both Y and N.
      No domination here.

Given this information, A will never choose NN because choosing YN always gives a better or equal payoff, regardless of B's choice.

So, we've narrowed down to:

A	Y
YN	$5,5$

From this, the Nash Equilibrium is indeed (YN, Y).

C: Watson 24.3

	U	D
LL'	$2,0$	$2,0$
LR'	$1,0$	$3,1$
RL'	$1,2$	$3,0$
RR'	$0,2$	$4,1$

Let's identify Nash Equilibria in the given payoff matrix.

A chooses from {LL', LR', RL', RR'} and B chooses from {U, D}.

For each of A's strategies, let's find out if B has a best response:

1. For LL':
    - If A chooses LL', B gets 0 from U and 0 from D.
      Both are equivalent; hence, both U and D are best responses for B.
2. For LR':
    - If A chooses LR', B gets 0 from U and 1 from D.
      Hence, D strictly dominates U for B when A chooses LR'.
3. For RL':
    - If A chooses RL', B gets 2 from U and 0 from D.
      Hence, U strictly dominates D for B when A chooses RL'.
4. For RR':
    - If A chooses RR', B gets 2 from U and 1 from D.
      Hence, U strictly dominates D for B when A chooses RR'.

Now, for each of B's strategies, let's find out if A has a best response:

1. For U:

- A gets 2 from LL', 1 from LR', 1 from RL', and 0 from RR'.
  Hence, LL' dominates all other strategies for A when B chooses U.

2. For D:
   - A gets 2 from LL', 3 from LR', 3 from RL', and 4 from RR'.
     Hence, RR' strictly dominates all other strategies for A when B chooses D.

Given this information, we can identify the Nash Equilibria:

1. (LL', U) is a Nash Equilibrium because when B chooses U, A's best response is LL' and when A chooses LL', both U and D are best responses for B.
2. (RR', D) is a Nash Equilibrium because when B chooses D, A's best response is RR' and when A chooses RR', U is the best response for B.

Thus, the game has two Nash Equilibria: (LL', U) and (RR', D).

D: Watson 24.4

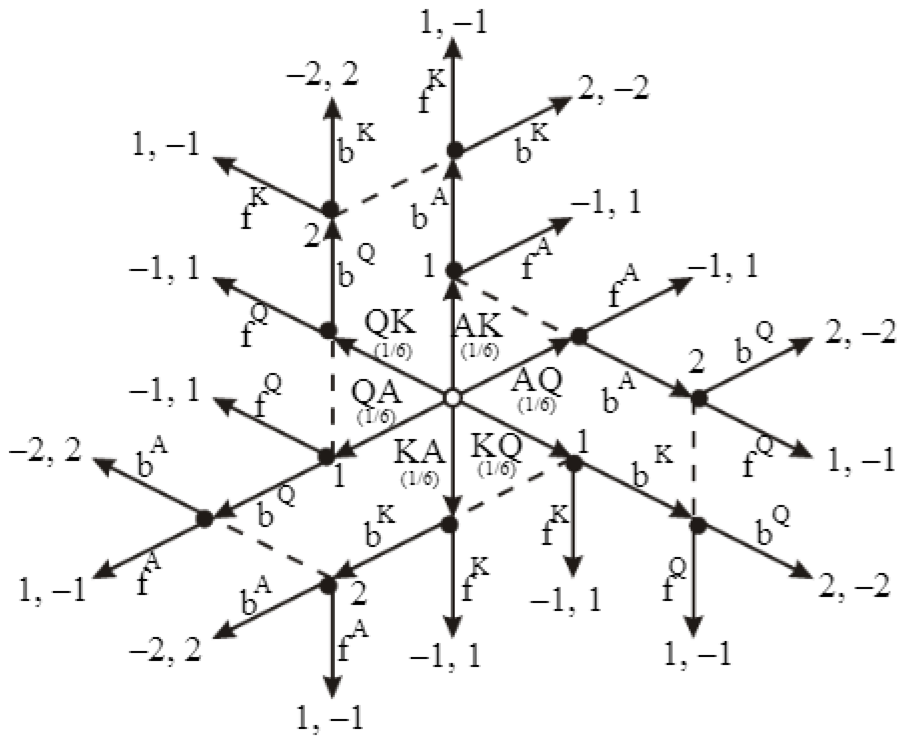
How to:

To represent this three-card poker game in extensive form, you'd create a game tree, where each node represents a decision point and each branch represents an action. Let's break it down step-by-step:

1. **Initial Deal**:
   - There are 6 equally likely possibilities:
     1. Player 1 gets Ace, Player 2 gets King
     2. Player 1 gets Ace, Player 2 gets Queen
     3. Player 1 gets King, Player 2 gets Ace
     4. Player 1 gets King, Player 2 gets Queen
     5. Player 1 gets Queen, Player 2 gets Ace
     6. Player 1 gets Queen, Player 2 gets King
2. **Player 1's Decision**:
   - For each of the six situations above, Player 1 can either Fold or Bid. This creates two branches off each of the 6 original branches.
3. **Player 2's Decision** (if Player 1 Bids):
   - If Player 1 decided to Fold, the game ends with payoffs of (-1,1).
   - If Player 1 Bids, then for each of the situations, Player 2 can either Fold or Bid. This creates two more branches off each of the "Bid" branches of Player 1's decision.
4. **Reveal Cards** (if both Bid):
   - If Player 2 Folds, the game ends with payoffs of (1,-1).
   - If both players Bid, the cards are compared and payoffs are given. Depending on the original deal, either Player 1 gets 2 and Player 2 gets -2, or vice versa.

To draw this:

- Start with a single node for the initial deal, branching into the 6 possible outcomes.
- From each of those branches, create two more branches for Player 1's decision.
- From each Bid decision of Player 1, create two more branches for Player 2's decision.
- From each Bid decision of Player 2, label the end of the branches with the result of the card comparison.



E: Watson 24.5

a)

	R	H
IEIF	$7, 5$	$0, 15$
IESF	$3.5, 0.5$	$0, 3.5$
SGIF	$8.5, 4$	$3.5, 0$
SESF	$5, -0.5$	$3.5, 2$

For Celera:

1. Against **IEIF**:
    - **R** gives 5
    - **H** gives 15

    H strictly dominates **R**.
2. Against **IESF**:
    - **R** gives 0.5
    - **H** gives 3.5

    H strictly dominates **R**.
3. Against **SGIF**:
    - **R** gives 4
    - **H** gives 0

    R strictly dominates **H**.
4. Against **SESF**:
    - **R** gives -0.5
    - **H** gives 2

    H strictly dominates **R**.

For Microsoft:

1. If Celera chooses **R**:
   - **IEIF** gives 7
   - **IESF** gives 3.5
   - **SGIF** gives 8.5
   - **SESF** gives 5

   SGIF strictly dominates the other strategies.
2. If Celera chooses **H**:
   - **IEIF** gives 0
   - **IESF** gives 0
   - **SGIF** gives 3.5
   - **SESF** gives 3.5

   Here, both **SGIF** and **SESF** give the same payoff, and neither strictly dominates the other. However, both do strictly dominate **IEIF** and **IESF**.

To find the Nash Equilibria, we look for strategy combinations where both players are playing strict best responses to each other.

From the above analysis:

- **SGIF** is Microsoft's best response when Celera chooses **R**. Also, when Microsoft chooses **SGIF**, **R** is Celera's best response. Hence, (**SGIF**, **R**) is a Nash Equilibrium.

For the other strategies:

- Although **H** is the best response for Celera when Microsoft plays **IEIF** or **IESF**, neither **IEIF** nor **IESF** is a best response for Microsoft when Celera chooses **H**.

The only Nash Equilibrium for the game is (**SGIF**, **R**).

b)

	SRBR	SRBH	SHBR	SHBH
FSES	$10, -4$	$10, -4$	$2.5, -1$	$2.5, -1$
FSEB	$10, -4$	$5, 3.5$	$5, -3.5$	$5, 0$
FBES	$15, -1$	$7, -6.5$	$7.5, 5.5$	$-5, 0$
FBEB	$10, 3$	$2, 1$	$10, 3$	$2, 1$