Seminar 5

This solution set is written and made by Jarle Kvile. It should not be taken as the full solution, and should not be thought to be sufficient to get any type of grade on an exam or a test of any kind.

A: The standard bargaining solution

In game theory, a **bargaining solution** refers to an outcome or a set of outcomes that players might expect to achieve after negotiating over a certain issue. One of the most well-known solutions is the **Nash bargaining solution**, developed by John Nash. The Nash bargaining solution is a way of determining an outcome in a two-player bargaining situation. It satisfies a number of desirable properties, including **Pareto efficiency**, **Symmetry**, **Invariance to equivalent utility representations**, and **Independence of irrelevant alternatives**.

Given two players trying to divide some benefits (e.g., money), the Nash bargaining solution selects the point that maximizes the product of the players' gains over their disagreement (or threat) points.

Formally: Let's assume two players are bargaining over a set S, where any point in S represents a possible agreement. Each player has a utility function, and there is a disagreement point d. The Nash bargaining solution is the point in S that maximizes the product of the players' excess utilities over their disagreement utilities.

Mathematically, for players 1 and 2:

$$\text{Maximize } (u_1(s) - u_1(d)) \times (u_2(s) - u_2(d))$$

Where s is a point in S.

Examples:

1. **Dividing a Cake**: Imagine two friends, Alice and Bob, trying to share a cake. Both want as much cake as possible. If they can't agree on how to share, neither gets any cake (disagreement point). Using the Nash bargaining solution, the most likely outcome is that Alice and Bob will each get half the cake, because 0.5 × 0.5 (the product of their relative gains) is larger than any other division.
2. **Splitting a Prize**: Two coworkers, Jack and Jill, have won a $100 prize for a joint project. If they can't decide how to split the money, they'll have to give it up. Assuming they value the money equally and their disagreement point is $0 each, the Nash solution would be for each to get $50.
3. **Salary Negotiation**: Emma is negotiating her salary with a company. Emma won't accept less than $50,000 (her disagreement point), and the company doesn't want to pay more than $70,000. The set S of possible agreements is [$50,000, $70,000]. Using the Nash bargaining solution (and simplifying a lot), a likely agreed salary might be $60,000, as this roughly maximizes the 'gain' each party receives over their respective disagreement points.

Bargaining power plays a critical role in determining outcomes in bargaining situations. In the context of the Nash bargaining solution, the disagreement or threat point is a key determinant of each player's bargaining power.

1. **Influence on Disagreement Point**: The point at which a player is willing to walk away from negotiations (the disagreement point) reflects their outside options or alternative offers. A player with a more favorable outside option has a higher disagreement point and therefore more bargaining power. The further a player's disagreement point is from the worst outcome for them, the more bargaining power they have.
2. **Impact on Bargaining Outcome**: In the Nash bargaining framework, the outcome is directly influenced by the players' disagreement points. If one player has more bargaining power (reflected in a more favorable disagreement point), the negotiated outcome will be closer to their ideal point.

Illustrative Example:

Consider a labor negotiation. A union is negotiating wage increases with an employer.

- If the union members have strong job alternatives elsewhere (maybe the job market is booming), they might have a high disagreement point, because they can walk away from the job and still be okay. This gives the union considerable bargaining power.
- Conversely, if the job market is weak and alternative jobs are scarce, the union's disagreement point (the wage at which they're willing to walk away) might be lower. This decreases the union's bargaining power.

In the Nash bargaining solution, the final negotiated wage will be influenced by these disagreement points. A union with more bargaining power (higher disagreement point) will likely negotiate a higher wage for its members than a union with less bargaining power.

3. **Unequal Bargaining Power**: If there's a significant disparity in bargaining power between the two parties, the outcome can be skewed heavily in favor of the party with more power. For example, if one party has a very high disagreement point and the other has a very low one, the party with the high disagreement point will capture a much larger share of the bargaining surplus.

Overall, bargaining power, as manifested through the disagreement point and outside options, is central to determining outcomes in bargaining situations, including those predicted by the Nash bargaining solution.

Adding barganing power to the examples above would yield:

1. Dividing a Cake:

Previously: Alice and Bob both wanted to get as much cake as possible, and if they didn't agree, neither would get any cake.

New Scenario: Suppose Bob has another friend, Charlie, who has offered Bob 40% of a different cake if Alice doesn't agree. So, Bob's disagreement point has shifted from 0% of the cake to 40%.

Outcome: With Nash bargaining, Alice and Bob would divide the cake such that the product of their gains from their disagreement points is maximized.

Let x be the fraction Alice gets. Bob gets $1 - x$.

Alice's gain $= x - 0 = x$

Bob's gain $= 1 - x - 0.4 = 0.6 - x$

Maximize: $x(0.6 - x)$

The solution would be different from the 50-50 split because of Bob's increased bargaining power.

2. Splitting a Prize:

Previously: Jack and Jill won \$100. If they couldn't decide how to split, they'd lose the prize.

New Scenario: Jill has another opportunity to participate in a different project that guarantees her \$40. So, her disagreement point is now \$40.

Outcome:

Let y be the amount Jack gets. Jill gets $100 - y$.

Jack's gain $= y - 0 = y$

Jill's gain $= 100 - y - 40 = 60 - y$

Maximize: $y(60 - y)$

Again, the split will lean more in Jill's favor because of her increased bargaining power.

3. Salary Negotiation:

Previously: Emma won't accept less than \$50,000, and the company doesn't want to pay more than \$70,000.

New Scenario: Emma has another job offer for \$55,000. So, her disagreement point is now \$55,000.

Outcome:

Let s be the salary Emma and the company agree on.

Emma's gain $= s - 55,000$

Company's gain (in terms of savings from the maximum they're willing to pay) $= 70,000 - s$

Maximize: $(s - 55,000)(70,000 - s)$

Given Emma's increased bargaining power, the final negotiated salary will likely be higher than in the previous scenario.

In each of these examples, the exact solution would require further mathematical optimization. But the general principle is that as the disagreement point of one player rises due to increased outside options or other factors, their share of the bargaining outcome also rises.

Watson 18.1

Let's do a step-by-step approach here, to make sure we have a sufficient tool chest in order to solve these problems.

a)

1. Graphing the Bargaining Set:

We'll plot the individual value functions for Jerry and Rosemary for the strategies $x \in \{0, 1\}$.

2. Maximizing Joint Value:

We will evaluate the joint value for each strategy x and select the one that gives the highest value.

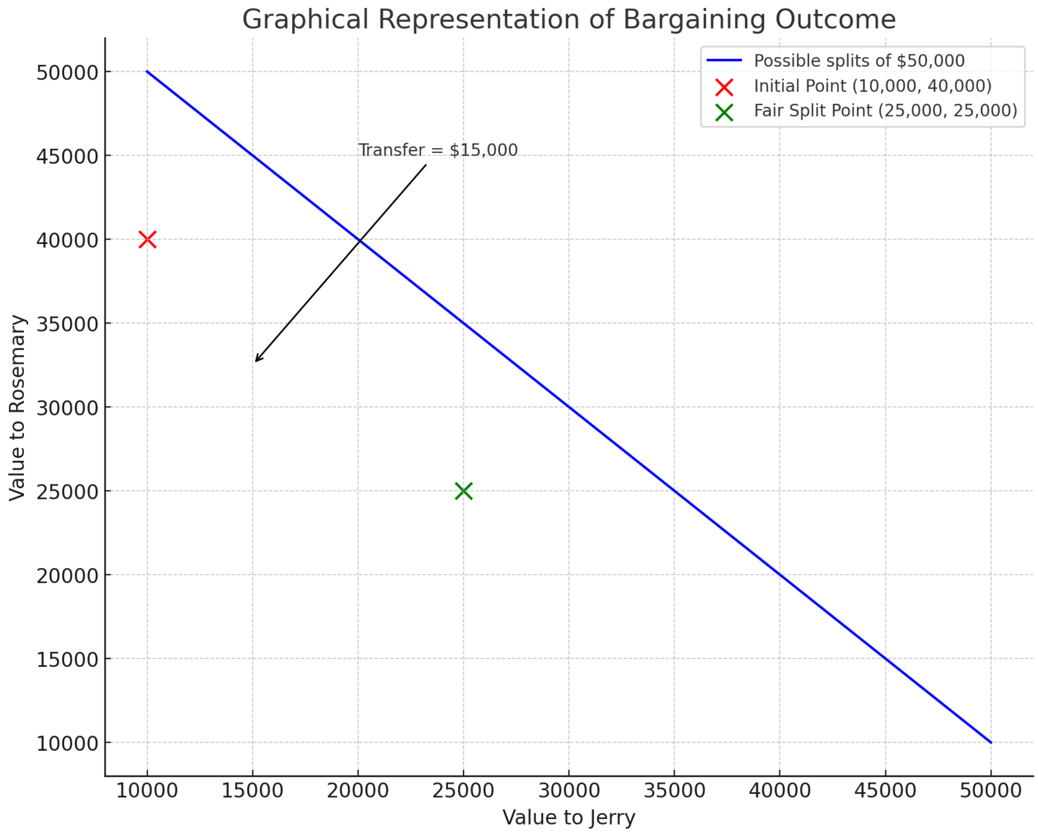
3. Determine Players' Individual Values:

Given the maximizing strategy x^*, we'll evaluate $v_j(x^*)$ and $v_R(x^*)$ to determine Jerry's and Rosemary's individual values.

4. Compute the Transfer t:

Using the correct approach, we'll compute the transfer t that Jerry needs to make to Rosemary.

Let's begin by graphing the bargaining set.



Here's the bargaining set for Jerry and Rosemary based on the two strategy options $x \in \{0, 1\}$.

Next, we'll compute the maximizing joint value and determine the individual values for Jerry and Rosemary.

Maximizing Joint Value:

The joint value for each strategy x is the sum of the individual values of Jerry and Rosemary. We'll evaluate the joint value for each strategy and then select the one that maximizes it.

Maximizing Joint Value:

The maximized joint value is $50,000, achieved when $x = 0$.

Individual Values:

- Jerry's value is $10,000.
- Rosemary's value is $40,000.

Next, we'll compute the transfer t that Jerry needs to make to Rosemary to ensure that both players receive a payoff of $25,000 each.

Using the relationship:

$$u_J = v_J(0) + t$$

Given that $v_J(0) = 10,000$ and $v_R(0) = 40,000$, the total joint value is $10,000 + 40,000 = 50,000$. Given $p_J = p_R = 1/2$, they should ideally split the 50,000 equally, which means each gets 25,000.

Jerry already has 10,000, so he needs a transfer of $25,000 - 10,000 = 15,000$ to get his fair share. Similarly, Rosemary needs to transfer 15,000 to Jerry to ensure they both get equal value.

So, the transfer $t = 15{,}000$ from Rosemary to Jerry.

In the standard bargaining solution:

- Both Jerry and Rosemary have a payoff of $25,000 each.
- Jerry transfers $15,000 to Rosemary to achieve this outcome.

b)

Given:

1. $d_J = 0$ (disagreement point for Jerry)
2. $d_R = 0$ (disagreement point for Rosemary)
3. $v_J(x) = 60,000 - x^2$
4. $v_R(x) = 800x$

5. x is any positive number
6. $p_J = \frac{1}{2}$ (probability of Jerry's proposal being accepted)
7. $p_R = \frac{1}{2}$ (probability of Rosemary's proposal being accepted)

To solve this problem:

1. Determine the maximized joint value:
   - To maximize the joint value, we need to maximize $v_J(x) + v_R(x)$.
2. Determine the players' individual values at the maximized joint value.
3. Compute the transfer t that the players select.

We'll start by determining the value of x that maximizes the joint value of $v_J(x) + v_R(x)$.

Given the procedure you've outlined, we should first find x^* by maximizing the sum of $v_J(x)$ and $v_R(x)$.

Step 1: Find the value x^* that maximizes the joint value v^*.

$$v^*(x) = v_J(x) + v_R(x)$$

$$v^*(x) = (60,000 - x^2) + 800x$$

To maximize this function, we differentiate it with respect to x and then set the derivative to zero. (Couldnt get the prime correct here but they are differentiated)

$$v^*(x) = \frac{d}{dx}(60,000 - x^2 + 800x)$$

$$v^*(x) = -2x + 800$$

Setting $v^*(x)$ to zero:

$$-2x + 800 = 0$$

$$x = 400$$

This gives $x^* = 400$.

Step 2: Find v^* using x^*.

$$v^*(400) = 60,000 - 400^2 + 800(400)$$

$$v^*(400) = 60,000 - 160,000 + 320,000$$

$$v^*(400) = 220,000$$

Now, using the standard bargaining solution:

For Player J:

$$d_J + p_J(v^* - d_J - d_R) = v_J(400) + t$$

For Player R:

$$d_R + p_R(v^* - d_J - d_R) = v_R(400) - t$$

Given that $d_J = d_R = 0$ and $p_J = p_R = \frac{1}{2}$:

Player J:

$$t = \frac{1}{2} \times 220,000 - (60,000 - 400^2)$$

$$t = 110,000 - 60,000 + 160,000$$

$$t = 210,000$$

This is the transfer from Player J to Player R. Note that this value doesn't make sense in context (since it's more than Player J's value at x^*).

At $x = 400$:

1. The value for Jerry, $v_J(400)$, is $-100,000$. However, this indicates that Jerry incurs a loss or cost at this point, and is not a possibile solution due to the disagreement point for Jerry.
2. The value for Rosemary, $v_R(400)$, is $320,000$.

Given the disagreement points $d_J = 0$ and $d_R = 0$, and the probabilities $p_J = \frac{1}{2}$ and $p_R = \frac{1}{2}$, the players should ideally split the maximized joint value (which is positive) equally.

Indeed, my apologies for overlooking that detail. Let's re-evaluate the scenario:

Given $v_J(x) = 60,000 - x^2$ and $v_R(x) = 800x$:

For $x^* = 400$:

$$v_J(400) = 60,000 - 400^2 = 60,000 - 160,000 = -100,000$$

$$v_R(400) = 800(400) = 320,000$$

So, the joint value at $x^* = 400$ is:

$$v^*(400) = -100,000 + 320,000 = 220,000$$

Given the bargaining power is equal for both players (i.e., $p_J = p_R = \frac{1}{2}$), each player is entitled to half of the surplus over their disagreement payoff, which is 0 for both:

$$Surplus = 220,000 - 0 = 220,000$$

$$Share = \frac{1}{2} \times 220,000 = 110,000$$

Since Jerry's value for $x = 400$ is -100,000, in order to ensure Jerry gets his share of 110,000 from the total surplus:

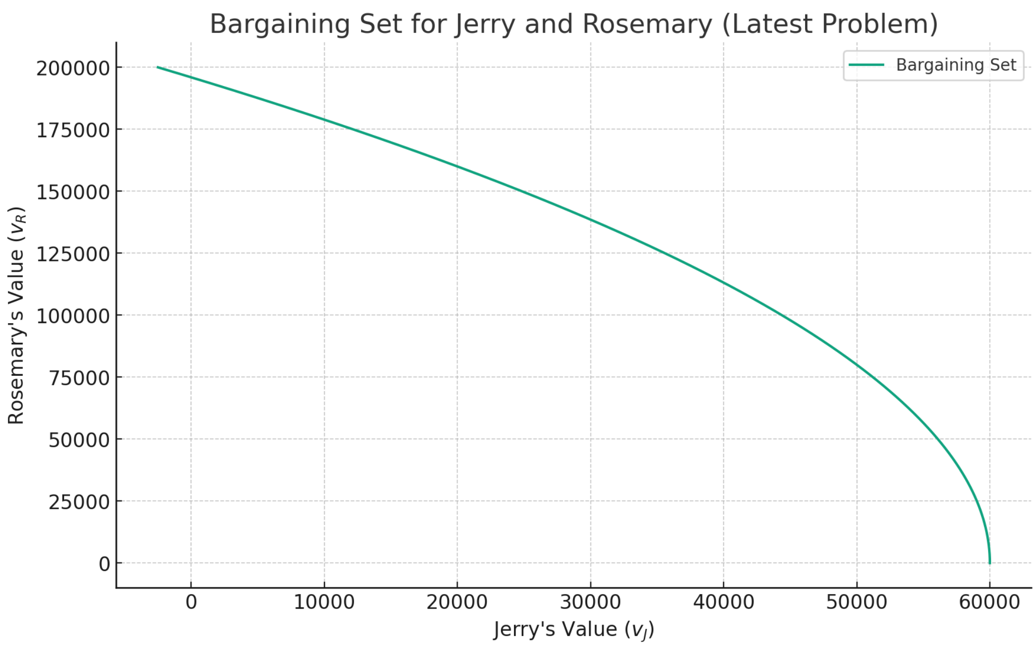
$$Transfer, t = 110,000 - (-100,000) = 210,000$$

No graph for this one. My pythonic skills where not good enough this morning. :)

c)

Given:

- $d_j = 40,000$ and $d_R = 20,000$ are the disagreement points for Jerry and Rosemary, respectively.
- $v_j(x) = 60,000 - x^2$ and $v_R(x) = 800x$ are the individual value functions for Jerry and Rosemary, respectively.
- x can be any positive number.
- $\pi_j = \frac{1}{4}$ and $\pi_R = \frac{3}{4}$ are the probabilities with which Jerry and Rosemary get to make a take-it-or-leave-it offer.



Here's the bargaining set for Jerry and Rosemary based on their respective value functions over the range of x from 0 to 250.

Next, we'll compute the maximizing joint value.

Maximizing Joint Value:

The joint value for each x is the sum of the individual values of Jerry and Rosemary. As before, we'll find the value of x that maximizes this joint value.

To do this, we'll differentiate the joint value function with respect to x and set it to zero to find the critical points. Then, we'll identify which of these points gives the maximum value.

Maximizing Joint Value:

The joint value is maximized at $x^* = 400$, yielding a maximum joint value of $v^* = 220,000$.

Next, we'll determine the individual values for Jerry and Rosemary at this maximizing strategy x^*.

Determine Players' Individual Values:

Given the strategy $x^* = 400$, we'll evaluate $v_j(x^*)$ and $v_R(x^*)$ to determine the values for Jerry and Rosemary.

Individual Values at $x^* = 400$:

- Jerry's value is $-\$100,000$.
- Rosemary's value is \$320,000.

Given these values and the provided disagreement points:

- $d_J = 40,000$
- $d_R = 20,000$

And the bargaining weights:

- $\pi_J = \frac{1}{4}$
- $\pi_R = \frac{3}{4}$

We'll compute the transfer t that Jerry needs to make to Rosemary to ensure that both players reach an agreement according to the Nash bargaining solution.

Using the Nash bargaining solution, given the individual values we just found, the disagreement points, and the bargaining weights, we can compute the transfer t:

$$u_J = d_J + \pi_J \times (v^* - d_J - d_R)$$

$$u_R = d_R + \pi_R \times (v^* - d_J - d_R)$$

Given that:

$$u_J = v_J(x^*) + t$$

$$u_R = v_R(x^*) - t$$

We can determine t. Let's compute it.

The computed transfer, t, is \$180,000.

In the standard bargaining solution for this problem:

- Jerry will transfer \$180,000 to Rosemary to achieve the agreement.

This means:

- Jerry's net value after the transfer is $-\$100,000 + \$180,000 = \$80,000$.
- Rosemary's net value after the transfer is $\$320,000 - \$180,000 = \$140,000$.

Given their respective disagreement points and bargaining weights, this solution ensures that both players are better off compared to their disagreement points, and the division of the surplus aligns with the Nash bargaining solution.

Watson 18.3)

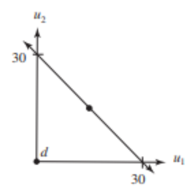
a)

Bargaining Problem :

Given:

- $x \in \{5, 10, 15\}$ are the strategy options.
- $v_1(x) = x$ and $v_2(x) = x$ are the individual value functions for players 1 and 2, respectively.
- $d_1 = 0$ and $d_2 = 0$ are the disagreement points for players 1 and 2.
- $\pi_1 = \frac{1}{2}$ and $\pi_2 = \frac{1}{2}$ are the probabilities with which players 1 and 2 get to make a take-it-or-leave-it offer.

Player 1's payoff is $u_1 = v_1(x) + t$ and player 2's payoff is $u_2 = v_2(x) - t$.



From the graph, we can see that the joint value is maximized when $x = 15$. This will be our chosen strategy x^*.

Next, we'll compute the transfer t that player 1 makes to player 2 to ensure both players receive an equal share of the surplus, given the Nash bargaining solution with equal bargaining weights.

Given that both disagreement points are 0, the surplus is:

$$\text{surplus} = v_1(x^*) + v_2(x^*) - d_1 - d_2$$

Using the Nash bargaining solution with equal bargaining weights (since both players have a probability of $\frac{1}{2}$ of making an offer), the payoffs for each player are:

$$u_i = d_i + \pi_i \times \text{surplus}$$

Calculate the Surplus:
The surplus is the difference between the joint value at the chosen strategy x^* and the sum of the disagreement values for both players.

$$\text{surplus} = v_1(x^*) + v_2(x^*) - d_1 - d_2$$

Determine Expected Payoffs from Surplus:
Given the Nash bargaining solution and the probabilities π_1 and π_2, the expected payoffs for the players from the surplus are:

$$u_1' = d_1 + \pi_1 \times \text{surplus}$$

$$u_2' = d_2 + \pi_2 \times \text{surplus}$$

Relate Expected Payoffs to Value Functions and Transfer:
Given the relationship between the players' payoffs, their value functions, and the transfer t:

$$u_1 = v_1(x^*) + t$$

$$u_2 = v_2(x^*) - t$$

We can equate the expected payoffs to these expressions to solve for t:

$$u_1' = v_1(x^*) + t$$

$$u_2' = v_2(x^*) - t$$

. **Solve for t**:
Using the above equations, we can derive the value of t.

Given the information provided:

- $x^* = 15$
- $d_1 = 0$ and $d_2 = 0$
- $v_1(x) = x$ and $v_2(x) = x$
- $\pi_1 = \frac{1}{2}$ and $\pi_2 = \frac{1}{2}$

Let's compute the transfer t step by step.

Following the step-by-step calculation:

1. The computed surplus for $x^* = 15$ is 30.
2. The expected payoffs from the surplus are:
    - $u_1' = 15$ for Player 1
    - $u_2' = 15$ for Player 2
3. Using the relationship between the expected payoffs, value functions, and the transfer t, we derived:
    - $u_1' = v_1(15) + t$
    - $u_2' = v_2(15) - t$
4. Solving for t, we find $t = 0$.

The computed transfer, t, is $0.

Given the nature of the problem where $v_1(x)$ and $v_2(x)$ are identical and the players have equal bargaining weights, it makes sense that no transfer is needed. Both players already receive an equal share of the surplus.

Players' Individual Payoffs:

Given $x^* = 15$ and $t = 0$:

- Player 1's payoff is $u_1 = v_1(15) + 0 = 15$.
- Player 2's payoff is $u_2 = v_2(15) - 0 = 15$.

Both players end up with a payoff of $15, which is half of the surplus from the chosen strategy $x^* = 15$.

b)

Given:

- $x \in \{5, 10, 15\}$ are the strategy options.
- $v_1(x) = x$ and $v_2(x) = x$ are the individual value functions for players 1 and 2, respectively.
- $d_1 = 2$ and $d_2 = 4$ are the disagreement points for players 1 and 2.
- $\pi_1 = \frac{1}{2}$ and $\pi_2 = \frac{1}{2}$ are the probabilities with which players 1 and 2 get to make a take-it-or-leave-it offer.

Player 1's payoff is $u_1 = v_1(x) + t$ and player 2's payoff is $u_2 = v_2(x) - t$.

Steps:

1. **Determine Chosen x**: We'll find the value of x that maximizes the joint value, which is the sum of the individual value functions.
2. **Compute the Transfer t**: Given the chosen x, the individual value functions, the disagreement points, and the probabilities, we'll compute the transfer t using the Nash bargaining solution.
3. **Players' Individual Payoffs**: We'll report the players' individual payoffs using the provided payoff functions.

Let's begin by determining the chosen x that maximizes the joint value.

The joint value is maximized when $x = 15$. This will be our chosen strategy x^*.

Next, we'll compute the transfer t that player 1 makes to player 2 to ensure both players reach an agreement according to the Nash bargaining solution.

Given:

- $x^* = 15$
- $d_1 = 2$ and $d_2 = 4$
- $v_1(x) = x$ and $v_2(x) = x$
- $\pi_1 = \frac{1}{2}$ and $\pi_2 = \frac{1}{2}$

We can compute the transfer t using the Nash bargaining solution:

1. Calculate the surplus as:

$$\text{surplus} = v_1(x^*) + v_2(x^*) - d_1 - d_2$$

2. Determine the expected payoff from the surplus for player 1:

$$u_1' = d_1 + \pi_1 \times \text{surplus}$$

3. Relate the expected payoff to the value function and transfer for player 1:

$$u_1' = v_1(x^*) + t$$

The computed transfer, t, is -1.

Given this bargaining solution:

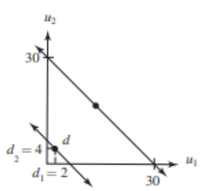
- Player 1 will transfer \$1 to Player 2 (since the transfer is negative).

Players' Individual Payoffs:

Given $x^* = 15$ and $t = -1$:

- Player 1's payoff is $u_1 = v_1(15) - 1 = 14$.
- Player 2's payoff is $u_2 = v_2(15) + 1 = 16$.

Both players end up dividing the surplus from the chosen strategy $x^* = 15$ according to the Nash bargaining solution, given their respective disagreement points and equal bargaining weights.



c)

Given:

- $x \in \{5, 10, 15\}$ are the strategy options.
- $v_1(x) = x$ and $v_2(x) = x$ are the individual value functions for players 1 and 2, respectively.
- $d_1 = 2$ and $d_2 = 4$ are the disagreement points for players 1 and 2.
- $\pi_1 = \frac{1}{4}$ and $\pi_2 = \frac{3}{4}$ are the probabilities with which players 1 and 2 get to make a take-it-or-leave-it offer.

Player 1's payoff is $u_1 = v_1(x) + t$ and player 2's payoff is $u_2 = v_2(x) - t$.

Steps:

1. **Determine Chosen x**: We'll find the value of x that maximizes the joint value, which is the sum of the individual value functions.
2. **Compute the Transfer t**: Given the chosen x, the individual value functions, the disagreement points, and the probabilities, we'll compute the transfer t using the Nash bargaining solution.
3. **Players' Individual Payoffs**: We'll report the players' individual payoffs using the provided payoff functions.

Let's begin by determining the chosen x that maximizes the joint value.

The joint value is again maximized when $x = 15$. This will be our chosen strategy x^*.

Next, we'll compute the transfer t that player 1 makes to player 2 to ensure both players reach an agreement according to the Nash bargaining solution with the updated bargaining weights.

Given:

- $x^* = 15$
- $d_1 = 2$ and $d_2 = 4$
- $v_1(x) = x$ and $v_2(x) = x$
- $\pi_1 = \frac{1}{4}$ and $\pi_2 = \frac{3}{4}$

t, is -7.

Given this bargaining solution:

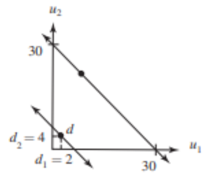
- Player 1 will transfer \$7 to Player 2 (since the transfer is negative).

Players' Individual Payoffs:

Given $x^* = 15$ and $t = -7$:

- Player 1's payoff is $u_1 = v_1(15) - 7 = 8$.
- Player 2's payoff is $u_2 = v_2(15) + 7 = 22$.

Both players divide the surplus from the chosen strategy $x^* = 15$ according to the Nash bargaining solution, given their respective disagreement points and updated bargaining weights.



d)

Given:

- $x \in \{5, 10, 15\}$ are the strategy options.
- $v_1(x) = 20x$ and $v_2(x) = -x^2$ are the individual value functions for players 1 and 2, respectively.
- $d_1 = 0$ and $d_2 = 0$ are the disagreement points for players 1 and 2.
- $\pi_1 = \frac{1}{4}$ and $\pi_2 = \frac{3}{4}$ are the probabilities with which players 1 and 2 get to make a take-it-or-leave-it offer.

Player 1's payoff is $u_1 = v_1(x) + t$ and player 2's payoff is $u_2 = v_2(x) - t$.

Steps:

1. **Determine Chosen x**: We'll find the value of x that maximizes the joint value, which is the sum of the individual value functions.
2. **Compute the Transfer t**: Given the chosen x, the individual value functions, the disagreement points, and the probabilities, we'll compute the transfer t using the Nash bargaining solution.
3. **Players' Individual Payoffs**: We'll report the players' individual payoffs using the provided payoff functions.

Let's begin by determining the chosen x that maximizes the joint value.

The joint value is maximized when $x = 10$. This will be our chosen strategy x^*.

Next, we'll compute the transfer t that player 1 makes to player 2 to ensure both players reach an agreement according to the Nash bargaining solution.

Given:

- $x^* = 10$
- $d_1 = 0$ and $d_2 = 0$
- $v_1(x) = 20x$ and $v_2(x) = -x^2$
- $\pi_1 = \frac{1}{4}$ and $\pi_2 = \frac{3}{4}$

We can compute the transfer t using the Nash bargaining solution (as did in a).

The computed transfer, t, is -175.

Given this bargaining solution:

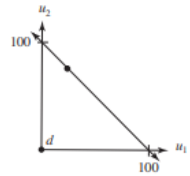
- Player 1 will transfer \$175 to Player 2 (since the transfer is negative).

Players' Individual Payoffs:

Given $x^* = 10$ and $t = -175$:

- Player 1's payoff is $u_1 = v_1(10) - 175 = 25$.
- Player 2's payoff is $u_2 = v_2(10) + 175 = 75$.

Both players divide the surplus from the chosen strategy $x^* = 10$ according to the Nash bargaining solution, given their respective disagreement points and updated bargaining weights.



e)

This might be a bit more tricky...

- x can be any real number (from $-\infty$ to $+\infty$).
- $v_1(x) = 16x + x^2$ and $v_2(x) = 8x - 2x^2$ are the individual value functions for players 1 and 2, respectively.
- $d_1 = 0$ and $d_2 = 0$ are the disagreement points for players 1 and 2.
- The bargaining weights π_1 and π_2 are arbitrary (we'll consider them as variables for now).

Player 1's payoff is $u_1 = v_1(x) + t$ and player 2's payoff is $u_2 = v_2(x) - t$.

Let's begin by determining the chosen x that maximizes the joint value.

The joint value is maximized when $x = 12$. This will be our chosen strategy x^*.

Next, we'll compute the transfer t that player 1 makes to player 2. However, since the bargaining weights π_1 and π_2 are arbitrary, the transfer will be a function of these weights.

Given:

- $x^* = 12$
- $d_1 = 0$ and $d_2 = 0$
- $v_1(x) = 16x + x^2$ and $v_2(x) = 8x - 2x^2$
- The bargaining weights π_1 and π_2 are arbitrary.

We can compute the transfer t using the Nash bargaining solution as a function of π_1 and π_2.

Of course! Let's break down the computation of the transfer t using mathematical notation:

Determining Chosen x:

Given the value functions:

$$v_1(x) = 16x + x^2$$

$$v_2(x) = 8x - 2x^2$$

The joint value function is:

$$v(x) = v_1(x) + v_2(x) = 24x - x^2$$

To maximize this joint value, we'll differentiate $v(x)$ with respect to x and set the result to 0:

$$\frac{dv(x)}{dx} = 24 - 2x = 0$$

Solving for x gives $x^* = 12$.

Computing the Transfer t:

Given the joint value function and the chosen x^*, the surplus is:

$$\text{surplus} = v_1(12) + v_2(12) = 288 - 144 = 144$$

Given the Nash bargaining solution and the arbitrary bargaining weights π_1 and π_2, the expected payoffs from the surplus are:

$$u_1' = d_1 + \pi_1 \times \text{surplus} = \pi_1 \times 144$$

$$u_2' = d_2 + \pi_2 \times \text{surplus} = \pi_2 \times 144$$

where $d_1 = 0$ and $d_2 = 0$.

From the relationship between the expected payoffs, value functions, and the transfer t:

$$u_1' = v_1(12) + t$$

Using this relationship, we derive the transfer t as a function of π_1:

Given:

$$u_1' = \pi_1 \times 144$$

and

$$v_1(12) = 16(12) + 12^2 = 192 + 144 = 336$$

We can express t as:

$$t(\pi_1) = u_1' - v_1(12)$$

$$t(\pi_1) = \pi_1 \times 144 - 336$$

$$t(\pi_1) = 144\pi_1 - 336$$

So, the transfer t as a function of π_1 is:

$$t(\pi_1) = 144\pi_1 - 336$$

Thank you for pointing it out, and I apologize for the confusion earlier.

Players' Individual Payoffs:

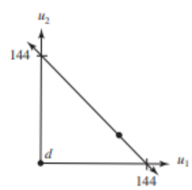
Given $x^* = 12$ and the transfer function $t(\pi_1)$:

- Player 1's payoff is $u_1 = v_1(12) + t(\pi_1)$.
- Player 2's payoff is $u_2 = v_2(12) - t(\pi_1)$.

Both players divide the surplus from the chosen strategy $x^* = 12$ according to the Nash bargaining solution, given their respective disagreement points and the arbitrary bargaining weights π_1 and π_2.

The computed transfer, t, as a function of the bargaining weight π_1 is:

$$t(\pi_1) = 138\pi_1 - 334$$



Watson 18.5

We are here touching on an essential insight related to the Nash bargaining solution. Given the nature of the Nash bargaining solution, the allocation of the surplus depends directly on the relative bargaining weights.

If your bargaining weight (π_1) is greater than 0.5:

- Increasing the maximal joint value v^* would be more beneficial to you, because your share of the increased value would be more than half. From a collective standpoint, this choice is efficient because it increases the overall welfare.

If your bargaining weight (π_1) is less than 0.5:

- Raising your disagreement payoff might be more advantageous for you personally. This is because, with a weight less than 0.5, you'd capture less than half of any increase in v^*. However, this choice isn't efficient from a total welfare standpoint, as it does not expand the "pie" but merely affects the division of it.

In essence, if you have strong bargaining power (weight > 0.5), it's both personally beneficial and efficient to increase v^*. If you have weaker bargaining power (weight < 0.5), raising your disagreement point might be more advantageous for you, but it's not an efficient choice for total welfare.

B The Coase Theorem

The Coase Theorem is a fundamental concept in law and economics. Named after Ronald Coase, who was awarded the Nobel Prize in Economics for his work on the topic, the theorem addresses how private economic actors can resolve disputes over property rights without the need for government intervention, given certain conditions.

The Coase Theorem states:

If trade in an externality is possible and there are sufficiently low transaction costs, bargaining will lead to an efficient outcome regardless of the initial allocation of property rights.

In simpler terms, if parties can negotiate without significant costs, they can achieve the most efficient outcome on their own, no matter who initially holds the rights.

Let's break this down a bit:

1. **Trade in an Externality**: The theorem often applies to situations where externalities (spillover effects from economic activity that affect third parties not directly involved in the activity) are present. A classic example is a factory that emits pollution affecting nearby residents.
2. **Low Transaction Costs**: Transaction costs include the costs of bargaining, drawing up contracts, and any other costs associated with the negotiation. For the Coase Theorem to hold, these costs (generally) need to be low.
3. **Initial Allocation Doesn't Matter**: Whether the factory has the right to pollute or the residents have the right to clean air, the parties will negotiate an outcome that maximizes total welfare. The initial rights just determine the direction of the payments (either the residents pay the factory to reduce pollution or the factory pays the residents for the right to pollute).

Watson 18.8

a)

Current Situation:

- If the confectioner operates the blender:
  - Confectioner's value: 400
  - Physician's value: 0 (because the noise prevents her from consulting with patients)
- If the confectioner does not operate the blender:
  - Confectioner's value: 0
  - Physician's value: 2000

Given that the confectioner has the legal right to operate the blender, the initial disagreement outcome (if no agreement is reached) is that the blender operates, and the confectioner earns 400 while the physician earns 0.

Using the Standard Bargaining Solution:

We can use the Nash bargaining solution to predict the outcome:

1. **Determine the Surplus**: The surplus is the difference between the joint value when they both maximize their combined earnings and the sum of their disagreement payoffs.
2. **Allocate the Surplus**: Given equal bargaining weights, they will split this surplus equally.
3. **Determine Payoffs**: Add half the surplus to each party's disagreement payoff to get their final payoffs.

1. Determine the Surplus

The surplus is the difference between the joint value when they both maximize their combined earnings (i.e., the confectioner doesn't operate the blender) and the sum of their disagreement payoffs (i.e., the confectioner operates the blender).

Given:

- Disagreement payoff for confectioner: 400 (he operates the blender)
- Disagreement payoff for physician: 0 (due to the noise)
- Cooperative payoff for confectioner: 0 (he doesn't operate the blender)
- Cooperative payoff for physician: 2000 (she can consult without noise)

$$\text{Surplus} = (\text{Cooperative payoff for confectioner} + \text{Cooperative payoff for physician}) - (\text{Disagreement payoff for confectioner} + \text{Disagreement payoff for}$$

$$\text{Surplus} = (0 + 2000) - (400 + 0) = 1600$$

2. Allocate the Surplus

Given equal bargaining weights, they will split this surplus equally:

$$\text{Confectioner's share of surplus} = \frac{1}{2} \times 1600 = 800$$

$$\text{Physician's share of surplus} = \frac{1}{2} \times 1600 = 800$$

3. Determine Final Payoffs

To find the final payoffs after bargaining, we add each party's share of the surplus to their disagreement payoff:

$$\text{Confectioner's final payoff} = \text{Disagreement payoff for confectioner} + \text{Confectioner's share of surplus} = 400 + 800 = 1200$$

$$\text{Physician's final payoff} = \text{Disagreement payoff for physician} + \text{Physician's share of surplus} = 0 + 800 = 800$$

Result:

- Confectioner's payoff: $\$1200$
- Physician's payoff: $\$800$

Given the situation and the bargaining solution, the parties would negotiate such that the confectioner would receive a payment of $\$1200$ to not operate the blender. In contrast, the physician would benefit by earning $\$800$ (from her original $\$2000$ minus the compensation paid to the confectioner). This ensures that the physician can consult with her patients without disturbance, and the confectioner is compensated for his loss of production. This outcome maximizes their combined value while taking into account their respective bargaining powers.

b)

The Scenario:

Given the new legal context, the confectioner cannot operate his blender without the physician's permission. This changes the dynamics of their disagreement:

Disagreement Outcome:

If no agreement is reached:

- The confectioner does not operate the blender (since it's illegal without the physician's permission).
- Confectioner's value: $\$0$
- Physician's value: $\$2000$ (she can consult without noise due to the absence of the blender's operation)

Determine the Surplus:

The surplus remains the difference between the joint value when they both maximize their combined earnings (i.e., the confectioner doesn't operate the blender) and the sum of their disagreement payoffs.

Allocate the Surplus:

Given equal bargaining weights, they will split this surplus equally. However, due to the change in property rights, the physician now has the stronger position in the negotiation.

Determine Final Payoffs:

Given the new property rights and the negotiation dynamics:

- The confectioner's payoff: $0
- The physician's payoff: $2000

This means that, given the legal framework where the confectioner cannot operate the blender without the physician's permission, the physician retains her full value of production, $2000, in the bargaining outcome. The confectioner, on the other hand, gets nothing. The physician essentially has the right to the entire surplus since she starts with the legal right to silence, which provides her with a strong negotiating position.

c)

Scenario:

If the confectioner uses his blender without contracting with the physician, the physician has the right to recover damages of z.

1. If the Parties Do Not Contract:

- **For** $z > 400$:
  - The confectioner will not use the blender.
  - Confectioner's payoff: $0
  - Physician's payoff: $2000
- **For** $z \leq 400$:
  - The confectioner will use the blender.
  - Confectioner's payoff: $\$400 - z$ (profit from using the blender minus the damages)
  - Physician's payoff: z (amount of damages)

2. If the Parties Contract:

The optimal outcome would be for the confectioner not to use the blender, and the physician to operate without noise disturbance. They would then negotiate to share the surplus, which is the difference between this cooperative outcome and their disagreement payoffs.

Given equal bargaining weights, they would split the surplus equally.

3. Final Payoffs:

- **For** $z > 400$:
  - The confectioner and the physician will contract (since it's in their best interest to do so).
  - They will equally split the surplus of $2000.
  - Confectioner's payoff: $1000
  - Physician's payoff: $1000
- **For** $z \leq 400$:
  - The confectioner's disagreement payoff is higher than half the surplus, so he prefers not to contract.
  - Confectioner's payoff: $\$400 - z$
  - Physician's payoff: z

In summary, the parties will contract when $z > 400$, and they will not contract when $z \leq 400$. The value of z determines the balance of power in the negotiation.

3 Bargaining games

Watson 19.1

a)

Ultimatum Game Setup:

1. **Superintendent's Move**: The superintendent chooses an offer x, where x is between 0 and 1.
2. **Union President's Move**: After observing the offer x, the president of the union can either accept ("yes") or reject ("no").

Outcomes:

- If the president accepts the offer, the superintendent gets $1 - x$ and the union president gets x.
- If the president rejects the offer, both parties get 0.

Backward Induction:

Step 1: Consider the union president's decision after observing the offer x.

- If the president believes that rejecting the offer is better, then she would get 0 (the disagreement payoff).
- If she accepts the offer, she gets x.
- Therefore, for any $x > 0$, the president is better off accepting the offer. If $x = 0$, she is indifferent between accepting and rejecting.

Step 2: Knowing the president's strategy, the superintendent makes his offer.

- The superintendent wants to keep as much of the "pie" as possible.
- Given that the president will accept any offer greater than 0, the superintendent will offer the smallest positive amount to the president and keep almost the entire "pie" for himself.

Prediction:

Using backward induction, the superintendent will offer an amount infinitesimally close to 0 to the president of the union (essentially the smallest possible positive amount). The union president, knowing that this is better than getting nothing, will accept the offer.

This result is a characteristic outcome of the ultimatum game, where the first mover can capture almost the entire value, especially when the second mover's disagreement payoff is low.

b)

Recap of the New Conditions:

1. The president of the union has committed to the teachers to hold out for a salary of at least z.
2. If the president accepts an offer $x < z$, she will be fired as the union leader.
3. Being fired results in a personal cost of y utility units for the president. So, her effective utility for an offer x where $x < z$ is $x - y$. For offers $x \geq z$, her utility remains x.

Decision Analysis:

Given an offer x by the superintendent, the president will evaluate her potential utility:

1. **If** $x < z$:
   - If she accepts, she gets $x - y$ (since she will be fired).
   - If she rejects, she gets 0.
   - She will accept if $x - y > 0$ or $x > y$.
2. **If** $x \geq z$:
   - If she accepts, she gets x (since she won't be fired).
   - If she rejects, she gets 0.
   - She will always accept offers in this range since x is always greater than 0.

Summary:

If the president accepts an offer x such that $x < z$, she will be fired. The effective utility she receives is $x - y$.

For her to be better off accepting the offer than rejecting it and getting 0, $x - y > 0$ or $x > y$.

However, if x reaches the threshold z, she doesn't face the risk of being fired. In this case, her utility is just x, and she would accept any $x \geq z$ because it's better than getting 0.

Now, when comparing the two thresholds y and z:

- If y is the smaller value, then $x > y$ implies that she will accept offers slightly greater than y and above.
- If z is the smaller value, then she would be willing to accept any offer $x \geq z$ because accepting an offer just below z would mean facing the firing penalty and getting less than 0 utility.

Thus, the president will accept offers x that are greater than or equal to the minimum of y and z. Therefore the president will accept x if $x \geq \min\{z, y\}$.

c)

Given our understanding of the president's decision-making, the superintendent will strategize to offer the smallest possible amount that the president will accept.

Outcome of the Game:

1. **Superintendent's Offer**: Knowing the president's decision rule, the superintendent will offer the smallest amount the president is willing to accept to maximize his own payoff. This amount will be $\min\{z, y\}$.
2. **President's Response**: Given the superintendent's offer of $\min\{z, y\}$, the president will accept it. This is because the offer will be either just high enough to cover her personal loss if she gets fired (in the case where $y < z$) or just high enough to meet her commitment to the teachers (in the case where $z \leq y$).

Dependency on y:

- If y is small (relative to z), then the superintendent will offer an amount slightly above y. The smaller y is, the less the union president is effectively willing to accept before she would prefer getting fired. In this case, the union's salary is largely determined by how much the president values her job relative to the salary.
- If y is large (and larger than z), then the superintendent will offer z, the amount the president promised the teachers. In this scenario, the union's salary is determined by the president's commitment to the teachers rather than her personal cost of being fired.

In essence, the union's final salary is directly influenced by y. The value of y determines whether the superintendent bases his offer on the president's personal cost or her commitment to the teachers.

d)

The president should just promise $z = y$.

1. **Maximizing Payoff with Minimal Risk**: By setting $z = y$, the president ensures that any offer she's willing to accept (because it's greater than or equal to y) also meets her commitment to the teachers. This eliminates the potential dilemma where she might have to accept an offer that meets her commitment but results in her being fired with a net negative utility.
2. **Simplicity and Transparency**: A promise of $z = y$ simplifies the negotiation dynamics. The superintendent knows that offering anything less than y would be fruitless since the president would reject it, both due to her personal cost and her commitment to the teachers.
3. **Avoiding the Dilemma**: As mentioned earlier, if $z > y$, and the superintendent offers something in the range $y < \text{offer} < z$, the president faces a dilemma. By setting $z = y$, this situation is entirely avoided.

Promising $z = y$ is a strategic move that aligns the president's personal interests with her professional commitment, making the negotiation straightforward and maximizing the union's potential payoff.

Watson 19.3

$\mathbf{T = 1}$ (One-stage game):

Player 1 knows that Player 2 will accept any positive amount since the alternative is zero. Therefore, Player 1 offers the least possible amount (0 in this case) to Player 2 and keeps the maximum for himself.

Equilibrium:

- Player 1: 1
- Player 2: 0

$\mathbf{T = 2}$ (Two-stage game):

Player 1 anticipates that in the next stage, Player 2 will offer him the smallest amount, δ. Therefore, to prevent moving to the next stage and to make Player 2 indifferent between accepting now or in the next stage, Player 1 offers δ to Player 2.

Equilibrium:

- Player 1: $1 - d$
- Player 2: d

$\mathbf{T = 3}$ (Three-stage game):

Now, Player 1 knows that in the next stage, Player 2 will offer him $1 - \delta$. Therefore, Player 1 needs to offer Player 2 an amount today that makes Player 2 indifferent between accepting today and waiting for tomorrow. That amount is $\delta(1 - \delta)$.

Equilibrium:

- Player 1: $1 - \delta(1 - d)$
- Player 2: $\delta(1 - \delta)$

And so on...

The pattern here is an alternating structure where each player, in their turn, tries to offer the other just enough to make them indifferent between accepting the current offer and waiting for the next round.

As T approaches infinity, the game will have an infinite number of stages, and the equilibrium payoffs will converge to the values you mentioned:

- Player 1: $\frac{1}{1+\delta}$
- Player 2: $\frac{d}{1+\delta}$

This is because the effect of future possible offers becomes negligible as we consider more and more stages, and the alternating structure means the players end up splitting the surplus in a manner reflective of their discount factors.

Watson 19.9)

a)

In this alternating-offer bargaining game, we have:

1. The Conservative Party (C) as the proposer in odd-numbered periods.
2. The Liberal Democratic Party (LD) always as the responder.
3. The Labor Party (L) is the bystander, which means they don't actively participate in the decision-making but will be affected by the outcome.

Given that $T = 1$, there's only one chance for the Conservative Party (C) to make an offer to the Liberal Democratic Party (LD).

Understanding LD's Strategy:

- The Liberal Democratic Party (LD) knows that if they reject the offer, the game ends and they get zero.
- Thus, LD would prefer any positive amount (even a tiny one) over getting nothing. They would also not mind an offer of zero since it's not worse than getting nothing.

- This gives LD two potential strategies:
  1. Accept if $x_{LD} \geq 0$ (i.e., they accept any non-negative offer).
  2. Accept if $x_{LD} > 0$ (i.e., they only accept strictly positive offers).

Understanding C's Strategy given LD's Strategy:

1. **If LD uses the first strategy** (accepting any non-negative offer):
   - The Conservative Party (C) knows that LD will accept even an offer of zero.
   - So, to maximize its own payoff, C will offer the least amount possible, which is zero, to LD and keep the rest for itself. This results in $x_C = 1$ and $x_{LD} = 0$ for the subgame perfect equilibrium.
2. **If LD uses the second strategy** (accepting only strictly positive offers):
   - Here's the dilemma: C wants to offer the smallest possible positive value to LD to make LD accept while keeping the maximum for itself.
   - However, there's no "smallest positive value" that C can distinctly identify. Thus, C doesn't have a clear best response. In other words, any small positive value C thinks of offering, they could think of an even smaller one. This results in a paradox.

Given these considerations, the more plausible equilibrium strategy for LD is to accept any non-negative offer. This provides clarity for C on how to make an offer, leading to the equilibrium where $x_C = 1$ and $x_{LD} = 0$.

b)

Given our earlier discussion, if the game reaches the last period T, the Conservative Party (C) is the proposer. Given that it's the last period, C knows that if the offer is rejected, both parties will end up with nothing. So, C will maximize its own payoff by offering 0 to LD and 1 to itself.

Now, let's consider the second-to-last period, $T - 1$. This is an even period, so the Labor Party (L) is the proposer. L knows that if its offer is rejected, in the next period C will offer 0 to LD. Therefore, L can offer a slightly positive amount, say ϵ, to LD and keep $1 - \epsilon$ for itself. LD will accept this offer because ϵ is better than the 0 they would get in the next period.

This logic can be used to work backward through all the periods of the game. In each period, the proposer will offer just slightly more than what the responder would get in the subsequent period.

However, there's a twist with the discount factor δ. Since players discount future payoffs, the value of a future offer decreases the further away it is.

When working with backward induction, for each period t, the proposer will consider what the LD will get in period $t + 1$ and offer them just slightly more than d times that amount.

Given this logic, let's determine the offer that will be accepted and in which period when T is odd.

Given that T is odd, the Conservative Party (C) is the proposer in the last period.

Let's work trough backward induction:

Period T (last period):

Proposer: Conservative Party (C)

- C offers $x_{LD} = 0$ to LD because if LD rejects the offer, both will end up with nothing.
- LD accepts this offer, because it's the last period, and it's better to accept 0 than to reject and get nothing.

Period $T - 1$ (second to last period):

Proposer: Labor Party (L)

- L knows that in the next period, LD will receive 0 from C.
- So, L offers slightly more than 0 (but less than what it would offer in a non-discounted setting, due to the discount factor δ). Specifically, L offers an amount slightly greater than $0 \times d$ (which is still 0). LD accepts this offer since it's better than waiting for the next period and getting 0.

Period $T - 2$:

Proposer: Conservative Party (C)

- C knows that in the next period, LD will accept an offer of 0.
- So, C will offer an amount slightly more than what LD would get in the next period discounted to the present, i.e., slightly more than $0 \times d$ (which is still 0).
- LD will compare this offer to what it expects to get in period $T - 1$ and the subsequent periods. LD will accept the offer if it's greater than its discounted expected future payoff.

Following this pattern backwards, in each period, the proposer will offer just slightly more than the discounted value of what the LD expects to get in the subsequent period.

Final Outcome:

Since the game is finite and players are forward-looking, they will anticipate this pattern from the start. LD will realize that waiting until the last period will get them a payoff of 0. As such, LD will accept the very first offer made in the first period, as long as it's non-negative.

The unique subgame perfect equilibrium for an odd T is:

- The Conservative Party (C) proposes an offer of $x_{LD} = 0$ in the first period.
- The Liberal Democratic Party (LD) accepts this offer.
- The game ends in the first period, without progressing to the subsequent periods.

In essence, the game's dynamics and the players' foresight ensure that the game ends immediately with the first offer, irrespective of the number of periods when T is odd.

c)

Since the dynamics of this game is somewhat different, given the infinite horizon and the fact that the Liberal Democratic Party (LD) always acts as the responder, its strategy will be shaped by the expected future payoffs. However, given the nature of the game and the fact that LD always knows it will be offered something in every period, it's always in its interest to accept any non-negative offer.

On the other side, knowing this, both the Conservative Party (C) and the Labor Party (L) have no incentive to offer anything more than the minimum, which is 0, to LD. They know LD will accept it, as waiting for a better offer in the future will never come.

Thus, you're absolutely correct:

1. The proposer, whether it's the Conservative Party (C) or the Labor Party (L), will always offer $x_{LD} = 0$.
2. LD will always accept any offer with $x_{LD} \geq 0$.

This forms the subgame perfect equilibrium for the infinite horizon game.

For future notes:

In most circumstances, we would look at it like this:

Given $T = \infty$ (i.e., the game goes on indefinitely), and we're looking for a subgame perfect equilibrium (SPE) in which an offer is accepted in the first period.

To determine the first-period offer in such an equilibrium, consider the following:

1. **Future Payoffs**: Since the game goes on indefinitely, both parties will have multiple opportunities to make and receive offers. However, with each passing period, the value of any potential future offer diminishes because of the discount factor d.
2. **Optimal Offers**: Given the structure of the game, each proposer knows that they will have another chance to propose an offer after some periods (for example, the Conservative Party (C) after every 2 periods). They will consider the discounted value of their future payoffs when determining their offers.
3. **Infinite Horizon Consideration**: When the game has an infinite horizon, the players will consider the entire future stream of payoffs. This contrasts with a finite T where the last period's dynamics heavily influence the entire game.

Given this, for an offer to be accepted in the first period, it should be such that the Liberal Democratic Party (LD) is indifferent between accepting that offer and rejecting it in hopes of a better offer in the future.

Now, let's determine the offer in the first period:

Proposer: Conservative Party (C)

C wants to make an offer such that LD is indifferent between accepting now or waiting. Given the alternating structure and the infinite horizon, LD's expectation of future payoffs will converge to a constant value over time.

If LD rejects the first offer, it anticipates receiving a certain amount x in the next period, discounted by d. But then, LD will again anticipate a similar amount in the subsequent periods, further discounted. Given the infinite horizon, the sum of these discounted future payoffs will converge to a value.

For LD to accept C's offer in the first period, C should offer LD a value equivalent to LD's expected discounted sum of future payoffs.

Using the concept of geometric series, the sum of infinite future payoffs for LD can be expressed as:

$$x = x \times d + x \times d^2 + x \times d^3 + \dots$$

From the above series, the present value (at period 1) of LD's expected payoffs from all future periods is:

$$x \times \frac{d}{(1-d)}$$

For LD to be indifferent between accepting the first-period offer and rejecting it for the anticipated future payoffs, C should offer:

$$x_{LD} = x \times \frac{d}{(1-d)}$$

This would be the first-period offer in the subgame perfect equilibrium where LD accepts the offer immediately.