# 利用bike-sharing数据集构建回归模型

首先加载和查看数据集

```
In [155]:
```

```
for line in open ('data/Readme.txt').readlines():
    print line.strip()
rawdata = sc. textFile('data/hour.csv')
data = rawdata.map(lambda x:x.split(','))
for i in data. take (5):
    print i
- instant: record index
- dteday : date
- season : season (1:springer, 2:summer, 3:fall, 4:winter)
- yr : year (0: 2011, 1:2012)
- mnth : month ( 1 to 12)
- hr : hour (0 to 23)
- holiday: weather day is holiday or not (extracted from http://dchr.dc.gov/page/ho
liday-schedule) (http://dchr.dc.gov/page/holiday-schedule))
- weekday: day of the week
- workingday: if day is neither weekend nor holiday is 1, otherwise is 0.
+ weathersit :
- 1: Clear, Few clouds, Partly cloudy, Partly cloudy
- 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist
- 3: Light Snow, Light Rain + Thunderstorm + Scattered clouds, Light Rain + Scattere
d clouds
- 4: Heavy Rain + Ice Pallets + Thunderstorm + Mist, Snow + Fog
- temp: Normalized temperature in Celsius. The values are divided to 41 (max)
- atemp: Normalized feeling temperature in Celsius. The values are divided to 50 (ma
_{\rm X})
- hum: Normalized humidity. The values are divided to 100 (max)
- windspeed: Normalized wind speed. The values are divided to 67 (max)
- casual: count of casual users
- registered: count of registered users
- cnt: count of total rental bikes including both casual and registered
[u'instant', u'dteday', u'season', u'yr', u'mnth', u'hr', u'holiday', u'weekday',
 u'workingday', u'weathersit', u'temp', u'atemp', u'hum', u'windspeed', u'casual',
 u'registered', u'cnt']
[u'1', u'2011-01-01', u'1', u'0', u'1', u'0', u'0', u'6', u'0', u'1', u'0.24', u'0.2
879', u'0.81', u'0', u'3', u'13', u'16']
[u'2', u'2011-01-01', u'1', u'0', u'1', u'0', u'6', u'0', u'1', u'0.22', u'0.2
727', u'0.8', u'0', u'8', u'32', u'40']
[u'3', u'2011-01-01', u'1', u'0', u'1', u'2', u'0', u'6', u'0', u'1', u'0.22', u'0.2
727', u'0.8', u'0', u'5', u'27', u'32']
[u'4', u'2011-01-01', u'1', u'0', u'1', u'3', u'0', u'6', u'0', u'1', u'0.24', u'0.2
879', u'0.75', u'0', u'3', u'10', u'13']
```

可以看到第一行为标签,第一列instance为序号可以删除,第二列dteday为日期,信息已包含在之后的yr,mnth等变量中,可以删除。此次预测只预测总数量,因此去除casual和registered。剩下的变量分别为:季节,年,月,小时,节假日,星期几,工作日,天气,温度,体感温度,湿度,风速。其中后四个变量已标准化。

# In [156]:

```
data = data.filter(lambda x: x[0] != 'instant')
data.cache()
print data.count()
```

17379

我打算用线性回归和决策树两种方法来尝试拟合数据集,在线性回归中,对类型变量进行二元编码。(决策树中不需要)建立辅助函数并转换数据。

In [157]:

```
def binary encode (rdd, i):
   return (rdd.
         map(lambda x : x[i])
         .distinct()
         .zipWithIndex()
         .collectAsMap())
#对第2-9列应用该函数得到转换的字典
mappings = [binary encode(data, i) for i in range(2, 10)]
print mappings
#将第2-9列转换为二元编码
from pyspark.mllib.regression import LabeledPoint
import numpy as np
def extract feature(line):
   1st = []
   for ind in range (2, 10):
      dic = mappings[ind-2]
      for i, j in dic. iteritems():
          if line[ind] == i:
             1st. append (1.0)
          else:
             1st. append (0)
   for ind in range (10, 14):
      1st. append(line[ind])
   return np. array (map(float, 1st))
def extract label(line):
   return float(line[-1])
binarydata = data.map(lambda x : LabeledPoint(extract_label(x), extract_feature(x)))
print binarydata.first()
print 'Raw data: '+ str(data.first()[2:])
print 'Raw Label: ' + str(data.first()[-1])
print 'Binary data: ' + str(binarydata.first().features)
print 'Binary label: '+ str(binarydata.first().label)
print 'Features number: ' + str(len(binarydata.first().features))
[\{u'1': 0, u'3': 1, u'2': 2, u'4': 3\}, \{u'1': 0, u'0': 1\}, \{u'11': 0, u'10': 1, u'1
2': 2, u'1': 3, u'3': 4, u'2': 5, u'5': 6, u'4': 7, u'7': 8, u'6': 9, u'9': 10,
u'8': 11}, {u'20': 0, u'21': 1, u'22': 2, u'23': 3, u'1': 4, u'0': 5, u'3': 6,
u'2': 7, u'5': 8, u'4': 9, u'7': 10, u'6': 11, u'9': 12, u'8': 13, u'11': 14, u'1
0': 15, u'13': 16, u'12': 17, u'15': 18, u'14': 19, u'17': 20, u'16': 21, u'19': 22,
u'18': 23}, {u'1': 0, u'0': 1}, {u'1': 0, u'0': 1, u'3': 2, u'2': 3, u'5': 4, u'4':
5, u'6': 6}, {u'1': 0, u'0': 1}, {u'1': 0, u'3': 1, u'2': 2, u'4': 3}]
1, 0.0 \rceil)
Raw data: [u'1', u'0', u'1', u'0', u'0', u'6', u'0', u'1', u'0.24', u'0.2879', u'0.8
1', u'0', u'3', u'13', u'16']
Raw Label: 16
0, 0. 0, 0. 0, 0. 0, 0. 0, 1. 0, 0. 0, 0. 0, 0. 0, 0. 0, 0. 0, 0. 0, 1. 0, 0. 0, 1. 0, 1. 0, 1. 0, 0. 0, 0. 0, 0. 0, 0. 24, 0. 287
9, 0.81, 0.0
Binary label: 16.0
Features number: 61
```

决策树中可以直接使用原始数据,因此只需要将u'1'格式转换为float,再用numpy封装一下

```
In [158]:

def extract_float(line):
    return np.array(line[2:14])

treedata = data.map(lambda x: LabeledPoint(extract_label(x), extract_float(x)))

print 'Raw data: '+ str(data.first()[2:])

print 'Raw Label: ' + str(data.first()[-1])

print 'Desicion tree data: ' + str(treedata.first().features)

print 'Desicion tree Label: ' + str(treedata.first().label)

Raw data: [u'1', u'0', u'1', u'0', u'0', u'6', u'0', u'1', u'0.24', u'0.2879', u'0.8

1', u'0', u'3', u'13', u'16']

Raw Label: 16

Desicion tree data: [1.0,0.0,1.0,0.0,0.0,0.0,0.0,1.0,0.24,0.2879,0.81,0.0]

Desicion tree Label: 16.0

训练线性回归模型和决策树模型
```

#### In [159]:

```
from pyspark.mllib.regression import LinearRegressionWithSGD

linear_model = LinearRegressionWithSGD.train(binarydata, iterations = 10, step = 0.1, intercept = Fa linear_pred_act = binarydata.map(lambda x: (linear_model.predict(x.features), x.label))

print 'linear predictions vs actual:'

print linear_pred_act.take(5)

linear predictions vs actual:

[(135.94648455498353, 16.0), (134.38058174607252, 40.0), (134.18407938613737, 32.0),
```

#### In [160]:

```
from pyspark.mllib.tree import DecisionTree
tree_model = DecisionTree.trainRegressor(treedata, {})
preds = tree_model.predict(treedata.map(lambda x:x.features))
actual = treedata.map(lambda x: x.label)
tree_pred_act = preds.zip(actual)
print tree_pred_act.take(5)
print tree_model.depth()
print tree_model.numNodes()

[54.913223140495866 16.0) (54.913223140495866 40.0) (53.171052631578945 32.0)
```

```
[(54.913223140495866, 16.0), (54.913223140495866, 40.0), (53.171052631578945, 32.0), (14.284023668639053, 13.0), (14.284023668639053, 1.0)]
5
63
```

评估模型的性能,采用均方误差,均方根误差,平均绝对误差,R平方系数

(133.88699144084512, 13.0), (133.77899037657545, 1.0)

# In [161]:

```
def squared_error(i, j):
    return (i-j)**2
def abs_error(i, j):
    return np. abs(i-j)
def square_log_error(i, j):
    return(np. log(i+1) - np. log(j+1))**2
```

# In [162]:

```
mse = linear_pred_act.map(lambda (i, j): squared_error(i, j)).mean()
mae = linear_pred_act.map(lambda (i, j): abs_error(i, j)).mean()
rmsle = np. sqrt(linear_pred_act.map(lambda (i, j): square_log_error(i, j)).mean())
print 'mean squared error: '+ str(mse)
print 'mean absolute error: ' + str(mae)
print 'rot mean squred log error: ' + str(rmsle)
```

mean squared error: 29897.3402015 mean absolute error: 130.532559912 rot mean squred log error: 1.48038670632

#### In [163]:

```
tree_mse = tree_pred_act.map(lambda (i, j): squared_error(i, j)).mean()
tree_mae = tree_pred_act.map(lambda (i, j): abs_error(i, j)).mean()
tree_rmsle = np. sqrt(tree_pred_act.map(lambda (i, j): square_log_error(i, j)).mean())
print 'desicion tree mean squared error: '+ str(tree_mse)
print 'desicion tree mean absolute error: ' + str(tree_mae)
print 'desicion tree rot mean squared log error: ' + str(tree_rmsle)
```

desicion tree mean squared error: 11611.4859995 desicion tree mean absolute error: 71.1501878649

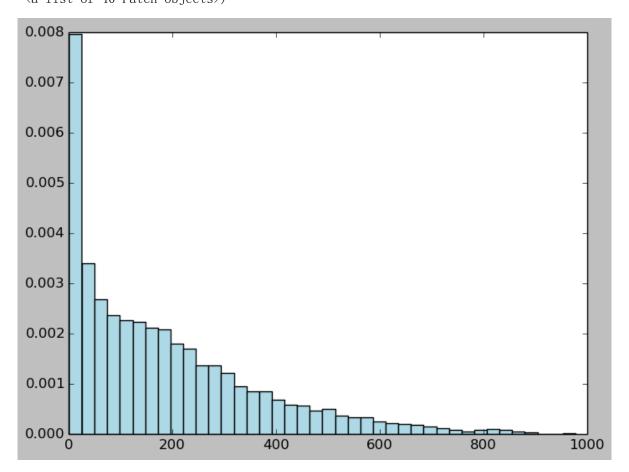
desicion tree rot mean squred log error: 0.625115258696

# In [164]:

```
import matplotlib.pyplot as plt
targets = data.map(lambda x:float(x[-1])).collect()
plt.hist(targets, bins = 40, normed = True, color = 'lightblue')
```

# Out[164]:

```
(array([ 7.96372860e-03,
                              3. 40527809e-03,
                                                  2.69781034e-03,
          2.37473340e-03,
                              2.27568792e-03,
                                                  2.24503098e-03,
          2. 12240324e-03,
                              2.09410453e-03,
                                                  1.80875921e-03,
          1.71207195e-03,
                              1.37720388e-03,
                                                  1. 37248743e-03,
          1. 23099388e-03,
                              9.64514361e-04,
                                                  8. 53677747e-04,
          8.65468877e-04,
                              6.83885488e-04,
                                                  5.94272907e-04,
          5.70690649e-04,
                              4. 78719842e-04,
                                                  5. 02302100e-04,
          3.72599680e-04,
                              3.49017422e-04,
                                                  3.39584518e-04,
          2.52330163e-04,
                              2.26389679e-04,
                                                  2.02807421e-04,
          1.88658066e-04,
                              1.50926453e-04,
                                                  1. 29702420e-04,
          9.90454845e-05.
                              6.36720972e-05,
                                                  8.72543554e-05.
          1. 01403710e-04,
                              8.25379038e-05,
                                                  5.65974197e-05,
          4.71645164e-05,
                              1.17911291e-05,
                                                  9. 43290329e-06,
          1.88658066e-05]),
array([
           1. ,
                   25. 4,
                            49.8,
                                     74. 2,
                                             98.6,
                                                     123. ,
                                                              147. 4,
                                                                       171.8,
         196.2,
                  220.6,
                           245. ,
                                            293.8,
                                                     318. 2,
                                                              342.6,
                                    269. 4,
                                                                       367.
         391.4.
                  415.8,
                           440.2,
                                    464.6,
                                            489. ,
                                                     513.4.
                                                              537. 8.
                                                                       562.2,
         586.6,
                  611. ,
                           635.4,
                                    659.8,
                                            684.2,
                                                     708.6,
                                                              733. ,
                                                                       757.4,
         781.8,
                  806.2,
                           830.6,
                                    855. ,
                                            879.4,
                                                     903.8,
                                                              928.2,
                                                                       952.6,
         977. ]),
<a list of 40 Patch objects>)
```



尝试取对数和平方根

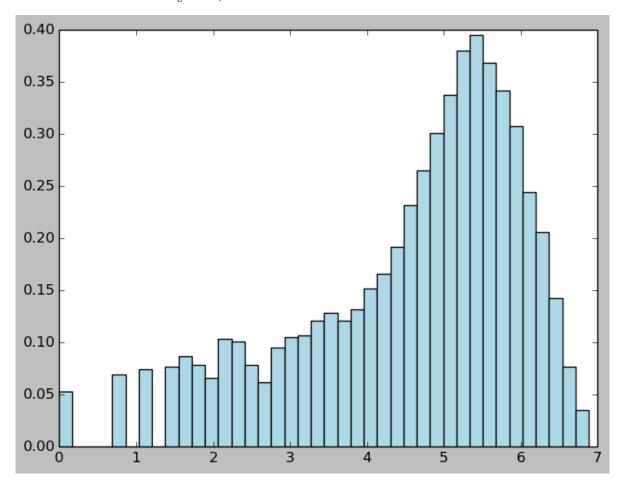
# In [165]:

```
plt. hist (np. log(targets), bins = 40, normed = True, color = 'lightblue')
```

# Out[165]:

```
(array([ 0.05282272,
                                      0.
                                                                  0.06953877,
                       0.
                                                    0.
                       0.0748879 ,
                                                    0.07722815,
                                                                  0.08692346,
         0.
                                      0.
                                                    0.10096494,
         0.07889975,
                       0.06619556,
                                      0. 10363951,
                                                                  0.07856543,
                                                                  0.1210242,
         0.0621837 ,
                       0.0956158 ,
                                      0.10497679,
                                                    0.1066484,
         0.12871358,
                       0.12135852,
                                      0.13172247,
                                                    0.15245037,
                                                                  0.16649185,
                                                                  0.3376642,
         0.19190025,
                       0.23235309,
                                      0.26545087,
                                                    0.30122321,
                                      0.36842173,
                                                    0.3423447,
                                                                  0.30824395,
         0.38012297,
                       0.39516741,
         0.24472297,
                       0.20661037,
                                      0. 14275506,
                                                    0.07689383,
                                                                  0.03543803),
array([ 0.
                       0. 17211217,
                                      0. 34422433,
                                                    0.5163365,
                                                                  0.68844867,
         0.86056083,
                       1.032673
                                      1. 20478516,
                                                    1.37689733,
                                                                  1.5490095,
         1.72112166,
                       1.89323383,
                                      2.065346 ,
                                                    2. 23745816,
                                                                  2.40957033,
         2.58168249,
                       2.75379466,
                                      2.92590683,
                                                    3.09801899,
                                                                  3. 27013116,
         3.44224333,
                       3.61435549,
                                      3.78646766,
                                                    3.95857982,
                                                                  4. 13069199,
         4. 30280416,
                       4. 47491632,
                                      4.64702849,
                                                    4.81914066,
                                                                  4.99125282,
         5. 16336499,
                       5. 33547716,
                                      5.50758932,
                                                    5.67970149,
                                                                  5.85181365,
         6.02392582,
                       6. 19603799,
                                      6. 36815015,
                                                    6. 54026232,
                                                                  6. 71237449,
         6.88448665]),
```

<a list of 40 Patch objects>)

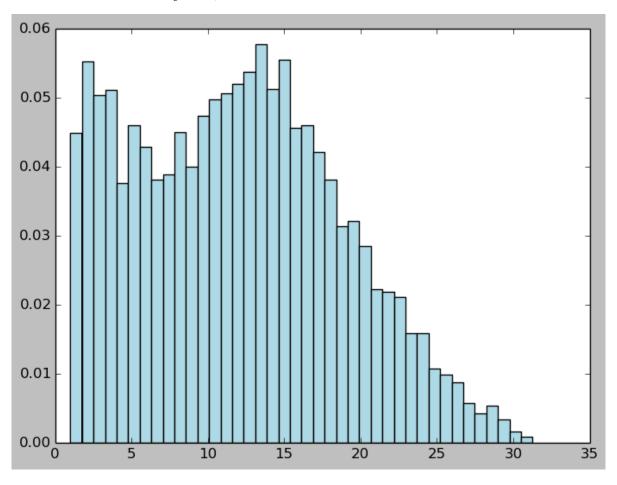


#### In [166]:

```
plt.hist(np.sqrt(targets), bins = 40, normed = True, color = 'lightblue')
```

```
Out[166]:
```

```
(array([ 0.04488088,
                       0.05530237,
                                      0.05043394,
                                                    0.05119463,
                                                                  0.0376543
         0.04609799,
                       0.04297915,
                                      0.03818678,
                                                    0.03887141,
                                                                  0.04510909,
         0.04001245,
                       0.04746724,
                                      0.04982538,
                                                    0.05073822,
                                                                  0.05203139,
         0.05385706,
                       0.05781266,
                                      0.0512707 ,
                                                    0.05553058,
                                                                  0.04564157,
         0.04609799,
                       0.04214239,
                                      0.03818678,
                                                    0.03149269,
                                                                  0.03217731,
         0.02852598,
                       0.0222883 ,
                                      0.02198402,
                                                    0.02122333,
                                                                  0.01589848,
                                                                  0.00585734,
         0.01597455,
                       0.01080184,
                                      0.00988901,
                                                    0.00882404,
         0.00425988,
                       0.00547699,
                                      0.00342312,
                                                    0.00167352,
                                                                  0.00091283),
array([
                          1.75642498,
                                         2. 51284996,
                                                        3. 26927494,
         1.
          4.02569992,
                          4.7821249,
                                         5. 53854988,
                                                        6. 29497486,
          7.05139984,
                          7.80782482,
                                         8.5642498,
                                                        9.32067478,
         10.07709976,
                         10.83352475,
                                        11.58994973,
                                                       12.34637471,
         13. 10279969,
                         13.85922467,
                                        14.61564965,
                                                       15. 37207463,
         16. 12849961,
                         16.88492459,
                                        17.64134957,
                                                       18. 39777455,
         19. 15419953,
                         19.91062451,
                                        20.66704949,
                                                       21. 42347447,
                                                       24.44917439,
         22. 17989945,
                         22. 93632443,
                                        23. 69274941,
         25. 20559937,
                        25.96202435,
                                        26. 71844933,
                                                       27.47487431,
         28. 23129929,
                        28. 98772427,
                                        29. 74414926,
                                                       30. 50057424,
                                                                       31.25699922]),
<a list of 40 Patch objects>)
```



接下来尝试用对数拟合模型

```
In [167]:
```

```
binarydata_log = binarydata.map(lambda x: LabeledPoint(np.log(x.label), x.features))
linear_model_log = LinearRegressionWithSGD.train(binarydata_log, iterations = 10, step = 0.1, interc
linear_pred_act_log = binarydata_log.map(lambda x: (np.exp(linear_model_log.predict(x.features)), np.
print 'linear predictions vs actual:'
print linear_pred_act_log.take(5)
mse = linear_pred_act_log.map(lambda (i, j): squared_error(i, j)).mean()
mae = linear_pred_act_log.map(lambda (i, j): abs_error(i, j)).mean()
rmsle = np. sqrt(linear_pred_act_log.map(lambda (i, j): square_log_error(i, j)).mean())
print 'mean squared error: '+ str(mse)
print 'mean absolute error: ' + str(mse)
print 'rot mean squared log error: ' + str(rmsle)
```

```
linear predictions vs actual:
[(40.91617570453451, 15.9999999999999), (39.374013505300653, 40.0), (38.9328734479 29204, 32.0), (38.068727061376421, 13.0), (37.834766952897375, 1.0)]
mean squared error: 47024.5721598
mean absolute error: 149.288618818
rot mean squared log error: 1.45256325985
```

#### In [168]:

```
treedata_log = treedata.map(lambda x:LabeledPoint(np.log(x.label), x.features))
tree_model_log = DecisionTree.trainRegressor(treedata_log, {})
preds_log = tree_model_log.predict(treedata_log.map(lambda x:x.features)).map(lambda x: np.exp(x))
actual_log = treedata_log.map(lambda x: x.label).map(lambda x: np.exp(x))
tree_pred_act_log = preds_log.zip(actual_log)

tree_mse_log = tree_pred_act_log.map(lambda (i, j): squared_error(i, j)).mean()
tree_mae_log = tree_pred_act_log.map(lambda (i, j): abs_error(i, j)).mean()
tree_rmsle_log = np.sqrt(tree_pred_act_log.map(lambda (i, j): square_log_error(i, j)).mean())
print 'desicion tree mean squared error: '+ str(tree_mse_log)
print 'desicion tree rot mean squared log error: ' + str(tree_rmsle_log)
```

```
desicion tree mean squared error: 14781.5759883
desicion tree mean absolute error: 76.4131099112
desicion tree rot mean squred log error: 0.640599610072
```

可以看到取对数对模型效果影响不大

创建训练集和测试集,测试不同参数对模型性能的影响

#### In [169]:

```
data_with_index = binarydata.zipWithIndex().map(lambda (i, j): (j, i))
test = data_with_index.sample(False, 0. 2, 42)
train = data_with_index.subtractByKey(test)
train_data = train.map(lambda (i, j):j)
test_data = test.map(lambda (i, j):j)
train_size = train_data.count()
test_size = test_data.count()
print "Training data size: %d" % train_size
print "Test data size: %d" % test_size
```

Training data size: 13843 Test data size: 3536

# In [170]:

```
tree_data_with_index = treedata.zipWithIndex().map(lambda (i, j): (j, i))
tree_test = tree_data_with_index.sample(False, 0. 2, 42)
tree_train = tree_data_with_index.subtractByKey(tree_test)
tree_train_data = train.map(lambda (i, j):j)
tree_test_data = test.map(lambda (i, j):j)
```

# In [171]:

```
def evaluate(train, test, iterations , step, regParam, regType, intercept):
    model = LinearRegressionWithSGD. train(train, iterations, step, regParam = regParam, regType = regTyp
    tp = test.map(lambda x: (x. label, model. predict(x. features)))
    rmsle = np. sqrt(tp. map(lambda(i, j): square_log_error(i, j)). mean())
    return rmsle
```

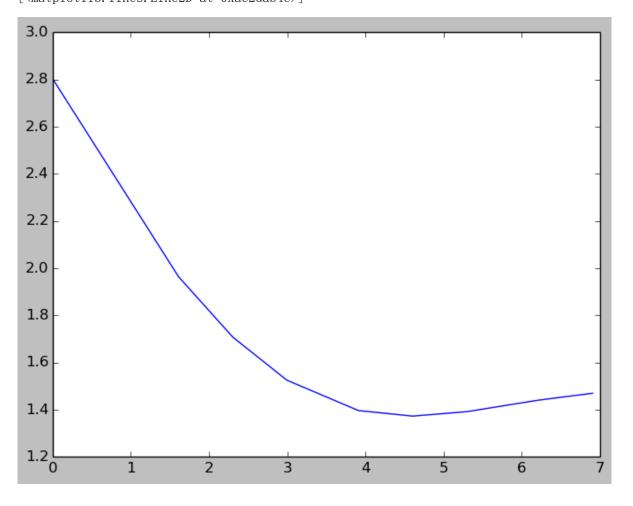
# In [172]:

```
params = [1,5,10,20,50,100,200,500,1000]
metrics = [evaluate(train_data, test_data, param, 0.01, 0.0, '12', False) for param in params]
print params
print metrics
plt.plot(np.log(params), metrics)
```

[1, 5, 10, 20, 50, 100, 200, 500, 1000] [2.8032563666537595, 1.9645810957004637, 1.7086221968290249, 1.5255705039801279, 1.3 966970293255028, 1.3733178276026903, 1.3921362057047302, 1.440916089834078, 1.470223 8639466663]

#### Out[172]:

[<matplotlib.lines.Line2D at 0xac2dab4c>]



# In [173]:

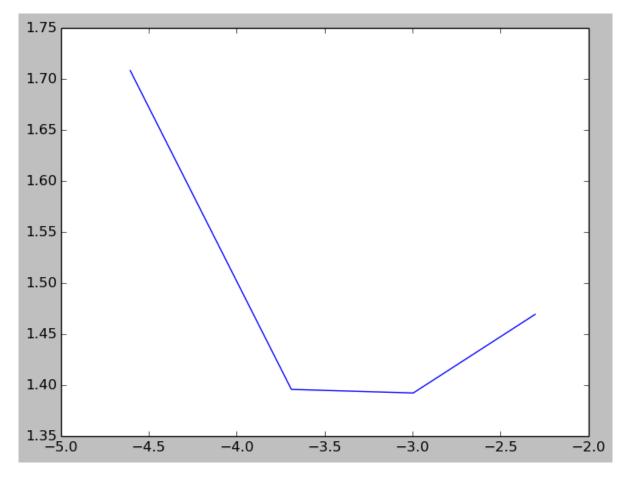
```
params = [0.01, 0.025, 0.05, 0.1, 1]
metrics = [evaluate(train_data, test_data, 10, param, 0.0, '12', False) for param in params]
print params
print metrics
plt.plot(np.log(params), metrics)
```

[0.01, 0.025, 0.05, 0.1, 1]

[1.7086221968290249, 1.3961771685298445, 1.392659943805455, 1.4697573635648185, nan]

# Out[173]:

[<matplotlib.lines.Line2D at 0xabb1a60c>]



# In [174]:

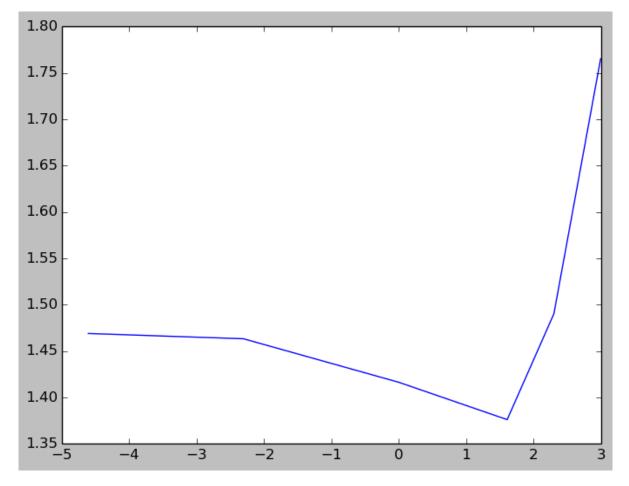
```
params = [0.0,0.01,0.1,1.0,5.0,10.0,20.0]
metrics = [evaluate(train_data,test_data,10,0.1,param,'12',False) for param in params]
print params
print metrics
plt.plot(np.log(params),metrics)
```

[0.0, 0.01, 0.1, 1.0, 5.0, 10.0, 20.0] [1.4697573635648185, 1.469113028540231, 1.4634188057285833, 1.4165237605797092, 1.376225790336346, 1.4904349424828804, 1.7653588820071182]

/usr/local/lib/python2.7/dist-packages/IPython/kernel/\_\_main\_\_.py:5: RuntimeWarning: divide by zero encountered in log

# Out[174]:

[<matplotlib.lines.Line2D at Oxabaf4c6c>]



# In [175]:

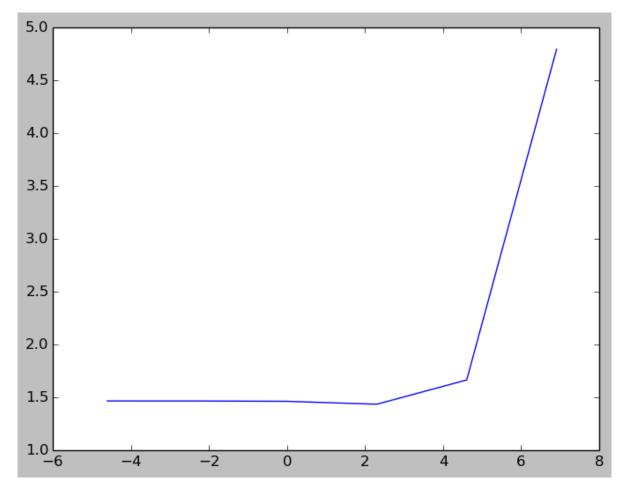
```
params = [0.0,0.01,0.1,1.0,10.0,100.0,1000.0]
metrics = [evaluate(train_data, test_data, 10, 0.1, param, '11', False) for param in params]
print params
print metrics
plt.plot(np.log(params), metrics)
```

[0.0, 0.01, 0.1, 1.0, 10.0, 100.0, 1000.0] [1.4697573635648185, 1.4697190854099258, 1.4693749895529875, 1.466217949786879, 1.43 86546455290785, 1.6683522645623436, 4.7975574647546564]

/usr/local/lib/python2.7/dist-packages/IPython/kernel/\_\_main\_\_.py:5: RuntimeWarning: divide by zero encountered in log

### Out[175]:

[<matplotlib.lines.Line2D at 0xaba53f4c>]



# In [176]:

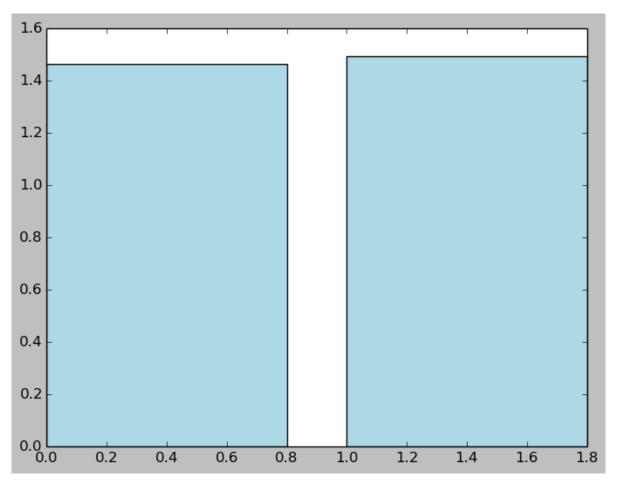
```
params = [False, True]
metrics = [evaluate(train_data, test_data, 10, 0.1, 1.0, '11', param) for param in params]
print params
print metrics
plt. bar(params, metrics, color = 'lightblue')
```

[False, True]

 $[1.\,466217949786879,\ \ 1.\,4931703614325664]$ 

### Out[176]:

<Container object of 2 artists>



可以发现线性回归模型的rmsle一直比较大,因此尝试优化决策树的参数

# In [177]:

```
def tree_evaluate(train, test, maxDepth, maxBins):
    model = DecisionTree.trainRegressor(train, {}, impurity = 'variance', maxDepth = maxDepth, maxBins
    preds = model.predict(test.map(lambda x:x.features))
    actual = test.map(lambda x:x.label)
    tp = actual.zip(preds)
    rmsle = np.sqrt(tp.map(lambda (i, j): square_log_error(i, j)).mean())
    return rmsle
```

# In [178]:

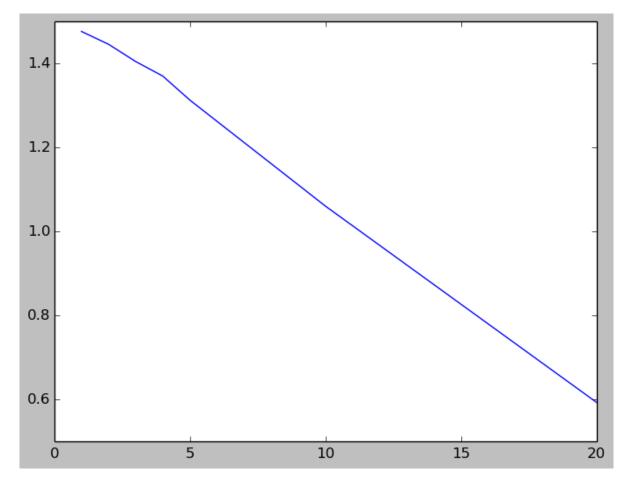
```
params = [1, 2, 3, 4, 5, 10, 20]
metrics = [tree_evaluate(tree_train_data, tree_test_data, param, 32) for param in params]
print params
print metrics
plt. plot(params, metrics)
```

[1, 2, 3, 4, 5, 10, 20]

 $\begin{bmatrix} 1.\ 4769680346251848, & 1.\ 4463996287263496, & 1.\ 4052970275150503, & 1.\ 3706730389096726, & 1.\ 3133800087056628, & 1.\ 06096810331115, & 0.\ 59356773203374946 \end{bmatrix}$ 

# Out[178]:

[<matplotlib.lines.Line2D at 0xab997c6c>]



# In [179]:

```
params = [2,4,8,16,32,64,100]
metrics = [tree_evaluate(tree_train_data, tree_test_data, 20, param) for param in params]
print params
print metrics
plt. plot(params, metrics)
```

[2, 4, 8, 16, 32, 64, 100]

 $[0.\ 54228074209636057,\ 0.\ 5392012822568224,\ 0.\ 58411806921283194,\ 0.\ 59365764578073632,$ 

0.59716251683048815, 0.62105899611168236, 0.6278350415424504

### Out[179]:

[<matplotlib.lines.Line2D at 0xab53462c>]

