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# ACM/ICPC Template Manual

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QUST

hxx

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## 0 Include

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define clr(a, x) memset(a, x, sizeof(a))
4 #define mp(x, y) make_pair(x, y)
5 #define pb(x) push_back(x)
6 #define X first
7 #define Y second
8 #define fastin \
9     ios_base::sync_with_stdio(0); \
10    cin.tie(0);
11 typedef long long ll;
12 typedef long double ld;
13 typedef pair<int, int> PII;
14 typedef vector<int> VI;
15 const int INF = 0x3f3f3f3f;
16 const int mod = 1e9 + 7;
17 const double eps = 1e-6;
18
19 int main()
20 {
21     #ifndef ONLINE_JUDGE
22         freopen("test.in", "r", stdin);
23         freopen("test.out", "w", stdout);
24     #endif
25
26     return 0;
27 }
```

# 1 Math

## 1.1 Prime

### 1.1.1 Eratosthenes Sieve

$O(n \log \log n)$  maxn  
 $notprime[i] = 0/1 \quad 0 \quad 1$

```

1 const int maxn = "Edit";
2 bool notprime[maxn] = {1, 1}; // 0 && 1
3 void GetPrime()
4 {
5     for (int i = 2; i < maxn; i++)
6         if (!notprime[i] && i <= maxn / i) //  $\sqrt{n}$ 
7             for (int j = i * i; j < maxn; j += i)
8                 notprime[j] = 1;
9 }
```

### 1.1.2 Euler Sieve

$O(n)$  phi[] prime[] tot  
n

```

1 const int maxn = "Edit";
2 bool vis[maxn];
3 int tot, phi[maxn], prime[maxn];
4 void CalPhi(int n)
5 {
6     clr(vis, 0);
7     phi[1] = 1;
8     tot = 0;
9     for (int i = 2; i < n; i++)
10     {
11         if (!vis[i])
12             prime[tot++] = i, phi[i] = i - 1;
13         for (int j = 0; j < tot; j++)
14         {
15             if (i * prime[j] > n) break;
16             vis[i * prime[j]] = 1;
17             if (i % prime[j] == 0)
18             {
19                 phi[i * prime[j]] = phi[i] * prime[j];
20                 break;
21             }
22             else
23                 phi[i * prime[j]] = phi[i] * (prime[j] - 1);
24         }
25     }
26 }
```

### 1.1.3 Prime Factorization

$fact[i][0]^{fact[i][1]}$  i

```

1 ll fact[100][2];
2 int getFactors(ll x)
3 {
4     int cnt = 0;
5     for (int i = 0; prime[i] <= x / prime[i]; i++)
6     {
7         fact[cnt][1] = 0;
8         if (x % prime[i] == 0)
9         {
10             fact[cnt][0] = prime[i];
11             while (x % prime[i] == 0) fact[cnt][1]++, x /= prime[i];
12             cnt++;
13         }
14     }
15     if (x != 1) fact[cnt][0] = x, fact[cnt++][1] = 1;
16     return cnt;
17 }

```

#### 1.1.4 Miller Rabin

$O(s \log n) \quad 2^{63}, s$

```

1 bool Miller_Rabin(ll n, int s)
2 {
3     if (n == 2) return 1;
4     if (n < 2 || !(n & 1)) return 0;
5     int t = 0;
6     ll x, y, u = n - 1;
7     while ((u & 1) == 0) t++, u >>= 1;
8     for (int i = 0; i < s; i++)
9     {
10         ll a = rand() % (n - 1) + 1;
11         ll x = Pow(a, u, n);
12         for (int j = 0; j < t; j++)
13         {
14             ll y = Mul(x, x, n);
15             if (y == 1 && x != 1 && x != n - 1) return 0;
16             x = y;
17         }
18         if (x != 1) return 0;
19     }
20     return 1;
21 }

```

#### 1.1.5 Segment Sieve

$[a, b)$

is\_prime[i-a]=true i  
 $a < b \leq 10^{12}, b - a \leq 10^6$

```

1 const int maxn = "Edit";
2 bool is_prime_small[maxn], is_prime[maxn];
3 int prime[maxn];
4 int segment_sieve(ll a, ll b)
5 {
6     int tot = 0;

```

```

7   for (ll i = 0; i * i < b; ++i)
8       is_prime_small[i] = true;
9   for (ll i = 0; i < b - a; ++i)
10      is_prime[i] = true;
11   for (ll i = 2; i * i < b; ++i)
12       if (is_prime_small[i])
13       {
14           for (ll j = 2 * i; j * j < b; j += i)
15               is_prime_small[j] = false;
16           for (ll j = max(2LL, (a + i - 1) / i) * i; j < b; j += i)
17               is_prime[j - a] = false;
18       }
19   for (ll i = 0; i < b - a; ++i)
20       if (is_prime[i]) prime[tot++] = i + a;
21   return tot;
22 }

```

## 1.2 Euler phi

### 1.2.1 Euler

```

1 ll Euler(ll n)
2 {
3     ll rt = n;
4     for (int i = 2; i * i <= n; i++)
5         if (n % i == 0)
6         {
7             rt -= rt / i;
8             while (n % i == 0) n /= i;
9         }
10    if (n > 1) rt -= rt / n;
11    return rt;
12 }

```

### 1.2.2 Sieve

```

1 const int N = "Edit";
2 int phi[N] = {0, 1};
3 void CalEuler()
4 {
5     for (int i = 2; i < N; i++)
6         if (!phi[i])
7             for (int j = i; j < N; j += i)
8             {
9                 if (!phi[j]) phi[j] = j;
10                phi[j] = phi[j] / i * (i - 1);
11            }
12 }

```

## 1.3 Basic Number Theory

### 1.3.1 Extended Euclidean

```

1 ll exgcd(ll a, ll b, ll &x, ll &y)
2 {
3     ll d = a;
4     if (b) d = exgcd(b, a % b, y, x), y -= x * (a / b);

```



```

5     else x = 1, y = 0;
6     return d;
7 }

```

### 1.3.2 $ax+by=c$

$$\begin{matrix} : X = x + k * dx, Y = y - k * dy \\ x \quad , \quad 0 \end{matrix}$$

```

1 #define Mod(a, b) (((a) % (b) + (b)) % (b))
2 bool solve(ll a, ll b, ll c, ll& x, ll& y, ll& dx, ll& dy)
3 {
4     if (a == 0 && b == 0) return 0;
5     ll x0, y0;
6     ll d = exgcd(a, b, x0, y0);
7     if (c % d != 0) return 0;
8     dx = b / d, dy = a / d;
9     x = Mod(x0 * c / d, dx);
10    y = (c - a * x) / b;
11    // y = Mod(y0 * c / d, dy); x = (c - b * y) / a;
12    return 1;
13 }

```

### 1.3.3 Multiplicative Inverse Modulo

$$\text{exgcd } a \quad m \quad , \quad \text{gcd}(a, m) == 1.$$

```

1 ll inv(ll a, ll m)
2 {
3     ll x, y;
4     ll d = exgcd(a, m, x, y);
5     return d == 1 ? (x + m) % m : -1;
6 }

```

$$a < p \quad p \quad ,$$

```

1 ll inv(ll a, ll p) { return Pow(a, p - 2, p); }

1 for (int i = 2; i < n; i++) inv[i] = inv[p % i] * (p - p / i) % p;

```

## 1.4 Modulo Linear Equation

### 1.4.1 Chinese Remainder Theory

$$\begin{matrix} X = r_i(\text{mod } m_i); \quad m_i \\ X = re + k * mo \end{matrix}$$

```

1 void crt(ll r[], ll m[], ll n, ll &re, ll &mo)
2 {
3     mo = 1, re = 0;
4     for (int i = 0; i < n; i++) mo *= m[i];
5     for (int i = 0; i < n; i++)
6     {
7         ll x, y, tm = mo / m[i];
8         ll d = exgcd(tm, m[i], x, y);
9         re = (re + tm * x * r[i]) % mo;

```

```

10     }
11     re = (re + mo) % mo;
12 }

```

### 1.4.2 ExCRT

$$X = r_i \pmod{m_i}; m_i$$

$$X = re + k * mo;$$

```

1 bool excrt(ll r[], ll m[], ll n, ll &re, ll &mo)
2 {
3     ll x, y;
4     mo = m[0], re = r[0];
5     for (int i = 1; i < n; i++)
6     {
7         ll d = exgcd(mo, m[i], x, y);
8         if ((r[i] - re) % d != 0) return 0;
9         x = (r[i] - re) / d * x % (m[i] / d);
10        re += x * mo;
11        mo = mo / d * m[i];
12        re %= mo;
13    }
14    re = (re + mo) % mo;
15    return 1;
16 }

```

## 1.5 Combinatorics

### 1.5.1 Combination

$$0 \leq m \leq n \leq 1000$$

```

1 const int maxn = 1010;
2 ll C[maxn][maxn];
3 void CalComb()
4 {
5     C[0][0] = 1;
6     for (int i = 1; i < maxn; i++)
7     {
8         C[i][0] = 1;
9         for (int j = 1; j <= i; j++) C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
10    }
11 }

```

$$0 \leq m \leq n \leq 10^5, \quad p$$

```

1 const int maxn = 100010;
2 ll f[maxn];
3 ll inv[maxn]; //
4 void CalFact()
5 {
6     f[0] = 1;
7     for (int i = 1; i < maxn; i++) f[i] = (f[i - 1] * i) % p;
8     inv[maxn - 1] = Pow(f[maxn - 1], p - 2, p);
9     for (int i = maxn - 2; ~i; i--) inv[i] = inv[i + 1] * (i + 1) % p;
10 }
11 ll C(int n, int m) { return f[n] * inv[m] % p * inv[n - m] % p; }

```

### 1.5.2 Lucas

$1 \leq n, m \leq 1000000000, 1 < p < 100000, p$

```

1  const int maxp = 100010;
2  ll f[maxn];
3  ll inv[maxn]; //
4  void CalFact()
5  {
6      f[0] = 1;
7      for (int i = 1; i < maxn; i++) f[i] = (f[i - 1] * i) % p;
8      inv[maxn - 1] = Pow(f[maxn - 1], p - 2, p);
9      for (int i = maxn - 2; ~i; i--) inv[i] = inv[i + 1] * (i + 1) % p;
10 }
11 ll Lucas(ll n, ll m, ll p)
12 {
13     ll ret = 1;
14     while (n && m)
15     {
16         ll a = n % p, b = m % p;
17         if (a < b) return 0;
18         ret = ret * f[a] % p * inv[b] % p * inv[a - b] % p;
19         n /= p, m /= p;
20     }
21     return ret;
22 }
```

### 1.5.3 Big Combination

$0 \leq n \leq 10^9, 0 \leq m \leq 10^4, 1 \leq k \leq 10^9 + 7$

```

1  vector<int> v;
2  int dp[110];
3  ll Cal(int l, int r, int k, int dis)
4  {
5      ll res = 1;
6      for (int i = l; i <= r; i++)
7      {
8          int t = i;
9          for (int j = 0; j < v.size(); j++)
10             {
11                 int y = v[j];
12                 while (t % y == 0) dp[j] += dis, t /= y;
13             }
14         res = res * (ll)t % k;
15     }
16     return res;
17 }
18 ll Comb(int n, int m, int k)
19 {
20     clr(dp, 0);
21     v.clear();
22     int tmp = k;
23     for (int i = 2; i * i <= tmp; i++)
24         if (tmp % i == 0)
25             {
26                 int num = 0;
27                 while (tmp % i == 0) tmp /= i, num++;

```

```

28         v.pb(i);
29     }
30     if (tmp != 1) v.pb(tmp);
31     ll ans = Cal(n - m + 1, n, k, 1);
32     for (int j = 0; j < v.size(); j++) ans = ans * Pow(v[j], dp[j], k) % k;
33     ans = ans * inv(Cal(2, m, k, -1), k) % k;
34     return ans;
35 }

```

### 1.5.4 Polya

$$N * N^{\frac{m^8+17m^4+6m^2}{24}}, c^{n^2} + 2c^{\frac{n^2+3}{4}} + c^{\frac{n^2+1}{2}} + 2c^{n\frac{n+1}{2}} + 2c^{\frac{n(n+1)}{2}}$$

```

1 // n c
2 ll solve(int c, int n)
3 {
4     if (n == 0) return 0;
5     ll ans = 0;
6     for (int i = 1; i <= n; i++) ans += Pow(c, __gcd(i, n));
7     if (n & 1) ans += n * Pow(c, n + 1 >> 1);
8     else ans += n / 2 * (1 + c) * Pow(c, n >> 1);
9     return ans / n / 2;
10 }

```

## 1.6 Fast Power

```

1 ll Mul(ll a, ll b, ll mod)
2 {
3     ll t = 0;
4     for (; b >= 1, a = (a << 1) % mod)
5         if (b & 1) t = (t + a) % mod;
6     return t;
7 }
8 ll Pow(ll a, ll n, ll mod)
9 {
10     ll t = 1;
11     for (; n; n >= 1, a = (a * a % mod))
12         if (n & 1) t = (t * a % mod);
13     return t;
14 }

```

## 1.7 Mobius Inversion

### 1.7.1 Mobius

$$F(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$$

$$F(n) = \sum_{n|d} f(d) \Rightarrow f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$$

```

1 ll ans;
2 const int maxn = "Edit";
3 int n, x, prime[maxn], tot, mu[maxn];
4 bool check[maxn];
5 void calmu()

```

```

6 {
7     mu[1] = 1;
8     for (int i = 2; i < maxn; i++)
9     {
10         if (!check[i]) prime[tot++] = i, mu[i] = -1;
11         for (int j = 0; j < tot; j++)
12         {
13             if (i * prime[j] >= maxn) break;
14             check[i * prime[j]] = true;
15             if (i % prime[j] == 0)
16             {
17                 mu[i * prime[j]] = 0;
18                 break;
19             }
20             else mu[i * prime[j]] = -mu[i];
21         }
22     }
23 }

```

### 1.7.2 Number of Coprime-pair

$n$  ( $n \leq 100000$ ),  $n$

```

1 ll solve()
2 {
3     int b[100005];
4     ll _max, ans = 0;
5     clr(b, 0);
6     for (int i = 0; i < n; i++)
7     {
8         scanf("%d", &x);
9         if (x > _max) _max = x;
10        b[x]++;
11    }
12    for (int i = 1; i <= _max; i++)
13    {
14        int cnt = 0;
15        for (ll j = i; j <= _max; j += i) cnt += b[j];
16        ans += 1LL * mu[i] * cnt * cnt;
17    }
18    return (ans - b[1]) / 2;
19 }

```

### 1.7.3 VisibleTrees

$\gcd(x, y) = 1$  ,  $x \leq n, y \leq m$

```

1 ll solve(int n, int m)
2 {
3     if (n < m) swap(n, m);
4     ll ans = 0;
5     for (int i = 1; i <= m; ++i) ans += (ll)mu[i] * (n / i) * (m / i);
6     return ans;
7 }

```

## 1.8 Fast Transformation

### 1.8.1 FFT

```

1  const double PI = acos(-1.0);
2  //
3  struct Complex
4  {
5      double x, y; // x+yi
6      Complex(double _x = 0.0, double _y = 0.0) { x = _x, y = _y; }
7      Complex operator-(const Complex& b) const { return Complex(x - b.x, y - b.y); }
8      Complex operator+(const Complex& b) const { return Complex(x + b.x, y + b.y); }
9      Complex operator*(const Complex& b) const { return Complex(x * b.x - y * b.y, x * b
        .y + y * b.x); }
10 };
11 /*
12  * FFT IFFT
13  * i (i )
14  * len 2
15  */
16 void change(Complex y[], int len)
17 {
18     for (int i = 1, j = len / 2; i < len - 1; i++)
19     {
20         if (i < j) swap(y[i], y[j]);
21         // ,i<j
22         //i +1,j +1, i j
23         int k = len / 2;
24         while (j >= k) j -= k, k /= 2;
25         if (j < k) j += k;
26     }
27 }
28 /*
29  * FFT
30  * len 2^k ,
31  * on==1 DFT,on== -1 IDFT
32  */
33 void fft(Complex y[], int len, int on)
34 {
35     change(y, len);
36     for (int h = 2; h <= len; h <= 1)
37     {
38         Complex wn(cos(-on * 2 * PI / h), sin(-on * 2 * PI / h));
39         for (int j = 0; j < len; j += h)
40         {
41             Complex w(1, 0);
42             for (int k = j; k < j + h / 2; k++)
43             {
44                 Complex u = y[k];
45                 Complex t = w * y[k + h / 2];
46                 y[k] = u + t, y[k + h / 2] = u - t;
47                 w = w * wn;
48             }
49         }
50     }
51     if (on == -1)
52         for (int i = 0; i < len; i++) y[i].x /= len;
53 }

```

## 1.8.2 NTT

$$P^G, G^P, G^{\frac{P-1}{n}}, w_n = e^{\frac{2i\pi}{n}} \quad P, G \quad 1.11$$

```

1  const int mod = 119 << 23 | 1;
2  const int G = 3;
3  int wn[20];
4  void getwn()
5  { //
6      for (int i = 0; i < 20; i++) wn[i] = Pow(G, (mod - 1) / (1 << i), mod);
7  }
8  void change(int y[], int len)
9  {
10     for (int i = 1, j = len / 2; i < len - 1; i++)
11     {
12         if (i < j) swap(y[i], y[j]);
13         int k = len / 2;
14         while (j >= k) j -= k, k /= 2;
15         if (j < k) j += k;
16     }
17 }
18 void ntt(int y[], int len, int on)
19 {
20     change(y, len);
21     for (int h = 2, id = 1; h <= len; h <<= 1, id++)
22     {
23         for (int j = 0; j < len; j += h)
24         {
25             int w = 1;
26             for (int k = j; k < j + h / 2; k++)
27             {
28                 int u = y[k] % mod;
29                 int t = 1LL * w * (y[k + h / 2] % mod) % mod;
30                 y[k] = (u + t) % mod, y[k + h / 2] = ((u - t) % mod + mod) % mod;
31                 w = 1LL * w * wn[id] % mod;
32             }
33         }
34     }
35     if (on == -1)
36     {
37         //
38         int inv = Pow(len, mod - 2, mod);
39         for (int i = 1; i < len / 2; i++) swap(y[i], y[len - i]);
40         for (int i = 0; i < len; i++) y[i] = 1LL * y[i] * inv % mod;
41     }
42 }

```

## 1.8.3 FWT

```

1  void fwt(int f[], int m)
2  {
3      int n = __builtin_ctz(m);
4      for (int i = 0; i < n; ++i)
5          for (int j = 0; j < m; ++j)
6              if (j & (1 << i))
7              {
8                  int l = f[j ^ (1 << i)], r = f[j];
9                  f[j ^ (1 << i)] = l + r, f[j] = l - r;

```

```

10         // or: f[j] += f[j ^ (1 << i)];
11         // and: f[j ^ (1 << i)] += f[j];
12     }
13 }
14 void ifwt(int f[], int m)
15 {
16     int n = __builtin_ctz(m);
17     for (int i = 0; i < n; ++i)
18         for (int j = 0; j < m; ++j)
19             if (j & (1 << i))
20             {
21                 int l = f[j ^ (1 << i)], r = f[j];
22                 f[j ^ (1 << i)] = (l + r) / 2, f[j] = (l - r) / 2;
23                 //
24                 // or: f[j] -= f[j ^ (1 << i)];
25                 // and: f[j ^ (1 << i)] -= f[j];
26             }
27 }

```

## 1.9 Numerical Integration

### 1.9.1 Adaptive Simpson's Rule

$$\int_a^b f(x)dx \approx \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$$

$$|S(a, c) + S(c, b) - S(a, b)|/15 < \epsilon$$

```

1 double F(double x) {}
2 double simpson(double a, double b)
3 { // Simpson
4     double c = a + (b - a) / 2;
5     return (F(a) + 4 * F(c) + F(b)) * (b - a) / 6;
6 }
7 double asr(double a, double b, double eps, double A)
8 { // Simpson ( ) [a,b] Simpson A
9     double c = a + (b - a) / 2;
10    double L = simpson(a, c), R = simpson(c, b);
11    if (fabs(L + R - A) <= 15 * eps) return L + R + (L + R - A) / 15.0;
12    return asr(a, c, eps / 2, L) + asr(c, b, eps / 2, R);
13 }
14 double asr(double a, double b, double eps) { return asr(a, b, eps, simpson(a, b)); }

```

### 1.9.2 Berlekamp-Massey

```

1 const int N = 1 << 14;
2 ll res[N], base[N], _c[N], _md[N];
3 vector<int> Md;
4 void mul(ll* a, ll* b, int k)
5 {
6     for (int i = 0; i < k + k; i++) _c[i] = 0;
7     for (int i = 0; i < k; i++)
8         if (a[i])
9             for (int j = 0; j < k; j++) _c[i + j] = (_c[i + j] + a[i] * b[j]) % mod;
10    for (int i = k + k - 1; i >= k; i--)
11        if (_c[i])
12            for (int j = 0; j < Md.size(); j++) _c[i - k + Md[j]] = (_c[i - k + Md[j]]
13            - _c[i] * _md[Md[j]]) % mod;
14    for (int i = 0; i < k; i++) a[i] = _c[i];

```



```

15 int solve(ll n, VI a, VI b)
16 {
17     ll ans = 0, pnt = 0;
18     int k = a.size();
19     assert(a.size() == b.size());
20     for (int i = 0; i < k; i++) _md[k - 1 - i] = -a[i];
21     _md[k] = 1;
22     Md.clear();
23     for (int i = 0; i < k; i++)
24         if (_md[i] != 0) Md.push_back(i);
25     for (int i = 0; i < k; i++) res[i] = base[i] = 0;
26     res[0] = 1;
27     while ((1LL << pnt) <= n) pnt++;
28     for (int p = pnt; p >= 0; p--)
29     {
30         mul(res, res, k);
31         if ((n >> p) & 1)
32         {
33             for (int i = k - 1; i >= 0; i--) res[i + 1] = res[i];
34             res[0] = 0;
35             for (int j = 0; j < Md.size(); j++) res[Md[j]] = (res[Md[j]] - res[k] * _md
[Md[j]]) % mod;
36         }
37     }
38     for (int i = 0; i < k; i++) ans = (ans + res[i] * b[i]) % mod;
39     if (ans < 0) ans += mod;
40     return ans;
41 }
42 VI BM(VI s)
43 {
44     VI C(1, 1), B(1, 1);
45     int L = 0, m = 1, b = 1;
46     for (int n = 0; n < s.size(); n++)
47     {
48         ll d = 0;
49         for (int i = 0; i <= L; i++) d = (d + (ll)C[i] * s[n - i]) % mod;
50         if (d == 0)
51             ++m;
52         else if (2 * L <= n)
53         {
54             VI T = C;
55             ll c = mod - d * Pow(b, mod - 2) % mod;
56             while (C.size() < B.size() + m) C.pb(0);
57             for (int i = 0; i < B.size(); i++) C[i + m] = (C[i + m] + c * B[i]) % mod;
58             L = n + 1 - L, B = T, b = d, m = 1;
59         }
60         else
61         {
62             ll c = mod - d * Pow(b, mod - 2) % mod;
63             while (C.size() < B.size() + m) C.pb(0);
64             for (int i = 0; i < B.size(); i++) C[i + m] = (C[i + m] + c * B[i]) % mod;
65             ++m;
66         }
67     }
68     return C;
69 }
70 int gao(VI a, ll n)
71 {
72     VI c = BM(a);

```

```

73     c.erase(c.begin());
74     for (int i = 0; i < c.size(); i++) c[i] = (mod - c[i]) % mod;
75     return solve(n, c, VI(a.begin(), a.begin() + c.size()));
76 }

```

### 1.10 Others

```

n, m
1 int josephus(int n, int m)
2 {
3     int r = 0;
4     for (int k = 1; k <= n; ++k) r = (r + m) % k;
5     return r + 1;
6 }

n^n
1 int leftmost(int n)
2 {
3     double m = n * log10((double)n);
4     double g = m - (ll)m;
5     return (int)pow(10.0, g);
6 }

n!
1 int count(ll n)
2 {
3     if (n == 1) return 1;
4     return (int)ceil(0.5 * log10(2 * M_PI * n) + n * log10(n) - n * log10(M_E));
5 }

```

### 1.11 Formula

1.  $n = \prod_{i=1}^k p_i^{a_i}$ 
  - (a)  $f(n) = \prod_{i=1}^k (a_i + 1)$
  - (b)  $g(n) = \prod_{i=1}^k (\sum_{j=0}^{a_i} p_i^j)$
2.  $n \varphi(n) / 2$
3.  $\gcd(n, i) = 1, \gcd(n, n - i) = 1 (1 \leq i \leq n)$
4.  $D(n) = (n - 1)(D(n - 2) + D(n - 1)) = \sum_{i=2}^n \frac{(-1)^k n!}{k!} = \lfloor \frac{n!}{e} + 0.5 \rfloor$
5.  $p \text{ is prime} \Rightarrow (p - 1)! \equiv -1 \pmod{p}$
6.  $\gcd(a, n) = 1 \Rightarrow a^{\varphi(n)} \equiv 1 \pmod{n}$
7.  $\gcd(n, p) = 1 \Rightarrow a^n \equiv a^{n \% \varphi(p)} \pmod{p}$
8.  $\pi(n), \lim_{n \rightarrow \infty} \pi(n) = \frac{n}{\ln n}$
9.  $x = N = \log_{10}(n) + 1$
10.  $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$
11.  $a > 1, m, n > 0, \gcd(a^m - 1, a^n - 1) = a^{\gcd(m, n)} - 1$
12.  $a > b, \gcd(a, b) = 1, \gcd(a^m - b^m, a^n - b^n) = a^{\gcd(m, n)} - b^{\gcd(m, n)}$

$$G = \gcd(C_n^1, C_n^2, \dots, C_n^{n-1}) = \begin{cases} n, & n \text{ is prime} \\ 1, & n \text{ has multy prime factors} \\ p, & n \text{ has single prime factor } p \end{cases}$$

$$\gcd(\text{Fib}(m), \text{Fib}(n)) = \text{Fib}(\gcd(m, n))$$

13.  $\gcd(m, n) = 1, :$

(a)  $m * n - m - n$

(b)  $N = \frac{(m-1)(n-1)}{2}$

14.  $(n+1)lcm(C_n^0, C_n^1, \dots, C_n^{n-1}, C_n^n) = lcm(1, 2, \dots, n+1)$

15.  $p \text{ , } (x+y+\dots+w)^p \equiv x^p + y^p + \dots + w^p \pmod{p}$

16.  $:1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012$

$$h(0) = h(1) = 1, h(n) = \frac{(4n-2)h(n-1)}{n+1} = \frac{C_{2n}^n}{n+1} = C_{2n}^n - C_{2n}^{n-1}$$

17.  $:B_n = -\frac{1}{n+1} \sum_{i=0}^{n-1} C_{n+1}^i B_i$

$$\sum_{i=1}^n i^k = \frac{1}{k+1} \sum_{i=1}^{k+1} C_{k+1}^i B_{k+1-i} (n+1)^i$$

18. FFT

$r \cdot 2^k + 1$	$r$	$k$	$g$
3	1	1	2
5	1	2	2
17	1	4	3
97	3	5	5
193	3	6	5
257	1	8	3
7681	15	9	17
12289	3	12	11
40961	5	13	3
65537	1	16	3
786433	3	18	10
5767169	11	19	3
7340033	7	20	3
23068673	11	21	3
104857601	25	22	3
167772161	5	25	3
469762049	7	26	3
998244353	119	23	3
1004535809	479	21	3
2013265921	15	27	31
2281701377	17	27	3
3221225473	3	30	5
75161927681	35	31	3
77309411329	9	33	7
206158430209	3	36	22
2061584302081	15	37	7
2748779069441	5	39	3
6597069766657	3	41	5
39582418599937	9	42	5
79164837199873	9	43	5
263882790666241	15	44	7
1231453023109121	35	45	3
1337006139375617	19	46	3
3799912185593857	27	47	5
4222124650659841	15	48	19
7881299347898369	7	50	6
31525197391593473	7	52	3
180143985094819841	5	55	6
1945555039024054273	27	56	5
4179340454199820289	29	57	3

## 2 String Processing

### 2.1 KMP

```

1 // yx
2 const int N = "Edit";
3 int next[N];
4 void initkmp(char x[], int m)
5 {
6     int i = 0, j = next[0] = -1;
7     while (i < m)
8     {
9         while (j != -1 && x[i] != x[j]) j = next[j];
10        next[++i] = ++j;
11    }
12 }
13 int kmp(char x[], int m, char y[], int n)
14 {
15     int i, j, ans;
16     i = j = ans = 0;
17     initkmp(x, m);
18     while (i < n)
19     {
20         while (j != -1 && y[i] != x[j]) j = next[j];
21         i++, j++;
22         if (j >= m) ans++, j = next[j];
23     }
24     return ans;
25 }

```

### 2.2 ExtendKMP

```

1 //next[i]:x[i...m-1] x[0...m-1]
2 //extend[i]:y[i...n-1] x[0...m-1]
3 const int N = "Edit";
4 int next[N], extend[N];
5 void pre_ekmp(char x[], int m)
6 {
7     next[0] = m;
8     int j = 0;
9     while (j + 1 < m && x[j] == x[j + 1]) j++;
10    next[1] = j;
11    int k = 1;
12    for (int i = 2; i < m; i++)
13    {
14        int p = next[k] + k - 1;
15        int L = next[i - k];
16        if (i + L < p + 1)
17            next[i] = L;
18        else
19        {
20            j = max(0, p - i + 1);
21            while (i + j < m && x[i + j] == x[j]) j++;
22            next[i] = j;
23            k = i;
24        }
25    }
26 }

```

```

27 void ekmp(char x[], int m, char y[], int n)
28 {
29     pre_ekmp(x, m, next);
30     int j = 0;
31     while (j < n && j < m && x[j] == y[j]) j++;
32     extend[0] = j;
33     int k = 0;
34     for (int i = 1; i < n; i++)
35     {
36         int p = extend[k] + k - 1;
37         int l = next[i - k];
38         if (i + l < p + 1)
39             extend[i] = l;
40         else
41         {
42             j = max(0, p - i + 1);
43             while (i + j < n && j < m && y[i + j] == x[j]) j++;
44             extend[i] = j, k = i;
45         }
46     }
47 }

```

## 2.3 Manacher

O(n)

```

1  const int N = "Edit";
2  char s[N], str[N << 1];
3  int p[N << 1];
4  void Manacher(char s[], int& n)
5  {
6      str[0] = '$', str[1] = '#';
7      for (int i = 0; i < n; i++) str[(i << 1) + 2] = s[i], str[(i << 1) + 3] = '#';
8      n = 2 * n + 2;
9      str[n] = 0;
10     int mx = 0, id;
11     for (int i = 1; i < n; i++)
12     {
13         p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
14         while (str[i - p[i]] == str[i + p[i]]) p[i]++;
15         if (p[i] + i > mx) mx = p[i] + i, id = i;
16     }
17 }
18 int solve(char s[])
19 {
20     int n = strlen(s);
21     Manacher(s, n);
22     return *max_element(p, p + n) - 1;
23 }

```

## 2.4 Aho-Corasick Automaton

```

1  const int maxn = "Edit";
2  struct Trie
3  {
4      int ch[maxn][26], f[maxn], val[maxn];
5      int sz, rt;

```

```

6   int newnode() { clr(ch[sz], -1), val[sz] = 0; return sz++; }
7   void init() { sz = 0, rt = newnode(); }
8   inline int idx(char c) { return c - 'A'; }
9   void insert(const char* s)
10  {
11      int u = 0, n = strlen(s);
12      for (int i = 0; i < n; i++)
13      {
14          int c = idx(s[i]);
15          if (ch[u][c] == -1) ch[u][c] = newnode();
16          u = ch[u][c];
17      }
18      val[u]++;
19  }
20  void build()
21  {
22      queue<int> q;
23      f[rt] = rt;
24      for (int c = 0; c < 26; c++)
25      {
26          if (~ch[rt][c])
27              f[ch[rt][c]] = rt, q.push(ch[rt][c]);
28          else
29              ch[rt][c] = rt;
30      }
31      while (!q.empty())
32      {
33          int u = q.front();
34          q.pop();
35          // val[u] += val[f[u]];
36          for (int c = 0; c < 26; c++)
37          {
38              if (~ch[u][c])
39                  f[ch[u][c]] = ch[f[u]][c], q.push(ch[u][c]);
40              else
41                  ch[u][c] = ch[f[u]][c];
42          }
43      }
44  }
45  //
46  int query(const char* s)
47  {
48      int u = rt, n = strlen(s);
49      int res = 0;
50      for (int i = 0; i < n; i++)
51      {
52          int c = idx(s[i]);
53          u = ch[u][c];
54          int tmp = u;
55          while (tmp != rt)
56          {
57              res += val[tmp];
58              val[tmp] = 0;
59              tmp = f[tmp];
60          }
61      }
62      return res;
63  }
64  };

```

## 2.5 Suffix Array

```

1 //      , 0(nlogn)
2 const int maxn = "Edit";
3 char s[maxn];
4 int sa[maxn], t[maxn], t2[maxn], c[maxn], rank[maxn], height[maxn];
5 //n      , 0~m-1
6 void build_sa(int m, int n)
7 {
8     n++;
9     int *x = t, *y = t2;
10    //
11    for (int i = 0; i < m; i++) c[i] = 0;
12    for (int i = 0; i < n; i++) c[x[i]] = s[i]++;
13    for (int i = 1; i < m; i++) c[i] += c[i - 1];
14    for (int i = n - 1; ~i; i--) sa[--c[x[i]]] = i;
15    for (int k = 1; k <= n; k <= 1)
16    {
17        // sa
18        int p = 0;
19        for (int i = n - k; i < n; i++) y[p++] = i;
20        for (int i = 0; i < n; i++)
21            if (sa[i] >= k) y[p++] = sa[i] - k;
22        //
23        for (int i = 0; i < m; i++) c[i] = 0;
24        for (int i = 0; i < n; i++) c[x[y[i]]]++;
25        for (int i = 0; i < m; i++) c[i] += c[i - 1];
26        for (int i = n - 1; ~i; i--) sa[--c[x[y[i]]]] = y[i];
27        // say x
28        swap(x, y);
29        p = 1;
30        x[sa[0]] = 0;
31        for (int i = 1; i < n; i++)
32            x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k] ? p
- 1 : p++;
33        if (p >= n) break; //      ,sa      ,
34        m = p; //
35    }
36    n--;
37    int k = 0;
38    for (int i = 0; i <= n; i++) rank[sa[i]] = i;
39    for (int i = 0; i < n; i++)
40    {
41        if (k) k--;
42        int j = sa[rank[i] - 1];
43        while (s[i + k] == s[j + k]) k++;
44        height[rank[i]] = k;
45    }
46 }
47
48 int dp[maxn][30];
49 void initrmq(int n)
50 {
51     for (int i = 1; i <= n; i++)
52         dp[i][0] = height[i];
53     for (int j = 1; (1 << j) <= n; j++)
54         for (int i = 1; i + (1 << j) - 1 <= n; i++)
55             dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
56 }

```

```

57 int rmq(int l, int r)
58 {
59     int k = 31 - __builtin_clz(r - l + 1);
60     return min(dp[l][k], dp[r - (1 << k) + 1][k]);
61 }
62 int lcp(int a, int b)
63 { //
64     a = rank[a], b = rank[b];
65     if (a > b) swap(a, b);
66     return rmq(a + 1, b);
67 }

```

## 2.6 Suffix Automation

```

1  const int maxn = "Edit";
2  struct SAM
3  {
4      int len[maxn << 1], link[maxn << 1], ch[maxn << 1][26];
5      int sz, rt, last;
6      int newnode(int x = 0)
7      {
8          len[sz] = x;
9          link[sz] = -1;
10         clr(ch[sz], -1);
11         return sz++;
12     }
13     void init() { sz = last = 0, rt = newnode(); }
14     void extend(int c)
15     {
16         int np = newnode(len[last] + 1);
17         int p;
18         for (p = last; ~p && ch[p][c] == -1; p = link[p]) ch[p][c] = np;
19         if (p == -1)
20             link[np] = rt;
21         else
22         {
23             int q = ch[p][c];
24             if (len[p] + 1 == len[q])
25                 link[np] = q;
26             else
27             {
28                 int nq = newnode(len[p] + 1);
29                 memcpy(ch[nq], ch[q], sizeof(ch[q]));
30                 link[nq] = link[q], link[q] = link[np] = nq;
31                 for (; ~p && ch[p][c] == q; p = link[p]) ch[p][c] = nq;
32             }
33         }
34         last = np;
35     }
36     int topcnt[maxn], topsam[maxn << 1];
37     void sort()
38     { //
39         clr(topcnt, 0);
40         for (int i = 0; i < sz; i++) topcnt[len[i]]++;
41         for (int i = 0; i < maxn - 1; i++) topcnt[i + 1] += topcnt[i];
42         for (int i = 0; i < sz; i++) topsam[--topcnt[len[i]]] = i;
43     }
44 };

```



### 3 Data Structure

#### 3.1 Binary Indexed Tree

$O(\log n)$

```

1 // 1
2 const int MAXN=100000;
3 struct BIT{
4     int n,c[MAXN<<1];
5     void init(int _n){n=_n;for(int i=0;i<=n;i++)c[i]=0;}
6     void add(int i,int v){for(;i<=n;i+=i&-i)c[i]+=v;}
7     int sum(int i){int s=0;for(;i>0;i-=i&-i)s+=c[i];return s;}
8 }bit;

```

#### 3.2 Segment Tree

```

1 #define lson rt << 1 //
2 #define rson rt << 1 | 1 //
3 #define Lson l, m, lson //
4 #define Rson m + 1, r, rson //
5 void PushUp(int rt); // lson rson rt
6 void PushDown(int rt[, int m]); // rt ,m ( )
7 void build(int l, int r, int rt); // rt , [l, r]
8 void update([...], int l, int r, int rt) // rt[l, r]
9 int query(int L, int R, int l, int r, int rt) // rt[l, r] [L, R]

```

##### 3.2.1 Single-point Update

```

1 const int maxn = "Edit";
2 int sum[maxn << 2]; // sum[rt]
3 void PushUp(int rt) { sum[rt] = sum[lson] + sum[rson]; }
4 void build(int l, int r, int rt)
5 {
6     if (l == r)
7     {
8         scanf("%d", &sum[rt]); //
9         return;
10    }
11    int m = (l + r) >> 1;
12    build(Lson);
13    build(Rson);
14    PushUp(rt);
15 }
16 void update(int p, int add, int l, int r, int rt)
17 {
18     if (l == r)
19     {
20         sum[rt] += add;
21         return;
22     }
23     int m = (l + r) >> 1;
24     if (p <= m)
25         update(p, add, Lson);
26     else
27         update(p, add, Rson);
28     PushUp(rt);

```

```

29 }
30 int query(int L, int R, int l, int r, int rt)
31 {
32     if (L <= l && r <= R) return sum[rt];
33     int m = (l + r) >> 1, s = 0;
34     if (L <= m) s += query(L, R, Lson);
35     if (m < R) s += query(L, R, Rson);
36     return s;
37 }

```

### 3.2.2 Interval Update

```

1  const int maxn = "Edit";
2  int seg[maxn << 2], sum[maxn << 2]; // seg[rt] , PushDown
3  void PushUp(int rt) { sum[rt] = sum[lson] + sum[rson]; }
4  void PushDown(int rt, int m)
5  {
6      if (seg[rt] == 0) return;
7      seg[lson] += seg[rt];
8      seg[rson] += seg[rt];
9      sum[lson] += seg[rt] * (m - (m >> 1));
10     sum[rson] += seg[rt] * (m >> 1);
11     seg[rt] = 0;
12 }
13 void build(int l, int r, int rt)
14 {
15     seg[rt] = 0;
16     if (l == r)
17     {
18         scanf("%lld", &sum[rt]);
19         return;
20     }
21     int m = (l + r) >> 1;
22     build(Lson);
23     build(Rson);
24     PushUp(rt);
25 }
26 void update(int L, int R, int add, int l, int r, int rt)
27 {
28     if (L <= l && r <= R)
29     {
30         seg[rt] += add;
31         sum[rt] += add * (r - l + 1);
32         return;
33     }
34     PushDown(rt, r - l + 1);
35     int m = (l + r) >> 1;
36     if (L <= m) update(L, R, add, Lson);
37     if (m < R) update(L, R, add, Rson);
38     PushUp(rt);
39 }
40 int query(int L, int R, int l, int r, int rt)
41 {
42     if (L <= l && r <= R) return sum[rt];
43     PushDown(rt, r - l + 1);
44     int m = (l + r) >> 1, ret = 0;
45     if (L <= m) ret += query(L, R, Lson);
46     if (m < R) ret += query(L, R, Rson);

```

```

47     return ret;
48 }

```

### 3.3 Splay Tree

```

1  #define key_value ch[ch[root][1]][0]
2  const int maxn = "Edit";
3  struct Splay
4  {
5      int a[maxn];
6      int sz[maxn], ch[maxn][2], fa[maxn];
7      int key[maxn], rev[maxn];
8      int root, tot;
9      int stk[maxn], top;
10 void init(int n)
11 {
12     tot = 0, top = 0;
13     root = newnode(0, -1);
14     ch[root][1] = newnode(root, -1);
15     for (int i = 0; i < n; i++) a[i] = i + 1;
16     key_value = build(0, n - 1, ch[root][1]);
17     pushup(ch[root][1]);
18     pushup(root);
19 }
20 int newnode(int p = 0, int k = 0)
21 {
22     int x = top ? stk[top--] : ++tot;
23     fa[x] = p;
24     sz[x] = 1;
25     ch[x][0] = ch[x][1] = 0;
26     key[x] = k;
27     rev[x] = 0;
28     return x;
29 }
30 void pushdown(int x)
31 {
32     if (rev[x])
33     {
34         swap(ch[x][0], ch[x][1]);
35         if (ch[x][0]) rev[ch[x][0]] ^= 1;
36         if (ch[x][1]) rev[ch[x][1]] ^= 1;
37         rev[x] = 0;
38     }
39 }
40 void pushup(int x) { sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1; }
41 void rotate(int x, int d)
42 {
43     int y = fa[x];
44     pushdown(y), pushdown(x);
45     ch[y][d ^ 1] = ch[x][d];
46     fa[ch[x][d]] = y;
47     if (fa[y]) ch[fa[y]][ch[fa[y]][1] == y] = x;
48     fa[x] = fa[y];
49     ch[x][d] = y;
50     fa[y] = x;
51     pushup(y);
52 }
53 void splay(int x, int goal = 0)

```

```

54     {
55         pushdown(x);
56         while (fa[x] != goal)
57         {
58             if (fa[fa[x]] == goal)
59                 rotate(x, ch[fa[x]][0] == x);
60             else
61             {
62                 int y = fa[x];
63                 int d = ch[fa[y]][0] == y;
64                 ch[y][d] == x ? rotate(x, d ^ 1) : rotate(y, d);
65                 rotate(x, d);
66             }
67         }
68         pushup(x);
69         if (goal == 0) root = x;
70     }
71     int kth(int r, int k)
72     {
73         pushdown(r);
74         int t = sz[ch[r][0]] + 1;
75         if (t == k) return r;
76         return t > k ? kth(ch[r][0], k) : kth(ch[r][1], k - t);
77     }
78     int build(int l, int r, int p)
79     {
80         if (l > r) return 0;
81         int mid = l + r >> 1;
82         int x = newnode(p, a[mid]);
83         ch[x][0] = build(l, mid - 1, x);
84         ch[x][1] = build(mid + 1, r, x);
85         pushup(x);
86         return x;
87     }
88     void select(int l, int r)
89     {
90         splay(kth(root, l), 0);
91         splay(kth(ch[root][1], r - l + 2), root);
92     }
93     //
94 };

```

### 3.4 Functional Segment Tree

$k$

```

1  //poj 2104
2  #include<cstdio>
3  #include<iostream>
4  #include<cmath>
5  #include<queue>
6  #include<stack>
7  #include<set>
8  #include<map>
9  #include<algorithm>
10 #include<vector>
11 #include<string>

```

```

12 #include<cstring>
13 using namespace std;
14 #define rep(i,a,b) for(int i=a;i<=b;i++)
15 #define per(i,a,b) for(int i=a;i>=b;i--)
16 #define pb push_back
17 #define mp make_pair
18 #define all(x) (x).begin(),(x).end()
19 typedef long long ll;
20 typedef vector<int> vi;
21 typedef pair<int,int> pii;
22 const int MAXN=1e5+6;
23 int n,m,cnt,x,y,k,root[MAXN],a[MAXN];
24 struct node{int l,r,sum;}T[MAXN*40];
25 vi v;
26 int getId(int x){return lower_bound(all(v),x)-v.begin()+1;}
27 void update(int l,int r,int &x,int y,int pos){
28     x++;cnt;
29     T[x]=T[y];
30     T[x].sum++;
31     if(l==r)return;
32     int mid=(l+r)>>1;
33     if(mid>=pos)update(l,mid,T[x].l,T[y].l,pos);
34     else update(mid+1,r,T[x].r,T[y].r,pos);
35 }
36 int query(int l,int r,int x,int y,int k){
37     if(l==r)return l;
38     int sum=T[T[y].l].sum-T[T[x].l].sum;
39     int mid=(l+r)>>1;
40     if(sum>=k)return query(l,mid,T[x].l,T[y].l,k);
41     else return query(mid+1,r,T[x].r,T[y].r,k-sum);
42 }
43 int work(){
44     scanf("%d%d",&n,&m);
45     v.clear();
46     rep(i,1,n)scanf("%d",&a[i]),v.pb(a[i]);
47     sort(all(v)),v.erase(unique(all(v)),v.end());
48     cnt=0;
49     rep(i,1,n)update(1,n,root[i],root[i-1],getId(a[i]));
50     rep(i,1,m)scanf("%d%d%d",&x,&y,&k),printf("%d\n",v[query(1,n,root[x-1],root[y],k)-1]);
51     return 0;
52 }
53 int main(){
54     #ifdef superkunn
55         freopen("input.txt","rt",stdin);
56     #endif
57     work();
58     return 0;
59 }

```

### 3.5 Sparse Table

```

1 const int maxn = "Edit";
2 int mmax[maxn][30], mmin[maxn][30];
3 int a[maxn], n, k;
4 void init()
5 {
6     for (int i = 1; i <= n; i++) mmax[i][0] = mmin[i][0] = a[i];

```

```

7     for (int j = 1; (1 << j) <= n; j++)
8         for (int i = 1; i + (1 << j) - 1 <= n; i++)
9             {
10                 mmax[i][j] = max(mmax[i][j - 1], mmax[i + (1 << (j - 1))][j - 1]);
11                 mmin[i][j] = min(mmin[i][j - 1], mmin[i + (1 << (j - 1))][j - 1]);
12             }
13 }
14 // op=0/1 [l,r] /
15 int rmq(int l, int r, int op)
16 {
17     int k = 31 - __builtin_clz(r - l + 1);
18     if (op == 0)
19         return max(mmax[l][k], mmax[r - (1 << k) + 1][k]);
20     return min(mmin[l][k], mmin[r - (1 << k) + 1][k]);
21 }

RMQ
1 void init()
2 {
3     for (int i = 0; (1 << i) <= n; i++)
4         for (int j = 0; (1 << j) <= m; j++)
5             {
6                 if (i == 0 && j == 0) continue;
7                 for (int row = 1; row + (1 << i) - 1 <= n; row++)
8                     for (int col = 1; col + (1 << j) - 1 <= m; col++)
9                         if (i)
10                             dp[row][col][i][j] = max(dp[row][col][i - 1][j],
11                                                         dp[row + (1 << (i - 1))][col][i - 1][j]);
12                         else
13                             dp[row][col][i][j] = max(dp[row][col][i][j - 1],
14                                                         dp[row][col + (1 << (j - 1))][i][j - 1]);
15             }
16 }
17 int rmq(int x1, int y1, int x2, int y2)
18 {
19     int kx = 31 - __builtin_clz(x2 - x1 + 1);
20     int ky = 31 - __builtin_clz(y2 - y1 + 1);
21     int m1 = dp[x1][y1][kx][ky];
22     int m2 = dp[x2 - (1 << kx) + 1][y1][kx][ky];
23     int m3 = dp[x1][y2 - (1 << ky) + 1][kx][ky];
24     int m4 = dp[x2 - (1 << kx) + 1][y2 - (1 << ky) + 1][kx][ky];
25     return max(max(m1, m2), max(m3, m4));
26 }

```

### 3.6 Heavy-Light Decomposition

```

1 const int maxn = "Edit";
2 struct HLD
3 {
4     int n, dfs_clock;
5     int sz[maxn], top[maxn], son[maxn], dep[maxn], fa[maxn], id[maxn];
6     vector<int> G[maxn];
7     void init(int n)
8     {
9         this->n = n, clr(son, -1), dfs_clock = 0;
10        for (int i = 0; i < n; i++) G[i].clear();
11    }
12    void add_edge(int u, int v) { G[u].pb(v), G[v].pb(u); }

```

```

13 void dfs(int u, int p, int d)
14 {
15     dep[u] = d, fa[u] = p, sz[u] = 1;
16     for (auto& v : G[u])
17     {
18         if (v == p) continue;
19         dfs(v, u, d + 1);
20         sz[u] += sz[v];
21         if (son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
22     }
23 }
24 void link(int u, int t)
25 {
26     top[u] = t, id[u] = ++dfs_clock;
27     if (son[u] == -1) return;
28     link(son[u], t);
29     for (auto& v : G[u])
30         if (v != son[u] && v != fa[u]) link(v, v);
31 }
32 //
33 int query_path(int u, int v)
34 {
35     int ret = 0;
36     while (top[u] != top[v])
37     {
38         if (dep[top[u]] < dep[top[v]]) swap(u, v);
39         ret += query(id[top[u]], id[u]);
40         u = fa[top[u]];
41     }
42     if (dep[u] > dep[v]) swap(u, v);
43     ret += query(id[u], id[v]);
44 }
45 };

```

### 3.7 Link-Cut Tree

```

1  const int maxn = "Edit";
2  struct LCT
3  {
4      int val[maxn], sum[maxn]; //
5      int rev[maxn], ch[maxn][2], fa[maxn];
6      int stk[maxn];
7      inline void init(int n)
8      { //
9          for (int i = 1; i <= n; i++) scanf("%d", val + i);
10     }
11     inline bool isroot(int x) { return ch[fa[x]][0] != x && ch[fa[x]][1] != x; }
12     inline bool get(int x) { return ch[fa[x]][1] == x; }
13     void pushdown(int x)
14     {
15         if (!rev[x]) return;
16         swap(ch[x][0], ch[x][1]);
17         if (ch[x][0]) rev[ch[x][0]] ^= 1;
18         if (ch[x][1]) rev[ch[x][1]] ^= 1;
19         rev[x] ^= 1;
20     }

```

```

21 void pushup(int x) { sum[x] = val[x] + sum[ch[x][0]] + sum[ch[x][1]]; }
22 void rotate(int x)
23 {
24     int y = fa[x], z = fa[fa[x]], d = get(x);
25     if (!isroot(y)) ch[z][get(y)] = x;
26     fa[x] = z;
27     ch[y][d] = ch[x][d ^ 1], fa[ch[y][d]] = y;
28     ch[x][d ^ 1] = y, fa[y] = x;
29     pushup(y), pushup(x);
30 }
31 void splay(int x)
32 {
33     int top = 0;
34     stk[++top] = x;
35     for (int i = x; !isroot(i); i = fa[i]) stk[++top] = fa[i];
36     for (int i = top; i; i--) pushdown(stk[i]);
37     for (int f; !isroot(x); rotate(x))
38         if (!isroot(f = fa[x])) rotate(get(x) == get(f) ? f : x);
39 }
40 void access(int x)
41 {
42     for (int y = 0; x; y = x, x = fa[x]) splay(x), ch[x][1] = y, pushup(x);
43 }
44 int find(int x) { access(x), splay(x); while (ch[x][0]) x = ch[x][0]; return x; }
45 void makeroot(int x) { access(x), splay(x), rev[x] ^= 1; }
46 void link(int x, int y) { makeroot(x), fa[x] = y, splay(x); }
47 void cut(int x, int y) { makeroot(x), access(y), splay(y), fa[x] = ch[y][0] = 0; }
48 void update(int x, int v) { val[x] = v, access(x), splay(x); }
49 int query(int x, int y) { makeroot(y), access(x), splay(x); return sum[x]; }
50 };

```



## 4 Graph Theory

### 4.1 Union-Find Set

```

1  const int MAXN=1e6+5;
2  struct DSU{
3      int p[MAXN];
4      void init(int n){for(int i=0;i<=n;i++)p[i]=i;}
5      int findp(int x){return x==p[x]?x:p[x]=findp(p[x]);}
6      void unite(int x,int y){x=findp(x);y=findp(y);if(x==y)return;p[y]=x;}
7      bool same(int x,int y){return findp(x)==findp(y);}
8  }dsu;

```

### 4.2 Minimal Spanning Tree

#### 4.2.1 Kruskal

```

1  //poj 1258
2  #include<cstdio>
3  #include<algorithm>
4  using namespace std;
5  const int MAXE=1e5+5;
6  const int MAXN=1e5+5;
7  struct DSU{
8      int p[MAXN];
9      void init(int n){for(int i=0;i<=n;i++)p[i]=i;}
10     int findp(int x){return x==p[x]?x:p[x]=findp(p[x]);}
11     void unite(int x,int y){x=findp(x);y=findp(y);if(x==y)return;p[y]=x;}
12     bool same(int x,int y){return findp(x)==findp(y);}
13 }dsu;
14 struct edge{int u,v,cost;}es[MAXE];
15 bool cmp(const edge &x,const edge &y){return x.cost<y.cost;}
16 int V,E;
17 int kruskal(){
18     sort(es,es+E,cmp);
19     dsu.init(V);
20     int res=0;
21     for(int i=0;i<E;i++){
22         if(!dsu.same(es[i].u,es[i].v)){
23             dsu.unite(es[i].u,es[i].v);
24             res+=es[i].cost;
25         }
26     }
27     return res;
28 }
29 int main(){
30     while(~scanf("%d",&V)){
31         E=0;
32         for(int i=1;i<=V;i++){
33             for(int j=1;j<=V;j++){
34                 int w;
35                 scanf("%d",&w);
36                 if(i==j)continue;
37                 es[E].u=i;
38                 es[E].v=j;
39                 es[E].cost=w;
40                 E++;
41             }
42         }
43     }

```

```

42     }
43     printf("%d\n",kruskal());
44 }
45 return 0;
46 }

```

## 4.3 Shortest Path

### 4.3.1 Dijkstra

```

1  //cf 610 A
2  #include<bits/stdc++.h>
3  using namespace std;
4  const int INF=1e9;
5  const int MAXV=5e3+50;
6  const int MAXE=1e5+50;
7  int V;
8  struct edge{int to,cost;};
9  vector<edge> G[MAXV];
10 typedef pair<int,int> P;
11 int d[MAXV];
12 void dijkstra(int s){
13     priority_queue<P,vector<P>,greater<P> > que;
14     fill(d,d+V+1,INF);
15     d[s]=0;
16     que.push(P(0,s));
17     while(!que.empty()){
18         P t=que.top();
19         que.pop();
20         int v=t.second;
21         if(d[v]<t.first)continue;
22         for(int i=0;i<G[v].size();i++){
23             edge e=G[v][i];
24             if(d[e.to]>d[v]+e.cost){
25                 d[e.to]=d[v]+e.cost;
26                 que.push(P(d[e.to],e.to));
27             }
28         }
29     }
30 }
31 int mat[405][405];
32 int main(){
33     int n,m;
34     scanf("%d%d",&n,&m);
35     V=n;
36     for(int i=1;i<=m;i++){
37         int u,v;
38         scanf("%d%d",&u,&v);
39         G[u].push_back(edge{v,1});
40         G[v].push_back(edge{u,1});
41         mat[u][v]=mat[v][u]=1;
42     }
43     dijkstra(1);
44     int ans;
45     if(d[n]==INF){
46         printf("-1");
47         return 0;
48     }
49     ans=d[n];

```

```

50     for(int i=1;i<=n;i++)G[i].clear();
51     for(int i=1;i<=n;i++){
52         for(int j=1;j<=n;j++){
53             if(i==j)continue;
54             if(mat[i][j]==0){
55                 G[i].push_back(edge{j,1});
56             }
57         }
58     }
59     dijkstra(1);
60     if(d[n]==INF){
61         printf("-1");
62         return 0;
63     }
64     printf("%d",max(ans,d[n]));
65     return 0;
66 }

```

#### 4.3.2 Spfa

```

1  //poj 3259
2  #include<cstdio>
3  #include<iostream>
4  #include<algorithm>
5  #include<queue>
6  #include<cstring>
7  using namespace std;
8  const int INF=1e9;
9  const int MAXV=500+5;
10 const int MAXE=2700+5;
11 int tot;
12 int head[MAXV];
13 struct node{
14     int to,cost,next;
15 }edge[MAXE<<1];
16 int d[MAXV];
17 queue<int> que;
18 bool inq[MAXV];
19 int qtime[MAXV];
20 void init(){
21     tot=0;
22     memset(head,-1,sizeof(head));
23 }
24 void add_edge(int u,int v,int x){
25     edge[tot].to=v;
26     edge[tot].cost=x;
27     edge[tot].next=head[u];
28     head[u]=tot++;
29 }
30 bool spfa(int n){
31     memset(d,-1,sizeof(d));
32     d[1]=0;
33     while(!que.empty())que.pop();
34     memset(inq,0,sizeof(inq));
35     memset(qtime,0,sizeof(qtime));
36     que.push(1);
37     inq[1]=1;
38     qtime[1]++;

```

```

39     while(!que.empty()){
40         int u=que.front();
41         que.pop();
42         inq[u]=0;
43         for(int i=head[u];i!=-1;i=edge[i].next){
44             int v=edge[i].to;
45             int w=edge[i].cost;
46             if(d[v]==-1||d[u]+w<d[v]){
47                 d[v]=d[u]+w;
48                 if(!inq[v]){
49                     inq[v]=1;
50                     que.push(v);
51                     qtime[v]++;
52                     if(qtime[v]>n){
53                         return false;
54                     }
55                 }
56             }
57         }
58     }
59     return true;
60 }
61 int main(){
62     int kase;
63     scanf("%d",&kase);
64     while(kase--){
65         init();
66         int n,m,w;
67         scanf("%d%d%d",&n,&m,&w);
68         while(m--){
69             int u,v,x;
70             scanf("%d%d%d",&u,&v,&x);
71             add_edge(u,v,x);
72             add_edge(v,u,x);
73         }
74         while(w--){
75             int u,v,x;
76             scanf("%d%d%d",&u,&v,&x);
77             add_edge(u,v,-x);
78         }
79         if(!spfa(n)){
80             puts("YES");
81         }else{
82             puts("NO");
83         }
84     }
85     return 0;
86 }

```

#### 4.4 Topo Sort

Ans ,G ,deg ,map  
1, 0

```

1  const int maxn = "Edit";
2  int Ans[maxn];
3  vector<int> G[maxn];

```

```

4  int deg[maxn];
5  map<PII, bool> S;
6  void init(int n)
7  {
8      S.clear();
9      for (int i = 0; i < n; i++) G[i].clear();
10     clr(deg, 0), clr(Ans, 0);
11 }
12 void add_edge(int u, int v)
13 {
14     if (S[mp(u, v)]) return;
15     G[u].pb(v), S[mp(u, v)] = 1, deg[v]++;
16 }
17 bool Toposort(int n)
18 {
19     int tot = 0;
20     queue<int> q;
21     for (int i = 0; i < n; ++i)
22         if (deg[i] == 0) q.push(i);
23     while (!q.empty())
24     {
25         int u = q.front();
26         q.pop();
27         Ans[tot++] = u;
28         for (auto& v : G[u])
29             if (--deg[v] == 0) q.push(v);
30     }
31     if (tot < n - 1) return false;
32     return true;
33 }

```

## 4.5 LCA

### 4.5.1 Tarjan

Tarjan

$O(n + q)$

```

1  const int maxn = "Edit";
2  int par[maxn];           //
3  int ans[maxn];          //
4  vector<int> G[maxn];     //
5  vector<PII> query[maxn]; //
6  bool vis[maxn];         //
7  inline void init(int n)
8  {
9      for (int i = 1; i <= n; i++)
10     {
11         G[i].clear(), query[i].clear();
12         par[i] = i, vis[i] = 0;
13     }
14 }
15 inline void add_edge(int u, int v) { G[u].pb(v); }
16 inline void add_query(int id, int u, int v)
17 {
18     query[u].pb(mp(v, id));
19     query[v].pb(mp(u, id));
20 }
21 void tarjan(int u)

```

```

22 {
23     vis[u] = 1;
24     for (auto& v : G[u])
25     {
26         if (vis[v]) continue;
27         tarjan(v);
28         unite(u, v);
29     }
30     for (auto& q : query[u])
31     {
32         int &v = q.X, &id = q.Y;
33         if (!vis[v]) continue;
34         ans[id] = find(v);
35     }
36 }

```

#### 4.5.2 DFS+ST

DFS+ST

$O(n \log n + q)$

```

1  const int maxn = "Edit";
2  vector<int> G[maxn], sp;
3  int dep[maxn], dfn[maxn];
4  PII dp[21][maxn << 1];
5  void init(int n)
6  {
7      for (int i = 0; i < n; i++) G[i].clear();
8      sp.clear();
9  }
10 void dfs(int u, int fa)
11 {
12     dep[u] = dep[fa] + 1;
13     dfn[u] = sp.size();
14     sp.push_back(u);
15     for (auto& v : G[u])
16     {
17         if (v == fa) continue;
18         dfs(v, u);
19         sp.push_back(u);
20     }
21 }
22 void initrmq()
23 {
24     int n = sp.size();
25     for (int i = 0; i < n; i++) dp[0][i] = {dfn[sp[i]], sp[i]};
26     for (int i = 1; (1 << i) <= n; i++)
27         for (int j = 0; j + (1 << i) - 1 < n; j++)
28             dp[i][j] = min(dp[i - 1][j], dp[i - 1][j + (1 << (i - 1))]);
29 }
30 int lca(int u, int v)
31 {
32     int l = dfn[u], r = dfn[v];
33     if (l > r) swap(l, r);
34     int k = 31 - __builtin_clz(r - l + 1);
35     return min(dp[k][l], dp[k][r - (1 << k) + 1]).Y;
36 }

```

## 4.6 Depth-First Traversal

### 4.6.1 Biconnected-Component

```

1  // bccno
2  const int maxn = "Edit";
3  int pre[maxn], iscut[maxn], bccno[maxn], dfs_clock, bcc_cnt;
4  vector<int> G[maxn], bcc[maxn];
5  stack<PII> s;
6  void init(int n)
7  {
8      for (int i = 0; i < n; i++) G[i].clear();
9  }
10 inline void add_edge(int u, int v) { G[u].pb(v), G[v].pb(u); }
11 int dfs(int u, int fa)
12 {
13     int lowu = pre[u] = ++dfs_clock;
14     int child = 0;
15     for (auto& v : G[u])
16     {
17         PII e = mp(u, v);
18         if (!pre[v])
19         {
20             // v
21             s.push(e);
22             child++;
23             int lowv = dfs(v, u);
24             lowu = min(lowu, lowv); // low
25             if (lowv >= pre[u])
26             {
27                 iscut[u] = true;
28                 bcc_cnt++;
29                 bcc[bcc_cnt].clear(); // !bcc 1
30                 for (;;)
31                 {
32                     PII x = s.top();
33                     s.pop();
34                     if (bccno[x.X] != bcc_cnt)
35                         bcc[bcc_cnt].pb(x.X), bcc[x.X] = bcc_cnt;
36                     if (bccno[x.Y] != bcc_cnt)
37                         bcc[bcc_cnt].pb(x.Y), bcc[x.Y] = bcc_cnt;
38                     if (x.X == u && x.Y == v) break;
39                 }
40             }
41         }
42         else if (pre[v] < pre[u] && v != fa)
43         {
44             s.push(e);
45             lowu = min(lowu, pre[v]); //
46         }
47     }
48     if (fa < 0 && child == 1) iscut[u] = 0;
49     return lowu;
50 }
51 void find_bcc(int n)
52 {
53     // s ,
54     clr(pre, 0), clr(iscut, 0), clr(bccno, 0);
55     dfs_clock = bcc_cnt = 0;

```

```

56     for (int i = 0; i < n; i++)
57         if (!pre[i]) dfs(i, -1);
58 }

```

#### 4.6.2 Strongly Connected Component

```

1  //cf 999 E
2  #include<bits/stdc++.h>
3  using namespace std;
4  typedef long long ll;
5  const int MAXN = 5005;//
6  const int MAXM = 5005;//
7  struct Edge{
8      int to,next;
9  } edge[MAXN];
10 int head[MAXN],tot;
11 int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN];//Belong 1~scc
12 int Index,top;
13 int scc;//
14 bool Instack[MAXN];
15 void init(){
16     tot = 0;
17     memset(head,-1,sizeof(head));
18 }
19 void addedge(int u,int v){
20     edge[tot].to = v;
21     edge[tot].next = head[u];
22     head[u] = tot++;
23 }
24 void Tarjan(int u){
25     int v;
26     Low[u] = DFN[u] = ++Index;
27     Stack[top++] = u;
28     Instack[u] = true;
29     for(int i = head[u]; i != -1; i = edge[i].next){
30         v = edge[i].to;
31         if( !DFN[v] ){
32             Tarjan(v);
33             if( Low[u] > Low[v] )Low[u] = Low[v];
34         }
35         else if(Instack[v] && Low[u] > DFN[v])
36             Low[u] = DFN[v];
37     }
38     if(Low[u] == DFN[u]){
39         scc++;
40         do{
41             v = Stack[--top];
42             Instack[v] = false;
43             Belong[v] = scc;
44         }
45         while( v != u);
46     }
47 }
48 void solve(int N){
49     memset(DFN,0,sizeof(DFN));
50     memset(Instack,0,sizeof(Instack));
51     Index = scc = top = 0;
52     for(int i = 1; i <= N; i++)

```



```

53         if(!DFN[i])
54             Tarjan(i);
55     }
56     int u[MAXM],v[MAXM],in[MAXN],vis[MAXN];
57     int n,m,s;
58     void dfs(int x){
59         Belong[x]=Belong[s];
60         vis[x]=true;
61         for(int i=head[x];i!=-1;i=edge[i].next){
62             int e=edge[i].to;
63             if(!vis[e])dfs(e);
64         }
65     }
66     int main(){
67         scanf("%d%d%d",&n,&m,&s);
68         init();
69         for(int i=1;i<=m;i++){
70             scanf("%d%d",&u[i],&v[i]);
71             addedge(u[i],v[i]);
72         }
73         solve(n);
74         dfs(s);
75         int ans=0;
76         for(int i=1;i<=m;i++){
77             if(Belong[u[i]]!=Belong[v[i]]){
78                 in[Belong[v[i]]]++;
79             }
80         }
81         set<int> ss;
82         for(int i=1;i<=n;i++){
83             ss.insert(Belong[i]);
84         }
85         set<int>::iterator it;
86         for(it=ss.begin();it!=ss.end();it++){
87             if(*it!=Belong[s]){
88                 if(in[*it]==0){
89                     ans++;
90                 }
91             }
92         }
93         printf("%d",ans);
94         return 0;
95     }

```

#### 4.6.3 2-SAT

```

1 //hdu 3062
2 #include<bits/stdc++.h>
3 using namespace std;
4 const int MAXV=1e4;
5 int V;
6 vector<int> G[MAXV];
7 vector<int> rG[MAXV];
8 vector<int> vs;
9 bool used[MAXV];
10 int Belong[MAXV];
11 void init(int x){
12     V=x;

```

```

13     for(int i=0;i<MAXV;i++){
14         G[i].clear();
15         rG[i].clear();
16     }
17 }
18 void add_edge(int u,int v){
19     G[u].push_back(v);
20     rG[v].push_back(u);
21 }
22 void dfs(int v){
23     used[v]=true;
24     for(int i=0;i<G[v].size();i++)
25         if(!used[G[v][i]]) dfs(G[v][i]);
26     vs.push_back(v);
27 }
28 void rdfs(int v,int k){
29     used[v]=true;
30     Belong[v]=k;
31     for(int i=0;i<rG[v].size();i++)
32         if(!used[rG[v][i]]) rdfs(rG[v][i],k);
33 }
34 int scc(){
35     memset(used,0,sizeof(used));
36     vs.clear();
37     for(int v=1;v<=V;v++){//from 1 to V
38         if(!used[v]) dfs(v);
39     }
40     int k=0;
41     memset(used,0,sizeof(used));
42     for(int i=vs.size()-1;i>=0;i--){
43         if(!used[vs[i]]) rdfs(vs[i],k++);
44     }
45     return k;
46 }
47 bool judge(){
48     for(int i=1;i<V;i+=2){
49         if(Belong[i]==Belong[i+1])return false;
50     }
51     return true;
52 }
53 int main(){
54     int n,m;
55     while(scanf("%d%d",&n,&m)!=EOF){
56         init(2*n);
57         for(int i=1;i<=m;i++){
58             int a1,a2,c1,c2;
59             scanf("%d%d%d%d",&a1,&a2,&c1,&c2);
60             add_edge(((a1*2+c1))+1,((a2*2+c2)^1)+1);
61             add_edge(((a2*2+c2))+1,((a1*2+c1)^1)+1);
62         }
63         scc();
64         printf("%s\n",judge()?"YES":"NO");
65     }
66     return 0;
67 }

```

## 4.7 Euler Path

- :

- : ( )
- :
- : ( , ),
- G
  - G
  - G ( ) 0 2.
- G
  - G
  - G
- G
  - G
  - u 1,v 1, (u ,v )
- G
  - G
  - G

#### 4.7.1 Fleury

```

,
1  const int maxn = "Edit";
2  int G[maxn][maxn];
3  int deg[maxn][maxn];
4  vector<int> Ans;
5  inline void init() { clr(G, 0), clr(deg, 0); }
6  inline void AddEdge(int u, int v) { deg[u]++, deg[v]++, G[u][v]++, G[v][u]++; }
7  void Fleury(int s)
8  {
9      for (int i = 0; i < n; i++)
10         if (G[s][i])
11             {
12                 G[s][i]--, G[i][s]--;
13                 Fleury(i);
14             }
15     Ans.pb(s);
16 }

```

## 4.8 Bipartite Graph Matching

- 1.
2.  $|G| = N \times N$  , , , ;  
 ( , ); , .  
 :  
 (a) ;  
 (b)  $p_1, p_2, \dots, p_k, p_1, p_k, , p_1, p_2, \dots, p_k$  .  
 ,  $G$  .  
 :  $|G| =$  ;  
 3.  $=$  -  
 :

## 4.8.1 Hungry(Matrix)

 $O(VE)$ .

0

```

1  const int maxn = "Edit";
2  int uN, vN;           //uN, vN
3  int g[maxn][maxn];    // g[i][j] i->j
4  int linker[maxn];
5  bool used[maxn];
6  bool dfs(int u)
7  {
8      for (int v = 0; v < vN; v++)
9          if (g[u][v] && !used[v])
10             {
11                 used[v] = true;
12                 if (linker[v] == -1 || dfs(linker[v]))
13                     {
14                         linker[v] = u;
15                         return true;
16                     }
17             }
18     return false;
19 }
20 int hungary()
21 {
22     int res = 0;
23     clr(linker, -1);
24     for (int u = 0; u < uN; u++)
25     {
26         clr(used, 0);
27         if (dfs(u)) res++;
28     }
29     return res;
30 }

```

## 4.8.2 Hungry(List)

init()

addege(u,v)

```

1  const int maxn = "Edit";
2  int n;
3  vector<int> G[maxn];
4  int linker[maxn];
5  bool used[maxn];
6  inline void init(int n)
7  {
8      for (int i = 0; i < n; i++) G[i].clear();
9  }
10 inline void addege(int u, int v) { G[u].pb(v); }
11 bool dfs(int u)
12 {
13     for (auto& v : G[u])
14     {
15         if (!used[v])
16         {
17             used[v] = true;

```

```

18         if (linker[v] == -1 || dfs(linker[v]))
19         {
20             linker[v] = u;
21             return true;
22         }
23     }
24 }
25 return false;
26 }
27 int hungary()
28 {
29     int ans = 0;
30     clr(linker, -1);
31     for (int u = 0; u < n; v++)
32     {
33         clr(used, 0);
34         if (dfs(u)) ans++;
35     }
36     return ans;
37 }

```

#### 4.8.3 Hopcroft-Carp

$$O(\sqrt{n} * E)$$

$uN$ , ( 0 )

```

1  const int maxn = "Edit";
2  vector<int> G[maxn];
3  int uN;
4  int Mx[maxn], My[maxn];
5  int dx[maxn], dy[maxn];
6  int dis;
7  bool used[maxn];
8  inline void init(int n)
9  {
10     for (int i = 0; i < n; i++) G[i].clear();
11 }
12 inline void addedge(int u, int v) { G[u].pb(v); }
13 bool bfs()
14 {
15     queue<int> q;
16     dis = INF;
17     clr(dx, -1), clr(dy, -1);
18     for (int i = 0; i < uN; i++)
19         if (Mx[i] == -1)
20             q.push(i), dx[i] = 0;
21     while (!q.empty())
22     {
23         int u = q.front();
24         q.pop();
25         if (dx[u] > dis) break;
26         for (auto& v : G[u])
27         {
28             if (dy[v] == -1)
29             {
30                 dy[v] = dx[u] + 1;
31                 if (My[v] == -1)
32                     dis = dy[v];

```

```

33         else
34         {
35             dx[My[v]] = dy[v] + 1;
36             q.push(My[v]);
37         }
38     }
39 }
40 }
41 return dis != INF;
42 }
43 bool dfs(int u)
44 {
45     for (auto& v : G[u])
46     {
47         if (!used[v] && dy[v] == dx[u] + 1)
48         {
49             used[v] = true;
50             if (My[v] != -1 && dy[v] == dis) continue;
51             if (My[v] == -1 || dfs(My[v]))
52             {
53                 My[v] = u, Mx[u] = v;
54                 return true;
55             }
56         }
57     }
58     return false;
59 }
60 int MaxMatch()
61 {
62     int res = 0;
63     clr(Mx, -1), clr(My, -1);
64     while (bfs())
65     {
66         clr(used, false);
67         for (int i = 0; i < uN; i++)
68             if (Mx[i] == -1 && dfs(i)) res++;
69     }
70     return res;
71 }

```

#### 4.8.4 Hungry(Multiple)

```

1  const int maxn = "Edit";
2  const int maxm = "Edit";
3  int uN, vN; //u,v ,
4  int g[maxn][maxm]; //
5  int linker[maxm][maxn];
6  bool used[maxm];
7  int num[maxm]; //
8  bool dfs(int u)
9  {
10     for (int v = 0; v < vN; v++)
11         if (g[u][v] && !used[v])
12         {
13             used[v] = true;
14             if (linker[v][0] < num[v])
15             {
16                 linker[v][++linker[v][0]] = u;

```

```

17         return true;
18     }
19     for (int i = 1; i <= num[0]; i++)
20         if (dfs(linker[v][i]))
21         {
22             linker[v][i] = u;
23             return true;
24         }
25     }
26     return false;
27 }
28 int hungary()
29 {
30     int res = 0;
31     for (int i = 0; i < vN; i++) linker[i][0] = 0;
32     for (int u = 0; u < uN; u++)
33     {
34         clr(used, 0);
35         if (dfs(u)) res++;
36     }
37     return res;
38 }

```

#### 4.8.5 Kuhn-Munkres

```

1  const int maxn = "Edit";
2  int nx, ny; //
3  int g[maxn][maxn]; //
4  int linker[maxn], lx[maxn], ly[maxn]; //y ,x,y
5  int slack[N];
6  bool visx[N], visy[N];
7  bool dfs(int x)
8  {
9      visx[x] = true;
10     for (int y = 0; y < ny; y++)
11     {
12         if (visy[y]) continue;
13         int tmp = lx[x] + ly[y] - g[x][y];
14         if (tmp == 0)
15         {
16             visy[y] = true;
17             if (linker[y] == -1 || dfs(linker[y]))
18             {
19                 linker[y] = x;
20                 return true;
21             }
22         }
23         else if (slack[y] > tmp)
24             slack[y] = tmp;
25     }
26     return false;
27 }
28 int KM()
29 {
30     clr(linker, -1), clr(ly, 0);
31     for (int i = 0; i < nx; i++)
32     {
33         lx[i] = -INF;

```

60 }

6 };

$$k \quad , \quad k$$



$u \rightarrow v, 1, -w$   $i \rightarrow i + 1, k, 0$ ,  
 $G()$ ,  
 $s, t, s, S - \{s\}$

#### 4.9.1 EdmondKarp

```

1  const int maxn = "Edit";
2  struct EdmondsKarp // O(v*E*E)
3  {
4      int n, m;
5      vector<Edge> edges; //
6      vector<int> G[maxn]; // ,G[i][j] i j e
7      int a[maxn]; // i
8      int p[maxn]; // p
9      void init(int n)
10     {
11         for (int i = 0; i < n; i++) G[i].clear();
12         edges.clear();
13     }
14     void AddEdge(int from, int to, int cap)
15     {
16         edges.pb(Edge(from, to, cap, 0));
17         edges.pb(Edge(to, from, 0, 0)); //
18         m = edges.size();
19         G[from].pb(m - 2);
20         G[to].pb(m - 1);
21     }
22     int Maxflow(int s, int t)
23     {
24         int flow = 0;
25         for (;;)
26         {
27             clr(a, 0);
28             queue<int> q;
29             q.push(s);
30             a[s] = INF;
31             while (!q.empty())
32             {
33                 int x = q.front();
34                 q.pop();
35                 for (int i = 0; i < G[x].size(); i++)
36                 {
37                     Edge& e = edges[G[x][i]];
38                     if (!a[e.to] && e.cap > e.flow)
39                     {
40                         p[e.to] = G[x][i];
41                         a[e.to] = min(a[x], e.cap - e.flow);
42                         q.push(e.to);
43                     }
44                 }
45                 if (a[t]) break;
46             }
47             if (!a[t]) break;
48             for (int u = t; u != s; u = edges[p[u]].from)
49             {
50                 edges[p[u]].flow += a[t];
51                 edges[p[u] ^ 1].flow -= a[t];

```

```

52         }
53         flow += a[t];
54     }
55     return flow;
56 }
57 };

```

#### 4.9.2 Dinic

```

1  const int maxn = "Edit";
2  struct Dinic
3  {
4      int n, m, s, t;          // , ( ),
5      vector<Edge> edges;      // edge[e] edge[e^1]
6      vector<int> G[maxn];    // ,G[i][j] i j e
7      bool vis[maxn];         //BFS
8      int d[maxn];            // i
9      int cur[maxn];          //
10     void init(int n)
11     {
12         this->n = n;
13         for (int i = 0; i < n; i++) G[i].clear();
14         edges.clear();
15     }
16     void AddEdge(int from, int to, int cap)
17     {
18         edges.pb(Edge(from, to, cap, 0));
19         edges.pb(Edge(to, from, 0, 0));
20         m = edges.size();
21         G[from].pb(m - 2);
22         G[to].pb(m - 1);
23     }
24     bool BFS()
25     {
26         clr(vis, 0);
27         clr(d, 0);
28         queue<int> q;
29         q.push(s);
30         d[s] = 0;
31         vis[s] = 1;
32         while (!q.empty())
33         {
34             int x = q.front();
35             q.pop();
36             for (int i = 0; i < G[x].size(); i++)
37             {
38                 Edge& e = edges[G[x][i]];
39                 if (!vis[e.to] && e.cap > e.flow)
40                 {
41                     vis[e.to] = 1;
42                     d[e.to] = d[x] + 1;
43                     q.push(e.to);
44                 }
45             }
46         }
47         return vis[t];
48     }
49     int DFS(int x, int a)

```

```

50     {
51         if (x == t || a == 0) return a;
52         int flow = 0, f;
53         for (int& i = cur[x]; i < G[x].size(); i++)
54         {
55             //
56             Edge& e = edges[G[x][i]];
57             if (d[x] + 1 == d[e.to] && (f = DFS(e.to, min(a, e.cap - e.flow))) > 0)
58             {
59                 e.flow += f;
60                 edges[G[x][i] ^ 1].flow -= f;
61                 flow += f;
62                 a -= f;
63                 if (a == 0) break;
64             }
65         }
66         return flow;
67     }
68     int Maxflow(int s, int t)
69     {
70         this->s = s;
71         this->t = t;
72         int flow = 0;
73         while (BFS())
74         {
75             clr(cur, 0);
76             flow += DFS(s, INF);
77         }
78         return flow;
79     }
80 };

```

#### 4.9.3 ISAP

```

1  const int maxn = "Edit";
2  struct ISAP
3  {
4      int n, m, s, t;          // , ( ),
5      vector<Edge> edges;      // edges[e] edges[e^1]
6      vector<int> G[maxn];    // ,G[i][j] i j e
7      bool vis[maxn];         //BFS
8      int d[maxn];            // i
9      int cur[maxn];          //
10     int p[maxn];             //
11     int num[maxn];           //
12     void init(int n)
13     {
14         this->n = n;
15         for (int i = 0; i < n; i++) G[i].clear();
16         edges.clear();
17     }
18     void AddEdge(int from, int to, int cap)
19     {
20         edges.pb(Edge(from, to, cap, 0));
21         edges.pb(Edge(to, from, 0, 0));
22         int m = edges.size();
23         G[from].pb(m - 2);
24         G[to].pb(m - 1);

```

```

25     }
26     int Augument()
27     {
28         int x = t, a = INF;
29         while (x != s)
30         {
31             Edge& e = edges[p[x]];
32             a = min(a, e.cap - e.flow);
33             x = edges[p[x]].from;
34         }
35         x = t;
36         while (x != s)
37         {
38             edges[p[x]].flow += a;
39             edges[p[x] ^ 1].flow -= a;
40             x = edges[p[x]].from;
41         }
42         return a;
43     }
44     void BFS()
45     {
46         clr(vis, 0);
47         clr(d, 0);
48         queue<int> q;
49         q.push(t);
50         d[t] = 0;
51         vis[t] = 1;
52         while (!q.empty())
53         {
54             int x = q.front();
55             q.pop();
56             int len = G[x].size();
57             for (int i = 0; i < len; i++)
58             {
59                 Edge& e = edges[G[x][i]];
60                 if (!vis[e.from] && e.cap > e.flow)
61                 {
62                     vis[e.from] = 1;
63                     d[e.from] = d[x] + 1;
64                     q.push(e.from);
65                 }
66             }
67         }
68     }
69     int Maxflow(int s, int t)
70     {
71         this->s = s;
72         this->t = t;
73         int flow = 0;
74         BFS();
75         clr(num, 0);
76         for (int i = 0; i < n; i++)
77             if (d[i] < INF) num[d[i]]++;
78         int x = s;
79         clr(cur, 0);
80         while (d[s] < n)
81         {
82             if (x == t)
83                 {

```

```

84         flow += Augument();
85         x = s;
86     }
87     int ok = 0;
88     for (int i = cur[x]; i < G[x].size(); i++)
89     {
90         Edge& e = edges[G[x][i]];
91         if (e.cap > e.flow && d[x] == d[e.to] + 1)
92         {
93             ok = 1;
94             p[e.to] = G[x][i];
95             cur[x] = i;
96             x = e.to;
97             break;
98         }
99     }
100     if (!ok) //Retreat
101     {
102         int m = n - 1;
103         for (int i = 0; i < G[x].size(); i++)
104         {
105             Edge& e = edges[G[x][i]];
106             if (e.cap > e.flow) m = min(m, d[e.to]);
107         }
108         if (--num[d[x]] == 0) break; //gap
109         num[d[x] = m + 1]++;
110         cur[x] = 0;
111         if (x != s) x = edges[p[x]].from;
112     }
113 }
114 return flow;
115 }
116 };

```

#### 4.9.4 MinCost MaxFlow

```

1  const int maxn = "Edit";
2  struct MCMF
3  {
4      int n, m;
5      vector<Edge> edges;
6      vector<int> G[maxn];
7      int inq[maxn]; //
8      int d[maxn]; //bellmanford
9      int p[maxn]; //
10     int a[maxn]; //
11     void init(int n)
12     {
13         this->n = n;
14         for (int i = 0; i < n; i++) G[i].clear();
15         edges.clear();
16     }
17     void AddEdge(int from, int to, int cap, int cost)
18     {
19         edges.pb(Edge(from, to, cap, 0, cost));
20         edges.pb(Edge(to, from, 0, 0, -cost));
21         m = edges.size();
22         G[from].pb(m - 2);

```

```

23     G[to].pb(m - 1);
24 }
25 bool BellmanFord(int s, int t, int& flow, ll& cost)
26 {
27     for (int i = 0; i < n; i++) d[i] = INF;
28     clr(inq, 0);
29     d[s] = 0;
30     inq[s] = 1;
31     p[s] = 0;
32     a[s] = INF;
33     queue<int> q;
34     q.push(s);
35     while (!q.empty())
36     {
37         int u = q.front();
38         q.pop();
39         inq[u] = 0;
40         for (int i = 0; i < G[u].size(); i++)
41         {
42             Edge& e = edges[G[u][i]];
43             if (e.cap > e.flow && d[e.to] > d[u] + e.cost)
44             {
45                 d[e.to] = d[u] + e.cost;
46                 p[e.to] = G[u][i];
47                 a[e.to] = min(a[u], e.cap - e.flow);
48                 if (!inq[e.to])
49                 {
50                     q.push(e.to);
51                     inq[e.to] = 1;
52                 }
53             }
54         }
55     }
56     if (d[t] == INF) return false; //
57     flow += a[t];
58     cost += (ll)d[t] * (ll)a[t];
59     for (int u = t; u != s; u = edges[p[u]].from)
60     {
61         edges[p[u]].flow += a[t];
62         edges[p[u] ^ 1].flow -= a[t];
63     }
64     return true;
65 }
66 int MincostMaxflow(int s, int t, ll& cost)
67 {
68     int flow = 0;
69     cost = 0;
70     while (BellmanFord(s, t, flow, cost));
71     return flow;
72 }
73 };

```

## 5 Computational Geometry

### 5.1 Basic Function

```

1  #define zero(x) ((fabs(x) < eps ? 1 : 0))
2  #define sgn(x) (fabs(x) < eps ? 0 : ((x) < 0 ? -1 : 1))
3
4  struct point
5  {
6      double x, y;
7      point(double a = 0, double b = 0) { x = a, y = b; }
8      point operator-(const point& b) const { return point(x - b.x, y - b.y); }
9      point operator+(const point& b) const { return point(x + b.x, y + b.y); }
10     //
11     bool operator==(point& b) { return zero(x - b.x) && zero(y - b.y); }
12     // ( )
13     double operator*(const point& b) const { return x * b.x + y * b.y; }
14     // ( )
15     double operator^(const point& b) const { return x * b.y - y * b.x; }
16     // P a
17     point rotate(point b, double a)
18     {
19         double dx, dy;
20         (*this - b).split(dx, dy);
21         double tx = dx * cos(a) - dy * sin(a);
22         double ty = dx * sin(a) + dy * cos(a);
23         return point(tx, ty) + b;
24     }
25     // a b
26     void split(double& a, double& b) { a = x, b = y; }
27 };
28 struct line
29 {
30     point s, e;
31     line() {}
32     line(point ss, point ee) { s = ss, e = ee; }
33 };

```

### 5.2 Position

#### 5.2.1 Point-Point

```

1  double dist(point a, point b) { return sqrt((a - b) * (a - b)); }

```

#### 5.2.2 Line-Line

```

1  // <0, *> ; <1, *> ; <2, P> P;
2  pair<int, point> spoint(line l1, line l2)
3  {
4      point res = l1.s;
5      if (sgn((l1.s - l1.e) ^ (l2.s - l2.e)) == 0)
6          return mp(sgn((l1.s - l2.e) ^ (l2.s - l2.e)) != 0, res);
7      double t = ((l1.s - l2.s) ^ (l2.s - l2.e)) / ((l1.s - l1.e) ^ (l2.s - l2.e));
8      res.x += (l1.e.x - l1.s.x) * t;
9      res.y += (l1.e.y - l1.s.y) * t;
10     return mp(2, res);
11 }

```

### 5.2.3 Segment-Segment

```
1 bool segxseg(line l1, line l2)
2 {
3     return
4         max(l1.s.x, l1.e.x) >= min(l2.s.x, l2.e.x) &&
5         max(l2.s.x, l2.e.x) >= min(l1.s.x, l1.e.x) &&
6         max(l1.s.y, l1.e.y) >= min(l2.s.y, l2.e.y) &&
7         max(l2.s.y, l2.e.y) >= min(l1.s.y, l1.e.y) &&
8         sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <= 0 &&
9         sgn((l1.s - l2.e) ^ (l2.s - l2.e)) * sgn((l1.e - l2.e) ^ (l2.s - l2.e)) <= 0;
10 }
```

### 5.2.4 Line-Segment

```
1 //l1 ,l2
2 bool segxline(line l1, line l2)
3 {
4     return sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <=
5         0;
6 }
```

### 5.2.5 Point-Line

```
1 double pointtoline(point p, line l)
2 {
3     point res;
4     double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
5     res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
6     return dist(p, res);
7 }
```

### 5.2.6 Point-Segment

```
1 double pointtosegment(point p, line l)
2 {
3     point res;
4     double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
5     if (t >= 0 && t <= 1)
6         res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
7     else
8         res = dist(p, l.s) < dist(p, l.e) ? l.s : l.e;
9     return dist(p, res);
10 }
```

### 5.2.7 Point on Segment

```
1 bool PointOnSeg(point p, line l)
2 {
3     return
4         sgn((l.s - p) ^ (l.e - p)) == 0 &&
5         sgn((p.x - l.s.x) * (p.x - l.e.x)) <= 0 &&
6         sgn((p.y - l.s.y) * (p.y - l.e.y)) <= 0;
7 }
```



## 5.3 Polygon

### 5.3.1 Area

```

1 double area(point p[], int n)
2 {
3     double res = 0;
4     for (int i = 0; i < n; i++) res += (p[i] ^ p[(i + 1) % n]) / 2;
5     return fabs(res);
6 }

```

### 5.3.2 Point in Convex

```

1 //      ,      (      <0 >0)
2 //      : [0,n)
3 // -1 :
4 // 0 :
5 // 1 :
6 int PointInConvex(point a, point p[], int n)
7 {
8     for (int i = 0; i < n; i++)
9         if (sgn((p[i] - a) ^ (p[(i + 1) % n] - a)) < 0)
10             return -1;
11         else if (PointOnSeg(a, line(p[i], p[(i + 1) % n])))
12             return 0;
13     return 1;
14 }

```

### 5.3.3 Point in Polygon

```

1 //      ,poly[]      3, 0~n-1
2 // -1 :
3 // 0 :
4 // 1 :
5 int PointInPoly(point p, point poly[], int n)
6 {
7     int cnt;
8     line ray, side;
9     cnt = 0;
10    ray.s = p;
11    ray.e.y = p.y;
12    ray.e.x = -1000000000000.0; // -INF,
13    for (int i = 0; i < n; i++)
14    {
15        side.s = poly[i], side.e = poly[(i + 1) % n];
16        if (PointOnSeg(p, side)) return 0;
17        //
18        if (sgn(side.s.y - side.e.y) == 0)
19            continue;
20        if (PointOnSeg(side.s, ray))
21            cnt += (sgn(side.s.y - side.e.y) > 0);
22        else if (PointOnSeg(side.e, ray))
23            cnt += (sgn(side.e.y - side.s.y) > 0);
24        else if (segxseg(ray, side))
25            cnt++;
26    }
27    return cnt % 2 == 1 ? 1 : -1;
28 }

```

### 5.3.4 Judge Convex

```

1 //
2 // 1~n-1
3 bool isconvex(point poly[], int n)
4 {
5     bool s[3];
6     clr(s, 0);
7     for (int i = 0; i < n; i++)
8     {
9         s[sgn((poly[(i + 1) % n] - poly[i]) ^ (poly[(i + 2) % n] - poly[i])) + 1] = 1;
10        if (s[0] && s[2]) return 0;
11    }
12    return 1;
13 }
```

## 5.4 Integer Points

### 5.4.1 On Segment

```

1 int OnSegment(line l) { return __gcd(fabs(l.s.x - l.e.x), fabs(l.s.y - l.e.y)) + 1; }
```

### 5.4.2 On Polygon Edge

```

1 int OnEdge(point p[], int n)
2 {
3     int i, ret = 0;
4     for (i = 0; i < n; i++)
5         ret += __gcd(fabs(p[i].x - p[(i + 1) % n].x), fabs(p[i].y - p[(i + 1) % n].y));
6     return ret;
7 }
```

### 5.4.3 Inside Polygon

```

1 int InSide(point p[], int n)
2 {
3     int i, area = 0;
4     for (i = 0; i < n; i++)
5         area += p[(i + 1) % n].y * (p[i].x - p[(i + 2) % n].x);
6     return (fabs(area) - OnEdge(n, p)) / 2 + 1;
7 }
```

## 5.5 Circle

### 5.5.1 Circumcenter

```

1 point waixin(point a, point b, point c)
2 {
3     double a1 = b.x - a.x, b1 = b.y - a.y, c1 = (a1 * a1 + b1 * b1) / 2;
4     double a2 = c.x - a.x, b2 = c.y - a.y, c2 = (a2 * a2 + b2 * b2) / 2;
5     double d = a1 * b2 - a2 * b1;
6     return point(a.x + (c1 * b2 - c2 * b1) / d, a.y + (a1 * c2 - a2 * c1) / d);
7 }
```

## 6 Dynamic Programming

### 6.1 Subsequence

#### 6.1.1 Max Sum

```

1 // a n,
2 int MaxSeqSum(int a[], int n)
3 {
4     int rt = 0, cur = 0;
5     for (int i = 0; i < n; i++)
6         cur += a[i], rt = max(cur, rt), cur = max(0, cur);
7     return rt;
8 }

```

#### 6.1.2 Longest Increase

```

1 // 1, LIS(), lis[]
2 const int N = "Edit";
3 int len, a[N], b[N], f[N];
4 int Find(int p, int l, int r)
5 {
6     while (l <= r)
7     {
8         int mid = (l + r) >> 1;
9         if (a[p] > b[mid])
10             l = mid + 1;
11         else
12             r = mid - 1;
13     }
14     return f[p] = l;
15 }
16 int LIS(int lis[], int n)
17 {
18     int len = 1;
19     f[1] = 1, b[1] = a[1];
20     for (int i = 2; i <= n; i++)
21     {
22         if (a[i] > b[len])
23             b[++len] = a[i], f[i] = len;
24         else
25             b[Find(i, 1, len)] = a[i];
26     }
27     for (int i = n, t = len; i >= 1 && t >= 1; i--)
28         if (f[i] == t) lis[--t] = a[i];
29     return len;
30 }
31
32 // ( 0 , )
33 int dp[N];
34 int LIS(int a[], int n)
35 {
36     clr(dp, 0x3f);
37     for (int i = 0; i < n; i++) *lower_bound(dp, dp + n, a[i]) = a[i];
38     return lower_bound(dp, dp + n, INF) - dp;
39 }

```

### 6.1.3 Longest Common Increase

```

1 // 1
2 int LCIS(int a[], int b[], int n, int m)
3 {
4     clr(dp, 0);
5     for (int i = 1; i <= n; i++)
6     {
7         int ma = 0;
8         for (int j = 1; j <= m; j++)
9         {
10             dp[i][j] = dp[i - 1][j];
11             if (a[i] > b[j]) ma = max(ma, dp[i - 1][j]);
12             if (a[i] == b[j]) dp[i][j] = ma + 1;
13         }
14     }
15     return *max_element(dp[n] + 1, dp[n] + 1 + m);
16 }

```

## 6.2 Digit Statistics

```

1 int a[20];
2 ll dp[20][state];
3 ll dfs(int pos, /*state*/, bool lead /* */, bool limit /* */)
4 {
5     // , , 0, pos== -1
6     if (pos == -1) return 1;
7     /* 1, , ,
8     pos, */
9     if (!limit && !lead && dp[pos][state] != -1) return dp[pos][state];
10    /* , */
11    int up = limit ? a[pos] : 9; // limit up
12    ll ans = 0;
13    for (int i = 0; i <= up; i++) // , ans
14    {
15        if () ...
16        else if () ...
17        ans += dfs(pos - 1, /* */, lead && i == 0, limit && i == a[pos])
18        //
19        /* i,
20        , state i */
21    }
22    // ,
23    if (!limit && !lead) dp[pos][state] = ans;
24    /* , lead, lead */
25    return ans;
26 }
27 }
28 ll solve(ll x)
29 {
30     int pos = 0;
31     do //
32         a[pos++] = x % 10;
33     while (x /= 10);
34     return dfs(pos - 1 /* */, /* */, true, true);
35     // , 0
36 }

```

## 7 Others

### 7.1 Matrix

#### 7.1.1 Matrix FastPow

```

1 typedef vector<ll> vec;
2 typedef vector<vec> mat;
3 mat mul(mat& A, mat& B)
4 {
5     mat C(A.size(), vec(B[0].size()));
6     for (int i = 0; i < A.size(); i++)
7         for (int k = 0; k < B.size(); k++)
8             if (A[i][k]) //
9                 for (int j = 0; j < B[0].size(); j++)
10                    C[i][j] = (C[i][j] + A[i][k] * B[k][j]) % mod;
11     return C;
12 }
13 mat Pow(mat A, ll n)
14 {
15     mat B(A.size(), vec(A.size()));
16     for (int i = 0; i < A.size(); i++) B[i][i] = 1;
17     for (; n; n >>= 1, A = mul(A, A))
18         if (n & 1) B = mul(B, A);
19     return B;
20 }

```

#### 7.1.2 Gauss Elimination

```

1 void gauss()
2 {
3     int now = 1, to;
4     double t;
5     for (int i = 1; i <= n; i++, now++)
6     {
7         /*for (to = now; !a[to][i] && to <= n; to++);
8         //
9         if (to != now)
10             for (int j = 1; j <= n + 1; j++)
11                 swap(a[to][j], a[now][j]);*/
12         t = a[now][i];
13         for (int j = 1; j <= n + 1; j++) a[now][j] /= t;
14         for (int j = 1; j <= n; j++)
15             if (j != now)
16             {
17                 t = a[j][i];
18                 for (int k = 1; k <= n + 1; k++) a[j][k] -= t * a[now][k];
19             }
20     }
21 }

```

### 7.2 Tricks

#### 7.2.1 Stack-Overflow

```

1 //
2 #pragma comment(linker, "/STACK:1024000000,1024000000")

```

### 7.2.2 Fast-Scanner

```

1 //
2 template <class T>
3 inline bool scan_d(T &ret)
4 {
5     char c;
6     int sgn;
7     if (c = getchar(), c == EOF) return 0; //EOF
8     while ((c != '-' && (c < '0' || c > '9'))) c = getchar();
9     sgn = (c == '-') ? -1 : 1;
10    ret = (c == '-') ? 0 : (c - '0');
11    while (c = getchar(), c >= '0' && c <= '9') ret = ret * 10 + (c - '0');
12    ret *= sgn;
13    return 1;
14 }
15 inline void out(int x)
16 {
17     if (x > 9) out(x / 10);
18     putchar(x % 10 + '0');
19 }

```

### 7.2.3 Strok-Sscanf

```

1 //
2 gets(buf);
3 int v;
4 char *p = strtok(buf, " ");
5 while (p)
6 {
7     sscanf(p, "%d", &v);
8     p = strtok(NULL, " ");
9 }

```

## 7.3 Mo Algorithm

, ,  $\sqrt{x}$  ,

```

1 //cf 671 E
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long ll;
5 const int MAXN=1<<20;
6 struct node{
7     int l,r,id;
8 }Q[MAXN];
9 int n,m,k;
10 int block;
11 int a[MAXN];
12 int pre[MAXN];
13 ll cnt[MAXN];
14 ll ANS,ans[MAXN];
15 bool cmp(node x,node y){
16     if(x.l/block==y.l/block)return x.r<y.r;
17     else return x.l/block<y.l/block;
18 }
19 void add(int x){

```

```

20     ANS+=cnt[pre[x]^k];
21     cnt[pre[x]]++;
22 }
23 void del(int x){
24     cnt[pre[x]]--;
25     ANS-=cnt[pre[x]^k];
26 }
27 int main(){
28     scanf("%d%d%d",&n,&m,&k);
29     block=(int)sqrt(n);
30     pre[0]=0;
31     for(int i=1;i<=n;i++){
32         scanf("%d",&a[i]);
33         pre[i]=a[i]^pre[i-1];
34     }
35     for(int i=1;i<=m;i++){
36         scanf("%d%d",&Q[i].l,&Q[i].r);
37         Q[i].id=i;
38     }
39     sort(Q+1,Q+1+m,cmp);
40     ANS=0;
41     memset(cnt,0,sizeof(cnt));
42     cnt[0]=1;
43     int L=1,R=0;
44     for(int i=1;i<=m;i++){
45         while(L>Q[i].l){L--;add(L-1);};
46         while(L<Q[i].l){del(L-1);L++;}
47         while(R<Q[i].r){R++;add(R);};
48         while(R>Q[i].r){del(R);R--};
49         ans[Q[i].id]=ANS;
50     }
51     for(int i=1;i<=m;i++){
52         printf("%lld\n",ans[i]);
53     }
54     return 0;
55 }

```

## 7.4 BigNum

### 7.4.1 High-precision

```

1  //
2  struct bint
3  {
4      int l;
5      short int w[100];
6      bint(int x = 0)
7      {
8          l = x == 0, clr(w, 0);
9          while (x) w[l++] = x % 10, x /= 10;
10     }
11     bool operator<(const bint& x) const
12     {
13         if (l != x.l) return l < x.l;
14         int i = l - 1;
15         while (i >= 0 && w[i] == x.w[i]) i--;
16         return (i >= 0 && w[i] < x.w[i]);
17     }
18     bint operator+(const bint& x) const

```

```

19     {
20         bint ans;
21         ans.l = l > x.l ? l : x.l;
22         for (int i = 0; i < ans.l; i++)
23         {
24             ans.w[i] += w[i] + x.w[i];
25             ans.w[i + 1] += ans.w[i] / 10;
26             ans.w[i] = ans.w[i] % 10;
27         }
28         if (ans.w[ans.l] != 0) ans.l++;
29         return ans;
30     }
31     bint operator*(const bint& x) const
32     {
33         bint res;
34         int up, tmp;
35         for (int i = 0; i < l; i++)
36         {
37             up = 0;
38             for (int j = 0; j < x.l; j++)
39             {
40                 tmp = w[i] * x.w[j] + res.w[i + j] + up;
41                 res.w[i + j] = tmp % 10;
42                 up = tmp / 10;
43             }
44             if (up != 0) res.w[i + x.l] = up;
45         }
46         res.l = l + x.l;
47         while (res.w[res.l - 1] == 0 && res.l > 1) res.l--;
48         return res;
49     }
50     void print()
51     {
52         for (int i = l - 1; ~i; i--) printf("%d", w[i]);
53         puts("");
54     }
55 };

```

#### 7.4.2 Complete High-precision

```

1  #define N 10000
2  class bint
3  {
4  private:
5      int a[N]; // N
6      int len; //
7  public:
8      //
9      bint() { len = 1, clr(a, 0); }
10     // int -> bint
11     bint(int n)
12     {
13         len = 0;
14         clr(a, 0);
15         int d = n;
16         while (n)
17             d = n / 10 * 10, a[len++] = n - d, n = d / 10;
18     }

```



```

19 // char[] -> int
20 bint(const char s[])
21 {
22     clr(a, 0);
23     len = 0;
24     int l = strlen(s);
25     for (int i = l - 1; ~i; i--) a[len++] = s[i];
26 }
27 //
28 bint(const bint& b)
29 {
30     clr(a, 0);
31     len = b.len;
32     for (int i = 0; i < len; i++) a[i] = b.a[i];
33 }
34 // bint = bint
35 bint& operator=(const bint& n)
36 {
37     len = n.len;
38     for (int i = 0; i < len; i++) a[i] = n.a[i];
39     return *this;
40 }
41 // bint + bint
42 bint operator+(const bint& b) const
43 {
44     bint t(*this);
45     int res = b.len > len ? b.len : len;
46     for (int i = 0; i < res; i++)
47     {
48         t.a[i] += b.a[i];
49         if (t.a[i] >= 10) t.a[i + 1]++, t.a[i] -= 10;
50     }
51     t.len = res + a[res] == 0;
52     return t;
53 }
54 // bint - bint
55 bint operator-(const bint& b) const
56 {
57     bool f = *this > b;
58     bint t1 = f ? *this : b;
59     bint t2 = f ? b : *this;
60     int res = t1.len, j;
61     for (int i = 0; i < res; i++)
62         if (t1.a[i] < t2.a[i])
63         {
64             j = i + 1;
65             while (t1.a[j] == 0) j++;
66             t1.a[j--]--;
67             while (j > i) t1.a[j--] += 9;
68             t1.a[i] += 10 - t1.a[i];
69         }
70     else
71         t1.a[i] -= t2.a[i];
72     t1.len = res;
73     while (t1.a[len - 1] == 0 && t1.len > 1) t1.len--, res--;
74     if (f) t1.a[res - 1] = 0 - t1.a[res - 1];
75     return t1;
76 }
77 // bint * bint

```

```

78  bint operator*(const bint& b) const
79  {
80      bint t;
81      int i, j, up, tmp, tmp1;
82      for (i = 0; i < len; i++)
83      {
84          up = 0;
85          for (j = 0; j < b.len; j++)
86          {
87              tmp = a[i] * b.a[j] + t.a[i + j] + up;
88              if (tmp > 9)
89                  tmp1 = tmp - tmp / 10 * 10, up = tmp / 10, t.a[i + j] = tmp1;
90              else
91                  up = 0, t.a[i + j] = tmp;
92          }
93          if (up) t.a[i + j] = up;
94      }
95      t.len = i + j;
96      while (t.a[t.len - 1] == 0 && t.len > 1) t.len--;
97      return t;
98  }
99  //    bint / int
100 bint operator/(const int& b) const
101 {
102     bint t;
103     int down = 0;
104     for (int i = len - 1; ~i; i--)
105         t.a[i] = (a[i] + down * 10) / b, down = a[i] + down * 10 - t.a[i] * b;
106     t.len = len;
107     while (t.a[t.len - 1] == 0 && t.len > 1) t.len--;
108     return t;
109 }
110 //    bint ^ n (n , n)
111 bint operator^(const int n) const
112 {
113     bint t(*this), rt(1);
114     if (n == 0) return 1;
115     if (n == 1) return *this;
116     int m = n;
117     for (; m >= 1; m >>= 1, t = t * t)
118         if (m & 1) rt = rt * t;
119     return rt;
120 }
121 //    bint > bint
122 bool operator>(const bint& b) const
123 {
124     int p;
125     if (len > b.len) return 1;
126     if (len == b.len)
127     {
128         p = len - 1;
129         while (a[p] == b.a[p] && p >= 0) p--;
130         return p >= 0 && a[p] > b.a[p];
131     }
132     return 0;
133 }
134 //    bint > int
135 bool operator>(const int& n) const { return *this > bint(n); }
136 //

```

```

137     void out()
138     {
139         for (int i = len - 1; ~i; i--) printf("%d", a[i]);
140         puts("");
141     }
142 };

```

## 7.5 VIM

```

1  syntax on
2  set cindent
3  set nu
4  set tabstop=4
5  set shiftwidth=4
6  set background=dark
7  set mouse=a
8
9  map<C-A> gg vG"+y
10 map<F5> :call Run()<CR>
11
12 func! Run()
13     exec "w"
14     exec "!g++ -std=c++11 -O2 % -o %<"
15     exec "!time ./%<"
16 endfunc
17
18 autocmd BufNewFile *.cpp 0r ~/include.cpp
19 autocmd BufNewFile *.cpp normal G
20
21 inoremap ( (<Esc>i
22 inoremap [ [<Esc>i
23 inoremap { {<CR><Esc>O
24 inoremap ' '<Esc>i
25 inoremap " "<Esc>i
26
27 inoremap ) <c-r>=ClosePair('')<CR>
28 inoremap ] <c-r>=ClosePair('']<CR>
29
30 func ClosePair(char)
31     if getline('.')[col('.')-1]==a:char
32         return "\<Right>"
33     else
34         return a:char
35     endif
36 endfunc

```