W271 Group Lab 1

Due 11:59pm Pacific Time Sunday February 9 2020

Instructions (Please Read Carefully):

- 20 page limit (strict)
- Do not modify fontsize, margin or line-spacing settings
- One student from each group should submit the lab to their student github repo by the deadline; submission and revisions made after the deadline will not be graded
- Answers should clearly explain your reasoning; do not simply 'output dump' the results of code without explanation
- Submit two files:

  1. A pdf file that details your answers. Include all R code used to produce the answers. Do not suppress the codes in your pdf file
  2. The R markdown (Rmd) file used to produce the pdf file

  The assignment will not be graded unless **both** files are submitted
- Name your files to include all group members names. For example the students' names are Stan Cartman and Kenny Kyle, name your files as follows:
  - `StanCartman_KennyKyle_Lab1.Rmd`
  - `StanCartman_KennyKyle_Lab1.pdf`
- Although it sounds obvious, please write your name on page 1 of your pdf and Rmd files
- All answers should include a detailed narrative; make sure that your audience can easily follow the logic of your analysis. All steps used in modelling must be clearly shown and explained
- For statistical methods that we cover in this course, use the R libraries and functions that are covered in this course. If you use libraries and functions for statistical modeling that we have not covered, you must provide an explanation of why such libraries and functions are used and reference the library documentation. For data wrangling and data visualization, you are free to use other libraries, such as dplyr, ggplot2, etc
- For mathematical formulae, type them in your R markdown file. Do not e.g. write them on a piece of paper, snap a photo, and use the image file
- Incorrectly following submission instructions results in deduction of grades
- Students are expected to act with regard to UC Berkeley Academic Integrity.

Investigation of the 1989 Space Shuttle Challenger Accident

Carefully read the Dalal et al (1989) paper (Skip Section 5).

Part 1 (25 points)

Conduct a thorough EDA of the data set. This should include both graphical and tabular analysis as taught in this course. Output-dump (that is, graphs and tables that don't come with explanations) will result in a very low, if not zero, score. Since the report has a page-limit, you will have to be selective when choosing visuals to illustrate your key points, associated with a concise explanation of the visuals. This EDA should begin with an inspection of the given dataset; examination of anomalies, missing values, potential of top and/or bottom code etc.

```
path = "/Users/jeff/Documents/MIDS/W271/w271_lab1/challenger.csv"

challenger<-read.table(file =  path, header = TRUE, sep = ",") #Import table

knitr::kable(
  challenger[1:5,1:5 ], caption = 'Top 6 Rows of Admissions Dataset')
```

Table 1: Top 6 Rows of Admissions Dataset

Flight	Temp	Pressure	O.ring	Number
1	66	50	0	6
2	70	50	1	6
3	69	50	0	6
4	68	50	0	6
5	67	50	0	6

First, we read in the data set to an object called 'challenger'. From here, we see that we have 5 variables in a dataframe with 23 observations. We review each of the 5 below:

- Flight: Simply akin to counting the row of the dataframe. Will not be integral to the analysis
- Temp: An integer variable that records the takeoff temperature in degrees Fahrenheit
- Pressure: An integer variable measuring the amount of pressure on the O.rings, measured in pounds per square inch
- O.ring: An integer variable that counts the number of O.ring failures on the flight in question
- Number: An integer variable recording the number of O.rings on each flight

```
#output in text format for view in latex
#stargazer(challenger, header= F, title = "Summary Table of Wheat Data")

#output in text format for view in R
stargazer(challenger, header= F, title = "Summary Table of Wheat Data", type='text')
```

```
##
## Summary Table of Wheat Data
## ===========================================================
## Statistic N   Mean   St. Dev. Min Pctl(25) Pctl(75) Max
## -----------------------------------------------------------
```

```
## Flight    23 12.000   6.782    1    6.5      17.5   23
## Temp      23 69.565   7.057   53     67       75    81
## Pressure  23 152.174  68.221  50     75      200    200
## O.ring    23  0.391   0.656   0      0        1      2
## Number    23  6.000   0.000   6      6        6      6
## ----------------------------------------------------------
```

This summary tallies each variable by range, so we get the see quartiles, minimums, maximums, and medians/means. A few conclusions can be drawn:

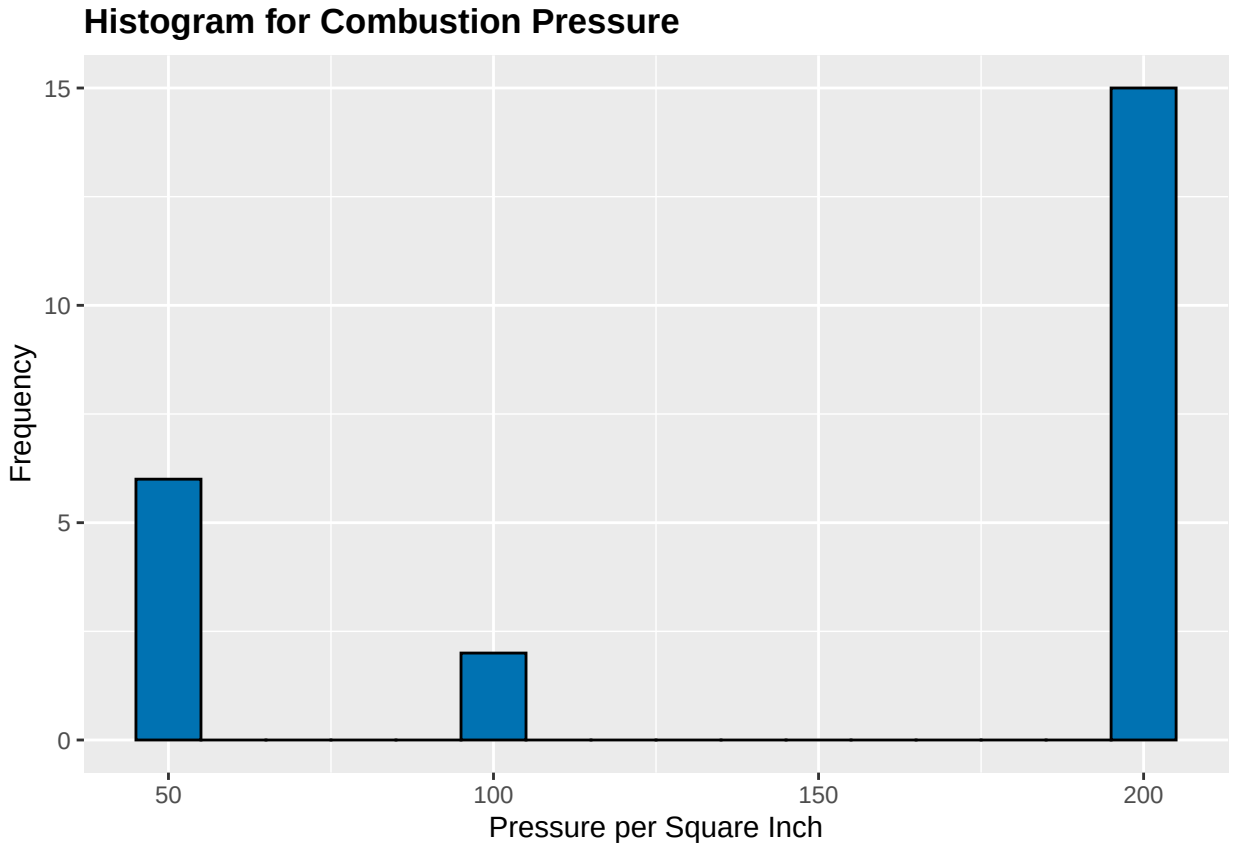
- Temp: Temperature in the dataset ranges between 53 and 81 degrees Fahrenheit
- Pressure: Pressure in the dataset ranges between 50 and 200 pounds per square inch, and appears to occur in increments of 50
- O.ring: It appears that in the dataset, most O.ring failures are 0 (meaning no failure), but there are some flights that did fail, and at least one flight that had as many as two O.ring failures.
- Number: This variable does not appear to be informative to the analysis, because it is 6 for all measurements.

```
sum(is.na(challenger))
```

```
## [1] 0
```

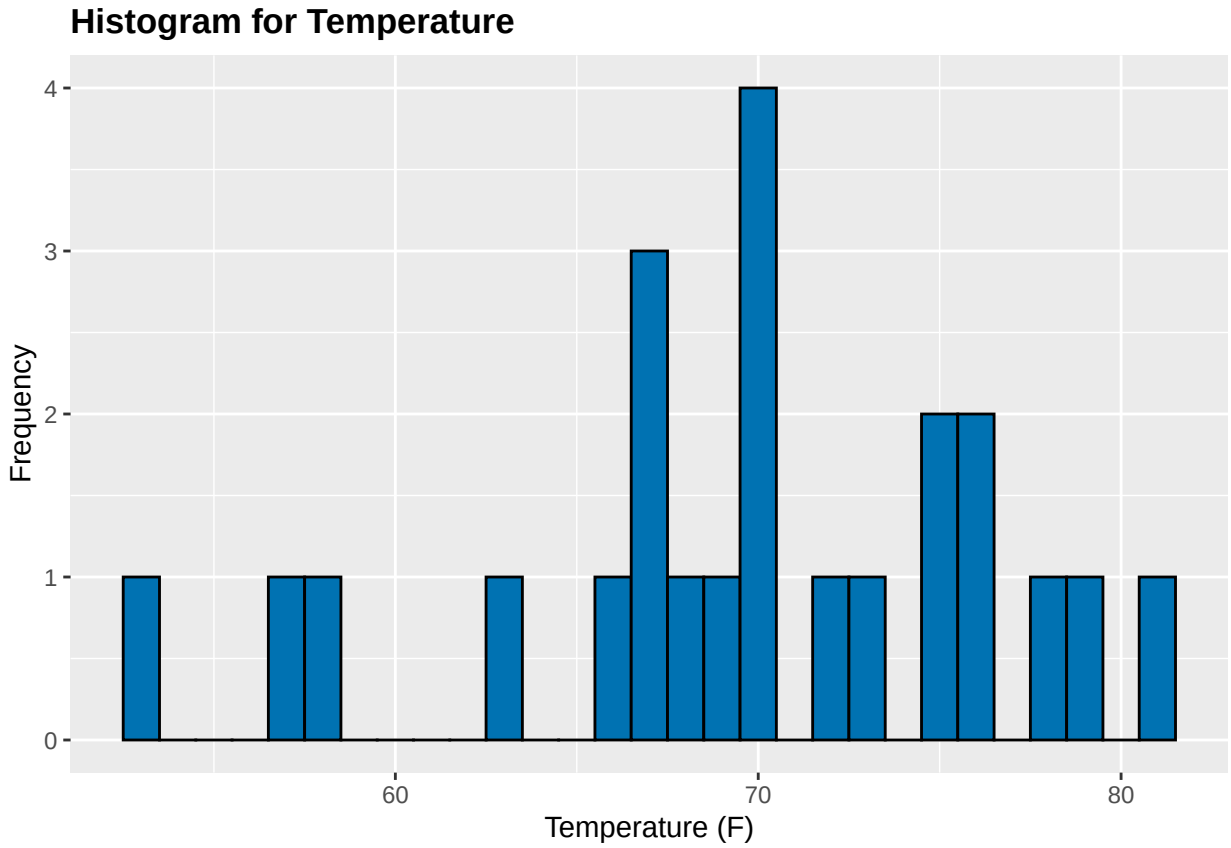
In the cell above, we examine the dataset for missing data, and find none. The EDA we have performed so far indicates that 'Flight' and 'Number' are not informative as explanatory variables. Temperature and Pressure are informative as explanatory variables, while O.ring will be our response variable. It may be worthwhile to transform the O.ring variable into a binary categorical variable such that any values over 0 all register as failures, and any zeros register as non-failures.

```
# Distribution of Pressure
ggplot(challenger, aes(x = Pressure)) +
  geom_histogram(aes(x = Pressure), binwidth = 10, fill="#0072B2", colour="black") +
  ggtitle("Histogram for Combustion Pressure") +
  xlab("Pressure per Square Inch") +
  ylab("Frequency") +
  theme(plot.title = element_text(lineheight=1, face="bold"))
```



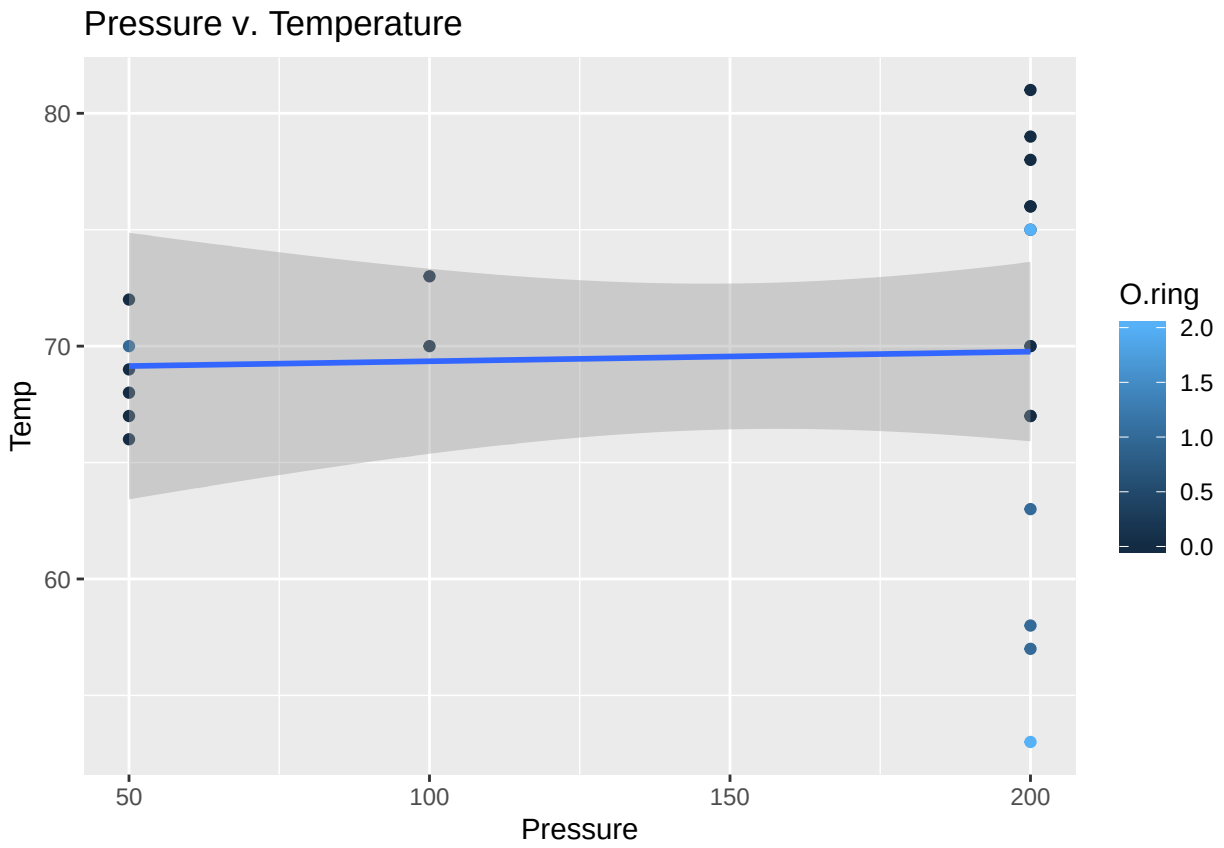
The histogram above confirms our notion that PSI measurements in the dataset occur in regular increments, with most measurements taking place at 200psi, but other measurements also taking place at 50 and 100 psi as well. This is a discrete integer variable.

```
# Distribution of Temp
ggplot(challenger, aes(x = Temp)) +
  geom_histogram(aes(x = Temp), binwidth = 1, fill="#0072B2", colour="black") +
  ggtitle("Histogram for Temperature") +
  xlab("Temperature (F)") +
  ylab("Frequency") +
  theme(plot.title = element_text(lineheight=1, face="bold"))
```



We extend our histogram analysis further, this time examining temperature. The distribution has two major peaks, right around upper 60s, and mid 70s, with only a few observations warmer or cooler than those areas. This is a continuous variable.

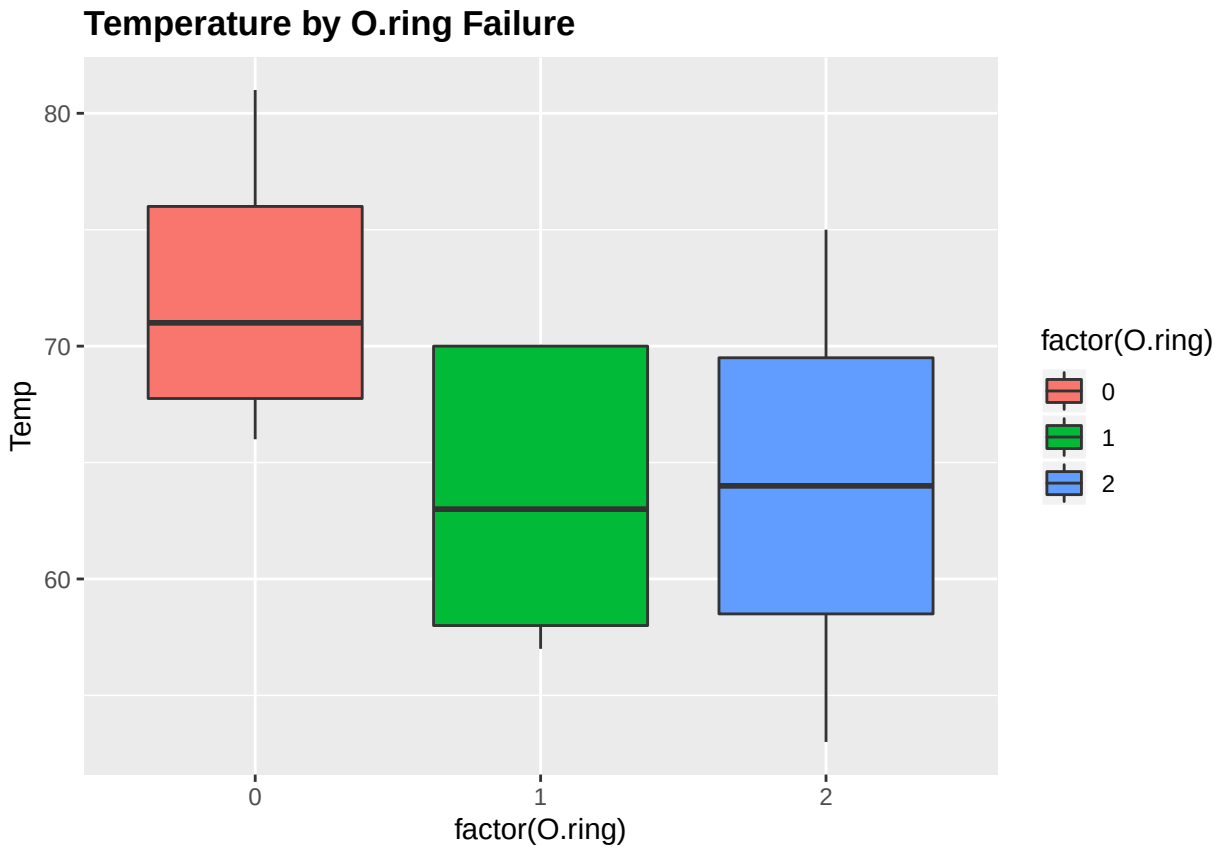
```
ggplot(challenger, aes(x=Pressure, y=Temp, color=O.ring)) + geom_point() +
  geom_smooth(method='lm') +
  ggtitle('Pressure v. Temperature')
```



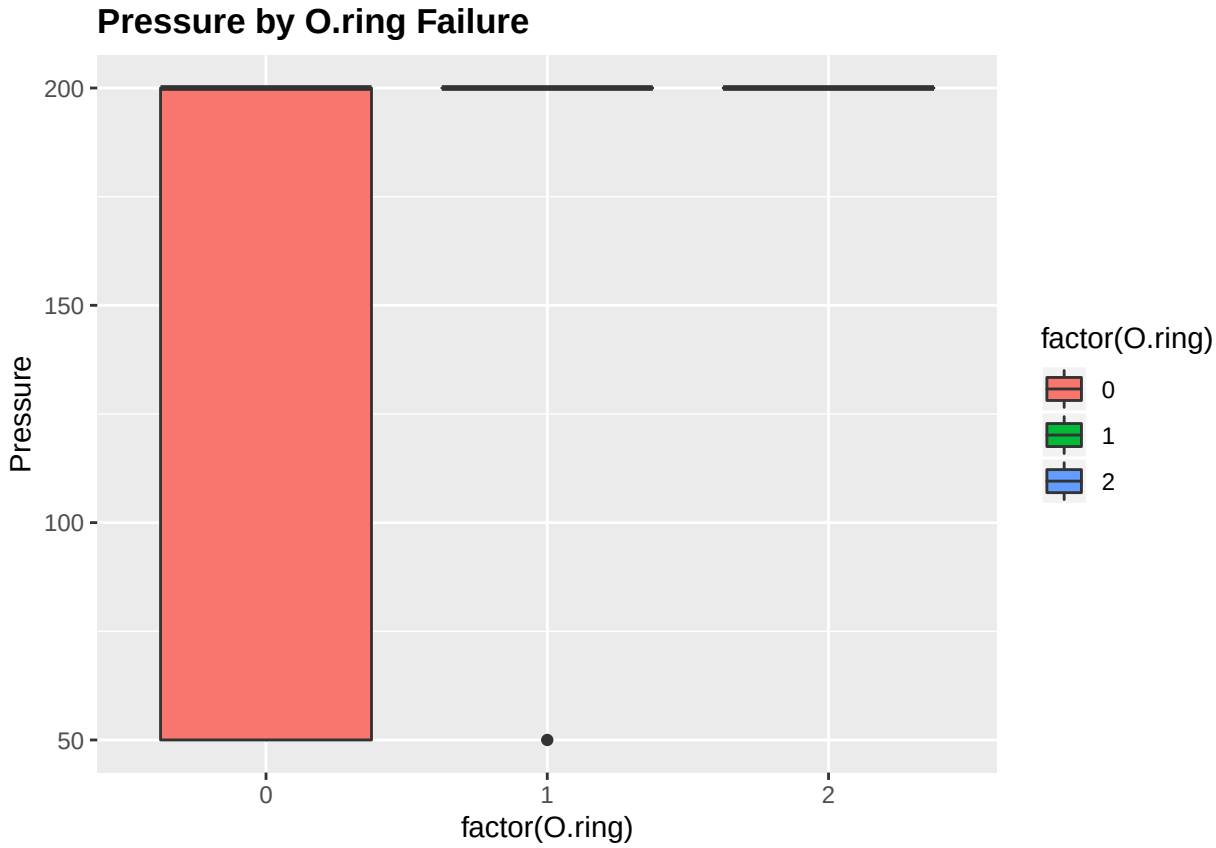
Taking into consideration the two explanatory variables that matter for our analysis, we create a scatterplot involving Temperature and Pressure. In addition to being a simple scatterplot, we include two other features. The first is 'best-fit' line with confidence bands provided by R. This is not terribly informative, because of the fact that Pressure is a discrete variable, so a linear model is of limited use. However, the second feature, in which we color the points according to the value of the response variable, is highly useful.

From this chart, we can draw several conclusions. First, most O-ring failures seem to occur at higher pressures. Second, most O-ring failures seem to occur at lower temperatures, with the only datapoint having two failures occuring at the lowest recorded temperatures.

```
ggplot(challenger, aes(factor(O.ring), Temp)) +
  geom_boxplot(aes(fill = factor(O.ring))) +
  ggtitle("Temperature by O.ring Failure") +
  theme(plot.title = element_text(lineheight=1, face="bold"))
```




```
ggplot(challenger, aes(factor(O.ring), Pressure)) +
  geom_boxplot(aes(fill = factor(O.ring))) +
  ggtitle("Pressure by O.ring Failure") +
  theme(plot.title = element_text(lineheight=1, face="bold"))
```



The two boxplots we examine here further illustrate the distribution of our two key explanatory variables, but using color to bring out O.ring failure. The first boxplot serves to reaffirm our first hypothesis that O.ring failures tend to occur at lower temperatures, while the second boxplot indicates that apart from one outlier at 50psi, O.ring failures all occur at 200psi (higher pressure). Thus, without having constructed any formal models or analytics, an initial view of the data may seem to indicate that lower temperatures and higher pressures lead to O.ring failure.

Part 2 (20 points)

Answer the following from Question 4 of Bilder and Loughin Section 2.4 Exercises (page 129):

(a) The authors use logistic regression to estimate the probability an O-ring will fail. In order to use this model, the authors needed to assume that each O-ring is independent for each launch. Discuss why this assumption is necessary and the potential problems with it. Note that a subsequent analysis helped to alleviate the authors' concerns about independence.

The authors use two types of regression to estimate the probability that an O-ring will fail. The first model that they use is a binomial logistic regression model, which $p(t, s)$ denotes the probability per joint of some thermal distress, t being the temperature and s being the pressure.

We can replicate the first model of the paper using the following R code (see below):

```
challenger_binomial <- read.table(path, header=TRUE, sep=",")
O.ring_binomial <- glm(formula=cbind(O.ring,6-O.ring) ~ Temp + Pressure, family=binomial, data=
summary(O.ring_binomial)
```

```
##
```

```
## Call:
## glm(formula = cbind(O.ring, 6 - O.ring) ~ Temp + Pressure, family = binomial,
##     data = challenger_binomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0361  -0.6434  -0.5308  -0.1625   2.3418
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.520195   3.486784   0.723   0.4698
## Temp        -0.098297   0.044890  -2.190   0.0285 *
## Pressure     0.008484   0.007677   1.105   0.2691
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 24.230  on 22  degrees of freedom
## Residual deviance: 16.546  on 20  degrees of freedom
## AIC: 36.106
##
## Number of Fisher Scoring iterations: 5
```

A key feature of the binomial distribution is that each trial has to be independent. This may be problematic in trying to model out the likelihood of O.ring failure when using the first model. As noted in the paper (p. 947), the failure of the secondary O-ring may be conditional on the performance of the primary O-ring.

The second model for occurence of O-ring incidents, creates a binary response variable for where there is 1 if there is an incident in the flight and 0 otherwise. In this scenario, there is some "information loss" associated with reducing our dependent variable to a binary response, but we do not make the assumption that the independence of each joint is required.

(b) Estimate the logistic regression model using the explanatory variables in a linear form.

```
challenger2 <- read.table(path, header=TRUE, sep=",")
challenger2$O.ring = ifelse(challenger2$O.ring > 0, 1, 0)

O.ring <- glm(formula=O.ring ~ Temp, family=binomial(link="logit"), data=challenger2)
summary(O.ring)
```

```
##
## Call:
## glm(formula = O.ring ~ Temp, family = binomial(link = "logit"),
##     data = challenger2)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0611  -0.7613  -0.3783   0.4524   2.2175
```

```
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  15.0429     7.3786   2.039   0.0415 *
## Temp         -0.2322     0.1082  -2.145   0.0320 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 28.267  on 22  degrees of freedom
## Residual deviance: 20.315  on 21  degrees of freedom
## AIC: 24.315
##
## Number of Fisher Scoring iterations: 5
```

(c) Perform LRTs to judge the importance of the explanatory variables in the model.

```
Anova(O.ring)
```

```
## Analysis of Deviance Table (Type II tests)
##
## Response: O.ring
##      LR Chisq Df Pr(>Chisq)
## Temp    7.952  1   0.004804 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(d) The authors chose to remove Pressure from the model based on the LRTs. Based on your results, discuss why you think this was done. Are there any potential problems with removing this variable?

The authors most likely removed Pressure due to its p-value being above 0.21 (with 0.05 being the generic cutoff for statistical significance). The problem with this is that there could be interactions between temperature and pressure that their models did not consider, and we also have a relatively small sample size to begin with. EDA did seem to show that there might be some sort of relationship.

Part 3 (35 points)

Answer the following from Question 5 of Bilder and Loughin Section 2.4 Exercises (page 129-130):

Continuing Exercise 4, consider the simplified model $logit(\pi) = \beta_0 + \beta_1 Temp$, where π is the probability of an O-ring failure. Complete the following:

(a) Estimate the model.

```
O.ring.temp <- glm(formula= O.ring ~ Temp, family=binomial(link=logit), data=challenger2)
summary(O.ring.temp)
```

```
##
## Call:
## glm(formula = O.ring ~ Temp, family = binomial(link = logit),
##     data = challenger2)
```

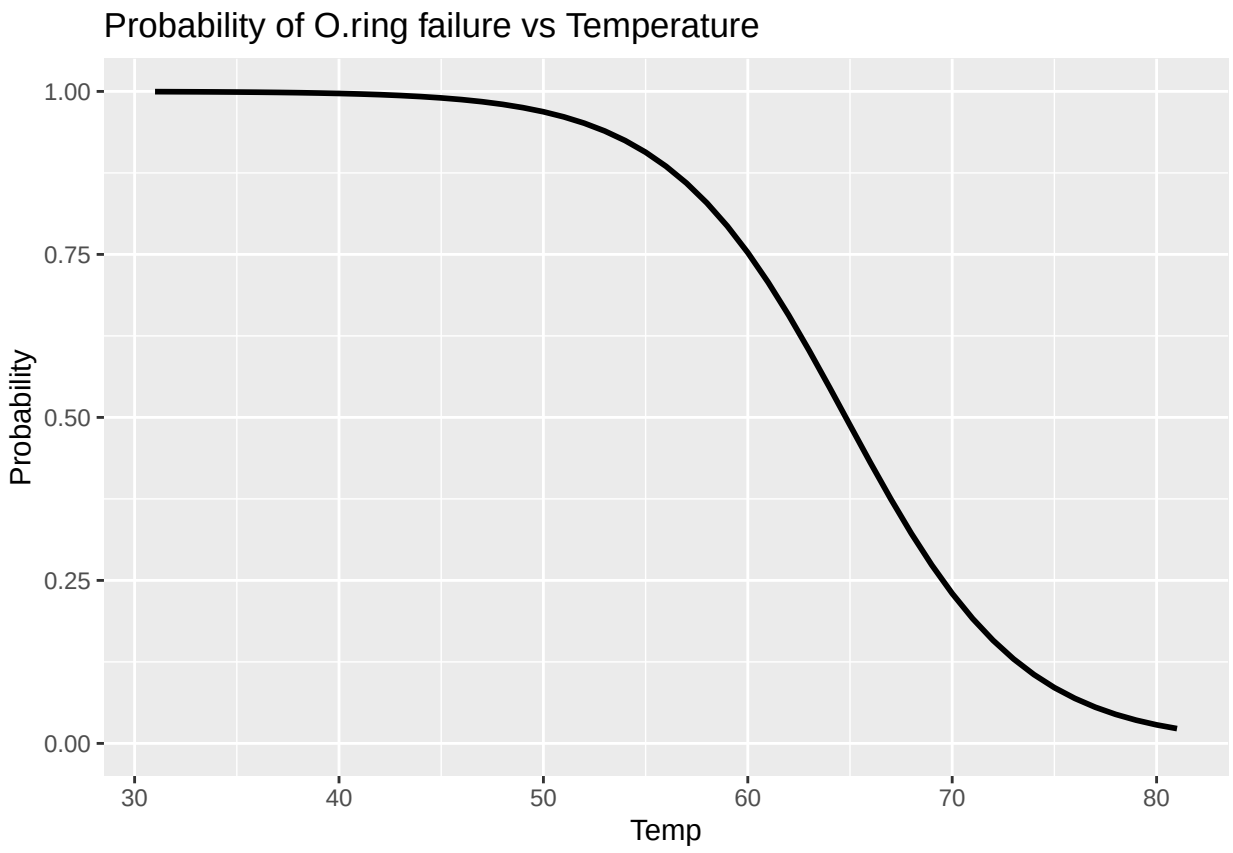
```
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0611  -0.7613  -0.3783   0.4524   2.2175
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  15.0429     7.3786   2.039   0.0415 *
## Temp         -0.2322     0.1082  -2.145   0.0320 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 28.267  on 22  degrees of freedom
## Residual deviance: 20.315  on 21  degrees of freedom
## AIC: 24.315
##
## Number of Fisher Scoring iterations: 5
```

(b) Construct two plots: (1) π vs. Temp and (2) Expected number of failures vs. Temp. Use a temperature range of 31° to 81° on the x-axis even though the minimum temperature in the data set was 53°.

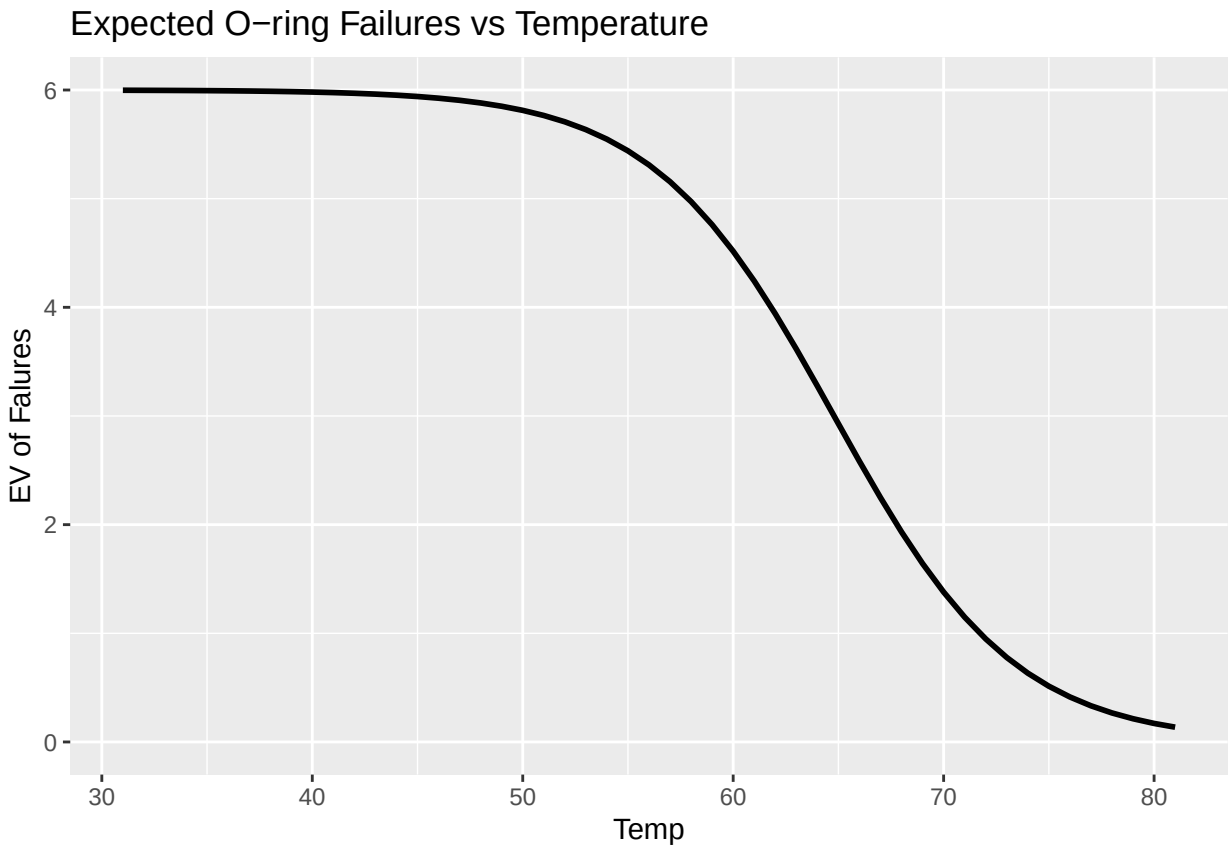
(p.91 and 92 of textbook)

```
###Jeff's code
predict.data <- as.data.frame.table(array(31:81))
predict.data$Temp <- predict.data$Freq

linear.pred<-predict(object = O.ring.temp, newdata = predict.data, type = "link", se = TRUE)
#linear.pred
alpha=.05
CI.lin.pred.lower<-linear.pred$fit - qnorm(p = 1-alpha/2)*linear.pred$se
CI.lin.pred.upper<-linear.pred$fit + qnorm(p = 1-alpha/2)*linear.pred$se
CI.pi.lower<-exp(CI.lin.pred.lower) / (1 + exp(CI.lin.pred.lower))
CI.pi.upper<-exp(CI.lin.pred.upper) / (1 + exp(CI.lin.pred.upper))
CI.pi<-exp(linear.pred$fit) / (1 + exp(linear.pred$fit))

df3b <- bind_cols(temp = array(31:81), pi = CI.pi, lowerpiCI = CI.pi.lower, upperpiCI = CI.pi.u

w <- aggregate(formula = O.ring ~ Temp, data= challenger2, FUN=sum)
n <- aggregate(formula = O.ring ~ Temp, data = challenger2, FUN=length)
w_n <- data.frame(Temperature=w$Temp, success=w$O.ring, trials=n$O.ring, proportion = round(w$O
w_n$combine <- w$O.ring / n$O.ring
#w_n

p = ggplot() +
  #geom_point(data = w.n, aes(x = Temperature, y = w$O.ring / n$O.ring), size=1) +
```

```
  geom_line(data = df3b, aes(x = temp, y = pi), size=1) +
  xlab('Temp') +
  ylab('Probability') +
  ggtitle("Probability of O.ring failure vs Temperature") +
  scale_x_continuous(limits = c(31,81)) +
  scale_y_continuous(limits = c(0, 1))
print(p)
```




```
q = ggplot() +
  geom_line(data = df3b, aes(x = temp, y = pi*6), size=1) +
  xlab('Temp') +
  ylab('EV of Falures') +
  ggtitle("Expected O-ring Failures vs Temperature") +
  scale_x_continuous(limits = c(31,81)) +
  scale_y_continuous(limits = c(0, 6))
print(q)
```

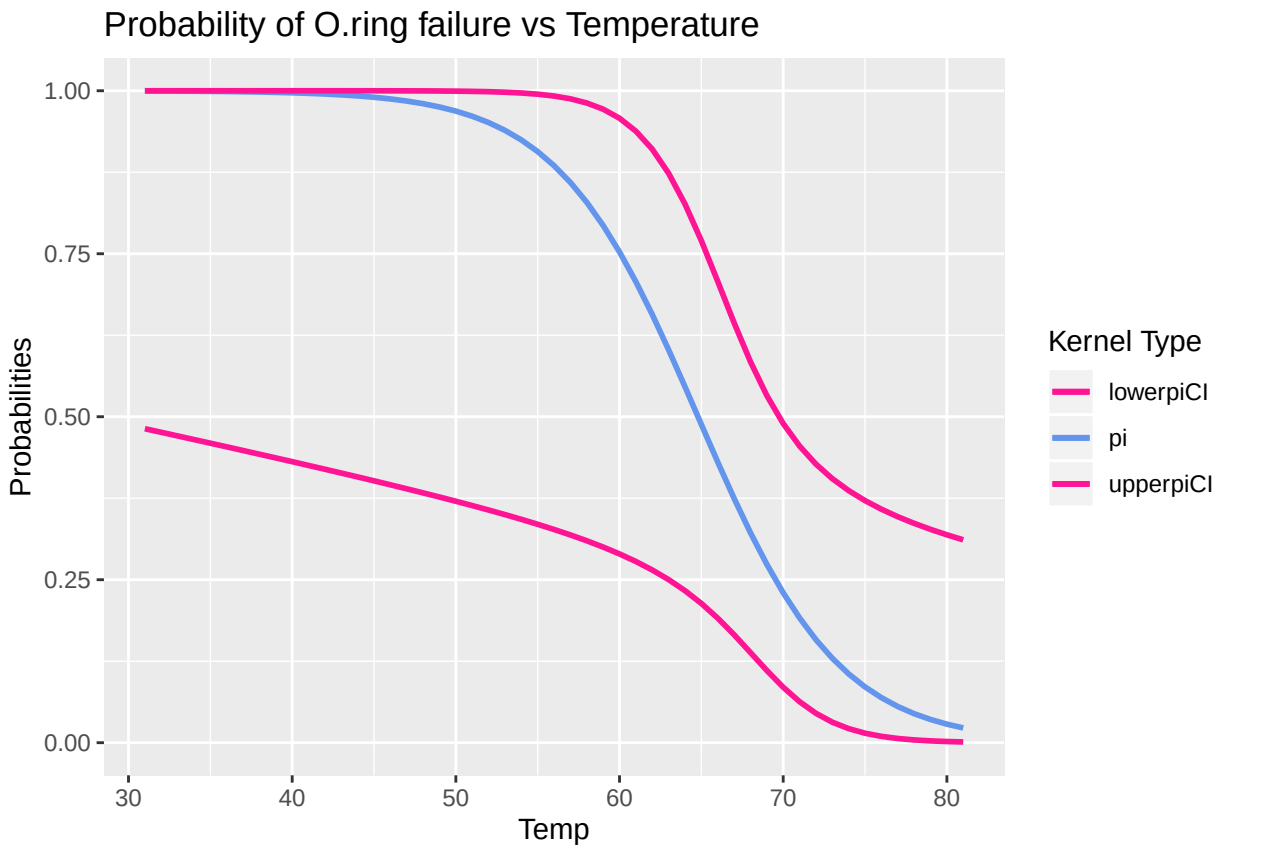



```
#plot(x = w$Temp, y = w$O.ring / n$O.ring, xlab="Temperature (F)", ylab="Estimated Prob", pane
#curve(expr = predict(object=O.ring.temp, newdata=data.frame(Temp = x), type="response"), col=



###Jeff's code
```

(c) Include the 95% Wald confidence interval bands for π on the plot. Why are the bands much wider for lower temperatures than for higher temperatures?

```
p = ggplot() +
  geom_line(data = df3b, aes(x = temp, y = pi, color = "pi"), size=1) +
  geom_line(data = df3b, aes(x = temp, y = lowerpiCI, color = "lowerpiCI"), size=1) +
  geom_line(data = df3b, aes(x = temp, y = upperpiCI, color = "upperpiCI"), size=1) +
  xlab('Temp') +
  ylab('Probabilities') +
  ggtitle("Probability of O.ring failure vs Temperature") +
  scale_x_continuous(limits = c(31,81)) +
  scale_y_continuous(limits = c(0, 1))  +
  scale_color_manual(values = c(
    'pi' = 'cornflowerblue',
    'lowerpiCI' = 'deeppink',
    'upperpiCI' = 'deeppink')
    ) +
```

```
  labs(color = 'Kernel Type')
print(p)
```



(d) The temperature was 31° at launch for the Challenger in 1986. Estimate the probability of an O-ring failure using this temperature, and compute a corresponding confidence interval. Discuss what assumptions need to be made in order to apply the inference procedures.

```
predict.data <- data.frame(Temp = 31)
predict(object=O.ring.temp, newdata=predict.data, type="response")
```

```
##         1
## 0.9996088
```

```
library(package=mcprofile)
K <- matrix(data = c(1,31), nrow=1, ncol=2)
linear.combo <- mcprofile(object=O.ring.temp, CM=K)
ci.logit.profile <- confint(object = linear.combo, level=0.95)
ci.logit.profile
```

```
##
##    mcprofile - Confidence Intervals
##
## level:        0.95
## adjustment:   single-step
##
```

```
##    Estimate lower upper
## C1     7.85  1.41  18.4
```

```
#exp(ci.logit.profile$confint) / (1+exp(ci.logit.profile$confint))
```

(e) Rather than using Wald or profile LR intervals for the probability of failure, Dalal et al. (1989) use a parametric bootstrap to compute intervals. Their process was to (1) simulate a large number of data sets (n = 23 for each) from the estimated model of Temp; (2) estimate new models for each data set, say and (3) compute at a specific temperature of interest. The authors used the 0.05 and 0.95 observed quantiles from the simulated distribution as their 90% confidence interval limits. Using the parametric bootstrap, compute 90% confidence intervals separately at temperatures of 31° and 72°.27

(f) Determine if a quadratic term is needed in the model for the temperature.

```
O.ring.H0 <- glm(formula= O.ring ~ Temp + Pressure, family=binomial(link=logit), data=challenge
O.ring.HA <- glm(formula= O.ring ~ Temp + Pressure + I(Temp^2), family=binomial(link=logit), da
anova(O.ring.H0, O.ring.HA, test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: O.ring ~ Temp + Pressure
## Model 2: O.ring ~ Temp + Pressure + I(Temp^2)
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1        20     18.782
## 2        19     18.549  1  0.23287   0.6294
```

Based on the p-value of 0.63, there does not appear to be a need for a quadratic term for temperature.

Part 4 (10 points)

With the same set of explanatory variables in your final model, estimate a linear regression model. Explain the model results; conduct model diagnostic; and assess the validity of the model assumptions. Would you use the linear regression model or binary logistic regression in this case? Explain why.

```
O.ring.linear <- lm(formula = O.ring ~ Temp + Pressure, data=challenger2)
summary(O.ring.linear)
```

```
##
## Call:
## lm(formula = O.ring ~ Temp + Pressure, data = challenger2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49600 -0.18816 -0.07650  0.08027  0.91267
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.658765   0.824297   3.225  0.00424 **
## Temp        -0.038136   0.011602  -3.287  0.00369 **
## Pressure     0.001962   0.001200   1.635  0.11779
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3837 on 20 degrees of freedom
## Multiple R-squared:  0.3952, Adjusted R-squared:  0.3347
## F-statistic: 6.534 on 2 and 20 DF,  p-value: 0.006549
```

Linearity in parameters holds for this model. Random sampling, the next assumption behind classical linear models, is something that is dependent on the methodology behind constructing the data set. Because we are dealing with space shuttle launches, which occur infrequently, there are likely to not be an over-abundance of shuttle launches, and we may actually be dealing with a dataset that is close to or exactly matches the population of total observations available.

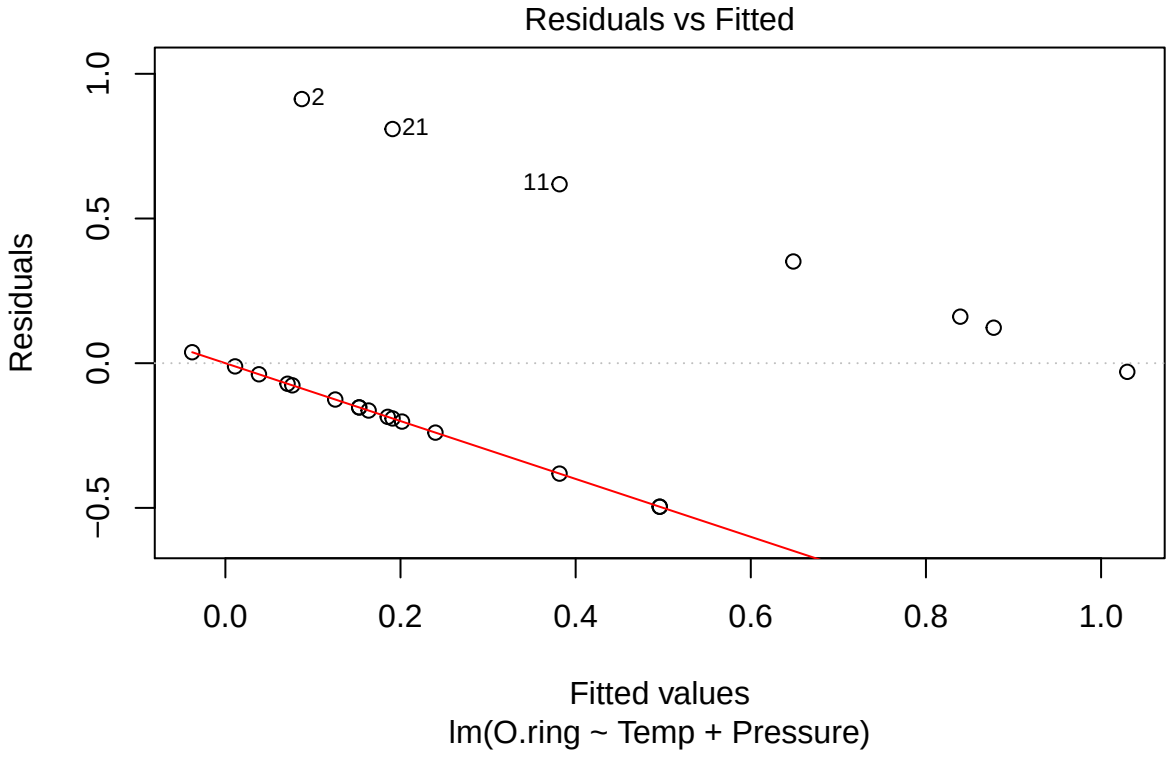
Below, the variance inflation factor shows that none of the values by variable are low, meaning the assumption of no perfect multicollinearity holds.

```
vif(O.ring.linear)
```

```
##     Temp Pressure
## 1.001588 1.001588
```

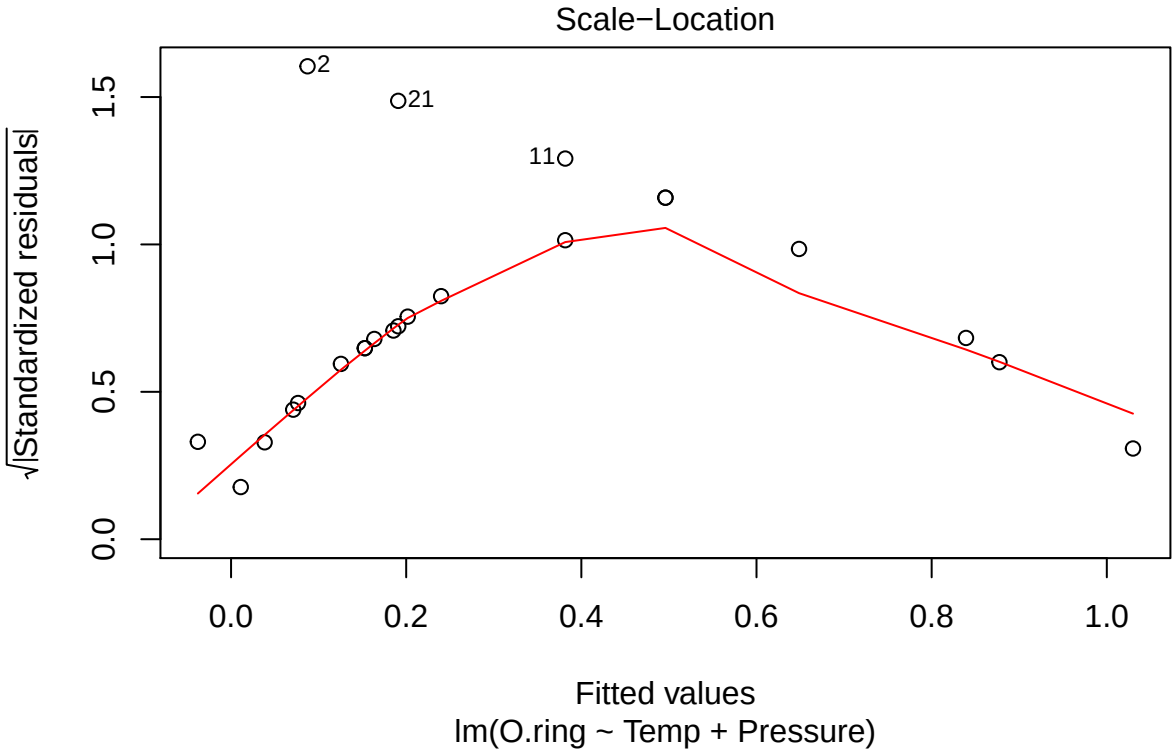
The following chart shows that the assumption of zero conditional mean is violated, because the line strays quite far from zero.

```
plot(O.ring.linear, which=1)
```



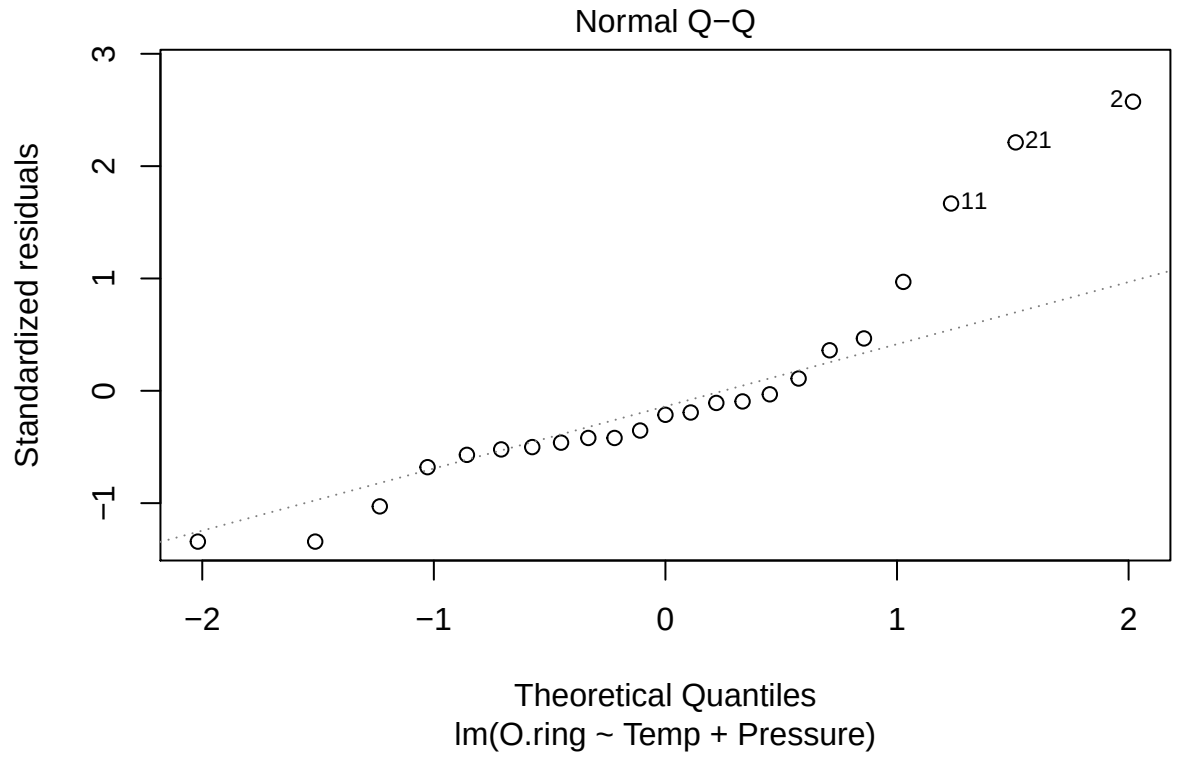
The Scale-Location chart below is not flat which shows this regression violates the assumption of homoskedasticity. Heteroskedasticity does not cause biased estimates in itself, but it does the formulas for standard errors to be inaccurate, which means we would need to use robust standard errors to avoid problems here.

```
plot(O.ring.linear, which = 3)
```

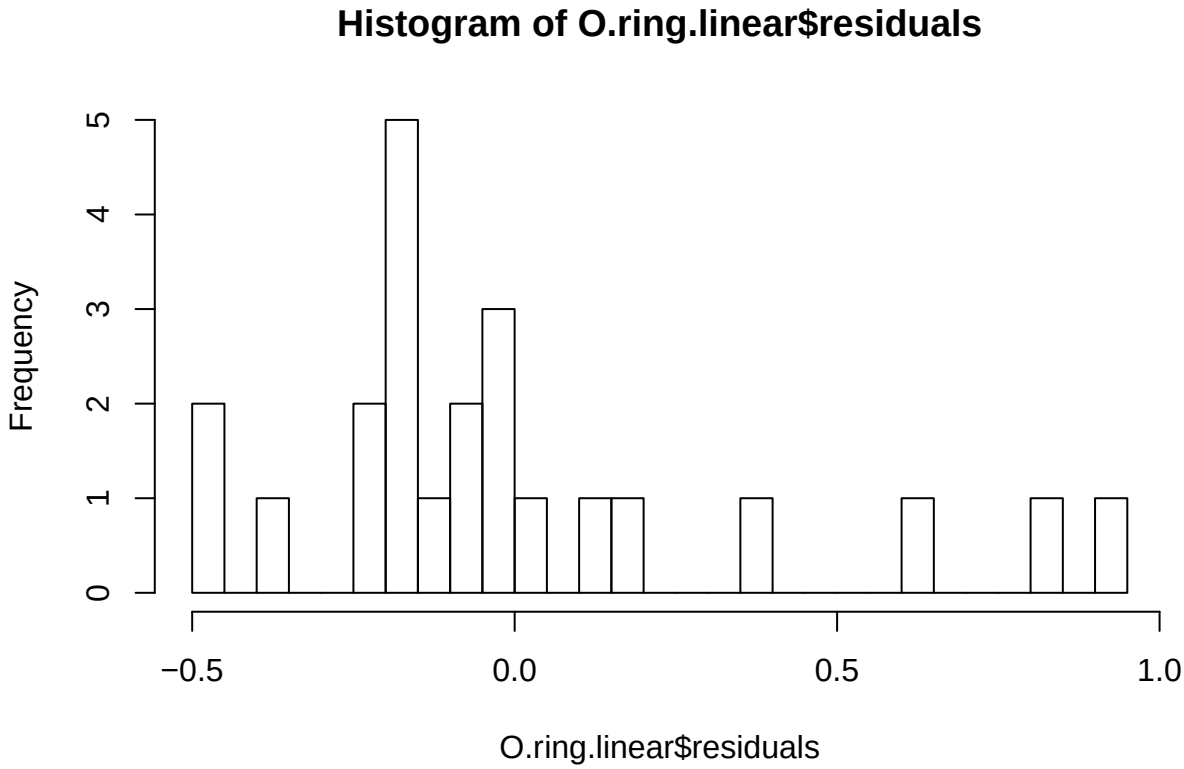


The charts below show that the normality of error assumption is not met because the residuals deviate from the straight line in the first chart (they look like a logistic function), and they do not follow a normal distribution in the second.

```
plot(O.ring.linear, which = 2)
```




```r
hist(O.ring.linear$residuals, breaks = 50)
```



Part 5 (10 points)

Interpret the main result of your final model in terms of both odds and probability of failure.

Summarize the final result with respect to the question(s) being asked and key takeaways from the analysis.

```
O.ring.linear <- lm(formula = O.ring ~ Temp + Pressure, data=challenger2)
summary(O.ring.linear)
```

```
##
## Call:
## lm(formula = O.ring ~ Temp + Pressure, data = challenger2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49600 -0.18816 -0.07650  0.08027  0.91267
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.658765   0.824297   3.225  0.00424 **
## Temp        -0.038136   0.011602  -3.287  0.00369 **
## Pressure     0.001962   0.001200   1.635  0.11779
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3837 on 20 degrees of freedom
## Multiple R-squared:  0.3952, Adjusted R-squared:  0.3347
## F-statistic: 6.534 on 2 and 20 DF,  p-value: 0.006549
```