Dynamic Programming Collection

The following is a set of problems that covers a variety of simple settings where dynamic programming (DP) can be used. Most of these examples come from the Winston or Taha textbooks, though many have been adapted.

DP1 Knapsack Problem

We have a container of size 20 units, and want to pack it with the following valuable items:

Item j	Size v_j	Value t_j
1	7	25
2	4	12
3	3	8

How many of each item should we pack in order to maximize the total value?

DP2 Knapsack Alternative

There is not a unique dynamic programming solution to a given problem. Consider again the knapsack problem, where we have a volume of 20 units that we can fill with three types of items, each with volume and value as given in DP1.

To find the optimal packing we determine first how to fill a small knapsack optimally and then, using this information, how to fill a larger knapsack optimally. We define $g(w)$ to be the maximum value that can be gained with a knapsack of volume w. With g as the value function, use DP to find the optimal packing of the knapsack.

DP3 Nonlinear Objective

Find non-negative integers x_1, x_2, x_3 to maximize $z = x_1 x_2^2 x_3^3$ subject to $x_1 + x_2 + x_3 = 7$.

DP4 X-Files

ET is about to fly home. For the trip to be successful, the ship's solar relay, warp drive, and candy maker must all function properly. ET has found three unemployed actors who are willing to help get the ship ready for takeoff. The following table gives, as a function of the number of actors assigned to repair each component, the probability that each component will function properly during the trip home.

	Actors Assigned			
Component	0	1	2	3
Solar relay	0.40	0.50	0.70	0.90
Warp drive	0.30	0.55	0.65	0.95
Candy maker	0.45	0.55	0.80	0.98

Use dynamic programming to help ET maximize the probability of having a successful trip home.

DP5 Another Nonlinear Objective

Consider the following nonlinear integer programming problem:

Maximize $(x_1+5)(x_2+1)(x_3+2)$ subject to $3x_1 + 2x_2 + x_3 \leq 6$, with x_1, x_2, x_3 all nonnegative integers.

Use dynamic programming to find all optimal solutions to this problem.

DP6 Yum! Strawberries!

The owner of a chain of three grocery stores has purchased five crates of fresh strawberries. The estimated potential sales of the strawberries before spoilage differs among the three stores. Therefore the owner wishes to know how she should allocate the five crates to the three stores to maximize expected profit.

For administrative reasons, the owner does not wish to split crates between stores. However, she is willing to distribute zero crates to any of her stores.

The following table gives the estimated expected profit at each store when it is allocated various numbers of crates:

Crates	*Store 1*	*Store 2*	*Store 3*
0	$0	$0	$0
1	$3	$5	$4
2	$7	$10	$6
3	$9	$11	$11
4	$12	$11	$12
5	$13	$11	$12

Use dynamic programming to determine how many of the five crates should be assigned to each of the three stores to maximize the total expected profit.

DP7 Come Fly with Me

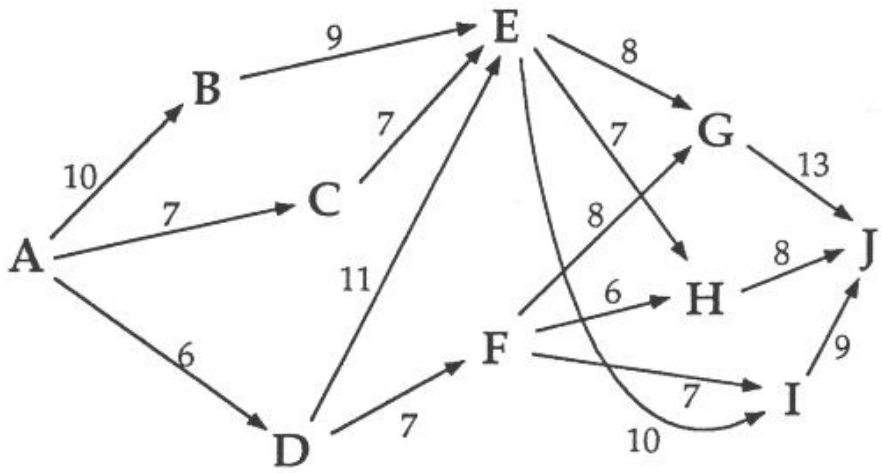
Daly Airlines has been told that it may schedule six flights per day departing from Brisbane. The destination of each flight may be Hervey Bay, Coffs Harbour, or Rockhampton. The following table shows the contribution to the company's profits from any given number of daily flights from Brisbane to each possible destination.

	Number of Planes					
Destination	1	2	3	4	5	6
Hervey Bay	$80	$150	$210	$250	$270	$280
Coffs Harbour	$100	$195	$275	$325	$300	$250
Rockhampton	$90	$180	$265	$310	$350	$320

Using dynamic programming, find the optimal number of flights that should depart Brisbane for each destination. How would the answer change if the airline were restricted to only four daily flights?

DP8 Altitude Sickness

Joe Cougar needs to drive from A to J. Due to a medical condition he wants to avoid high altitude. The following figure gives the maximum altitude of roads between intermediate cities on the way to J.



Use dynamic programming to find a route which minimizes the maximum altitude of Joe's journey.

DP9 Minimal Studying

In order to graduate from State University, Angie Warner needs to pass at least one of the three subjects she is taking this semester. She is now enrolled in Algebra, Calculus, and Statistics. Angie's busy schedule of extra-curricular activities allows her to spend only 4 hours per week on studying. Angie's probability of passing each course depends on the number of hours she spends studying for the course, as follows:

Hours of study per week	Probability of Passing		
	Algebra	Calculus	Statistics
0	.20	.25	.10
1	.30	.30	.30
2	.35	.33	.40
3	.38	.35	.45
4	.40	.38	.50

Use dynamic programming to determine how many hours per week Angie should spend studying each subject. (Hint: Explain why maximizing the probability of passing at least one course is equivalent to minimizing the probability of failing all three courses).

DP10 Dracula's Democracy

The state of Transylvania consists of three cities with the following populations: city 1, 1.2 million people; city 2, 1.4 million people; city 3, 400,000 people. The Transylvania House of Representatives consists of three representatives. Given proportional representation, city 1 should have $d_1 = 3(1.2/3) = 1.2$ representatives; city 2 should have $d_2 = 1.4$ representatives; and city 3 should have $d_3 = 0.4$ representatives. Since each city must receive an integral number of representatives, this is impossible.

Transylvania has therefore decided to allocate xi representatives to city i, where the allocation x_1, x_2, x_3 minimizes the maximum discrepancy between the desired and actual number of representatives received by a city (i.e. max{$|x_i - d_i|$}). Use dynamic programming to solve Transylvania's problem.

DP11 Budget Allocation

A corporation is entertaining proposals from its four production plants for possible expansion of facilities. The corporation is budgeting $8 million for allocation to all four plants. Each plant j submits its proposals giving total cost (c_j) and total revenue (R_j) for each proposal, as summarized in the table below. The goal of the corporation is to maximize the total revenue resulting from the allocation of the $8 million to the four plants.

	Plant 1		Plant 2		Plant 3		Plant 4	
Proposal	c_1	R_1	c_2	R_2	c_3	R_3	c_4	R_4
1	0	0	1	1.5	0	0	0	0
2	3	5	3	5	1	2.1	2	2.8
3	4	7	4	6	-	-	3	3.6

Use dynamic programming to obtain the optimal allocation.

DP12 Sales Reps

A company has five sales representatives available for assignment to three sales districts. The sales in each district during the current year depend on the number of sales representatives assigned to the district and on whether the national economy has a bad or good year, as given in the following table.

Sales reps assigned to district	*Sales (millions)*		
	District 1	District 2	District 3
0	$1, $4	$2, $5	$3, $4
1	$2, $6	$4, $6	$5, $5
2	$3, $7	$5, $6	$6, $7
3	$4, $8	$6, $6	$7, $7

In the Sales column for each district, the first number represents sales if the national economy had a bad year, and the second number represents sales if the economy had a good year. There is a .3 chance that the national economy will have a good year and a .7 chance that it will have a bad year. Use dynamic programming to determine an assignment of sales representatives to districts that maximizes the company's expected sales.

DP13 Chess Strategy

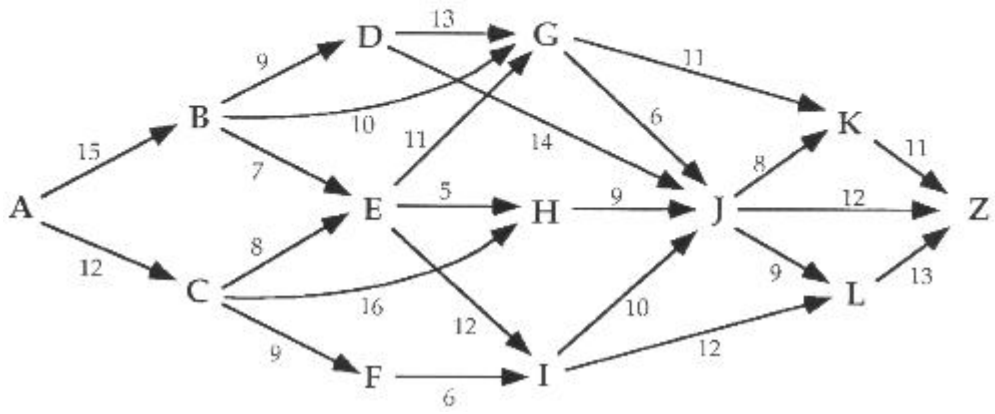
Vladimir Bronowsky is playing Keith Smithson in a two-game chess match. Winning a game scores one match point, and drawing a game scores a half match point. After the two games are played, the player with more match points is declared the champion. If the two players are tied after two games, they continue playing until somebody wins a game (the winner of that game will be the champion). During each game, Bronowsky can play one of two ways: boldly or conservatively. If he plays boldly, he has a 45% chance of winning the game and a 55%

chance of losing the game. If he plays conservatively, he has a 90% chance of drawing the game and a 10% chance of losing the game.

Bronowsky's goal is to maximize his probability of winning the match. Use dynamic programming to help him accomplish this goal.

DP14 Shortest Path

Consider the network schematic in the figure below, where the numbers indicate distances between nodes. Use dynamic programming to find the shortest path from A to Z.



DP15 Parking Problem

Jenny drives along a straight road towards a particular shop, looking for a vacant parking space. Vacancies occur at random, on average once in every 10 places. Once past the shop, Jenny will take the next available space. But at what point in approaching the shop should she accept a vacant space?

DP16 Inventory

A company knows that the demand for its product during each of the next four months will be as follows:

Month	1	2	3	4
Demand	2	3	2	4

At the beginning of each month, the company must determine how many units should be produced during the current month. During a month in which units are produced, a setup cost of $3 is incurred. In addition, there is a variable cost of $1 for every unit produced. At the end of each month, a holding cost of 50 cents per unit on hand is incurred. Capacity limitations allow a maximum of 5 units to be produced during each month. The size of the company's warehouse restricts the ending inventory of each month to at most 4 units.