Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2015

MATH3202 Operations Research and Mathematical Planning

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - writing is not permitted at all

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

There are **40** marks available on this exam from **3** questions.

Provide your answers in the booklet provided.

For Examiner Use Only

Question	Mark

Total ________

Question 1

12 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = x_1 + 2x_2 + 3x_3$$

Subject to:

$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 + x_4 &= 12 \\ 6x_1 + x_2 + 5x_3 + x_5 &= 9 \\ x_1 + 3x_2 + 3x_3 + x_6 &= 7 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Assume we have a current basis of x_3, x_4, x_6. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective?

b) What is the entering variable for the next step of the Revised Simplex Algorithm?

c) What is the leaving variable?

d) What is the new value of the objective? Verify that the new solution is optimal.

e) If the right hand side of the second constraint is changed to $9 + \delta$ for some value of $\delta > 0$ will the value of z increase or decrease? By how much?

f) Assuming no other data changes, what value does the objective function coefficient of x_1 have to exceed so that x_1 is non-zero in the optimal solution?

Hint The following information may be useful:

$$\begin{bmatrix} 4 & 1 & 0 \\ 5 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1/5 & 0 \\ 1 & -4/5 & 0 \\ 0 & -3/5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 1 & 5 & 0 \\ 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1/4 & 5/12 \\ 0 & 1/4 & -1/12 \\ 1 & -1/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2

14 marks total

AustVol is a volunteer organisation which needs to assign a pool of volunteers to teams so as to meet targets for known skill types in each team.

AustVol have allocated a score to each of their volunteers for each skill type. For each team/skill type combination they have also determined target scores. These targets are of the form:

[minimum score, preferred score, maximum score]

where minimum score $\leq$ preferred score $\leq$ maximum score.

These targets apply to the sum of the scores of volunteers assigned to the team. For example, if one skill type was "driver", with a value of 1 for any volunteer who can drive and 0 otherwise, a target could be used to set the minimum, maximum and preferred number of drivers on each team.

Additional rules also need to be considered, as follows:

- Some volunteers are pre-assigned to specific teams;
- Some volunteers are excluded from specific teams;
- Some pairs of volunteers may not be assigned to the same team.

AustVol also have an importance score for each team, which is a measure of the weight to apply to the team. Their aim is to assign volunteers to teams so that:

- Each volunteer is assigned to exactly one team;
- The minimum/maximum targets are satisfied;
- All the additional rules are satisfied; and
- The total extent to which teams fall short of preferred scores for skills, weighted by the importance of the teams, is minimised.

a) Develop an integer programming model to determine the optimal team assignments. Clearly define all sets, data, variables, objective function and constraints.
[11 marks]

b) Modify your model so that the objective is to minimise the maximum amount any team falls short of its preferred skill targets (summed over all skills). You need only specify the new objective and any new variables and constraints.
[3 marks]

Question 3

14 marks total

a) A company is planning how to allocate 4 salespeople to 3 sales districts. The extra profit $r_n(x_n)$ obtained per month by assigning x_n salespeople to district n is indicated in the following table:

	x_n				
n	0	1	2	3	4
1	0	16	25	30	32
2	0	12	17	21	22
3	0	10	14	16	17

Use dynamic programming to determine the allocation of salespeople that will maximize the total monthly profit. Clearly define your value function.
[7 marks]

b) A firm is planning its advertising strategy for a period of four weeks. In each week the sales level will be either *High* or Low and the firm will receive profits on sales of $800 or $600, respectively.

If the sales were *High* in the previous week then there is a 60% chance that sales will be *High* again in the current week if they do not advertise in the current week or 80% if they do advertise. If the sales were *Low* in the previous week then there is a 20% chance that sales will be *High* in the current week if they do not advertise in the current week or 60% if they do advertise.

The cost of advertising in one week is $70. An extra cost of $80 is incurred if the level of sales (and thus production) is changed from one week to the next.

Use stochastic dynamic programming to model this problem. What advertising strategy should the firm pursue?
[7 marks]

END OF EXAMINATION