An algorithm for replicating multi-threaded applications as done in replicated Popcorn. The application is made deterministic through the use of logical time. Any inter-thread synchronization operation must be protected by calls to *EnterSync* and *ExitSync*. Reads of the socket *API* are modeled by *EnterRead*. The scheduler processes (one per kernel) makes sure that the different copies of the application are consistent.

EXTENDS Naturals, Sequences, Integers, Library

CONSTANTS Pid, MaxTime, Kernel, SchedulerPID, Requests

Assume $SchedulerPID \notin Pid$

 $InitLogTime \triangleq 1$

 $LogTime \stackrel{\Delta}{=} InitLogTime ... MaxTime$ The set of logical time values

Processes are of the form $\langle k, pid \rangle$, where k is the kernel the process is running on.

```
P \triangleq Kernel \times Pid

Primary \triangleq CHOOSE \ k \in Kernel : TRUE

Ker(p) \triangleq p[1]

PID(p) \triangleq p[2]
```

 $OnKernel(kernel) \triangleq \{kernel\} \times Pid$

Logical time comparison, using PIDs to break ties.

```
Less(p, tp, q, tq) \stackrel{\triangle}{=} tp < tq \lor (tp = tq \land PID(p) \le PID(q))
```

The sequence of TCP packets that will be received. No duplicates allowed (therefore the set TcpData must be big enough) so that any misordering of the threads will lead to a different data read. For TCPMultiStream, each stream has different data.

```
StreamLength \triangleq 3

TcpData \triangleq 1 \dots StreamLength * Requests

TcpStream \triangleq [i \in 1 \dots StreamLength \mapsto i]

TcpMultiStream \triangleq [r \in 1 \dots Requests \mapsto

([i \in 1 \dots StreamLength \mapsto i + (StreamLength * (r - 1))])]

ASSUME NoDup(TcpStream)

ASSUME Len(TcpStream) = StreamLength
```

Shifts a sequence by 1: $Shift(\langle 1,\, 2,\, 3\rangle)=\langle 2,\, 3\rangle$ and $Shift(\langle \rangle)=\langle \rangle$

```
Shift(s) \stackrel{\triangle}{=}
IF Len(s) > 1
THEN [i \in 1 ... (Len(s) - 1) \mapsto s[i + 1]]
ELSE \langle \rangle
Shiftn(s) \stackrel{\triangle}{=}
IF Len(s) > 1
```

```
THEN [i \in 1 ... (Len(s) - 1) \mapsto s[i + 1]]
ELSE \langle -1 \rangle
```

The algorithm *ReadAppend* models a set of worker threads being scheduled by the deterministic scheduler and executing the following code.

```
Code of worker w: While(true){
x = read(socket);
append(\langle w, x \rangle, file);
}
```

Variables:

The variable bumps records all logical time bumps executed by the primary in order for the secondaries to do the same, i.e. the initial logical time, the new logical time, and the value read from the tcp buffer. $\langle t1,\,t2,\,d\rangle\in bumps[pid]$ means that the primary bumped process pid from logical time t1 to t2 and delivered the data d. Note that the scheduler set bumps[pid] to a value that depends on the logical time of the processes on all replicas, and this value is then immediately available to all replicas. A more detailed model would instead include a distributed implementation of the choice of the logical time to bump the process to.

reads[p] stores the last value read by p from the socket.

tcpBuff[k] represents the state of the tcp buffer on kernel k. Each time a process reads from the buffer, the buffer shrinks by 1.

Definitions:

Bumped(kernel) is the set of processes running on the kernel "kernel" which are waiting to execute a "bump" decided by the primary.

If $p \in Bumped(Ker(p))$ then BumpedTo(p) is the logical time to which p should be bumped to.

If $p \in WaitingSync(Ker(p))$ then IsNextProc(kernel, p) is true iff p is the process to be scheduled next, that is: (1) p has the lowest ltime among running and waiting-sync processes and (2) if q is on the same kernel and q is waiting for a read and the primary has already decided to which logical time tq to bump q, then ltime[p] must be less than tq.

Bump To is the logical time to which to bump a process that needs bumping. It is some logical time greater than all the logical times reached by any process on any kernel.

```
--algorithm ReadAppend\{
variables
status = [p \in P \mapsto \text{"running"}],
ltime = [p \in P \mapsto InitLogTime],
file = [k \in Kernel \mapsto \langle \rangle],
bumps = [p \in Pid \mapsto \{\}],
reads = [p \in P \mapsto -1],
tcpBuff = [k \in Kernel \mapsto TcpMultiStream],
Queue for accepted connections
socketQueue = [k \in Kernel \mapsto \langle \rangle],
Queue for unhandled connections
requestQueue = [k \in Kernel \mapsto [r \in 1 ... Requests \mapsto r]],
The socket that is handled by a process
handledSocket = [p \in P \mapsto -1],
```

```
connections = [k \in Kernel \mapsto \langle \rangle]
define {
     \begin{array}{ll} Run(p) \; \stackrel{\triangle}{=} \; status[p] = \text{"running"} \\ Running(kernel) \; \stackrel{\triangle}{=} \; \{ p \in OnKernel(kernel) : Run(p) \} \end{array}
     WaitingSync(kernel) \triangleq
         \{p \in OnKernel(kernel) : status[p] = "waiting sync"\}
     WaitingRead(kernel) \triangleq
         \{p \in OnKernel(kernel) : status[p] = "waiting read"\}
     Bumped(kernel) \triangleq \{ p \in OnKernel(kernel) : \}
          \land status[p] = "waiting read"
          \land \exists t \in LogTime : \exists d \in TcpData : \langle ltime[p], t, d \rangle \in bumps[PID(p)] \}
     BumpedTo(p) \triangleq
         CHOOSE t \in LogTime : \exists d \in TcpData : \langle ltime[p], t, d \rangle \in bumps[PID(p)]
     BumpData(p) \triangleq
         CHOOSE d \in TcpData : \exists t \in LogTime : \langle ltime[p], t, d \rangle \in bumps[PID(p)]
    IsNextProc(kernel, p) \stackrel{\Delta}{=}
          \land \forall q \in Running(kernel) \cup WaitingSync(kernel) :
                 q \neq p \Rightarrow Less(p, ltime[p], q, ltime[q])
          \land \forall q \in Bumped(kernel) : Less(p, ltime[p], q, BumpedTo(q))
    BumpTo \stackrel{\triangle}{=} CHOOSE \ i \in LogTime : \forall \ p \in P : ltime[p] < i
}
macro EnterRead(p){
     status[p] := "waiting read";
macro EnterSync(p){
     status[p] := "waiting sync";
macro ExitSync(p){
     ltime[p] := ltime[p] + 1;
 Processes consume a connection
process (worker \in P){
           ww1: while (TRUE){
                       EnterSync(self);
           ww2:
                      await Run(self);
                      if (Len(requestQueue[Ker(self)]) > 0){
           ww3:
                            handledSocket[self] := requestQueue[Ker(self)][1];
                            requestQueue[Ker(self)] := Shift(requestQueue[Ker(self)]);
                      };
           ww4:
                       ExitSync(self);
                      if (handledSocket[self] \neq -1){
           ww5:
                                 while (Len(tcpBuff[Ker(self)][handledSocket[self]]) > 0){
                                       EnterRead(self);
```

```
await Run(self);
                          ww10:
                                    EnterSync(self);
                          ww11:
                                   await Run(self);
                                    file[Ker(self)] :=
                                       Append(file[Ker(self)], \langle PID(self), reads[self] \rangle);
                                   ExitSync(self);
                          ww12:
             ww13: handledSocket[self] := -1;
             };
       }
   process (scheduler \in \{\langle k, SchedulerPID \rangle : k \in Kernel\}) \{
       s1: while (TRUE){
                 either { Schedule a process waiting for synchronization:
                     with (p \in \{p \in WaitingSync(Ker(self)) : \}
                              IsNextProc(Ker(self), p)\})\{
                         status[p] := "running" \} 
                 or { Bump a process waiting for a read:
                     with (p \in WaitingRead(Ker(self))){
                         if (Ker(self) = Primary) On the primary.
                           Record the bump for the secondaries.
                             bumps[PID(p)] :=
                                 bumps[PID(p)] \cup \{\langle ltime[p], BumpTo, tcpBuff[Ker(self)][handledSocket[p]][1]\}\}
                             ltime[p] := BumpTo;
                         }else { On a replica:
 Wait until the primary has bumped p and the data to be delivered to p in at the head of the tcp buffer
                           await p \in Bumped(Ker(self)) \land BumpData(p) = tcpBuff[Ker(self)][handledSocket]
                           ltime[p] := BumpedTo(p); Bump the process
                       };
                       reads[p] := tcpBuff[Ker(self)][handledSocket[p]][1];
                       tcpBuff[Ker(self)][handledSocket[p]] :=
                           Shift(tcpBuff[Ker(self)][handledSocket[p]]);
                       status[p] := "running"; Let p run.
               }
       }
BEGIN TRANSLATION
Variables status, ltime, file, bumps, reads, tcpBuff, socketQueue,
            requestQueue, handledSocket, connections, pc
define statement
```

 $Run(p) \stackrel{\Delta}{=} status[p] = "running"$

```
Running(kernel) \triangleq \{ p \in OnKernel(kernel) : Run(p) \}
WaitingSync(kernel) \triangleq
     \{p \in OnKernel(kernel) : status[p] = "waiting sync"\}
WaitingRead(kernel) \stackrel{\Delta}{=}
     \{p \in OnKernel(kernel) : status[p] = "waiting read"\}
Bumped(kernel) \stackrel{\Delta}{=} \{ p \in OnKernel(kernel) : \}
     \land status[p] = "waiting read"
      \land \exists t \in LogTime : \exists d \in TcpData : \langle ltime[p], t, d \rangle \in bumps[PID(p)] \}
BumpedTo(p) \triangleq
     CHOOSE t \in LogTime : \exists d \in TcpData : \langle ltime[p], t, d \rangle \in bumps[PID(p)]
BumpData(p) \triangleq
     CHOOSE d \in TcpData : \exists t \in LogTime : \langle ltime[p], t, d \rangle \in bumps[PID(p)]
IsNextProc(kernel, p) \triangleq
      \land \forall q \in Running(kernel) \cup WaitingSync(kernel) :
              q \neq p \Rightarrow Less(p, ltime[p], q, ltime[q])
      \land \ \forall \ q \in Bumped(kernel) : Less(p, \ ltime[p], \ q, \ BumpedTo(q))
BumpTo \stackrel{\triangle}{=} CHOOSE \ i \in LogTime : \forall \ p \in P : ltime[p] < i
vars \triangleq \langle status, ltime, file, bumps, reads, tcpBuff, socketQueue, \rangle
             requestQueue, handledSocket, connections, pc \rangle
ProcSet \triangleq (P) \cup (\{\langle k, SchedulerPID \rangle : k \in Kernel\})
Init \stackrel{\Delta}{=} Global variables
            \land status = [p \in P \mapsto "running"]
            \land ltime = [p \in P \mapsto InitLogTime]
            \land file = [k \in Kernel \mapsto \langle \rangle]
            \land bumps = [p \in Pid \mapsto \{\}]
            \land reads = [p \in P \mapsto -1]
            \land tcpBuff = [k \in Kernel \mapsto TcpMultiStream]
            \land socketQueue = [k \in Kernel \mapsto \langle \rangle]
            \land requestQueue = [k \in Kernel \mapsto [r \in 1 .. Requests \mapsto r]]
            \land handledSocket = [p \in P \mapsto -1]
            \land connections = [k \in Kernel \mapsto \langle \rangle]
            \land pc = [self \in ProcSet \mapsto CASE \ self \in P \rightarrow \text{``ww1''}]
                                                  \Box self \in \{\langle k, SchedulerPID \rangle : k \in Kernel\} \rightarrow "s1" \}
ww1(self) \stackrel{\triangle}{=} \wedge pc[self] = \text{"ww1"}
                    \land status' = [status \ EXCEPT \ ![self] = "waiting sync"]
                    \land pc' = [pc \text{ EXCEPT } ![self] = \text{"ww2"}]
                    \land UNCHANGED (ltime, file, bumps, reads, tcpBuff, socketQueue,
                                           requestQueue, handledSocket, connections
ww2(self) \stackrel{\Delta}{=} \wedge pc[self] = \text{``ww2''}
                    \wedge Run(self)
                    \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww3"}]
```

```
\land UNCHANGED \langle status, ltime, file, bumps, reads, tcpBuff,
                                       socketQueue, requestQueue, handledSocket,
                                       connections \rangle
ww3(self) \stackrel{\triangle}{=} \wedge pc[self] = \text{"ww3"}
                  \land IF Len(requestQueue[Ker(self)]) > 0
                         THEN \land handledSocket' = [handledSocket except ![self] = requestQueue[Ker(self)][1]]
                                  \land requestQueue' = [requestQueue EXCEPT ![Ker(self)] = Shift(requestQueue[Ker(self)]) = Shift(requestQueue)
                         ELSE ∧ TRUE
                                  \land UNCHANGED \langle requestQueue, handledSocket \rangle
                  \land pc' = [pc \text{ EXCEPT } ![self] = \text{"ww4"}]
                  \land UNCHANGED \langle status, ltime, file, bumps, reads, tcpBuff,
                                      socketQueue, connections \rangle
ww4(self) \stackrel{\triangle}{=} \wedge pc[self] = \text{``ww4''}
                  \land ltime' = [ltime \ EXCEPT \ ![self] = ltime[self] + 1]
                  \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww5"}]
                  \land UNCHANGED \langle status, file, bumps, reads, tcpBuff, socketQueue,
                                       requestQueue, handledSocket, connections
ww5(self) \triangleq \land pc[self] = \text{``ww5''}
                  \land IF handledSocket[self] \neq -1
                         THEN \wedge pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww9"}]
                         ELSE \land pc' = [pc \text{ EXCEPT } ! | self] = \text{"ww13"}]
                  \land UNCHANGED \langle status, ltime, file, bumps, reads, tcpBuff,
                                       socketQueue, requestQueue, handledSocket,
                                       connections \rangle
ww9(self) \stackrel{\Delta}{=} \wedge pc[self] = \text{"ww9"}
                  \land IF Len(tcpBuff[Ker(self)][handledSocket[self]]) > 0
                         THEN \land status' = [status \ EXCEPT \ ![self] = "waiting read"]
                                 \land pc' = [pc \text{ EXCEPT } ![self] = \text{"ww10"}
                         ELSE \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww13"}]
                                  \land UNCHANGED status
                  ∧ UNCHANGED ⟨ltime, file, bumps, reads, tcpBuff, socketQueue,
                                      requestQueue, handledSocket, connections
ww10(self) \triangleq \wedge pc[self] = \text{``ww10''}
                   \wedge Run(self)
                   \land status' = [status \ EXCEPT \ ![self] = "waiting sync"]
                   \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww11"}]
                   ∧ UNCHANGED ⟨ltime, file, bumps, reads, tcpBuff, socketQueue,
                                        requestQueue, handledSocket, connections
ww11(self) \triangleq \land pc[self] = \text{``ww11''}
                   \wedge Run(self)
                   \land file' = [file EXCEPT ! [Ker(self)] = Append(file[Ker(self)], \langle PID(self), reads[self] \rangle)]
```

```
\land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww12"}]
                     \land UNCHANGED \langle status, ltime, bumps, reads, tcpBuff,
                                           socketQueue, requestQueue, handledSocket,
                                           connections \rangle
ww12(self) \triangleq \wedge pc[self] = \text{``ww12''}
                     \wedge ltime' = [ltime \ EXCEPT \ ![self] = ltime[self] + 1]
                     \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww9"}]
                     \land UNCHANGED \langle status, file, bumps, reads, tcpBuff, socketQueue,
                                           requestQueue, handledSocket, connections
ww13(self) \stackrel{\triangle}{=} \wedge pc[self] = \text{``ww13''}
                     \land handledSocket' = [handledSocket \ EXCEPT \ ![self] = -1]
                     \land pc' = [pc \text{ EXCEPT } ! [self] = \text{"ww1"}]
                     \land UNCHANGED \langle status, ltime, file, bumps, reads, tcpBuff,
                                           socketQueue, requestQueue, connections \rangle
worker(self) \stackrel{\triangle}{=} ww1(self) \lor ww2(self) \lor ww3(self) \lor ww4(self)
                             \vee ww5(self) \vee ww9(self) \vee ww10(self) \vee ww11(self)
                             \vee ww12(self) \vee ww13(self)
s1(self) \stackrel{\Delta}{=} \wedge pc[self] = "s1"
                \land \lor \land \exists p \in
                                       \{p \in WaitingSync(Ker(self)) : \}
                                   IsNextProc(Ker(self), p):
                             status' = [status \ EXCEPT \ ![p] = "running"]
                        \land UNCHANGED \langle ltime, bumps, reads, tcpBuff \rangle
                    \vee \wedge \exists p \in WaitingRead(Ker(self)):
                              \wedge IF Ker(self) = Primary
                                     THEN \land bumps' = [bumps \ \text{EXCEPT} \ ![PID(p)] = bumps[PID(p)] \cup \{\langle ltime[p], Bumps[PID(p)] \} \}
                                              \wedge ltime' = [ltime \ EXCEPT \ ![p] = BumpTo]
                                     ELSE \land p \in Bumped(Ker(self)) \land BumpData(p) = tcpBuff[Ker(self)][handledSociety]
                                              \land ltime' = [ltime \ EXCEPT \ ![p] = BumpedTo(p)]
                                              \land bumps' = bumps
                             \land reads' = [reads \ EXCEPT \ ![p] = tcpBuff[Ker(self)][handledSocket[p]][1]]
                              \land \mathit{tcpBuff'} = [\mathit{tcpBuff} \ \ \mathsf{Except} \ \ ![\mathit{Ker}(\mathit{self})][\mathit{handledSocket}[\mathit{p}]] = \mathit{Shift}(\mathit{tcpBuff}[\mathit{Ker}(\mathit{self})]) 
                              \land status' = [status \ EXCEPT \ ![p] = "running"]
                 \land pc' = [pc \text{ EXCEPT } ![self] = "s1"]
                 \land UNCHANGED \langle file, socketQueue, requestQueue, handledSocket,
                                       connections
scheduler(self) \stackrel{\triangle}{=} s1(self)
Next \stackrel{\triangle}{=} (\exists self \in P : worker(self))
               \vee (\exists self \in \{\langle k, SchedulerPID \rangle : k \in Kernel\} : scheduler(self))
Spec \stackrel{\Delta}{=} Init \wedge \Box [Next]_{vars}
```

END TRANSLATION

RequestSynchronized $\stackrel{\triangle}{=} \forall k1, k2 \in Kernel$:

- $\lor \ \textit{Prefix}(socketQueue[k1], socketQueue[k2])$
- $\lor Prefix(socketQueue[k2], socketQueue[k1])$

 $FilesSynchronized \stackrel{\Delta}{=} \forall k1, k2 \in Kernel:$

- \vee Prefix(file[k1], file[k2])
- $\vee Prefix(file[k2], file[k1])$

 $ConnectionSynchronized \ \stackrel{\Delta}{=} \ \forall \, k1, \, k2 \in Kernel:$

- $\vee Prefix(connections[k1], connections[k2])$
- $\vee Prefix(connections[k1], connections[k1])$