Technische Universität Kaiserslautern
Fachbereich Mathematik
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Multicriteria Optimization

Exercise Sheet 9

IN-CLASS EXERCISES
((To be done in the tutorial on December 20th, 2011))

Problem 9.1 – SUM-MAX-SPP

Let $G = (V, A)$ be a directed graph, $s, t \in V$ and denote by $\mathcal{P}^{s,t}$ the set of all s-t paths in G, where a path $P \in A^{|P|}$ is defined by its edges. Consider the following shortest path problem with one sum and $p - 1$ bottleneck objectives:

$$\begin{aligned} \min \quad & \left(\sum_{a \in P} c^1(a),\ \max_{a \in P} c^2(a),\ \ldots,\ \max_{a \in P} c^p(a) \right) \\ \text{s.t.} \quad & P \in \mathcal{P}^{s,t}. \end{aligned} \tag{SUM-MAX-SPP}$$

a) Find a good upper bound on the number of nondominated points for SUM-MAX-SPP.

b) Find an algorithm to find a complete minimal set of efficient solutions. You do not have to give a rigorous proof of the correctness. *Hint: Try to first find a solution for* $p = 2$*, then generalize.*

c) How can we solve the problem if there are bottleneck objectives only?

TURN-IN EXERCISES
(Please hand in by January 5th, 2012)

Problem 9.2

a) Using the multicriteria label-setting algorithm from the lecture, compute a minimal complete set of efficient paths from v_1 to v_7 in the graph of Figure 1.

b) Consider a directed graph $G = (V, A)$ with cost function $c : A \to \mathbb{R}^p$ not containing negative cycles, i. e., for all cycles Z in G we have $c(Z) = \sum_{a \in Z} c(a) \geq 0$.

   1. Prove the following statement: Let $P^{s,t}$ be an efficient s-t path on G. Then any subpath $P^{u,v}$ from u to v, where u and v are vertices on $P^{s,t}$, is an efficient path from u to v.

   2. Show that the converse is not true in general, i. e. compositions of efficient paths are not necessarily efficient.

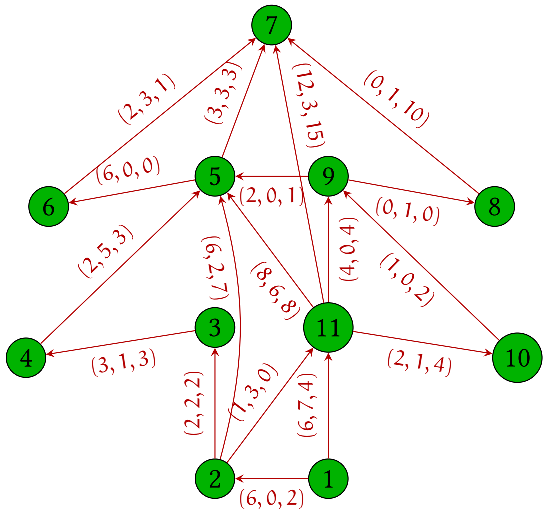

Figure 1: The graph for Problem 2

Problem 9.3 – Multicriteria FloydWarshall

Recall how the algorithm of Floyd and Warshall computes all-to-all shortest paths in a directed graph under the assumption that it does not contain negative dicycles. The objective of this exercise is to extend that algorithm so as to find a minimal complete efficient set for a multicriteria shortest path problem (which also must not contain any negative cycle in the sense of Problem 2).

To solve this exercise, assign a list of labels to each node, similar as for the multicriteria label-setting algorithm in the lecture. Then use the generalized Bellman principle of the previous exercise to restrict the set of possible efficient paths in the inner `FOR` loop of FloydWarshall.

Download of exercises at
`http://optimierung.mathematik.uni-kl.de/teaching/ws1112/multicriteria-opt.html`