

Multicriteria Optimization

Exercise Sheet 9

IN-CLASS EXERCISES

(To be done in the tutorial on December 20th, 2011)

Problem 9.1 – SUM-MAX-SPP

Let $G = (V, A)$ be a directed graph, $s, t \in V$ and denote by $\mathcal{P}^{s,t}$ the set of all s - t paths in G , where a path $P \in \mathcal{P}^{s,t}$ is defined by its edges. Consider the following shortest path problem with one sum and $p - 1$ bottleneck objectives:

$$\min \left(\sum_{a \in P} c^1(a), \max_{a \in P} c^2(a), \dots, \max_{a \in P} c^p(a) \right) \quad (\text{SUM-MAX-SPP})$$

s. t. $P \in \mathcal{P}^{s,t}$.

- Find a good upper bound on the number of nondominated points for SUM-MAX-SPP.
- Find an algorithm to find a complete minimal set of efficient solutions. You do not have to give a rigorous proof of the correctness. *Hint: Try to first find a solution for $p = 2$, then generalize.*
- How can we solve the problem if there are bottleneck objectives only?

TURN-IN EXERCISES

(Please hand in by January 5th, 2012)

Problem 9.2

- Using the multicriteria label-setting algorithm from the lecture, compute a minimal complete set of efficient paths from v_1 to v_7 in the graph of Figure 1.
- Consider a directed graph $G = (V, A)$ with cost function $c : A \rightarrow \mathbb{R}^p$ not containing negative cycles, i. e., for all cycles Z in G we have $c(Z) = \sum_{a \in Z} c(a) \geq 0$.
 - Prove the following statement: Let $P^{s,t}$ be an efficient s - t path on G . Then any subpath $P^{u,v}$ from u to v , where u and v are vertices on $P^{s,t}$, is an efficient path from u to v .
 - Show that the converse is not true in general, i. e. compositions of efficient paths are not necessarily efficient.

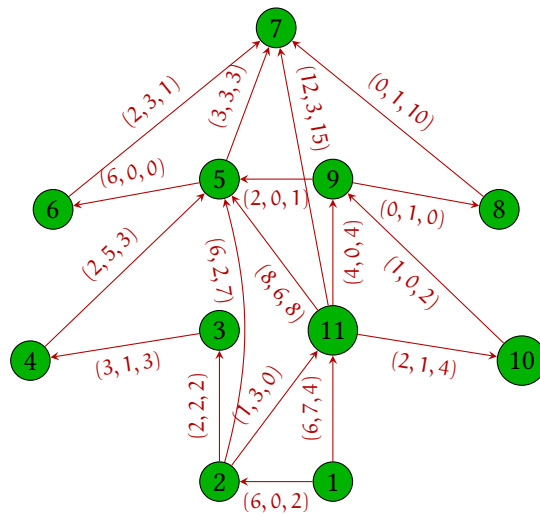


Figure 1: The graph for Problem 2

Problem 9.3 – Multicriteria FLOYDWARSHALL

Recall how the algorithm of Floyd and Warshall computes all-to-all shortest paths in a directed graph under the assumption that it does not contain negative dicycles. The objective of this exercise is to extend that algorithm so as to find a minimal complete efficient set for a multicriteria shortest path problem (which also must not contain any negative cycle in the sense of Problem 2).

To solve this exercise, assign a list of labels to each node, similar as for the multicriteria label-setting algorithm in the lecture. Then use the generalized Bellman principle of the previous exercise to restrict the set of possible efficient paths in the inner FOR loop of FLOYDWARSHALL.

Download of exercises at
<http://optimierung.mathematik.uni-kl.de/teaching/ws1112/multicriteria-opt.html>