EE5027 Adaptive Signal Processing
Final Project: Adaptive Beamforming

Notice

- **Due at 9:00pm, January 17, 2022 (Monday)** $= T_d$ for the electronic copy of your final report.
- Please submit **your report (.pdf file)**, **all your MATLAB files (.m files)**, and **your results (.mat file)** to NTU COOL (`https://cool.ntu.edu.tw/courses/7920`)
- Please **type your final report**. This report can be typed either in English or in Chinese.
- No extensions, unless granted by the instructor one day before T_d.

Introduction

In this course, we went through several adaptive filters, such as LMS adaptive filters, RLS adaptive filters, and Kalman filters. We also introduced the array data model and adaptive beamforming. In this final project, we will explore a numerical example that combines all these topics together.

Recently, communications with *moving sources* have become a significant research topic. This scenario finds applications in self-driving vehicles, satellite communications, and unmanned aerial vehicles (UAV). In this applications, it is common that the signals are emitted from a moving source. If we design adaptive algorithms and adaptive beamformers properly, we may recover the source signals emitted from moving objects.

Problem Formulation

We consider an uniform linear array with N elements and the inter-element spacing $d = \lambda/2$. The output $\mathbf{x}(t)$ of this array is modeled as

$$\mathbf{x}(t) = \mathbf{a}(\theta_{\mathrm{s}}(t))\mathfrak{s}(t) + \mathbf{a}(\theta_{\mathrm{i}}(t))\mathfrak{i}(t) + \mathbf{n}(t), \tag{1}$$

where the time index $t = 1, 2, \ldots, L$. The steering vector is defined as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{j\pi \sin\theta} & e^{j2\pi \sin\theta} & \ldots & e^{j(N-1)\pi \sin\theta} \end{bmatrix}, \tag{2}$$

where $-90^\circ \leq \theta < 90^\circ$. The source signal is $\mathfrak{s}(t)$, the interference signal is $\mathfrak{i}(t)$, the noise vector is $\mathbf{n}(t)$. We have some prior knowledge about (1):

- The DOA of the signal $\theta_{\mathrm{s}}(t)$ changes with time t. The value of $\theta_{\mathrm{s}}(t)$ is unknown. But we know that $0^\circ < \theta_{\mathrm{s}}(t) < 10^\circ$ for all t. (In practice, we have some prior information about the source DOA, but the exact source DOA is unknown.)

- The DOA of the interference $\theta_{\mathrm{i}}(t)$ also changes with time t. The value of $\theta_{\mathrm{i}}(t)$ is unknown. (In practice, the interference is the unwanted part. We might not have prior information about the interference.)
- The source signal $\mathfrak{s}(t)$, the interference signal $\mathfrak{i}(t)$, and the noise $\mathbf{n}(t)$ are uncorrelated.
- The Doppler effect is omitted in the source signal.

According to (1) and the associated assumptions, *synthetic data* is generated and collected into the matrix `matX` in `ASP_Final_Data.mat`:

$$\texttt{matX} = \begin{bmatrix} \widetilde{\mathbf{x}}(1) & \widetilde{\mathbf{x}}(2) & \widetilde{\mathbf{x}}(3) & \dots & \widetilde{\mathbf{x}}(L) \end{bmatrix}, \tag{3}$$

where $\widetilde{\mathbf{x}}(t)$ denotes the *array measurement* at time t.

Objective

Given the array measurements in `matX`, recover the source signals.

Final Report (80%)

Methods (70%)

In this part, you have to address ***at least*** the following items:

1. Review the details of the following beamformers
   - The beamformer with uniform weights
   - The beamformer with array steering
   - The MVDR beamformer
   - The LCMV beamformer
2. Estimate the direction of the source $\theta_{\mathrm{s}}(t)$ and the direction of the interference $\theta_{\mathrm{i}}(t)$. Describe the details of your estimator.
   Hint: You may use adaptive filters to denoise your estimates.
3. Plot the estimated angles $\widehat{\theta}_{\mathrm{s}}(t)$ and $\widehat{\theta}_{\mathrm{i}}(t)$ over the time index t.
4. Design a beamformer that is suitable for the data model in (1). Elaborate the details and the rationale of your beamformer.
   Hint: You may start with the adaptive algorithms (LMS, RLS, Kalman, etc.) or the adaptive beamformers introduced in this course, and then look for their variants in the literature. Your beamformer should be different from those in Item 1.
5. Plot the estimated source signal $\widehat{\mathfrak{s}}(t)$ over the time index t, by using the beamformers in Item 1 and your beamformer.
6. **Submit all the related MATLAB files**. Make sure these files are executable.

Additional Discussions (10%)

Comment on the ***computational cost*** of

- Your DOA estimators in Item 2.
- The beamformers in Item 1
- Your beamformer

You are encouraged to conduct extra experiments to validate your arguments.

Performance Evaluation (20%)

We will assess your estimation performance according to **the source DOA (5%)**, **the interference DOA (5%)**, and the **source signal (10%)**. **Please submit your data in a mat file**. This file contains three vectors `theta_s_hat`, `theta_i_hat`, and `s_t_hat`. These vectors are defined as

$$\texttt{theta\_s\_hat} \triangleq \begin{bmatrix} \widehat{\theta}_{\mathrm{s}}(1) & \widehat{\theta}_{\mathrm{s}}(2) & \dots & \widehat{\theta}_{\mathrm{s}}(L) \end{bmatrix} \qquad \text{(numbers in degrees)}, \tag{4}$$

$$\texttt{theta\_i\_hat} \triangleq \begin{bmatrix} \widehat{\theta}_{\mathrm{i}}(1) & \widehat{\theta}_{\mathrm{i}}(2) & \dots & \widehat{\theta}_{\mathrm{i}}(L) \end{bmatrix} \qquad \text{(numbers in degrees)}, \tag{5}$$

$$\texttt{s\_t\_hat} \triangleq \begin{bmatrix} \widehat{\mathfrak{s}}(1) & \widehat{\mathfrak{s}}(2) & \dots & \widehat{\mathfrak{s}}(L) \end{bmatrix}, \tag{6}$$

where $\widehat{\theta}_{\mathrm{s}}(t)$ is the estimated source DOA, $\widehat{\theta}_{\mathrm{i}}(t)$ is the estimated interference DOA, and $\widehat{\mathfrak{s}}(t)$ is the estimated source signal. We will compare your results with the *true* DOAs and true source signal. Let $\theta_{\mathrm{s}}(t)$, $\theta_{\mathrm{i}}(t)$, and $\mathfrak{s}(t)$ be the true parameters. We define three vectors

$$\texttt{theta\_s} \triangleq \begin{bmatrix} \theta_{\mathrm{s}}(1) & \theta_{\mathrm{s}}(2) & \dots & \theta_{\mathrm{s}}(L) \end{bmatrix} \qquad \text{(numbers in degrees)}, \tag{7}$$

$$\texttt{theta\_i} \triangleq \begin{bmatrix} \theta_{\mathrm{i}}(1) & \theta_{\mathrm{i}}(2) & \dots & \theta_{\mathrm{i}}(L) \end{bmatrix} \qquad \text{(numbers in degrees)}, \tag{8}$$

$$\texttt{s\_t} \triangleq \begin{bmatrix} \mathfrak{s}(1) & \mathfrak{s}(2) & \dots & \mathfrak{s}(L) \end{bmatrix}. \tag{9}$$

But **these true answers are unavailable to you**. We will evaluate your grade in this part by the MATLAB script

```matlab
% theta_s_hat: 5%
max([ round( 5 * ( 1 - norm(theta_s_hat(:) - theta_s(:)) / norm(theta_s) ) ), 0])
% theta_i_hat: 5%
max([ round( 5 * ( 1 - norm(theta_i_hat(:) - theta_i(:)) / norm(theta_i) ) ), 0])
% s_t_hat: 10%
max([ round( 10 * ( 1 - norm(s_t_hat(:) - s_t(:)) / norm(s_t) ) ), 0])
```

In other words, if your results are closer to the true parameters, then you get more grades.

Last updated November 29, 2021.