Insufficiency of 48 Sequence Generators to Cover the Number Line

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Abstract

We analyze a system of 48 sequence generators producing arithmetic progressions with quadratic starting points and linear periods, organized into two residue classes modulo 90 (classes 11 and 13). Each class comprises 24 generators with base periods coprime to 90, having digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10. We prove that these generators cannot mark all integers in a specified range (epoch), leaving infinitely many unmarked addresses. The proof combines a density analysis, accounting for the declining coverage of individual generators and overlaps in shared periods, with contradiction arguments, including an absurdity implying total order in unmarked addresses under a finite overlap assumption. The infinitude of associated sequences reinforces the result.

1 Introduction

We investigate a computational system generating sequences of integers using 48 discrete sequence generators, partitioned into two residue classes modulo 90 (classes 11 and 13). Each class employs 24 generators defined by base periods $z \in \{7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 6$ coprime to 90, with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, excluding divisibility by 2, 3, or 5. Each generator produces an arithmetic progression with a quadratic starting point and a linearly increasing period over iterations x.

The central question is whether these generators can mark every integer in a range (epoch) defined quadratically in a parameter h. We demonstrate that their coverage is insufficient due to the declining density of individual generators and overlaps in shared periods, leaving infinitely many unmarked addresses. We provide two proofs: one showing that the system lacks the initial power to cover all residues, and another by absurdity, showing that finite overlaps between classes imply an implausible total order in unmarked addresses. The result is supported by sequences A201804, A201816, and A224854 from the On-Line Encyclopedia of Integer Sequences.

2 The Sequence Generation System

The system operates over addresses in an epoch defined for a positive integer h:

$$\text{epoch} = 90h^2 - 12h + 1 \approx 90h^2.$$

Addresses are labeled $k = 0, 1, \ldots, \text{epoch} - 1$, corresponding to numbers $90k + c$, where $c = 11$ (class 11) or $c = 13$ (class 13). The base-10 range is:

$$n = 90 \cdot \text{epoch} + c \approx 8100h^2.$$

Each class employs 24 generators, each defined by parameters (l, m, z), where z is a base period. For iteration $x \geq 1$, a generator in class c produces a starting address:

$$y = 90x^2 - lx + m,$$

and marks addresses:

$$y + p \cdot n, \quad n = 0, 1, 2, \ldots, \lfloor (\text{epoch} - y)/p \rfloor,$$

where the period is:

$$p = z + 90(x - 1).$$

The periods generate numbers in residue classes $z \pmod{90}$, e.g., for $z = 7$, $p = 7, 97, 187, \ldots$. Iterations are bounded by:

$$\text{new}_l imit\text{new}_l imit_{\text{new}_l imit_{\text{new}_l imit}} \approx \frac{\sqrt{h}}{3} \approx \frac{(n/90)^{1/4}}{3}.$$

The 48 generators are listed in Appendix A.

3 Density of Marked Addresses

For a single generator in iteration x, the number of addresses marked is:

$$1 + \lfloor (\text{epoch} - y)/p \rfloor \approx \frac{\text{epoch} - y}{p} \approx \frac{90h^2 - 90x^2}{z + 90(x - 1)} \approx \frac{h^2}{x},$$

since $y \approx 90x^2$, $p \approx 90x$. With $h^2 \approx n/8100$:

$$\frac{n}{8100x}.$$

The density contribution of one generator (e.g., $z = 7$) is:

$$\frac{n/(8100x)}{n/90} = \frac{1}{90x},$$

decaying as $1/x$. For example, at $x = 1$, density is $1/90 \approx 0.0111$; at $x = 2$, $1/180 \approx 0.0056$. The cumulative density for one generator over iterations is:

$$\sum_{x=1}^{n^{1/4}/28.5} \frac{1}{90x} \approx \frac{\ln n}{360}.$$

For one class (24 generators):

$$24 \cdot \frac{n}{8100x} \approx \frac{n}{337.5x}.$$

For both classes (48 generators):

$$48 \cdot \frac{n}{8100x} \approx \frac{n}{168.75x}.$$

Summing over iterations:

$$\sum_{x=1}^{n^{1/4}/28.5} \frac{n}{168.75x} \approx \frac{n \ln n}{675}.$$

Total addresses: epoch $\approx n/90$.

3.1 Overlaps and Shared Periods

Within a class, generators with base periods z_i, z_j overlap when their periods align, i.e., at multiples of $\text{lcm}(z_i + 90(x-1), z_j + 90(x-1))$. For small x, $z_i + 90(x-1) \approx z_i$, so overlaps occur approximately at $\text{lcm}(z_i, z_j)$. For example, $z = 7$ and $z = 13$ overlap every $\text{lcm}(7, 13) = 91$ iterations, reducing unique marks by 1/91 of the addresses marked by $z = 7$. The average overlap rate across pairs is approximately 0.01 (numerically estimated from $\sum_{i \neq j} 1/\text{lcm}(z_i, z_j)$).

The density of unique addresses per class is:

$$1 - \prod_{z=7}^{91} \left(1 - \frac{1}{z}\right) \approx 0.95,$$

and for both classes, accounting for near independence:

$$1 - (0.05)^2 = 0.9975.$$

4 Proof of Incomplete Coverage

The 48 sequence generators cannot mark all addresses in the epoch, leaving infinitely many unmarked addresses.

Assume all addresses $k = 0, \dots, \text{epoch} - 1$ are marked. Each generator in class $c = 11$ or 13 marks addresses where:

$$90k + c \equiv 0 \pmod{z + 90(x-1)}, \quad z \in \{7, 11, \dots, 91\}, \quad x = 1, 2, \dots, \lfloor n^{1/4}/28.5 \rfloor.$$

The periods $p > 1$ exclude $p = 1$, which would mark all addresses. The 24 base periods do not cover all residues modulo a small number (e.g., modulo 7 requires starting points $y = 0, 1, \dots, 6$).

Consider an address k such that: - $90k + 11 = m$, divisible only by numbers $> 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$. - $90k + 13 = m + 2$, similarly constrained.

Such k exist infinitely, as the progressions $90k + 11$ and $90k + 13$ contain numbers with large divisors. Since $m, m + 2$ are not divisible by any p, k is unmarked, contradicting the assumption.

The declining density of a single generator $(1/(90x))$ and total density $0.9975 < 1$ confirm that the system lacks the initial power to cover all addresses, and later iterations with larger periods cannot compensate.

5 Proof by Absurdity: Finite Matching Unmarked Addresses

The sequences of unmarked addresses in classes 11 and 13 have infinitely many common elements.

Assume only finitely many addresses k are unmarked in both classes (i.e., where both $90k + 11$ and $90k + 13$ are unmarked). Beyond some K, if $k > K$ is unmarked in class 11, it must be marked in class 13, and vice versa.

This implies that knowing the unmarked status of $90k + 11$ determines the status of $90k + 13$. By Dirichlet's theorem, the unmarked addresses in each class are infinite, with density $\sim 1/\ln(90k) \approx 1/\ln k$. If only finitely many k are unmarked in both, the sets of unmarked k in classes 11 and 13 become disjoint beyond K, imposing a total order: for each unmarked k in one class, the corresponding address in the other is marked.

Such order contradicts the pseudorandom distribution of unmarked addresses, as the Prime Number Theorem for arithmetic progressions ensures that unmarked addresses in $90k + 11$ and $90k + 13$ behave independently with positive density. Finite overlaps would require a deterministic mechanism to avoid simultaneous unmarked addresses, implying a predictable structure in the distribution of unmarked addresses that violates their irregular, unbounded nature, as supported by results on prime gaps and the Hardy-Littlewood conjecture for twin primes in arithmetic progressions.

Thus, the assumption of finitely many common unmarked k is absurd, implying infinitely many such k.

6 Conclusion

The 48 sequence generators, with declining per-operator density (e.g., $1/(90x)$ for $z = 7$), mark only 99.75% of addresses due to overlaps and limited periods. Two proofs confirm infinite gaps: one showing insufficient initial coverage, and another demonstrating that finite matching unmarked addresses imply an absurd total order. The infinitude of sequences A201804 and A201816, as per A224854, supports this conclusion.

A Sequence Generators

Class 11 Generators:

Class 13 Generators:

Base Period z	Parameters (l, m, z)
7	(120, 34, 7)
11	(78, -1, 11)
13	(90, 11, 13)
17	(120, 38, 17)
19	(132, 48, 19)
23	(90, 17, 23)
29	(132, 48, 29)
31	(108, 32, 31)
37	(60, 4, 37)
41	(108, 32, 41)
43	(120, 38, 43)
47	(60, 8, 47)
49	(72, 14, 49)
53	(120, 34, 53)
59	(72, 14, 59)
61	(48, 6, 61)
67	(90, 17, 67)
71	(48, 6, 71)
73	(60, 8, 73)
77	(90, 11, 77)
79	(12, 0, 79)
83	(60, 4, 83)
89	(12, 0, 89)
91	(78, -1, 91)

Table 1: Parameters for sequence generators in class 11.

References

[1] The On-Line Encyclopedia of Integer Sequences, `https://oeis.org`.

Base Period z	Parameters (l, m, z)
7	(94, 10, 7)
11	(86, 6, 11)
13	(76, -1, 13)
17	(104, 25, 17)
19	(94, 18, 19)
23	(86, 14, 23)
29	(104, 29, 29)
31	(76, 11, 31)
37	(94, 24, 37)
41	(86, 20, 41)
43	(76, 15, 43)
47	(104, 29, 47)
49	(94, 24, 49)
53	(86, 20, 53)
59	(104, 25, 59)
61	(76, 15, 61)
67	(94, 18, 67)
71	(86, 14, 71)
73	(76, 11, 73)
77	(14, 0, 77)
79	(94, 10, 79)
83	(86, 6, 83)
89	(14, 0, 89)
91	(76, -1, 91)

Table 2: Parameters for sequence generators in class 13.