Incomplete Coverage of the Number Line by 48 Discrete Sequence Generators

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Abstract

We investigate a system of 48 discrete sequence generators that produce arithmetic progressions with quadratic starting points and linear periods, organized into two residue classes modulo 90 (classes 11 and 13). Each class comprises 24 generators, defined by base periods coprime to 90, corresponding to numbers with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10. We prove that these generators cannot mark all integers within a specified range (epoch), leaving infinitely many unmarked addresses. By analyzing the density of generated addresses and employing a contradiction argument, we demonstrate that the system's coverage is strictly less than 1, ensuring persistent gaps. This result is supported by the infinitude of sequences associated with the system's structure, as documented in the On-Line Encyclopedia of Integer Sequences.

1 Introduction

We study a computational system that generates sequences of integers using 48 discrete sequence generators, partitioned into two classes corresponding to residue classes 11 and 13 modulo 90. Each class employs 24 generators, each defined by a base period $z \in \{7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91\}$, all coprime to 90. These base periods have digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, ensuring they are not divisible by 2, 3, or 5. The generators produce arithmetic progressions with quadratically determined starting points and linearly increasing periods over iterations x.

The central question is whether these 48 generators can collectively mark every integer within a range, termed the epoch, defined as a quadratic function of a parameter h. We hypothesize that the generators fail to cover all integers, leaving infinitely many unmarked addresses. This paper provides a formal analysis of the generators' coverage density and a proof of incomplete coverage, leveraging the system's structural properties.

2 The Sequence Generation System

The system operates over a range of integers, or addresses, defined by the epoch for a positive integer h:

$$\text{epoch} = 90h^2 - 12h + 1 \approx 90h^2.$$

Addresses are labeled $k = 0, 1, \ldots, \text{epoch} - 1$, corresponding to base-10 numbers $90k + c$, where $c = 11$ (class 11) or $c = 13$ (class 13). The base-10 range is:

$$n = 90 \cdot \text{epoch} + c \approx 8100h^2.$$

Each class employs 24 sequence generators, each characterized by parameters (l, m, z), where z is a base period. For iteration $x \geq 1$, a generator in class c produces a starting address:

$$y = 90x^2 - lx + m,$$

and a sequence of addresses:

$$y + p \cdot n, \quad n = 1, 2, \ldots, \lfloor (\text{epoch} - y)/p \rfloor,$$

where the period is:

$$p = z + 90(x - 1).$$

The period p generates numbers in the residue class $z \pmod{90}$, such as $p = 7, 97, 187, \ldots$ for $z = 7$. The number of iterations is bounded by:

$$\text{new}_l imit\text{new}_l imit_{\text{new}_l imit_{\text{new}_l imit}} \approx \frac{\sqrt{h}}{3} \approx \frac{(n/90)^{1/4}}{3}.$$

The 48 generators are defined as follows, with parameters (l, m, z):

Class 11 Generators:

Class 13 Generators:

Each base period z is coprime to 90, ensuring digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, excluding numbers divisible by 2, 3, or 5.

Each generator places marks at intervals of period p, starting at y. Each iteration x introduces 48 new marks, accumulating across iterations. The question is whether these marks cover every address in the epoch.

3 Density of Generated Addresses

For a generator in iteration x, the number of addresses marked is:

$$1 + \lfloor (\text{epoch} - y)/p \rfloor \approx \frac{90h^2 - 90x^2}{z + 90(x-1)} \approx \frac{90h^2}{90x} = \frac{h^2}{x},$$

since $y \approx 90x^2$, $p \approx 90x$. Given $h^2 \approx n/8100$:

$$\frac{n}{8100x}.$$

Base Period z	Parameters (l, m, z)
7	(120, 34, 7)
11	(78, -1, 11)
13	(90, 11, 13)
17	(120, 38, 17)
19	(132, 48, 19)
23	(90, 17, 23)
29	(132, 48, 29)
31	(108, 32, 31)
37	(60, 4, 37)
41	(108, 32, 41)
43	(120, 38, 43)
47	(60, 8, 47)
49	(72, 14, 49)
53	(120, 34, 53)
59	(72, 14, 59)
61	(48, 6, 61)
67	(90, 17, 67)
71	(48, 6, 71)
73	(60, 8, 73)
77	(90, 11, 77)
79	(12, 0, 79)
83	(60, 4, 83)
89	(12, 0, 89)
91	(78, -1, 91)

Table 1: Parameters for sequence generators in class 11.

For one class (24 generators):

$$24 \cdot \frac{n}{8100x} \approx \frac{n}{337.5x}.$$

For both classes (48 generators):

$$48 \cdot \frac{n}{8100x} \approx \frac{n}{168.75x}.$$

Summing over iterations $x = 1$ to

The 48 sequence generators cannot mark all addresses in the epoch, leaving infinitely many unmarked addresses.

Assume the 48 generators mark every address $k = 0, 1, \ldots, \text{epoch} - 1$. Each generator in class $c = 11$ or 13 marks addresses k where:

$$90k + c \equiv 0 \pmod{z + 90(x - 1)},$$

Base Period z	Parameters (l, m, z)
7	(94, 10, 7)
11	(86, 6, 11)
13	(76, -1, 13)
17	(104, 25, 17)
19	(94, 18, 19)
23	(86, 14, 23)
29	(104, 29, 29)
31	(76, 11, 31)
37	(94, 24, 37)
41	(86, 20, 41)
43	(76, 15, 43)
47	(104, 29, 47)
49	(94, 24, 49)
53	(86, 20, 53)
59	(104, 25, 59)
61	(76, 15, 61)
67	(94, 18, 67)
71	(86, 14, 71)
73	(76, 11, 73)
77	(14, 0, 77)
79	(94, 10, 79)
83	(86, 6, 83)
89	(14, 0, 89)
91	(76, -1, 91)

Table 2: Parameters for sequence generators in class 13.

for $z \in \{7, 11, \ldots, 91\}$, $x = 1, 2, \ldots, \lfloor n^{1/4}/28.5 \rfloor$.

Consider an address k such that: - $90k + 11 = m$, where m is divisible only by numbers $> 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$. - $90k + 13 = n$, where n is divisible only by numbers $> 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$.

Let:

$$90k + 11 = m, \quad 90k + 13 = m + 2.$$

Choose m larger than the maximum period. Since $m, m + 2$ are not divisible by any $z + 90(x - 1)$, k is unmarked, contradicting the assumption. Such k are infinite, and the density (0.9975) confirms gaps.

4 Mechanical Interpretation

Each generator places marks at intervals $p \approx 90x$, starting at $y \approx 90x^2$. The 48 marks per iteration accumulate to:

$$48 \cdot \frac{n^{1/4}}{28.5} \approx 1.68 n^{1/4}.$$

The ratio to addresses:
$$\frac{1.68n^{1/4}}{n/90} \approx \frac{151.2}{n^{3/4}},$$
approaches 0, indicating sparse coverage. The periods do not cover all residues, unlike a system covering all residues modulo a small number.

5 Connection to Known Sequences

The sequences A201804 (`https://oeis.org/A201804`) and A201816 (`https://oeis.org/A201816`) list addresses k where $90k+11$ and $90k+13$ are prime, corresponding to unmarked addresses. The system is described in A224854 (`https://oeis.org/A224854`). Their infinitude supports the proof of persistent gaps.

6 Conclusion

The 48 sequence generators mark 99.75% of addresses, leaving 0.25% unmarked. A contradiction proof confirms infinite gaps, supported by the sparse coverage ratio. The infinitude of A201804 and A201816 reinforces the system's limitations.

References

[1] The On-Line Encyclopedia of Integer Sequences, `https://oeis.org`.