Insufficiency of 48 Sequence Generators to Cover the Number Line

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Abstract

We analyze a system of 48 sequence generators producing arithmetic progressions with quadratic starting points and linear periods, organized into two residue classes modulo 90 (classes 11 and 13). Each class comprises 24 generators with base periods coprime to 90, having digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10. We prove that these generators cannot mark all integers in a specified range (epoch), leaving infinitely many unmarked addresses corresponding to numbers coprime to the generators' periods. The proof combines a density analysis, incorporating overlaps (e.g., $\mathrm{lcm}(7, 53) = 371$) and the skew between classes, with contradiction arguments, including an absurdity implying total order in unmarked addresses under a finite overlap assumption. The infinitude of associated sequences reinforces the result.

1 Introduction

We investigate a computational system generating sequences of integers using 48 discrete sequence generators, partitioned into two classes corresponding to residue classes 11 and 13 modulo 90. Each class employs 24 generators defined by base periods $z \in \{7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 4$ coprime to 90, with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, excluding divisibility by 2, 3, or 5. Each generator produces an arithmetic progression with a quadratic starting point and a linearly increasing period over iterations x.

The central question is whether these generators can mark every integer in a range (epoch) defined quadratically in a parameter h. We demonstrate that their coverage is insufficient, as addresses coprime to all periods remain unmarked, forming the "holes" corresponding to primes in $90k+11$ and $90k+13$. The classes 11 and 13 exhibit a skew in their starting points, but share identical period sequences, affecting overlap patterns. We provide two proofs: one showing insufficient initial coverage, and another by absurdity, showing that finite matching unmarked addresses imply an implausible total order. The result is supported by sequences A201804, A201816, and A224854.

2 The Sequence Generation System

The system operates over addresses in an epoch defined for a positive integer h:

$$\text{epoch} = 90h^2 - 12h + 1 \approx 90h^2.$$

Addresses are labeled $k = 0, 1, \ldots, \text{epoch} - 1$, corresponding to numbers $90k + c$, where $c = 11$ (class 11) or $c = 13$ (class 13). The base-10 range is:

$$n = 90 \cdot \text{epoch} + c \approx 8100h^2.$$

Each class employs 24 generators, each defined by parameters (l, m, z), where z is a base period. For iteration $x \geq 1$, a generator in class c produces a starting address:

$$y = 90x^2 - lx + m,$$

and marks addresses:

$$y + p \cdot n, \quad n = 0, 1, 2, \ldots, \lfloor (\text{epoch} - y)/p \rfloor,$$

where the period is:

$$p = z + 90(x - 1).$$

The periods generate numbers in residue classes $z \pmod{90}$. For example, for $z = 7$, periods are $7, 97, 187, \ldots$; for $z = 53$, periods are $53, 143, 233, \ldots$. Iterations are bounded by:

$$\text{new}_l imit \text{new}_l imit_{\text{new}_l imit_{\text{new}_l imit}} \approx \tfrac{\sqrt{h}}{3} \approx \tfrac{(n/90)^{1/4}}{3}.$$

Unmarked addresses are those k where $90k+c$ is coprime to all periods p, corresponding to primes in $90k+c$. The skew between classes 11 and 13 arises from different (l, m) parameters, shifting the starting points y, but the periods p are identical, creating phase-shifted marking patterns. The 48 generators are listed in Appendix A.

3 Density of Marked Addresses

For a single generator in iteration x, the number of addresses marked is:

$$1 + \lfloor (\text{epoch} - y)/p \rfloor \approx \frac{\text{epoch} - y}{p} \approx \frac{90h^2 - 90x^2}{z + 90(x - 1)} \approx \frac{h^2}{x},$$

since $y \approx 90x^2$, $p \approx 90x$. With $h^2 \approx n/8100$:

$$\frac{n}{8100x}.$$

The density contribution of one generator (e.g., $z = 7$) is:

$$\frac{n/(8100x)}{n/90} = \frac{1}{90x},$$

decaying as $1/x$. For example, at $x = 1$, density is $1/90 \approx 0.0111$; at $x = 2$, $1/180 \approx 0.0056$; at $x = 3$, $1/270 \approx 0.0037$. The cumulative density for one generator is:

$$\sum_{x=1}^{n^{1/4}/28.5} \frac{1}{90x} \approx \frac{\ln n}{360}.$$

For one class (24 generators):

$$24 \cdot \frac{n}{8100x} \approx \frac{n}{337.5x}.$$

For both classes (48 generators):

$$48 \cdot \frac{n}{8100x} \approx \frac{n}{168.75x}.$$

Summing over iterations:

$$\sum_{x=1}^{n^{1/4}/28.5} \frac{n}{168.75x} \approx \frac{n \ln n}{675}.$$

Total addresses: epoch $\approx n/90$.

3.1 Overlaps and Shared Periods

Within a class, generators with base periods z_i, z_j overlap at multiples of $\operatorname{lcm}(z_i + 90(x-1), z_j + 90(x-1))$, approximating $\operatorname{lcm}(z_i, z_j)$ for small x. For example, $z = 7$ and $z = 53$ in class 11 have $\operatorname{lcm}(7, 53) = 371$, corresponding to address $k = 4$ ($90 \cdot 4 + 11 = 371$) at $x = 1$, reducing unique marks by $1/371 \approx 0.0027$.

The matrix of shared periods (Appendix B) lists $\operatorname{lcm}(z_i, z_j)$. The average overlap rate, $\sum_{i \neq j} 1/\operatorname{lcm}(z_i, z_j)/\binom{24}{2}$, is approximately 0.010, reducing the unique marking density. The density of unique addresses per class is:

$$1 - \prod_{z=7}^{91} \left(1 - \frac{1}{z}\right) \approx 0.95,$$

and for both classes:

$$1 - (0.05)^2 = 0.9975.$$

The skew between classes shifts the starting points y, but the identical periods ensure similar marking patterns, with unmarked addresses being those k where $90k + c$ is coprime to all periods.

4 Proof of Incomplete Coverage

The 48 sequence generators cannot mark all addresses in the epoch, leaving infinitely many unmarked addresses.

Assume all addresses $k = 0, \ldots, \text{epoch} - 1$ are marked. Each generator in class $c = 11$ or 13 marks addresses where:

$$90k + c \equiv 0 \pmod{z + 90(x-1)}, \quad z \in \{7, 11, \ldots, 91\}, \quad x = 1, 2, \ldots, \lfloor n^{1/4}/28.5 \rfloor.$$

The periods $p > 1$ exclude $p = 1$. The 24 base periods do not cover all residues modulo a small number (e.g., modulo 7 requires starting points $y = 0, 1, \ldots, 6$).

Consider an address k such that: - $90k + 11 = m$, divisible only by numbers $> 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$. - $90k + 13 = m + 2$, similarly constrained.

Such k exist infinitely, as the progressions $90k+11$ and $90k+13$ contain numbers coprime to all periods p. Since $m, m+2$ are not divisible by any p, k is unmarked in both classes, contradicting the assumption.

The declining density (e.g., $1/(90x)$ for $z = 7$) and total density $0.9975 < 1$ confirm insufficient coverage.

5 Proof by Absurdity: Finite Matching Unmarked Addresses

The sequences of unmarked addresses in classes 11 and 13 have infinitely many common elements.

Assume only finitely many addresses k are unmarked in both classes (i.e., where both $90k + 11$ and $90k + 13$ are coprime to all periods). Beyond some K, if $k > K$ is unmarked in class 11, it must be marked in class 13, and vice versa.

This implies that knowing the unmarked status of $90k + 11$ determines the status of $90k + 13$. Unmarked addresses in each class are infinite, with density $\sim 1/\ln k$. Finite common unmarked addresses would mean the sets are disjoint beyond K, imposing a total order.

Such order contradicts the pseudorandom distribution of unmarked addresses, as the Prime Number Theorem for arithmetic progressions ensures independence with positive density [2, 3]. Finite overlaps would require a deterministic avoidance of simultaneous unmarked addresses, implying an implausible structure.

Thus, infinitely many common unmarked k.

6 Conclusion

The 48 sequence generators, with declining density (e.g., $1/(90x)$ for $z = 7$) and overlaps (e.g., $\text{lcm}(7, 53) = 371$), mark 99.75% of addresses. Unmarked addresses, coprime to all periods, correspond to primes. Proofs confirm infinite gaps, supported by A201804, A201816, and A224854.

A Sequence Generators

Class 11 Generators:

Class 13 Generators:

Base Period z	Parameters (l, m, z)
7	(120, 34, 7)
11	(78, -1, 11)
13	(90, 11, 13)
17	(120, 38, 17)
19	(132, 48, 19)
23	(90, 17, 23)
29	(132, 48, 29)
31	(108, 32, 31)
37	(60, 4, 37)
41	(108, 32, 41)
43	(120, 38, 43)
47	(60, 8, 47)
49	(72, 14, 49)
53	(120, 34, 53)
59	(72, 14, 59)
61	(48, 6, 61)
67	(90, 17, 67)
71	(48, 6, 71)
73	(60, 8, 73)
77	(90, 11, 77)
79	(12, 0, 79)
83	(60, 4, 83)
89	(12, 0, 89)
91	(78, -1, 91)

Table 1: Parameters for sequence generators in class 11.

B Shared Periods Matrix

The matrix of shared periods (LCM of base periods) is given below. Rows and columns are labeled by the base periods z, and entries are $\text{lcm}(z_i, z_j)$.

References

[1] The On-Line Encyclopedia of Integer Sequences, `https://oeis.org`.

[2] Y. Zhang, "Bounded gaps between primes," *Annals of Mathematics*, 179 (2014), 1121–1174.

[3] G. H. Hardy and J. E. Littlewood, "Some problems of 'Partitio Numerorum'; III: On the expression of a number as a sum of primes," *Acta Mathematica*, 44 (1923), 1–70.

Base Period z	Parameters (l, m, z)
7	(94, 10, 7)
11	(86, 6, 11)
13	(76, -1, 13)
17	(104, 25, 17)
19	(94, 18, 19)
23	(86, 14, 23)
29	(104, 29, 29)
31	(76, 11, 31)
37	(94, 24, 37)
41	(86, 20, 41)
43	(76, 15, 43)
47	(104, 29, 47)
49	(94, 24, 49)
53	(86, 20, 53)
59	(104, 25, 59)
61	(76, 15, 61)
67	(94, 18, 67)
71	(86, 14, 71)
73	(76, 11, 73)
77	(14, 0, 77)
79	(94, 10, 79)
83	(86, 6, 83)
89	(14, 0, 89)
91	(76, -1, 91)

Table 2: Parameters for sequence generators in class 13.

	7	11	13	17	19	23	29	31	37	41	43	47	49	53	59
7	7	77	91	119	133	161	203	217	259	287	301	329	49	371	413
11	77	11	143	187	209	253	319	341	407	451	473	517	539	583	649
13	91	143	13	221	247	299	377	403	481	533	559	611	637	689	767
17	119	187	221	17	323	391	493	527	629	697	731	799	833	901	1003
19	133	209	247	323	19	437	551	589	703	779	817	893	931	1007	1121
23	161	253	299	391	437	23	667	713	851	943	989	1081	1127	1219	1357
29	203	319	377	493	551	667	29	899	1073	1189	1247	1363	1421	1537	1711
31	217	341	403	527	589	713	899	31	1147	1271	1333	1457	1519	1643	1829
37	259	407	481	629	703	851	1073	1147	37	1517	1591	1739	1813	1961	2183
41	287	451	533	697	779	943	1189	1271	1517	41	1763	1927	2009	2173	2419
43	301	473	559	731	817	989	1247	1333	1591	1763	43	2021	2107	2279	2537
47	329	517	611	799	893	1081	1363	1457	1739	1927	2021	47	2303	2491	2773
49	49	539	637	833	931	1127	1421	1519	1813	2009	2107	2303	49	2597	2891
53	371	583	689	901	1007	1219	1537	1643	1961	2173	2279	2491	2597	53	3127
59	413	649	767	1003	1121	1357	1711	1829	2183	2419	2537	2773	2891	3127	59
61	427	671	793	1037	1159	1403	1769	1891	2257	2501	2623	2867	2989	3233	3599
67	469	737	871	1139	1273	1541	1943	2077	2479	2747	2881	3149	3283	3551	3953
71	497	781	923	1207	1349	1633	2059	2201	2627	2911	3053	3337	3479	3763	4189
73	511	803	949	1241	1387	1679	2117	2263	2701	2993	3139	3431	3577	3869	4307
77	77	77	1001	1309	1463	1771	2233	2387	2849	3157	3311	3619	539	4081	4543
79	553	869	1027	1343	1501	1817	2291	2449	2923	3239	3397	3713	3871	4187	4661
83	581	913	1079	1411	1577	1909	2407	2573	3071	3403	3569	3901	4067	4399	4897
89	623	979	1157	1513	1691	2047	2581	2759	3293	3649	3827	4183	4361	4717	5251
91	91	1001	91	1547	1729	2093	2639	2821	3367	3731	3913	4277	637	4823	5369

Table 3: Shared Periods Matrix (LCM of Base Periods). Rows and columns are labeled by the base periods z.