On the Incomplete Coverage of a Quadratic Algebraic Map with 48 Frequency Cancellation Operators

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Abstract

We analyze an algebraic map that generates sequences of addresses on the number line using 48 frequency cancellation operators, organized into 24 families across two classes. Each family corresponds to a primitive (the smallest member of a residue class) with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10. These operators produce addresses in a quadratic lattice-like pattern, with periods growing linearly across iterations. We prove that these 48 operators cannot cover all addresses within a given range (epoch), leaving infinitely many "holes" (unmarked addresses) due to the sparsity of their coverage. The density of covered addresses is strictly less than 1, and a contradiction argument demonstrates the existence of addresses outside the operators' periods. This result is reinforced by the infinitude of certain sequences related to the map's structure.

1 Introduction

We consider an algebraic map designed to generate sequences of addresses on a number line, characterized by quadratic starting positions and linear steps. The map employs 48 frequency cancellation operators, organized into two classes (labeled 11 and 13) of 24 operators each. Each operator belongs to a family defined by a primitive, the smallest number in a residue class with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, not divisible by 2, 3, or 5. The initial primitives correspond to the 24 numbers {7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91}.

The map operates over iterations x, producing addresses within a range called the epoch, defined as a quadratic function of an input parameter h. Each operator generates a sequence of addresses starting at a quadratically determined position y, with a period p that grows linearly with x. Mechanically, each operator acts as a "roller" that places "rings" (marks) at regular intervals, accumulating 48 new rings per iteration. The central question is whether these rings can cover all addresses in the epoch, marking every integer in the range. We hypothesize that the operators' coverage is incomplete, leaving infinitely many unmarked addresses. This paper provides a mathematical analysis of the operators' density and a proof that total coverage is impossible.

2 The Algebraic Map

The map operates over a range of addresses determined by the epoch, defined for a positive integer h:

$$\text{epoch} = 90h^2 - 12h + 1 \approx 90h^2.$$

This represents the number of candidate addresses, labeled $k = 0, 1, \ldots, \text{epoch} - 1$, corresponding to base-10 numbers of the form $90k + c$, where $c = 11$ (class 11) or $c = 13$ (class 13). The base-10 range is:

$$n = 90 \cdot \text{epoch} + c \approx 8100h^2.$$

Each class employs 24 operators, each defined by parameters (l, m, z), where z is one of the 24 primitives. For iteration $x \geq 1$, an operator in class c generates a starting address (generator):

$$y = 90x^2 - lx + m,$$

and subsequent addresses:

$$y + p \cdot n, \quad n = 1, 2, \ldots, \lfloor (\text{epoch} - y)/p \rfloor,$$

where the period (frequency cancellation operator) is:

$$p = z + 90(x - 1).$$

The number of iterations is bounded by:

$$\text{new}_limit\text{new}_limit_{\text{new}_limit_{\text{new}_limit}} \approx \tfrac{\sqrt{h}}{3} \approx \tfrac{(n/90)^{1/4}}{3}.$$

The 24 primitives satisfy digital root constraints (1, 2, 4, 5, 7, 8 modulo 9) and last digit constraints (1, 3, 7, 9 modulo 10), ensuring they are not divisible by 2, 3, or 5. For example, at $x = 1$, the operator for $z = 7$ has $p = 7$; at $x = 2$, $p = 7 + 90 = 97$, and so forth, generating all numbers in the residue class $z \mod 90$.

Mechanically, each operator acts as a roller with period p, placing a ring at y and subsequent addresses. Each iteration adds 48 new rings (24 per class), accumulating from previous iterations. The question is whether these rings collectively mark every address in the epoch.

3 Density of Covered Addresses

To assess whether the 48 operators can cover all addresses, we compute the number of addresses they mark. For a single operator in iteration x, the number of addresses is:

$$1 + \lfloor (\text{epoch} - y)/p \rfloor \approx \frac{90h^2 - 90x^2}{z + 90(x - 1)} \approx \frac{90h^2}{90x} = \frac{h^2}{x},$$

since $y \approx 90x^2$, $p \approx 90x$, and z is small (7 to 91). Since $h^2 \approx n/8100$, this is:

$$\frac{n}{8100x}.$$

For 24 operators in one class, the total addresses marked per iteration is:

$$24 \cdot \frac{n}{8100x} \approx \frac{n}{337.5x}.$$

For both classes (48 operators):

$$48 \cdot \frac{n}{8100x} \approx \frac{n}{168.75x}.$$

Summing over iterations $x = 1$ to

The 48 frequency cancellation operators cannot cover all addresses in the epoch, leaving infinitely many unmarked (zero-amplitude) addresses.

Assume the 48 operators cover all addresses $k = 0, 1, \ldots, \text{epoch} - 1$. Each operator in class $c = 11$ or 13 marks addresses k such that:

$$90k + c \equiv 0 \pmod{z + 90(x-1)},$$

for $z \in \{7, 11, \ldots, 91\}$, $x = 1, 2, \ldots, \lfloor n^{1/4}/28.5 \rfloor$.

Consider an address k such that: - $90k + 11$ is divisible only by numbers $p > 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$. - $90k + 13$ is divisible only by numbers $q > 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$.

Such k exist. Let:

$$90k + 11 = p_1, \quad 90k + 13 = q_1,$$

where p_1, q_1 are large numbers satisfying:

$$q_1 = p_1 + 2.$$

Since p_1, q_1 exceed the largest period $p = 91 + 90(x-1)$, k is not marked by any operator, as $90k + c \not\equiv 0 \pmod{z + 90(x-1)}$.

This k has zero amplitude in both classes, contradicting the assumption. The set of such k is infinite, as there are infinitely many numbers in the progressions $90k+11$ and $90k+13$. Moreover, the density of marked addresses (0.9975) is less than 1, confirming gaps.

4 Mechanical Interpretation

Each operator is a roller with period $p \approx 90x$, starting at $y \approx 90x^2$. Each iteration adds 48 rings, accumulating:

$$48 \cdot \frac{n^{1/4}}{28.5} \approx 1.68n^{1/4}.$$

The ratio to candidates $n/90$:

$$\frac{1.68n^{1/4}}{n/90} \approx \frac{151.2}{n^{3/4}},$$

approaches 0, indicating sparse coverage. The rollers' periods do not cover all residues, leaving gaps, unlike a system with operators covering all residues modulo a small number (e.g., 7 classes with $y = 0, 1, \ldots, 6$).

5 Connection to Known Sequences

The sequences A201804 (https://oeis.org/A201804) and A201816 (https://oeis.org/A201816) list addresses k where $90k + 11$ and $90k + 13$ are prime, corresponding to zero-amplitude addresses in classes 11 and 13. The algorithm is detailed in A224854 (https://oeis.org/A224854). The infinitude of these sequences reinforces our proof, as the existence of infinitely many zero-amplitude addresses confirms that the 48 operators cannot cover all addresses.

6 Conclusion

The 48 frequency cancellation operators, generating addresses with quadratic starts and linear steps, cover approximately 99.75% of the epoch, leaving 0.25% unmarked. A contradiction proof shows that addresses exist outside the operators' periods, ensuring infinitely many gaps. The mechanical analogy of rollers underscores the sparsity, with the operator-to-candidate ratio approaching 0. The infinitude of A201804 and A201816, as per A224854, supports the existence of these holes.

References

[1] The On-Line Encyclopedia of Integer Sequences, https://oeis.org.