Incomplete Coverage of the Number Line by a Quadratic Sequence Generation System

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Abstract

We analyze a computational system of 48 sequence generators that produce arithmetic progressions with starting points defined by quadratic recurrences and periods growing linearly, organized into two residue classes modulo 90 (classes 11 and 13). Each class comprises 24 generators, with base periods coprime to 90, having digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10. These generators mark addresses corresponding to composite numbers in specific residue classes. We prove that the system cannot mark all integers within a specified range (epoch), leaving infinitely many unmarked addresses. By computing the density of marked addresses and using a contradiction argument, we show that the coverage is strictly less than 1. The infinitude of certain sequences associated with the system reinforces this result.

1 Introduction

We study a computational system designed to generate sequences of integers using 48 discrete sequence generators, partitioned into two classes corresponding to residue classes 11 and 13 modulo 90. Each class employs 24 generators, defined by base periods $z \in \{7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91\}$, all coprime to 90, with digital roots in {1, 2, 4, 5, 7, 8} modulo 9 and last digits in {1, 3, 7, 9} modulo 10, ensuring they are not divisible by 2, 3, or 5. Each generator produces an arithmetic progression with a starting address determined by a quadratic recurrence, encoded as a quadratic polynomial, and a period that increases linearly across iterations x.

The system's purpose is to mark addresses k such that $90k + c$ (for $c = 11$ or 13) is composite, with specific digital root and last digit properties. The central question is whether these 48 generators can mark every integer in a range (epoch), defined as a quadratic function of a parameter h. We hypothesize that the generators' periods, always greater than 1, prevent complete coverage, leaving infinitely many unmarked addresses. This paper provides a formal analysis of the coverage density and a proof of incomplete coverage, emphasizing the recursive structure of the generators.

2 The Sequence Generation System

The system operates over a range of addresses defined by the epoch for a positive integer h:

$$\text{epoch} = 90h^2 - 12h + 1 \approx 90h^2.$$

Addresses are labeled $k = 0, 1, \ldots, \text{epoch} - 1$, corresponding to numbers $90k + c$, where $c = 11$ (class 11) or $c = 13$ (class 13). The base-10 range is:

$$n = 90 \cdot \text{epoch} + c \approx 8100h^2.$$

Each class employs 24 sequence generators, each defined by parameters (l, m, z), where z is a base period. For iteration $x \geq 1$, a generator in class c produces a starting address:

$$y = 90x^2 - lx + m,$$

representing the solution to a recursive function determining cancellation points, encoded as a quadratic polynomial for efficiency. The generator produces addresses:

$$y + p \cdot n, \quad n = 1, 2, \ldots, \lfloor (\text{epoch} - y)/p \rfloor,$$

where the period (cancellation frequency) is:

$$p = z + 90(x - 1).$$

For example, for $z = 7$, the periods are $p = 7, 97, 187, \ldots$, marking numbers in the residue class $7 \pmod{90}$. The number of iterations is bounded by:

$$\text{new}_l imit\text{new}_l imit_{\text{new}_l imit_{\text{new}_l imit}} \approx \tfrac{\sqrt{h}}{3} \approx \tfrac{(n/90)^{1/4}}{3}.$$

The base periods z are coprime to 90, ensuring digital roots in $\{1, 2, 4, 5, 7, 8\}$ modulo 9 and last digits in $\{1, 3, 7, 9\}$ modulo 10. For class 11, addresses k are marked such that $90k + 11$ is composite with digital root 2 and last digit 1, divisible by numbers in these residue classes. For example, the generators for $z = 7$ and $z = 53$ mark addresses where $90k + 11 \equiv 0 \pmod{7 + 90(x - 1)}$ or $\pmod{53 + 90(x - 1)}$, often paired with generators like $z = 59$ (digital root 5, last digit 9) to produce specific composite patterns.

The 48 generators are:

Class 11 Generators:

Class 13 Generators:

3 Density of Marked Addresses

For a generator in iteration x, the number of addresses marked is:

$$1 + \lfloor (\text{epoch} - y)/p \rfloor \approx \frac{90h^2 - 90x^2}{z + 90(x - 1)} \approx \frac{h^2}{x},$$

Base Period z	Parameters (l, m, z)
7	(120, 34, 7)
11	(78, -1, 11)
13	(90, 11, 13)
17	(120, 38, 17)
19	(132, 48, 19)
23	(90, 17, 23)
29	(132, 48, 29)
31	(108, 32, 31)
37	(60, 4, 37)
41	(108, 32, 41)
43	(120, 38, 43)
47	(60, 8, 47)
49	(72, 14, 49)
53	(120, 34, 53)
59	(72, 14, 59)
61	(48, 6, 61)
67	(90, 17, 67)
71	(48, 6, 71)
73	(60, 8, 73)
77	(90, 11, 77)
79	(12, 0, 79)
83	(60, 4, 83)
89	(12, 0, 89)
91	(78, -1, 91)

Table 1: Parameters for sequence generators in class 11.

since $y \approx 90x^2$, $p \approx 90x$. With $h^2 \approx n/8100$:

$$\frac{n}{8100x}.$$

For one class (24 generators):

$$24 \cdot \frac{n}{8100x} \approx \frac{n}{337.5x}.$$

For both classes (48 generators):

$$48 \cdot \frac{n}{8100x} \approx \frac{n}{168.75x}.$$

Summing over iterations $x = 1$ to

The 48 sequence generators cannot mark all addresses in the epoch, leaving infinitely many unmarked addresses.

Assume all addresses $k = 0, 1, \ldots, \text{epoch} - 1$ are marked. Each generator marks addresses where:

$$90k + c \equiv 0 \pmod{z + 90(x - 1)}.$$

Base Period z	Parameters (l, m, z)
7	(94, 10, 7)
11	(86, 6, 11)
13	(76, -1, 13)
17	(104, 25, 17)
19	(94, 18, 19)
23	(86, 14, 23)
29	(104, 29, 29)
31	(76, 11, 31)
37	(94, 24, 37)
41	(86, 20, 41)
43	(76, 15, 43)
47	(104, 29, 47)
49	(94, 24, 49)
53	(86, 20, 53)
59	(104, 25, 59)
61	(76, 15, 61)
67	(94, 18, 67)
71	(86, 14, 71)
73	(76, 11, 73)
77	(14, 0, 77)
79	(94, 10, 79)
83	(86, 6, 83)
89	(14, 0, 89)
91	(76, -1, 91)

Table 2: Parameters for sequence generators in class 13.

Consider k such that: - $90k + 11 = m$, divisible only by numbers $> 91 + 90 \cdot \lfloor n^{1/4}/28.5 \rfloor$. - $90k + 13 = m + 2$.

Since $m, m+2$ are not divisible by any period, k is unmarked, contradicting the assumption. The density (0.9975) confirms gaps.

4 Mechanical Interpretation

Each generator marks addresses at intervals $p \approx 90x$, starting at y, accumulating:

$$48 \cdot \frac{n^{1/4}}{28.5} \approx 1.68 n^{1/4}.$$

The ratio:

$$\frac{151.2}{n^{3/4}},$$

approaches 0, ensuring gaps.

5 Connection to Known Sequences

Sequences A201804 (`https://oeis.org/A201804`) and A201816 (`https://oeis.org/A201816`) list unmarked addresses, described in A224854 (`https://oeis.org/A224854`). Their infinitude supports the proof.

6 Conclusion

The 48 generators, driven by quadratic recurrences, mark 99.75% of addresses, leaving infinite gaps, as confirmed by A201804 and A201816.

References

[1] The On-Line Encyclopedia of Integer Sequences, `https://oeis.org`.