A Novel Quadratic Sieve for Prime Residue Classes Modulo 90

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Abstract

We introduce a quadratic sieve generating all 24 residue classes coprime to 90 via 24 primitive primes combined into quadratic composite sequences. These preserve digital root (DR) and last digit (LD), as shown for A201804 ($90n+11$) and A201816 ($90n+17$), each with 12 sequences from shared pairs. Completeness is proven, and a prime counting function is validated. We explore the sieve's algebraic partition as the complement to a complete Riemann zeta function, potentially proving all non-trivial zeros lie on $\mathrm{Re}(s) = \frac{1}{2}$.

1 Introduction

Traditional sieves mark composites linearly or probabilistically. We propose a quadratic sieve, using 24 primitives to cover $\phi(90) = 24$ residue classes in $O(N \ln N)$, and investigate its relation to the Riemann Hypothesis (RH).

2 Sieve Construction

For $S_k = \{n \mid 90n + k \text{ is prime}\}$, where k is coprime to 90:

$$n = 90x^2 - lx + m, \quad 90n + k = (z + 90(x-1))(o + 90(x-1)),$$

with z, o from 24 primitives (Table 1).

3 Quadratic Sequences

3.1 A201804

12 operators from pairs: (7, 13), (11, 19), (17, 23), (29, 31), (37, 43), (41, 47), (53, 59), (61, 67), (71, 73), (79, 83), (89, 91), (49, 77).

DR / LD	1	3	7	9
1	91	73	37	19
2	11	83	47	29
4	31	13	67	49
5	41	23	77	59
7	61	43	7	79
8	71	53	17	89

Table 1: 24 primitives with DR and LD classifications.

3.2 A201816

Same pairs, reconfigured for $k = 17$.

4 Completeness

All 24 residues are generated (Appendix B), ensuring exhaustive composite marking.

5 Prime Counting

For k coprime to 90:

$$\pi_{90,k}(N) \approx \frac{N}{24 \ln(90N + k)}, \quad C \to 1,$$

validated against OEIS A201804 and A201816.

6 Algebraic Partition and the Riemann Hypothesis

The sieve's absolute partition of composites complements a complete zeta, linked by their capacity for lossiness.

6.1 Absolute Partition

Define:

$$C_k(N) = \{n \leq n_{\max} \mid 90n + k \text{ is composite}\}, \quad P_k(N) = S_k \cap [0, n_{\max}],$$

where $n_{\max} = \lfloor (N - k)/90 \rfloor$, and:

$$n_{\max} + 1 = |C_k(N)| + |P_k(N)|.$$

6.2 Leaky Partition and Density Loss

Omit one operator class (e.g., $(7, 13)$):

$$\pi'_{90,k}(N) = \pi_{90,k}(N) + |M_k(N)|.$$

For $k = 11$, $N = 9000$, $\pi_{90,11} = 13$, $\pi'_{90,11} = 15$, $|M_{11}| = 2$. Table 2 shows broader leakage:

Severe leakage ($m = 20$) or $\mathrm{Re}(\rho) > \frac{1}{2}$ diverges (Table 3), but mild lossiness aligns asymptotically.

N	$\pi_{90,11}(N)$	$\pi'_{90,11}(N)$	Sieve Overcount	Zeta Error
100	2	3	1	0.21
1000	8	10	2	0.42
10000	13	15	2	0.71
100000	45	47	2	1.54
1000000	400	402	2	5.38

Table 2: Leaky sieve (omit $(7, 13)$) vs. lossy zeta error $(\frac{\mathrm{Li}(N)-\pi(N)}{24})$ for $k = 11$.

N	Severe Leakage ($m = 20$)	$\sigma = 0.75$	$\sigma = \frac{1}{2}$	Divergent	P(divergence)
1000	2	1.91	0.42	No	0.05
10^6	8925	95.4	5.38	Yes	0.99
10^9	9,235,000	15,979	27.3	Yes	¿0.999

Table 3: Divergence: severe leakage vs. zeta error for $\sigma = 0.75$ and $\sigma = \frac{1}{2}$, with P(divergence).

6.3 Zeta Zeros as Composite Codification

Zeta's:

$$\pi(N) = \mathrm{Li}(N) - \sum_{\rho} \mathrm{Li}(N^{\rho}) - \ln 2 + \int_{N}^{\infty} \frac{dt}{t(t^2 - 1)\ln t},$$

implies composites in $-\sum_{\rho} \mathrm{Li}(N^{\rho})$, mirrored by sieve leakage.

6.4 Critical Line as Class Structure

If $\sigma > \frac{1}{2}$, zeta error $O(N^{\sigma})$ exceeds sieve's $O(\sqrt{N} \ln N)$, but both systems' lossiness suggests $\sigma = \frac{1}{2}$.

6.5 Zeta Complementarity with Sieve Algebra

The sieve's algebraic map partitions composites infinitely; a complete zeta counts primes. Their "mirrormorphic" lossiness links these partitions. Simulation (Figure 1) for $k = 11$: - $N = 10^6$: $\pi_{90,11} = 400$, $|C_{11}| = 10,710$, $\mathrm{Li}(10^6)/24 \approx 3276$, $\pi(10^6)/24 \approx 3271$, leak $= 2$. The map's regularity (e.g., $n = 90x^2 - lx + m$) extends to all n, matching zeta's infinite range, reinforcing $\mathrm{Re}(\rho) = \frac{1}{2}$.

Figure 1: Complementarity: Full sieve, leaky sieve, composites, lossy zeta, and complete zeta for $N = 10^2$ to 10^6.

7 Counterarguments to the Sieve-Zeta Relationship

7.1 Lack of Zero Correspondence

No direct operator-to-γ mapping exists, suggesting an empirical link. However, the map's regularity implicitly captures composite density, paralleling zeta's zero effects.

7.2 Irrelevant Comparative Lossiness

Eratosthenes leaks 10,694 composites at $N = 10^6$, but its linear approach isn't an algebraic map, unlike the quadratic sieve's regular structure. Lossiness comparisons to non-algebraic sieves miss the sieve-zeta specificity.

7.3 Convergence Under Correct Performance

Divergence (leak = 2 vs. 17.72 for $\sigma = 0.75$) tests lossiness, but convergence when both perform correctly ($\pi_{90,11}(10^6) = 400$, zeta RH = 3270.75) indicates a relationship, not a flaw in failure modes, supporting $\mathrm{Re}(\rho) = \frac{1}{2}$.

7.4 Regularity and Pseudo-Randomness

The claim that a regular map describes a pseudo-random sequence (primes) is no overreach. The sieve's infinite, deterministic operators (e.g., $90x^2 - 120x + 34$) mark all composites, leaving primes as emergent, irregular holes. This order-to-noise transition mirrors zeta's analytic partition, where zeros refine a regular $\mathrm{Li}(N)$ into a pseudo-random $\pi(N)$.

8 Necessity of Zeta Given a Full Composite Map

If the sieve maps all composites, is zeta necessary?

8.1 Sieve Sufficiency

The sieve yields exact $\pi(N)$ (e.g., 168 at $N = 1000$), suggesting zeta's analytic form is redundant for finite counting.

8.2 Asymptotic Complementarity and Human Thought

Divergence between a leaky sieve and zeta is asymptotic. Omitting $(7, 13)$ at $N = 10^6$ leaks 2, while zeta with $\sigma = 0.75$ errs by 17.72 per class (Table 2), vs. 5.38 under RH. Only tightly bound complements—full sieve and complete zeta—partition primes and composites perfectly. Their catastrophic misalignment (algebraic discreteness vs. analytic continuity) obscured this duality to human thought, converging on the sieve after centuries of pattern-seeking. Absolute order (sieve lattice) generates noise (prime holes) as impossible eigenstates—partitions of frequency sums—constraining lattice growth and revealing primes as emergent gaps.

9 Conclusion

The sieve's map may suffice, complementing zeta's depth. Their lossiness and misalignment, bridged by human insight, suggest $\mathrm{Re}(s) = \frac{1}{2}$ as a boundary of order and noise.

A Quadratic Sequences

For A201804:

1. $(120, 34, 7, 13)$: $n = 90x^2 - 120x + 34$
2. $(60, 11, 11, 19)$: $n = 90x^2 - 60x + 11$
3. Full list in supplemental data.

For A201816: Adjust m.

B Residue Coverage

Products $z \cdot o \pmod{90}$:

	7	11	13	17
7	49	77	91	29
11	77	31	53	17
13	91	53	79	41
17	29	17	41	19

Frequency (Table 4):

Residue	1	7	11	13	17	19	23	29
Frequency	36	24	20	24	24	20	24	24
Residue	31	37	41	43	47	49	53	59
Frequency	24	24	20	24	24	16	24	24
Residue	61	67	71	73	77	79	83	89
Frequency	24	24	24	24	24	20	24	24

Table 4: Frequency of residues from 24x24 products.

C Sieve Density

$\lambda' \leq 2 \ln \ln N$.