# COMPUTING APPLICATIONS FOR ENGINEERS

## PALS 2 Solutions

① (i)

<u>1st order</u> (use $3 < x < 5$ since finding for $f(4)$)

$x_0 = 3$, $x_1 = 5$

$$\frac{f_1(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f_1(4) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$= 5.25 + \frac{19.75 - 5.25}{5 - 3}(4 - 3)$$

$$= 12.5$$

<u>2nd order</u> (choose $x_0 = 2$, $x_1 = 3$ & $x_2 = 5$)

$$f_2(4) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0) = 4$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{5.25 - 4}{3 - 2} = 1.25$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$= \frac{\dfrac{19.75 - 5.25}{5 - 3} - \dfrac{5.25 - 4}{3 - 2}}{5 - 2}$$

$$= 2$$

$$f_2(4) = b_0 + b_1(4-2) + b_2(4-2)(4-3)$$
$$= 4 + 1.25(2) + 2(2)(1)$$
$$= 10.5$$

(ii)

<u>1st order</u>

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$f_1(4) = \frac{4-5}{3-5}(5.25) + \frac{4-3}{5-3}(19.75)$$

$$= 12.5$$

<u>2nd order</u>

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$f_2(4) = \frac{(4-3)(4-5)}{(2-3)(2-5)}(4) + \frac{(4-2)(4-5)}{(3-2)(3-5)}(5.25) + \frac{(4-2)(4-3)}{(5-2)(5-3)}(19.75)$$

$$= 10.5$$

(2) (ii)

Linear splines

Since the objective is to determine the value of $o$ when $T = 27°C$

$T_0 = 24$ , $o(24) = 8.418$

$T_1 = 32$ , $o(32) = 7.305$

$$o(T) = o(T_0) + \frac{o(T_1) - o(T_0)}{T_1 - T_0}(T - T_0)$$

$$o(27) = 8.418 + \frac{7.305 - 8.418}{32 - 24}(27 - 24)$$

$$= 8.000625 \text{ mg/L}$$

# Quadratic splines

$$\theta(T) = a_1T^2 + b_1T + C_1 \quad , \quad 0 \le T \le 8$$
$$= a_2T^2 + b_2T + C_2 \quad , \quad 8 \le T \le 16$$
$$= a_3T^2 + b_3T + C_3 \quad , \quad 16 \le T \le 24$$
$$= a_4T^2 + b_4T + C_4 \quad , \quad 24 \le T \le 32$$
$$= a_5T^2 + b_5T + C_5 \quad , \quad 32 \le T \le 40$$

$$a_1(0)^2 + b_1(0) + C_1 = 14.621 \quad \text{—— } ①$$
$$a_1(8)^2 + b_1(8) + C_1 = 11.843 \quad \text{—— } ②$$
$$a_2(8)^2 + b_2(8) + C_2 = 11.843 \quad \text{—— } ③$$
(11)
$$a_2(16)^2 + b_2(16) + C_2 = 9.870 \quad \text{—— } ④$$
$$a_3(16)^2 + b_3(16) + C_3 = 9.870 \quad \text{—— } ⑤$$
(12)
$$a_3(24)^2 + b_3(24) + C_3 = 8.418 \quad \text{—— } ⑥$$
$$a_4(24)^2 + b_4(24) + C_4 = 8.418 \quad \text{—— } ⑦$$
(13)
$$a_4(32)^2 + b_4(32) + C_4 = 7.305 \quad \text{—— } ⑧$$
$$a_5(32)^2 + b_5(32) + C_5 = 7.305 \quad \text{—— } ⑨$$
(14)
$$a_5(40)^2 + b_5(40) + C_5 = 6.413 \quad \text{—— } ⑩$$

* Differentiating eqn 2 & 3 with respect to T should should give the same answer at the same point since both equation should give the same gradient value at the same point

$$2a_1(8) + b_1 - 2a_2(8) - b_2 = 0 \qquad (11)$$

$$2a_2(8) + b_2 - 2a_3(8) - b_3 = 0 \qquad (12)$$

$$2a_3(8) + b_3 - 2a_4(8) - b_4 = 0 \qquad (13)$$

$$2a_4(8) + b_4 - 2a_5(8) - b_5 = 0 \qquad (14)$$

Assume 2nd derivative is zero at the first point

$$2a_1 = 0 \qquad (15)$$

* Putting the 15 equations into the MATLAB script, the equations below were obtained.

$$
\begin{aligned}
o(T) &= 0 - 0.3473T + 14.621, & 0 \le T \le 8 \\
&= 0.0126T^2 - 0.5485T + 15.4260, & 8 \le T \le 16 \\
&= 0.0069T^2 - 0.4577T + 15.4260, & 16 \le T \le 24 \\
&= 0.0052T^2 - 0.4305T + 15.753, & 24 \le T \le 32 \\
&= 0.0042T^2 - 0.4146T + 16.2616, & 32 \le T \le 40
\end{aligned}
$$

Since we are estimating $o(27)$, hence the equation for the range of $T$ of $24 \leq T \leq 32$ was used

$$o(27) = 0.0052(27)^2 - 0.4305(27) + 15.753$$
$$= 7.9203$$