ENG 60104 COMPUTING APPLICATIONS FOR ENGINEERS

PALS 1 Solutions

Q1

$$0 = 1 - \frac{Q^2}{g A_c^3} B$$

$$0 = 1 - \frac{20^2}{9.81(3x + \frac{x^2}{2})^3}(3+x)$$

$$0 = 9.81(3x + \frac{x^2}{2})^3 - 20^2(3+x)$$

$\boxed{\text{BISECTION METHOD}}$

$x_l = 0.5$

$x_u = 2.5$

$$x_r = \frac{x_l + x_u}{2} = \frac{0.5 + 2.5}{2} = 1.5$$

$$f x_l = 9.81(3(0.5) + \frac{0.5^2}{2})^3 - 400(3+0.5) = -1357.9$$

$$f x_r = 9.81(3(1.5) + \frac{1.5^2}{2})^3 - 400(3+1.5) = -54.0308$$

since $f x_l \times f x_r > 0$

$\therefore x_l = x_r = 1.5$

$$x_r = \frac{1.5 + 2.5}{2} = 2$$

$$\varepsilon_r = \left| \frac{2 - 1.5}{2} \right| \times 100\% = 25\%$$

$$f_{x_l} = 9.81\left(3(1.5) + \frac{1.5^2}{2}\right)^3 - 400(3+1.5) = -54.0308$$

$$f_{x_r} = 9.81\left(3(2) + \frac{2^2}{2}\right)^3 - 400(3+2) = 3022.7$$

Since $f_{x_l} * f_{x_r} < 0$

$\therefore \quad X_u = X_r = 2$

$$X_r = \frac{1.5 + 2}{2} = 1.75$$

$$\varepsilon_r = \left| \frac{1.75 - 2}{2} \right| \times 100\% = 12.5\%$$

$$f_{x_l} = 9.81\left(3(1.5) + \frac{1.5^2}{2}\right)^3 - 400(3+1.5) = -54.0308$$

$$f_{x_r} = 9.81\left(3(1.75) + \frac{1.75^2}{2}\right)^3 - 400(3+1.75) = 1159.1$$

Since $f_{x_l} * f_{x_r} < 0$

$\therefore \quad X_u = X_r = 1.75$

$$X_r = \frac{1.5 + 1.75}{2} = 1.625$$

$$\varepsilon_r = \left| \frac{1.625 - 1.75}{1.625} \right| \times 100\% = 7.69\%$$

$$f_{x_l} = 9.81\left(3(1.5) + \frac{1.5^2}{2}\right)^3 - 400(3+1.5) = -54.0308$$

$$f_{x_r} = 9.81\left(3(1.625) + \frac{1.625^2}{2}\right)^3 - 400(3+1.625) = 482.6988$$

Since $f_{x_l} * f_{x_r} < 0$

$\therefore \quad X_u = X_r = 1.625$

$$X_r = \frac{1.5 + 1.625}{2} = 1.5625$$

$$\varepsilon_r = \left| \frac{1.5625 - 1.625}{1.5625} \right| \times 100\% = 4\%$$

FALSE POSITION METHOD

$x_l = 0.5$

$x_u = 2.5$

$$f_{x_l} = 9.81\left(3(0.5) + \frac{0.5^2}{2}\right)^3 - 400(3+0.5)$$
$$= -1357.9$$

$$f_{x_u} = 9.81\left(3(2.5) + \frac{2.5^2}{2}\right)^3 - 400(3+2.5)$$
$$= 9566.7$$

$$x_r = x_u - \frac{f_{x_u}\,(x_l - x_u)}{f_{x_l} - f_{x_u}}$$

$$= 2.5 - \frac{9566.7\,(0.5-2.5)}{-1357.9 - 9566.7} = 0.7486$$

$$f_{x_r} = 9.81\left(3(0.7486) + \frac{0.7486^2}{2}\right)^3 - 400(3+0.7486)$$
$$= -1341.3$$

Since $f_{x_l} \times f_{x_r} > 0$

$\therefore\ x_l = x_r = 0.7486$

$\boxed{x_l = 0.7486\ ,\quad x_u = 2.5}$

$f_{x_l} = -1341.3$

$f_{x_u} = 9566.7$

$$x_r = 2.5 - \frac{9566.7\,(0.7486 - 2.5)}{-1341.3 - 9566.7} = 0.9640$$

$$\varepsilon_r = \left|\frac{0.9640 - 0.7486}{0.9640}\right| \times 100\% = 22.3\%$$

$f_{x_r} = -1214.6$

Since $f_{x_l} \times f_{x_r} > 0$

$\therefore\ x_l = x_r = 0.9640$

$\boxed{x_l = 0.9640 , \ x_u = 2.5}$

$f_{x_l} = -1214.6$

$f_{x_u} = 9566.7$

$$x_r = 2.5 - \frac{9566.7(0.9640 - 2.5)}{-1214.6 - 9566.7} = 1.137$$

$$\varepsilon_r = \left|\frac{1.137 - 0.9640}{1.137}\right| \times 100\% = 15.21\%$$

$f_{x_r} = -999.559$

Since $f_{x_l} \times f_{x_r} > 0$

$\therefore \ x_l = x_r = 1.137$

$\boxed{x_l = 1.137, \ x_u = 2.5}$

$f_{x_l} = -999.559$

$f_{x_u} = 9566.7$

$$x_r = 2.5 - \frac{9566.7(1.137 - 2.5)}{-999.559 - 9566.7} = 1.266$$

$$\varepsilon_r = \left|\frac{1.266 - 1.137}{1.266}\right| \times 100\% = 10.19\%$$

$f_{x_r} = -751.921$

Since $f_{x_l} \times f_{x_r} > 0$

$\therefore \ x_l = x_r = 1.266$

$\boxed{x_l = 1.266, \ x_u = 2.5}$

$f_{x_l} = -751.92$

$f_{x_u} = 9566.7$

$$x_r = 2.5 - \frac{9566.7(1.266 - 2.5)}{-751.92 - 9566.7} = 1.356$$

$$\varepsilon_r = \left|\frac{1.356 - 1.266}{1.356}\right| \times 100\% = 6.637\%$$

$fx_r = -525.42$

Since $fx_l \times fx_r > 0$

$\therefore \quad x_l = x_r = 1.356$

$$\boxed{x_l = 1.356, \quad x_u = 2.5}$$

$fx_l = -525.42$

$fx_u = 9566.7$

$$x_r = 2.5 - \frac{9566.7\,(1.356 - 2.5)}{-525.42 - 9566.7} = 1.416$$

$$\varepsilon_r = \left| \frac{1.416 - 1.356}{1.416} \right| \times 100\% = 4.237\%$$

Q2

$\boxed{\text{NEWTON - RAPHSON METHOD}}$

$$f(x) = 0.95x^3 - 5.9x^2 + 10.9x - 6$$

$$f'(x) = 2.85x^2 - 11.8x + 10.9$$

$$X_0 = 3.5$$

$$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$$

$$= 3.5 - \frac{0.95(3.5)^3 - 5.9(3.5)^2 + 10.9(3.5) - 6}{2.85(3.5)^2 - 11.8(3.5) + 10.9}$$

$$= 3.366$$

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

$$= 3.366 - \frac{0.95(3.366)^3 - 5.9(3.366)^2 + 10.9(3.366) - 6}{2.85(3.366)^2 - 11.8(3.366) + 10.9}$$

$$= 3.345$$

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$$

$$= 3.345 - \frac{0.95(3.345)^3 - 5.9(3.345) + 10.9(3.345) -}{2.85(3.345)^2 - 11.8(3.345) + 10.9}$$

$$= 3.364$$

SECANT METHOD

$X_0 = 2.5$

$X_1 = 3.5$

$f(x_0) = 0.95(2.5)^3 - 5.9(2.5)^2 + 10.9(2.5) - 6$

$= -0.78125$

$f(x_1) = 0.95(3.5)^3 - 5.9(3.5)^2 + 10.9(3.5) - 6$

$= 0.60625$

$$x_2 = x_1 - f(x_1)\left(\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right)$$

$$= 3.5 - 0.60625\left(\frac{3.5 - 2.5}{0.60625 - (-0.78125)}\right)$$

$= 3.063$

$f(x_2) = 0.95(3.063)^3 - 5.9(3.063)^2 + 10.9(3.063) - 6$

$= -0.666$

$$x_3 = 3.063 - (-0.666)\left(\frac{3.063 - 3.5}{-0.666 - 0.60625}\right)$$

$= 3.292$

$f(x_3) = 0.95(3.292)^3 - 5.9(3.292)^2 + 10.9(3.292) - 6$

$= -0.165$

$$x_4 = 3.292 - (-0.165)\left(\frac{3.292 - 3.063}{-0.165 - (-0.666)}\right)$$

$= 3.367$

Q3

$$\begin{bmatrix} 0 & -7 & 5 \\ 0 & 4 & 7 \\ 4 & -3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ -40 \end{bmatrix}$$

$\boxed{\text{CRAMER's RULE}}$

$$D = \begin{vmatrix} 0 & -7 & 5 \\ 0 & 4 & 7 \\ 4 & -3 & 7 \end{vmatrix}$$

$$= 0\,\left(4(7) - (7)(-3)\right) - (-7)\left(0(7) - 7(4)\right)$$

$$+ 5\,\left(0(-3) - 4(4)\right)$$

$$= 0 + 7\,(-28) + 5\,(-16)$$

$$= -276$$

$$x_1 = \frac{\begin{vmatrix} 50 & -7 & 5 \\ -30 & 4 & 7 \\ -40 & -3 & 7 \end{vmatrix}}{-276}$$

$$= \frac{50(49) - (-7)(70) + 5(250)}{-276}$$

$$= -15.181$$

$$X_2 = \frac{\begin{vmatrix} 0 & 50 & 5 \\ 0 & -30 & 7 \\ 4 & -40 & 7 \end{vmatrix}}{-276}$$

$$= \frac{0 - 50(-28) + 5(120)}{-276}$$

$$= -7.246$$

$$X_3 = \frac{\begin{vmatrix} 0 & -7 & 50 \\ 0 & 4 & -30 \\ 4 & -3 & -40 \end{vmatrix}}{-276}$$

$$= \frac{0 - (-7)(120) + 50(-16)}{-276}$$

$$= -0.145$$

GAUSS ELIMINATION METHOD

$$\left[\begin{array}{ccc|c} 0 & -7 & 5 & 50 \\ 0 & 4 & 7 & -30 \\ 4 & -3 & 7 & -40 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 4 & -3 & 7 & -40 \\ 0 & 4 & 7 & -30 \\ 0 & -7 & 5 & 50 \end{array}\right] \quad 4R_3 + 7R_2$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 4 & -3 & 7 & -40 \\ 0 & 4 & 7 & -30 \\ 0 & 0 & 69 & -10 \end{array}\right]$$

$\therefore$

$$69x_3 = -10$$
$$x_3 = -0.1449$$

$$4x_2 + 7x_3 = -30$$
$$x_2 = \frac{-30 - 7(-0.1449)}{4} = -7.246$$

$$4x_1 - 3x_2 + 7x_3 = -40$$
$$x_3 = \frac{-40 - 7(-0.1449) + 3(-7.246)}{4}$$
$$= -15.181$$