Sliding Mode Controller
where R, L, C, V0 > 0. The control signal u can only take on binary values, i.e., u {0, 1}.

This is a test!

Look at this.

$x_2 = -2x_1$

$x_2 = 2x_1$

Sliding Mode Controller
where R, L, C, V0 > 0. The control signal u can only take on binary values, i.e., u {0, 1}.

Test
consider s = x1 x 1 and u = 1 2 (1 sgn(s))
Investigate the behavior on s = 0 using equivalent control.

$$\dot{\underline{x}} = A\underline{x} + \underline{b}(\underline{x})u, \quad \underline{x} \in \mathbb{R}^2,\ u \in \mathbb{R}$$

$$A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} \sin^2 y \\ \cos^2 y \end{bmatrix}, \quad y = [1\ 1]\,\underline{x}.$$

This is a test!

Look at this nice formula

Colored text

Exercise:

$\dot{\underline{x}} = 0$

Determine EP for unforced System: $u = 0$

$\dot{\underline{x}}^* = 0$

Filippovs method:

$\dot{x}_{av} = \alpha \underline{f}^+ + (1-\alpha)\underline{f}^-$

$\alpha = \dfrac{ds}{\dfrac{dx}{ds} f^-}$

$\dfrac{}{dx}(f^- - f^+)$

$s = x_1 - x_1^*$

ds

$$\overline{dx} = [1 \quad 0]^T$$

new controller $u^+ = \frac{1}{2}(1-1) = 0$

$u^- = \frac{1}{2}(1+1) = 1$

$$f^+ = \begin{bmatrix} -\frac{1}{L}x_2 \\ \frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{bmatrix}$$

$$f^- = \begin{bmatrix} -\frac{1}{L}x_2 + \frac{V_0}{L} \\ \frac{1}{C}x_1 - \frac{1}{R_L}x_2 \end{bmatrix}$$

$$\alpha = \frac{-\frac{1}{L}x_2 + \frac{V_0}{L}}{\underline{V_0}}$$

$\begin{bmatrix} eq_1 \\ eq_2 \end{bmatrix}^L$

u^-, u^+

$= \ldots$

$= s > 0 \quad u = \frac{1}{2}(1 - \text{sign}(s))$

$s < 0 \quad u$

⋮

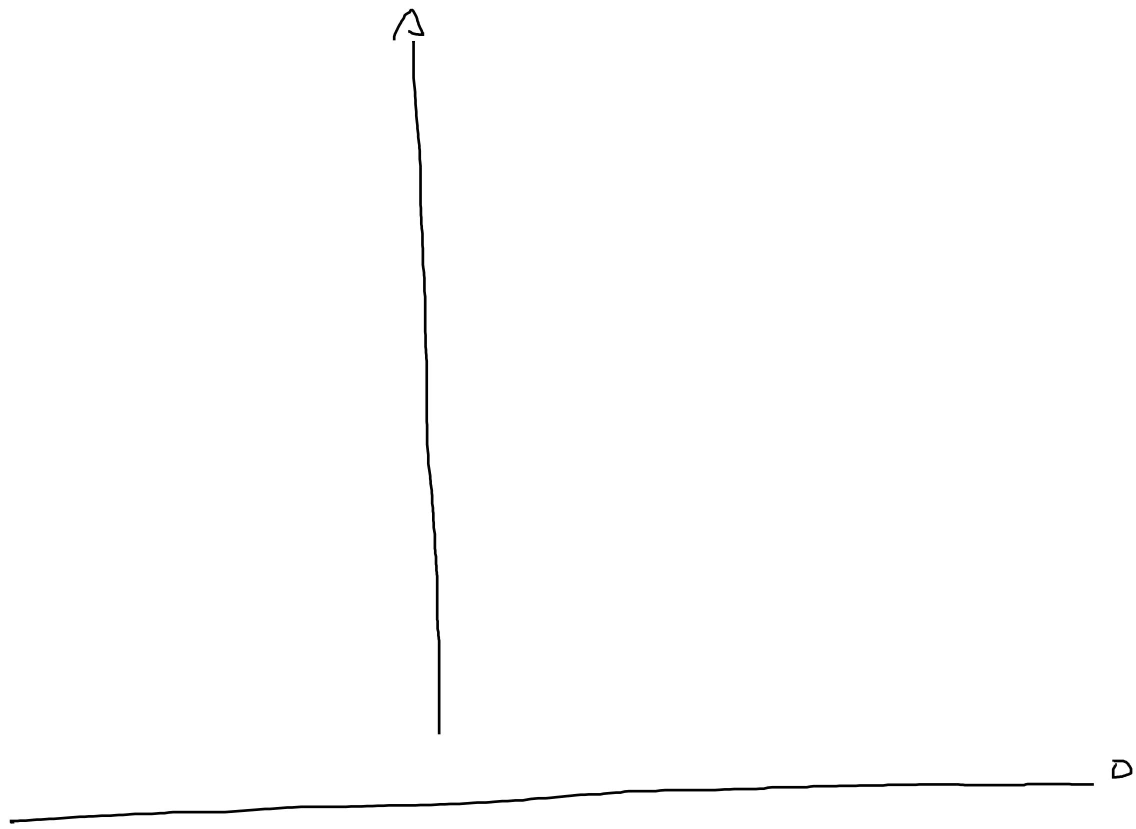
$= \frac{1}{2}\left(1 - \text{sing}(s)\right)$

1.3. Sketch behaviour of $x_2(t)$

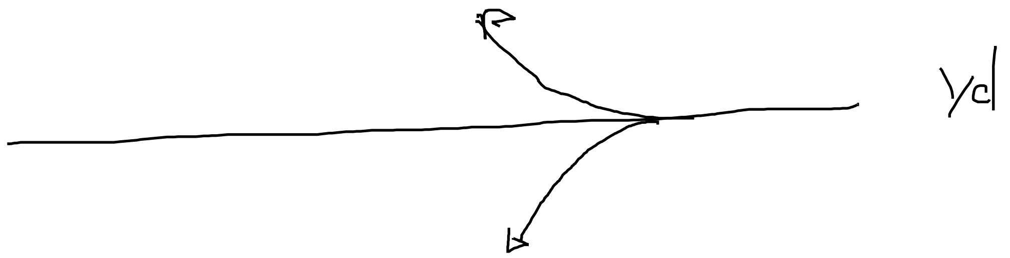
$s = 0: \quad x_1 = \lambda_1^* = \frac{V_p}{R}$

$\dot{x}_2 = \frac{1}{C} x_1 - \frac{1}{RC} x_2 = \frac{V_p}{RC} - \frac{1}{RC} x_2$

$x_2(t) = V_d + \left(x_2(0) - V_d\right) e^{-\frac{t}{RC}}$



$t \to \infty$, $x_2 = y_d$



1

$x_2(0) < y_d$

$x_2(0) > y_d$

$x_2(t)$

1.4 Case study for existence

$$s > 0 :$$
$$\dot{s} = \dot{x}_1 = -\frac{1}{L}x_2 + \frac{V_0}{L}u^+ \qquad u^+ = 0$$
$$= -\frac{1}{L}x_2 < 0$$
$$\Omega_1 = \{x_2 : x_2 > 0\}$$
$$s < 0 : \dot{s} = \dot{x}_1 = -\frac{1}{L}x_2 + \frac{V_0}{L}u^-, \quad u^- = 0$$
$$= -\frac{1}{L}x_2 + \frac{V_0}{L} > 0$$
$$\Omega_2 = \{x_2 : x_2 < V_0\}$$

$$\lim_{s \to 0^+} -\frac{1}{L} x_2 < 0 \qquad \Omega_3 = \Omega_1$$

$$\lim_{s \to 0^-} -\frac{1}{L} x_2 + \frac{V_0}{L} > 0 \qquad \Omega_4 = \Omega_2$$

$$\Omega = \Omega_1 \wedge \Omega_2 \wedge \Omega_3 \wedge \Omega_4 \quad \Big| \quad x_2 : 0 < x_2 < V_0$$

|

$u \quad \Omega \quad s$

$=$

iding mode exists
limit value
equals to
itself

Additional excerase:

$$-2x_1^* - 6u x_1^* = 0 \qquad \text{if } u \neq \frac{1}{3} \quad x_1^* = 0$$

$$x_2^* = 0$$

1)

2) $u = \frac{1}{3}$

second equation holds

- orever -> no information

2

$x_1^* + 3x_2^* = 0$

$x_1^* = t, \; x_2^* = -\frac{2}{3}t \quad x^* = \begin{bmatrix} t \\ -\frac{2}{3}t \end{bmatrix}$

2.2 - Lyapunov indirect method

if $Re(\lambda(A)) < 0$ then the system

(

would be stable

2. Approximation with Taylor

1. Determine matrix A

2. Determine $\lambda(A)$ solving $|\lambda I - A| = 0$

3. Check weather all $Re(\lambda(A)) < 0$

Z.3. $S = x_1 + x_2$, $u = sign(x_1 s)$

$s > 0$: $\dot{s} = \dot{x}_1 + \dot{x}_2 = \dots$

$x_1 < 0$, $x_1 s < 0$, $u = sign(x_1 s) = -1$

Then $\dot{s} =$

Ω_1

$s < 0$ Ω_2: $\dot{s} = \dots$

Limits:

3.3 Investigate the existence and stability of sliding mode for the controller $u = -u_0\,\mathrm{sign}(s_2)$ with $s_2 = i - i^*$.

Z4. α:

This formula is super important!

$x_2 = -2x_1$

$x_2 = 2x_1$