The Three Body Problem

Michael Dymek

Western Washington University

June 2nd 2021

The Three Body Problem

Given the initial positions and velocities of three point-masses, determine how the positions of each body evolves over time.

This problem does not have a general analytic solution the way that two bodies does.

Theory

When solving for the motion of an object under given forces, we use Newton's second law

$$\vec{a} = \frac{\vec{F}}{m}.$$

Between any two bodies, the gravitational force between them is

$$F_G = \frac{Gm_1 m_2}{r^2}\hat{r},$$

where $G = 6.67 \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2$ is the gravitational constant.

Theory

Then, for an arbitrary n number of bodies, we can calculate the x and y components of acceleration using the following formula. The gravitational force acting on the u^{th} body is

$$\vec{F}_{x,u} = \sum_{u \neq v} \frac{Gm_v m_u}{r_{u,v}^2} \frac{\Delta x_{u,v}}{r_{u,v}} = \sum_{u \neq v} \frac{Gm_v m_u \Delta x_{u,v}}{r_{u,v}^3}$$
$$F_{y,u} = \sum_{u \neq v} \frac{Gm_v m_u}{r_{u,v}^2} \frac{\Delta y_{u,v}}{r_{u,v}} = \sum_{u \neq v} \frac{Gm_v m_u \Delta y_{u,v}}{r_{u,v}^3}$$

This is shown for two dimensions, but it generalizes up nicely.

Theory

Then we'll use the Euler-Cromer method for solving the equation of motion. That is

$$v_{x,y}[n+1] = v_{x,y}[n] + a_{x,y}[n]\Delta t$$
$$s_{x,y}[n+1] = s_{x,y}[n] + v_{x,y}[n+1]\Delta t$$

This differs from the Euler method in how we use the $(n+1)^{\text{th}}$ velocity in calculating the $(n+1)^{\text{th}}$ position.

Stable Solution

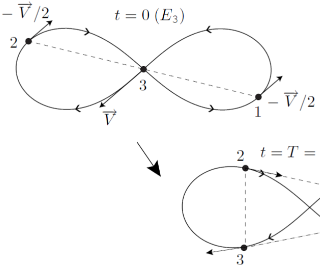

Figure: Analytically proven stable solution to three body problem. Calculated under the conditions where $G = 1$, $r_1 = -r_2 = (0.970, -0.243)$, $r_3 = (0, 0)$, $v_3 = -2v_1 = -2v_2 = (-0.932, -0.865)$, and $m_1 = m_2 = m_3 = 1$.

The main goal of my project is to numerically duplicate these results that have been mathematically shown to exist.

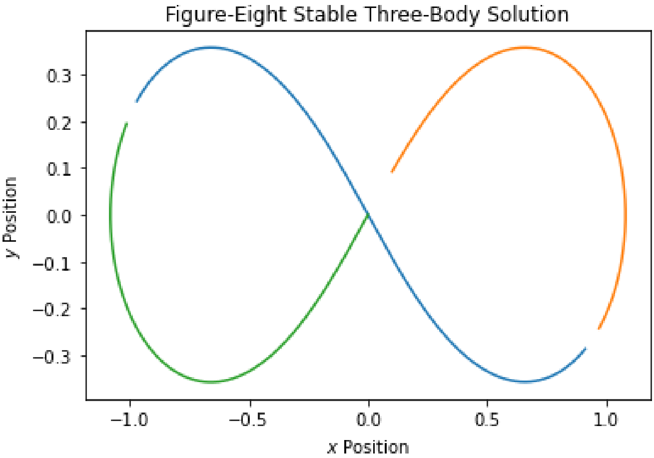

Figure: Simulation run under the conditions where $G = 1$, $r_1 = -r_2 = (0.970, -0.243)$, $r_3 = (0, 0)$, $v_3 = -2v_1 = -2v_2 = (-0.932, -0.865)$, and $m_1 = m_2 = m_3 = 1$.

Extensions

Where I want to go from here

- Investigating whether or not this orbit is stable.
- Use this simulation to determine the average distance of each planet from each other.