Hierarchical population systems

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March 1, 2016

Modeling population dynamics

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- Three species: Foxes $\rightarrow$ rabbits $\rightarrow$ grass

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- Three cases: Deterministic, stochastic, spatial

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- Three species: Foxes $\rightarrow$ rabbits $\rightarrow$ grass
- Three cases: Deterministic, stochastic, spatial
- Parameters principally equivalent, although numerically different

Equations

$$\frac{\partial F}{\partial t} = \alpha_1 \frac{R(1-K_1F)}{1+K_2R}F - \alpha_2 F$$
$$\frac{\partial R}{\partial t} = -\alpha_3 \frac{R(1-K_1F)}{1+K_2R}F + \alpha_4 \frac{G(1-K_3R)}{1+K_4G}R - \alpha_5 R$$
$$\frac{\partial G}{\partial t} = -\alpha_6 \frac{G(1-K_3R)}{1+K_4G}R + \alpha_7(1-K_5G)G$$

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\end{aligned}
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- Theme: Logarithmic growth proportional to population size, limited derivatives

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\end{aligned}
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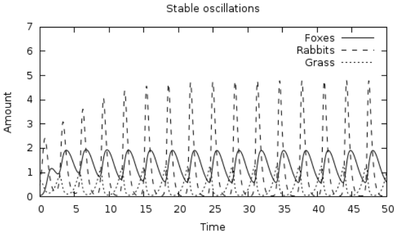
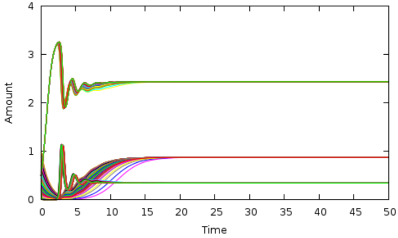
- Theme: Logarithmic growth proportional to population size, limited derivatives
- Also diffusion of animals in spatial model

Results – Deterministic

- Oscillations and convergence





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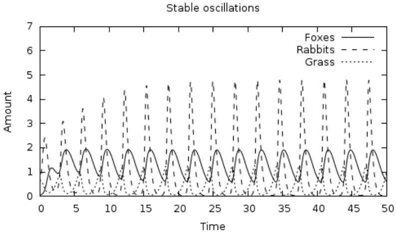
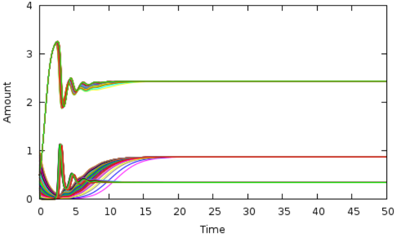
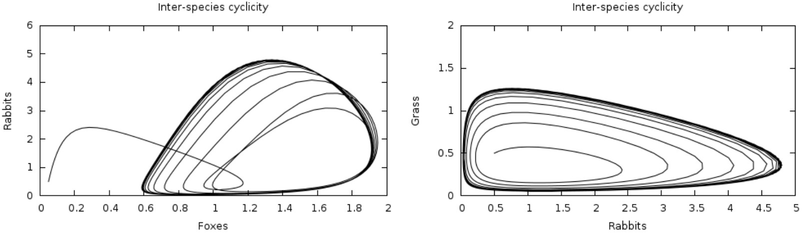




- Sanity check: Convergence abides to our equations

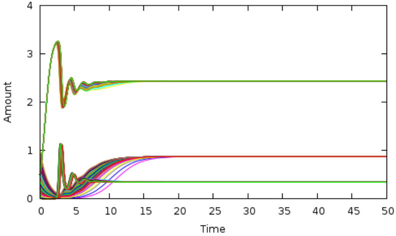
Results – Deterministic

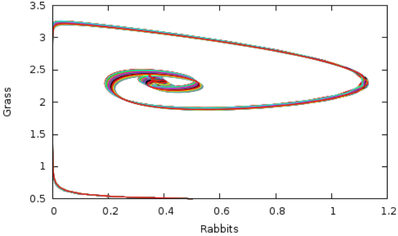
- Cyclic attractors (oscillations)



Results – Deterministic

- Spiral attractors (convergence)





Results – Stochastic

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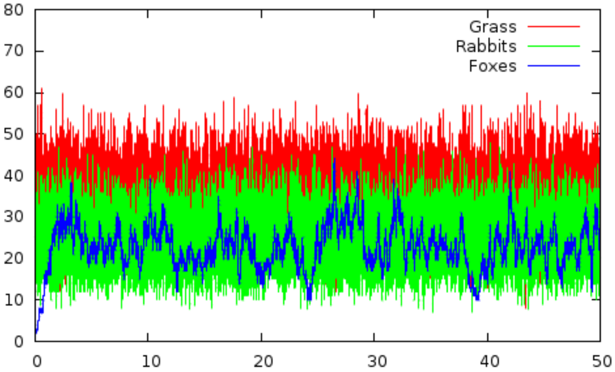
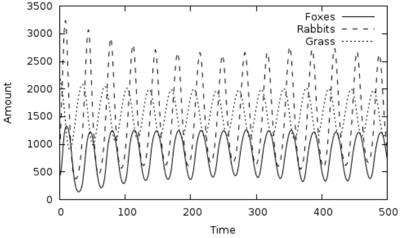
Results – Stochastic

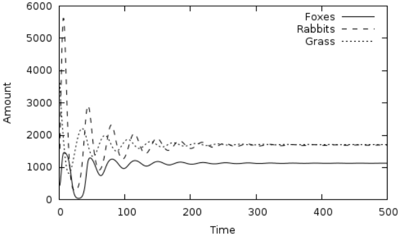
- "Stable" oscillations



Results – Spatial

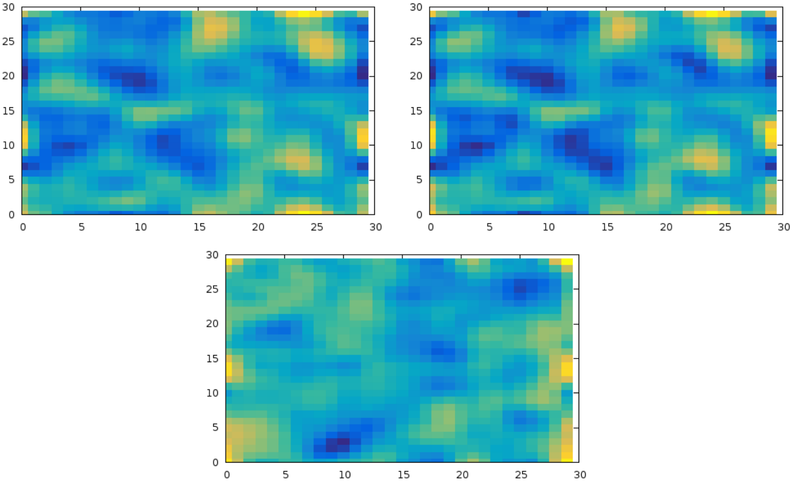
- Stable oscillations and convergence





Results – Spatial

- Typical case: Foxes, rabbits, grass



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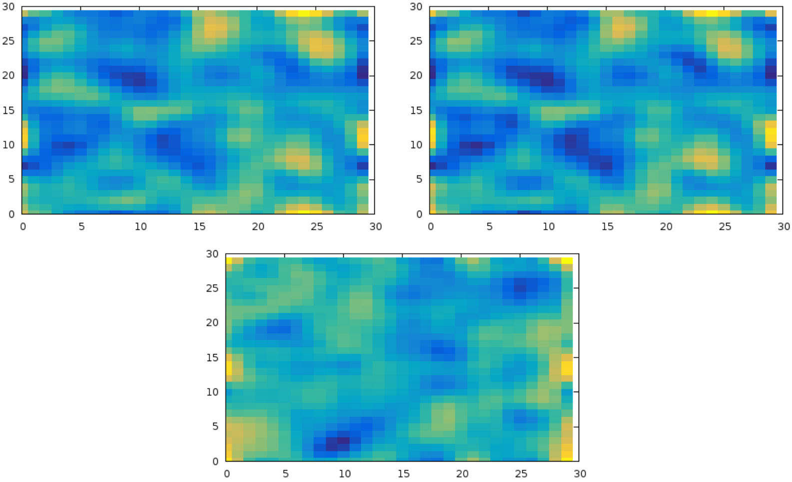

- Stabilization in value tends to mean stabilization in patterning

Code

- Implemented in Java
- Deterministic: RK-4
- Stochastic: Gillespie
- Spatial: Forward + Forward-Central Euler