Energy-Flow Cosmology (EFC)
Formal Mathematical Specification

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Abstract

This document provides the complete mathematical specification of Energy-Flow Cosmology (EFC). It defines the entropy coordinate, the energy-flow field, the structural and dynamical equations, and all related constructs used in the computational implementation found in `src/efc/`.

Contents

1 Notation

The Energy-Flow Cosmology (EFC) framework uses the following central quantities:

- $S(\mathbf{x})$: entropy field or entropy coordinate.
- ∇S : entropy gradient, source of structural asymmetry.
- $E_f(\mathbf{x})$: energy-flow field; local density of directional energy flux.
- $\Phi(E_f, S)$: energy-flow potential governing structure.
- $H(E_f, S)$: expansion mapping derived from the flow-strength.
- $v(r)$: circular velocity used in SPARC validation.
- z : cosmological redshift.
- $\rho(\mathbf{x})$: matter/energy distribution (only used for comparison).

Coordinates:

- $\mathbf{x}$: spatial 3-vector.
- r : radial coordinate for rotation curve modelling.

2 Model Parameters

EFC uses a small set of structural and dynamical parameters. These are conceptually defined here; numerical defaults appear in `src/efc/parameters.py`.

2.1 Entropy Parameters

- $S_{\min}$, $S_{\max}$: bounds of the entropy coordinate.
- k_S : scaling factor controlling the strength of entropy gradients.

2.2 Energy-Flow Parameters

- $E_{f,0}$: reference scale for the energy-flow field.
- A_Φ : normalization constant for the energy-flow potential.

2.3 Expansion Parameters

- A_H : normalization constant for the expansion mapping H.
- α : optional exponent controlling non-linearity of the expansion law.

These parameters form the bridge between theory and numerics and determine the scaling of all observables.

3 EFC-S: Entropy Field and Gradient

The EFC-S sector defines the entropy field $S(\mathbf{x})$. This field encodes local structural organization and acts as a non-mass-based driver of curvature-like behavior.

3.1 Entropy Field

The entropy field is treated as a scalar function:

$$S : \mathbb{R}^3 \to [S_{\min}, S_{\max}]. \tag{1}$$

The field can be interpreted as a measure of information or organizational density in configuration space.

3.2 Entropy Gradient

The entropy gradient is:

$$\nabla S(\mathbf{x}) = \left(\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z} \right). \tag{2}$$

The scaled physical gradient used in the model is:

$$\mathbf{g}_S = k_S \nabla S. \tag{3}$$

This gradient introduces directional asymmetry which couples to the energy-flow field and affects structure formation.

4 EFC-D: Energy-Flow Dynamics

The energy-flow field $E_f(\mathbf{x})$ represents the directional flow of energy across entropy gradients.

4.1 Energy-Flow Field

We define:

$$E_f : \mathbb{R}^3 \to \mathbb{R}. \tag{4}$$

Physically, E_f measures how strongly energy is "channeled" along structures created by gradients in S.

4.2 Energy-Flow Potential

EFC defines an effective potential:

$$\Phi(E_f, S) = A_\Phi \, E_f \, (1 + S). \tag{5}$$

This is the baseline form used in the Python implementation:

$$\Phi \propto E_f S + E_f. \tag{6}$$

More general forms (not yet implemented) include:

$$\Phi = A_\Phi E_f (1 + S^\beta), \tag{7}$$

where β controls non-linear coupling.

4.3 Circular Velocity Relation

For rotation curves:

$$v(r) = \sqrt{r \frac{\partial \Phi}{\partial r}}. \tag{8}$$

This connects the potential directly to SPARC data.

5 EFC-H: Expansion Mapping

The expansion rate H in EFC is not derived from density as in ΛCDM, but from the interaction of energy-flow and entropy.

5.1 Baseline Expansion Law

The current implementation defines:

$$H(E_f, S) = A_H \sqrt{|E_f|}\,(1 + S). \tag{9}$$

This links:

- entropy to redshift evolution,
- energy-flow to effective curvature,
- structural conditions to expansion behaviour.

5.2 Generalized Form

A more general version can be written as:

$$H(E_f, S) = A_H |E_f|^{\alpha/2} (1 + S)^{\gamma}, \tag{10}$$

with tunable exponents α, γ.

6 EFC-Structure: Halo Formation and Φ-Equilibria

Structure formation in EFC is governed by equilibrium conditions in the energy-flow potential.

6.1 Structural Equation

Stable configurations satisfy the condition:

$$\nabla \Phi(E_f, S) = 0. \tag{11}$$

This defines EFC halos.

6.2 Entropy–Flow Coupling

Including the entropy gradient gives:

$$\nabla \Phi + S \nabla S = 0. \tag{12}$$

This yields profiles of the form:

$$\Phi(r) = A_\Phi E_f(r)(1 + S(r)). \tag{13}$$

These profiles predict rotation curves without dark matter and large-scale distribution without a cosmological constant.

7 Observables and Validation Mapping

7.1 JWST Predictions

The structural growth function $G(E_f, S)$ determines the abundance and luminosity of early galaxies. Higher entropy gradients accelerate early structure formation.

7.2 DESI / BAO

The expansion law produces:

$$H(z) = H(E_f(z), S(z)). \tag{14}$$

This maps directly to DESI measurements of the expansion curve.

7.3 SPARC Rotation Curves

Given:

$$\Phi(r) = A_\Phi E_f(r)(1 + S(r)), \tag{15}$$

rotation curves follow:

$$v(r) = \sqrt{r \frac{d\Phi}{dr}}. \tag{16}$$

This is computed numerically in `efc_validation.py`.