THE BEST ACT PREP COURSE EVER

COMPLEX AND RATIONAL NUMBERS

ACT Math: Problem Set

1. i is a complex number and n is an integer. Which of the following is not a possible value of i^{n+1}?

A. 1
B. i
C. 0
D. -1
E. $-i$

2. Maria is finding the zeroes of a polynomial, and the quadratic formula gives $x = 9 \pm \sqrt{-16c^2}$. If c is a non-negative number, what is x written as a complex number?

A. $9 \pm 4c$
B. $9 \pm 4ci$
C. $9 + 4c$
D. $9 - 4ci$
E. $9 + 4ci$

3. If $i = \sqrt{-1}$, then what does $\frac{1-i}{1+i} * \frac{-1}{1-i}$ =?

A. $\frac{1+i}{2}$
B. $1-i$
C. $\frac{-1-i}{2}$
D. $\frac{i-1}{2}$
E. $-1+i$

4. For all pairs of nonzero real numbers x and y, the product of the complex number $x - yi$ and which of the following complex numbers is a real number?

A. $x + yi$
B. $x - yi$
C. xyi
D. $y - xi$
E. $x + i$

5. The product of two numbers is 41. One of the numbers is the complex number $5+4i$. What is the other number?

A. $5-4i$
B. $5+4i$
C. $-5-4i$
D. $-5+4i$
E. $\frac{41}{5-4i}$

6. Which equation given in factored form has the roots $\frac{1}{4}, \frac{2}{3}, i$, and $-i$?

A. $(4x-1)(3x-2)(x^2-1)$
B. $(4x+1)(3x-2)(x^2-1)$
C. $(4x-1)(3x-2)(x^2+1)$
D. $(4x-1)(3x+2)(x^2+1)$
E. $(4x-1)(3x+2)(x^2-1)$

7. For the complex number i such that $i^2 = -1$, what is the value of $i^8 - 2i^2 - 1$?

A. -4
B. -2
C. 0
D. 2
E. 4

8. What is the sum of $\sqrt{-20}$ and $\sqrt{-125}$?

A. $-7i\sqrt{5}$
B. $7i\sqrt{5}$
C. $-21i\sqrt{5}$
D. $21i\sqrt{5}$
E. $i\sqrt{105}$

9. What is the square of the complex number $(2i-4)$?

A. $12-16i$
B. $20-16i$
C. -20
D. 12
E. 20

10. For $i^2 = -1$, $(3-i)^2 = ?$

A. 8
B. 10
C. $8-6i$
D. $8+6i$
E. $10-6i$

11. The solution set for the equation $3^{x^2+3}-1=0$ contains:

A. Only 1 imaginary numbers
B. Only 2 imaginary number
C. 1 imaginary and 1 real number
D. 1 negative real number and 1 imaginary number
E. 1 real number, which is 0.

12. For all $x<0$, which of the following expressions is equivalent to $\dfrac{\sqrt{x}}{\sqrt{x}-i}$?

A. $\dfrac{x-\sqrt{x}}{x-1}$

B. $\dfrac{x+\sqrt{x}}{x-1}$

C. $\dfrac{-x-\sqrt{x}}{-x+1}$

D. $\dfrac{x+\sqrt{-x}}{x+1}$

E. $\dfrac{\left(x-\sqrt{x}\right)}{x+1}$

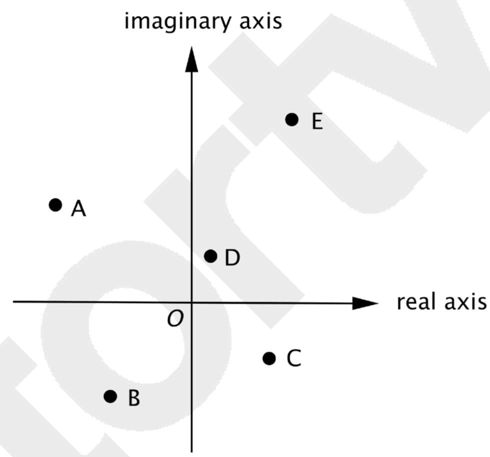
13. Which of the following expressions is equivalent to $9x^2+169$?

A. $(3x+13)^2$
B. $(3x+13i)^2$
C. $(3x-13i)^2$
D. $(3x-13)(3x+13)$
E. $(3x-13i)(3x+13i)$

14. What complex number equals $(3-4i)(\pi+3i)$?

A. $(12+3\pi)i+(9-4\pi)$
B. $(12-3\pi)+(9-4\pi)i$
C. $(12+3\pi)+(9+4\pi)i$
D. $(12+3\pi)+(9-4\pi)i$
E. $(12-3\pi)i+(9-4\pi)i$

15. The figure below depicts a complex plane with the horizontal axis representing real values and the vertical axis representing imaginary values. The modulus of a complex number $a+bi$ is $\sqrt{a^2+b^2}$. By looking at the points below, which point has the smallest modulus?



A. A
B. B
C. C
D. D
E. E

ANSWER KEY

1. C **2. B** **3. D** **4. A** **5. A** **6.C** **7. D** **8. B** **9. A** **10. C** **11. B** **12. E** **13. D** **14. D** **15. D**

ANSWER EXPLANATIONS

1. **C.** By definition, i is the complex number equal to $\sqrt{-1}$, and when taken to a power, equals one of four possible answers $(i, -1, -i, \text{ or } 1)$. For **A.** $i^4 = \sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1} = (-1)(-1) = 1$. **B.** is wrong because $i^1 = \sqrt{-1} = i$; **D.** $i^2 = \sqrt{-1}\sqrt{-1} = -1$ **E.** ; $i^3 = \sqrt{-1}\sqrt{-1}\sqrt{-1} = -1\sqrt{-1} = -i$. As such, the answer is **C.** Also, the only number that can equal 0 when taken to any power is zero. Therefore, 0 cannot be the answer.

2. **B.** $x = 9 \pm \sqrt{-16c^2} = 9 \pm 4c\sqrt{-1}$. Since $i^2 = -1$, we can rewrite $\sqrt{-1} = i$. This gives us $x = 9 \pm 4ci$.

3. **D.** Canceling out $1-i$ from the numerator and denominator, we get $-\frac{1}{1+i}$. To get the i term out of the denominator, we multiply the fraction by the conjugate on the top and bottom to get $-\frac{1}{1+i}\left(\frac{1-i}{1-i}\right) = -\frac{1-i}{1+i-i+1} = -\frac{1-i}{2} = \frac{i-1}{2}$.

4. **A.** The product of a complex number and its conjugate is a real number because the $+$ and $-$ in front of the imaginary terms cancel out when foiled. So, $(x-yi)(x+yi)$ will be equal to a real number. To verify this, we foil the factors and get $x^2 - xyi + xyi - y^2i^2 = x^2 - (-1)y^2 = x^2 + y^2$. This is a real number because x and y are real numbers.

5. **A.** The product of a complex number and its conjugate is a real number because the $+$ and $-$ in front of the imaginary terms cancel out when foiled. So, the only way $(5-4i)$ multiplied by something can yield a real number will be when it is multiplied by its conjugate. To verify, we multiply $(5-4i)(5+4i)$ and get $25 - 4i(5) + 5(4i) - 16i^2 = 25 - (-1)16$ $\to 25 + 16 \to 41$.

6. **C.** If a polynomial has roots equal to $\frac{1}{4}, \frac{2}{3}, i$, and $-i$, it means that these terms make the polynomial equal to zero when plugged in. So, the following equations must be true: $x - \frac{1}{4} = 0 \to 4x - 1 = 0$, $x - \frac{2}{3} = 0 \to 3x - 2 = 0$, $x - i = 0$, and $x + i = 0$. So, the polynomial can be written as $(4x-1)(3x-2)(x-i)(x+i)$. Multiplying the last two factors by foil, we get $(4x-1)(3x-2)(x^2 - xi + xi -\!\!-1) = (4x-1)(3x-2)(x^2+1)$.

7. **D.** The first term, $i^8 = (i^2)^4 = (-1)^4 = 1$. The second term, $-2i^2 = -2(-1) = 2$. By substituting these into the equation, we get $1 + 2 - 1$, which equals 2.

8. **B.** We can break down the square roots to $\sqrt{(-1)(4)(5)}$ and $\sqrt{(-1)(25)(5)}$. The $\sqrt{-1}$'s become i, and the perfect squares become their square roots. Thus, we get $2i\sqrt{5}$ and $5i\sqrt{5}$. Their sum is $7i\sqrt{5}$.

9. **A.** By FOILing, we get $(2i-4)^2 = 2i * 2i - 8i - 8i + 16$. $2i * 2i$ is equal to $4 * -1 = -4$, so simplifying, we get $-4 - 16i + 16$. We combine the integers to get $12 - 16i$.

10. C. By FOILing, we get $(3-i)^2 = 3*3-3i-3i-i*i$. Simplifying gets us $9-6i+i^2$. We are given that $i^2=-1$, so plugging that in: $9-6i-1=8-6i$.

11. A. In order to satisfy the equation, 3^{x^2+3} must equal 1. An exponential function only equals 1 when its exponent is equal to 0. Solve for $x^2+3=0$. This becomes $x^2=-3$. x must then be equal to $i\sqrt{3}$ and $-i\sqrt{3}$. There are 2 imaginary numbers in the solution set.

12. D. In order to simplify the expression, we multiply the top and bottom of the fraction by the conjugate of the denominator. Since the denominator is $\sqrt{x}-i$, its conjugate is $\sqrt{x}+i$. Our expression now becomes $\frac{\sqrt{x}\left(\sqrt{x}+i\right)}{\left(\sqrt{x}-i\right)\left(\sqrt{x}+i\right)}$. Distributing the top and FOILing the bottom gives us $\frac{x+i\sqrt{x}}{x+i\sqrt{x}-i\sqrt{x}-i^2}$, which we can simplify to $\frac{x+\sqrt{-x}}{x+1}$.

13. E. $9x^2+169$ can be expressed as the product of complex conjugates. The first term is the square of $3x$, and the second term is the product of $13i$ and $-13i$. Thus, we can set up our equation as $(3x+13i)(3x-13i)$. The 'O' and 'I' of FOILing cancel each other out, leaving us with $9x^2+169$.

14. D. Multiplying the expression out using foil, we get $(3-4i)(\pi+3i)=3\pi-4i\pi+9i-(-1)12$. Now, separating the real and imaginary terms, we get $3\pi+12-4\pi i+9i=(12+3\pi)+(9-4\pi)i$.

15. D. The modulus of the complex number is essentially the distance from the origin to the point. This can be seen since the value of the modulus, $\sqrt{a^2+b^2}$, is the Pythagorean theorem, which is used to find the distance of a point from the origin. Thus, we can tell what the smallest modulus is by seeing which point is the closest to the origin. In this case, it's D.