

THE BEST ACT PREP COURSE EVER

SEQUENCES AND SERIES

ACT Math: Problem Set

1. Stacy and Stephanie are building a 4-level square pyramid out of wooden blocks for a school project. Each of the levels consists of a consecutive, perfect square amount of blocks with the top level having 4 blocks. How many blocks must Stacy and Stephanie use?
A. 54
B. 90
C. 29
D. 28
E. 14
2. Christine started a savings account in order to save up money to get laser eye surgery to correct her vision. She deposits \$25 into her account the 1st month. Each following month, she deposits \$15 more than the amount she deposited the previous month. So, Christine deposits \$40 the 2nd month, \$55 the 3rd month, and so on and so forth. She makes her final deposit of \$235 the 15th month. What is the total amount for Christine's 15 deposits?
A. \$1950
B. \$1715
C. \$2200
D. \$2465
E. \$600
3. The first term of an arithmetic sequence is 10 and its common difference is -3. What is the sum of the first 10 terms of this sequence?
A. -35
B. -50
C. -32
D. 0
E. -3
4. How many terms are there between 23 and 72, inclusive, in the given arithmetic sequence?
2, 9, 16, 23, ..., 72
A. 9
B. 6
C. 8
D. 7
E. 10
5. The first 3 terms of an arithmetic sequence are $11\frac{1}{2}$, $9\frac{5}{16}$, $7\frac{1}{8}$ respectively. What is the fifth term of the sequence?
A. $4\frac{15}{16}$
B. $\frac{9}{16}$
C. $2\frac{3}{4}$
D. 5
E. $5\frac{15}{16}$
6. The 9th, 10th, and 11th terms of an arithmetic sequence are 47, 55, and 63, respectively. What are the first 3 terms of the sequence?
A. -25, -16, -7
B. -24, -17, -9
C. -9, -1, 7
D. -9, -2, 5
E. -17, -9, -1
7. What is the sum of the first 55 terms of the arithmetic sequence 2, 4, 6 ... ?
A. 3,080
B. 3,192
C. 1,540
D. 110
E. 38
8. During his first week as a salesman, Barney sold 33 products. He challenged himself to sell 3 more products each successive week than he had sold the week before. If Barney meets, but does not surpass his goal, how many products does he sell during his 15th week?
A. 145
B. 78
C. 72
D. 75
E. 63

9. What is the sum of the first 3 terms of an arithmetic sequence in which the 6th term is 23 and the 11th term is 30.5?
- A. 46.5
B. 55.5
C. 51
D. 53.5
E. 1.5
10. Consecutive terms of a certain arithmetic sequence have a positive common difference. The sum of the first 3 terms of the sequence is 135. Which of the following values CANNOT be the first term of the arithmetic sequence?
- A. 44.7
B. 42.5
C. 35
D. 44
E. 50
11. The first 4 terms of a geometric sequence are 0.66, -1.98, 5.94 and -17.82. What is the ninth term?
- A. 4330.26
B. -1443.42
C. -12990.78
D. 18247.68
E. -9.9
12. If 3 is the first term and 243 is the fifth term of a geometric sequence, which of the following is the third term?
- A. 81
B. 27
C. 9
D. 54
E. $\frac{3}{5}$
13. The first and second terms of a geometric sequence are x and ∇x , respectively. What would be the 999th term in the sequence?
- A. $\nabla^{1000} x$
B. $\nabla^{999} x$
C. $\nabla^{998} x$
D. $(\nabla x)^{998}$
E. $(\nabla x)^{999}$
14. The first 3 terms of a geometric sequence are 2, 7, and 24.5. What is the next term in the sequence?
- A. 85.75
B. 49
C. 42
D. 29.5
E. 36.75
15. The formula to find the sum of an infinite geometric series with the first term a and common ratio $|r| < 1$ is $\frac{a}{1-r}$. The sum of a given infinite geometric series is $\frac{2}{3}$ and the common ratio is $-\frac{1}{2}$. What is the fourth term of this series?
- A. $-\frac{1}{8}$
B. 1
C. $-\frac{1}{2}$
D. $\frac{1}{4}$
E. $-\frac{1}{32}$
16. In a certain number sequence, each term after the 1st term is the result of adding 3 to the previous term and then multiplying the sum by 4. The 4th term in the sequence is 700. What is the first term?
- A. 40
B. 9.95
C. 20.2
D. 10
E. 7
17. The decimal representation of $\frac{5}{14}$ repeats and can be expressed as 0.357142857142857142857 What is the 500th digit of this decimal?
- A. 1
B. 2
C. 8
D. 7
E. 3

18. A triangular number, T_n , when $n > 0$, is a triangular array of dots with n points on each side. The figure below shows the first 4 triangular numbers. What is the value of T_{50} ?

Term	Value
1	1
2	3
3	6
4	10

- A. 1275
B. 2750
C. 2550
D. 125
E. 147
19. The sum of the first 20 positive odd integers is 400. Which of the following is the sum of the first 40 positive odd integers?
- A. 1,580
B. 1,600
C. 800
D. 1,523
E. 4,400
20. Which of the following terms is not in the geometric sequence 4, -10, 25 ... ?
- A. -62.5
B. -390.625
C. 2441.406
D. 156.25
E. 976.5625

ANSWER KEY

1. A 2. A 3. A 4. C 5. C 6. E 7. A 8. D 9. C 10. E 11. A 12. B 13. C 14. A
 15. A 16. E 17. D 18. A 19. B 20. C

ANSWER EXPLANATIONS

- A.** The first level has 4 blocks, and $4=2^2$. So, the next level has $(2+1)^2=3^2 \rightarrow 9$. The third level has 4^2 blocks and the final 4th level has 5^2 blocks. Adding the total number of blocks together to make the pyramid, we get $2^2+3^2+4^2+5^2=4+9+16+25 \rightarrow 54$ blocks.
- A.** $n_1=25$ and after n months, Christine deposits $25+15(n-1)$. So, $n_{15}=25+15(15-1) \rightarrow 235$. We want to calculate the sum of all 15 deposits, which is $25+40+55 \dots +235$. We can calculate this with the formula $Sum = \frac{(F+L)n}{2}$ where F = the first number in the sequence, L = the last number in the sequence, and n = the number of numbers in the sequence. Plugging in $F=25$, $L=235$ and $n=15$, we get $Sum = \frac{(25+235)15}{2} \rightarrow 1950$.
- A.** The term in the sequence can be calculated by $10-3(n-1)$. So, the 10th term in the sequence is $10-3(10-1)=10-27 \rightarrow -17$. We can calculate the sum of the first 10 terms with the formula $Sum = \frac{(F+L)n}{2}$ where F = the first number in the sequence, L = the last number in the sequence, and n = the number of numbers in the sequence. Plugging in $F=10$, $L=-17$, and $n=10$ we get $Sum = \frac{(10-17)10}{2} \rightarrow -35$.
- C.** The arithmetic sequence 2, 9, 16, 23...72 has a common difference of $9-2=7$. Since each term is separated by 7, the number of terms from 23 to 72 can be calculated as $n = \frac{(72-23)}{7} \rightarrow 7$. In other words, if we start with the number 23 and add 7 to each term, 72 would be the 7th term. However, we must add 1 to the count because 23 is also included. So, the number of terms between 23 and 79 inclusive is 8.
- C.** The arithmetic sequence $11\frac{1}{2}, 9\frac{5}{16}, 7\frac{1}{8} \dots$ has a common difference of $9\frac{5}{16} - 11\frac{1}{2} = 9\frac{5}{16} - 11\frac{8}{16} \rightarrow -2\frac{3}{16}$. So the 4th term in the sequence is $7\frac{1}{8} - 2\frac{3}{16} = 4\frac{15}{16}$ and the 5th term is $4\frac{15}{16} - 2\frac{3}{16} = 2\frac{12}{16} \rightarrow 2\frac{3}{4}$.
- E.** The common difference between the numbers in the series is $55-47=8$. So, if the 9th term in the series is 47, then the first term in the sequence is eight terms before the 9th term. This means the first term is $47-8(8)=47-64 \rightarrow -17$. Then, the first three terms are -17, $-17+8$, and $-17+8(2)$ which are -17, -9, and -1.
- A.** The common difference of the arithmetic sequence is $4-2=2$, so the 55th term of the sequence can be calculated as $2+2(55-1)=2+2(54) \rightarrow 110$. So we can calculate the sum of the first 55 terms by using the formula $Sum = \frac{(F+L)n}{2}$ where F = the first number in the sequence, L = the last number in the sequence, and n = the number of numbers in the sequence. Plugging in $F=2$, $L=110$, and $n=55$, we get $Sum = \frac{(2+110)55}{2} \rightarrow 3080$.

8. **D.** The first term in the sequence is 33 and the common difference in the sequence is 3. So, during his 15th week, Barney's sales can be calculated as $n_{15} = 33 + 3(15 - 1) \rightarrow 33 + 42 \rightarrow 75$.
9. **C.** Since the 6th term is 23, the 11th term is 30.5, and the 11th and 6th terms are 5 common differences away, the common difference in the sequence is $\frac{30.5 - 23}{5} = \frac{7.5}{5} \rightarrow 1.5$. So, the first term of the sequence—which is 5 common differences away from the 6th term—can be calculated by $23 - 1.5(5) = 23 - 7.5 \rightarrow 15.5$. The 2nd and 3rd terms are then $15.5 + 1.5 = 17$ and $15.5 + 1.5(2) = 18.5$. So, the sum of the first three terms of the sequence are $15.5 + 17 + 18.5 = 51$.
10. **E.** The first three terms of the sequence sum up to be 135, so if we take $\frac{135}{3} = 45$, we know that the first term cannot be greater than or equal to 45, or else there are no values for the second and third term that will make the sum of the first three terms equal to 135. So, the answer choice that is greater than 45 cannot be the first term of the sequence. $50 > 45$ so the answer choice E is correct.
11. **A.** The common ratio of this geometric sequence is $-\frac{1.98}{0.66} = -3$. Since the 4th term is -17.82 and the 9th term is 5 ratios greater than the 4th term, the 9th term can be calculated as $n_9 = -17.82 * (-3)^5 \rightarrow 4330.26$.
12. **B.** Since 3 is the first term and 243 is the 5th term, the common ratio of the geometric sequence can be calculated as d in the equation $3d^4 = 243$. So, $d^4 = 81 \rightarrow d = 3$. So, the 3rd term can be calculated using the common ratio 3. That is, $n_3 = 3 * 3^2 \rightarrow 27$.
13. **C.** Since the first and second terms are separated by a factor of ∇ , ∇ is the common ratio of the geometric sequence. This means that the 999th term, which is 998 terms away from the 1st term, is $\nabla^{998}x$.
14. **A.** The common ratio of the geometric sequence can be calculated as $\frac{7}{2} = 3.5$. So, the next term in the sequence is $24.5(3.5) = 85.75$.
15. **A.** We solve for the first term, a , by plugging in $S = \frac{2}{3}$ and $r = -\frac{1}{2}$ into the formula $S = \frac{a}{1-r}$.

$$\frac{2}{3} = \frac{a}{1 - \left(-\frac{1}{2}\right)} \rightarrow \frac{2}{3} = \frac{a}{\frac{3}{2}} \rightarrow \frac{2}{3} = \frac{2}{3}a \rightarrow a = 1$$

So, the 4th term of the sequence, which is 3 common ratios greater than the first term, is $1\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$.

16. **E.** Each term in the sequence can be calculated as $n_i = 4(n_{i-1} + 3)$. Isolating the n_{i-1} we get the formula $n_{i-1} = \frac{n_i}{4} - 3$

$$\text{We are given that } n_4 = 700. \text{ So, } n_3 = \frac{700}{4} - 3. \text{ Performing this operation on } n_3 \text{ and } n_2 \text{ we get } n_1 = \frac{\frac{700}{4} - 3}{4} - 3$$

$$\rightarrow \frac{\frac{172}{4} - 3}{4} - 3 \rightarrow \frac{40}{4} - 3 \rightarrow 7.$$

17. **D.** We must recognize that the decimal value of $\frac{5}{14}$ is represented by the digit 3 followed by the 6 digits 571428 repeating in that order. So, the 2nd term and every 7 terms away will be the digit 5. This can be described as every $(1+6n)^{th}$ term is the digit 5. The largest value $1+6n$ less than 500 can be calculated by finding $\frac{500-1}{6} = 83.17$, rounding down to get 83 and then calculating $1+6(83) = 499$. This means that the 499th term will be a 5. From there, we can count 1 more terms to get to the 500th term which lands on the digit 7.
18. **A.** Each triangle number $T_n = n + (n-1) + (n-2) \dots + (n-n)$, so the value of T_{50} can be calculated using the formula $Sum = \frac{(F+L)n}{2}$ where F = the first number in the sequence, L = the last number in the sequence, and n = the number of numbers in the sequence. Plugging in $F = 1$, $L = 50$, and $n = 50$ we get $Sum = \frac{(1+50)50}{2} = 1275$.
19. **B.** The sequence of odd numbers is 1,3,5,7... and the n^{th} term can be calculated as $2n-1$. So, the 21st term in the sequence is $n_{21} = 2(21) - 1 \rightarrow 41$ and the 40th term in the sequence is $n_{40} = 2(40) - 1 \rightarrow 79$. So, the sum of the 21st to 40th terms can be calculated by the formula $Sum = \frac{(F+L)n}{2}$ where F = the first number in the sequence, L = the last number in the sequence, and n = the number of numbers in the sequence. Plugging in $F = 41$, $L = 79$, and $n = 20$, we get $Sum = \frac{(41+79)20}{2} \rightarrow 1200$.
- Since we are given that the sum of the first 20 odd numbers is 400, we can add that value to 1200 to get the sum of the first 40 odd numbers $= 1200 + 400 = 1600$. We can also arrive at the same answer if we directly calculate the sum of the first 40 odd terms by plugging in $F = 1$, $L = 79$, and $n = 40$. This yields $\frac{(1+79)40}{2} = 1600$.
20. **C.** The common ratio of the geometric sequence can be calculated as $-\frac{10}{4} = -2.5$. So, the terms in the geometric sequence can all be represented in the form $4(-2.5)^n$ for some integer n . The only answer choice that can not be represented in that form is 2441.406. This is because if we set $2441.406 = 4(-2.5)^n$ and solve for n , we get $310.3515 = (-2.5)^n \rightarrow \log_{-2.5} 310.3515 = \log_{-2.5} (-2.5)^n \rightarrow \log_{-2.5} 310.3515 = n$. Since $(-2.5)^6 = 244.1406$ and $(-2.5)^7 = -610.3516$ we know the value that satisfies $\log_{-2.5} 310.3515 = n$ is not an integer.