LAW OF SINES AND COSINES

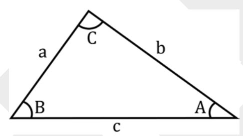
ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- Law of sines
- Law of cosines

These laws work for any triangle (not just a right triangle). You will want to memorize these rules to quickly solve trig problems.

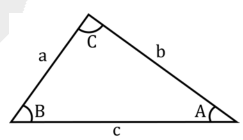
LAW OF SINES



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This rule states that the length of a side over the opposite angle equals another side over that respective opposite angle.

LAW OF COSINES



$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

TIP: As mentioned in previous chapters, always draw out a trig problem if it does not already give you a diagram.

APPLYING THESE LAWS

You can determine which law to use in a problem by looking at what information was given in the problem. For example, if only the lengths are given, use the law of cosines.

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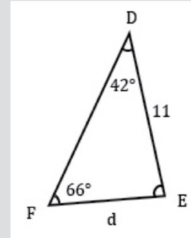
Example:

What is the approximate length of d in triangle DEF ?

A. 6
B. 7
C. 7.4
D. 8
E. 8.7



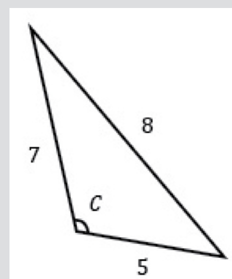
We know the angle of F, which is $66°$, the angle of D, which is $42°$, and side $\overline{DE}$, which is 11. With these, we can set up an equation using the law of sines:

$$\frac{d}{\sin 42°} = \frac{11}{\sin 66°} \rightarrow d = 8.057$$

Answer: **D.**

Using the diagram below, find the approximate angle C.

A. 78.2°
B. 81.8°
C. 88.1°
D. 96.2°
E. 112.7°



*not drawn to scale

Using the law of cosines, we can plug in the numbers into the equation:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$8^2 = 5^2 + 7^2 - 2(5)(7)\cos(C)$$

This gives us:

$$64 = 25 + 49 - 70\cos(C)$$

(Note: as long as you correlate the angle with the opposite side, it doesn't matter which side is which variable.)

Now, we can solve for C. Isolating the variable, we get:

$$-10 = -70\cos(C)$$

Dividing both sides by -70, we have:

$$\frac{1}{7} = \cos(C)$$

Multiplying both sides by $\cos^{-1}$ (the inverse of cosine), we get $C \approx 81.787°$.

Answer: **B.**

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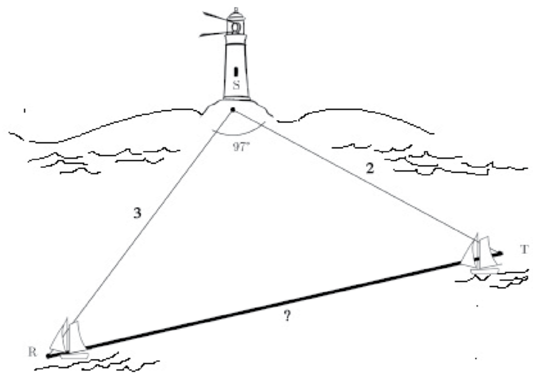
1. Two sailboats, at points R and T as shown in the figure below, are 3 miles and 2 miles away from a lighthouse at point S (respectively). If $\angle RST = 97°$, which of the following is equal to the distance between the two boats at R and T?
(Note: For any $\triangle ABC$, where a is the length of the side opposite $\angle A$, b is the length of the side opposite $\angle B$, and c is the length of the side opposite $\angle C$, $\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b} = \frac{\sin(\angle C)}{c}$ and $c^2 = a^2 + b^2 - 2ab\cos\angle C$.)



A. $2\left(\frac{\sin(97°)}{3}\right)$

B. $2^2 + 3^2 - (2)(2)(3)\cos(97°)$

C. $3\left(\frac{\sin(97°)}{2}\right)$

D. $\sqrt{2^2 + 3^2 - (2)(3)\cos(97°)}$

E. $\sqrt{2^2 + 3^2 - (2)^2(3)\cos(97°)}$

2. In $\triangle XYZ$, the measure of $\angle Z$ is $39°$, the measure of $\angle Y$ is $65°$, and the length of $\overline{ZY}$ is 13 inches. Which of the following is an expression for the length of $\overline{XZ}$?
(Note: The law of sines states that for any triangle, the ratios of the lengths of the sides to the sines of the angels opposite those sides are equal.)

A. $\frac{13\sin 39°}{\sin 76°}$

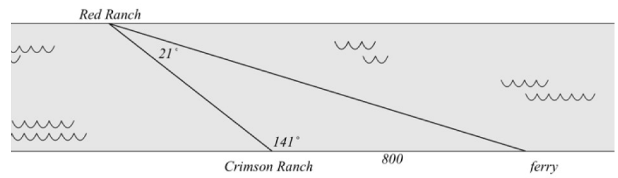
B. $\frac{13\sin 39°}{\sin 65°}$

C. $\frac{13\sin 65°}{\sin 39°}$

D. $\frac{507}{76}$

E. $\frac{13\sin 65°}{\sin 76°}$

3. Red Ranch and Crimson Ranch lie on opposite sides of a river. The nearest ferry is located 800 meters from Crimson ranch. The ranchers estimate the angles between these locations to be as shown on the map below. Using these estimates, which of the follow expressions gives the distance, in meters, between Red Ranch and Crimson Ranch?



A. $\frac{800}{21}$

B. $\frac{800}{\sin 21^\circ}$

C. $\frac{800\sin 18^\circ}{\sin 21^\circ}$

D. $1000\tan 18^\circ$

E. $\frac{800}{\cos 141^\circ}$

4. In the figure below, a radar screen shows 2 boats. Boat A is located at a distance of 45 nautical miles and bearing 110°, and Boat B is located at a distance of 60 nautical miles and bearing 220°. Which of the following is an expression for the straight line distance, in nautical miles, between the two boats?

A. $\sqrt{45^2 + 60^2}$

B. $45^2 + 60^2 - 2(45)(60)(\cos 110^\circ)$

C. $45^2 + 60^2 - 2(45)(60)(\sin 110^\circ)$

D. $\sqrt{45^2 + 60^2 - 2(45)(60)(\cos 110^\circ)}$

E. $\sqrt{45^2 + 60^2 - 2(45)(60)(\cos 220^\circ)}$

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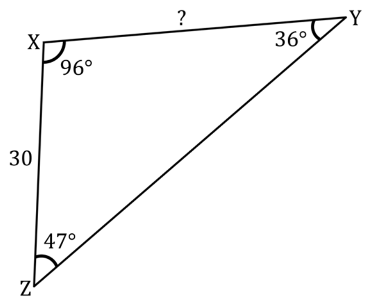
5. For $\triangle XYZ$ below, the length of $\overline{XZ}$ is 30 yards. Which of the following equations, when solved, will give the length, in yards, of $\overline{XY}$?
(Note: The law of sines states that given $\triangle ABC$,
$\frac{\sin(\angle A)}{a}=\frac{\sin(\angle B)}{b}=\frac{\sin(\angle C)}{c}$.)



A. $\frac{\sin 36^\circ}{30}=\frac{\sin 47^\circ}{XY}$

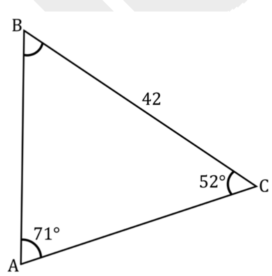
B. $\frac{\sin 47^\circ}{30}=\frac{\sin 36^\circ}{XY}$

C. $\frac{\sin 97^\circ}{30}=\frac{\sin 47^\circ}{XY}$

D. $\frac{\sin 97^\circ}{30}=\frac{\sin 36^\circ}{XY}$

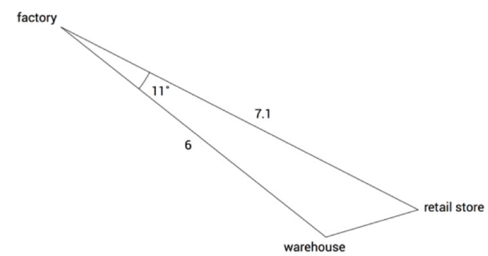
E. $\frac{\sin 36^\circ}{30}=\frac{\sin 97^\circ}{XY}$

6. In $\triangle ABC$ below, $\angle A$ measures 71° , $\angle C$ measures 52° , and the length of $\overline{BC}$ is 42 feet. To the nearest foot, what is the length of $\overline{AB}$?
(Note: The law of sines states that the lengths of the sides of a triangle are proportional to the singes of the opposite angles. Note also that $\sin 71^\circ \approx .946$ and $\sin 52^\circ \approx .788$.)



A. 50
B. 45
C. 37
D. 35
E. 57

7. A warehouse is 6 miles from a factory, and a retail store is 7.1 miles from the factory, as shown below. The angle between straight lines from the warehouse to the factory and store is 11° . The approximate distance, in miles, from the warehouse to the retail store is given by which of the following expressions?
(Note: The law of cosines states that for any triangle with vertices A , B , and C and sides opposite those vertices with lengths a , b , and c , respectively, $c^2=a^2+b^2-2ab\cos C$.)



A. $\sqrt{6^2+7.1^2+2(6)(7.1)(\cos 11^\circ)}$

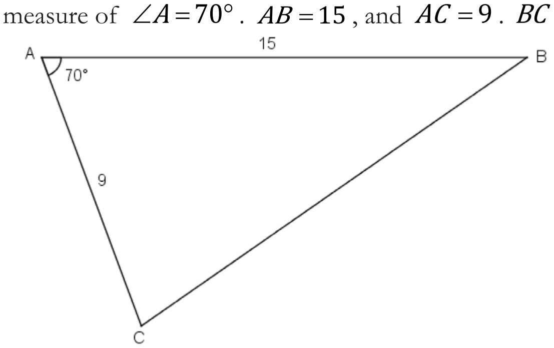
B. $\sqrt{6^2+7.1^2-2(6)(7.1)(\cos 79^\circ)}$

C. $\sqrt{6^2+7.1^2-2(6)(7.1)(\cos 11^\circ)}$

D. $\sqrt{7.1^2+6^2}$

E. $\sqrt{7.1^2-6^2}$

8. Triangle $\triangle ABC$ is shown in the figure below. The measure of $\angle A=70^\circ$. $AB=15$, and $AC=9$. $BC=?$



A. $15\sin 70^\circ$

B. $9\cos 70^\circ$

C. $\sqrt{15^2-9^2}$

D. $\sqrt{9^2+15^2}$

E. $\sqrt{9^2+15^2-2(9)(15)\cos 70^\circ}$

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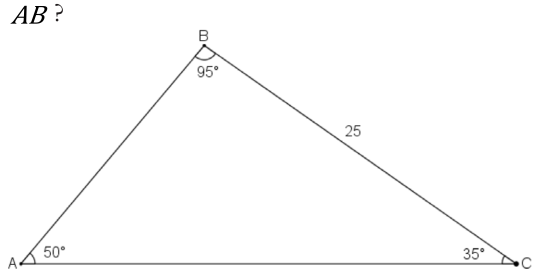
9. In $\triangle ABC$, shown below, angle measures are as marked. Which of the following is an expression for the length of $\overline{AB}$?



A. $\dfrac{25\sin 50°}{\sin 95°}$

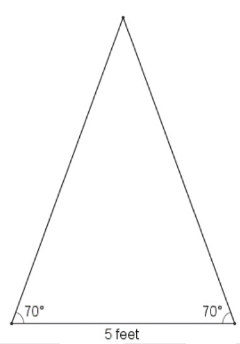
B. $\dfrac{25\sin 35°}{\sin 50°}$

C. $\dfrac{25\sin 35°}{\sin 95°}$

D. $\dfrac{25\sin 95°}{\sin 50°}$

E. $\dfrac{25\sin 95°}{\sin 35°}$

10. Austin is pitching a tent for a camping trip. He wants the entrance to the tent to be an isosceles triangle with a base of 5 feet and base angles measuring $70°$, as shown below. Which of the following expressions gives the perimeter of the entrance in feet?



A. $5+3\left(\dfrac{5\sin 70°}{\sin 40°}\right)$

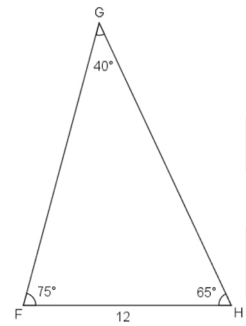
B. $5+2\left(\dfrac{5\sin 70°}{\sin 40°}\right)$

C. $5+2\left(5\sin 70°\right)$

D. $5+2\left(5\tan 70°\right)$

E. $3\left(\dfrac{5\sin 70°}{\sin 40°}\right)$

11. In $\triangle FGH$ below, $FH=12$ meters. To the nearest tenth of a meter, how many meters long is $\overline{FG}$?
(Note: $\sin 75° \approx 0.966$, $\sin 65° \approx 0.906$, $\sin 40° \approx 0.643$)



A. 8.0

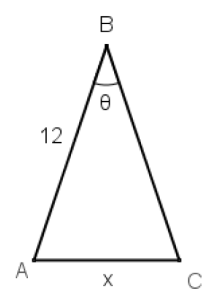
B. 8.5

C. 11.3

D. 16.9

E. 18.0

12. An isosceles triangle has legs of equal length 12 furlongs, and a third side of x furlongs. The degree measure between the sides that are 12 furlongs long is θ. In terms of x, $\theta =$?



A. $cos^{-1}\dfrac{x^2+288}{288}$

B. $\dfrac{-x^2+288}{288}$

C. $\dfrac{x^2-24}{288}$

D. $cos^{-1}\dfrac{288-x^2}{288}$

E. $\dfrac{24-x^2}{288}$

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ANSWER KEY

1. E 2. E 3. C 4. D 5. A 6. D 7. C 8. E 9. B 10. B 11. D 12. D

ANSWER EXPLANATIONS

1. **E.** We are given two side lengths and the angle opposite the mystery side, so we should use the law of cosines. Plug in $a=3, b=2$, and $\angle C=97°$ to get the equation $c^2=3^2+2^2-2(3)(2)cos°\angle C$. Taking the square root of both sides, we get $c=\sqrt{3^2+2^2-2(3)(2)\cos(97°)}=\sqrt{2^2+3^2-2^2(3)\cos(97°)}$.

2. **E.** We know that the angles in a triangle always add up to $180°$, so we can find $\angle X$ by subtracting $\angle Z$ and $\angle Y$ from $180°$. We get $\angle X=180°-39°-65°=76°$. Using the law of sines, we can solve for the length of $\overline{XZ}$ by plugging in $\angle Y=65°$ as the opposite angle of $\overline{XZ}$, and $\angle X=76°$ as the opposite angle of $\overline{ZY}$, which is given to be a length of 13. Since the law of sines is a ratio, as long as the numerators and denominators correspond consistently, we can arrange the proportion any way (although traditionally the opposite angle is the numerators and its corresponding side is the denominator). In this case, however, we will put the side length on top, giving us $\frac{13}{\sin 76°}=\frac{\overline{XY}}{\sin 65°}$. Multiplying both sides by $\sin 65°$, we get $\overline{XY}=\frac{13(\sin 65°)}{\sin 76°}$.

3. **C.** Because this is not a right triangle and you are told to solve for a distance, you must use either the law of sines or the law of cosines. You have all three angles of a triangle (since two are given, you can solve for the third with the triangle sum theorem) and one side length, which means you cannot use the law of cosines (as that requires you to have 2 side lengths). You must use the law of sines, and the only answer choice that even somewhat resembles what your answer would be if you did is C. Just to check, if we set up the proportion properly, $\frac{\sin 21^\circ}{800}=\frac{\sin 17^\circ}{d}$, we get $d=\frac{800\sin 17^\circ}{\sin 21^\circ}$, which is indeed answer C.

4. **D.** In this problem, again it is not told that we need to use the law of sines or the law of cosines, but because it is NOT a right triangle and we are given two sides lengths, the question most likely wants us to use the law of cosines: $c^2=a^2+b^2-2ab\cos\angle C$. The angle between the two boats is found by subtracting the smaller angle from the larger angle: $220^\circ-110^\circ=110^\circ$. Looking at the answer choices, you don't have to have the law of cosines memorized, but you do have to be familiar enough to recognize it. In this case, in order to know that D is the right answer, you must know to use cosine, not sine, and to use the angle between the two sides whose lengths are given. You must also remember to take the square root of the final expression, because of the c^2 part of the formula. Answer D is the only possible answer.

5. **A.** Using the law of sines, where the ratio of the sides and the angles opposite to each side are equal, we see that $\frac{\sin(\angle X)}{YZ}=\frac{\sin(\angle Y)}{XZ}=\frac{\sin(\angle Z)}{XY}$, which means $\frac{\sin 97^\circ}{YZ}=\frac{\sin 36^\circ}{30}=\frac{\sin 47^\circ}{XY}$. Thus, our answer is A, $\frac{\sin 36^\circ}{30}=\frac{\sin 47^\circ}{XY}$.

6. **D.** Using the law of sines, which states that given $\triangle ABC$, $\frac{\sin(\angle A)}{a}=\frac{\sin(\angle B)}{b}=\frac{\sin(\angle C)}{c}$, we get $\frac{\sin 71^\circ}{42}=\frac{\sin\angle B}{AC}=\frac{\sin 52^\circ}{AB}$. If we only look at part of this equation that is relevant to us, the part with the angles and side length plugged in, and plug in the sine values given to us in the note, we get $\frac{.946}{42}=\frac{.788}{AB}$. Solving for $\overline{AB}$, we get $\overline{AB}=34.9852\ldots\approx 35$.

7. **C.** Because it is not a right triangle and you are given the formula for the law of cosines, you simply need to plug in the appropriate values. If you let the retail store be vertex A, the warehouse be vertex B, and the factory be vertex C, then a is the distance between the factory and the warehouse, b is the distance between the factory and the store, and c is the distance between the warehouse and retail store, which is what we are solving for. You can plug in the given values into the law of cosines: $c^2=6^2+7.1^2-2(6)(7.1)\cos 11^\circ$. Taking the square root of both sides, we get $c=\sqrt{6^2+7.1^2-2(6)(7.1)\cos 11^\circ}$.

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8. **E.** The law of cosines states that $c^2 = a^2 + b^2 - 2ab\cos\angle C$. In our case, we want to find the measure of $\overline{BC}$. We plug in the other legs of the triangle for b and c, and the angle opposite from $\overline{BC}$ for $\angle C$. Our formula is $(BC)^2 = 9^2 + 15^2 - 2(9)(15)\cos 70°$. Square root both sides to isolate our answer: $BC = \sqrt{9^2 + 15^2 - 2(9)(15)\cos 70°}$.

9. **B.** The law of sines states that for a triangle with lengths a, b, and c opposite angles $\angle A, \angle B$, and $\angle C$ respectively, $\frac{\sin a}{\angle A} = \frac{\sin b}{\angle B} = \frac{\sin c}{\angle C}$. We want to find AB, so we will use the measure of the angle opposite it and a given opposite angle-side pair to plug into our formula: $\frac{\sin 35°}{AB} = \frac{\sin 50°}{25}$. Cross multiply and divide by $\sin 50°$ to get $AB = \frac{25\sin 35°}{\sin 50°}$.

10. **B.** The law of sines states that for a triangle with lengths a, b, and c opposite angles $\angle A, \angle B$, and $\angle C$ respectively, $\frac{\sin a}{\angle A} = \frac{\sin b}{\angle B} = \frac{\sin c}{\angle C}$ .We know that the missing angle of the isosceles triangle is $40°$ because all of the angles must sum to $180°$. To find the measure of the other two legs of the isosceles triangle, n, we plug into the law of sines: $\frac{\sin 70°}{n} = \frac{\sin 40°}{5}$. Rearranging the equation yields $n = \frac{5\sin 70°}{\sin 40°}$. The perimeter is equal to 5, the bottom leg, plus $2n$, the other two legs: $5 + 2\left(\frac{5\sin 70°}{\sin 40°}\right)$.

11. **D.** The law of sines states that for a triangle with lengths a, b, and c opposite angles $\angle A, \angle B$, and $\angle C$ respectively, $\frac{\sin a}{\angle A} = \frac{\sin b}{\angle B} = \frac{\sin c}{\angle C}$. Plugging in our values, including the approximations of the sines, we get $\frac{0.906}{FG} = \frac{0.643}{12}$. Rearranging gives us $FG = \frac{12(0.906)}{0.643} \approx 16.9$.

12. **D.** The law of cosines states that for a triangle with lengths a, b, and c opposite angles $\angle A, \angle B$, and $\angle C$ respectively, $a^2 = b^2 + c^2 - 2ab\cos\angle A$. Plugging in the values we have $x^2 = 12^2 + 12^2 - 2(12)(12)\cos\theta$, where $a = x$, and $\angle A = \theta$. Simplify and isolate the θ term: $(-288)\theta = x^2 - 288$. Isolate θ: $\theta = cos^{-1}\frac{288 - x^2}{288}$.

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