CHAPTER

1 SCIENTIFIC NOTATION

SKILLS TO KNOW

- How to complete basic problems that ask for answers in scientific notation
- How to complete word problems involving scientific notation

THE BASICS

Most of you probably learned scientific notation back in elementary or middle school—it's one of those skills that you may have known once, but may be a bit rusty now.

So let's review some of the basics:

1. What Is It?

For a number in the form $a*10^n$ to be in scientific notation, it must contain a value (a) greater than or equal to 1 and less than 10 $(1 \le a < 10)$ multiplied by ten to power (n), where n is an integer.

Some ACT® problems may be easier to solve when you can eliminate answers NOT officially in scientific notation off that technicality.

This **IS** in scientific notation:

$5.5*10^{-7}$

This is **NOT** in scientific notation:

$55*10^{-6}$

2. Positive Exponents

When a number is multiplied by ten to a positive exponent of degree n, move the decimal point n places to the RIGHT.

Expand the following number:

$4.3*10^9$

Because the exponent is 9, Move the decimal points nine places to the right:

$4.3\,0\,0\,0\,0\,0\,0\,0\,0\,0.$

As you can see, I draw a little loop for each hop, and count all the loops.

4,300,000,000

You get 4.3 billion.

(To change the number back to scientific notation, move the decimal back to the left, and count. When you are to the point that only one number remains to the left, you're done. Count the number of moves—that is your exponent.)

3. Negative Exponents

When a number is multiplied by ten to a negative exponent of degree n, move the decimal point n places to the LEFT.

$$5.854\times10^{-5}+3.56\times10^{-3}=?$$

First we'll put the first number in regular form, then the second. Then we'll add the numbers and convert back to scientific notation.

$$\underset{5\ 4\ 3\ 2\ 1}{\bullet 0\,0\,0\,0\,5.854} \quad + \quad \underset{3\ 2\ 1}{\bullet 0\,0\,3.56}$$

Then line up the decimal points to add the numbers (don't forget to carry the "1"):

$$\begin{array}{r} {\scriptstyle 1\,\diagdown} \\ 0.00005854 \\ +0.00356000 \\ \hline 0.00361854 \end{array}$$

Now we convert back to scientific notation:

$$\underset{1\ 2\ 3}{0.0\,0\,3\bullet 61854}$$

As you can see, we need to move the decimal point three places to the right—so we add an exponent of negative 3 to our 10:

$$3.61854*10^{-3}$$

That's our answer!

WORD PROBLEMS

We'll also need to know how to solve word problems that involve scientific notation. In general, for these problems, first convert the scientific notation numbers into standard form. Then, solve the problem. Finally, convert your answer back to scientific notation.

The area of four oceans is given in the following chart in square miles. What is the total area that these four oceans cover?

Indian	$2.84*10^{7}$
Pacific	$6.25*10^{7}$
Atlantic	$4.11*10^{7}$
Arctic	$5.43*10^{6}$

For the problem, because three of the oceans have the same power of 10, we can add those integers first, factoring out the 10^7 term.

$$2.84*10^7+6.25*10^7+4.11*10^7=10^7*(2.84+6.25+4.11)$$

Using your calculator, you can find that:

$$2.84+6.25+4.11=13.2$$

Now we can convert that to standard form:

$$13.2\underset{1\,2\,3\,4\,5\,6\,7}{0\,0\,0\,0\,0\,0\,0.}$$

That's 132 million: 132,000,000.

Now we convert the Arctic Ocean to standard form:

$$5.4\underset{1\,2\,3\,4\,5\,6}{3\,0\,0\,0\,0\,0.}$$

That's 5,430,000 or 5.43 million.

Now we add 132 million plus 5.43 million:

$$\begin{array}{r} 132{,}000{,}000 \\ +5{,}430{,}000 \\ \hline 137{,}430{,}000 \end{array}$$

Finally, we convert back to scientific notation:

$$1.\underset{1\,2\,3\,4\,5\,6\,7\,8}{3\,7\,0\,0\,0\,0\,0\,0.}$$

or $1.3743*10^8$.

TRICKY PROBLEMS

Whenever x and y are both integers, what is the approximate value of $\left(4.56*10^x\right)\left(5.22*10^y\right)$ expressed in scientific notation?

A. $2.38*10^{yx+1}$

B. $2.38*10^{x+y}$

C. $2.38*10^{x+y+1}$

D. $2.38*10^{x+y-1}$

E. $2.38*10^{x-y-1}$

Occasionally, you may have scientific notation problems that involve variables in the answer choices. For these problems, you could make up numbers, find the pattern and plug in what you made up into each answer choice to discover the answer. You can also attempt algebraically.

Algebraic Method:

Here you can first rearrange to get:

$$\left(4.56*5.22\right)\left(10^{x}*10^{y}\right)$$

Applying our exponent rule:

$$\left(a^{x}*a^{y}=a^{x+y}\right)$$

That's approximately:

$$\left(23.8\right)\left(10^{x+y}\right)$$

Now I put the 23.8 in scientific notation:

$$\left(2.38*10^{1}\right)\left(10^{x+y}\right)$$

And finally, I combine the 10 with the other 10 powers:

$$\left(2.38\right)\left(10^{1}*10^{x+y}\right)$$

Again I apply the power rule $\left(a^{x}*a^{y}=a^{x+y}\right)$:

$$\left(2.38\right)\left(10^{x+y+1}\right)$$

Answer: **C.**

TIP: For more help with the problems that follow, check out Chapter 15 on Speed and Rates. Many word problems here overlap with that problem type.