THE BEST ACT PREP COURSE EVER

FOILING AND FACTORING

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- Multiplying polynomials (FOIL)/expanding expressions
- Factoring Monomials, Binomials, Trinomials (Quadratic Equations)
- The Zero Product Property/Solving Quadratics
- Special Products (Difference of Squares, Square of a Sum, Square of a Difference)
- Solve for "a," "h," or "k" in a quadratic equation by using factoring or FOIL
- "Rainbow" Distribution: expanding more complex products

FOIL

The expression $(5a+3)(a-4)$ is equivalent to:

A. $8a-4$ **B.** $5a^2-23a-12$ **C.** $5a^2+17a-12$
D. $5a^2-a-12$ **E.** $5a^2-17a-12$

In order to solve this problem, we need to use FOIL. **FOIL** is an acronym we use to describe **binomial expansion**: how we use the distributive property to solve for the product of a binomial expression (a two-term expression, such as $5a+3$) times a binomial (such as $a-4$). Each letter in FOIL represents the product of two terms. We find these four products and then add them together to find the total product of the binomials.

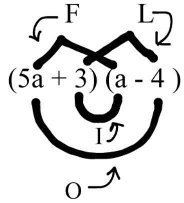
First –i.e. the FIRST TERMS: $(\mathbf{5a}+3)(\mathbf{a}-4)$

Outer – i.e. the OUTER TERMS: $(\mathbf{5a}+3)(a\mathbf{-4})$

Inner – i.e. the INNER TERMS: $(5a\mathbf{+3})(\mathbf{a}-4)$

Last—i.e. the LAST TERMS: $(5a\mathbf{+3})(a\mathbf{-4})$

This is the order in which we'll multiply the terms, creating four separate products. I draw a slightly sinister-looking smiley face to help me keep track of these four separate products:



FIRST	**O**UTER	**I**NNER	**L**AST
$(5a)(a)$	$(5a)(-4)$	$(3)(a)$	$(3)(-4)$
$5a^2$	$-20a$	$3a$	-12

Then we will add all four products together:

$$5a^2-20a+3a-12=5a^2-17a-12$$

The sum of each of the four products represented by F, O, I, and L is our answer: $5a^2-17a-12$.

Notice how the O and the I are "like terms" that can combine, in this case, $-20a$ and $3a$? This is true every time you FOIL a pair of binomials of the form $ax+b$ where x is any variable and a and b are constants: you will always combine the O term and the I term to get the center term in your final answer. In contrast, the first terms and last terms will not combine with any other terms.

Answer: $5a^2-17a-12$.

TIP: Remember to include the sign of terms like -4, or else you'll get the wrong answer! Negative signs always "hug" to the right: they travel with any number they are in front of when using the distributive property or FOIL.

FACTORING BASICS

Factoring Monomials

You'll need to be able to factor basic monomial elements out of any monomial, binomial, or polynomial.

For review:

A **monomial** is a single product such as $4x$, $7x^3$, or $8n^2$.
A **binomial** has two elements added together such as $4x+3$ or $5n^3+3n$.
A **polynomial** has multiple elements added together such as $5n^3+3n^2+7n+2$ or $5x^2+2x+4$.

These items can be factored by pulling out monomial factors.

All of the following monomials are factors over the integers of $24x^2y+16x^2y^3-8x^4y^2$ EXCEPT:

A. 8 **B.** $6x$ **C.** $2x^2$ **D.** $8x^2$ **E.** $4x^2y$

Here we need to understand what this means: we are looking for an answer choice that does not divide evenly into each "piece" of the original polynomial: $24x^2$, $16x^2y^3$, and $8x^4y^2$. Notice the word "EXCEPT!" We can use process of elimination by going through each answer, and checking whether the monomial answer choice divides evenly into each piece of our polynomial, i.e. if the monomial answer choice is a factor of each of $24x^2$, $16x^2y^3$, and $8x^4y^2$. I can keep track of each answer using Y for yes and N for no, so as not to be confused by the "EXCEPT." I'm looking for N (not a factor).

A. 8 goes into 24, 16 and -8 so that is a factor of each element. (Y)
B. 6 does go into 24—but not into 16 or -8—so this is NOT a factor of this polynomial. This is the answer. (N)
C. 2 goes into 24, 16 and -8 while x^2 goes into x^2, x^2, and x^4, so this is a factor of each element. (Y)
D. 8 goes into 24, 16 and -8 while x^2 goes into x^2, x^2, and x^4, so this is a factor of each element. (Y)

E. 4 goes into 24, 16 and -8 while x^2 goes into x^2, x^2, and x^4, and y goes into y, y^2, y^3. (Y)

Answer: **B**.

Factoring using FOIL (Quadratics with no leading coefficient):

You also need to know how to factor a typical quadratic expression into the product of two binomials. In other words, you need to know how to "undo" FOIL. These problems are simplest when the leading coefficient, i.e. the coefficient of x^2 (the number in front of x^2) is "invisible" or is one (Note: we don't actually write out the coefficient one, i.e. x^2 has a coefficient of one, or no leading coefficient, whereas $2x^2$ has a coefficient of two.)

What is the factored form of the expression $x^2+5x-36$?

To solve this problem, we are essentially performing FOIL backwards.

In general, these types of problems will have solutions that look like this:

$(x+a)(x+b)$—what we start with when we FOIL.

Thinking back to FOIL, we can match up parts of the expression that relate to specific parts of the model solution, and FOIL the above model to see how the parts relate. Remember how the sum of the O and the I always form the middle term? That is an important fact when factoring:

Model: $(x+a)(x+b)$

FIRST	OUTER + INNER	LAST
x^2	$+5x$	-36
$(ax)(cx)$	$(ax)(d)+(b)(cx)$	$(b)(d)$
acx^2	$(ad+bc)(x)$	bd

As you can probably see from the above, our simplest terms to deal with are always the first and the last—they have fewer variables and are less complicated, which means they are the best place to start.

Let's take that first term. We can see that that these two terms equal each other. Because there is no leading coefficient (i.e. the coefficient is "one" in this case), this step is easy. We basically don't have to do anything.

$$x^2=x^2$$

Now let's look at our last terms:

$$-36=ab$$

If we set the model product equal to the "last" term in the original quadratic expression we get that the product of a and b is -36. Now we can start to understand what we'll need. We need two numbers that pair together to form -36.

Let's table this idea for a moment and look at our **middle term**.

From our "matchy matchy" set up we know:

$$5x = (ax) + (bx)$$

I can simplify this to:

$$5x = (a+b)x$$

At this point, dividing by x (assuming it can be any number, not simply zero) I know 5 must equal $a+b$:

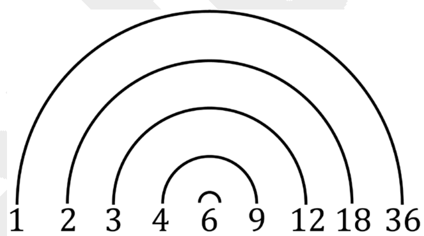
$$5 = a+b$$

In short: we are looking for numbers that multiply to the last term ($ab = -36$), and sum to the coefficient of the middle term ($a+b=5$). This sounds confusing, but the more you practice this process, the more you'll get the hang of it.

To get the answer from here:

STEP 1: Come up with factors of the last term, -36.
STEP 2: See if those factors sum to the middle term's coefficient, 5.

Step 1: Negative sign aside, we want a factor pair that forms 36. My factor rainbow below shows all the possible factor pairs of 36 (for more on factor rainbows, see Book 1 Chapter: LCM & GCF):



I also know that **to get a negative number as a product**, I'll need **one positive factor** and **one negative factor**. (If I wanted a positive product, both factors could be negative or both positive).

Now for Step 2: We are looking for a pair here such that when one of the pair is positive and one is negative, both sum to five. In other words, we want two factors in a pair that have a difference of 5.

Because 4 and 9 are five apart, let's try some combination of those:

$$4 \text{ and } -9$$

But $4+(-9)=-5$, not 5. We need positive 5.

If we flip the signs of the last pair to get -4 and 9, though we get $-4+9=5$, which works!

Thus, a and b are -4 and 9.

So, $ab=-4(9)=-36$ (The negative is very important!)

AND

$$-4+9=5.$$

Putting our numbers together, we get $(x-4)(x+9)$ as our answer.

TIPS:

1. To check your answer, FOIL it out and see if it matches the polynomial in the question.
2. You don't have to create an entire factor rainbow each time you factor. You can also list pairs of factors, or use your intuition given the idea of a sum and a product to deduce which factor pair might work best. Again, the more of these you do, the better you'll get. If you're ever stuck, though, a factor rainbow is a great way to ensure you haven't missed anything!

Answer: $(x-4)(x+9)$.

Factoring With Coefficients In Front Of The Largest Term

What is the factored form of the expression $3x^2+4x-4$?

The coefficient of 3 in front of x^2 makes this problem a little more complex than the last one.

Still, we'll run a similar process, setting up a template to understand what we want:

$$(?x+a)(?x+b)$$

Let's skip straight to finding the coefficients of our first terms. We're going to assume as we factor that we are only looking for integer coefficients. Here we can be pretty certain that our first terms will be 3 and 1, since those are the only whole numbers whose product is 3.

$$(3x+a)(x+b)$$

Now, imagine FOILing this binomial. We can line up that FOIL product with our original expression:

F: $3x^2$	**O:** $3bx$	**PLUS**	**I:** ax	**L:** ab
$3x^2$		$+4x$		-4

The middle ones, O and I, sum to the middle term of our polynomial, $4x$, so $3bx+ax=4x$, or (to make things simpler) $3b+a=4$.

Just as we did before, we can look for pairs of numbers that create -4 and would create O and I terms that sum to $4x$. We just need to guess and check to find the answer, plugging into the equation $3b+a=4$. (If this is confusing, just find the pairs that create -4 as a product, plug them in, and FOIL the O and I terms).

The difference here is that we aren't just adding a and b, the coefficient from the first term has an effect! Let's think of pairs of numbers whose product is -4.

-4 and 1: $3(-4)+1=-11$ **Incorrect**

-2 and 2: $3(-2)+2=-4$ **Closer**

2 and -2: $3(2)-2=4$ **Correct**

$b=2$ and $a=-2$ so if $(3x+a)(x+b)$, we can plug these in to form $(3x-2)(x+2)$.

Answer: $(3x-2)(x+2)$.

TIP: Sometimes figuring out the first term is not so easy. In these cases, you simply have more possibilities to guess and check. Remember your first term will be some combination of factors of your leading coefficient. It seems daunting, and it is somewhat time consuming, so if all else fails, and a factoring problem is very difficult, you can always use the **quadratic equation** to solve and then work back to the factors from that method. See the chapter in this book on **Quadratics and Polynomials** for more on this equation.

ZERO PRODUCT PROPERTY

Now that we're up to speed on factoring, let's talk about how to solve an equation with factored parts.

Which of the following is NOT a solution of $(x-9)(x+2)(x-4)(x+7)=0$?

A. –9 **B.** 9 **C.** –2 **D.** 4 **E.** –7

To solve this, we need to understand the Zero Product Property.

ZERO PRODUCT PROPERTY

If $ab=0$, then $a=0$, $b=0$, or $a=b=0$.

TIP: Sometimes the ACT calls answers to factored polynomials (such as this one) "roots" instead of solutions. Sometimes it will also call these answers "zeros."

If the product of any two or more items equals zero, then it follows that one of the items MUST be equal to zero. When we solve equations, we can use this idea to then simplify the entire equation down into further smaller equations, all of which are equal to zero:

$$(x-9)(x+2)(x-4)(x+7)=0 \text{ implies}$$

$$(x-9)=0 \text{ or } (x+2)=0 \text{ or } (x-4)=0 \text{ or } (x+7)=0$$

We can solve each equation and get all of the possible answers: $x=9$, $x=-2$, $x=4$, and $x=-7$.

The answer is thus **A.** -9

SPECIAL PRODUCTS

Certain patterns in math often emerge that we can memorize in order to speed up factoring. The idea with special products is to memorize the pattern and apply it, rather than using FOIL or traditional factoring methods.

The Difference of Squares

THE DIFFERENCE OF SQUARES

The product of the difference $a-b$ and the sum $a+b$ is equal to a squared minus b squared.

Pattern: $a^2-b^2=(a-b)(a+b)$

Why this works: Our middle term always cancels out:

Ex. 1: $(a-b)(a+b)$

$a^2+ab-ab-b^2$ FOIL

a^2-b^2 Simplify.

Ex. 2: Let $a=x$ and $b=3$:

$(x-3)(x+3)$

$x^2+3x-3x-3^2$

x^2-9

$9x^2-16=?$

To factor this, we apply this memorized pattern:

$$(a-b)(a+b)=a^2-b^2$$
$$\downarrow \quad \downarrow$$
$$9x^2-16$$
$$\downarrow \quad \downarrow$$
$$(3x)^2-(4)^2$$
$$a=3x,\ b=4$$

As you can see, this expression matches the second half of the above pattern—where $a=3x$ and $b=4$. Now that we know a and b, substitute in these values in the pattern's first half, and you're done:

$$(a-b)(a+b)$$
$$(3x-4)(3x+4)$$

Answer: $(3x-4)(3x+4)$.

The Square of a Sum

THE SQUARE OF A SUM

Pattern: $(a+b)^2 = a^2 + 2ab + b^2$

Why this works:

$$(a+b)(a+b)$$
$$a^2 + ab + ab + b^2$$
$$a^2 + 2ab + b^2$$

Example: Let $a = x$ and $b = 3$:

$$(x+3)^2$$
$$(x+3)(x+3)$$
$$x^2 + 3x + 3x + 3^2$$
$$x^2 + 2(3)(x) + 3^2$$
$$x^2 + 6x + 9$$

$(4x+6)^2 = ?$

Apply the pattern: $(ax+b)^2 = a^2x^2 + 2abx + b^2$

In our polynomial: $a = 4x$; $b = 6$

Plug in: $(4x)^2 + 2(4x)(6) + (6)^2 = 16x^2 + 48x + 36$

Answer: $16x^2 + 48x + 36$.

MISTAKE ALERT: Don't forget to distribute the exponent on the $4x$ term—you need to square both the 4 and the x!

The Square of a Difference

THE SQUARE OF A DIFFERENCE

Pattern: $(a-b)^2 = a^2 - 2ab + b^2$

Why this works:

$$(a-b)(a-b)$$
$$a^2 - ab - ab + b^2$$
$$a^2 - 2ab + b^2$$

Example: Let $a = x$ and $b = 3$:

$$(x-3)^2 = (x-3)(x-3)$$
$$x^2 - 3x - 3x + 3^2$$
$$x^2 - 6x + 9$$

$(2x-4)^2 = ?$

In our polynomial:

$$a = 2x\ ,\ b = 4$$
$$a^2 - 2ab + b^2$$
$$(2x)^2 - 2(2x)(-4) + (-4)^2 = 4x^2 + 16x + 16$$

Answer: $4x^2 + 16x + 16$.

SOLVING FOR PARTS OF A QUADRATIC EQUATION

Consider the equation $x^2 + 11x + b$. One solution to this equation is -4. What is the value of b?

Here, we can use the idea of factors to set up a FOIL pattern and solve for b. We could also plug in the solution into x, set the equation equal to 0, and solve for b. Let's try the first way:

When we know a solution is equal to -4, we also know one of its factors will be $(x+4)$. We know the other factor will be x plus some other number, because the leading coefficient of x^2 is one:

$$x = -4 \rightarrow (x+4)(x+n)$$

Now we can FOIL this binomial product:

$$\begin{aligned}(x+4)(x+n) &= x^2 + 4x + nx + 4n \\ &= x^2 + (4+n)x + 4n\end{aligned}$$

Now we play "matchy-matchy":

$$\begin{array}{c} x^2 + (4+n)x + 4n \\ \downarrow \qquad \swarrow \\ x^2 + 11x + b \end{array}$$

We can then solve for n and subsequently b:

$$4 + n = 11$$
$$n = 7$$

$$b = 4n$$
$$b = 4(7)$$
$$b = 28$$

Answer: 28.

Again, we can also solve this by plugging in -4 for x and setting the equation equal to zero:

$$x^2 + 11x + b = 0$$
$$4^2 + 11(-4) + b = 0$$
$$16 - 44 + b = 0$$
$$-28 + b = 0$$
$$b = -28$$

"RAINBOW" DISTRIBUTION: EXPANDING TERMS LARGER THAN BINOMIALS

$$(x-2)^3 = ?$$

When we have to expand more than a single product of two binomials, we must understand what is at play when we distribute terms.

Here we can split this into pieces, first using foil to expand the product of the first two binomials:

$$(x-2)^2(x-2) = (x^2 - 4x + 4)(x-2)$$

Now we use the distributive property, thinking in terms of a "rainbow" we are going to take each term in $(x-2)$ and "rainbow" distribute it to each term in the quadratic $(x^2 - 4x + 4)$.

$$(x-2)(x^2 - 4x + 4)$$

Now let's "rainbow" the -2 term to each element in the quadratic:

$$(x-2)(x^2 - 4x + 4)$$
$$x(x^2 - 4x + 4) + (-2)(x^2 - 4x + 4)$$
$$(x^3 - 4x^2 + 4x) + (-2x^2 + 8x - 8)$$
$$x^3 - 4x^2 - 2x^2 + 4x + 8x - 8$$
$$x^3 - 6x^2 + 12x - 8$$

Answer: $x^3 - 6x^2 + 12x - 8$.

What is the absolute value of the difference of the solutions for the equation $2x^2+5x=-12$?

$$2x^2+5x=-12$$
$$2x^2+5x+12=0$$
$$(2x-3)(x+4)=0$$

Using the Zero Product:

$$2x-3=0 \qquad x+4=0$$
$$2x=3 \qquad x=-4$$
$$x=\frac{3}{2}$$

$$\left|\frac{3}{2}-(-4)\right|=\left|\frac{3}{2}+4\right|=\left|5\frac{1}{2}\right|=5\frac{1}{2}$$

Answer: 5.5

What expression is equivalent to $\frac{3x^2+21x+18}{x^2+3x-18}$?

$$\frac{3x^2+21x+18}{x^2+3x-18}$$
$$\frac{(3x+?)(x+?)}{(x+6)(x-3)}$$
$$\frac{(3x+3)(x+6)}{(x+6)(x-3)}$$
$$\frac{(3x+3)}{(x-3)}$$

Answer: $\frac{3x+3}{x-3}$.

1. What is the magnitude of the difference of the solutions for the equation $(x+5)(x-1)=0$?

A. 3
B. 6
C. 5
D. 4
E. 2

2. Which value of x is a solution to the equation $x^2+7x-14=-26$?

A. 2
B. 5
C. −2
D. 3
E. −4

3. Whatisthecompletefactorizationof $4x+8xy+16x^3y$?

A. $4x+8xy\left(1+2x^2\right)$
B. $4x\left(1+2y+4x^2y\right)$
C. $4x\left(2+y+2x^2y\right)$
D. $2x\left(2+4y+8x^2y\right)$
E. $x\left(4+8y+16x^2y\right)$

4. If $(x+9)$ is a factor of $3x^2+kx+36$, what is the value of k?

A. −31
B. 31
C. 9
D. −9
E. 15

5. The equation $x^2-16x+k=0$ has exactly one real solution for what value of k?

A. 16
B. −16
C. −64
D. 64
E. 8

6. Which of the following quadratic expressions has a solution at $n=4a$ and $n=-5b$?

A. $n^2+n\left(4a-5b\right)-20ab$
B. $n^2-n\left(4a-5b\right)-20ab$
C. $n^2+n\left(4a+5b\right)-20ab$
D. $n^2+n\left(4a-5b\right)+20ab$
E. $n^2-n\left(4a+5b\right)+20ab$

7. For the quadratic equation $3x^2+20x+K$, what value of K will yield $\frac{1}{3}$ and -7 as solutions for x?

A. 7
B. −7
C. 21
D. 12
E. −14

8. What expression is equivalent to $\left(\frac{2}{3}x-\frac{1}{2}y\right)^2$?

A. $\frac{4}{9}x^2+\frac{2}{3}xy+\frac{1}{4}y^2$
B. $\frac{4}{9}x^2-\frac{2}{3}xy-\frac{1}{4}y^2$
C. $\frac{4}{9}x^2+\frac{2}{3}xy-\frac{1}{4}y^2$
D. $\frac{4}{9}x^2-\frac{4}{3}xy+\frac{1}{4}y^2$
E. $\frac{4}{9}x^2-\frac{2}{3}xy+\frac{1}{4}y^2$

9. If $x^2-y^2=169$ and $x+y=13$, then $y=$?

A. 13
B. −13
C. 1
D. 0
E. 6

10. How many pairs of real numbers (x,y) satisfy $xy=5$ and $(x+y)^2=20$?

A. 0
B. 1
C. 2
D. 3
E. 4

11. What expression is equivalent to $(-4x-11)(x+2)$?

A. $(4x-11)(x+2)$

B. $(-4x+11)(-x-2)$

C. $(4x+11)(-x-2)$

D. $-(4x+11)(x-2)$

E. $(4x+11)(x+2)$

12. The trinomial x^2+8x-9 can be factored as the product of 2 linear factors in the form $(x+a)(x+b)$. What is the polynomial sum of these 2 factors?

A. $2x-8$
B. $2x+8$
C. $2x-9$
D. $2x+9$
E. $2x-1$

13. The expression $(4x+2)(x-3)$ is equivalent to:

A. $5x-1$

B. $5x+1$

C. $4x^2-6$

D. $4x^2-10x-6$

E. $4x^2+5x-6$

14. $(7a+2b)(3b-a)$ is equivalent to:

A. $11ab$
B. $7ab$
C. $21a^2-ab-2b^2$
D. $-7a^2+19ab+6b^2$
E. $7a^2+19ab+6b^2$

15. Which of the following is the factored form of the expression $3x^2-14x+8$?

A. $(x-2)(3x+4)$

B. $(x-4)(3x-2)$

C. $(x-4)(3x+2)$

D. $(x+4)(3x-2)$

E. $(x+2)(3x-4)$

16. Let a and b be real numbers. If $(a-b)^2=a^2-b^2$, it *must* be true that:

A. Either a or b is zero.
B. Both a and b are zero.
C. Both a and b are positive.
D. Both a and b are negative
E. $a=b$

17. The trinomial x^2+x-20 can be factored as the product of 2 linear factors, in the form $(x+a)(x+b)$. What is the polynomial sum of these 2 factors?

A. $2x+1$
B. $2x-1$
C. $2x-9$
D. $2x+9$
E. $2x-20$

ANSWER KEY

1. B 2. E 3. B 4. B 5. D 6. B 7. B 8. E 9. D 10. C 11. C 12. B 13. D 14. D 15. B 16. E 17. A

ANSWER EXPLANATIONS

1. **B.** $(x+5)(x-1)=0$ means that $(x+5)=0 \rightarrow x=-5$ or $(x-1)=0 \rightarrow x=1$. So, the difference between the possible values for x is $1-(-5)=1+5 \rightarrow 6$, or $-5-1=-6$. Only 6 is an answer given, so B is correct.

2. **E.** To factor $x^2+7x-14=-26$, we first move everything to the same side (the left side) of the equal sign, so that the sum of the terms equals zero. By adding 26 on both sides, we get $x^2+7x+12=0$. Now, we factor the expression by finding two integers that add up to 7 and multiply to be 12. The numbers 3 and 4 work. We factor to $(x+3)(x+4)=0$. Thus, the values -3 and -4 are solutions to the equation. Only -4 is one of the choices given, so that is the answer.

3. **B.** Every term in the expression is a multiple of $4x$, so we can factor out $4x$ to get:
$4x+8xy+16x^3y=4x\left(1+2y+4x^2y\right)$.

4. **B.** If $(x+9)$ is a factor of $3x^2+kx+36$, then that means $(x+9)(ax+b)=3x^2+kx+36$. Foiling out $(x+9)(ax+b)$, we get $ax^2+9ax+bx+9b=3x^2+kx+36$. Comparing the two sides, we can conclude that $ax^2=3x^2 \rightarrow a=3$ and $9b=36 \rightarrow b=4$. Lastly, $9ax+bx=kx$, and plugging in $a=3$ and $b=4$, we get $9ax+bx=9(3)x+4x=(27+4)x=31x=kx$. So, $k=31$.

5. **D.** $x^2-16x+k=0$ has exactly one real solution if it is a perfect square. So, we can write $x^2-16x+k=0$ in the form $(x+a)(x+a)=0$ where $a^2=k$ and $2ax=-16x$. Solving for a, we get $2ax=-16x \rightarrow 2a=-16 \rightarrow a=-8$. Now we can plug in $a=-8$ to solve for k. $a^2=k \rightarrow (-8)^2=k \rightarrow 64=k$.

6. **B.** If $n=4a$ and $n=-5b$ are two solutions to the equation, that means that when $n=4a$ or $n=-5b$, the equation equals zero. This means the equation can be written as $(n-4a)(n-(-5b)) \rightarrow (n-4a)(n+5b)$. Foiling this out, we get $n^2-4an+5bn-20ab=n^2-n(4a-5b)-20ab$.

7. Using the quadratic formula, we plug in $a=3$, $b=20$, and $c=K$ to get :
$$x=\frac{-20\pm\sqrt{20^2-4(3)(K)}}{2(3)}=\frac{-20\pm\sqrt{400-12K}}{6}=-\frac{10}{3}\pm\frac{\sqrt{400-12K}}{6}.$$
Setting this expression (with the radical in its positive form since $\frac{1}{3}>-7$) to $\frac{1}{3}$, we get:
$$\frac{1}{3}=-\frac{10}{3}+\frac{\sqrt{400-12K}}{6} \rightarrow \frac{11}{3}=+\frac{\sqrt{400-12K}}{6} \rightarrow 22=\sqrt{400-12K} \rightarrow 484=400-12K \rightarrow 84=-12K \rightarrow -7=K.$$
To check that this value of K is correct, we check if this value of K also makes the second solution true. Plugging in $K=-7$ and $x=-7$, we get $3(-7)^2+20(-7)+(-7)=0 \rightarrow 3(49)-140-7=0 \rightarrow 147-140-7=0 \rightarrow 0=0$ is true.

8. **E.** Foiling this expression out, we get $\left(\frac{2}{3}x-\frac{1}{2}y\right)^2=\frac{4}{9}x^2-\frac{1}{3}xy-\frac{1}{3}xy+\frac{1}{4}y^2=\frac{4}{9}x^2-\frac{2}{3}xy+\frac{1}{4}y^2$.

9. **D.** $x^2 - y^2$ is a difference of squares, so it can be written as $(x+y)(x-y)$. This gives us $(x+y)(x-y)=169$. Plugging in our given value that $(x+y)=13$, we get $13(x-y)=169 \rightarrow x-y=13$. Now we have two equations $x-y=13$ and $x+y=13$. We can solve for x and substitute: $x=13+y$. $(13+y)+y=13$. $13+2y=13$. $2y=0$. This can only be true if This can only be true if $y=0$.

10. **C.** We expand the polynomial to $x^2+2xy+y^2=20$. Since $xy=5$, $y=\frac{5}{x}$. Plugging this in gives us $x^2+2x\left(\frac{5}{x}\right)+\frac{25}{x^2}=20$. We can simplify this $x^2+\frac{25}{x^2}-10=0$. Multiplying both sides by x^2 gives us $x^4-10x^2+25=0$. We can express this as $\left(x^2-5\right)^2=0$, and taking the square root of both sides gives us $x^2-5=0$. From this, it's simple to find that $x=\pm\sqrt{5}$, so there are 2 roots, each with their own respective y-values, leading us to our answer of 2 pairs.

11. **C.** We can factor out the negative sign from $(-4x-11)(x+2)$ to get $-(4x+11)(x+2)$. Now, we can redistribute the negative sign to the second factor and get $(4x+11)(-x-2)$.

12. **B.** We factor to get $(x+9)(x-1)$. If we sum the two polynomials, we get: $(x+9)+(x-1) \rightarrow x+9+x-1 \rightarrow 2x+8$.

13. **D.** Simple FOILing gives us $4x^2-12x+2x-6=4x^2-10x-6$.

14. **D.** FOILing gives us $21ab-7a^2+6b^2-2ab$. We rearrange this to $-7a^2+21ab-2ab+6b^2=-7a^2+19ab+6b^2$.

15. **B.** Our expression for $3x^2-14x+8$ will look like $(3x+n)(x+m)$. We need two numbers that multiply to 8 and sum to -14 when one of them is multiplied by 3. Whole number pairs that multiply to 8 are 8 and 1, -8 and -1, 4 and 2, and -4 and -2. Since they have to sum to a negative number, we can discard the positive sets, as the middle term coefficient, -14, is negative, leaving us with -8 and -1 and -4 and -2. If we sum the -8 and -1 pair while multiplying one of them by 3, we either get $-8+3(-1)=-8-3=-11$, or $-1+3(-8)=-1-24=-25$. Neither satisfies our equation. Using -4 and -2, we get either $-4+3(-2)=-4-6=-10$ or $-2+3(-4)=-2-12=-14$. The latter satisfies our equation. In order to have the -4 be multiplied by 3 when we FOIL, we have to put it in the opposite group from the $3x$. Thus, our solution will be $(x-4)(3x-2)$.

16. **E.** FOILing out the left side, we get $a^2-2ab+b^2=a^2-b^2$. This simplifies to $-2ab=-2b^2 \rightarrow ab=b^2 \rightarrow a=b$.

17. **A.** -20 is the product of -4 and 5 or -5 and 4. Only -4 and 5 sum to 1. Thus, x^2+x-20 can factor to $(x+5)(x-4)$. Adding these linear factors together gives us $x+5+x-4=2x+1$.