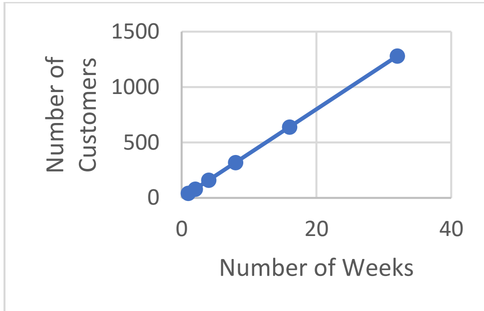
Linear and Exponential Growth Answer Key

1. **A.** Suppose *w* is the week number and *C* is the number of customers and we plot the information in the table on a *wC*-plane. The first point given is (1, 40). Whenever the week number is doubled, the number of customers arriving on that week is also doubled. Therefore, we get the following data points: (1, 40) (2, 80) (4, 160) (8, 320) (16, 640) (32, 1280). Plotting these points, we get the graph:



It is clear that the growth is linear and that the slope of this line is $\frac{1280-40}{32-1} = \frac{1240}{31} = 40.$. In other words, week number *w+1* will have approximately 40 more customers than in week *w*. The number of customers grew linearly due to an increase of approximately 40 customers every week. Choices B and D are incorrect because the number of customers did not grow exponentially. Choice C is incorrect because the number of customers per week did not increase by 100 percent every week, which would represent exponential growth.

2. **C.** The amount of chemical remaining after fifteen seconds in the first reaction is $A = 22 - 0.4(15) = 22 - 6 = 16$. In the second reaction, the amount of chemical loses half of itself every five seconds, or the total amount divides by 2 every five seconds. This is an exponential decay, so the number of chemical divides by 4 every ten seconds, and by 8 every fifteen seconds. The amount remaining after fifteen seconds in the second reaction is: $A_1 = \frac{22}{8} =.$ 2.75. The different between the amounts of chemical remaining is: $16 - 2.75 = 13.25$. Therefore, A is 13.25 grams greater than A_1 after 15 seconds. Choice A is incorrect because that is the amount remaining after fifteen seconds. Choice B is incorrect because that is the amount lost after fifteen seconds. Choice D is incorrect because that is the amount remaining in the first reaction after fifteen seconds.

3. **A.** For the old program, it takes 0.1 milliseconds to compute the first digit of pi, 2*0.1 milliseconds to compute the second digit of pi, 2^2*0.1 milliseconds to compute the third number of pi and so on. Because the time is being doubled when the index increases by a constant, 1, this is an exponential growth. $Old\ time = 0.1 \times 2^{n-1} = 0.1 \times \left(\frac{2^n}{2}\right) = 0.05 \times 2^n$. When $n = 6$, the time it takes for the old program to run is equal to $0.05 \times 2^6 = 0.1 \times 2^5 = 3.2.$ milliseconds to compute the 6th digit of π. For the new program, it takes 0.5 milliseconds to compute the first digit of π, $0.5 + 0.3$ milliseconds for the seconds digit of π, $0.5 + 2 \times 0.3$ milliseconds to compute the third digit of $\pi$and so on. This is an example of a linear growth. $New\ time = 0.5 + 0.3 \times (n - 1) = 0.5 + 0.3n - 0.3 = 0.2 + 0.3n$. When $n = 6$, the time it takes for the new program to run is equal to $0.2 + 0.3 \times 6 = 2.0.$ milliseconds. The difference in run time between these two programs is $3.2 - 2 = 1.2$ milliseconds. Choices B, C, and D are incorrect because none of them give the correct value of how much longer it takes the old program than the new program. Choice B is how long it takes for the new program to run. Choice D is approximately how long it takes for the old program to run.

4. **A.** Linear equations can be written in the form $y = mx + b.$ where $m.$ is the rate of change and $b.$ is the initial value. The given equation follows this form, where the initial value is 200 and for every $1\frac{\mu g}{dl}$ increase in t, there is a $135\frac{\mu g}{dl}$ increase in s. If we divide both values by 10, we do not change the meaning of the equation, and we get for every $0.1\frac{\mu g}{dl}$ increase in t, there is a $13.5\frac{\mu g}{dl}$ increase in s. Choices C and D are incorrect because the equation is not exponential. B is incorrect because there is not a $20\frac{\mu g}{dl}$ in s for every $\frac{\mu g}{dl}$ increase in t.

5. **B**. The slope-intercept form of equations is $y = mx + b$ where b is the initial amount and m is the rate of change. The relationship given fits this form, where $m = 1.59$ and $b = 25.4$. Thus, the rate of change shows us

that for each week that passes, the growth rate, as a percent, increases by 1.59. It is linear because the urbanization rate increases 1.59 each year. Choices C and D are incorrect because the increase in growth rate is not exponential. Choice A is incorrect because 25.4 is the initial value, not the growth rate.

6. **D.** We know that the price is dropping by 14% each week. Equivalently, the price is only $1 - 0.14 = 0.86$, or 86% of the original price. Choices A and C are incorrect because the prices are dropping so r should be less than 1. Choice B is incorrect because the new price is not 14% of the original price, it is 14% off the original price.

7. **C.** To find the price, plug in the numbers from the formula. We get $p = 150(0.86)^3 = 95.4084 \approx 95.41$. A, B, and D are incorrect and may result from error in plugging in the correct numbers.

8. **D**. According to the formula, the number of avocado trees one year from now will be $200 + 0.1(200)\left(1 - \left(\frac{200}{300}\right)\right) = 207$. Then using the formula again for the number of avocado trees two years from now will be $207 + 0.1(207)\left(1 - \left(\frac{207}{300}\right)\right) = 213.417$. Rounding this we get 213. Then, using the formula one last time for the number of avocado trees three years from now will be $213 + 0.1(213)\left(1 - \left(\frac{213}{300}\right)\right) = 219.177$. Rounding this value to the nearest whole number gives 219 trees. Choice A is incorrect and is the initial number of avocado trees. Choice B is incorrect and is the number of avocado trees one year from now. Choice C is incorrect and is the number of avocado trees two years from now.

9. **C**. If the number of trees is to be increased from 200 to 230 next year, then the number of trees that the farm can support, K, must satisfy the equation $230 = 200 + 0.3(200)\left(1 - \left(\frac{200}{K}\right)\right)$. Solving for K, we get that $K = 400$. Choices A, B, and D are incorrect and may result from using the equation incorrectly.

10. **D**. An exponential equation has the form of $y = a(r)^t$ where a is the initial value, r is the rate of growth (or loss), and t is time. If we set t = 0, 1, 2, we get population values of $P(t) = 219$, 459.9, 965.79 respectively. From these values, we can see that the population of bacteria is 2.1 times larger than it was in the previous hour. Choices A and B are incorrect because the relationship is not linear. Choice C is incorrect because 219 is the initial value and not the growth rate of the bacteria.