SCIENTIFIC NOTATION

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- How to complete basic problems that asks for answers in scientific notation
- How to complete word problems involving scientific notation

THE BASICS

Most of you probably learned scientific notation back in elementary or middle school—it's one of those skills that you may have known once, but may be a bit rusty now.

So let's review some of the basics:

1) For a number in the form $a*10^n$ to be in scientific notation, it must contain a value (a) greater than or equal to 1 and less than 10 $(1 \le a < 10)$ multiplied by ten to power (n), where n is an integer.

Some ACT problems may be easier to solve when you can eliminate answers NOT officially in scientific notation off that technicality.

This **IS** in scientific notation:

$$5.5*10^{-7}$$

This is **NOT** in scientific notation:

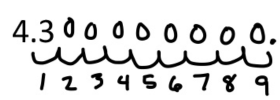
$$55*10^{-6}$$

2) When a number is multiplied by ten to a positive exponent of degree n, move the decimal point n places to the RIGHT.

Expand the following number:

$$4.3*10^9$$

Because the exponent is 9, Move the decimal points nine places to the right:



As you can see, I draw a little loop for each hop, and count all the loops.

$$4{,}300{,}000{,}000$$

You get **4.3** billion.

(To change the number back to scientific notation, move the decimal back to the left, and count. When you are to the point that only one number remains to the left, you're done. Take the number of moves—that is your exponent).

3) When a number is multiplied by ten to a negative exponent of degree n, move the decimal point n places to the LEFT.

$$5.854\times10^{-5}+3.56\times10^{-3}=?$$

First we'll put the first number in regular form, then the second. Then we'll add the numbers and convert back to scientific notation.

$$.00005.854 \qquad + \qquad .003.56$$

5 4 3 2 1   3 2 1

Then line up the decimal points to add the numbers (don't forget to carry the "1"):

$$\begin{array}{r} 1 \\ 0.00005854 \\ +\ 0.00356 \\ \hline 0.00361854 \end{array}$$

Now we convert back to scientific notation:

.00361854

1 2 3

As you can see, we need to move the decimal point three places to the right—so we add an exponent of negative 3 to our 10:

$$3.61854*10^{-3}$$

That's our answer!

WORD PROBLEMS:

We'll also need to know how to solve word problems that involve scientific notation. In general, for these problems, first convert the scientific notation numbers into standard form. Then, solve the problem. Finally, convert your answer back to scientific notation.

The area of four oceans is given in the following chart in square miles. What is the total area that these four oceans cover?

Indian	$2.84*10^7$
Pacific	$6.25*10^7$
Atlantic	$4.11*10^7$
Arctic	$5.43*10^6$

The area of four oceans is given in the following chart in square miles. What is the total area that these four oceans cover?

For the problem, because three of the oceans have the same power of 10, we can add those integers first, factoring out the 10^7 term.

$$2.84*10^7+6.25*10^7+4.11*10^7=10^7*(2.84+6.25+4.11)$$

Using your calculator, you can find that $2.84+6.25+4.11=13.2$

Now we can convert that to standard form:

13.2000000.
1 2 3 4 5 6 7

That's 132 million: $132{,}000{,}000$

Now we convert the Arctic Ocean to standard form and add:

5,430000.
1 2 3 4 5 6

That's $5{,}430{,}000$ or 5.43 million.

Now we add 132 million plus 5.43 million:

$$\begin{array}{r} 132{,}000{,}000 \\ +\quad 5{,}430{,}000 \\ \hline 137{,}430{,}000 \end{array}$$

Finally, we convert back to scientific notation:

137,430,000.
8 7 6 5 4 3 2 1

or $1.3743*10^8$

TRICKY PROBLEMS:

Whenever x and y are both integers, what is the approximate value of $\left(4.56*10^{x}\right)\left(5.22*10^{y}\right)$ expressed in scientific notation?

A. $2.38*10^{yx+1}$

B. $2.38*10^{x+y}$

C. $2.38*10^{x+y+1}$

D. $2.38*10^{x+y-1}$

E. $2.38*10^{x-y-1}$

Occasionally, you may have scientific notation problems that involve variables in the answer choices. For these problems, you could make up numbers, find the pattern and plug in what you made up into each answer choice to discover the answer. You can also attempt algebraically.

Algebraic Method:

Here you can first rearrange to get: $\left(4.56*5.22\right)\left(10^{x}*10^{y}\right)$

Applying our exponent rule: $(a^{x}*a^{y}=a^{x+y})$

That's approximately:

$$(23.8)(10^{x+y})$$

Now I put the 23.8 in scientific notation:

$$(2.38*10^{1})(10^{x+y})$$

And finally, I combine the 10 with the other 10 powers:

$$(2.38)(10^{1}*10^{x+y})$$

Again I apply the power rule $(a^{x}*a^{y}=a^{x+y})$

$$(2.38)(10^{x+y+1})$$

Answer: **C.**

1. The platelet count of a healthy adult is about 2.5×10^{-5} parts per 1 million parts blood. In a patient with thrombocytopenia, a disorder in which the body produces fewer platelets, the platelet level is 100 times lower than this average. What is the approximate patient's platelet count in parts per million?

A. 2.5×10^{-7}

B. 2.5×10^{-3}

C. 2.5×10^{-105}

D. -9.9×10

E. 2.5×10^{7}

2. The average diameter of a monocyte, a type of white blood cell, is 2.00×10^{-5} meters. A monocyte's diameter is about how many times the diameter of a red blood cell, whose average diameter is 6.50×10^{-6}?

A. 3.25×10

B. 3.08×10

C. 3.08

D. 3.25

E. 3.08×10^{-11}

3. A hummingbird beats its wings approximately 14 times per second. After 5 hours and 20 minutes, how many times has the bird beat its wings?

A. 7.47×10^{1}

B. 2.62×10^{5}

C. 4.36×10^{3}

D. 2.70×10^{5}

E. 4.48×10^{2}

4. At sea level, the speed of sound is $3.40*10^{2}$ meters per second. After 3 hours and 30 minutes, about how many kilometers could a sound wave travel?

A. 4.28×10^{3} km

B. 7.14×10^{3} km

C. 7.14×10^{4} km

D. 6.73×10^{4} km

E. 4.28×10^{5} km

5. According to the EPA, water is no longer safe to drink once the level of lead in the water reaches $15{,}000{,}000$ parts per billion. What is this level of lead water contamination written in scientific notation?

A. 1.5×10^{-1}

B. 1.5×10^{-2}

C. 1.5×10^{-3}

D. 1.5×10

E. 1.5×10^{2}

6. Which is the closest to 6.5×10^{-4}?

A. 0.065

B. 0.0065

C. 0.00065

D. 0.000065

E. 0.0000065

7. Whenever x and y are both integers, what is $\dfrac{3.5\times10^{y}}{6.2\times10^{x}}$ expressed in scientific notation?

A. $5.6\times10^{y-x-1}$

B. $5.6\times10^{y-x}$

C. $5.6\times10^{\frac{y}{x}-1}$

D. $5.6\times10^{y+x}$

E. $5.6\times10^{x-y-1}$

ANSWERS

1. A **2. C** **3. D** **4. A** **5. B** **6. C** **7. A**

ANSWER EXPLANATIONS

1. **A.** If the platelet count is 100 times lower than the healthy amount of $2.5*10^{-5}$ parts, then it has a count of $\frac{2.5*10^{-5}}{100}=\frac{2.5*10^{-5}}{10^2}\to 2.5*10^{-7}$ parts per million.

2. **C.** 2.00×10^{-5} is x times $6.50*10^{-6}$. Solving for x, we get

$$2.00\times10^{-5}=x\left(6.50\times10^{-6}\right)\to x=\frac{2.00\times10^{-5}}{6.50\times10^{-6}}=\frac{2}{6.50}\times10^{-5-(-6)}=0.308\times10^{1}=3.08.$$

3. **D.** A hummingbird's wing beats $\frac{14\,beats}{sec}\times\frac{60\text{sec}}{1\,min}=840$ beats per minute. We calculate 5 hours and 20 minutes is equal to $5\left(60\right)+20=320$ min. So, a hummingbird beats its wings approximately $\frac{840\,beats}{min}\times320min=268800$ beats in 5 hours and 20 minutes. Writing this in scientific notation, we have 2.70×10^{5}.

4. **A.** At sea level, the speed of sound is $\frac{3.4\times10^{2}\,meters}{sec}\times\frac{60\text{sec}}{1\,min}=60\left(3.4\right)\times10^{2}=204\times10^{2}=2.04\times10^{4}$ meters per minute. We calculate 3 hours and 30 minutes is equal to $3\left(60\right)+30=210$ min. So, the speed of sound is approximately $\frac{2.04\times10^{4}\,meters}{min}\times\frac{1\,kilometer}{1000\,meters}\times210min=428.4\times10=4.28\times10^{3}$ kilometers in 3 hours and 30 minutes.

5. **B.** $15{,}000{,}000$ parts per billion is equal to $\frac{15{,}000{,}000}{1{,}000{,}000{,}000}$. Canceling out the zeros on the numerator and denominator, we get $\frac{15}{1000}=0.015=1.5\times10^{-2}$.

6. **C.** $6.5\times10^{-4}=0.00065$. So, the value closest to that, with the same number of zeros preceding the first non-zero integer, is 0.00065.

7. **A.** $\frac{3.5\times10^{y}}{6.2\times10^{x}}$ is $\frac{3.5}{6.2}\times10^{y-x}=0.56\times10^{y-x}=5.6\times10^{-1}\times10^{y-x}=5.6\times10^{y-x-1}$.