SAT 41 Complex Numbers Answers and Explanations

1) D

$i = \sqrt{-1}$, which means that $i^2 = -1$. First, we must FOIL the complex number, as follows:

$$(3 - 4i)(3 - 4i)$$
$$9 - 12i - 12i + 16i^2$$
$$9 - 24i + 16i^2$$

Now we substitute $i^2 = -1$.

$$9 - 24i + 16(-1)$$
$$-7 - 24i$$

2) C

When adding complex numbers, we simply add up the real numbers within the complex numbers. In this case, we can derive the answer as follows:

$$(2 + 3i) + (4 + 9i)$$
$$(2 + 4) + (3 + 9)i$$
$$6 + 12i$$

3) B

For this problem, we must multiply the top and bottom of the fraction by the conjugate of the denominator which in this case would be $3 + 2i$.

$$\frac{4 + 5i}{3 - 2i} \times \frac{3 + 2i}{3 + 2i}$$
$$\frac{12 + 8i + 15i + 10i^2}{9 - 4i^2}$$
$$\frac{12 + 23i - 10}{9 + 4}$$
$$\frac{2}{13} + \frac{23}{13}i$$

4) C

First let's FOIL the denominator of this expression.

$$(2 + 2i)^2$$
$$4 + 4i + 4i + 4i^2$$
$$4 - 4 + 8i$$
$$8i$$

If the denominator of the expression is 8i, we must multiply the numerator and denominator by 8i as follows.

$$\frac{3 + 8i}{8i} \times \frac{8i}{8i}$$

$$\frac{24i + 64i^2}{64i^2}$$
$$-\frac{3}{8}i + 1$$

5) D

For this problem, we must treat i as a variable when we multiply the complex numbers as shown below.

$$(3 + 3i)(4 - 10i)$$
$$12 - 30i + 12i - 30i^2$$
$$12 - 18i + 30$$
$$42 - 18i$$

6) A

For this problem, we must multiply the top and bottom of the fraction by the conjugate of the denominator which in this case would be $8 + 2i$.

$$\frac{10 + 4i}{8 - 2i} \times \frac{8 + 2i}{8 + 2i}$$
$$\frac{80 + 20i + 32i + 8i^2}{64 - 4i^2}$$
$$\frac{80 + 20i + 32i - 8}{64 + 4}$$
$$\frac{72 + 52i}{68}$$

Now we simplify to get

$$\frac{72}{68} + \frac{52}{68}i$$
$$\frac{18}{17} + \frac{13}{17}i$$

7) C

For complex numbers, we add the real numbers inside as normal as follows

$$(2 + 3i) + (-3 + 8i)$$
$$2 - 3 + 3i + 8i$$
$$-1 + 11i$$

8) A

First let's foil the expression $(9 - 4i)^2$

$$(9 - 4i)(9 - 4i)$$
$$81 - 36i - 36i + 16i^2$$
$$81 - 72i - 16$$
$$65 - 72i$$

Now we subtract $65 - 72i$ from $6 + 3i$.

$$(6+3i)-(65-72i)$$
$$6+3i-65+72i$$
$$-59+75i$$

9)B

We simply multiply the two complex numbers.

$$(-5+4i)(7+7i)$$
$$-35-35i+28i+28i^2$$
$$-35-7i-28$$
$$-63-7i$$

10) A

To change the expression into the form $a+bi$, we must multiply the numerator and denominator by the conjugate of the denominator. In this case, the conjugate would be $3+4i$.

$$\frac{12+i}{3-4i}\times\frac{3+4i}{3+4i}$$
$$\frac{36+48i+3i+4i^2}{9-16i^2}$$
$$\frac{32+51i}{25}$$
$$\frac{32}{25}+\frac{51}{25}i$$

In this standard form, a is $\frac{32}{25}$.