Manipulating quadratic and exponential expressions answers and explanations

Answers
1.B 2.C 3.C 4.B 5.C 6.A 7.C 8.C 9.D 10.D

Answer Explanations

1. **B.** In this problem, we are given the expression $A(t) = 100(1.75)^{\frac{t}{4}}$ and asked to find the best numerical expression that represents the profit gained after one year. Note that interest grows at an exponential rate because the loan will gain more interest in the last week than the first week. Since there are 52 weeks in a year, we plug in 52 for t and solve.
$$A(t) = 100(1.75)^{\frac{t}{4}}$$
$$A(52) = 100(1.75)^{\frac{52}{4}}$$
$$A(52) = 100(1.75)^{13}$$
After one year, the total amount of money owed will equal $100(1.75)^{13}$. However, the initial amount owed when $t = 0$ was 100. So, we find that the total profit gained through interest is the final amount owed minus the initial amount owed, or $100(1.75)^{13} - 100$. By factoring out the 100, we get $100((1.75)^{13} - 1)$.

2. **C.** In this problem, we are given the expression $p = 0.5(m - 2.5)(m - 8) + 12$ and asked to find when the deer population reaches 12,000 again. Here we want $p = 12$. Therefore:
$$12 = 0.5(m - 2.5)(m - 8) + 12$$
$$0 = (m - 2.5)(m - 8)$$
$$m = 2.5 \text{ and } m = 8$$
Because the problem is asking when the deer population reaches twelve thousand for the second time in the year, we know the answer is 8, because m represents months after January 1. The first time the deer population reaches 12,000 is 2.5 months after January 1, and the second time is 8 months after January 1.

3. **C.** In this problem, we are given the expression $h(t) = -4.9t^2 + 14.7t$ and asked to find when a vertically tossed baseball hits the ground. Note that when $h(t) = 0$ the height of the baseball is at ground level. Therefore:
$$h(t) = -4.9t^2 + 14.7t$$
$$0 = -4.9(t^2 - 3t)$$
$$0 = -4.9(t)(t - 3)$$
We know that $t = 0$ and $t = 3$. Since the ball was thrown at $t = 0$, we can assert that the ball hits the ground at $t = 3$.

4. **B.** In this problem, we are given the expression $h(t) = -4t^2 + 24t + 64$ and asked to find when the ball reaches its maximum height. Note that we must rewrite our equation in vertex form to determine the maximum height.
$$h(t) = -4t^2 + 24t + 64$$
$$h(t) = -4(t^2 - 6t) + 64$$
$$h(t) = -4(t^2 - 6t + 9) + 100$$
$$h(t) = -4(t - 3)^2 + 100$$
Note that the value of a is negative; therefore, the parabolic structure is inverted, and our vertex point is the maximum value. Thus, our maximum value comes from the vertex coordinates $(h, k) \rightarrow (3,100)$. From this, we know that the maximum value is 100 feet and it is reached at 3 seconds.

5. **C.** In this problem, we are given the expression $E = 28 + 0.5(t - 8)^2$ and asked to find the elevation in meters of the volume of the storm clouds at the start of the recording. Given this information, we know that the *start* of recording can be represented by $t = 0$. $E = 28 + 0.5\left((0) - 8\right)^2 \rightarrow E = 28 + 0.5(64) \rightarrow E = 60$. Thus, the elevation in meters of the volume of air at the start of recording is 60 meters.

6. **A.** In this problem, we are given the expression $V(x) = \pi(6x^2 + 36x + 54)$ and asked to find the approximate radius of the old silo. In order to find this, we must first simplify our equation:

$$V(x) = \pi(6x^2 + 36x + 54)$$
$$V(x) = 6\pi(x^2 + 6x + 9)$$
$$V(x) = 6\pi(x + 3)^2$$

Since the height of the new silo is the same as the old one, the only dimension that changes is the radius. Therefore, if the change in radius, x, equals 0, then the value inside the parenthesis is the radius of the old silo. Since $(0 + 3) = 3$, $3in$ is the radius of the old silo.

7. **C.** In this problem, we are given the expression $P = \frac{1}{4}s^2 - 6s + 24$ and asked to find the coin market price where Jenny will incur the greatest loss. We can determine the minimum value by rewriting the equation in vertex form:

$$P = a(s - s_0)^2 + P_0$$
$$P = \frac{1}{4}s^2 - 6s + 24$$
$$P = \frac{1}{4}(s^2 - 24s) + 24$$
$$P = \frac{1}{4}(s - 12)^2 - \frac{1}{4}(144) + 24$$
$$P = \frac{1}{4}(s - 12)^2 - 36 + 24$$
$$P = \frac{1}{4}(s - 12)^2 - 12$$

Remembering that equations in vertex form can be written as $a(x - h)^2 + k$, since a is positive, P achieves a minimum at $(12, -12)$. Therefore, a maximum loss of $12,000 after one year occurs if the stock price is $12.

8. **C.** In this problem, we are given the expression $P(x) = -\frac{1}{5}x^2 + 11.6x$ and asked to find the maximum height of the *Arc de Triomphe.* Note to find the maximum height we will put the equation in vertex form.

$$P(x) = -\frac{1}{5}x^2 + 11.6x$$
$$P(x) = -\frac{1}{5}(x^2 - 58x)$$
$$P(x) = -\frac{1}{5}(x - 29)^2 + 168.2$$

Therefore, the maximum height of the *Arc de Triomphe* is 168.2 meters, when $x = 29$.

9. **D.** In this problem, we are given the expression $h(x) = -0.25x^2 + 12x$ and asked to find the width of the arch at ground level. In order to determine the width, we must know where the arch hits the ground.

$$h(x) = -0.25x^2 + 12x$$
$$h(x) = -0.25x(x - 48)$$

Looking at the factored form, we can determine the x -intercepts are $(0,0)$ and $(48,0)$. So, the width of the arch at ground level is $48 - 0 = 48\ m$.

10. **D.** In this problem, we are given the expression $P(x) = -\frac{1}{5}x^2 + 5.8x$ and asked to find the number of units that need to be sold to reach the break-even point.

$$P(x) = -\frac{1}{5}x^2 + 5.8x$$
$$P(x) = -\frac{1}{5}x(x - 29)$$
$$0 = -\frac{1}{5}x(x - 29)$$
$$x = 0 \text{ and } x = 29$$

Thus, the breakeven point is at 29 units.