

THE BEST ACT PREP COURSE EVER

FRACTIONS

ACT Math: Problem Set

1. Which of the following is less than $\frac{2}{3}$?
- A. $\frac{3}{4}$
B. $\frac{3}{5}$
C. $\frac{6}{9}$
D. $\frac{7}{10}$
E. $\frac{9}{13}$
2. Seta has a long list of numbers that she first must divide by $\frac{2}{3}$ and then multiply by $\frac{8}{9}$ for a school assignment. She could achieve the same result in one step by dividing each number on the list by which of the following numbers?
- A. $\frac{3}{4}$
B. $\frac{1}{2}$
C. $\frac{1}{4}$
D. 3
E. 4
3. Mitch is making trail mix that needs $3\frac{1}{3}$ cups of raisins. However, he only has $\frac{1}{2}$ cups of raisins at home. The amount of raisins Mitch has at home is what fraction of the amount he needs for the recipe?
- A. $\frac{2}{3}$
B. $\frac{1}{2}$
C. $\frac{1}{4}$
D. $\frac{2}{15}$
E. $\frac{3}{20}$
4. Sidra is building a birdhouse. She drills a hole with a drill bit and tries to find a corresponding screw size. The $\frac{1}{4}$ inch screw is too big, and the $\frac{1}{8}$ inch screw is too small. Which of the following could be the size of the drill bit Sidra used?
- A. $\frac{1}{2}$ -inch
B. $\frac{3}{16}$ -inch
C. $\frac{5}{16}$ -inch
D. $\frac{4}{32}$ -inch
E. $\frac{8}{32}$ -inch
5. Sugar Rush Candy Company has x chocolate bars and places \$25 gift certificates for their stores in y of them as a way to attract more customers. Which of the following is a general expression for the fraction of the chocolate bars that do NOT have gift certificates?
- A. $\frac{y}{x}$
B. $\frac{x}{y}$
C. $\frac{x-y}{x}$
D. $\frac{x-y}{y}$
E. $\frac{y-x}{x}$
6. Of the 907 juniors in Walker High School, approximately $\frac{4}{5}$ of them are taking a foreign language, and approximately $\frac{3}{4}$ of those are taking Spanish. Which of the following is the closest estimate for how many juniors are taking Spanish?
- A. 180
B. 430
C. 550
D. 750
E. 825

7. The expression $\frac{6 - \frac{1}{8}}{2 + \frac{1}{16}}$ is equal to:
- A. $\frac{94}{33}$
 B. $\frac{1551}{128}$
 C. 3
 D. 6
 E. 10
8. Which of the following is equivalent to $\frac{28x}{9} + \frac{4x}{3}$?
- A. $2\frac{2}{9}$
 B. $4x$
 C. $4\frac{2}{3}x$
 D. $4\frac{4}{9}x$
 E. $13\frac{1}{3}x$
9. The expression $\frac{\frac{1}{5} + \frac{1}{6}}{\frac{3}{4} - \frac{1}{3}}$ equals:
- A. $\frac{11}{144}$
 B. $\frac{9}{2}$
 C. $\frac{11}{20}$
 D. $\frac{18}{11}$
 E. $\frac{54}{11}$
10. $\frac{1}{4} * \frac{2}{5} * \frac{3}{6} * \frac{4}{7} * \frac{5}{8} * \frac{6}{9} * \frac{7}{10} * \frac{8}{11} = ?$
- A. $\frac{1}{82}$
 B. $\frac{3}{55}$
 C. $\frac{1}{165}$
 D. 1
 E. $\frac{1}{110}$
11. In what order should $\frac{5}{2}, \frac{9}{3}, \frac{5}{4}$ and $\frac{7}{5}$ be listed to be arranged by increasing size?
- A. $\frac{5}{4} < \frac{7}{5} < \frac{5}{2} < \frac{9}{3}$
 B. $\frac{5}{4} < \frac{7}{5} < \frac{9}{3} < \frac{5}{2}$
 C. $\frac{9}{3} < \frac{5}{4} < \frac{9}{3} < \frac{5}{2}$
 D. $\frac{5}{2} < \frac{5}{4} < \frac{9}{3} < \frac{7}{5}$
 E. $\frac{7}{5} < \frac{5}{4} < \frac{5}{2} < \frac{9}{3}$
12. Suppose $z < -1$. Which of the following has the greatest value?
- A. $\frac{1}{z}$
 B. z^2
 C. z^3
 D. $\sqrt{-z}$
 E. $-\frac{z}{2}$
13. What is the least common denominator of the fractions $\frac{3}{8}, \frac{4}{9}$, and $\frac{7}{30}$?
- A. 47
 B. 220
 C. 360
 D. 500
 E. 2,160
14. What is the least common denominator for adding fractions $\frac{2}{45}, \frac{4}{63}$, and $\frac{3}{14}$?
- A. 126
 B. 315
 C. 630
 D. 2,835
 E. 39,690

15. Which of the following is the least common denominator for the expression $\frac{1}{29^2 \cdot 89 \cdot 907^2} + \frac{1}{29 \cdot 89^3} + \frac{1}{29^3 \cdot 907}$?
- A. $29 \cdot 89$
B. $29 \cdot 89 \cdot 907$
C. $29 \cdot 89 \cdot 907^2$
D. $29^3 \cdot 89^3 \cdot 907^2$
E. $29^6 \cdot 89^4 \cdot 907^3$
16. What fraction of $3\frac{1}{2}$ is $1\frac{1}{6}$?
- A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{3}{1}$
D. $\frac{1}{6}$
E. $\frac{5}{12}$
17. What fraction is halfway between $\frac{1}{7}$ and $\frac{1}{5}$?
- A. $\frac{1}{3}$
B. $\frac{2}{35}$
C. $\frac{1}{6}$
D. $\frac{12}{35}$
E. $\frac{6}{35}$
18. The value of x that solves $\frac{x}{3} + 1 = \frac{5}{6}$ lies between which of the following numbers?
- A. -3 and -1
B. -1 and 0
C. 0 and 1
D. 1 and 3
E. 3 and 6
19. Which of the following gives the range of numbers that are within $\frac{4}{3}$ of the number $\frac{3}{4}$?
- A. $-\frac{25}{12}$ to $\frac{7}{12}$
B. $-\frac{25}{12}$ to $\frac{25}{12}$
C. $-\frac{7}{12}$ to $\frac{7}{12}$
D. $-\frac{7}{12}$ to $\frac{25}{12}$
E. -1 to 1
20. When $1 \leq x \leq 6$ and $18 \leq y \leq 24$, the largest possible value for $\frac{4}{y-x}$ is:
- A. $\frac{4}{23}$
B. $\frac{2}{9}$
C. $\frac{4}{17}$
D. $\frac{1}{3}$
E. 3
21. For American Literature class, Liz must read *As I Lay Dying* in 11 days. She reads $\frac{1}{16}$ of the book each day for the first 4 days. For the remaining 7 days, on average, what fraction of the book must Liz read per day?
- A. $\frac{15}{112}$
B. $\frac{3}{28}$
C. $\frac{3}{4}$
D. $\frac{1}{4}$
E. $\frac{1}{16}$

ANSWER KEY

1. B 2. A 3. E 4. B 5. C 6. C 7. A 8. D 9. D 10. C 11. A 12. B 13. C 14. C
 15. D 16. B 17. E 18. B 19. D 20. D 21. B

ANSWER EXPLANATIONS

- B.** We can use decimal notation as an easy way to compare values. Remember that $\frac{2}{3} \approx 0.66$. Answer choice A, $\frac{3}{4} = 0.75$, is greater, so it is not the answer. For B, $\frac{3}{5} = \frac{6}{10} = 0.6$, which is less than 0.66 , so B is the correct answer.
- A.** Dividing by a fraction is the same as multiplying by the reciprocal of that fraction. Thus, we can say that the function is multiplying by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$, and multiplying again by $\frac{8}{9}$. Combined, we multiply by $\frac{3}{2} * \frac{8}{9} = \frac{4}{3}$. However, the question asks for what we divide by, so our answer is the reciprocal, $\frac{3}{4}$.
- E.** We can find the fraction Mitch already has by dividing the amount he already has by how much he needs:

$$\frac{\frac{1}{2}}{3\frac{1}{3}} = \frac{\frac{1}{2}}{\frac{10}{3}} = \frac{1}{2} * \frac{3}{10} = \frac{3}{20}$$
- B.** It is simple to solve this problem by finding the least common multiple of all of the fractions, which will be 16. Be careful to only use this technique in problems where we can tell by looking that the least common multiple is easy to find, otherwise it may become too large. In this case, we have $\frac{1}{4} = \frac{4}{16}$, the size that is too big, and $\frac{1}{8} = \frac{2}{16}$, the size that is too small. Answer A, $\frac{1}{2} = \frac{8}{16}$, too big. Answer B, $\frac{3}{16}$, falls between $\frac{4}{16}$ and $\frac{2}{16}$, so it is our answer.
- C.** Since x is the total number of bars and y is the number of bars that do have gift certificates, $x - y$ must be the number of bars that do not have certificates. To express this as a fraction of the whole, we divide by the whole number of chocolate bars, x . Thus our final answer is $\frac{x - y}{x}$.
- C.** To find the fraction of a population as a whole number, we multiply the whole number by the fraction. First find the number of students taking a foreign language: $907 * \frac{4}{5} = 725.6 \approx 726$. Next, find the fraction of these students who are taking Spanish: $726 * \frac{3}{4} = 544.5 \approx 545$. The closest answer choice is 550, C.
- A.** First, express the top and bottom of the overall fraction in terms of their least common denominators. For the top, $6 - \frac{1}{8}$ is equal to $\frac{48}{8} - \frac{1}{8}$. For the bottom, $2 + \frac{1}{16}$ is equal to $\frac{32}{16} + \frac{1}{16}$. Substituting these in for the top and bottom gives us

$$\frac{\frac{48-1}{8}}{\frac{32+1}{16}} = \frac{\frac{47}{8}}{\frac{33}{16}}$$
 Since dividing by a fraction is the same as multiplying by its reciprocal, this is $\frac{47}{8} * \frac{16}{33} = \frac{47}{1} * \frac{2}{33} = \frac{94}{33}$.
- D.** First, express the fractions in terms of their least common denominator, 9. We get $\frac{28x}{9} + \frac{12x}{9} = \frac{40x}{9}$. None of the answer choices has an improper fraction, so we turn this into a mixed number: $\frac{40x}{9} = \left(\frac{36}{9} + \frac{4}{9}\right)x = \left(4 + \frac{4}{9}\right)x = 4\frac{4}{9}x$.

9. D. First, express the addition and subtractions of fractions in terms of their least common denominator. $\frac{1}{5} + \frac{1}{6}$ becomes

$$\frac{6}{30} + \frac{5}{30} = \frac{11}{30}, \quad \frac{3}{4} - \frac{1}{3} \text{ becomes } \frac{9}{12} - \frac{4}{12} = \frac{5}{12}. \text{ Now our expression looks like } \frac{\frac{11}{30}}{\frac{5}{12}}.$$

We will divide by the fractions, starting with the numerator, by multiplying by the reciprocal of the denominator. This gives

$$\text{us } \frac{1}{4} \cdot \frac{30}{5} = \frac{30}{20} = \frac{3}{2}, \quad \frac{30}{44} = \frac{30}{44} \cdot \frac{12}{5} = \frac{6}{11} \cdot \frac{12}{5} = \frac{6}{11} \cdot \frac{3}{1} = \frac{18}{11}.$$

10. C. We can solve this problem by plugging it into a calculator, but it is actually easier to start by cancelling out equal terms on the top and bottom. There is a 4, 5, 6, 7, and 8 on the top and the bottom, and they cancel out, leaving us with $\frac{1 \cdot 2 \cdot 3}{9 \cdot 10 \cdot 11}$.

We can further simplify this by cancelling out the 2 and 10 to get 1 and 5, and by cancelling out the 3 and 9 to get 1 and 3. We are now left with $\frac{1}{3 \cdot 5 \cdot 11}$. This equals $\frac{1}{165}$.

11. A. This problem is simpler to solve when we express the fractions as decimals. $\frac{5}{2}$ becomes 2.5, $\frac{9}{3}$ becomes 3, $\frac{5}{4}$ becomes

1.25. $\frac{7}{5}$ is not immediately obvious, but recognize that it equals $1\frac{2}{5} = 1\frac{4}{10}$. 4 out of 10 is 0.4, so overall this is 1.4. We now can order our decimals: 2.5, 3, 1.25, 1.4 with ease. In increasing size, the order is 1.25, 1.4, 2.5, 3. Now, by matching the decimals with the corresponding fractions from the beginning, we get $\frac{5}{4}, \frac{7}{5}, \frac{5}{2}, \frac{9}{3}$.

12. B. In a question like this, we can start by considering which answers will be positive and which will be negative, and setting aside the negative ones. Since z is negative, answer choice A, $\frac{1}{z}$ will also be negative. Answer choice B, z^2 will be positive. Answer choice C, z^3 , will be negative. Answer choice D, $\sqrt{-z}$, will be positive. Answer choice E, $-\frac{z}{2}$, will be positive. The positive answers are thus B, D, and E. To find the largest one, consider that answer C, z^2 , because it is greater than -1 and therefore *not* a fraction, produces a number larger than $|z|$, while D and E produce numbers smaller than $|z|$, taking the square root of the absolute value and dividing by 2, respectively. Note that if the problem said only $z < 0$, this answer could not be determined. But because we are told that $z < -1$, answer choice C works.

13. C. In finding the least common denominator, the only important parts of the fraction are the denominator. The least common denominator is the least common multiple of the denominators: 8, 9, and 30. We can express these as their prime factorizations: $2 \cdot 2 \cdot 2$, $3 \cdot 3$, and $2 \cdot 3 \cdot 5$. The maximum number of 2's in the prime factorization of any of the numbers is three, so 2^3 must be part of the prime factorization of the LCD. The maximum number of 3's in the prime factorization of any of the numbers is two, so 3^2 is also part of our LCD. Finally, The maximum number of 5's in the prime factorization of any of the numbers is 1, giving us 5^1 . Our least common multiple is the product of all of these: $2^3 3^2 5^1 = 8 \cdot 9 \cdot 5 = 360$.

14. **C.** The only important parts of the fraction when finding the least common multiple are the denominators. The least common denominator is the least common multiple of the denominators: 45, 63, and 14. We look their prime factorizations to find the LCD: $3 \cdot 3 \cdot 5$, $3 \cdot 3 \cdot 7$, and $2 \cdot 7$, respectively. Every single distinct number in these prime factorizations must appear in our LCD, and the number must at least appear the maximum number of times it appears in any one factorization. To elaborate, the maximum number of 2's in the prime factorization of any of the numbers is 1, so 2^1 is in our LCD. The maximum number of 3's in the prime factorization of any of the numbers is two, giving us 3^2 for our LCD. The maximum number of 5's in the prime factorization of any of the numbers is one, which tells us 5^1 is in our LCD. Finally, the maximum number of 7's in the prime factorization of any of the numbers is one, giving us 7^1 . The least common multiple is the product of these: $2^1 3^2 5^1 7^1 = 2 \cdot 9 \cdot 5 \cdot 7 = 630$.
15. **D.** The least common denominator is the least common multiple of the denominators: $29^2 \cdot 89 \cdot 907^2$, $29 \cdot 89^3$, and $29^3 \cdot 907$. The number 29 appears three times at most in any one of the denominators, giving us 29^3 . The number 89 also appears three times at most in any one of the denominators, giving us 89^3 . The number 907 appears two times at most in any single one of the denominators, giving us 907^2 . Our least common multiple is the product of this: $29^3 89^3 907^2$.
16. **B.** If $1\frac{1}{6}$ is a certain fraction of $3\frac{1}{2}$, that means that $3\frac{1}{2}$ multiplied by that fraction will equal $1\frac{1}{6}$. We set up our equation as $3\frac{1}{2}n = 1\frac{1}{6}$. Convert from mixed fractions to improper fractions: $\frac{7}{2}n = \frac{7}{6}$. Next, isolate n by multiplying both sides by the conjugate of n 's coefficient: $\frac{2}{7}\left(\frac{7}{2}n\right) = \left(\frac{7}{6}\right)\frac{2}{7}$. This becomes $n = \frac{2}{6} = \frac{1}{3}$.
17. **E.** To find the number halfway between the two numbers, we add them together and halve the sum.

$$\frac{1}{2}\left(\frac{1}{7} + \frac{1}{5}\right) = \frac{1}{2}\left(\frac{5}{35} + \frac{7}{35}\right) = \frac{1}{2}\left(\frac{12}{35}\right) = \frac{6}{35}.$$
18. **B.** Solve for x . $\frac{x}{3} + 1 = \frac{5}{6} \rightarrow \frac{x}{3} = -\frac{1}{6} \rightarrow x = -\frac{1}{2}$. $\frac{1}{2}$ lies between -1 and 0 .
19. **D.** To find the left bound of the range of numbers, find $\frac{3}{4} - \frac{4}{3} = \frac{9}{12} - \frac{16}{12} = -\frac{7}{12}$. To find the right bound of the range of numbers, find $\frac{3}{4} + \frac{4}{3} = \frac{9}{12} + \frac{16}{12} = \frac{25}{12}$. Thus, the bounds of the range are $-\frac{7}{12}$ and $\frac{25}{12}$.
20. **D.** The largest possible value will have the smallest possible denominator. Thus, y will be its smallest possible value, and x will be its largest possible value. $\frac{4}{18-6} = \frac{4}{12} = \frac{1}{3}$.
21. **B.** After reading $\frac{1}{16}$ of the book for 4 days, she has completed $\frac{4}{16} = \frac{1}{4}$ of the book. Now, she has 7 days left to finish the remaining $1 - \frac{1}{4} = \frac{3}{4}$ of the book. This means that each day she has to read $\frac{\frac{3}{4}}{7} = \frac{3}{28}$ of the book in order to finish on time.