THE BEST ACT PREP COURSE EVER

# ABSOLUTE VALUE

*ACT Math: Lesson and Problem Set*

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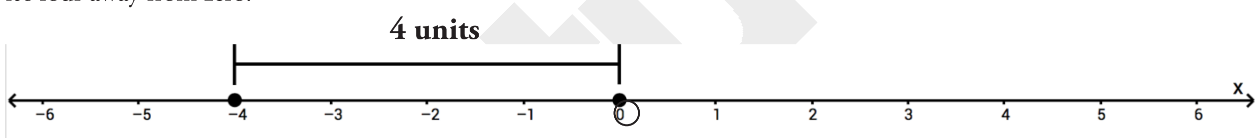
## SKILLS TO KNOW

- Basic definition of absolute value and how to apply it
- How to solve basic absolute value equations with a single variable
- How to solve "properties of numbers" problems involving absolute value (Could be true, must be true, etc.)
- How to set up word problems using the idea of "open sentence" equations and inequalities
- How to graph absolute value equations -- both on a number line, and in the coordinate plane (See Coordinate Geometry)
- How to solve and graph absolute value inequalities (See Coordinate Geometry)

## 1. THE BASICS

Remember, absolute value means the distance between a number and zero.

So if you are asked to solve: $|-4|$ you ask yourself, "how far from zero is $-4$?" As you can see in the number line below, it's four away from zero.




In practice, when you see those bars, make whatever value is between them positive. That's it. If the value between the bars is positive already, the answer is the same as the number between the bars. $(|7| = 7)$ If the value is negative, you make it positive. $(|-2| = 2)$. In any case, any absolute value **is always zero or positive—it cannot be negative.** Absolute value signs are like a form of parentheses—in PEMDAS you treat them on the same level as you would treat parentheses, i.e., complete all work within them first, then simplify once you have a single numeric value between the bars (if possible). For example:

$$|-2-3| - 4 = ?$$

$$|-5| - 4 =$$

$$5 - 4 = 1$$

## 2. SOLVING ABSOLUTE VALUE EQUATIONS

To solve absolute value equations, follow these four steps:

### STEP 1: ISOLATE

Isolate the part of the equation that has absolute value in it (often it will already be isolated).

How many real solutions exist for the equation: $|x^2 - 5| - 9 = 0$

$\left|x^2-5\right|-9=0$ The absolute value part is not isolated yet!

$\left|x^2-5\right|=9$ Add 9 to both sides to isolate the absolute value

**STEP 2: Check for the impossible.**

Remember <u>absolute values cannot be negative</u>! If your answer is negative at this stage, you're done. The answer is no solution.

> For example, $\left|x^2-4\right|=-12$ is impossible! No absolute value can ever be negative!

Our problem above, however, $\left|x^2-5\right|=9$ sets an absolute value equal to a positive—so we move on.

**STEP 3: SPLIT THE PROBLEM**

Split the problem into two cases. Case 1: When the element between the bars is positive. Think about it—what you're doing is changing the sign on what the answer would be if there were no bars—as such you need to simply REMOVE the bars, and **Case 2:**, Multiply ONE side of the equation by $-1$. I typically multiply whichever side is simplest (i.e. if a plain integer is on one side, that's the side I'll choose so I don't have to distribute the negative).

> Again our equation is $\left|x^2-5\right|-9$

| **Case 1: Remove the bars** | **Case 2: Multiply one side of the equation by $-1$**<br>**Don't forget to distribute the negative if necessary |
|---|---|
| $x^2-5=9$<br>$x^2=14$ Add 5 to both sides<br>$x=\pm\sqrt{14}$ Take the square root of both sides<br><br>We have two real, irrational solutions:<br>$\sqrt{14}$ ***and*** $-\sqrt{14}$ | $x^2-5=(9)(-1)$ Multiply by negative one<br>$x^2-5=-9$ Add 5 to both sides<br>$x^2=-4$ No real numbers squared equal a negative.<br><br>Given that the question does not want to consider non-real solutions, there are no solutions from this case. |

**STEP 4. DOUBLE CHECK!**

It's always a good idea to double check absolute value questions if you have time—though if you don't forget step 2, you'll probably be fine. In any case, I find that plugging in at the end of these problems helps eliminate extraneous solutions and careless errors, particularly on more challenging problems (i.e. in the last 15 or so questions on the test.) For this example, when I plug in $\sqrt{14}$ and $-\sqrt{14}$, I get $14-5=9$—that's correct so we're good.

The answer to the question is 2—there are **two real solutions. Remember to always reread the question** before you put an answer. Often you'll be asked not for the numeric solution, but rather the *number* of possible solutions, the sum of the solutions or some other value!

This **"safety net"** step is particularly important on absolute value problems!

## 2. ABSOLUTE VALUE INEQUALITIES

Here's the deal—when approaching absolute value inequalities you can usually turn them into equations and then test points after you've solved the thing down. That method works and there's nothing really wrong with it—though testing points is sometimes time consuming. The great thing about this method is that it's easy to learn—and your prep time is limited. These come up less often than other problems in this area, so this is the method I recommend for its simplicity (it's just less likely you'll mess it up!) for anyone aiming for under a 31-32:

Step 1: Make the inequality sign into an equals sign
Step 2: Solve as an equality
Step 3: Take your "hinge points" create regions, then test regions

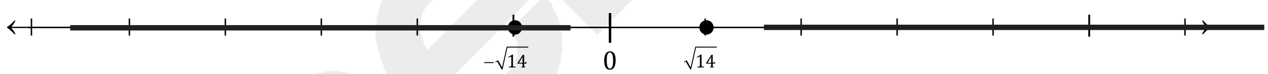
For simplicity's sake, let's take that problem above and make it an inequality: $\left|x^2-5\right|-9\geq 0$

We make the greater than sign an equals sign, solve it down in the same way—and get our "hinge" points as $\sqrt{14}$ *and* $-\sqrt{14}$ (see above example if you want to see how I got these numbers).

At this point we need to test three regions, find a test point in each, and plug it into $\left|x^2-5\right|\geq 9$

| **Region** | | **Test Point** | | **Plug into** $\left|x^2-5\right|\geq 9$ | | | |
|---|---|---|---|---|---|---|---|
| Less than $-\sqrt{14}$ | → | test point of $-\sqrt{16}=-4$ | → | $\left|(-4)^2-5\right|=\left|16-5\right|=11$ | → | $11>9$ | YES |
| Between $-\sqrt{14}$ *and* $\sqrt{14}$ | → | test point of $0$ | → | $\left|0^2-5\right|=\left|-5\right|=5$ | → | $5>9$ | NO |
| More than $\sqrt{14}$ | → | test point of $\sqrt{16}=4$ | → | $\left|4^2-5\right|=\left|16-5\right|=11$ | → | $11>9$ | YES |

As you can see, the first and third test regions are greater than $9$, so these adhere to the inequality and make it true. As such the answer is $x\leq-\sqrt{14}\ or\ x\geq\sqrt{14}$. We can also write this as $x\leq-\sqrt{14}\cup x\geq\sqrt{14}$.




Sometimes, that little method won't work. Occasionally, on very tough problems, you're stuck using your brain, using logic (or memorization) to keep the inequality sign in—but that takes more study time to learn. Aim to do these if your goal is a 32+ on the math section.

The solution set for the inequality $\left|3x+a\right|<5$ is $\left\{x\,|-\frac{1}{3}<x<3\right\}$. What is the value of $a$?

Here I can set this equal to $5$ and then solve down my two cases:

$3x+a=5$ or $3x+a=-5$ but now with that last method I get stuck—I could try to plug in the two x "hinge points" but I'm not sure which equation each goes with and the whole process could get time consuming if I have to test each of the two values in each of the two equations—(that's four tests!) not to mention we don't know the sign direction.
Instead I'll keep the inequality sign in and instead think about what absolute value means—

$\left|3x+a\right|<5\ \textit{means that}\ \left|SOMETHING\right|<5$—in other words, that SOMETHING is "less than $5$ away from zero"—the distance between that something and zero is less than five. With that meaning in mind, I know that that SOMETHING is either five away to the left (at negative five) or five away to the right (at five) or somewhere in that zone even closer to zero. I get to all this through logical reasoning, and now I have something to solve down—with the goal of isolating the $x$ in the middle.

$$-5<3x+a<5$$
$$-5-a<3x<5-a$$

$$\frac{-5-a}{3} < x < \frac{5-a}{3}$$ This looks familiar—let's match it up with our condition

$$\downarrow \qquad\qquad \downarrow$$

$$-\frac{1}{3} < x < 3$$ Now we'll turn this coincidence into a couple of equations

| | |
|---|---|
| $\frac{-5-a}{3} = -\frac{1}{3}$ | $\frac{5-a}{3} = 3$ |
| $-5-a=-1$ | $5-a=9$ |
| $-a=4$ | $-a=4$ |
| $a=-4$ | $a=-4$ |

**NOTE: it's very important that you solve out BOTH equations—the two $a$ values might not be equal and in that case there would be no solution (or you made a mistake)!**

In any case, here, $a=-4$

## 3. PROPERTIES OF NUMBERS ABSOLUTE VALUE QUESTIONS

These problems are the kind that you don't likely see much in math at school, and that is what makes them a challenge.

In these problems you often have a parameter—such as "$\textit{For all } x > 0$" **that limits the input** of what values we're talking about. When you have that limitation, you can't simply "solve" the problem algebraically—you can either make up a number or use a combination of algebra and logic.

**TECHNIQUE #1: Make up a number**

**Make up a number when you have a parameter, have variables in the answer choice, and <u>have no algebraic way to simplify.</u>**

Though in other parts of the book, I may warn you that making up numbers is a slow way to go, with absolute value problems, there is often no straight algebraic way out—only a logical one—and logic can strain your brain—even if it's often faster if you're good at it or it's possible. If you're aiming for a 33+ I recommend you learn both strategies—otherwise you can likely get away with making up numbers.

If $x \ge 7$, then $|7-x| = ?$

**A.** $7-x$
**B.** $x-7$
**C.** $x+7$
**D.** $0$
**E.** $-7-x$

**Here the easiest way to the answer is likely to plug in $8$. First we plug into the expression $|7-x|$ in the question stem:**

$|7-x| = |7-8| = |-1| = 1$

Now I plug in $8$ to all the answers and look for a result of "$1$" – I have to try EVERY answer though in case I randomly chose a number that works twice (it happens—and is a good reason to avoid choosing any number already in the problem).

**A.** $7-x=7-8=-1$ *NO*
**B.** $x-7=8-7=1$ *YES*
**C.** $x+7=8+7=15$ *NO*
**D.** $0$ *NO*
**E.** $-7-x=-7-8=-15$ *NO*

B is the correct answer, as it equals $1$.

**TECHNIQUE #2**
**You can also do these problems with a combination of algebra & logic— it's sometimes faster, but only if your brain is practiced enough to see the logical connections quickly.**
**Let's try splitting the problem into two cases—**

$$|7-x|$$

is either going to be equal to $7-x$ or $-(7-x)$
When I split this into two cases, I don't have the luxury of an equation—I only have an expression—and that means I must distribute the negative in the second case. $-(7-x) = -7+x = x-7$
Now I have my two cases:

$$7-x \quad \text{and} \quad x-7$$

Instantly we know the answer is either (A) $7-x=7-8=-1$ or (B) $x-7=8-7=1$.
But here I'm not done and I need to use logic, considering the restriction on x:
First, I consider the idea that in BOTH of these two cases, if these are what this expression equals, each MUST BE non-negative. That means for the first case, $x$ has to be smaller than $7$ to create a non-negative answer. Once $x$ grows to a larger number, the expression would be negative, and an absolute value can't be negative.
In the second case, I can imagine how $x$ MUST BE positive and at least $7$—else subtracting $7$ is going to send the number into the region of negatives. Since I know the question states that $x \geq 7$, that means the second case is the correct case.
As you can see, all this thinking is a bit confusing—true, I get an algebraic answer—true, if you're aiming for a 36 it's good to understand these problems this well, but for many this method is overkill.
The bottom line with doing these the logical way is that you're looking to see the general **behavior of numbers and expressions.**

## 4. Absolute Value Inequalities: Word Problems

NOTE: These don't show up all that often on the test—I would recommend practicing them if you're aiming for above a 30 and prepping for at least a few weeks.

TIME SAVER TIP:
The other thing to remember about absolute value is that it can also be used to represent the phrase: **"the difference between x and a number is"** or in coordinate geometry, "**the distance between *a* and *b* is**."
For example:

$$|x-3| \leq 7$$

Can be translated as
"The distance between $x$ and $3$ is no more than $7$." Or "$x$ and $3$ are no more than $7$ apart."

Think about it—when you subtract $3$ from $x$, you do what you do when you calculated slope. You find the difference in two numbers—one of the $y$'s and the other $y$, which is the same as the distance. Remember rise over run? You can use the slope formula, or you can measure it out with little boxes on the page as a distance. **When you take the absolute value of a difference, you find the physical distance between two points.** Slope formulas still use the sign (you're figuring out a rate, really) but the concept of distance is the same.
You can also think of these as a way to represent margin of error.
Let's say you're building a bookshelf and it's supposed to be $36''$ high, but can have a margin of error of up to a quarter inch. **The difference between** the ideal, $36$, and the actual height, $d$, can be **no more than** a quarter inch.

$$|36-d| \leq .25$$

Because absolute value subtracts one thing from another thing, but then gets rid of the "negative" sign, **it's the same as finding the DISTANCE between two values or numbers**. This idea can come in handy on the occasional word problem. I recommend you commit to memory that phrase—**"the distance/difference between"** —to think of absolute value subtraction problems!

The diameter, $d$, of the plastic pipes that a hardware stores sells must satisfy the inequality $|4-d| \le .005$. What is the maximum diameter, in centimeters, that a plastics pipe may have?

Let's talk about what all this represents—

The part in the absolute value sign means that the **distance between $4$ and $d$ is no greater than $.005$. $4$ and $d$ are no more than $.005$ apart.**

That means we can be $.005$ more than $4$, $.005$ less than $4$ or in between hanging out even closer to $4$ and get $d$.

If we need the greatest diameter, that would mean we'd add that miniscule margin of error to our "ideal" of $4$—

$$4+.005=4.005$$

Answer: $4.005$

Now you could back-solve these type of problems, meaning you could plug in all the answer choices and see what works—but that method is slow. These are called "open sentence" inequalities. They're found in most Algebra I books (I know!) so in all likelihood you learned these things back in 8th grade. Seriously, if you want more practice on these, find your younger sister's math book… The best advice is just to memorize the phrase—

**THE DISTANCE BETWEEN ( ) and ( ) is (no more than / less than / greater than / at least ).**

If you can translate these equations to that idea, you'll be able to solve these problems extremely quickly.

1. $|8-3|-|3-8|=?$

A. $-10$
B. $10$
C. $0$
D. $-6$
E. $-5$

2. What is the value of $|x-y|-(x-2y)^2$ when $x=2$ and $y=4$?

A. 34
B. 38
C. 2
D. $-4$
E. 4

3. If $x \geq 9$, then $|9-x|=$ ?

A. $9-x$
B. $x-9$
C. $x+9$
D. 0
E. $-9-x$

4. $-4|-13+2|=$?

A. 44
B. $-15$
C. $-60$
D. 60
E. $-44$

5. If $|x+7|=32$ what are all the possible values for $x$?

A. 25 and $-25$
B. 39 and 25
C. $-7$ and 7
D. 25 and $-7$
E. 25 and $-39$

6. If $|x-3|=14$ what are all the possible values for $x$?

A. 17 and $-11$
B. $-17$ and $-11$
C. 17 and 11
D. 17 and $-17$
E. 3 and $-3$

7. If $|x-7|=|-2|$ how many different values are possible for $x$?

A. 0
B. 1
C. 2
D. 4
E. Infinitely many

8. How many real solutions are possible for $m$ in the equation $|3m+5|=-4$?

A. 0
B. 1
C. 2
D. 4
E. Infinitely many

9. $-6|-7+8|=?$

A. $-6$
B. $-5$
C. $-90$
D. 90
E. 6

10. For all $n<0$, $\left|-n^5\right|-\left(-|2|^3\right)=$?

A. $n^5+6$
B. $-n^5+8$
C. $n^5-8$
D. $n^5+8$
E. $-n^5+6$

11. What are all the real solutions to the equation $\left|x^2\right|+3|x|-10=0$?

A. $\pm 2$
B. $\pm 5$
C. $-5$ and 2
D. $-5$ and $-2$
E. $\pm 2$ and $\pm 5$

12. If $a>b$ then $|a-b|=$

A. $\sqrt{a-b}$
B. $-(a-b)$
C. $-(a+b)$
D. $a-b$
E. 0

**13.** For all non-zero real numbers $a$ and $b$ such that $\left|\frac{a}{b}\right| = \left|\frac{b}{a}\right|$, which of the following COULD be TRUE?

I. $a = -\sqrt{b^2}$
II. $a - b = 2$
III. $ab = -ab$

**A.** I only
**B.** II only
**C.** I & II only
**D.** I & III only
**E.** I, II, and III

**14.** At a newsstand, it costs $n$ dollars for a newspaper, and $m$ dollars for a magazine. The difference between the cost of $15$ newspapers and $18$ magazines is $\$48$. Which of the following equations represents the relationship between $n$ and $m$?

**A.** $\frac{15n}{18m} = 48$
**B.** $270nm = 48$
**C.** $|15n - 18m| = 48$
**D.** $|15n + 18m| = 48$
**E.** $18n - 15m = 48$

**15.** For all real values of $x$, $y$, and all values of $a$ such that $a \geq 0$, $|x| = |y| = -a$ for how many $(x, y)$ solutions?

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

**16.** For how many pairs $(a, b)$ is the following equation true?

$$\left|\frac{a}{b} - \frac{b}{a}\right| = \left|\frac{b}{a} - \frac{a}{b}\right|$$

**A.** 0
**B.** 1
**C.** 2
**D.** 4
**E.** Infinitely many

**17.** If $x \geq 5$, then $|x - 5| = ?$

**A.** 0
**B.** $x - 5$
**C.** $x + 5$
**D.** $-x + 5$
**E.** $-x - 5$

**18.** If $x - |x| = 0$ then $x$ is:

**A.** always negative
**B.** sometimes positive
**C.** always positive
**D.** always zero
**E.** sometimes negative

**19.** If $|x| = -x$, $|-y| = y$, and $xy \neq 0$, which of the following must be negative?

**A.** $x^y$
**B.** $y^x$
**C.** $x - y$
**D.** $x + y$
**E.** $y - x$

**20.** If $|a| - b = |b| - a$ and $a > 0$, which of the following statements must be true?

**A.** $a + b = 0$
**B.** $ab < 0$
**C.** $a = b$
**D.** $a = 0$ or $b = 0$, but not both
**E.** $a = -b$

**21.** If $|x| > x$, then which of the following must be true?

**A.** $-x \leq x$
**B.** $x = 0$
**C.** $x^3 < 0$
**D.** $x \geq 0$
**E.** $2x > x$

**22.** Which of the following equations could be used to represent "the distance between $x$ and $n$" if there are two solutions for $x$, if $n$ is the mean of the two solutions, and the two solutions are 10 units apart on the number line?

**A.** $|x + n| = 10$
**B.** $|x - n| = 10$
**C.** $|x - n| = 5$
**D.** $|x + n| = 5$
**E.** $|x - 5| = n$

**23.** On an amusement park ride, riders must be between $42$ and $72$ inches in order to ride. Which of the following equations could be used to represent the possible heights, $h$, of a potential rider, in inches?

**A.** $|h-57| \geq 15$

**B.** $|h-57| \leq 15$

**C.** $|h-72| \leq 30$

**D.** $|h-42| \leq 30$

**E.** $|h+42| \leq 30$

**24.** At an apparel factory, a pair of medium shorts has an average waist circumference of $29$ inches. These shorts must not vary from the average waist circumference by more than $.25$ inch to pass quality control inspections. Which of the following equations could be used to represent the range of possible waist circumferences, $c$, that would pass quality control inspections for a pair of medium shorts?

**A.** $|29-c| \leq 0.25$

**B.** $|29-c| \geq 0.25$

**C.** $|c-0.25| \geq 29$

**D.** $\dfrac{c}{29} \leq 0.25$

**E.** $|29+c| \geq 0.25$

**25.** The solution to which of the following equations is the set of real numbers that are $3$ units away from $7$?

**A.** $|x-7| = 3$

**B.** $|x+3| = 7$

**C.** $|x-3| = 7$

**D.** $|x+7| = 3$

**E.** $|x-7| = -3$

**26.** Which irrational number is the solution to $|x^2-18|-7=0$?

**A.** $\sqrt{11}$

**B.** $\sqrt{5}$

**C.** $2.5$

**D.** $3\sqrt{2}$

**E.** $4\sqrt{2}$

**27.** Which of the following expressions, if any, are equal to each other for all real numbers $x$?

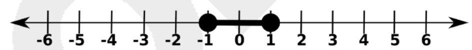
I. $-\sqrt{(-x)^2}$

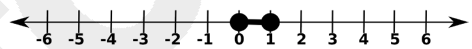
II. $|-x|^3$

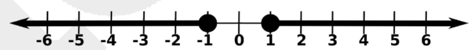
III. $-|-x|$

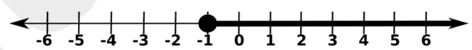
**A.** I and II only
**B.** II and III only
**C.** I and III only
**D.** I, II, and III
**E.** None of the above

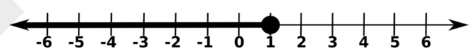
**28.** Which of the following graphs represents the solution set of the inequality $|x| \leq 1$?

**A.** 

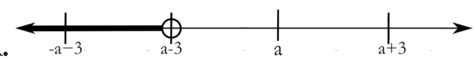

**B.** 

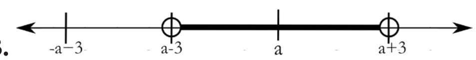

**C.** 

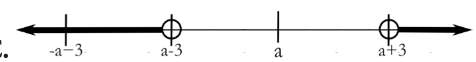

**D.** 


**E.** 

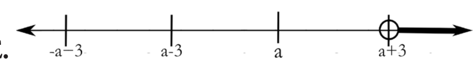

**29.** Whenever $a > 0$, which of the following real number line graphs represents the solution for $x$ to the inequality $|x-a| > 3$?

**A.** 


**B.** 


**C.** 


**D.** 


**E.** 


**30.** Which of the following is the solution set for $-2|x+4| \geq -12$?

**A.** 


**B.** 


**C.** 


**D.** 


**E.** 

**ANSWER KEY**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1. C** | **2. B** | **3. B** | **4. E** | **5. E** | **6. A** | **7. C** | **8. A** | **9. A** | **10. D** | **11. A** | **12. D** | **13. C** | **14. C** | **15. B** |
| **16. E** | **17. B** | **18. B** | **19. C** | **20. C** | **21. C** | **22. C** | **23. B** | **24. A** | **25. A** | **26. A** | **27. C** | **28. A** | **29. C** | **30. E** |

**ANSWER EXPLANATIONS**

1. **C.** $|8-3|-|3-8|=|5|-|-5|\to 5-5=0$

2. **B.** Plugging in $x=2$ and $y=4$, we get $|2-4|+(2-2(4))^2 \to |-2|+(2-8)^2 \to 2+(-6)=2+36\to 38.$

3. **B.** $|9-x|$ is the positive difference between $x$ and 9. $x-9$ is also equal to the positive difference between $x$ and 9 because $x\geq 9$. Notice that $9-x$ will yield a negative answer and will not be equivalent to $|9-x|$.

4. **E.** $-4|-13+2|=-4|-11|\to -4(11)\to -44$

5. **E.** $|x+7|=32$ means that $x+7=32$ or $x+7=-32$. Solving these two inequalities separately, we get $x=32-7\to 25$ or $x=-32-7\to -39$. So, the possible values for $x$ are $25$ and $-39$.

6. **A.** $|x-3|=14$ means that $x-3=14$ or $x-3=-14$. Solving these two inequalities separately, we get $x=14+3\to 17$ or $x=-14+3\to -11$. So, the possible values for $x$ are $17$ and $-11$.

7. **C.** $|x-7|=|-2|$ means $|x-7|=2$. So, $x-7=2$. or $x-7=-2$. Solving these two inequalities separately, we get $x=7+29$ or $x=7-2\to 5$. So, there are two possible solutions.

8. **A.** The absolute value of something is always positive, so there is no solution for $|3m+5|=-4$ because $-4$ is negative.

9. **A.** $-6|-7+8|=-6|1|\to -6$

10. **D.** $|-n^5|-\left(-|2|^3\right)=n^5-(-8)\to n^5+8.$

11. **A.** We first solve this without the absolute values $x^2+3x-10=0$. This factors out to be $(x+5)(x-2)$. So, the solutions that make this equal to zero are $x=-5$ and $x=2$. Since the solution $|x|$ must be positive, $|x|$ must equal $2$, so $x=\pm 2$.

12. **D.** If $a>b$ then $a-b$ is positive. So, $|a-b|=a-b$.

13. **C.** If $\left|\frac{a}{b}\right|=\left|\frac{b}{a}\right|$ then multiplying both sides by $|a||b|$, we get $|a^2|=|b^2|$. Since all squares are positive, $a^2=b^2$. Taking the square root on both sides, we get $a=\pm\sqrt{b^2}$. So, I could be true. For II, we have $a-b=2\to a=b+2$ and plugging in $a=b+2$ to $\left|\frac{a}{b}\right|=\left|\frac{b}{a}\right|$ we get $\left|\frac{b+2}{b}\right|=\left|\frac{b}{b+2}\right|$. This statement could be true if $b=-1\to\left|\frac{-1+2}{-1}\right|=\left|\frac{-1}{-1+2}\right|\to\left|\frac{1}{-1}\right|=\left|-\frac{1}{1}\right|\to 1=1$. For III, we have $ab=-ab\to a=-a$ or $b=-b$. This is only true if $a$ or $b$ equals zero. However, it is given that $a$ and $b$ are non-zero, so III cannot be true.

**14. C.** The cost of $15$ newspapers can be represented by $15n$ and the cost of $18$ magazines can be represented by $18m$. The difference between these two prices is $48$, but we don't know if $15n$ is greater or if $18m$ is greater. So, $|15n-18m|=48$.

**15. B.** $|x|$ and $|y|$ are always positive or zero, so $|x|=|y|=-a$ is only true when $x$, $y$ and $a$ are zero. $|0|=|0|=-0=0$. There is only one solution.

**16. E.** Because $|x|=|-x|$, we can manipulate one side of the expression by multiplying it by $-1$. This gives us $\left|\frac{a}{b}-\frac{b}{a}\right|=\left|(-1)\left(\frac{b}{a}-\frac{a}{b}\right)\right|$ and therefore that $\left|\frac{a}{b}-\frac{b}{a}\right|=\left|\frac{a}{b}-\frac{b}{a}\right|$. This is true for all values of $a$ and $b$ given that neither of them are $0$, so there are infinitely many solutions.

**17. B.** If $x\geq 5$ then $x-5$ is positive. So, $|x-5|=x-5$.

**18. B.** If $x-|x|=0$ then $x=|x|$. So, $x\geq 0$. Since $0$ is not positive, $x$ is only sometimes positive.

**19. C.** If $|x|=-x$ then $x$ must be negative because any absolute value is positive, and the negative of a negative number is positive. Likewise, $|-y|=y$ implies that $y$ is positive. So, the only answer choice that gives us a negative value is $x-y$ because it is a negative number minus a positive number. Note that all other answer choices may yield negative values but may also yield positive values. $x-y$ is the only choice that guarantees a negative answer.

**20. C.** If $a$ is positive, then $|a|=a$. So, we can rewrite the equation as $a-b=|b|-a$. Adding $a$ on both sides and adding $b$ on both sides, we get $2a=b|b|$. Since we know $2a$ is positive and $|b|$ has to be positive because it is an absolute value, we know that $b$ also has to be positive in order for the statement to be true. This means that $2a=2b\rightarrow a=b$.

**21. C.** If $|x|>x$, then that means $x$ is negative. Hence, $x^3$ is also negative.

**22. C.** The distance between $x$ and $n$ is $|x-n|=d$. If the average of the two solutions is $n$, then their sum is $2n$. The solutions are, $x-n=d$ or $x-n=-d\rightarrow x=n+d$ or $xn-d$. Their sum is verified to equal $2n$ because $n+d+n-d=2n$. Now, we are given that the two solutions are $10$ units apart, so $n+d-(n-d)=10\rightarrow 2d=10\rightarrow d=5$. So, the distance between $x$ and $n$ can be expressed as $|x-n|=5$.

**23. B.** The heights must not be over $72$ and not be below $42$, so the heights can be represented as within the range of the mean of the heights $\pm$ half the range of the heights. This can be expressed as $h\leq\frac{72+42}{2}\pm\frac{72-42}{2}\leq\frac{114}{2}\pm\frac{30}{2}\leq 57\pm 15$. This is re-written as $h-57\leq\pm 15$ or $|h-57|\leq 15$.

**24. A.** The range of waist circumferences that will pass the inspections is within the range of the average $29$ inches $\pm\frac{1}{4}$ inches. So, $c-29\leq\pm\frac{1}{4}$. This means $|c-29|\leq\frac{1}{4}$. This is equivalent to $|29-c|\leq\frac{1}{4}$.

**25.** **A.** The set of numbers that are $3$ units away from $7$ can be expressed with the expression $x = 7 \pm 3$. Subtracting $7$ on both sides, we have $x - 7 = \pm 3$ . This can be written with absolute values as $|x-7| = 3$.

**26.** **A.** Adding $7$ on both sides of the equation, we get $|x^2 - 18| = 7$ so $x^2 - 18 = 7$ or $x^2 - 18 = -7$. Adding $18$ on both sides of both equations, we get $x^2 = 23$ or $x^2 = 11$. So, $x = \pm\sqrt{23}$ or $\pm\sqrt{11}$. $\pm\sqrt{11}$ is the only solution for $x$ that is in the answer choices.

**27.** **C.** I. simplifies to $-\sqrt{(-x)^2} = -\sqrt{x^2} \to -x$. II. Simplifies to $|-x|^3 = x^3$, and III. Simplifies to $-|-x| = -x$. So, only expressions I and III are equal for all $x$.

**28.** **A.** $|x| \le 1$ means $-1 \le x \le 1$, which is represented by the graph in answer choice A.

**29.** **C.** $|x-a| > 3$ means that $-3 > x - a > 3$, which when we add $a$ to all three parts becomes: $-3 + a > x > 3 + a$ (this can also be written as $x < a + 3$ and $x > a + 3$). The graph in answer choice C correctly reflects this.

**30.** **E.** First isolate the absolute value sign by dividing both side by $-2$ to get $|x+4| \ge 6$. Now we can treat it like a regular inequality and say that $-6 \ge x + 4 \ge 6$. When we subtract $4$ from all three parts, we get $-10 \ge x \ge 2$, which is the same as $x \ge 2$ and $x \le -10$, which is shown in answer E.