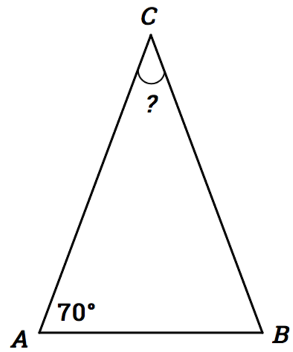
TRIANGLES

ACT Math: Problem Set

1. In $\triangle ABC$ below, $\overline{AC}=\overline{CB}$ and $\angle$ A measures 70°. What is the measure of $\angle C$?



A. 35°
B. 40°
C. 45°
D. 50°
E. 55°

2. In isosceles triangle $\triangle RST$, $\overline{RS}$ is congruent to $\overline{ST}$ and the measure of one base angle $\angle R$ is 67.5°. What is the measure of vertex angle $\angle S$?

A. 13.5°
B. 45°
C. 67.5°
D. 85°
E. 135°

3. What is the length, in feet, of the hypotenuse of a right triangle with legs that are 3 meters long and 7 meters long, respectively?

A. $\sqrt{10}$
B. $\sqrt{58}$
C. 10
D. 14
E. 21

4. Which of the following sets of 3 numbers could be the side lengths, in feet, of a right triangle?

A. 1,1,1
B. 2,6,8
C. 3,7,10
D. 5,12,13
E. 7,10,29

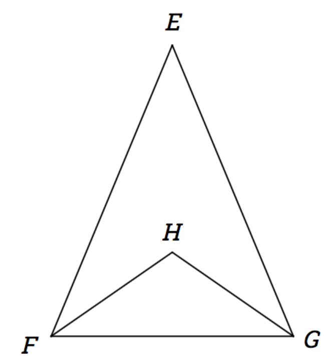
5. What is the area, in square meters, of a right triangle with sides of length 7 meters, 24 meters, and 25 meters?

A. 84
B. 87.5
C. 168
D. 300
E. 336.5

6. In $\triangle XYZ$, $\overline{XY} \cong \overline{XZ}$, and the measure of $\angle X$ is 44°. What is the measure of the sum of $\angle X$ and $\angle Y$, in degrees?

A. 44°
B. 68°
C. 112°
D. 136°
E. Cannot be determined from the given information.

7. Triangles $\triangle EFG$ and $\triangle HFG$, shown below, are isosceles with base $\overline{FG}$. Segments $\overline{FH}$ and $\overline{GH}$ bisect $\angle EFG$ and $\angle EGF$, respectively. If $\angle$ EFH is 61°, what is the measure, in degrees, of $\angle$ FGH?



A. 29°
B. 58°
C. 61°
D. 119°
E. 122°

8. In ÄDEF, $\angle D$ and $\angle F$ are congruent, and the measure of $\angle E$ is 56°. What is the measure of $\angle D$?

A. 28°
B. 56°
C. 62°
D. 100°
E. 124°

9. In $\triangle ABC$, the measure of $\angle B$ is twice the measure of $\angle A$, and $\angle C$ is three times the measure of $\angle A$. What is the measure, in degrees, of the sum of $\angle B$ and $\angle C$?

A. $30°$
B. $60°$
C. $90°$
D. $150°$
E. $180°$

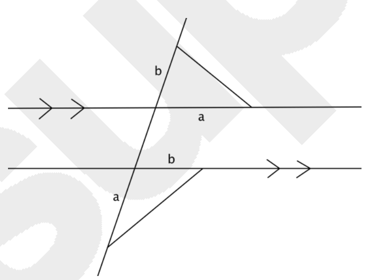
10. In $\triangle XYZ$, the measure of $\angle X$ is $57°$ and the measure of $\angle Y$ is $32°$. Which of the following inequalities involving the lengths of the sides of $\triangle XYZ$ is FALSE?

A. $\overline{YZ} > \overline{XZ}$
B. $\overline{XY} > \overline{YZ}$
C. $\overline{XY} > \overline{XZ}$
D. $\overline{XZ} > \overline{YZ}$
E. Not enough information

11. In isosceles triangle $\triangle DEF$, base angles $\angle D$ and $\angle F$ each measure $36°$. Points D, E, and G are collinear points, with E between D and G. What is the measure of $\angle$ GEF?

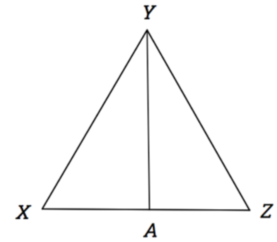
A. $18°$
B. $24°$
C. $36°$
D. $68°$
E. $72°$

12. Two triangles are presented in the diagram below, each with sides of lengths a and b. If the area of the top triangle is 30 square centimeters, what is the area of the bottom triangle?



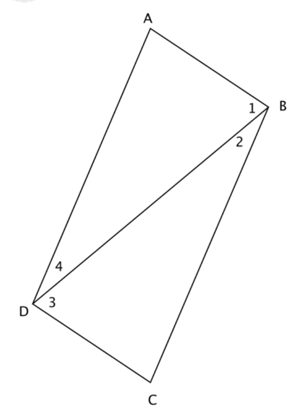
A. 15
B. 19
C. 25
D. 30
E. 33

13. In the figure below, $\overline{YA}$ is an altitude of equilateral triangle $\triangle XYZ$. If $\overline{YX}$ is 8 units long, how many units long is $\overline{YA}$?



A. 4
B. $4\sqrt{3}$
C. 8
D. $8\sqrt{3}$
E. 16

14. In the figure below, $\overline{AB} \cong \overline{CD}$. Kayla wants to apply the Side-Angle-Side (SAS) congruence theorem to prove that $\triangle ABD \cong \triangle CBD$. Which of the following congruences, if established, is sufficient?

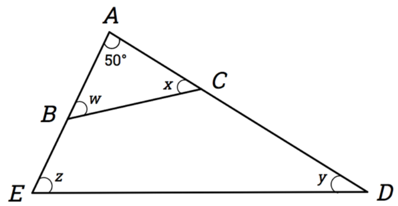


A. $\angle A \cong \angle C$
B. $\angle 1 \cong \angle 4$
C. $\angle 1 \cong \angle 3$
D. $\angle 2 \cong \angle 4$
E. $\angle 2 \cong \angle 3$

15. The measure of a vertex angle of an isosceles triangle is $(x-12)°$. The base angles each measure $(3x+40)°$. What is the measure in degrees of the vertex angle?

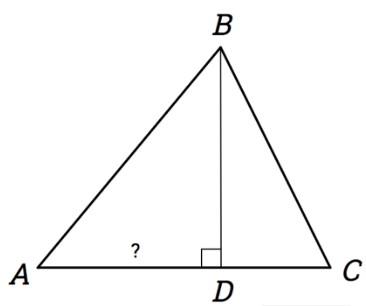
A. $4°$
B. $13°$
C. $16°$
D. $35°$
E. $88°$

16. In $\triangle ADE$ below, B lies on $\overline{AE}$; C lies on $\overline{AD}$; and w, x, y, and z are angle measures, in degrees. The measure of $\angle A$ is 50°. Which of the following must be true?



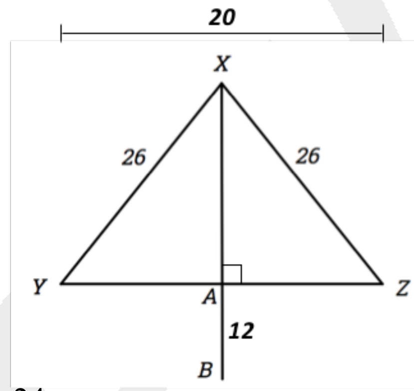
A. $w + x < 50°$
B. $x + y > 50°$
C. $z = w$
D. $z + y > w + x$
E. $w + x = z + y$

17. The area of $\triangle ABC$ below is 16 square inches and the area of $\triangle ABD$ is 10 square inches. If $\overline{AC}$ is 8 inches long, how long is length $\overline{AD}$, in inches?



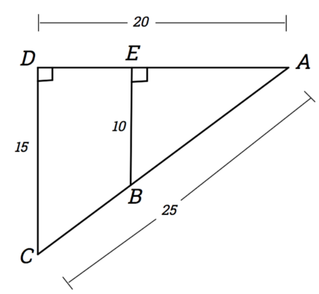
A. 3
B. 4
C. 5
D. 6
E. 7

18. Harry is examining the shape of a spear for his history class. In the figure below, the arrowhead is represented by $\triangle XYZ$. Base $\overline{YZ}$ is 20 inches long, and sides $\overline{XY}$ and $\overline{XZ}$ are each 26 inches long. The shaft, represented by $\overline{XA}$, is perpendicular to the base and extends 12 inches below the bottom of the arrowhead. How many inches long is the shaft?



A. 24
B. 36
C. 43
D. 130
E. 142

19. Shown below are right triangles $\triangle ACD$ and $\triangle ABE$ with lengths given in feet. What is the length, in feet, of $\overline{AE}$?

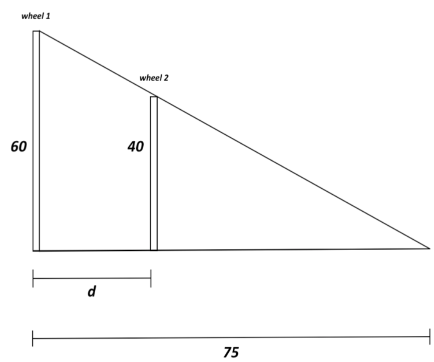


A. $10\frac{1}{5}$
B. $10\frac{1}{4}$
C. $10\frac{3}{4}$
D. $13\frac{1}{3}$
E. $37\frac{1}{2}$

20. The lengths, in meters, of all 3 sides of a triangle are positive integers, and 1 side is 9 inches long. If the area of this triangle is positive, what is the smallest possible perimeter for this triangle, in meters?

A. 11
B. 12
C. 18
D. 19
E. 27

21. Mark is shopping to replace his bicycle's wheels. He places two wheels, one with a radius of 60 centimeters and one with a radius of 30 centimeters, parallel with each other as if they were the two front tires of a car on level ground. As he does so, he notices that in the late afternoon light their shadows end in the same below, as shown in the diagram below. How far apart did he place the wheels, measuring from the wheel's left edges? (note: the wheels' widths are negligent)



A. 25
B. 37.5
C. 40
D. 50
E. 112.5

22. In order to paint the window of her house, Shelley needs to find a ladder of appropriate length. The side of her house is perpendicular to the level ground so that the base of the ladder is 15 feet away from the base of the building. How long does the ladder need to be to reach 20 feet up the house?

A. 5
B. 15
C. 17
D. 25
E. 35

ANSWERS

1. B	2. B	3. B	4. D	5. B	6. C	7. C	8. C	9. D	10. D	11. E	12. D	13. B	14. C
15. A	16. E	17. C	18. B	19. E	20. D	21. A	22. D						

ANSWER EXPLANATIONS

1. B. Since $AC \cong CB$, $\angle A$ and $\angle B$ are also congruent. Since all angles in a triangle must equal 180°, and the sum of $\angle A$ and $\angle B$ is 140°, $\angle C$ must be 40°.
2. B. In an isosceles triangle, the two base angles are congruent. Since $\angle R$ is given to us to be 67.5°, $\angle T$, the other base angle, must also be 67.5°. All angles in a triangle must sum to 180°, so it follows that $\angle S$ must be 45°.
3. B. This question depends on our knowledge of the Pythagorean Theorem: $a^2 + b^2 = c^2$. Since a and b represent the legs of the right triangle and c represents the hypotenuse, the equation would look like: $3^2 + 7^2 = c^2$. After some algebraic calculation, we find c to be $\sqrt{58}$.
4. D. If we know what to look for and recognize the Pythagorean triple in answer D, then we've found the correct answer and can move on. But if we don't recognize the Pythagorean triple, we can find the answer another way. In any triangle, the sum of two side lengths must always be greater than the third side length, which eliminates answer choices B and C. Now look for which set of side lengths is a right triangle, which means that the lengths satisfy the Pythagorean Theorem. Answer A is an equilateral triangle, whose angles by definition are all $60°$, while answer E doesn't satisfy the Pythagorean Theorem. Answer D, (5, 12, 13) does.
5. B. In a right triangle, the hypotenuse will always have the greatest length. Therefore, 7 and 24 must be the legs of the triangle, which in a right triangle are also our base and height. Thus, we can plug these numbers into the triangle area formula, A = $\left(\frac{1}{2}\right)(7)(24)$, and find the area to be 84 square meters.
6. C. It always helps to sketch a quick diagram. We can determine that $\angle X$ is the vertex angle by looking at the graph or because it appears in both the congruent line segments. Since $\angle X$ is $44°$, and all angles in a triangle must sum to $180°$, the sum of base angles $\angle Y$ and $\angle Z$ must be $136°$. Because the triangle is isosceles, we can divide 136° by 2 in order to find the measure of $\angle Y$, or 68°. Thus the addition of $\angle X$ and $\angle Y$ is $112°$.
7. C. Since $\triangle EFG$ is an iscoceles triangle, $\angle EFG$ and $\angle EGF$ are congruent to each other. Secondly, since segments $\overline{FH}$ and $\overline{GH}$ bisect these angles, the 4 angles created ($\angle EFH$, $\angle HFG$, $\angle EGH$, and $\angle HGF$ are all congruent. Thus, $\angle EFH$ is congruent to $\angle FGH$, making the answer 61°.
8. C. The sum of all angles in a triangle is 180°. If we subtract the measure of $\angle E$ from the total degrees, we see that the sum of angle $\angle D$ and $\angle F$ is 124°. Because these two angles are congruent, we can divide 124° by 2 to find the measure of $\angle D$.
9. D. We can start by assigning variables to the various angles. Let $\angle A = x$, because the other angles are described in terms of $\angle A$. Because $\angle B$ is twice the value of $\angle A$ and $\angle C$ is three times that value, we can let $\angle B = 2x$ and $\angle C = 3x$. Since all angles in a triangle must sum to 180°, we can set the sum of our assigned variables equal to 180°: $x + 2x + 3x = 180$. Solving this, we find that x, or $\angle A$, equals 30°. Thus, $\angle B$ is $60°$ and $\angle C$ is 90°. The question asks for the sum of $\angle B$ and $\angle C$, so the answer is D, $150°$.
10. D. If $\angle X$ is 57° and $\angle Y$ is 32°, then $\angle Z$ must be 91°. Since the side opposite the largest angle is the largest side, and the side opposite the smallest angle is the smallest side, $\overline{XY} > \overline{YZ} > \overline{XZ}$. Only choice D, stating that $\overline{XZ} > \overline{YZ}$, is not true.
11. E. This problem is testing for knowledge of geometrical theorems. We must remember that an exterior angle in a triangle is equal to the sum of the 2 remote interior angles. The interior angles remote to exterior angle $\angle GEF$ would be base angles $\angle D$ and $\angle F$. The sum of these base angles is $72°$.
12. D. Since the angles of the triangles formed by the transversal across the parallel lines are supplementary, the values of the sine of the angle between sides a and b are the same. Since the area of a triangle is equal to the sine of an angle times the lengths of the adjacent sides, and the sides of the two triangles are equal, the areas of the triangles are equal.
13. B. This question is concerned with the $30° - 60° - 90°$ Special Right Triangle. This theorem says that the side length opposite the 30° angle is x units, the side length opposite the 60° angle is x$\sqrt{3}$ units, and the side length opposite the 90° angle is 2x units. Since $\triangle XYZ$ is an equilateral triangle, $\angle YXZ$ must be 60°, bisected angle $\angle XYA$ must be 30°, and because $\overline{YA}$ is the altitude, by the definition of an altitude $\angle YAX$ must be 90°. Therefore, side $\overline{YX}$, which is 8 units, constitutes as side 2x. That tells us that $x = 4$, so it follows that $\overline{YA}$ is x$\sqrt{3}$, in our case 4$\sqrt{3}$.

14. C. Since $\overline{AB} \cong \overline{CD}$, and triangles $\triangle ABD$ and $\triangle CBD$ share $\overline{BD}$ as a side (and it is congruent to itself by definition), the only congruence necessary for the Side-Angle-Side congruence theorem is the angle between these congruent sides. The angle between $\overline{AB}$ and $\overline{BD}$ in $\triangle ABD$ is $\angle 1$, and in $\triangle CBD$, between $\overline{CD}$ and $\overline{BD}$ is $\angle 3$. Thus, proving a congruence between $\angle 1$ and $\angle 3$ is sufficient for the Side-Angle-Side congruence theorem.
15. A. First we must find the value of unknown variable x. Since all angles in a triangle sum to 180°, we can set the sum of the given expressions to the that as well: $(x-12)+(3x+40)+(3x+40)=180$. Solving, we find that x equals 16. Since the question is asking for the vertex angle, we plug x into the vertex angle expression $(x-12)$ and find that the vertex angle is $4°$.
16. E. Both the angles in $\triangle ADE$ and the angles in $\triangle ABC$ must sum to 180°. The two triangles share one common angle, $\angle A$, so for both triangles the sum of the remaining angles must be equal to $180-\angle A$. $180-\angle A = w+x = z+y$, so Choice E is correct.
17. C. We can first use the length of $\overline{AC}$ and the area of $\triangle ABC$ in order to find the length of altitude $\overline{DB}$. Using the formula for the area of a triangle: $16=\left(\frac{1}{2}\right)(8)h=\left(\frac{1}{2}\right)(8)\left(\overline{DB}\right)$, we can find the height $\overline{DB}$ to be 4. Next we can use the area of $\triangle ABD$ and its height to find length $\overline{AD}$. Using the formula for the area of the triangle: $10=\left(\frac{1}{2}\right)b(4)=\left(\frac{1}{2}\right)\left(\overline{AD}\right)(4)$, we find that the base length $\overline{AD}$ is 5.
18. B. Since $\triangle XYA$ and $\triangle XZA$ have 2 sides of common length (their hypotenuses and $\overline{XA}$), we can deduce that their third sides, $\overline{YA}$ and $\overline{AZ}$ must be congruent, each 10 inches long. Now we can see that both triangles have legs of 10 and 24. If we can recognize that the doubled Pythagorean triple of 5-12-13, we can save time by finding the length of $\overline{XA}$ to be 24. If not, we can the Pythagorean theorem to solve for $\overline{XA}$. Either way, we must add $\overline{XA}$, 24 inches, to $\overline{AB}$, given as 12 inches, in order to find the total length of the shaft, which is 36 inches.
19. E. $\angle A$ in $\triangle ABE$ and $\angle A$ in $\triangle ACD$ are congruent to each other. $\angle BEA$ in $\triangle ABE$ and $\angle CDA$ in $\triangle ACD$ are congruent to each other. Therefore, since the 2 angles are congruent, the third must also be congruent, and the triangles are similar. Thus, the corresponding sides must be proportional to each other. We can solve this by setting up a proportional equation, placing the smaller triangle's sides over the larger triangle's corresponding side: $\frac{\overline{EB}}{\overline{DC}}=\frac{\overline{AE}}{\overline{AD}}$ $\frac{10}{15}=\frac{\overline{AE}}{20}$. Solve and we find that $\overline{AE}=13\frac{1}{3}$.
20. D. The sum of the lengths of any two sides of a triangle must be greater than the third side. Therefore, if one side of the triangle is 9 inches, then the sum of the two other sides must be at the least 10 inches, since the sides must be integers. Thus, the smallest possible perimeter is 19 inches.
21. A. If two angles in a triangle are congruent to two corresponding angles in a second triangle, then these triangles are similar. The triangles formed by Block A and Block B and their shadows each have a right angle and a shared angle. Thus, the two triangles are similar. In two similar triangles, the corresponding sides are proportional, so we can solve by setting up a proportional equation relating these corresponding sides. Let d equal the distance between the bikes. Relating the corresponding side of the two trianges, our equations is $\frac{40}{60}=\frac{75-d}{75}$. Solve and we find that $d=25$. Note that it doesn't matter which value is in the numerator or denominator, so long as the relationship is consistent. For example, we could have set our equation as $\frac{40}{75-d}=\frac{60}{75}$, and the answer would have been the same because we are putting the same triangle's side's in the same fraction both times.
22. D. When the ladder rests against the side of a house that is perpendicular to the ground, it creates a right triangle with legs of 15 and 20 feet. In order to find the length of the ladder, we can use Pythagorean Theorem, $15^2+20^2=c^2$. After algebraic calculation, we can see that c, or the length of the ladder is closest to 25.