CHAPTER

20 SOHCAHTOA

SKILLS TO KNOW

- SOHCAHTOA Basics
- How to solve for a missing side or angle using SOHCAHTOA
- Right Triangle Word problems
- Choosing which trig function to use

This chapter is all about basic trigonometry involving right triangles. Most problems of this type can be solved by applying that wonderful mnemonic, SOHCAHTOA!

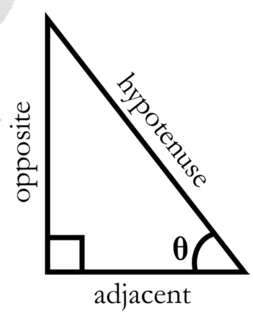
SOHCAHTOA BASICS

So what is "SOHCAHTOA"? It's an acronym to remember the three respective ratios described by Sine, Cosine, and Tangent in a right triangle:

Trigonometric function	Abbreviation	Acronym	Is equal to
Sine	sin	SOH	**O**pposite leg divided by **H**ypotenuse
Cosine	cos	CAH	**A**djacent leg divided by **H**ypotenuse
Tangent	tan	TOA	**O**pposite leg divided by **A**djacent leg

The diagram below shows how these sides are oriented. As you can see:

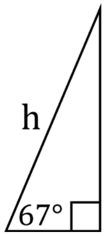
- The Hypotenuse is always the longest side of the triangle
- The Opposite is the side farthest from the angle of interest (here labelled θ)
- The Adjacent is the shorter side closest to the angle of interest (i.e. NOT the hypotenuse)



$$\sin(\theta) = \frac{opposite}{hypotenuse}$$

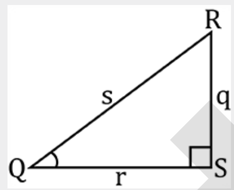
$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\tan(\theta) = \frac{opposite}{adjacent}$$



So, for example, when a question asks for $\sin(67°)$, it is asking for the ratio of the length of the opposite side from an angle of $67°$ to the length of the hypotenuse in a right triangle with those particular angles. Essentially, when we know two angles in a triangle ($67°$ and $90°$), we actually know all three angles, as all measures in a triangle sum to $180°$, and we can subtract to calculate the last angle. When we know all the angles, we also know that all triangles with those angles are similar. All sides of all similar triangles have the same ratios to each other, so it makes sense that we could store in our calculators all possible values for these ratios for the specific case of right triangles.

For right triangle QRS below, which of the following expressions is equal to $\tan Q$?



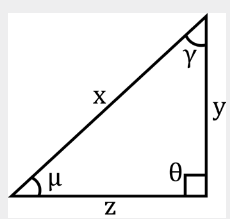
A. $\frac{q}{r}$ **B.** $\frac{q}{s}$ **C.** $\frac{r}{s}$ **D.** $\frac{s}{r}$ **E.** $\frac{r}{q}$

Here, we first think tangent is TOA. We need the Opposite over the Adjacent. We find the side opposite point Q: q is our numerator. Then we find the adjacent side (the short leg that touches Q), which is r, our denominator.

$\frac{q}{r}$, or answer choice A, is correct.

Answer: **A**.

Right triangle XYZ below has angle measures, (μ, θ, and γ). Which of the following is true about the quotient of $\cos\mu$ and $\cos\gamma$?



A. It equals 1. **B.** It equals $\frac{y}{z}$. **C.** It equals $\frac{z}{y}$. **D.** It equals $\frac{y^2}{x^2}$.

E. The value cannot be determined from the given information.

For this problem, it's important you don't get overwhelmed by the weird Greek letters. Remember we can use Greek letters for angles and they are simply variables. Also remember quotient means divide. Finally, BE CAREFUL, the Greek letter γ looks a lot like y, and there are two other labels that say "y" that are different! This is written this way to try to fool you!

Now let's think of what $\cos\mu$ is: CAH tells us to find adjacent over hypotenuse, or $\frac{z}{x}$.

Now let's find $\cos\gamma$: adjacent over hypotenuse is $\frac{y}{x}$.

When we divide, order matters, so we must take $\frac{z}{x}$ and multiply by the reciprocal of $\frac{y}{x}$:

$$\frac{z}{x}\times\frac{x}{y}=\frac{z}{y}$$

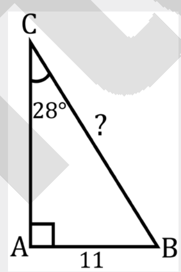
Answer: **C**.

FINDING A MISSING SIDE

Often we encounter problems that ask us to solve for a missing side in a right triangle. Luckily, we can make use of our calculators, knowledge of right triangle anatomy, and SOHCAHTOA to find any missing sides or angles.

On the ACT®, though, you won't need your calculator's built in values for sin/cos/tan. Typically, answer choices for these problems will NOT be simplified; i.e. the ACT® will only ask you to set up the problem, leaving expressions such as "sin" or "arctan" in the answer choices. Other times, the ACT® may give you a value for the sin, cos, or tan, and ask you to approximate an answer.

Find the missing side $\overline{BC}$ in the triangle given below.



A. $\frac{11}{\cos(28)}$ **B.** $\frac{\cos(28)}{11}$ **C.** $\frac{11}{\sin(28)}$ **D.** $\frac{\sin(28)}{11}$ **E.** $11\sin(28)$

When I see this problem, the first step I take is to mark side $\overline{BC}$ with a question mark. Then I look at what I have: angle of 28 degrees, its OPPOSITE (O) side, 11, and I need $\overline{BC}$, the HYPOTENUSE (H). By first marking what I need, it is easy to figure out which trig function I need, whether it's $\sin$, $\cos$, or $\tan$. Given O and H, I need $\sin$ because SOH calls for O and H. Now let's set up our equation:

$$\sin(x°)=\frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(28°)=\frac{11}{\overline{BC}}$$

We're not done yet, as all the answer choices solve for an expression *equal* to $\overline{BC}$. We need to solve for $\overline{BC}$, and that takes a bit of algebraic manipulation. First, I multiply both sides by $\overline{BC}$:

$$\sin(28°)(\overline{BC})=11$$

And then I divide by $\sin(28°)$:

$$\overline{BC}=\frac{11}{\sin(28°)}$$

Answer: **C**.

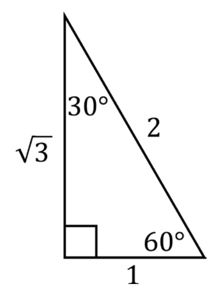
FINDING A MISSING ANGLE

Sometimes in a right triangle, the ACT® will give you two or more side lengths of a right triangle and ask you to solve for an angle. Though in math class you typically solve these using your calculator's built in trig functions, on the ACT® you won't need a calculator. As with missing side problems, the ACT® will ask you to simplify everything while leaving answers in trig notation (arcsin, $\tan^{-1}$, etc.), or will give you select values necessary to solve even if you don't have a calculator. Now you might be wondering? What is arcsin!?

Let's talk **inverse trigonometric functions**.

INVERSE TRIG FUNCTIONS

Inverse trig functions help us turn our sin, cos, or tan value into the angle it represents. For instance, $\sin(30°)=\frac{1}{2}$. If we know that $\sin x=\frac{1}{2}$, we can use the inverse of sin (called arcsin or $\sin^{-1}$) to get back to the angle, $30°$. If I type $\arcsin\left(\frac{1}{2}\right)$ in my calculator, it will return the answer: 30 .

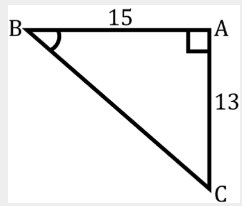
Trigonometric Function	Inverse Function	
sin (ex. $\sin(30°)=\frac{1}{2}$)	$\sin^{-1}$ or arcsin (ex. $\arcsin\left(\frac{1}{2}\right)=30°$)	
cos (ex. $\cos(60°)=\frac{1}{2}$)	$\cos^{-1}$ or arccos (ex. $\cos^{-1}\left(\frac{1}{2}\right)=60°$)	
tan (ex. $\tan(60°)=\frac{\sqrt{3}}{1}$)	$\tan^{-1}$ or arctan (ex. $\arctan\left(\sqrt{3}\right)=60°$)	

TIP: If using a calculator for inverse trig calculations, always be sure your calculator is in degree mode for degree problems or radian mode for radian problems!

Inverse functions are denoted by a "-1" superscript or an "arc" prefix. The ACT and your calculator may use these interchangeably. Both notations means the same thing.

Now let's go through an example to see how we apply these functions in the context of a real problem:

Find $\angle B$ in the triangle given below.



A. $\arcsin(B) = \frac{13}{15}$ **B.** $\arccos(B) = \frac{13}{15}$ **C.** $\arccos(B) = \frac{15}{13}$

D. $\arctan(B) = \frac{15}{13}$ **E.** $\arctan(B) = \frac{13}{15}$

First, we know the two legs of the triangle. These are the OPPOSITE (O) and ADJACENT (A) sides to the angle in question. O and A are part of TOA: thus, we'll use the tangent function.

$$\tan(B) = \frac{opposite}{adjacent}$$
$$= \frac{13}{15}$$

We just solved for the tangent of the angle. But we need to solve for an expression that equals the angle itself. To translate a tangent into the angle it corresponds to, we use the inverse function, $\arctan$ or $\tan^{-1}$.

The above expression can be rewritten as:

$$\arctan\left(\frac{13}{15}\right) = B \text{ or } \tan^{-1}\left(\frac{13}{15}\right) = B$$

We now simply scour the answer choices for one of these choices.

Answer: **E**.

COSECANT, SECANT AND COTANGENT

In addition to using sine, cosine and tangent to describe the ratios in a triangle, we can also write trigonometric ratios using cosecant, secant, and cotangent, all of which are the reciprocals of sine, cosine and tangent, respectively.

cosecant	$\csc(\theta)=\frac{hypotenuse}{opposite}$	$\frac{1}{\sin(\theta)}$
secant	$\sec(\theta)=\frac{hypotenuse}{adjacent}$	$\frac{1}{\cos(\theta)}$
cotangent	$\cot(\theta)=\frac{adjacent}{opposite}$	$\frac{1}{\tan(\theta)}$

Cotangent sounds like tangent, so that one is easy to remember. To remember secant, I see that the "s" and the "c" have flipped positions so it sort of looks like cos backwards, thus I remember that it is $\frac{1}{\cos}$ or cos flipped around. Csc has "s" in the second position not the first, so I imagine that means it represents sin but flipped around also, or $\frac{1}{\sin}$.

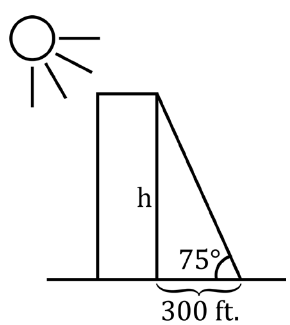
We use these ratios in the same way we use sin, cos and tan. Know these appear rarely on the ACT, and remember, if you forget which is which, you can always use your calculator and a made up number or two to double check.

RIGHT TRIANGLE WORD PROBLEMS

TIP: Trigonometric word problems tend to be much easier to solve if you draw it out so we don't get any numbers mixed up. Always draw a picture if there isn't one already! Always label what you need if it isn't already labeled!

At a time when the sun's rays are striking the ground at $75°$, a skyscraper casts a shadow that is 300 feet long. To the nearest foot, how tall is the skyscraper?

Let's draw this out first:



Now that we can visualize everything, we know we're trying to find height "h" of the building. The angle given, 75°, is OPPOSITE (O) to "h" and ADJACENT (A) to the given shadow length, 300 ft. Thus, knowing O and A of TOA, we can solve for "h" by using tangent. Remember, tan is opposite over adjacent:

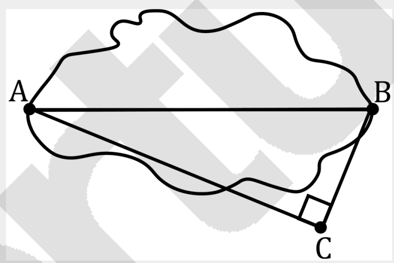
$$\tan(75^\circ) = \frac{h}{300}$$

Isolating the h, we get $h = 300\tan(75^\circ)$.

Answer: $300\tan(75)$.

Again, for most ACT® problems, this is as far as you need to go! If you look at the answers and need to solve, look for a table of values that includes $\tan(75)$ OR use your calculator. Be sure to round according to instructions.

Scientists attempting to measure the width of a mountain, represented in the figure below by $\overline{AB}$, use the measurements represented in the figure below. The distance from A to C is 5 km. The measure of $\angle CBA$ is 63°. What is the width of the mountain to the nearest tenth, if $\sin 63 \approx 0.891$, $\cos 63 \approx 0.454$, and $\tan 63 \approx 1.963$?



$$\sin(63^\circ) = \frac{AC}{AB}$$

$$\sin(63^\circ) = \frac{5 \text{ km}}{AB}$$

$$AB = \frac{5 \text{ km}}{\sin(63^\circ)}$$

Plugging in from our values, and using our calculator, we get:

$$\frac{5}{0.891} = 7.3 \text{ rounded to the nearest tenth}$$

Answer: 7.3.