CHAPTER

5 PERCENTS

SKILLS TO KNOW

- Basic Percent Translation Problems
- Percent Increase and Decrease
- Percent Short Cuts
- Percent Word Problems
  - The "Macy's" Problem (Multiple percent manipulations)
  - Start with Percents / End with Percents/Ratios problems
  - The Percent Proportion

PERCENT BASICS

What is a percent anyhow? A percent is a part out of one hundred that helps us understand numbers better. Percent is always PART over WHOLE (below, x represents our percent). It's like a fraction whose denominator is always 100.

$$\frac{Part}{Whole}=\frac{x}{100}$$

We'll use this proportion idea in word problems. For now let's dive into some percent-speak:

TRANSLATING PERCENT PROBLEMS

The basic type of percent problem involves translating English expressions involved in percent calculations into "math" language. Many percent problems involve words like "what," "is," "percent," and "of." These words can be translated to math:

"**WHAT**" means an unknown. Put a question mark or a new* variable (i.e. x or n).

"**PERCENT**" means divide by 100. Always divide **the number mentioned before the word percent** by 100 (i.e. 5 percent $=\frac{5}{100}$).

"**WHAT PERCENT**" means divide a new* variable by 100 (i.e. "what percent" means you should write $\frac{x}{100}$).

"x **PERCENT**" whenever you see a variable next to "percent" write the variable over 100 (i.e $x\%=\frac{x}{100}$).

"**IS**" means equals (i.e. $=$).

"**OF**" means multiply (i.e. write $*$ or $\times$).

**If you see the word "WHAT" DO NOT reuse a variable already used in the problem somewhere. Make up a new variable!*

When you see these words, simply translate them one at a time to form equations.

If 10 is x percent of 50, what is x percent of 30?

$$\begin{array}{ccccc} 10 & \text{is} & x \text{ percent} & \text{of} & 50 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 10 & = & \dfrac{x}{100} & \times & 50 \end{array}$$

First Equation: $10=\frac{x}{100}(50)$

$$\begin{array}{ccccc} \text{what} & \text{is} & x \text{ percent} & \text{of} & 30 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ ? & = & \dfrac{x}{100} & \times & 30 \end{array}$$

Second Equation: $?=\frac{x}{100}(30)$

To solve, we first simplify the first equation and get:

$$10=\frac{x}{100}(50)$$
$$10=\frac{50x}{100}$$
$$10=\frac{x}{2}$$
$$20=x$$

Now we plug in $20=x$:

$$?=\frac{20}{100}(30)$$
$$?=\frac{1}{5}(30)$$
$$?=6$$

PERCENT INCREASE AND DECREASE

PERCENT INCREASE OR DECREASE FORMULA

$$\boxed{\frac{\textit{New Number} - \textit{Original Number}}{\textit{Original Number}}\times 100}$$

A quick way to think of it: **NEW minus OLD over OLD.**

If the answer is **POSITIVE** it is a percent **increase**. If the answer is **NEGATIVE** it is a percent **decrease**.

Ashley worked a total of 80 hours last week and 56 hours this week. By what percentage did her work hours change?

Because her number of hours went down, we know this question is asking for a percent decrease. We can plug in the numbers given (80 is the original number, 56 is the new number) into our percent decrease equation and solve:

$$\frac{56-80}{80}\times 100=-30\%$$

Thus, our answer is a 30% **decrease**.

Another way to think of percent changes is: $\frac{\textit{Numeric Change}}{\textit{Original}}$

If the number of shirts increased by 5 and there were originally 10 shirts, then what is the percent increase in the number of shirts?

$$\frac{\textit{Numeric Change}}{\textit{Original}}=\frac{5}{10} \text{ or } 50\%$$

PERCENT SHORT CUTS

Most students learn the traditional set up for percent equations and handle a typical word problem in the following fashion:

Marley bought four bandanas for $\$3.00$ each, and two shorts for $\$15.00$ each. If she pays 8.5% sales tax on her purchase, what is her total cost?

Step 1: Calculate the cost of the items before the tax:

Four bandanas times $\$3.00$ each $=\$4\times 3=\$12$ total for bandanas

Two shorts for $\$15.00$ each $=\$15\times 2=\$30$ total for shorts

$\$12+\$30=\$42$ total for all items.

Step 2: Calculate the sales tax:

Sales tax equals 8.5% of the total, $\$42$:

$$\frac{8.5}{100}\times \$42=\$3.57 \text{ or } 0.085\times \$42=\$3.57$$

Step 3: Add the sales tax to the pre-tax total:

$$\$42+\$3.57=\$45.57$$

BUT THERE IS A FASTER WAY!

The above problem utilizes the following formula (albeit in steps):

$$Original\ Amount + \frac{Percent\ Increase}{100}\left(Original\ Amount\right) = Total\ Amount$$

Let's imagine that our original pre-tax amount is "x":	$x + \frac{Percent\ Increase}{100}\left(x\right) = Total$
Once we know that percent increase, let's fill it in:	$x + \frac{8.5}{100}\left(x\right) = Total$ or $x + 0.085x = Total$
AHA! Do you see that we can simplify that expression on the right?	$x + 0.085x = 1.085x$

The thing is, this idea holds for EVERY percent increase (and decrease) problem. Thus, we can add "1" to the percent, put that in front of "x" and do this problem not in three steps, but in two.

At Step 2, we simple calculate $\left(1+0.085\right)\left(\$42\right)$ or $1.085 \times \$42$ and get our answer with one calculator entry!

The same principle holds for percent decrease—but then you subtract from 1.

Marni buys 4 shirts originally priced at n dollars each, but has a 20% off discount coupon that is applied at checkout. Which of the following represents her total cost before tax?

A. $4.8n$
B. $4n$
C. $3.2n$
D. $0.8n$
E. $0.2n$

Step 1: Find her pre-discount cost: the number of shirts times the cost per shirt: $4n$

Step 2: Multiply by $1-d$ where d is the discount expressed as the numeric percent value (i.e. in hundredths or divided by 100 already):

$$\left(1-0.2\right)\left(4n\right) \text{ or } 0.8\left(4n\right)$$

Step 3: Simplify:

$$8\left(4n\right) = 3.2n$$

Answer: **C.**

WORD PROBLEMS

The "Macy's" Problem

Many word problems involve multiple percent discounts. At first glance these might seem most easily solved by adding together all the discount percents and then reducing the original number by that amount. However, that strategy will often backfire: you must reduce according to how the problem is worded.

I call these "Macy's" problems, because they reflect a common advertising trick that Macy's (and other stores) often use. Rather than give customers a flat 50% off, Macy's will advertise 30% and then and additional 10% off with coupon and 10% off for using your Macy's card! Sounds like a great deal, right? Like 50% off, right? Wrong!

With these kinds of incentives, the additional discounts come off of the intermediate, already reduced prices, NOT off of the original. As a result, you need to do these problems in steps.

Sweta buys a sweater originally priced at $\$120$. The sweater is subject to a 30% off sale, and Sweta has a coupon for an additional 10% off any sale item. She additionally gets a 10% off discount off the coupon discounted price as part of an early bird promotion. What does she pay, before tax, for her sweater?

First take the 30% off—I'll use my shortcut method described above:

$$0.7(\$120)$$

Then take the additional 10% off:

$$0.9(0.7(\$120))$$

Then take another 10% off:

$$0.9(0.9(0.7(\$120)))=\$68.04$$

Answer: $\$68.04$

As you can see, she didn't pay $\$60$ (or 50% of the original price)—the store eked another $\$8.04$ out of her!

On June 1, a used car was priced at $\$11{,}000$. On June 15, the price was reduced by 15%. After a month, the price was reduced 10% of the June 15 price. What was the final price?

A. $\$7425$
B. $\$8250$
C. $\$8415$
D. $\$8505$
E. $\$9350$

I'll solve this one with the traditional equation method:

$$\$11{,}000 - 0.15(\$11{,}000) = \$9350$$

$$\$9{,}350 - 0.10(\$9{,}350) = \$8415$$

Always read your word problems carefully! You need to know what the percent is coming off of—be VERY CAREFUL to know whether it is the original price, discounted one, etc.—to ensure you don't make a careless error!

Starts with Percents (or Ratios)/ Ends with Percents (or Ratios)

Whenever you have a percent or ratio problem that has NO REAL VALUES—i.e. you have no idea regarding the actual amounts involved—make up a number to make the problem easier to solve! I prefer to make up 100 or 10 or some multiple thereof to make the calculations easier. The example below shows this idea with percents, but this method also works with ratio problems.

A class has an activities fund; 20% of the fund is spent in September, and 10% of the remaining monies are spent in October. What percent of the original fund is left by the end of October?

A. 30%
B. 42%
C. 67%
D. 70%
E. 72%

Let's make up a number: $\$100$

If the fund has $\$100$, then they spent $\$20$ in September:

$$0.2(\$100) = \$20$$

And have $\$80$ left at the start of October:

$$\$100 - \big(0.2(\$100)\big) = \$80 \text{ or } (0.8)(\$100) = \$80$$

Now we must be careful. The problem referenced 10% "of remaining monies." The most common error, again, that students make is to take this percent off the original money amount.

During October, they spent 10% of the remaining $\$80$, which is $\$8$:

$$0.10(\$80) = \$8$$

Thus, $\$80 - \$8 = \$72$ of the original fund remains.

We could also find this by thinking that we need 0.9 or 90% of the $\$80$ to remain after 10% was spent:

$$0.9(\$80) = \$72$$

$\$72$ is then what percent of the original amount, $\$100$? That's easy because we choose 100 as our starting number! It's the final number over the original:

$$\frac{72}{100} = 72\%$$

Answer: **E**.

Percent Proportions

Sometimes you need to think about what percent means to solve a problem: part over whole.

> Julie has won 52 out of 80 tennis matches times. If she wins every game from now, what is the least number of games she must win to improve her win percentage to at least 70%?

For this problem we start with part over whole:

$$\frac{52}{80} = \frac{\textit{Matches Won}}{\textit{Total Matches}} = \textit{Her Current Percentage} = 65\%$$

We want to get that up to $70\%+$—to do that she'll need to play more games and win them. We'll call the number of games she plays and wins "g." We'll add g to the top of the fraction because it adds to the number won—but also to the bottom of the fraction—as it adds to the total number of matches. We'll also not worry about the "minimum" idea for now and just figure out how many wins get her to 70—she needs at least that many and she can't have a fraction of a win so in the end we'll round up—but again we can deal with that in a minute.

$$\frac{52+g}{80+g} = 0.7$$
$$52+g = 0.7(80+g)$$
$$52+g = 56+0.7g$$
$$0.3g = 56-52$$
$$0.3g = 4$$
$$g \approx 13.3333$$

We now round—she needs at least 13.3333 matches, so that means at 14 matches she'll finally be above 70%. *If you're not sure, plug in 13 and you'll find that it creates an answer below 70%!*

Remember these maximum/minimum problems can be confusing. When in doubt, double check your work and test your answer!

Answer: 14.