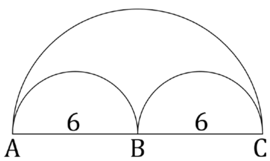
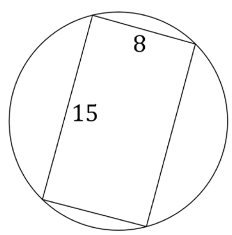
1. In the figure shown below, two congruent semicircles are adjacent to one another inside a bigger semicircle. The diameter of each congruent semicircle is 6 cm. What is the sum of the lengths, in centimeters, of the three arcs of these semicircles $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, and $\overset{\frown}{AC}$?



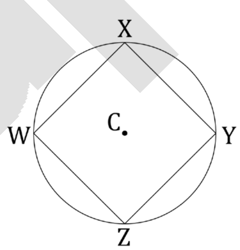
A. 6π
B. 9π
C. 12π
D. 15π
E. 18π

2. In the figure below, the corners of the rectangle with sides of length 8 and 15 is inscribed in a circle. What is the area of the circle?



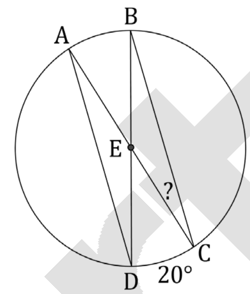
A. 289π
B. $\dfrac{289\pi}{4}$
C. $\dfrac{289\pi}{2}$
D. $2\sqrt{161}\pi$
E. $\dfrac{161\pi}{4}$

3. In the figure below, square $WXYZ$ with side lengths $=4$ is inscribed in a circle with center C. What is the area of the circle?



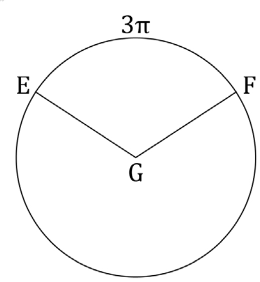
A. 4π
B. 8π
C. 12π
D. 16π
E. 64π

4. In the circle shown below, chords $\overline{AC}$ and $\overline{BD}$ intersect at E, which is the center of the circle, and the measure of minor arc $\overset{\frown}{CD}$ is $20°$. What is the measure of $\angle BCE$?



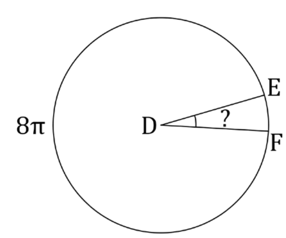
A. $10°$
B. $15°$
C. $20°$
D. $25°$
E. $30°$

5. In the figure below, central angle $\angle EGF$ is $120°$ and the arc $\overset{\frown}{EF}$ is 3π. What is the circle's diameter?



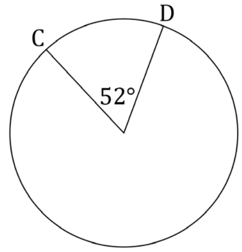
A. 3
B. 3π
C. 9
D. 9π
E. 5π

6. In the circle below centered at D with radius 5, the arc length of the obtuse sector is 8π. What is the value of angle $\angle EDF$ in degrees?



A. $12°$
B. $25°$
C. $36°$
D. $72°$
E. $144°$

7. In the circle below with diameter 8, the central angle intercepting arc $\overset{\frown}{CD}$ is $52°$. What is the length of arc $\overset{\frown}{CD}$?



A. 52π

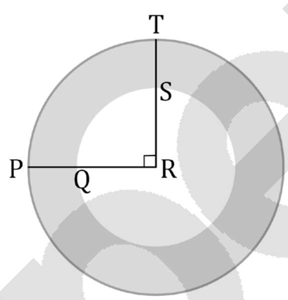
B. $\frac{52\pi}{45}$

C. $\frac{7\pi}{6}$

D. $\frac{10\pi}{9}$

E. $\frac{104\pi}{45}$

8. The diagram below shows a quarter of each of 2 circles both having point R as their center. Point S lies of $\overline{TR}$ and point Q lies on $\overline{PR}$. The length of $\overline{PQ}$ is $x-2$ centimeters and the length of $\overline{QR}$ is $x+1$ centimeters. What is the area, in square centimeters, of the shaded portion of the entire circle?



A. $(3x^2-2x)\pi$

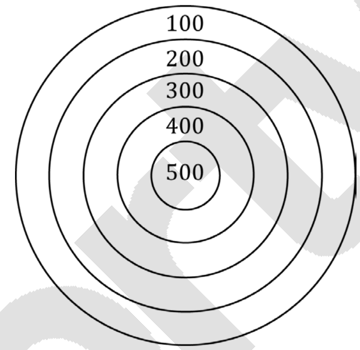
B. $(3x^2-6x)\pi$

C. $(3x^2-6x+2)\pi$

D. $(3x^2-2x+2)\pi$

E. $(3x^2+4x+2)\pi$

9. Becca is making a bet with her friend that she can throw a dart and hit the bull's eye (innermost circle). If the outer most circle has a diameter of 20 inches and there is a 2-inch difference in radius for each inner circle, what are the chances that Becca will win the bet?



A. $\frac{49}{100}$

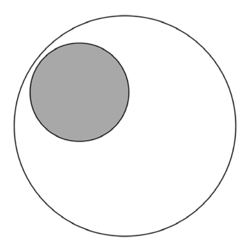
B. $\frac{21}{50}$

C. $\frac{1}{25}$

D. $\frac{4}{25}$

E. $\frac{9}{25}$

10. To make one of a pair of googly eyes, Alex pastes a dark circular piece of felt onto a larger circular piece of plastic, as shown below. The radius of the larger circle is 5 centimeters. If the smaller circle's area is $\frac{1}{3}$ the of the uncovered area of the white circle, what is the circumference, in centimeters, of the smaller, dark circle?



A. $\frac{5\pi}{2}$

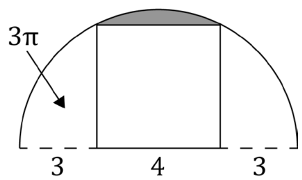
B. $\frac{10\pi}{3}$

C. $\frac{10\pi}{\sqrt{3}}$

D. $\frac{25\pi}{4}$

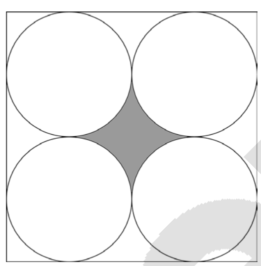
E. 5π

11. A square with side length 4 is inscribed in a half circle with diameter 10. The area of the space in the half circle to the left of the square is equal to 3π. What is the area of the shaded region?



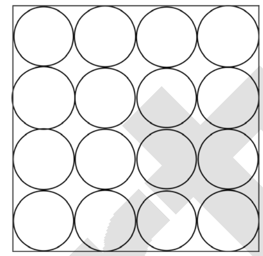
A. $94\pi - 16$
B. $19\pi - 16$
C. $41\pi - 16$
D. $\frac{13\pi}{2} - 16$
E. $13\pi - 16$

12. In the figure below, the square has side length of 16 inches. The circles within the square are congruent, and each circle is tangent to 2 of the other circles. The region that is interior to the square and exterior to all 4 circles is shaded. What is the perimeter, to the nearest inch, of the shaded region?



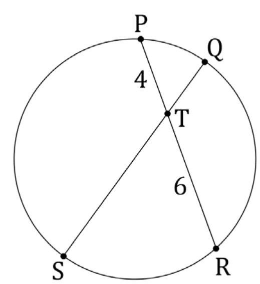
A. 16
B. 32
C. 4π
D. 8π
E. 16π

13. In the figure below, each of the congruent circles are tangent to another circle or the edge of the square with side length 18. There are 16 circles. What is the circumference of one of these circles?



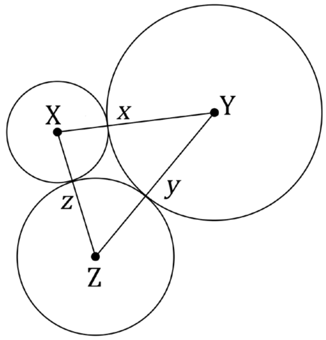
A. 9π
B. $\frac{9\pi}{2}$
C. $\frac{9\pi}{4}$
D. $\frac{81\pi}{4}$
E. $\frac{81\pi}{16}$

14. In the figure below, chord $\overline{PR}$ intersects chord $\overline{SQ}$ at point T. If segment $\overline{ST}$ is equal to 8, what is the length of segment $\overline{TQ}$? (Note: The product of segments of intersecting chords are equal: $PT \cdot TR = QT \cdot TS$)



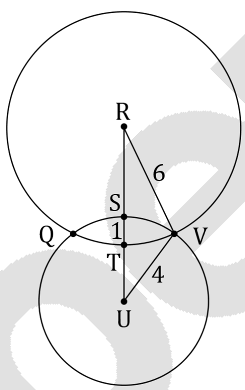
A. 1
B. 2
C. 3
D. $\frac{3}{2}$
E. 4

15. In the figure below, three tangent circles with centers $X, Y,$ and Z and radii $x, y,$ and z are shown. The perimeter of the triangle ΔXYZ is 30. What is $x + y + z$?



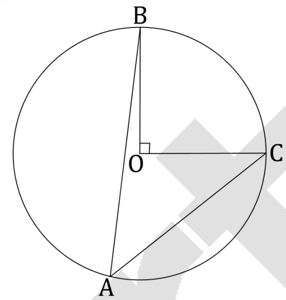
A. 30
B. 15
C. 10
D. $\frac{15}{2}$
E. 15π

16. In the figure below, the circles centered at R and U intersect at points Q and V. The points R, S, T and U are collinear. If the lengths of $\overline{ST}$, $\overline{RV}$, and $\overline{VU}$ are $1, 6,$ and 4 respectively, what is the length of the segment $\overline{RU}$?



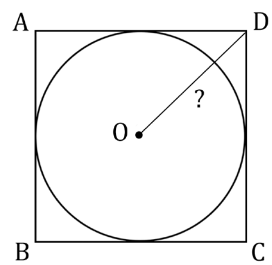
A. 9
B. 10
C. 8
D. 7
E. $\sqrt{20}$

17. In the figure below, the points $A, B,$ and C lie on the circle centered at point O. If angle $\angle ABO = 8°$, what is $\angle OCA$?



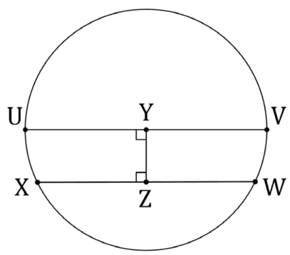
A. 37
B. 42
C. 45
D. 47
E. 50

18. In the figure below, a circle is inscribed in a square with side s, what is the length of segment $\overline{OD}$ in terms of s?



A. s
B. $2s$
C. $s\sqrt{2}$
D. $s\sqrt{3}$
E. $\frac{s\sqrt{2}}{2}$

19. In the figure below, the chord $\overline{XW}$ of length 20 is parallel to the diameter $\overline{UV}$ of length 30. If the circle is centered at point Y, what is the length of the segment $\overline{YZ}$?



A. 5
B. $5\sqrt{5}$
C. $10\sqrt{5}$
D. $\frac{5\sqrt{5}}{2}$
E. $\sqrt{5}$

20. A circle with radius 6 has a sector with central angle of $48°$. What is the area of the sector?

A. $\frac{12}{5}\pi$
B. $\frac{4}{5}\pi$
C. $\frac{3}{4}\pi$
D. $\frac{24}{5}\pi$
E. $\frac{24}{5}\pi$

21. The Ferris wheel at the amusement park has a radius of 50 feet, rotates at a constant speed, and completes 1 rotation in 5 minutes. How many feet does a Ferris wheel passenger car travel along the circular path in 30 seconds?

A. 12π
B. 10π
C. 20π
D. 10
E. 20

22. Jenny is sewing a tablecloth for a circular table with a radius of 2 feet for a Girl Scouts merit badge. The finished tablecloth must hang down 8 inches over the edge of the table. The tablecloth's edges must be hemmed to be considered finished, which means she will have to fold over 1 inch of the material at the edge. Jenny wants to use only one piece of fabric, a rectangular piece that is 6 feet wide. What is the shortest length of fabric, in inches, Jenny could use without needing to use a second piece of fabric?

A. 16
B. 40
C. 42
D. 64
E. 66

23. Given two circles of different diameters, what is the maximum number of points of intersection that the two circles can have?

A. 0 only
B. 0 or 1 only
C. 2 only
D. 0, 1, or 2
E. Infinitely many

24. Stacy is designing a new logo for Candy Cane Cable Company. Her design consists of two concentric circles. The bigger circle has a radius of 8 feet. The smaller circle is one fourth the area of the bigger circle. Which of the following is an expression for the area, in square feet, of the smaller circle?

A. $\frac{8}{4}\pi$
B. $\left(\frac{8}{4}\right)^2\pi$
C. $\frac{64}{4}\pi$
D. 4π
E. 2π

25. What is the area of a circle having the points $(5,3)$ and $(-7,-13)$ as endpoints of a diameter?

A. 10π
B. 100π
C. 400π
D. 20π
E. 200π

26. In the standard (x, y) coordinate plane, the graph of the equation $x^2 + 8x - 5y^2 - 10y + 12 = 0$ is a/an:

A. Linear line
B. Parabola
C. Circle
D. Ellipse
E. Hyperbola

27. What is the largest value of x for which there exists a real value of y such that $x^2 + y^2 = 100$?

A. 6
B. 8
C. 10
D. 99
E. 100

ANSWER KEY

1. C **2. B** **3. B** **4. A** **5. C** **6. D** **7. B** **8. B** **9. C** **10. E** **11. D** **12. D** **13. B** **14. C**
15. B **16. A** **17. A** **18. E** **19. B** **20. D** **21. B** **22. E** **23. D** **24. C** **25. D** **26. E** **27. C**

ANSWER EXPLANATIONS

1. **C.** The two smaller semi circles have diameter $=6$ while the larger semi-circle has diameter $=6+6=12.$ The length of an arc is equal to half the length of the full circle's circumference. So, using the formula $C=\pi d$, the sum of arc-lengths $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, and $\overset{\frown}{AC}$ is equal to $\frac{1}{2}\pi(6)+\frac{1}{2}\pi(6)+\frac{1}{2}\pi(12)=3\pi+3\pi+6\pi\to12\pi.$

2. **B.** Since the rectangle is inscribed in the circle, the diagonal of the rectangle is equal to the circle's diameter. We find the length of the rectangle's diagonal using the Pythagorean Theorem $a^2+b^2=c^2$. Plugging in $a=8$ and $b=15,$ we get $8^2+15^2=c^2\to c=\sqrt{8^2+15^2}\to c=\sqrt{64+225}\to c=\sqrt{289}=17.$ Now, we know that the diameter of the circle is equal to 17, so the radius of the circle is $\frac{17}{2}.$ We now plug in $\frac{17}{2}$ for the radius of the circle to find the area of the circle using the formula $A=\pi r^2$. $A=\left(\frac{17}{2}\right)^2\pi\to\frac{289\pi}{4}$.

3. **B.** We are given the side lengths of square $WXYZ$: 4. We can draw out a triangle using the points XCW. We know that $XC=CW$ because they are both radii of the circle. Thus, using the Pythagorean theorem, we can set up the equation $r^2+r^2=4^2\to2r^2=16\to r^2=8\to r=\sqrt{8}$. Using the formula to find the area of a circle ($A=\pi r^2$), we can solve for the area: $A=\pi\left(\sqrt{8}\right)^2\to A=8\pi$.

4. **A.** Since $\angle DBC$ is the inscribed angle of $\overset{\frown}{DC}$, it measures half of $\overset{\frown}{DC}$'s central angle, which makes $\angle DBC=10°$. $\overline{BE}$ and CE are equal since they are both radii of the circle, which makes the angles opposite them in the triangle equal. Thus, $\angle BCE=\angle DBC=10°$.

5. **C.** We wish to first find the circumference by finding the arc length that would be proportional to $360°$ as 3π is proportional to the central angle of $\angle EGF=120°$. To solve this, we set up the proportions $\frac{3\pi}{120°}=\frac{x}{360°}$ and solve for x. Cross multiplying the equation, we get $3\pi(360)=120x$. Dividing by 120 on both sides, we get 9π is the circumference of the circle. Since we know that $C=\pi d$, we can solve for the diameter of the circle by solving $9\pi=\pi d$. Dividing π on both sides, we get $d=9.$

6. **D.** Using the formula $C=2\pi r,$ we get $C=2(5)\pi=10\pi.$ Since we know the arc length of the obtuse sector is $8\pi,$ we know the arc length of the acute sector is $10\pi-8\pi=2\pi.$ We wish to first find the angle that would be proportional to 2π as the circumference 10π is proportional to $360°$. To solve this, we set up the proportions $\frac{2\pi}{x°}=\frac{10\pi}{360°}$ and solve for x. Cross-multiplying the equation, we get $2\pi(360)=10\pi x$. Dividing by 10π on both sides, we get $72°=x$.

7. **B.** Using the formula $C=\pi d$, we find $C=8\pi.$ We wish to first find the arc length that would be proportional to $52°$ as the circumference 8π is proportional to $360°$. To solve this, we set up the proportions $\frac{x}{52°}=\frac{8\pi}{360°}$ and solve for x. Cross multiplying the equation, we get $360x=8(52)\pi$. Dividing both sides by 360, we get $x=\frac{8(52)\pi}{360}\to\frac{52\pi}{45}.$

8. **B.** The area of the shaded region is equal to the area of the big circle minus the area of the small circle. The big circle has radius $PR=x-2+x+1=2x-1.$ The small circle has radius $QR=x+1.$ So, the area of the big circle is $A=\pi r^2\to(2x-1)^2\pi\to\left(4x^2-4x+1\right)\pi$. The area of the small circle is $A=\pi r^2\to(x+1)^2\pi\to\left(x^2+2x+1\right)\pi$. Now, we find the shaded area by subtracting the two areas we just found: $=\left(4x^2-4x+1\right)\pi-\left(x^2+2x+1\right)\pi=\left(4x^2-4x+1-x^2-2x-1\right)\pi\to\left(4x^2-x^2-4x-2x+1-1\right)\pi\to(3x^2-6x)\pi$.

9. **C.** The probability that Becca will hit the bulls eye can be calculated by the area of the innermost circle divided by the area of the outermost circle. We know that the diameter of the outermost circle is 20 inches, which means the radius is $r=\frac{1}{2}d=\frac{1}{2}(20)\rightarrow 10.$ Since each inner circle has a radius 2 inches smaller than the next bigger circle, we know that the second outermost circle has radius $=10-2=8$ inches, the third outermost circle has radius $=8-2=6$ inches, the fourth outermost circle has radius $=6-2=4$ inches, and the innermost circle has radius $=4-2=2$ inches. Using the formula $A=\pi r^2$, we calculate the area of the innermost circle to be $A=\pi 2^2 \rightarrow 4\pi$ and the area of the outermost circle to be $A=\pi 10^2 \rightarrow 100\pi$. The probability can then be calculated as $\frac{4\pi}{100\pi}=\frac{4}{100}\rightarrow\frac{1}{25}.$

10. **E.** $A=\pi r^2$, we calculate the area of the big circle to be $\pi 5^2=25\pi.$ Using the formula Let A be the area of the smaller circle. We are told that the area of the smaller circle is equal to $\frac{1}{3}$ the area of the uncovered area of the white circle, which means it is equal to $\frac{1}{3}$ (area of big circle – area of small circle). Plugging in the values we know, we get $A=\frac{1}{3}(25\pi-A).$ Multiplying by 3 on both sides gives us $3A=25\pi-A.$ Adding A on both sides gives us $4A=25\pi$. Finally, dividing by 4 on both sides gives us $A=\frac{25\pi}{4}.$ Using the formula $A=\pi r^2$ again, we find the radius of the small circle is $\frac{25\pi}{4}=\pi r^2\rightarrow\frac{25}{4}=r^2\rightarrow r=\frac{5}{2}.$ Using the formula $C=2\pi r$, we find $C=2\pi\left(\frac{5}{2}\right)\rightarrow 5\pi$.

11. **D.** We first calculate the area of the half circle by using the formula $A=\frac{\pi r^2}{2}.$ We are given that the diameter of the half circle is 10 , so the radius is $\frac{10}{2}=5.$ Plugging in $r=5.$ We get $A=\frac{\pi(5)^2}{2}\rightarrow\frac{25\pi}{2}.$ Now, we can calculate the area of the shaded region as the area of the half circle minus the areas of the non-shaded areas in the half circle. We are given that the space to the left of the square is 3π and the side of the square is 4 , so the non-shaded areas add up to equal $3\pi+4(4)+3\pi=6\pi+16.$ Subtracting this from the area of the half circle, we get $\frac{25\pi}{2}-(6\pi+16)=\frac{25\pi}{2}-6\pi-16\rightarrow\frac{25\pi}{2}-\frac{12\pi}{2}-16\rightarrow\frac{13\pi}{2}-16.$

12. **D.** The perimeter of the shaded region is equal to $4\left(\frac{1}{4}\right)C$ where $C=\pi d.$ We are given that the side of the square has length of 16 inches, which means that the diameter of a circle is $\frac{16}{2}=8$ inches. Plugging in $d=8$, we get $C=\pi d=8\pi.$ Plugging in $C=8\pi$, we get $4\left(\frac{1}{4}\right)C=4\left(\frac{1}{4}\right)(8\pi)\rightarrow 8\pi.$

13. **B.** The side of the square is 18 , so the diameter of each of the circles is $\frac{18}{4}=\frac{9}{2}.$ Using the formula $C=\pi d$ and plugging in $d=\frac{9}{2}$, we get $C=\frac{9}{2}\pi$.

14. **C.** When two chords of a circle intersect within the circle, the product of the two segments of one chord is equal to the product of the other two segments of the other chord. So, the product of the segments of chord $\overline{PR}$ is equal to $4(6)=24.$ Setting the product of the segments of chord $\overline{SQ}$ also equal to 24 , we get $8x=24$ where $x=$ the length of segment $\overline{TQ}.$ Thus, $\overline{TQ}=\frac{24}{8}\rightarrow 3.$

15. **B.** The perimeter of the triangle is equal to $\overline{XY}+\overline{YZ}+\overline{ZX}=x+y+y+z+z+x\rightarrow 2x+2y+2z\rightarrow 2(x+y+z).$ We are given this is equal to 30 , so $2(x+y+z)=30\rightarrow x+y+z=\frac{30}{2}\rightarrow 15.$

16. **A.** Since $\overline{RV}$ of length 6 connects the center of circle R to the side of the circle, it is the radius of circle R . We can observe that segment $\overline{RT}$ is also a radius of circle R and thus must also have length 6 . Similarly, Since $\overline{VU}$ of length 4 connects the center of circle U to the side of the circle, it is the radius of circle U .We can observe that segment $\overline{US}$ is also a radius of circle U and thus must also have length 4 . The length of segment $\overline{RU}$ is $\overline{RT}+\overline{US}-\overline{ST}$ because the middle segment $\overline{ST}$ is counted twice (once as part of $\overline{RT}$ and again as part of $\overline{US}$). Plugging in the values for the segments, we get $\overline{RU} = \overline{RT} + \overline{US} - \overline{ST} \rightarrow 6+4-1=9$. Note that we cannot use the Pythagorean theorem to solve for the third side of the triangle in this case because it is not stated that the triangle is a right triangle.

17. A. Since angle $\angle BOC$ is at the midpoint of circle O, any angle inscribed in circle O and tangent to points B and C and any other point on the circle between the larger arc $\overset{\frown}{BC}$ is equal to half of angle $\angle BOC$. So, angle $\angle BAC$ is equal to $\frac{90}{2} = 45°$. The sum of the angles in any 4-sided polygon is $360°$, so plugging in the angles in polygon $ABOC$ that we know, we get $45+8+(360-90)+\angle OCA = 360 \rightarrow 45+8+270+\angle OCA = 360 \rightarrow \angle OCA = 37$.

18. E. We can calculate the length of segment $\overline{OD}$ by imagining a right triangle formed by the midpoint of segment $\overline{AD}$ and points O and D. This triangle has sides of length $\frac{s}{2}, \frac{s}{2}$ and the hypotenuse $\overline{OD}$. Using the Pythagorean theorem or the 45-45-90 triangle rule, we can calculate the hypotenuse $\overline{OD}$ is equal to $\sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2} = \sqrt{\frac{2s^2}{4}} \rightarrow \frac{s\sqrt{2}}{2}$.

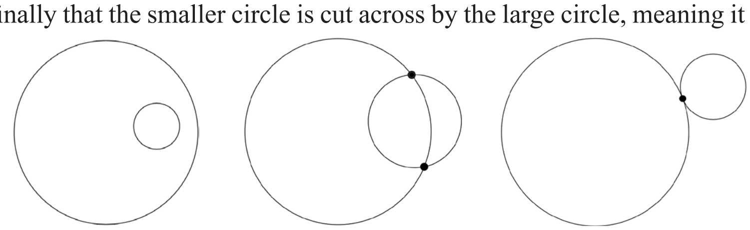
19. B. Since points Y and Z are midpoints of the segments $\overline{UV}$ and $\overline{XW}$ respectively, the length of $\overline{YV} = \frac{\overline{UV}}{2} \rightarrow \frac{30}{2} = 15$ and the length of $\overline{ZW} = \frac{\overline{XW}}{2} \rightarrow \frac{20}{2} = 10$. Since $\overline{UV}$ is the diameter of the circle, we know that $\overline{YV}$ is the radius. If we draw a segment connecting points Y and W, the segment $\overline{YW}$ will also be a radius and have length 15. Now, we can imagine a triangle formed by the points $Y, Z,$ and W. Using the Pythagorean theorem, we can solve for segment $\overline{YZ}$ by plugging in $a=10$ and $c=15$ and solving for b. We get $10^2 + b^2 = 15^2 \rightarrow 100 + b^2 = 225 \rightarrow b^2 = 125 \rightarrow b = 5\sqrt{5}$.

20. D. Using the formula $A = \pi r^2$, we can first find the area of the entire circle to be $A = \pi(6)^2 \rightarrow 36\pi$. This area corresponds to $360°$ of the circle. We now want to find the sector that corresponds to $48°$. We set up the equation $\frac{36\pi}{360°} = \frac{x}{48°}$ where $x =$ the area of the sector. Cross multiplying, we get $48(36\pi) = 360x \rightarrow 48\pi = 10x \rightarrow x = \frac{48\pi}{10} = \frac{24\pi}{5}$.

21. B. The Ferris wheel has a perimeter of 100π. If the Ferris wheel completes 1 rotation in 5 minutes, that means that it travel at the following rate: $\frac{100\pi}{300\,seconds}$. If you divide top and bottom by 10, you find how much it rotates in 30 seconds: $\frac{100\pi/10}{300/10\;seconds} = \frac{10\pi}{30\,seconds}$. Thus, our answer is 10π. Alternatively, you could use the arc length formula, $s = \theta r$, where θ is the angle rotated in radians.

22. E. The table has a 2 foot radius, meaning it is 4 feet, or 48 inches, across. In addition, the tablecloth must stretch down 9 inches on either side, 8 inches that hang and 1 inch for the hem. As such, the total width of the fabric must at least be $48+2(9)=66$ inches.

23. D. The circles have different radii, which means they cannot be congruent and thus have infinitely many points of intersection. Additionally, nowhere is it said that the two circle share the same center point. One must be larger, and one must be smaller, so there are three possible scenarios: that the smaller circle is completely inside or outside the larger circle, meaning there are 0 points of intersection, that the smaller circle and larger circle are tangent to one another, thus intersecting at one point only, and finally that the smaller circle is cut across by the large circle, meaning it has 2 points of intersection.



24. **C.** The area of the bigger circle can be calculated by the formula $A=\pi r^2=\pi 8^2=64\pi$. The smaller circle's area is $\frac{1}{4}$ of the big circle's area, so it is $\frac{64\pi}{4}$.

25. **B.** Because we are given the points of the circle's diameter, we can calculate the distance of the diameter: $\sqrt{\left(5-(-7)\right)^2+\left(3-(-13)\right)^2} \rightarrow \sqrt{12^2+16^2} \rightarrow \sqrt{144+256} \rightarrow \sqrt{400}=20$. Now that we know the diameter is 20, we also know that the radius is 10. Using the formula to find the area of a circle, $A=\pi 10^2 \rightarrow 100\pi$.

26. **E.** Completing the square for the equation, we get $x^2+8x+16-5\left(y^2+2y+1\right)+12-16+5=0$. This is simplified to be $\left(x+4\right)^2-5\left(y+1\right)^2=1$. The equation of a hyperbola is in the form $\frac{\left(x-a\right)^2}{c^2}-\frac{\left(y-b\right)^2}{d^2}=1$, so our equation matches this form.

27. **C.** If we plug in $y=0$, we maximize the value of x. This value is found to be $x^2+0^2=100 \rightarrow x^2=100 \rightarrow x=10$.