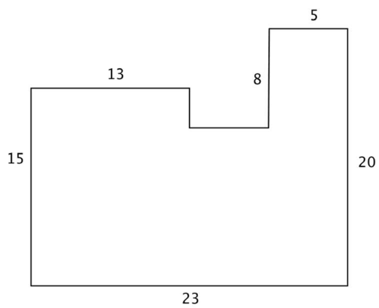
POLYGONS

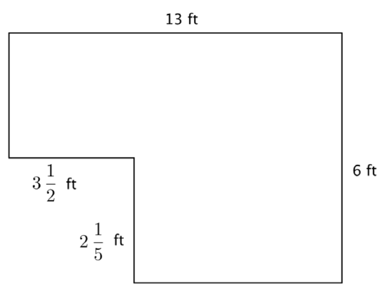
ACT Math: Problem Set

1. A polygon has the dimensions below. All angles are right angles, and all the dimensions given are in inches. What is the area, in square inches, of the polygon?



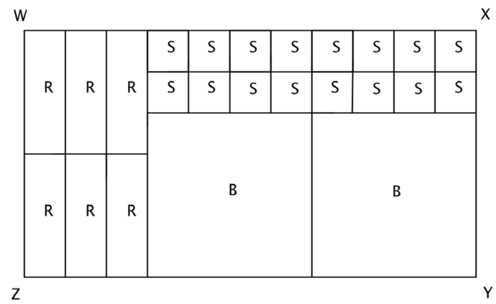
A. 86
B. 295
C. 355
D. 370
E. 460

2. A polygon has the dimensions below. All angles are right angles. What is the area, in square feet, of the figure below?



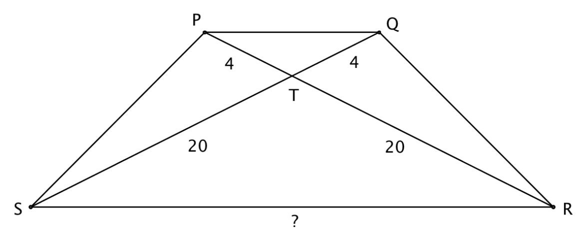
A. 38
B. $70\dfrac{3}{10}$
C. $71\dfrac{9}{10}$
D. $72\dfrac{9}{10}$
E. 78

3. As shown below, rectangle $WXYZ$ is divided into 2 big squares (labeled B) each x inches on a side, 16 small squares (labeled S) each y inches on a side, and 6 rectangles (labeled R) each x inches by y inches. What is the total area, in square inches of $WXYZ$?



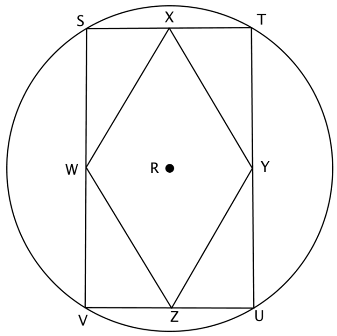
A. $2x+6xy+16y$
B. $4x^2+16y^2$
C. $2x^2+34y^2$
D. $2x^2+6xy+16y^2$
E. $2x^2+6x^2y^2+16y^2$

4. In the isosceles trapezoid $PQRS$ below, T is the intersection of the diagonals. What is the length of $\overline{SR}$ in terms of $\overline{PQ}$?



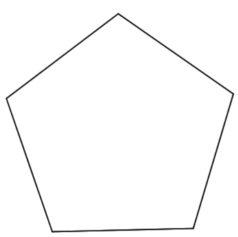
A. $5PQ$
B. $10PQ$
C. $16PQ$
D. $20PQ$
E. $24PQ$

5. In the figure below, the vertices of rectangle $STUV$ lie on the circle and the vertices of rhombus $WXYZ$ are the midpoints of the sides of rectangle $STUV$. Point R is at the center of the circle, rectangle, and rhombus. The radius of the circle is 15 units. What is the perimeter of the rhombus, in units?



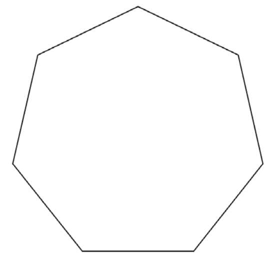
A. 15
B. 30
C. 40
D. 60
E. 80

6. What is the maximum number of distinct diagonals that can be drawn in the pentagram shown below?



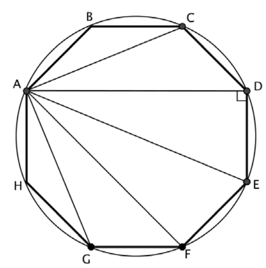
A. 2
B. 5
C. 7
D. 10
E. 15

7. The heptagon shown below has 7 sides of equal length. What is the sum of the measures of the interior angles in the heptagon?



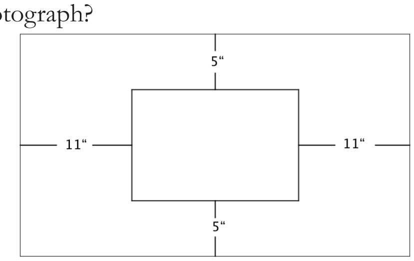
A. 360°
B. 540°
C. 720°
D. 900°
E. 1080°

8. Inscribed in the circle below is a regular octagon $ABCDEFGH$ with some diagonals shown. What is the measure of $\angle DAE$?



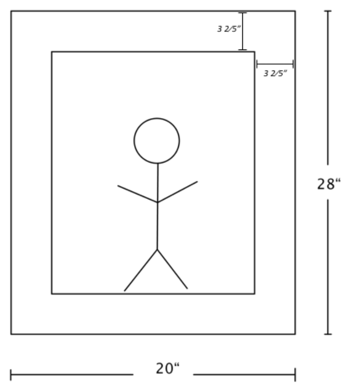
A. 11.25°
B. 15°
C. 17.5°
D. 22.5°
E. 25°

9. The area of a rectangular card is 544 square inches. A rectangular photograph that is twice as wide as it is tall is glued to the card as shown below so that there are 5 inches of clearance above and below it and 11 inches of clearance to either side. How many inches tall is the photograph?



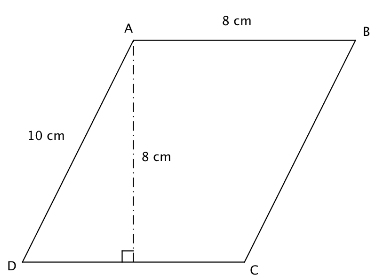
A. 5
B. 6
C. 8
D. 12
E. 27

10. Depicted here is a painting in its frame. The frame has a uniform width of $3\frac{2}{5}$ inches. What is the area, in square inches, of the visible part of the painting?



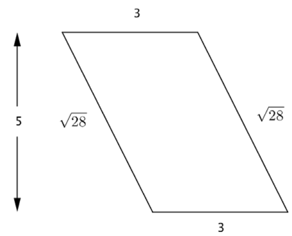
A. 233.6
B. 279.84
C. 326.08
D. 408.36
E. 560

11. What is the area, in square centimeters, of the parallelogram below?



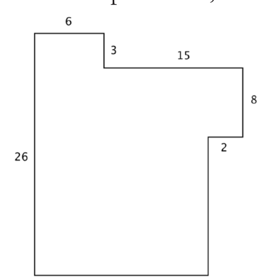
A. 60
B. 64
C. 76
D. 80
E. 104

12. In the parallelogram below, lengths are given in centimeters. What is the area of the parallelogram, in square centimeters?



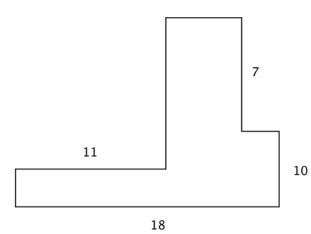
A. 13.1
B. 15
C. 16
D. 16.3
E. Cannot be determined.

13. In the figure below, each pair of intersecting line segments meets at a right angle, and all the lengths are given in inches. What is the perimeter, in inches, of the figure?



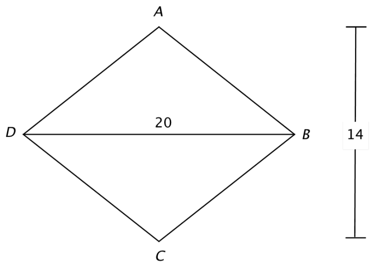
A. 60
B. 81
C. 94
D. 99
E. 107

14. All line segments in the figure below are horizontal or vertical, and units are given in inches. What is the perimeter, in inches, of the figure below?



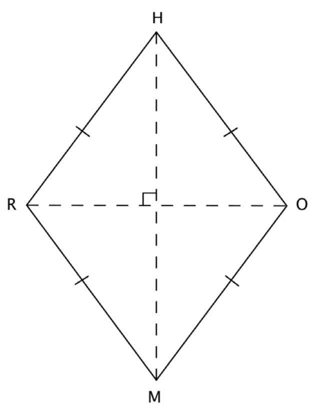
A. 46
B. 53
C. 63
D. 70
E. 77

15. In rhombus $ABCD$, shown below, $\overline{BD}$ is 20 inches long and $\overline{AC}$ is 14 inches long. What is the area, in square inches, of the rhombus?



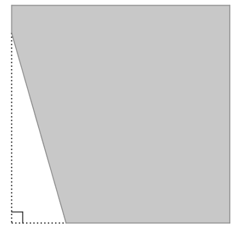
A. $2\sqrt{149}$
B. $4\sqrt{149}$
C. 70
D. 140
E. 280

16. The area of rhombus $RHOM$ is 230 square inches. If RO =5 inches, what is the length of HM in inches?



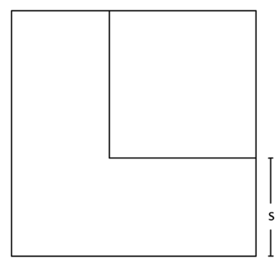
A. 18.4
B. 23
C. 25
D. 46
E. 92

17. A right triangle, with legs of length 5 and 12 inches, was cut off of a square with 13 inch sides, resulting in the shaded portion of the figure below. What is the approximate perimeter, in inches, of the shaded portion?



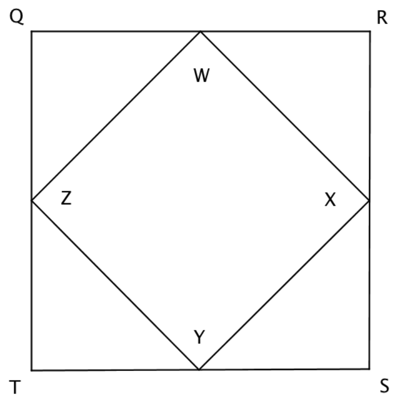
A. 39
B. 43
C. 47
D. 48
E. 52

18. In the figure below, the area of the larger square is 162 square centimeters, and the area of the smaller square is 72 square centimeters. What is s, in centimeters?



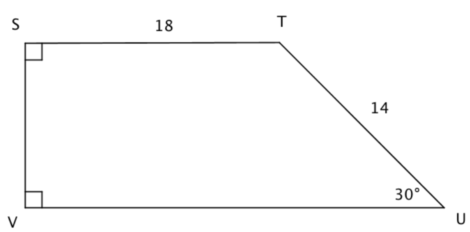
A. 3
B. $3\sqrt{2}$
C. $3\sqrt{10}$
D. $6\sqrt{2}$
E. 45

19. In the figure below, $QRST$ is a square and W, X, Y, and Z are the midpoints of the square's sides. If $RS = 16$ inches, what is the perimeter of $WXYZ$, in inches?



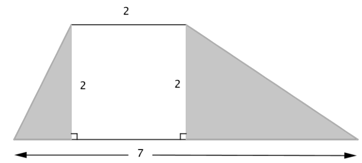
A. 16
B. $16\sqrt{2}$
C. 32
D. $32\sqrt{2}$
E. $64\sqrt{2}$

20. In trapezoid $STUV$ below, $\angle U$ is $30°$ and $\angle S = \angle V = 90°$. Side lengths are given in meters. Which of the following values is closest to the area, in square meters, of $STUV$?



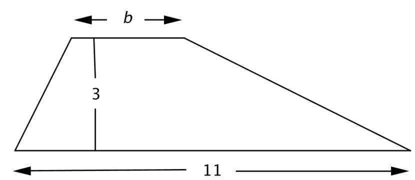
A. 83.6
B. 103.4
C. 126
D. 150.5
E. 168.5

21. The trapezoid below is formed by two shaded triangles and an unshaded square. Lengths are given in feet. What is the combined area, in square feet, of the two shaded triangles?



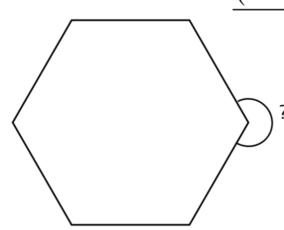
A. 4
B. 5
C. 6
D. 7
E. 10

22. The area of the trapezoid below is 24 square inches. The altitude is 3 inches. One base measures 11 inches. What is the length of the other base, b, in inches?



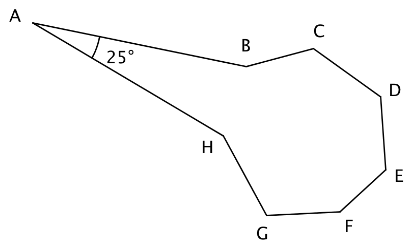
A. 4
B. 5
C. 6
D. 7
E. 8

23. A regular hexagon is shown below. What is the measure of thedesignated angle? (The measure of each interior angle of a regular *n*-sided polygon is $\frac{(n-2)180°}{n}$).



A. 120°
B. 270°
C. 225°
D. 240°
E. 250°

24. In octagon $ABCDEFGH$, shown below, $\angle A$ measures $25°$. What is the total measure of all 8 interior angles?



A. $360°$
B. $135°$
C. $1260°$
D. $1440°$
E. $1080°$

25. If the area of a rectangle that is twice as wide as it is long is 128 square feet, which of the following could be the perimeter of the rectangle?

A. 8
B. 16
C. 48
D. 128
E. 256

26. A rectangle has a diagonal of 13 units and a side of 5 units. What is the perimeter, in units, of the rectangle?

A. 34
B. 36
C. 50
D. 20
E. 48

27. A rectangular animal pen is 3 times a wide as it is long. If the area of the pen is 867 square meters, what is the perimeter of the pen?

A. 51 meters
B. 117.6 meters
C. 136 meters
D. 140 meters
E. 143.6 meters

28. The area of a large wall tapestry is 126 square feet. The length of the tapestry is 4 more than twice the width. What is the perimeter of the tapestry?

A. 252 feet
B. 73 feet
C. 62 feet
D. 56.1 feet
E. 50 feet

29. George is putting tile flooring in his living room. The living room is rectangular and measures 10 feet by 25 feet. The tiles measure 6 inches by 10 inches. What is the minimum number of tiles that George will need?

A. 417
B. 500
C. 570
D. 600
E. 630

30. A rectangular poster has a width that is 5 centimeters longer than its length and an area of 500 square centimeters. What is the length of the poster in centimeters?

A. 17.5
B. 20
C. 25
D. 40
E. 100

31. A parallelogram has a perimeter of 68 centimeters and a side that measure 22 centimeters. If it can be determined, what are the lengths of the other 3 sides?

A. 22. 3.1, 3.1
B. 22, 12, 12
C. 22, 14.5, 14,5
D. 22, 15, 15
E. Cannot be determined from the given information

32. Jimmy has 240 feet of fence to build an enclosure around his garden. If the enclosure is to be rectangular with a length 20 feet longer than its width, what will be the approximate dimensions of the enclosure?

A. 50 feet long, 30 feet wide
B. 50 feet long, 70 feet wide
C. 70 feet long, 50 feet wide
D. 75 feet long, 55 feet wide
E. 90 feet long, 70 feet wide

33. A rhombus has sides that measure 9 inches and two of its interior angles measure $120°$. How many square inches is the area of the rhombus?

B. $\frac{81}{4}$
C. $\frac{81}{2}$
D. $\frac{81\sqrt{3}}{2}$
E. 81

E. $81\sqrt{3}$

34. What is the area, in square meters, of a trapezoid with parallel bases of 10 feet and 13 feet and a height of 6 feet?

A. 60
B. 69
C. 72
D. 78
E. 138

35. A trapezoid has an area of 77 square inches and parallel bases that measure 9 inches and 5 inches. What is the height in inches of the trapezoid?

A. 7
B. 8.6
C. 11
D. 15.4
E. 16

36. Lorenzo has a lawn in his backyard that forms a rectangle that is 25 meters wide and 30 meters long. Around the entirety of the lawn is a walkway with a width of 3 meters. What is the total area of the lawn and the walkway in square meters?

A. 750
B. 915
C. 924
D. 1008
E. 1116

37. A rectangular wall that measures 20 feet by 34 feet is going to be painted. In the middle of the wall, there is a circular mirror with a diameter of 8 feet. If the wall will not be painted under the mirror, what is the approximate area of the wall that will be painted in square feet?

A. 479
B. 590
C. 615
D. 630
E. 655

38. A quadrilateral has sides that are all n inches long. Which of the following cannot be this quadrilateral?

A. Rhombus
B. Square
C. Trapezoid
D. Parallelogram
E. Rectangle

39. All of the following quadrilaterals have diagonals that are not always congruent except:

A. Rhombus
B. Rectangle
C. Parallelogram
D. Trapezoid
E. Kite

ANSWERS

1. C	**2.** B	**3.** D	**4.** A	**5.** D	**6.** B	**7.** E	**8.** D	**9.** B	**10.** B	**11.** B	**12.** B	**13.** C	**14.** D
15. D	**16.** D	**17.** D	**18.** B	**19.** D	**20.** E	**21.** B	**22.** B	**23.** D	**24.** E	**25.** B	**26.** A	**27.** C	**28.** E
29. D	**30.** B	**31.** B	**32.** C	**33.** C	**34.** B	**35.** C	**36.** E	**37.** D	**38.** C	**39.** B			

ANSWER EXPLANATIONS

1. **C.** We divide the polygon into 3 rectangles by drawing a line down from the right end of the 13 unit segment and another down from the left end of the 5 unit segment. The leftmost rectangle has a width of 13 and a height of 15, giving it an area of 195. The rightmost rectangle has a width of 5 and a height of 20, giving it an area of 100. The center rectangle is the width of the total width of the polygon, 23, minus the other horizontal line segments, 13 and 5: $23-(13+5)=5$. We can find the height of the center rectangle by starting with 20 from the segment to the right and subtracting 8 due to the segment that stretches from the top of the tallest part to the top of the center rectangle: $20-8=12$. The area of the center rectangle is $12*5=60$. Summing the areas of the rectangles gives us $195+100+60=355$.
2. **B.** In contrast to the answer to question 1, we can also find the area by subtracting the area of what is cut off from a larger polygon. In this case, we can subtract the $3\frac{1}{2}$ by $2\frac{1}{5}$ feet rectangle from the 13 by 6 feet rectangle: $(13*6)-\left(3\frac{1}{2}*2\frac{1}{5}\right)=78-7.7=70.3$ or $70\frac{3}{10}$.
3. **D.** The area of each of the big squares is its side length squared: x^2. The area of each of the small squares is its respective side length square: y^2. The area of each of the rectangles is x times y: xy. There are 2 big squares, 6 rectangles, and 16 small squares, so to get their total area we multiply their respective areas by the number of shapes to get $2x^2+6xy+16y^2$.
4. **A.** $\triangle PQT$ and $\triangle SRT$ are similar. (This can be demonstrated with the vertical angles theorem and parallel lines theorems.) Thus, the ratio of PQ to SR is equal to the ratio of PT to ST: $\frac{PQ}{SR}=\frac{4}{20}$. This simplifies to $\frac{PQ}{SR}=\frac{1}{5}$. Cross multiply to get $5PQ=SR$.
5. **D.** Draw a line from X to Z and from W to Y, dividing the rectangle into 4 smaller rectangles. We can see that each of the sides of the rhombus is a diagonal of one of the smaller rectangles. The diagonals of a rectangle are congruent. If we draw the other matching diagonals that are not pictured but are nevertheless congruent, we see that they stretch from the center to points on the circle, and thus equal the radius, 15. Since the 4 sides of the rhombus are congruent to the 4 diagonals, the 4 sides of the rhombus added together, the perimeter, are equal to 4 times the radius. Therefore the perimeter of the rhombus equals $4\cdot 15=60$.
6. **B.** All of the distinct diagonals in the pentagon form a pentagram, a 5-pointed star. It is made up of 5 lines, thus 5 distinct diagonals. The general equation for the number of distinct diagonals in a polygon is $\frac{n(n-3)}{2}$.
7. **D.** The sum of the measures of the interior angles in a polygon equals $(n-2)*180°$ where n is the number of sides of the polygon. For a heptagon, that's $(7-2)*180°=5*180°=900°$
8. **D.** We're going to find the measure of $\angle DAE$ by finding the measure of $\angle DEA$ and using the Triangular Sum Theorem (which states that the measure of the interior angles of a triangle sums to $180°$). We know that the total sum of the interior angles of any octagon is $(8-2)\cdot 180°=1080°$, and that if all the angles are congruent (as they are in a regular octagon), each interior angle equals $\frac{1080°}{8}=135°$. Now we proceed by cutting the octagon half along $\overline{AE}$. In cutting the octagon in half, the angle $\angle BAH$ and $\angle DEF$ are both bisected, so the resultant angles created are congruent: $\angle BAE\cong\angle DEA$. $\angle DEF$ originally equaled $135°$, so $\angle DEA=\frac{135°}{2}=67.5°$. $\angle ADE$ is a right angle, sby substitution $\angle DAE+90°+67.5°=180°$. Simplifying, $\angle DAE=22.5°$.
9. **B.** Let x be the height of the photograph. Consequently, the width of the photograph can be expressed as $2x$. The height of the card is $x+2(5)=x+10$. The width of the card is $2x+2(11)=2x+22$. The area of the card is the product of the width and height: $(x+10)(2x+22)=544$. Multiply the binomials to get $2x^2+42x+220=544$. This simplifies to $x^2+21x-162=0$. Factoring this, we get $(x-6)(x+27)=0$. We solve to find that the positive root is $x=6$. We must use the positive root because it is the measure of a line segment, which must have a positive length. Thus, the photograph is 6 inches.
10. **B.** The height of the painting is the height of the frame minus two times the width of the frame, since the frame runs along the top and the bottom: $28-2\left(3\frac{2}{5}\right)=21\frac{1}{5}$. The width of the painting is found by applying the same concept to the

width of the rectangle formed by the frame: $20-2\left(3\frac{2}{5}\right)=13\frac{1}{5}$. The area of the visible portion of the painting is thus $21\frac{1}{5}\cdot 13\frac{1}{5}=279.84$.

11. **B.** The area of a parallelogram is the base times the height. Here, the base is 8 cm and the height is 8 cm, so the area is $8\cdot 8=64$.
12. **B.** The area of a parallelogram is the base times the height. Here, the base is 3 cm and the height is 5 cm, as indicated on the left, so the area is $3\cdot 5=15$.
13. **C.** The perimeter of a polygon like this is equal to the 2 times the total horizontal length plus 2 times the total vertical length. It is easier to solve using this method instead of finding the length of every line segment and adding them together. The total vertical length is 26, seen in the long vertical line segment on the left. The total horizontal length is 21, seen in the 6 and 15 inch long segments on the top that together span the entire horizontal length (or that segments of 19 and 2 along the bottom). Thus, the perimeter is $2(26)+2(21)=94$.
14. **D.** Like the solution to #13, we will find the total horizontal and vertical lengths of the polygon to find the perimeter. The total horizontal length is 18 inches, from the horizontal line segment that spans the entire horizontal length along the bottom. The total vertical length is 17, from the 7 and 10 inch segments that span the entire vertical length on the right. The perimeter of the figure is thus $2(18)+2(17)=70$.
15. **D.** All rhombuses are kites, and the area of a kite is equal to half the product of its diagonals. Thus, the area of the rhombus is $\frac{20\cdot 14}{2}=\frac{280}{2}=140$.
16. **E.** The area of a rhombus, like any kite, is equal to half the product of its diagonals. We set up our equation as $\frac{5\cdot HM}{2}=230$. Simplifying, $HM=92$.
17. **D.** We can find two of the five sides of the shaded portion easily as they are the sides of the square, so they are both 13 inches. Another 2 sides can be found as the sides of the square minus the legs of the triangle that are cut off: 13 minus 5 and 12, respectively, giving sides of lengths 8 and 1. The final side is the hypotenuse of the right triangle, which we find, using the Pythagorean theorem, is 13(note: it could also be found by remembering that 5, 12, and 13 is a Pythagorean triple). The total perimeter of the shaded portion is $13+13+8+1+13=48$.
18. **B.** If the area of the smaller square is equal to the 72, then the side length of the smaller square is $\sqrt{72}=6\sqrt{2}$. Since the area of the larger square is equal to 162, the side length of the larger square is $\sqrt{162}=9\sqrt{2}$. s is the difference between the larger side and the smaller side: $9\sqrt{2}-6\sqrt{2}=3\sqrt{2}$.
19. **D.** Each of the sides of the interior square are the hypotenuse of an isosceles right triangle with legs that are each equal to half of the side length of the larger square. The leg of each 45-45-90 triangle formed is 8(since the legs are bisected segments of the outer square's length). Using our knowledge of the ratio of sides of a 45-45-90 triangle, we can conclude that the hypotenuse of our triangles, which is the side of the smaller square is $8\sqrt{2}$. Since there are 4 hypotenuses/interior square sides, the total perimeter of the smaller square is $32\sqrt{2}$.
20. **E.** Drawing an altitude down from T, we divide the figure into a rectangle and a 30-60-90 triangle. We can find the height of both using the ratio of sides of a 30-60-90 triangle. Since a segment of length 14 is directly across from the $90°$ angle, the side directly across from the $30°$ must be 7, and the base of the triangle (across from the $60°$ angle) is $7\sqrt{3}$. Thus, the area of the triangle is $\frac{1}{2}\cdot 7\cdot 7\sqrt{3}\approx 42.4$. We know that the area of the rectangle is $7\cdot 18=126$, so adding the areas together, the area of the entire figure is $126+42.4=168.4$, which is closest to 168.5.
21. **B.** The combined area of the shaded triangles is equal to the total area of the trapezoid minus the area of the unshaded square. The area of the trapezoid is $\frac{2+7}{2}\cdot 2=9$. The area of the unshaded square is $2\cdot 2=4$. Thus the area of the shaded triangles is $9-4=5$.
22. **B.** We solve this problem by plugging the values given into the equation for the area of a trapezoid and solving for b: $\frac{b+11}{2}\cdot 3=24$. This becomes $b+11=16$. Thus $b=5$.
23. **D.** We are given the formula for finding the interior angle of a regular n-sided polygon, so we plug in 6 for a hexa-

gon: $\frac{(6-2)180°}{6} = 120°$. The exterior angle and the interior angle must sum to $360°$, so the exterior angle is $360° - 120° = 240°$.

24. **E.** The sum of the measures of the interior angles of any polygon is only dependent on the number of sides. Because this is an octagon, the sum of the measures of the interior angles is $(8-2)180° = 1080°$
25. **C.** Let n be the length of the rectangle. $2n$ is the width. The area given is equal to the product of the length and the width: $2n \cdot n = 128$. This simplifies to $n^2 = 64$, so $n = 8$. The perimeter is $2(2 \cdot 8) + 2(8) = 48$.
26. **A.** The diagonal of the rectangle is the hypotenuse of a right triangle, where the side is one of its legs. The other leg, and so the other side, can be found as $\sqrt{13^2 - 5^2} = 12$ (note: it could also be found by remembering that 5, 12, and 13 is a Pythagorean triple). The perimeter is thus $2(5) + 2(12) = 34$.
27. **C.** Let l be the length of the pen. The width is $3l$. The relationship between l and the area is $l * 3l = 867$. From this, we can isolate l to get $l = 17$. The perimeter is thus $2(17) + 2(3 \cdot 17) = 136$.
28. **E.** Let w be the width of the tapestry. Per the description, the length of the tapestry is $2w + 4$. The relationship to the area is $w(2w+4) = 126$. We distribute to get $2w^2 + 4w = 126$, which becomes $2w^2 + 4w - 126 = 0$. We simplify to $w^2 + 2w - 63 = 0$. Factoring gives us $(w-7)(w+9) = 0$. We take the positive root, $w = 7$, since w is a real worth length and thus must be positive. The perimeter of the tapestry is thus $2w + 2(2w+4) = 2(7) + 2(2 \cdot 7 + 4) = 14 + 36 = 50$.
29. **D.** First, we convert our measurements into the same units. To avoid working in fractions, let us convert dimensions given in feet into inches. The living room measures 10 feet by 25 feet, which is 120 inches by 3000 inches. The total area, in inches square, is $120 \cdot 300 = 36000$. An individual tiles covers $6 \cdot 10 = 60$ square inches. Thus, we need at least $\frac{36000 in^2}{60 in^2} = 600$ tiles.
30. **B.** Let l be the length of the poster. The width is $l + 5$. The area is $l(l+5) = 500$. We distribute to get $l^2 + 5l = 500$, which rearranged becomes $l^2 + 5l - 500 = 0$. Factor and we find that $(l-20)(1+25) = 0,$ meaning $l = 20$ or $l = -25$. l cannot be negative, so l must be 20.
31. **B.** We know that one of the other sides must be 22, since the opposite sides of a parallelogram are equal. The other 2 sides plus the sides with length 22 must sum to 68. We express this as $2x + 44 = 68$, where x is the length of the other sides. Solving, we find that $x = 12$. Thus, 2 of the other sides have length 12. The 3 sides we are not given measure 22, 12, and 12, respectively.
32. **C.** Let w be the width of the enclosure. The length is $w + 20$. The total perimeter is 240, so $2(w) + 2(w+20) = 240$. This becomes $4w + 40 = 240$. Solving, we determine that $w = 50$, and so $l = 70$. The dimensions are 50 feet wide and 70 feet long.
33. **C.** Because two of the interior angles are $120°$, the other two must each be $60°$ (the sume of all angles in a quadrilateral is $360°$ and rhombuses have pairs of congruent angles). If we sketch the rhombus and draw an altitude down from one of the $120°$ vertices, we see that it forms a right triangle with one angle that is equal to $60°$. The $120°$ angle that was split by drawing the altitude now must be $30°$ within the circle, while forming a right angle on the other side. Since we now have a 30-60-90 triangle, we can use our knowledge of the ratios of the sides of a 30-60-90 triangle to find the height of the rhombus, which is the altitude we drew. The 9 inch segment is directly across from the $90°$ angle, so the height, the segment across from the $60°$ angle, is $4.5\sqrt{3}$. The area of the rhombus is $9 \cdot 4.5\sqrt{3}$, or $9 \cdot \frac{9}{2}\sqrt{3} = \frac{81\sqrt{3}}{2}$.
34. **B.** The area of a trapezoid is half the sum of the bases times the height: $\frac{10+13}{2} \cdot 6 = 69$.
35. **C.** We set up our relationship between the height of the trapezoid and the area as $\frac{9+5}{2}h = 77$. By isolating h we get $h = \frac{154}{14} = 11$.

36. **E.** To get the length and width of the combined lawn and walkway, we add 2 times the width of the walkway to the length and width of the lawn, respectively. The length is $30+2(3)=36$ m long. The width is $25+2(3)=31$ m. The area of the combined lawn and walkway is $36 \cdot 31 = 1116$ m^2.
37. **D.** The area that is to be painted is equal to the total area of the wall minus the area of the mirror: $20 \cdot 34 - \pi(4)^2$ (note that the radius is 4, the question only indicates the diameter). This is equal to $680 - 16\pi \approx 630$.
38. **C.** The quadrilateral cannot be a trapezoid because trapezoids by definition have 2 sides that are of unequal length.
39. **B.** The diagonals of a rectangle are already congruent. There are many proofs of this; one is that the diagonals are hypotenuses of right triangles with congruent legs, and so must themselves be congruent.