The Best ACT Prep Course Ever

DIRECT AND INVERSE VARIATION

ACT Math: Problem Set

1. Which of the following expressions correctly translates the statement: a varies inversely as the quotient of b and c^2, and directly as d^3?

A. $a=\frac{kd^3c^2}{b}$

B. $a=\frac{kb}{d^3c^2}$

C. $a=\frac{kd^3}{bc^2}$

D. $a=\frac{kd^3b}{c^2}$

E. $a=\frac{k}{bc^2d^3}$

2. Which of the following equations correctly shows the statement: p varies directly as the square root of q and inversely as the product of r and s?

A. $p=\frac{kr\sqrt{q}}{s}$

B. $p=\frac{krs}{\sqrt{q}}$

C. $p=\frac{ks\sqrt{q}}{r}$

D. $p=\frac{k\sqrt{q}}{rs}$

E. $p=krs\sqrt{q}$

3. In physics, Power is the amount of work done divided by the time it takes, or the rate of work. Power (P) varies directly as the product of Force (F) and Distance (D) and inversely as Time (T). Which of the following expressions correctly shows the formula for Power?

A. $P=\frac{kFT}{D}$

B. $P=\frac{kFD}{T}$

C. $P=\frac{kDT}{F}$

D. $P=\frac{kT}{FD}$

E. $P=kFDT$

4. A certain recipe produces 7 cups of tea and requires $2\frac{1}{4}$ cups of water. If all ingredients are increased proportionally, how many cups of water are required for the recipe to yield $45\frac{1}{2}$ cups of tea?

A. $5\frac{3}{8}$

B. 8

C. $8\frac{1}{2}$

D. $14\frac{5}{8}$

E. $20\frac{1}{2}$

5. Given that $x=5$ when $y=16$ for the proportion $\frac{2}{x}=\frac{y}{k}$, what is y when $x=3$?

A. $\frac{3}{40}$

B. $\frac{1}{4}$

C. 16

D. $26\frac{2}{3}$

E. $40\frac{1}{5}$

6. At a certain college, the acceptance rate of admitted students varies indirectly with the number of applicants. When $5{,}000$ students applied, the rate was 37%. What is the acceptance rate next year, rounded to the nearest tenth, if $8{,}300$ students decide to apply?

A. 8.3%

B. 18.5%

C. 22.3%

D. 44.6%

E. 61.4%

7. The temperature of a gas T varies directly with the square root of the pressure, P. The current pressure of the gas is 15 atm. What is the pressure required to have double the temperature as it currently has?

A. $\frac{2}{\sqrt{15}}$

B. $\sqrt{15}$

C. $4\sqrt{15}$

D. 30

E. 60

8. The dollars d that Katie earns at a certain company is directly proportional to the hours h that she works and inversely proportional to the cube of vacation days v. With k as the constant of proportionality, which equation correctly represents Katie's earnings?

A. $\frac{k}{hv^3}$

B. $\frac{kh}{v^3}$

C. $\frac{kv^3}{h}$

D. $\frac{hv^3}{k}$

E. khv^3

9. The amount of time t needed to build a company's hotels is directly proportional with the number of bricks b used and inversely proportional to the number of workers w employed. The current hotel building system employs 850 workers and 1275 bricks. Without changing the amount of bricks b used, what is the minimum amount of workers w necessary in order to halve time t?

A. 425

B. 637

C. 666

D. 1700

E. 1912

10. Ethan discovers that his weight on Earth and Mars are directly proportional. Ethan weighs 324 lbs. on Earth and 122.4 lbs. on the planet of Mars. If the constant of proportionality remains the same, and his brother is 98 lbs on Earth, how much does his brother weigh on Mars, to the nearest whole number?

A. 4

B. 19

C. 37

D. 259

E. 405

11. For all $n>0$, what happens to the value of $\frac{n^4-n^2}{n^2}+1$ as n increases?

A. It increases proportionally to n^2

B. It decreases proportionally to n^2

C. It increases proportionally to n^3

D. It decreases proportionally to n^3

E. It remains constant.

12. For the equation $E=\frac{h}{n}$, h is a proportionality constant. When $n=14$, $E=20$. So, if $n=7$, what is the corresponding value of E?

A. 40

B. 0.1

C. 10

D. 0.025

E. 0.25

13. Consider the exponential equation $y=Bx^z$, where B and x are positive real constants and z is a negative real number. The value of y increases as the value of z decreases if and only if which of the following statements about x is true?

A. $-5<x$

B. $0<x$

C. $0\le x\le 1$

D. $0<x<1$

E. $1<x<2$

14. A driving instructor charges 43 per lesson, plus a fee for the use of his car. The charge for the use of the car varies directly with the cube root of the time spent driving the car. If a driving lesson with 64 minutes of driving time costs 55, how much does a lesson with 27 minutes of driving time cost?

A. $52
B. $46
C. $27
D. $58
E. $43

15. Grant has a map of the city. The key states that each 3 inches on the map equals 50 feet in the actual city. Because Grant is 6 feet tall, how long would he be on the map in inches?

A. 0.12
B. 0.36
C. 0.72
D. 18
E. 100

16. If it takes Heather 4 hours to finish painting 3 rooms, how long would it take her to paint 10 rooms, to the nearest minute?

A. 12
B. 13
C. 200
D. 400
E. 800

ANSWER KEY

1. A 2. D 3. B 4. D 5. D 6. C 7. E 8. B 9. D 10. C 11. A 12. A 13. D 14. A

ANSWER EXPLANATIONS

1. A. The quotient of b and c^2 is $\frac{b}{c^2}$. So if a varies inversely with $\frac{b}{c^2}$ and directly with d^3, then $a=\frac{kd^3}{\frac{b}{c^2}}=\frac{kd^3c^2}{b}$.

2. D. The square root of q is $\sqrt{q}$ and the product of r and s is rs. So, if p varies directly as $\sqrt{q}$ and inversely as rs, then $p=\frac{k\sqrt{q}}{rs}$.

3. B. The product of Force (F) and Distance (D) is FD. So, if P varies directly as FD and inversely as T, then $P=\frac{kFD}{T}$.

4. D. Since the tea and water have a directly proportional relationship, $t=kw$. Plug in the first set of numbers given, $7=\frac{9}{4}k$ and solve to find that $k=\frac{28}{9}$. Using the same equation plug in 45.5 for t and solve for the unknown amount of water, w. The equation $\frac{91}{2}=\frac{28}{9}w$ yields $w=\frac{112}{14}$, which is $14\frac{5}{8}$.

5. D. First, plug in the given values for x and y in the equation to solve for k. Once we find that $k=40$, we can use the second set of given variables in order to isolate y. When we solve the equation $\frac{2}{3}=\frac{y}{40}$, we find that $y=26\frac{2}{3}$.

6. C. Because the relationship between the acceptance rate and number of applicants is indirect, we can use the formula $y=\frac{k}{x}$. When we plug in the given values of x and y, $37=\frac{k}{5000}$, we can find that $k=185{,}000$. Then plug in the values of k and the new number of applicants into the same formula to find that the acceptance rate is approximately 22.3%.

7. E. First, we must determine the formula that corresponds to the information. Since this is direct variation, the formula is $T=k\sqrt{P}$. Since we are looking for double the temperature, T does not have to be a precise number, so for our purposes let $T=1$. When we plug in these variables, we find $k=\frac{1}{\sqrt{15}}$. With this value of k and a new equation, let $T=2$, which is the original temperature doubled. Once we plug variables T and k into the equation, $2=\left(\frac{1}{\sqrt{15}}\right)\left(\sqrt{P}\right)$, we can solve for pressure P and find that $P=60$.

8. B. Since the problem tells us hours h is directly proportional to dollars d, we know that $d=kh$. If this is inversely proportional to v^3, then we can find that $d=\frac{kh}{v^3}$.

9. D. We can summarize the given relationships with the equation $t = \frac{kb}{w}$. Since the problem is not concerned with the actual value of time, but rather a change in t, we don't need to plug in any specific values. Halving the time means that we need to get to $\frac{1}{2}t$, which is the same as halving the left side of the equation above. Given that the constant of proportionality, k, and the number of bricks, b, does not change, the number of workers must double in order to make the right side of the equation halve as well. $850 * 2 = 1700$.

10. C. Since the relationship between weight on Earth and weight on Mars are directly proportional, we can set up the equation $324 = 122.\ 4k$ to model the relationship and solve to find that $k \approx 2.647$. Then use this value of k to solve for the weight w of Ethan's brother. Using this equation, $98 = (2.647)(w)$, we find the brother's weight to be 37 lbs. on Mars.

11. A. Simplifying $\frac{(n^4 - n^2)}{n^2} + 1 = \frac{n^4 - n^2}{n^2} + \frac{n^2}{n^2} = \frac{n^4}{n^2} = n^2$. So, as n increases, the expression increases proportionally to n^2.

12. A. Plugging in $n = 14$ and $E = 20$, we get $20 = \frac{h}{14} \rightarrow h = 20(14) = 280$. Now, plugging in $n = 7$ and $h = 280$, we get $E = \frac{280}{7} = 40$.

13. D. The value of y increases while the power of x decreases only when x is between 0 and 1. So, the value of x is within the bounds $0 < x < 1$. Note that x cannot equal 0 or 1 because $0^z = 0$ and $1^z = 1$ for any negative real z.

14. A. If t represents the time spent driving the car, $f =$ the car use fee, and $C =$ the total car for the lesson, then the charge for car use can be represented as $f = C - 43$ and $f = k\sqrt[3]{t}$ where k is a constant. We know that when $t = 64$, $C = 55$. So, $f = C - 43 \rightarrow 55 - 4 \rightarrow 12$. We plug in $f = 12$ and $t = 64$ in the equation $f = k\sqrt[3]{t}$ to solve for the constant k. We get $12 = k(\sqrt[3]{64}) \rightarrow 12 = 4k \rightarrow k = 3$. So, the relationship between f and t is $f = 3\sqrt[3]{t}$. Plugging in $t = 27$, we can solve for the car use cost $f = 3\sqrt[3]{27} \rightarrow 3(3) \rightarrow 9$. So, the total cost for the 27-minute lesson is $C = 43 + f \rightarrow 43 + 9 \rightarrow 52$.