1. The domain of the function $f(a)=\dfrac{1}{3-\dfrac{1}{|a|}}$ includes all real values of a except:

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. $3,0$

D. $-3,0$

E. $0, \frac{1}{3}$, and $-\frac{1}{3}$

2. The y-values of $g(x)$ vary directly with the square of $(x-3)$ for all real numbers. The graph of $y=g(x)$ in the standard Cartesian plane is which of the following?

A. A line

B. A hyperbola

C. An ellipse

D. A parabola

E. A circle

3. Which of the following lists the real values of x that make the expression $\dfrac{5x-1}{x^3-6x^2-55x}$ undefined?

A. 0 only

B. -5 only

C. $-5,11$ only

D. $-11,0,5$ only

E. $-5,0,11$ only

4. In the standard (x,y) coordinate plane, when $w\neq 0$ and $z\neq 0$, the graph of $f(x)=\dfrac{5x^2-w}{x^2+z}$ has a *horizontal* asymptote at:

A. $y=5$

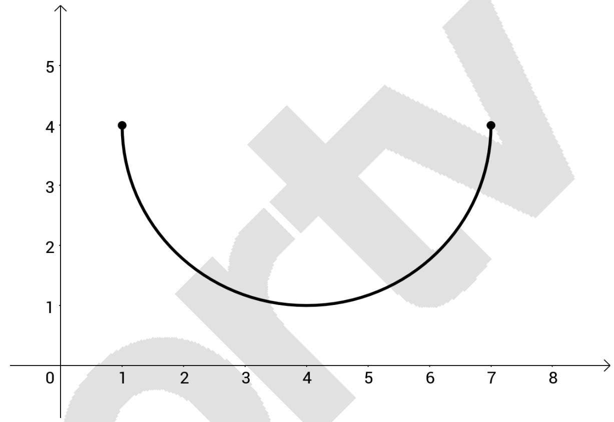
B. $y=\dfrac{-w}{z}$

C. $y=0$

D. $y=z$

E. There is no horizontal asymptote.

5. The domain of a function is the set of all values of x for which $f(x)$ is defined. One of the following sets is the domain for the function graphed below. Which set is it?



A. $\{1,2,3,4,5,6,7\}$

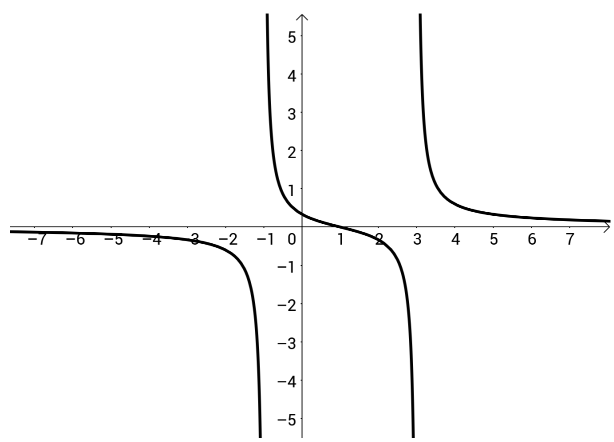
B. $\{1,2,3,4\}$

C. $\{x:1\leq x\leq 7\}$

D. $\{x:1\leq x\leq 4\}$

E. $\{x:1<x<7\}$

6. The equation $\dfrac{(x-1)}{x^2-2x-3}$ is graphed on the standard (x,y) coordinate plane below. No point on the graph has which of the following x-coordinate?



A. -2

B. -1

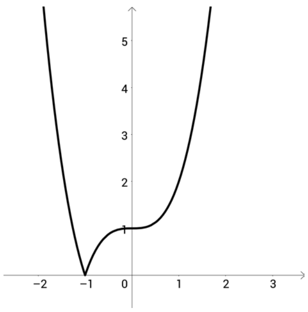
C. 0

D. 1

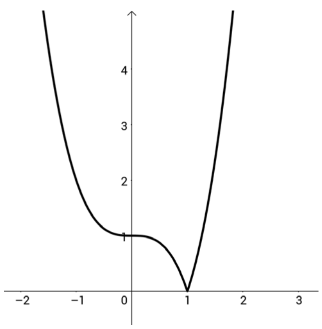
E. 2

7. Consider the functions $f(x)=|x|+2$ and $g(x)=x^3-1$. Which of the following graphs is the graph of $y=f(g(x))$ in the standard coordinate plane?

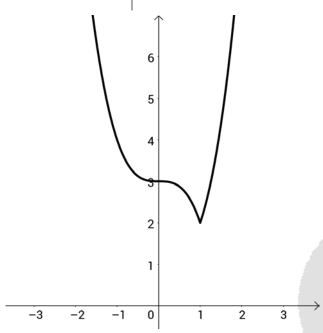
A.



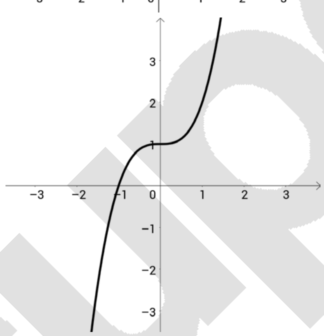
B.



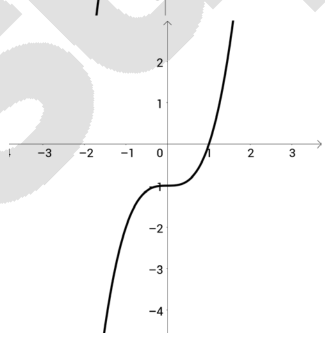
C.



D.

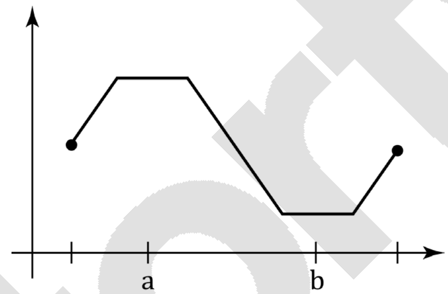


E.

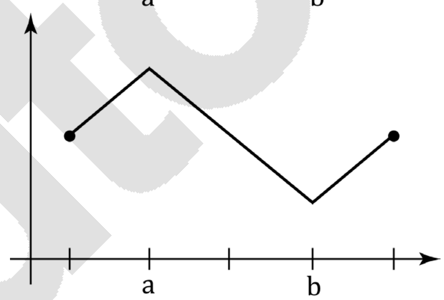


8. A person pedaling a bicycle is pedaling at a constant speed. Let t represent the time that has elapsed since the person started pedaling and let h represent the height above ground of one of the pedals. Below are graphs of one full cycle of the pedals. The pedal is at its maximum height at $t=a$, and is at its minimum height at $t=b$. Which of the following graphs represents the relationship between t and h during this rotation?

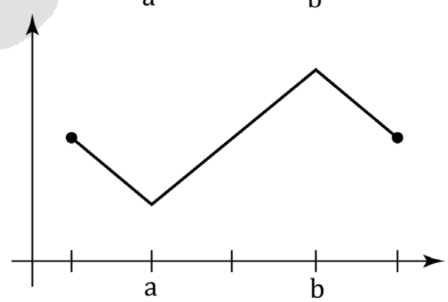
A.



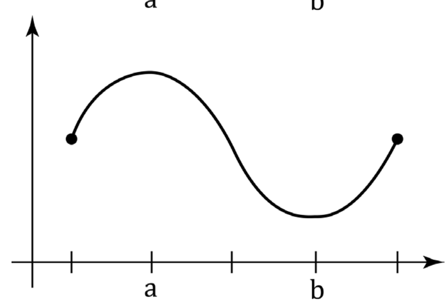
B.



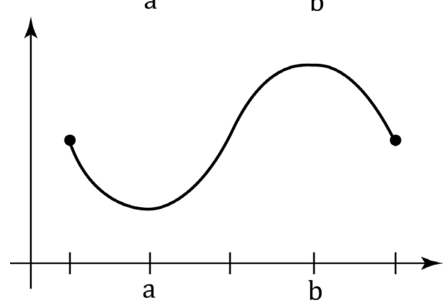
C.



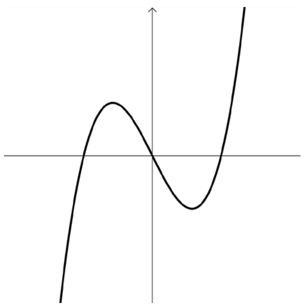
D.

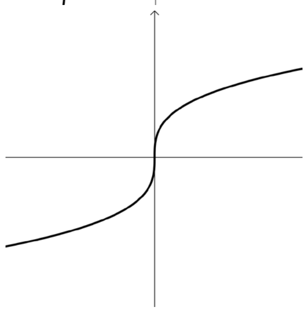


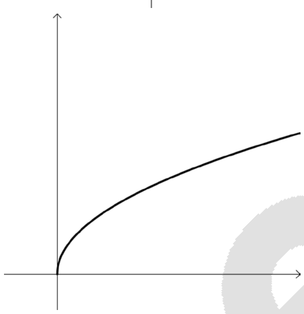
E.

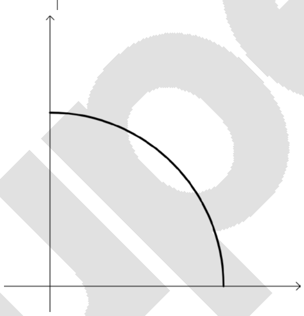


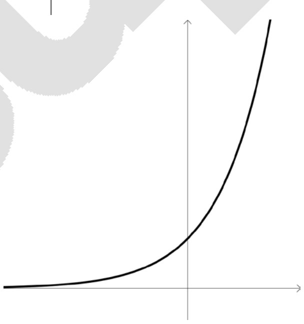
9. A function is a *one-to-one* function if and only if each x in the domain of $f(x)$ corresponds to a unique $f(x)$ and each $f(x)$ corresponds to a unique x. Which of the following graphs is *not* a *one-to-one* function?

A.


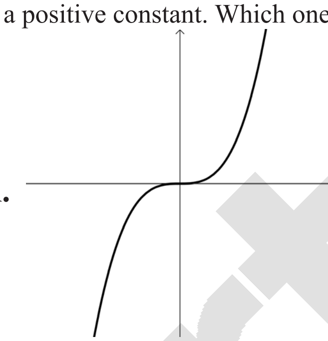
B.


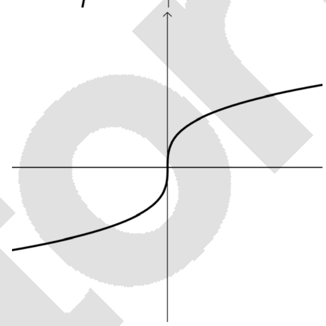
C.


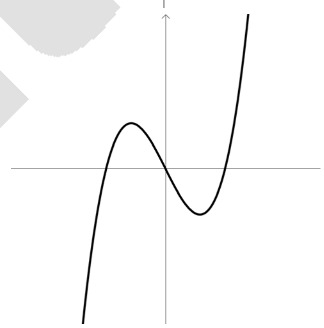
D.


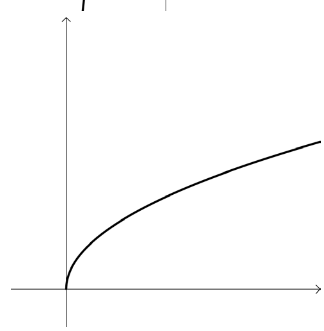
E.


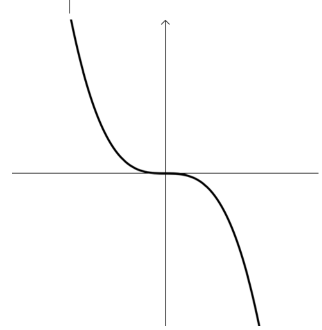
10. One of the graphs below is that of $y = -Kx^3$, where K is a positive constant. Which one?

A.


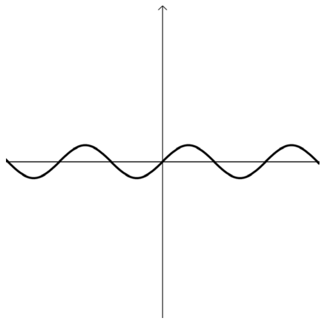
B.


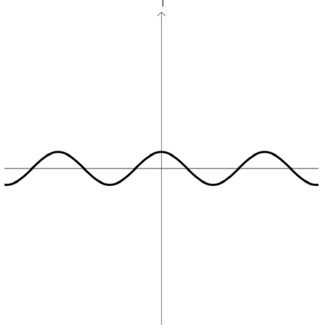
C.


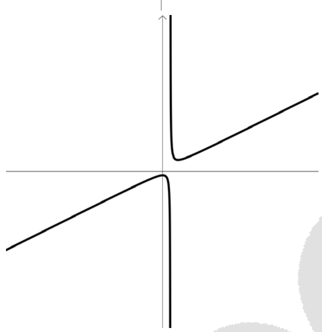
D.


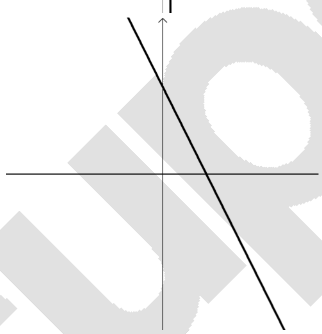
E.


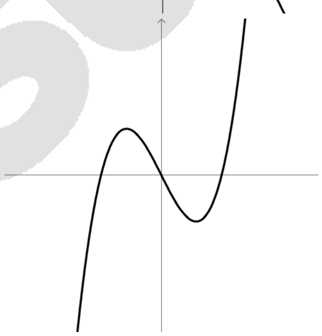
11. A function f is an even function if and only if $f(-x)=f(x)$ for every value of x in the domain of f. One of the functions graphed in the standard (x,y) coordinate plane below is an even function. Which one?

A. 

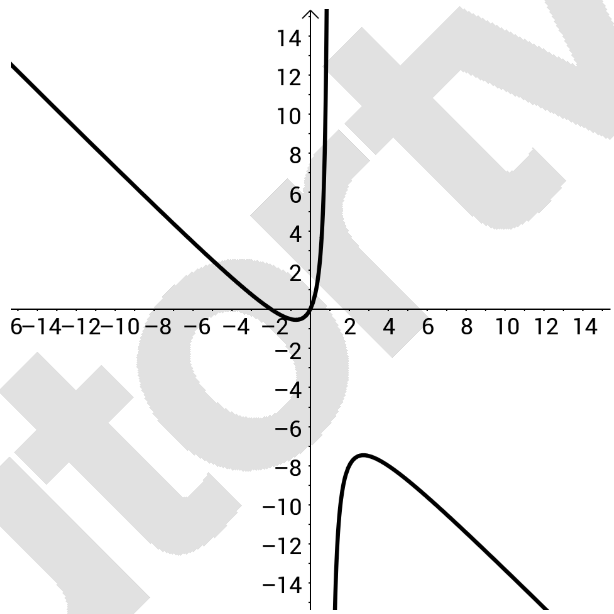
B. 

C. 

D. 

E. 

12. The graph of the function $f(x)=\dfrac{-x^3-3x^2-2x}{x^2-1}$ is shown in the standard (x,y) coordinate plane below. Which of the following, if any, is a list of each of the vertical asymptotes of $f(x)$?

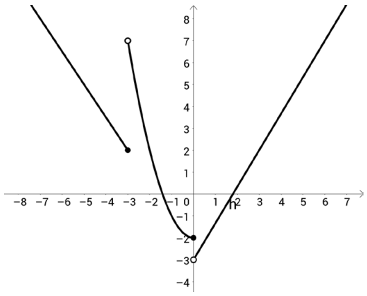


A. $x=-1$ and $x=-2$
B. $y=-x-3$
C. $x=-1$ and $x=1$
D. $x=1$
E. This function has no vertical asymptote.

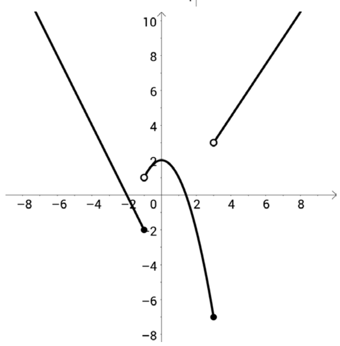
13. Which of the following is the graph of the function $f(x)$ defined below?

$$f(x)=\begin{cases}-x+2 & x\le -1\\ x^2-1 & -1<x\le 3\\ 2x-3 & x>3\end{cases}$$

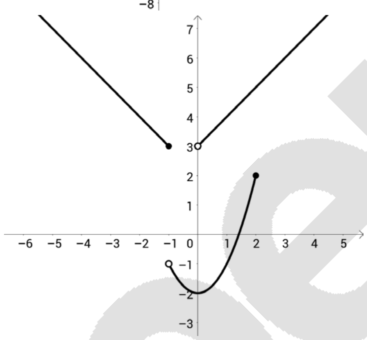
A.



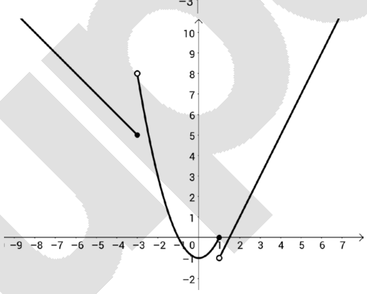
B.



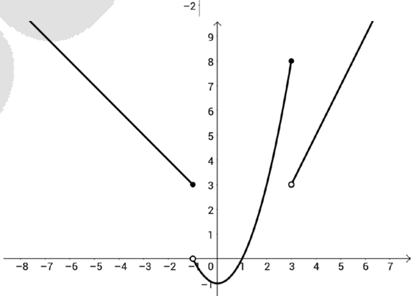
C.



D.



E.

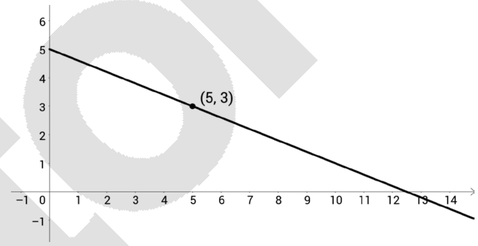


14. At what value(s) of x is $\dfrac{(x+2)(x-1)^2}{x(x+1)}$ undefined?

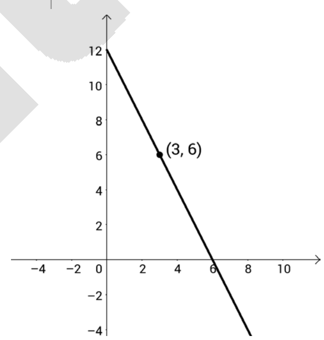
A. 0 and -1
B. 0 and 1
C. -2 and 1
D. -1 only
E. 0 only

15. One of the following graphs in the standard (x,y) coordinate plane represents the equation $5y-20=-2x$ for $x\ge 0$. Which one is it?

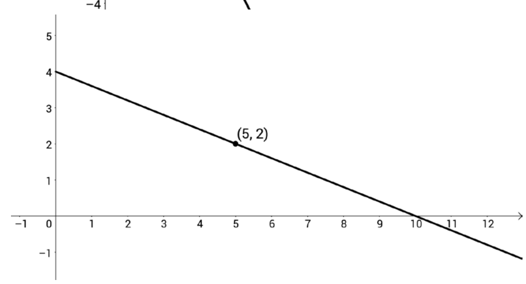
A.



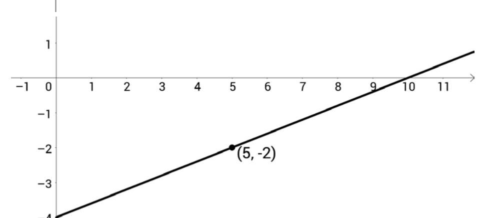
B.



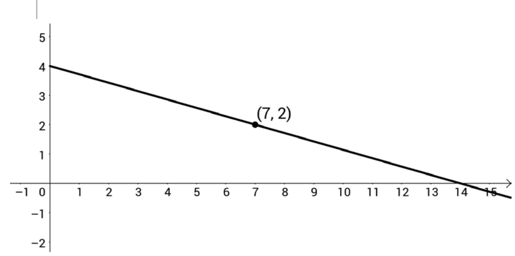
C.



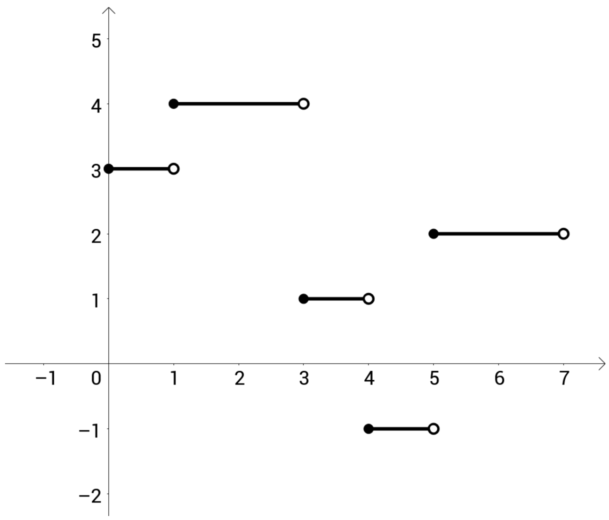
D.



E.



16. The entire graph of $y = f(x)$ in shown in the standard (x, y) coordinate plane below. One of the following sets is the domain of f. Which set?



A. $\{-1,1,2,3,4\}$

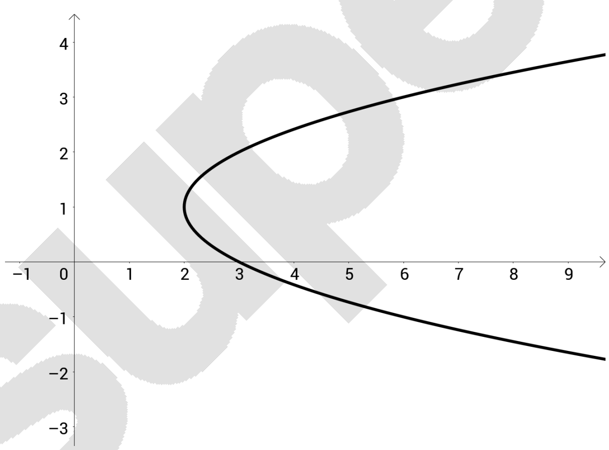
B. $\{0,1,2,3,4,5,6,7\}$

C. $\{-1,0,1,2,3,4,\}$

D. $\{x \mid 0 \le x < 7\}$

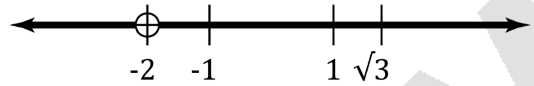
E. $\{x \mid -1 \le x < 4\}$

17. Shown in the standard (x, y) coordinate plane below, the graph $x = (y-1)^2 + 2$ is restricted by one of the following conditions. Which one?



A. $x \le 2$
B. $x \le 1$
C. $x \ge 2$
D. $y \ge 1$
E. $y \ge 2$

18. If $f(x) = \dfrac{5}{x+2}$ and $g(x) = x^2 - 3$, which of the following number lines shows the domain of $f(g(x))$?

A.


B.


C.


D.

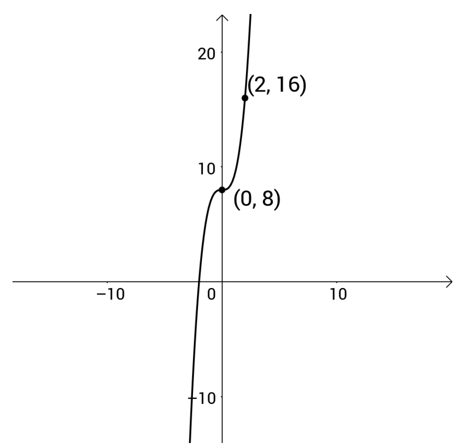

E.


19. For all $0 < n < 1$, which of the following statements describes the function f defined by $f(x) = n^x$?

A. f is constant for all x
B. f is increasing for all $x \ge 0$
C. f is decreasing for all $x \ge 0$
D. f is increasing for $0 \le x < 1$ and decreasing for $x \ge 1$
E. f is decreasing for $0 \le x < 1$ and increasing for $x \ge 1$

For questions 20-22:

The graph of $y = g(x)$ is shown in the standard (x, y) coordinate plane below with two points labeled.



20. What is the x-intercept of the graph of $y = g(x)$?

A. -8
B. -2
C. 0
D. 2
E. 8

21. The function $y = g(x)$ can be classified as one of the following types of functions. Which one is it?

A. Constant
B. Linear
C. Quadratic
D. Cubic
E. Trigonometric

22. Which of the following equations corresponds to the reflection of $y = g(x)$ across the line $y = x$?

A. $y = x^3 - 8$

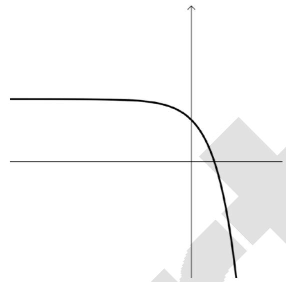
B. $y = (x-8)^3$

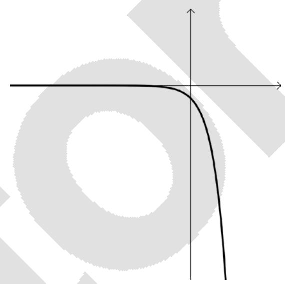
C. $y = \sqrt[3]{x+8}$

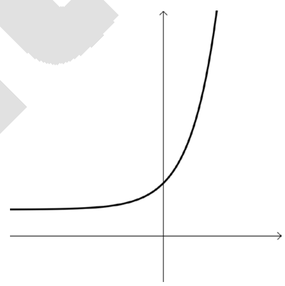
D. $y = \pm\sqrt[3]{x-8}$

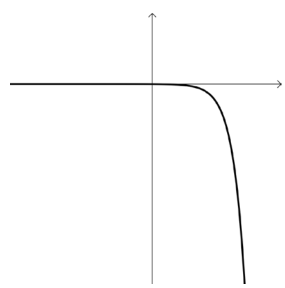
E. $y = \sqrt[3]{x-8}$

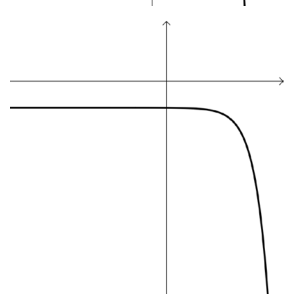
23. One of the graphs below is that of $y = -e^x + K$, where K is a positive constant. Which one?

A.


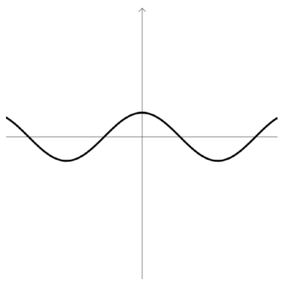
B.


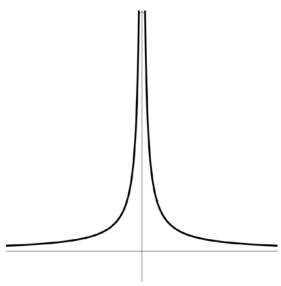
C.


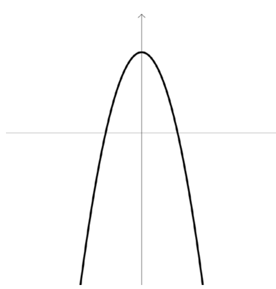
D.


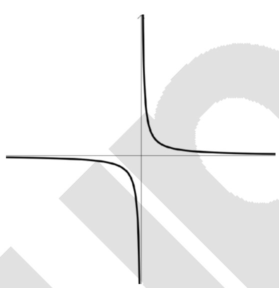
E.


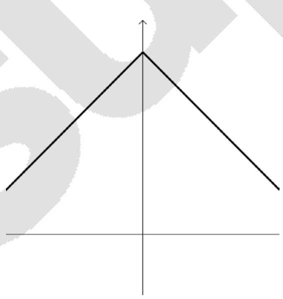
24. A function f is an even function if and only if $f(-x) = f(x)$ for every value of x in the domain of f. Which of the functions graphed in the standard (x, y) coordinate plane below is not even function?

A. 

B. 

C. 

D. 

E. 

ANSWER KEY

1. E **2. D** **3. E** **4. A** **5. C** **6. B** **7. C** **8. D** **9. A** **10. E** **11. B** **12. D** **13. E** **14. A**
15. C **16. D** **17. C** **18. B** **19. C** **20. B** **21. D** **22. E** **23. A** **24. D**

ANSWER EXPLANATIONS

1. **E.** The domain of the function $f(a)=\dfrac{1}{3-\left(\dfrac{1}{|a|}\right)}$ does not include the values of a that make $3-\left(\dfrac{1}{|a|}\right)=0$ or $|a|=0$ because if the denominator of a fraction is 0, the fraction is undefined. Solving this equation for the values of a, we get $3=\dfrac{1}{|a|}$ so $3|a|=1\rightarrow|a|=\dfrac{1}{3}$. This means $a=\pm\dfrac{1}{3}$. Solving $|a|=0$, we get $a=0$. So, the values of a that are not in the domain of the function are 0 and $\pm\dfrac{1}{3}$.

2. **D.** Since $g(x)$ varies directly as the square of $(x-3)$, we can write $g(x)=k(x-3)^2$ for some constant k. This is the expression for a parabola with center at $(3,0)$.

3. **E.** The expression is undefined if the denominator equals zero. So, setting $x^3-6x^2-55x=0$, we wish to find the values of x that make the expression undefined. Factoring out an x, we get $x\left(x^2-6x-55\right)=0$. Since we factored out an x, we know that the value $x=0$ makes the expression undefined. We solve for the rest of the values by using the quadratic equation. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-55)}}{2}=\dfrac{6\pm\sqrt{36+220}}{2}=\dfrac{6\pm\sqrt{256}}{2}=\dfrac{6\pm16}{2}\rightarrow x=-5,11.$ So the values of x that make the denominator equal to zero and the expression undefined are $x=0,-5,11$.

4. **A.** To find the horizontal asymptote, we evaluate x at a very large value and see where the expression approaches to. For very large x, the constants w and z become negligible, and so the expression approaches $f(x)=\dfrac{5x^2}{x^2}=5$. So, the horizontal asymptote is at $y=5$.

5. **C.** Looking at the graph, we can see that the curve starts where $x=1$, and only extends to the point where $x=7$. Thus, the answer is $\{x:1\le x\le 7\}$.

6. **B.** Looking at the graph there appears to be asymptotes at $x=-1$ and $x=3.3$ is not an option, but -1, answer (B), is. We can also simplify the equation to confirm. $\dfrac{(x-1)}{x^2-2x-3}=\dfrac{(x-1)}{(x+1)(x-3)}$. None of the terms cancel. Since when the denominator equals zero, there is a vertical asymptote, the equation tells us that there are asymptotes at $x=-1$ and $x=3$.

7. **C.** $y=f(g(x))=\left|x^3-1\right|+2$. The graph of x^3 in translated one unit down, then when the absolute value is taken, only the parts of the graph that have negative y values are reflected over the x-axis. Finally, this form is translated an additional 2 units up.

8. **D.** This question is more common sense than graphing skills. Imagine the pedal of a bicycle as someone pedals with a constant speed. The pedal follows a circular path as it rises and falls, so the graph should somewhat resemble a circle. That eliminates answers (A), (B), and (C). The question says that the pedal reached a maximum at $t=a$, and of the two left, only (D) has a maximum at $t=a$, so the answer is (D).

9. **A.** The graph is of a cubic function but because it spikes upward, we can see that for a couple values of y, there are multiple potential x's. To prove this, take a pencil, parallel with the x-axis and slide it upward. We see that along one horizontal line, the graph intersects multiple times. The rest of the answers are all *one-to-one* functions because they pass the 'pencil' rule both horizontally and vertically.

10. **E.** The equation $y=-Kx^3$ is a basic cubic function. Because the constant in the equation $y=-Kx^3$ is positive, the negative sign makes the graph be reflected over the x-axis.

11. **B.** Even functions are symmetrical across the y-axis. The only answer given that complies is answer (B), a cosine function.

12. **D.** From the graph we can see that the function appears to approach an asymptote at $x=1$, but we can analyze the equation as well to confirm. Simplify the function by factoring $f\left(x\right)=\dfrac{-x^3-3x^2-2x}{x^2-1}=\dfrac{\left(-x\right)\left(x+1\right)\left(x+2\right)}{\left(x+1\right)\left(x-1\right)}=\dfrac{\left(-x\right)\left(x+1\right)\left(x+2\right)}{\left(x+1\right)\left(x-1\right)}$. Vertical asymptotes occur when the denominator is equal to zero, which in this equation is when $x=1$. At $x=-1$, because the terms cancel, there is only a hole, but not an asymptote.

13. **E.** Answer (E) is the only one that has the jumps at the right places and the correct functions for each segment. For example, answer (B) jumps between parts at $x=-1$ and $x=3$, but the middle part, the parabola piece, is the wrong function (it is facing downward instead of up).

14. **A.** The function is undefined when the denominator is equal to zero, which is only possible when $x=0$ or $x=-1$.

15. **C.** Re-arrange the equation given to a more recognizable form. $5y-20=-2x \rightarrow y=-\dfrac{2}{5}x+4$. The new equation tells us that the graph has a y-intercept at the point $\left(0,4\right)$ and a slope of $-\dfrac{2}{5}$. Plug in 0 for y and solve to find our x-intercept, which is $\left(10,0\right)$. The only graph that contains these two points is answer (C).

16. **D.** This function has a range of integers only, as its pieces are horizontal lines at integer y-value, but the domain concerns horizontal progression, which in this graph is actually continuous from its beginning, where $x=0$, to its end point, where x approaches but does not equal (implied by the open circle) 7. Every time a segment ends, the next segment begins at the same x value just a different y-value.

17. **C.** Looking at the equation alone we know that the y term must at least be 0 if not greater, so when the 2 is added we know that the entire expression, x, must be equal to 2 or greater. Looking at the graph tells us the same time, because it is a sideways facing parabola that opens to the right and from the graph we can see that the vertex is at 2. Thus by either method we determine that $x\leq 2$.

18. **B.** The domain of a composite function is limited by the domain of the inner function, as well as any domain restriction of the new, composite function. The inner function, $g\left(x\right)=x^2-3$, has no domain restrictions, but the composite function does. $f\left(g\left(x\right)\right)=\dfrac{5}{\left(x^2-3\right)+2}=\dfrac{5}{x^2-1}$, so the domain is undefined when $x=\pm 1$, which is what answer B's graph shows.

19. **C.** We know that n will be a fraction less than 1, such as $\dfrac{1}{4}$ or $\dfrac{5}{32}$. These fractions are the same as a greater-than-one number to the power of negative one. For example, $\dfrac{1}{4}=4^{-1}$ and $\dfrac{5}{32}=\left(\dfrac{32}{5}\right)^{-1}$. Thus, the function f is equal to a greater-than-one number to the power of negative one to the power of x or, simplified, to the power of negative x. A negative sign in front of x flips the graph horizontally, so this graph will look like a greater-than-one number to the power of x flipped horizontally. Sketching this, we see that the function approaches zero asymptotically in the positive direction, and grows exponentially in the negative direction. This graph is always decreasing as we move to the right.

20. B. The graph shown is a cubic function. We can determine its exact formula by starting with our basic cubic function $y = x^3$. We then move it up 8 in order to match the graph's y-intercept. We now have the equation $y = x^3 + 8$. This equation matches the graph perfectly, since at $x = 2$, we get $y = 16$. The x-intercept of this graph can be found by finding where the y-value of the equation is 0: $0 = x^3 + 8$. This becomes $x^3 = -8$. Simplify this to yield $x = -2$. In addition, because $y = x^3$ has rotational symmetry, the point $(-2,16)$ can be rotated about $(0,8)$ by 180° to reveal that the point $(-2,0)$ is also on the function.

21. D. The function looks like $y = x^3$, so we can tell it is a cubic function with just a visual inspection.

22. E. A reflection across the line $y = x$ is the inverse function, where the x and corresponding y-values of a function are switched. Thus, from our original equation $y = x^3 + 8$ we get $x = y^3 + 8$. Solving this equation for y gives us $y = \sqrt[3]{x-8}$.

23. A. First, we know that C is not a viable answer as the graph will have a negative slope. Furthermore, we know that K is a positive constant which indicates that the graph will have a positive y-intercept. The only answer choice that satisfies all these requirements is A.

24. D. An easy way to visualize this problem is that each x value and its corresponding negative value, $-x$, will have the same y value. If this is the case, all choices except D have a single y value for the both the positive and negative x value.