FUNCTION AS A MODEL

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- How to understand function notation
- How to plug in values and solve for other values when given equations
- How to understand given equations—what means what, and how do the parts work together?

Oftentimes on the ACT you'll see **word problems that include a formula.** We call these problems "Function as a Model." The equation models some particular circumstance; most often, you're given some values that you can plug into that equation, and then you're asked to solve for something given the information. At other times, you'll be asked to interpret these functions and what particular variables mean.

A few things to remember when working on these types of problems:

FUNCTION NOTATION

Function notation confuses many students. Don't let that be you!

A microscope company's profit, P dollars, when m microscopes are made and sold can be modeled by $P(m) = m^2 - 440m - 30{,}000$. What is the least number of microscopes the company must make and sell in order for the company to not lose money on this production run?

Function notation (P(m), F(x), etc.) means one value—it's just like a "y"!

$P(m)$ is a single value—the PROFIT—it doesn't mean multiply *P* times some number m! It's just a fancy way of writing what would be *y* in an equation you would graph. You can even scribble out the entire ugly function notation and write "y" for myself to keep things straight. Same goes for times when a problem uses a letter such as *P*—without the function notation—if it's isolated on one side of the equation, treat it like a y!

If function notation confuses you, check out our chapter on functions in our Algebra II book, or look up "function notation" on the internet for a refresher.

Function notation is interchangeable with a single letter!

$F(x)$ can also be written as F, $C(n)$ can also be written as C, etc. As you see in the problem above P dollars is the same as $P(m)$. This is confusing, but it's a fact you need to be aware of. A problem can ask for the value of $P(m)$ or P but it's asking for the same thing.

PLUG IN

One of the most common tactics when you see these given equation problems is to plug in! You have to solve and simplify, and as formulas have multiple variables you'll typically need to eliminate one or more by figuring out what number to plug in.

First plug in any given values—or do any simple calculations necessary to come up with a value to plug in. These problems often have given values (the initial investment was $\$300$, she swam for two weeks, etc.). Always plug in given values, or figure out how to use those values to find the value you need to plug in, first.

TIPS: **When in doubt, try zero!** If you're short on what number to plug in, think about whether there's a "zero" inherent in the question—are you trying to make a profit? Show a loss? Both of those happen when some value crosses zero. Is there a variable that is the "number of years" an investment has grown—think about it—if your investment just started, that's zero years or investment periods. In other words, if you think there's nothing to plug in, zero may help you out—experiment, play around with the numbers until the problem clicks.

Make up numbers! Even if you have five equations in the answer choices, plugging in can help you understand how the numbers work together—make the problem real and understandable. You can even make up other numbers (try using 1, 2, etc.) to understand how a function works.

The number, N, of students at Fitzgerald High School who will catch a cold through week t of school is modeled by the function $N(t)=\frac{600t^2-450}{t^2+5}$. If there are 3000 students in the school and the semester is 15 weeks, according to the model approximately what percent of students will catch the flu by the end of the semester?

To solve, we simply identify what "t" is—15 weeks of school—and plug that number in. Don't worry about the 3000, since that number doesn't matter.

$$N(15)=\frac{600(15)^2-450}{(15)^2+5}$$

$$\frac{600(225)-450}{225+5}$$

$$\frac{135000-450}{230}$$

$$\frac{134550}{230}=585$$

$$\frac{585}{3000}=0.195=19.5\%$$

Answer: **20%** (notice the question says "approximately")

A colony of bacteria grows exponentially, as described by the equation $y = y_0(3)^t$, where t represents the number of days, y is the number of bacteria, and y_0 is the original population when $t = 0$. When the colony is initially placed in a petri dish, there are 10 bacteria. According to this formula, how many cells will be in the group at the end of the week?

Now we must identify what t is (number of days) and what the original population is (10). To find the number of days—use your brain—how many days are in a week? 7! So $t = 7$.

$$t = 7; y_0 = 10$$

$$y = y_0(3)^t$$

$$y = (10)(3)^7$$

$$y = 10(2187) = 21870$$

Answer: 21870

Always double check your logic

What makes these problems tough is that they're integrated with real world problems. Even if something is modeled by an equation, if that something is a physical distance, it's can't be negative. If it's a number of items sold, it also can't be negative—make sure your answer at the end makes sense.

A microscope company's profit, P dollars, when m microscopes are made and sold can be modeled by $P(m) = m^2 - 440m - 30{,}000$. What is the least number of microscopes the company must make and sell in order for the company to not lose money on this production run?

Let's take the problem above—we want to make a profit—how do you make a profit? By getting more than "0" for the letter P—because P is profit! How do we get P above zero? Well, imagining $P = y$ and $m = x$, this is an upwards facing parabola, so it's going to sink down and then rise back up. What we want to know is when P rises above 0—so to solve for that I set P equal to 0 and solve:

$$0 = m^2 - 440m - 30{,}000$$

This is a basic quadratic equation—we can solve by factoring or the quadratic equation. I'll factor, but you can always use the quadratic equation (you can even program your calculator to do the quadratic equation for you), or solve by graphing.

$$0 = (m - 500)(m + 60)$$

$$m = 500 \text{ or } -60$$

Now here's the tough part—because this is a word problem, you're not actually looking for -60, even though it's the smaller answer of the two. You have to realize that you can't make a negative number of microscopes. That doesn't work, so the answer is 500. If you make between $0 - 499$ microscopes, you'll lose money, because at 500, they neither lose nor earn money.

TIP: **When in doubt, you're looking for "zeros," points of intersection, and vertexes**

At the heart of all these problems are the same ideas that you're solving for in regular problems involving quadratics, exponential functions, and linear functions. If you're having trouble figuring out what the problem is looking for, ask yourself if any of these ideas (zeros, points of intersection, vertexes) will push you forward.

UNDERSTANDING EQUATIONS

Finally, you'll confront questions that test your ability to understand which numbers mean what in a given equation. You'll need to be able to decipher where each variable and constant came from and what they mean. Many of the same tips that we just mentioned will come into play in these types of problems. One of the best things you can do on these is to start plugging in values and understand the equation by working with it.

A paper airplane is thrown from the second story balcony of a building. The flight of a paper airplane can be modeled by the equation $h(t) = -\frac{t^2}{5} + t + 10$, where t is the time in seconds after the paper airplane has been launched and h is the height of the paper airplane in feet. According to this equation, which of the following statements is true about the paper airplane?

A. After 10 seconds, the paper airplane reached its maximum height.
B. After 5 seconds, the paper airplane reached its maximum height.
C. After 11.25 seconds the paper airplane reaches its maximum height.
D. After 10 seconds the paper airplane hits the ground.
E. After 2.5 seconds the paper airplane hits the ground.

Notice that ALL the choices give us a "seconds" to deal with—we can solve this out by backsolving and plugging in the seconds for the value t. Let's start with 10 (answers A & D) because it's in two different answer choices AND in the problem.

$$h(t) = -\frac{t^2}{5} + t + 10$$

$$h(10) = -\frac{10^2}{5} + 10 + 10$$

$$h(10) = -\frac{100}{5} + 20$$

$$h(10) = -20 + 20 = 0$$

The height (h) is zero when we're at 10 seconds. That's answer choice D and we're done. If we weren't so lucky, we could continue to plug in, look for other intercepts (plug in zero), or look for the vertex value to help us narrow the field. Remember, the greatest height is often the vertex, the ground or landing is a zero, etc.

Answer: **D.** After 10 seconds the paper airplane hits the ground.

1. In the desert biome, the temperature can range greatly from day to night. A typical range in Celsius would be $-18° \le C \le 20°$. Given the formula $F = \frac{9}{5} \times C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius, what would this biome's temperature range be in Fahrenheit?

A. $1.4° \le F \le 72°$
B. $22° \le F \le 43.11°$
C. $64.4° \le F \le 68°$
D. $-0.4° \le F \le 68°$
E. $22° \le F \le 68°$

2. Candice deposits $\$2{,}000$ at 5% annual interest compounded yearly at a bank which use the formula $A = P(1+r)^n$, where A is the current value; P is the amount deposited initially; r is the rate of interest for one compounding period, expressed as a decimal; and n is the number of periods, to determine the current value of the account. Which of the following would be the approximate value of the account after 10 years?

A. $\$35$
B. $\$1228$
C. $\$3258$
D. $\$21000$
E. $\$115330$

3. The number of clients Candy Cane Cable acquired in each year can be modeled by the function $C(t) = 800t + 250$, where $t = 0$ corresponds to the year 1999. Using the given model, how many clients would you expect Candy CandyCable to acquire in the year 2007?

A. 1605850
B. 64250
C. 7450
D. 5850
E. 6650

4. The time, t seconds, required for a simple pendulum x feet long to make 1 complete swing can be modeled by $t = 4\pi\sqrt{\frac{x}{12}}$. How many seconds will it take for a simple pendulum that is 3 feet long to make one complete swing?

A. Between 7 and 8 seconds
B. Between 6 and 7 seconds
C. Between 2 and 3 seconds
D. Between 1 and 2 seconds
E. Less than 1 second

5. For a certain species of frog, the optimal temperature range for the tadpoles in degrees Celsius is $20° \le C \le 35°$. The conversion from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. What is the optimal temperature range for the tadpoles in degrees Fahrenheit?

A. $68° \le F \le 95°$
B. $68° \le F \le 51.44°$
C. $43.11° \le F \le 95°$
D. $43.11° \le F \le 51.44°$
E. $36° \le F \le 63°$

6. Jose invested $\$1400$ in a high-yield savings account. In 7 months, his investment had earned a $\$62$ interest. To the nearest tenth, what is Jose's annual interest rate? (Use $I = Prt$, where I is the amount of interest earned, P is the initial investment, r is the interest rate, and t is the time, in years.)

A. 0.6%
B. 7.6%
C. 14.3%
D. 8.9%
E. 10.6%

7. $y = a(1-r)^x$ is an equation that models decay. a is the initial amount before decay, r is the rate of decay, and x is the number of time intervals that have passed. A certain element decays by 12% every hour. If there are 500 grams of this element at $5:00$ AM, which expression shows how many grams will be left at $11:00$ AM the same day?

A. $500(1+.12)^6$
B. $500(1-12)^6$
C. $500(12)^6$
D. $500(.12)^6$
E. $500(1-.12)^6$

8. The length L, in centimeters, of a simple pendulum is given by the equation $L = \frac{gT^2}{4\pi^2}$, where g is the acceleration of gravity in meters per second squared. What is the length of the pendulum, knowing that the acceleration due to gravity is $9.8 m/s^2$ and the period of the pendulum is $3.5s$?

A. 2.63
B. 3.04
C. 3.63
D. 4
E. 5.21

9. A ball is kicked into the air from ground. Its height, h feet above ground, t seconds after it is launched, is given by $h = -16t^2 + 64t$. During the ball's descent, at what value of t is the ball 48 feet off of the ground?

A. 3
B. 1
C. 2
D. 4
E. 5

10. A microwave company's profit, P dollars, when m microwaves are made and sold can be modeled by $P = m^2 - 400m - 120{,}000$. What is the least number of microwaves the company must make and sell in order for the company to not lose money on this production run?

A. 400
B. 500
C. 550
D. 600
E. 650

11. The amount, in kilograms, of usable wood left in a wood-burning furnace starting with 25 kilograms of wood can be approximated by the equation $y = 25 - 2.5h$, where h is the number of hours the stove has been burning for $0 \le h \le 10$, and y is the number of kilograms of wood remaining. According to this equation, which of the following statements is true about the stove?

A. After burning for 1 hour, the furnace still has 22.5 kilograms of wood left.
B. After burning for 1 hour, the furnace has used 22.5 kilograms of wood.
C. After burning for 2.5 hours, the furnace has used 2.5 kilograms of wood.
D. After burning for 9 hours, the furnace still has 22.5 kilograms of wood.
E. After burning for 0.5 hours, the furnace has burned 5 kilograms of wood.

12. Temperatures measured in degrees Fahrenheit (F) are related to temperatures measured in degrees Celsius (C) by the formula $F = \frac{9}{5}C + 32$. There is 1 value of x for which x degrees Fahrenheit equals $x - 30$ degrees Celsius. What is that value?

A. -77.5
B. -2.5
C. 5.75
D. 27.5
E. 77.5

13. A triangle has an angle measuring $60°$ and the leg opposite that measures 30 inches. One of the legs adjacent to the $60°$ angle has a changing length corresponding to $f(t) = 80 + 2t - 2t^2$ inches, starting from $t = 0$ where t is the time in seconds. What is the time, t, at which the resulting triangle will have an area that is $\frac{1}{2}$ that of the original triangle?

A. 4 seconds
B. 5 seconds
C. 6 seconds
D. 7 seconds
E. 8 seconds

14. The distance, d in meters, that an accelerating object travels is given by $d = \frac{1}{2}at^2$, where a is the acceleration rate in meters per second per second and t is the time in seconds. A motorbike accelerates from a stop with an acceleration of 40 meters per second squared over a distance of 60 meters. About how many seconds did the motorbike travel?

A. Between 4 and 5
B. Between 2 and 3
C. 1
D. Between 1 and 2
E. 2

15. A children's wind-up car toy travels a distance, d inches, as modeled by the function $d = 12 + 8t$ where t is the number of seconds after the car has been wound up and released. If the car is released at 12 feet, to the nearest second, how many seconds would it take to reach 21 feet?

A. 10
B. 11
C. 12
D. 13
E. 14

ANSWER KEY

1. D 2. C 3. E 4. B 5. A 6. B 7. E 8. B 9. A 10. D 11. A 12. D 13. B 14. D 15. C

ANSWER EXPLANATIONS

1. **D.** Converting the lower bound of the range from Celsius to Fahrenheit, we get $F=\frac{9}{5}(-18)+32\rightarrow-\frac{162}{5}+32=$ $=-32.4+32\rightarrow-0.4$ Converting the upper bound of the range from Celsius to Fahrenheit, we get $F=\frac{9}{5}(20)+32\rightarrow\frac{180}{5}+32=36+32\rightarrow68.$ So, the range of temperature in Fahrenheit is $-0.4\le F\le68.$

2. **C.** We are given that $P=2000, r=0.05,$ and $n=10.$ Plugging these values into the equation $A=P(1+r)^n$, we solve for A. $A=2000(1+0.05)^{10}\rightarrow2000(1.05)^{10}\rightarrow3257.79\approx3258.$

3. **E.** Since $t=0$ corresponds to the year 1999, the year 2007 corresponds to when $t=2007-1999=8$. So, plugging in $t=8,$ we get $C(8)=800(8)+250\rightarrow6400+250=6650.$

4. **B.** Plugging in $x=3,$ we get $t=4\pi\sqrt{\frac{3}{12}}=4\pi\sqrt{\frac{1}{4}}=4\pi\left(\frac{1}{2}\right)=2\pi$. The value of 2π is ≈6.28 which is between 6 and 7 seconds.

5. **A.** Converting the lower bound of the range from Celsius to Fahrenheit, we get $F=\frac{9}{5}(20)+32\rightarrow\frac{180}{5}+32\rightarrow36+32=68.$ Converting the upper bound of the range from Celsius to Fahrenheit, we get $F=\frac{9}{5}(35)+32\rightarrow\frac{315}{5}+32\rightarrow63+32=95.$ So, the range of temperature in Fahrenheit is $68\le F\le95.$

6. **B.** We first need to know that 7 months is equal to $\frac{7}{12}$ years. Plugging in $I=62, P=1400,$ and $t=\frac{7}{12}$ into $I=Prt$, we get $62=1400r\left(\frac{7}{12}\right).$ Dividing both sides by $(1400)\left(\frac{7}{12}\right),$ we get $r=0.0759$. So, the interest rate r is approximately 7.6%.

7. **E.** $r=0.12, a=500,$ and $x=6.$ Plugging these values into the given formula, we get $y=500(1-0.12)^6$.

8. **B.** This problem is as simple as plugging numbers into their respective variables and putting the expression into your calculator. Plugging the values given for time and acceleration due to gravity, we get $L=\frac{9.8m/s^2*(3.5s)^2}{4*\pi^2}\approx3.04.$

9. **A**. The question gives us the value of t and wants us to find the value of h that makes the equation true. So, plugging in $h=48,$ we get $-16x^2+64x=-48.$ Moving all the values to the right side of the equation, we get $0=-16x^2+64x-48.$ Factoring out, 16 we get $16(-x^2+4x-3).$ Now, factoring the polynomial inside the parenthesis, we get $16(-x+1)(x-3)=0.$ This gives us two t values of 1 and $3,$ but the problem asks for the time at which the ball has a height of 48 feet AND is descending. Because the ball is kicked from rest, rises, and then falls, the time closer to 0 $(t=1)$ will be when the ball is rising, so the time further from 0 $(t=3)$ will be when the ball is falling and is the correct answer.

10. **D.** For the company to not lose money, the profit P must be zero or positive. Setting $P=0$ and solving for m would give us the least amount of microwaves the company must sell for the company to not lose money. We have $0=m^2-400m-120{,}000.$ To solve for $n,$ we must find two numbers that add up to equal -400 and multiply to be $-120{,}000.$ The numbers -600 and 200 satisfy these conditions, so we can factor the equation to be $(m-600)(m+200)=0.$ This means $m-600=0\rightarrow m=600$ or $m+200=0\rightarrow m=-200.$ The positive value for m is our answer because the company cannot sell a negative number of microwaves.

11. **A.** The equation $y=25-2.5h$ has a y -intercept of 25 and a slope of $-2.5.$ This means that for every hour (h), the amount of wood (y) is decreasing by 2.5. So, running through the answer choices, we see that for answer choice A, after burning for 1 hour, the furnace still has 22.5 kilograms of wood left. This is true because the amount of wood left after 1 hour is equivalent to the equation evaluated at $h=1.$ This gives us $y=25-2.5*1=22.5.$ So there are 22.5 kilograms of wood left after 1 hour.

12. **D.** If x degrees Fahrenheit equals $x-30$ degrees Celsius, we can plug these values into our formula as $x=\frac{9}{5}(x-30)+32.$. Distributing the $\frac{9}{5}$ gets us $x=\frac{9}{5}x-54+32$. This simplifies to $\frac{4}{5}x=22.$ Multiplying each side by $\frac{5}{4}$ isolates the x to get $x=\frac{55}{2}=27.5.$

13. **B.** The area of a triangle is equal to any of the bases times the corresponding height (the height is the length of the altitude drawn from the point opposite the base). Since the point opposite the leg with a changing length does not move, neither does the altitude, so the height is constant. Therefore, if the base doubles its area or halves its length, the area will also double or halve, respectively. At $t=0$, the length is 80. To find when the area is $\frac{1}{2}$ that of the original triangle, we need to find the time when the length of the leg is $\frac{1}{2}*80=40$. Set up our equation as $80+2t-2t^2=40$ and rearrange to $2t^2-2t-40=0$. This simplifies to $t^2-t-20=0$. This factors into $(t+4)(t-5)=0$. The solutions are $t=-4$ and $t=5$. t can only be a positive value, so the solution is 5 seconds.

14. **D.** Plugging in $d=60$ and $a=40,$ we have $60=\frac{1}{2}(40)t^2.$ Dividing by 20 on both sides, we get $3=t^2$ or $t=\sqrt{3}.$ Since $\sqrt{1}<\sqrt{3}<\sqrt{4},\sqrt{3}$ is between 1 and $2.$

15. **C.** The car must travel 9 feet, which is $9*12=108$ inches. So, we plug in 108 for the distance d and solve for the time $t.$ We get $108=12+8t\rightarrow 96=8t\rightarrow t=12.$