SPEEDS AND RATES

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- How to solve distance problems
- Understand that rates are like proportions and per means divide
- How to solve wheel distance problems (Circumference= 1 revolution)

THE DISTANCE FORMULA

$$\boxed{d = rt}$$

Memorize this. When in doubt set up as many equations as necessary using sub numbers or letters (subscript i.e. $d_a = r_a t_a$) to denote who/what you're talking about. Then start a chain of substitution until you get to the answer you need.

Nathaniel drove 360 miles in 9 hours of driving time. How much faster would he have to drive than his average speed to cut his driving time by 3 hours?

A. $20\ mph$ **B.** $30\ mph$ **C.** $40\ mph$ **D.** $50\ mph$ **E.** $60\ mph$

Use the distance formula (I use subscript to keep each case straight): $d_a = r_a t_a$

For the actual trip (trip a), we let $d_a = 360$ and $t_a = 9hr$, and plug in to the distance formula: $360\,mi = r_a\left(9\,hr\right)$

Then solve for r_a:

$$\frac{360\,mi}{9\,hr} = r_a$$

$$40\frac{mi}{hr} = r_a \text{ is his speed for the actual trip}$$

Now make a formula for the hypothetical trip (trip h) and fill in the pieces we know: $d_h = r_h t_h$

We know we want to cut the time by three hours—so $9-3=6$; $t_h = 6$

In terms of the distance, he's trying to take the same trip, so $d_h = d_a = 360$

$$360\,mi = r_h\left(6\,hr\right)$$

$$\frac{360\,mi}{6\,hr} = r_h$$

$$60\,mph = r_h$$

Now read the question—we need how much faster this answer is than our speed for the actual trip, or the difference between the hypothetical faster trip and the actual one:

$$r_h - r_a$$

$$60\,mph - 40\,mph = 20\,mph$$

Answer: **A.**

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WORK FORMULA

$$w = rt$$

Where w is the amount of work (ex. number of pages), r is the rate of work (ex. pages per hour) and t is the time (ex: hours).

COMBINED WORK FORMULA

$$w = (r_1 + r_2)t$$

Where r_1 is the rate of one worker or machine, and r_2 is the rate of another worker or machine working for the same amount of time.

If they don't work for the same amount of time, then calculate each amount of work separately and add them together. Remember the word RATE is an equation in disguise. It's some quantity over another that equals the rate. Rates are essentially proportions.

TIP: PER MEANS DIVIDE.
Whenever you see "per" or could phrase a relationship between two items with the word "per," think "divide"!

A whatsit machine makes 35 whatsits per minute. A second whatsit machine makes 50 whatsits per minute. The second whatsit machine starts making whatsits 5 minutes after the first whatsit machine starts. Both machines stop 10 minutes after the first machine starts. How many whatsits were produced by the two machines together?

A. 212.5 **B.** 425 **C.** 600 **D.** 675 **E.** 850

If work equals rate times time $w = rt$, and the number of whatsits produced represents the "work" we now need to figure out rate and time and solve for w.

Remember PER means DIVIDE. So 35 whatsits per minute means divide 35 by 1 minute! 50 whatsits per minute means divide 50 by 1 minute! "Per" also indicates a rate.
Thus here are our two rates:

First machine: $\dfrac{35 \ whatsits}{1 \ min}$

Second machine: $\dfrac{50 \ whatsits}{1 \ min}$

From the problem we know the first machine stops after 10 minutes.
The second machine works five minutes less as it starts 5 minutes later, but ends at the same time $10 \ min - 5 \ min = 5 \ min$
Now we can either analyze the rate and know that multiplying it by minutes will make the minutes cancel, and give us what we need (whatsits) or we can know to multiply rate times time from the formula.

Remember, $w = rt$

The first machine will produce:

$$\underbrace{\frac{35 \ whatsits}{1 \ \cancel{min}}}_{\text{(rate)}} \underbrace{(10 \ \cancel{min})}_{\text{(time)}} = \underbrace{35 \ whatsits}_{\text{work}}$$

The second machine will produce:

$$\underbrace{\frac{50 \ whatsits}{1 \ \cancel{min}}}_{\text{(rate)}} \underbrace{(10-5) \ \cancel{min}}_{\text{(time)}} = \underbrace{250 \ whatsits}_{\text{work}}$$

We can then add together the sum of each machine's production.

$$350 \ whatsits + 250 \ whatsits = 600 \ whatsits$$

Answer: **C.**

Notice how the "minutes" on the bottom of the rate cancel with the time in "minutes"—you can always use labels to help you set up rate word problems correctly.

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WHEEL PROBLEMS

Sarah rode a bike with wheels 36 inches in diameter. During 4 minutes of her ride, the wheels made 180 revolutions. At what average speed, in *feet per second*, did Sarah travel, rounded to the nearest *foot per second*?

A. 2 **B.** 7 **C.** 15 **D.** 135 **E.** 424

With problems like this, we first figure out what we NEED. **We need FEET PER SECOND.**

Here we use the fact that the word PER means DIVIDE. So we need $\frac{FEET}{SECONDS}$**.**

To find that, we need to find the TOTAL FEET she travelled and the TOTAL SECONDS she travelled.

TIP: Average Rate is **always** TOTAL DISTANCE over TOTAL TIME.

Let's start with the feet. We have a 36" diameter wheel, but that's not the feet we need. We need the distance travelled. When a wheel travels, the distance it completes is equal to the circumference of the wheel times the number of rotations. With each rotation, the entire circumference touches the ground and equals the amount travelled. Imagine if your wheel was made of a piece of string. Cut it and lay it flat—you'll realize that's also the distance your wheel has travelled.

Again we need $C\times(180\,revolutions)$.

To find C (circumference): $C=\pi d$ where d is the diameter. Let's first make our 36" diameter into feet $36\,in=3\,ft$
Now we'll find the circumference:

$$C=\pi d$$
$$C=\pi(3)=3\pi$$

This is the number of **feet** we travel **per revolution** (remember PER means DIVIDE!) Take that idea and plug into our equation $C\times(180\,revolutions)$.

$$3\pi\frac{ft}{\cancel{rev}}*180\,\cancel{rev}\approx 1696\,ft$$

See how the revolutions "cancel" just as variables would? We call that cancellation "dimensional analysis"—making sure your units cancel is a great way to ensure you've set up the problem right.

Let's go back to our original need: $\frac{FEET}{SECONDS}$. We can now fill in the feet: $\frac{1696\,ft}{SECONDS}$.

To find the seconds, we convert 4 minutes to seconds using **dimensional analysis (if you've never done it, Google it!** In short, it's converting between different units.):

First come up with the conversion factor: $\frac{60\,sec}{1\,min}$.

Because there are 60 seconds in 1 minute, these are equivalent values. We can multiply ANY number by this conversion rate and doing so won't change the "value" of what we find—i.e. it stands for the same amount, but with different labels. It's similar to multiplying by a giant " 1 "—when you multiply by one you don't change the value of something. If necessary, flip your conversion rate upside down—but make sure the denominator (bottom) has the same label as the number you're multiplying by (here "minutes" is that label, so we want minutes on the bottom so it will cancel):

$$(4\,\cancel{min})\frac{60\,sec}{1\,\cancel{min}}=240\,sec$$

Now we complete our original need item by filling in the missing seconds**:** $\frac{1696\,ft}{SECONDS}$ is $\frac{1696\,ft}{240\,sec}\approx 7$

Answer: **B.**

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TIPS IF YOU'RE STUCK:

1. Remember that oftentimes you have hidden equal quantities—for example a round trip flight is the same distance both directions; if two people start at the same time, their time is the same, etc.
2. Check your units—sometimes these problems will mix measurements (i.e. inches / feet, seconds / minutes). If you never learned **dimensional analysis**, learn it! (Google can help you!)
3. Re-read the question—there may be one more detail you've missed. Don't forget these sometimes show up in three part questions. Look up if you're lost to find more information.
4. Seeing the word "distance" doesn't mean it's actually a distance/rate problem. It may be a Function as a Model problem—see that chapter (chapter 16) for identifying that question type. If you have a given equation, forget about the distance formula and use what the test gives you. If you have a given chart of information, forget about the formulas and use the given information first! Then turn to memorized formulas if you need more.

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1. Doug and Emily are driving cars on a track, and they both do 20 laps around a 1-mile track. Doug drives at an average of 60 miles per hour around the track while Emily drives at an average of 80 miles per hour. When Emily finishes her 20 laps, how many laps does Doug have left?

A. 0
B. 5
C. 10
D. 12
E. 4

2. Bryan and David go running in the park every morning. Bryan starts before David and is 30 meters ahead when David starts. Bryan runs at a rate of 2.5 meters per seond, while David runs at a rate of 4.0 meters per second. Which function shows the time t it takes David to catch up to Bryan?

A. $t = 30$
B. $4.0t = 30$
C. $2.5t = 30$
D. $1.5t = 30$
E. $1.5t = 60$

3. Ronny can walk 4 miles in t minutes. At that rate, how long will it take her to walk 11 miles?

A. $\frac{11t}{4}$
B. $t + 7$
C. $\frac{4t}{11}$
D. $\frac{t}{11}$
E. $\frac{4}{t}$

4. Two delivery trucks leave from a warehouse at the same time. The first one moves at 45 miles per hour and drives 1 hour west and 2 hours south. The second one moves at 60 miles per hour and drives 2 hours west and 1 hour south. What expression gives the distance between the two delivery trucks 3 hours after they leave the warehouse?

A. $\sqrt{(45+120)^2+(90+60)^2}$
B. $\sqrt{(45-120)^2-(90-60)^2}$
C. $\sqrt{(45-120)^2+(90-60)^2}$
D. $\sqrt{(45-60)^2+(90-120)^2}$
E. $\sqrt{(45-60)^2+(90-45)^2}$

5. Neela ran uphill to her friend's house and it took her $\frac{t}{6}$ minutes. When she ran downhill back home, she ran 2.5 times as fast as she had uphill. How many minutes did her journey home take?

A. $\frac{5t}{12}$
B. $\frac{t}{15}$
C. $\frac{7}{30t}$
D. $\frac{12}{7t}$
E. $2.5t$

6. A monkey eats 13 bananas in 8 days. At this rate, how many bananas does the monkey eat in $8+n$ days where n is every additional day?

A. $\frac{13n}{8}$
B. $13+\frac{n}{8}$
C. $13+\frac{13n}{8}$
D. $13+n$
E. $\frac{8+n}{13}$

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7. A car's windshield washer fluid reservoir holds 540 ounces of washing fluid, and there is a small leak causing the fluid to leak out at a constant rate of 9 ounces per minute. The car is travelling at 50 miles per hour. If the reservoir starts full, in how many miles will the washer fluid reservoir be empty?

A. 45
B. 50
C. 55
D. 60
E. 65

8. A cylinder in a diesel engine displaces 2×10^3 cubic centimeters and there are 6×10^5 oxygen molecules in the cylinder, what is the average number of oxygen molecules per cubic centimeter?

A. 3×10^3
B. 3×10^1
C. 3×10^0
D. 3×10^2
E. 3

For Questions 9-10, refer to the table below.

Monthly payment per $500 borrowed for various annual rates and numbers of payments			
Annual Interest Rate	Number of monthly payments		
	48	60	72
4%	11.29	9.21	7.82
7%	11.79	9.9	8.52
11%	12.92	10.87	9.52
13%	13.41	11.38	10.04

Paul is planning on purchasing a motorcycle, and he will need to borrow some money. The chart shows different monthly payments based on interest rates and loan terms.

9. Paul finds a motorcycle that costs $10,365. He will have to borrow $7,000. What will his monthly payment be if he borrows the money for 60 months at 11% interest?

A. $152.18
B. $76.09
C. $304.36
D. $159.32
E. $13.28

10. Paul visits another dealership that is offering an end-of-the-year deal with 4% annual interest on loans for 48, 60, or 72 months. Paul can only manage to pay $100 per month. What is the largest loan he can afford with his budget?

A. $4000
B. $5000
C. $6000
D. $7000
E. $8000

11. A wheel of a tricycle is 5 inches in diameter. If it rolls along a line without slipping, how many inches has the wheel traveled after 50 revolutions?

A. 250π
B. 250
C. 500π
D. 500
E. 750π

12. Greg pushes a cart so that one wheel rotates $\frac{2\pi}{3}$ radians. What fraction of the circumference of the wheel has the wheel traveled?

A. $\frac{2\pi}{3}$
B. $\frac{1}{6}$
C. $\frac{1}{3}$
D. $\frac{2}{3}$
E. $\frac{3}{2}$

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13. A cable has to be 7 mm thick for every 10 kg it supports. Which of the following expressions gives the thickness of the cable *in* centimeters required to support a weight of 300 stones? (Note: 1 stone $\approx$ 6.35 kg)

A. $\frac{(7)(6.35)(10)}{30}$

B. $\frac{(7)(6.35)(30)}{10}$

C. $\frac{(7)(6.35)}{(10)(10)(30)}$

D. $\frac{(7)(10)}{(6.35)(30)}$

E. $\frac{7}{(6.35)(30)}$

14. Chrissie is learning how to ride a unicycle with a wheel 30 inches in diameter. She can ride for 2 minutes before falling over. If she only moved in one direction and her wheel made 100 rotations during this time, what was her average speed *in feet per second* over that time interval?

A. 2.083π

B. 3.125π

C. 6.25π

D. 12.5π

E. 25π

15. A recipe for a turkey says to cook at $375°F$ for 35 minutes per 1.5 pounds. How long should a 9-pound turkey be cooked?

A. 3 hours

B. 2 hours 55 minutes

C. 3 hours 15 minutes

D. 3 hours 30 minutes

E. 4 hours

16. A motorcycle has wheels 24 inches in diameter. During 3 minutes, it makes 1620 revolutions. What is the average speed of the motorcycle, to the nearest mph? (Note: 5280 ft=1 mile)

A. 37 mph

B. 38 mph

C. 39 mph

D. 40 mph

E. 41 mph

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ANSWER KEY

1. B 2. D 3. A 4. C 5. B 6. C 7. B 8. D 9. A 10. C 11. A 12. C 13. B 14. A 15. D 16. C

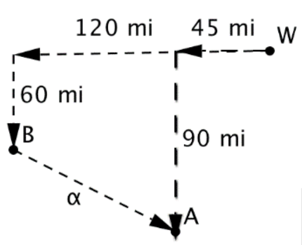
ANSWER EXPLANATIONS

1. **B.** When Emily finishes her 20 laps (20 miles), she would have spent $20\,miles * \frac{1\,hour}{80\,miles} = 0.25$ hours driving. In that same amount of time, Doug would have driven $0.25\,hours * \frac{60\,miles}{1\,hour} = 15$ miles (15 laps). So, at that time, Doug will still have $20 - 15 = 5$ laps left to drive.

2. **D.** If we set $t = 0$ to be the time at which David starts to run, then we can write Bryan's distance (B) as the formula $B = 30 + 2.5t$. David's distance (D) would be written as $D = 4t$. We set these distances to be equal and then solve for the time t that makes the equality true. The equality is when $B = D \rightarrow 30 + 2.5t = 4t$. Subtracting $2.5t$ on both sides, we get $30 = 1.5t$.

3. **A.** $11\,miles * \frac{t\ minutes}{4\,miles} = \frac{11t}{4}$ minutes for Ronny to walk 11 miles.

4. **C.** The first truck's distance traveled is calculated using the formula $distance = speed * time$. So, the first truck's distance traveled west is $d = 45\frac{mi}{hr} * 1hr = 45mi$ and his distance traveled south is $d = 45\frac{mi}{hr} * 2hrs = 90mi$. Likewise, the second truck's distance traveled west is $d = 60\frac{mi}{hr} * 2hrs = 120mi$ and his distance traveled south is $d = 60\frac{mi}{hr} * 1hr = 60mi$. This gives us the following diagram.



The picture above illustrates the first truck's route starting from the warehouse (point W) to point A and the second truck's route from point W to point B. The distance between their locations after three hours is represented by $\propto$. To find $\propto$, we find the horizontal and vertical differences from point A to point B and use the Pythagorean Theorem to find $\propto$. The horizontal distance is $120\,mi - 45\,mi$ and the vertical distance is $90\,mi - 60\,mi$. So, plugging these values into the Pythagorean Theorem, we get $\propto = \sqrt{(120-45)^2 + (90-60)^2} = \sqrt{(45-120)^2 + (90-60)^2}$.

5. **B.** If Neela takes $\frac{t}{6}$ minutes to run uphill, and she runs 2.5 times as fast as her uphill speed on her way home, then she takes 2.5 times less time to complete the same commute. Thus, it will take her $\frac{t}{6}\left(\frac{1}{2.5}\right) = \frac{t}{15}$ minutes to get home.

6. **C.** $(8+n)days * \frac{13}{8}\frac{bananas}{days} = \frac{(8+n)13}{8} = \frac{13(8)+13n}{8} = 13 + \frac{13n}{8} bananas$.

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7. B. First calculating the amount of fluid that is lost due to the leakage, we calculate $540-9t$ where t is in minutes. The fluid reservoir will be completely empty when $540-9t=0$. Adding $9t$ to both sides, we get $540=9t$. Dividing both sides by 9, we get $60=t$. So, in one hour, the reservoir will be empty. In one hour, the car will have traveled $1\,hour*50\frac{miles}{hour}=50\,miles$.

8. D. $\frac{6\times10^5\,molecules}{2\times10^3\,cubic\ centimeters}=3\times10^2\frac{molecules}{cubic\ centimeters}$.

9. A. Looking at the chart, we see that if Paul borrows the money for 60 months at 11% interest, he will have to pay 10.87 every month per $\$500$ borrowed. Paul is borrowing $\$7{,}000$, so his monthly payment can be calculated as $\$7{,}000*\frac{\$10.87}{\$500}=\$152.18$ per month.

10. C. We are looking for the maximum loan amount Paul can afford, so we want to look at the minimum rate of monthly payments. In this case, the minimum monthly payment amount offered with 4% annual interest for 48, 60, or 72 months is $\$7.82$ (the 72 month loan).We want to find the maximum loan amount x that would yield a monthly payment of less than $\$100$. This can be calculated by $\frac{\$7.82}{\$500}*x\le100$. $x\le100*\frac{\$500}{\$7.82}\to x\le\$6393.86$. So, the maximum loan amount he can afford is $\$6000$.

11. A. The circumference of the circle is calculated by the formula $C=\pi d$. Plugging in $d=5$, we get $C=5\pi$. So, for every revolution, the wheel travels 5π inches. After 50 revolutions, the wheel would have traveled $5\pi*50=250\pi$ inches.

12. C. One complete circle is 2π radians, so $\frac{2\pi}{3}$ radians is $\left(\frac{1}{3}\right)2\pi$ or $\frac{1}{3}$ of the circle.

13. B. We start with $7\,cm=10\,kg$. The distance isn't literally equivalent to the weight, but they are equivalent in their proportion to each other. This makes it easy to perform the upcoming calculations. First, find how thick of a cable is needed for $1\,kg$ by dividing both sides by 10: $\frac{7\,cm}{10}=1\,kg$. Next, convert the left side to centimeters by multiplying the left side by the proportion $\frac{1\,cm}{10\,mm}$. This gives us $\frac{7\,cm}{(10)(10)}=1\,kg$. Next, convert the left side to stone by multiplying it by the proportion $\frac{1\,stone}{6.35\,kg}$ to get $\frac{7\,cm}{(10)(10)}=\frac{1\,stone}{6.35}$. Isolate the unit of weight on the right side by multiplying both sides by 6.35 to get $\frac{(7)(6.35)}{(10)(10)}cm=1\,stone$. Finally, multiply both sides by 300 to get the right side equivalent to $300\,stone$. The final expression on the left side is $\frac{(7)(6.35)(300)}{(10)(10)}$. The only expression equivalent to this is answer choice (B).

14. A. Average speed is equal to the distance travelled divided by the time taken. In this case, the distance taken is equal to the number of revolutions times the circumference of the circle. The circumference of the circle is the diameter times π: 30π inches. The total distance covered is thus 100 rotations of this, or $100(30\pi)$ inches. The time taken was 2 minutes. However, we are looking for the speed in feet per second. Thus, we convert the distance by dividing it by 12, since there are

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12 inches in a foot: $D=\frac{100(30\pi)}{12}$ inches. Time is multiplied by 60, for the 60 seconds in a minute: $T=2(60)=120$ seconds.

The speed is thus $\frac{D}{T}=\frac{\frac{100(30\pi)}{12}}{120}$ seconds $\rightarrow \frac{100(30\pi)}{12(120)}=\frac{3000\pi}{1440}\rightarrow 2.083\pi$.

15. D. The turkey is cooked at a rate of $\frac{1.5\,pounds}{35\,minutes}=\frac{1\,pound}{23\frac{1}{3}\,minutes}$, so if we want to cook 9 pounds, multiply top and bottom by 9: $\frac{9(1\,pound)}{9\left(23\frac{1}{3}\,minutes\right)}\rightarrow\frac{9\,pound}{210\,minutes}$. 210 minutes is 3 hours and 30 minutes (divide 210 by 60 to find the time in hours).

16. C. Here we use the fact that the word PER means DIVIDE. So we need $\frac{Miles}{Hour}$. Let's first convert from inches to miles. Find the unit equivalents and put the labels in the right places to make the labels cancel: $24\,inches\times\frac{1\,foot}{12\,inches}\times\frac{1\,mile}{5280\,feet}=\frac{1}{2640}\,miles$.
Now I need to turn this diameter into the miles travelled. To do so, multiply by π to get the circumference $C=d\pi=\frac{1}{2640}mi\times\pi$ and then multiply the circumference times the number of revolutions $\frac{1}{2640}\times\pi\frac{mi}{rev}\times 1620\,rev\approx 1.93\,mi$. Now we can find our number of hours, converting from 3 minutes: $3min\left(\frac{1hr}{60min}\right)=\frac{1}{20}hr$. Finally we put these two pieces back into our original rate in miles per hour (miles divided by hours): $\frac{1.93mi}{\frac{1}{20}hr}=1.93*20\,mph=38.6\,mph$.

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