CHAPTER

10 FUNCTION AS A MODEL

SKILLS TO KNOW

- How to understand function notation
- How to plug in values and solve for other values when given equations
- How to understand given equations—what means what, and how do the parts work together?

Oftentimes on the ACT® you'll see **word problems that include a formula.** We call these problems "Function as a Model." The equation models some particular circumstance; most often, you're given some values that you can plug into that equation, and then you're asked to solve for something given the information. At other times, you'll be asked to interpret these functions and what particular variables mean.

A few things to remember when working on these types of problems:

FUNCTION NOTATION

Function notation confuses many students. Don't let that be you!

> A microscope company's profit, P dollars, when m microscopes are made and sold can be modeled by $P(m)=m^2-440m-30{,}000$. What is the least number of microscopes the company must make and sell in order for the company to not lose money on this production run?

Function notation ($P(m)$, $F(x)$, etc.) means one value—it's just like a "y"!
$P(m)$ is a single value—the PROFIT—it doesn't mean multiply P times some number m! It's just a fancy way of writing what would be y in an equation you would graph. You can even scribble out the entire ugly function notation and write "y" to keep things straight. Same goes for times when a problem uses a letter such as P—without the function notation—if it's isolated on one side of the equation, treat it like a y!

If function notation confuses you, check out our chapter on functions in our Advanced Algebra section (part two of this book), or look up "function notation" on the internet for a refresher.

Function notation is interchangeable with a single letter!
$F(x)$ can also be written as F, $C(n)$ can also be written as C, etc. As you see in the problem above P dollars is the same as $P(m)$. This is confusing, but it's a fact you need to be aware of. A problem can ask for the value of $P(m)$ or P but it's asking for the same thing.

PLUG IN

One of the most common tactics when you see these given equation problems is to plug in! You have to solve and simplify, and as formulas have multiple variables you'll typically need to eliminate one or more by figuring out what number to plug in. **First plug in any given values—or do any simple calculations necessary to come up with a value to plug in.** These problems often have given values (the initial investment was $\$300$, she swam for two weeks, etc.). Always plug in given values, or figure out how to use those values to find the value you need to plug in, first.

TIPS: **When in doubt, try zero!** If you're short on what number to plug in, think about whether there's a "zero" inherent in the question—are you trying to make a profit? Show a loss? Both of those happen when some value crosses zero. Is there a variable that is the "number of years" an investment has grown—think about it—if your investment just started, that's zero years or investment periods. In other words, if you think there's nothing to plug in, zero may help you out—experiment, play around with the numbers until the problem clicks.

Make up numbers! Even if you have five equations in the answer choices, plugging in can help you understand how the numbers work together—make the problem real and understandable. You can even make up other numbers (try using 1, 2, etc.) to understand how a function works.

The number, N, of students at Fitzgerald High School who will catch a cold through week t of school is modeled by the function $N(t)=\frac{600t^2-450}{t^2+5}$. If there are 3000 students in the school and the semester is 15 weeks, according to the model approximately what percent of students will catch the flu by the end of the semester?

To solve, we simply identify what "t" is—15 weeks of school—and plug that number in. Then we can find the number of students who caught the flu.

$$N(15)=\frac{600(15)^2-450}{(15)^2+5}$$

$$\frac{600(225)-450}{225+5}$$

$$\frac{135000-450}{230}$$

$$\frac{134550}{230}=585$$

Now we need to turn this number into a percent. Remember percent is part over whole. The whole is the total number of students in the school, 3000.

$$\frac{585}{3000}=0.195=19.5\%$$

Answer: 20% (notice the question says "approximately").

A colony of bacteria grows exponentially, as described by the equation $y = y_0(3)^t$, where t represents the number of days, y is the number of bacteria, and y_0 is the original population when $t = 0$. When the colony is initially placed in a petri dish, there are 10 bacteria. According to this formula, how many cells will be in the group at the end of the week?

Now we must identify what t is (number of days) and what the original population is (10). To find the number of days—use your brain—how many days are in a week? 7! So $t = 7$.

$$t = 7; y_0 = 10$$

$$y = y_0(3)^t$$

$$y = (10)(3)^7$$

$$y = 10(2187) = 21870$$

Answer: 21870.

TIP: Always double check your logic! What makes these problems tough is that they're integrated with real world problems. Even if something is modeled by an equation, if that something is a physical distance, it can't be negative. If it's a number of items sold, it also can't be negative—make sure your answer at the end makes sense.

A microscope company's profit, P dollars, when m microscopes are made and sold can be modeled by $P(m) = m^2 - 440m - 30{,}000$. What is the least number of microscopes the company must make and sell in order for the company to not lose money on this production run?

Let's take the problem above—we want to make a profit—how do you make a profit? By getting more than "0" for the letter P—because P is profit! How do we get P above zero? Well, imagining $P = y$ and $m = x$, this is an upwards facing parabola, so it's going to sink down and then rise back up. What we want to know is when P rises above 0—so to solve for that I set P equal to 0 and solve:

$$0 = m^2 - 440m - 30{,}000$$

This is a basic quadratic equation—we can solve by factoring or the quadratic equation. I'll factor, but you can always use the quadratic equation (you can even program your calculator to do the quadratic equation for you), or solve by graphing.

$$0 = (m - 500)(m + 60)$$

$$m = 500 \text{ or } -60$$

Now here's the tough part—because this is a word problem, you're not actually looking for -60, even though it's the smaller answer of the two. You have to realize that you can't make a negative number of microscopes. That doesn't work, so the answer is 500. If you make between $0 - 499$ microscopes, you'll lose money, because at 500, they neither lose nor earn money.

Answer: 500.

TIP: **When in doubt, you're looking for "zeros," points of intersection, and vertexes.** At the heart of all these problems are the same ideas that you're solving for in regular problems involving quadratics, exponential functions, and linear functions. If you're having trouble figuring out what the problem is looking for, ask yourself if any of these ideas (zeros, points of intersection, vertexes) will push you forward.

UNDERSTANDING EQUATIONS

Finally, you'll confront questions that test your ability to understand which numbers mean what in a given equation. You'll need to be able to decipher where each variable and constant came from and what they mean. Many of the same tips that we just mentioned will come into play in these types of problems. One of the best things you can do on these is to start plugging in values and understand the equation by working with it.

A paper airplane is thrown from the second story balcony of a building. The flight of a paper airplane can be modeled by the equation $h(t)=-\frac{t^2}{5}+t+10$, where t is the time in seconds after the paper airplane has been launched and h is the height of the paper airplane in feet. According to this equation, which of the following statements is true about the paper airplane?

A. After 10 seconds, the paper airplane reached its maximum height.
B. After 5 seconds, the paper airplane reached its maximum height.
C. After 11.25 seconds the paper airplane reaches its maximum height.
D. After 10 seconds the paper airplane hits the ground.
E. After 2.5 seconds the paper airplane hits the ground.

Notice that ALL the choices give us a "seconds" to deal with—we can solve this out by backsolving and plugging in the seconds for the value t. Let's start with 10 (answers A & D) because it's in two different answer choices AND in the problem.

$$h(t)=-\frac{t^2}{5}+t+10$$

$$h(10)=-\frac{10^2}{5}+10+10$$

$$h(10)=-\frac{100}{5}+20$$

$$h(10)=-20+20=0$$

The height (h) is zero when we're at 10 seconds. That's answer choice D and we're done. If we weren't so lucky, we could continue to plug in, look for other intercepts (plug in zero), or look for the vertex value to help us narrow the field. Remember, the greatest height is often the vertex, the ground or landing is a zero, etc.

Answer: **D.** After 10 seconds, the paper airplane hits the ground.