# RATIONAL EXPRESSIONS AND EQUATIONS

*ACT Math: Problem Set*

**1.** The expression $\dfrac{6x+12}{24x^3}$ is equivalent to which of the following?

**A.** $\dfrac{1}{x^2}+\dfrac{1}{2x^3}$

**B.** $\dfrac{1}{2x^2}+\dfrac{1}{4x^3}$

**C.** $\dfrac{1}{4x^2}+\dfrac{1}{2x^3}$

**D.** $\dfrac{1}{2x^2}+\dfrac{1}{2x^3}$

**E.** $\dfrac{1}{x^2}-\dfrac{1}{2x^3}$

**2.** For $x \neq 15$, $\dfrac{x^2-225}{(x-15)^3}=?$

**A.** $\dfrac{x+15}{(x-15)^2}$

**B.** $\dfrac{x-15}{(x+15)^2}$

**C.** $\dfrac{x+15}{(x+15)^2}$

**D.** $\dfrac{x-15}{(x-15)^2}$

**E.** $\dfrac{x+15}{(x-15)^3}$

**3.** When $x \neq 3$ and $x \neq -3$, $\dfrac{5x}{x^2-6x+9}+\dfrac{5x}{9-x^2}$ is equivalent to:

**A.** $0$

**B.** $\dfrac{30x}{(x-3)^2(x+3)}$

**C.** $\dfrac{-30x^2}{(x-3)^2(x+3)}$

**D.** $\dfrac{10x^2+30x}{(x-3)^2(x+3)}$

**E.** $\dfrac{15x}{(x-3)^2(x+3)}$

**4.** For all $x$ in the domain of the function $\dfrac{(x-3)^2}{x^4-9x^2}$, this function is equivalent to:

**A.** $\dfrac{2}{x^2}$

**B.** $\dfrac{1}{x^2}$

**C.** $\dfrac{(x-3)}{x^3+3x^2}$

**D.** $\dfrac{(x-3)}{(x+3)}$

**E.** $\dfrac{1}{x^4}-\dfrac{1}{9x^2}$

**5.** For all $x>6$, $\dfrac{(x^2+7x+10)(x-2)}{(x^2+x-6)(x+5)}=?$

**A.** $\dfrac{x+2}{x+3}$

**B.** $\dfrac{x+3}{x+2}$

**C.** $\dfrac{x-2}{x+5}$

**D.** $\dfrac{x+2}{x+5}$

**E.** $\dfrac{x+2}{x-5}$

**6.** If $\dfrac{A}{15}+\dfrac{B}{100}=\dfrac{20A+3B}{5x}$ and $A, B$, and $x$ are integers greater than $1$, then what must $x$ be equal to?

**A.** 23

**B.** 60

**C.** 115

**D.** 300

**E.** 1500

**7.** When $\frac{x+2}{-x^3-5x^2-6x}$ is defined, it is equivalent to which of the following expressions?

A. $\frac{-1}{x+3}$

B. $\frac{1}{x+3}$

C. $\frac{-x}{x+3}$

D. $\frac{-1}{x^2+3x}$

E. $\frac{1}{x^2+3x}$

**8.** If $m>0$, $\frac{4}{m}-\frac{m}{7}=$ ?

A. $\frac{28-4m}{7m}$

B. $\frac{28-4m}{m^2}$

C. $\frac{4-m}{7m}$

D. $\frac{28-m^2}{7m}$

E. $\frac{4-m}{m-7}$

**9.** When $x \neq 4$ and $x \neq -4$, $\frac{3x}{x^2-16}+\frac{3x}{4-x}$ is equivalent to:

A. $\frac{3x^2}{x^2-16}$

B. $\frac{-3x^2-12x}{x^2-16}$

C. $\frac{-3x^2-9x}{x^2-16}$

D. $\frac{3x^2-15x}{x^2-16}$

E. $\frac{-9x}{x^2-16}$

**10.** Which of the following expressions is equal to $\frac{5}{7-\sqrt{8}}$ ?

A. $\frac{35-\sqrt{40}}{41}$

B. $\frac{35+5\sqrt{8}}{41}$

C. $\frac{5}{33}$

D. $\frac{35-5\sqrt{8}}{33}$

E. $\frac{5}{41}$

**ANSWER KEY**

**1. C** **2. A** **3. B** **4. C** **5. A** **6. D** **7. D** **8. D** **9. C** **10. B**

**ANSWER EXPLANATIONS**

1. **C.** We can split the fraction up into two separate fractions to get $\frac{6x}{24x^3}+\frac{12}{24x^3}$. Simplifying each fraction, we get $\frac{1}{4x^2}+\frac{1}{2x^3}$.

2. **A.** We must first recognize that the numerator is a difference of two squares. We can then factor it to be $x^2-225=(x+15)(x-15)$. Rewriting the fraction, we get $\frac{(x+15)(x-15)}{(x-15)^3}=\frac{x+15}{(x-15)^2}$.

3. **B.** In order to find a common denominator for both fractions, we first factor both denominators. We get $x^2-6x+9=(x-3)(x-3)$ for the first denominator, and $9-x^2=(3-x)(x+3)$ for the second denominator. We can write the second denominator as $9-x^2=-(x-3)(x+3)$. So, the equation can be rewritten as $\frac{5x}{(x-3)(x-3)}-\frac{5x}{(x-3)(x+3)}$. The least common multiple of both denominators is $(x-3)(x-3)(x+3)$.

   Writing both fractions with the new denominator, we must multiply the fraction by $\frac{x+3}{x+3}$ and the second fraction by $\frac{x-3}{x-3}$ which gives us $\frac{5x}{(x-3)(x-3)}*\frac{x+3}{x+3}-\frac{5x}{(x-3)(x+3)}*\frac{x-3}{x-3}=\frac{5x(x+3)}{(x-3)(x-3)(x+3)}$ $-\frac{5x(x-3)}{(x-3)(x-3)(x+3)}$. Now, we can add the numerators to get $\frac{5x(x+3)-5x(x-3)}{(x-3)(x-3)(x+3)}$. Distributing $5x$, we get $\frac{5x^2+15x-(5x^2-15x)}{(x-3)(x-3)(x+3)}=\frac{5x^2+15x-5x^2+15x}{(x-3)(x-3)(x+3)}=\frac{30x}{(x-3)(x-3)(x+3)}=\frac{30x}{(x-3)^2(x+3)}$.

4. **C.** Factor out an $x^2$ from the denominator $\frac{(x-3)^2}{x^4-9x^2}=\frac{(x-3)^2}{x^2(x^2-9)}$. $x^2-9$ is the difference of squares, so your expression becomes $\frac{(x-3)^2}{x^2(x+3)(x-3)}$. Simplify to get $\frac{(x-3)}{x^3+3x^2}$, answer (C).

   Answer (A) incorrectly tries to square the two terms in the numerator instead of expanding, and then tries to split the sums and differences into two different fractions. Answer (B) again tries to square the numerator incorrectly, although it simplifies the resultant expression correctly. Answer (D) forgets the $x^2$ term that was factored out in the beginning, and answer (E) disregards the numerator and incorrectly assumes that the denominator sum is the same as the sum of fractions.

5. **A.** We want to factor $(x^2+7x+10)$ and $(x^2+x-6)$ into something we can cancel out. To factor $(x^2+7x+10)$, we need two numbers that add up to equal $7$ and multiply to equal $10$. The numbers $5$ and $2$ work, so we can factor it as $(x+2)(x+5)$. To factor $(x^2+x-6)$, we need two numbers that add up to equal $1$ and multiply to equal $-6$. The numbers $-2$ and $3$ work, so we can factor it as $(x+3)(x-2)$. Substituting these factored forms in, we get $\frac{(x+2)(x+5)(x-2)}{(x+3)(x-2)(x+5)}$. Canceling out terms, we get $\frac{x+2}{x+3}$.

6. **D.** This question is tricky because with numbers like these you must trust that it will work out. Instead of trying to find a common denominator to add the fractions, first find a way to set the numerators equal. Multiply the $A$ term top and bottom by $20$, and the $B$ term top and bottom by $3$: $\frac{A}{15}\left(\frac{20}{20}\right)+\frac{B}{100}\left(\frac{3}{3}\right)=\frac{20A+3B}{5x}\rightarrow\frac{20A}{300}+\frac{3B}{300}=\frac{20A+3B}{5x}$.

   Without even trying, we have a common denominator and can thus combine these two fractions!

   $\frac{20A}{300}+\frac{3B}{300}=\frac{20A+3B}{5x}\rightarrow\frac{20A+3B}{300}=\frac{20A+3B}{5x}$. Since the numerators are equal, the denominators must be equal as well, giving is the equation: $300=5x\rightarrow x=60$.

7. **D.** Because the expression is defined, we don't have to worry about any holes or asymptotes, we can just simplify the expression as far as we can go. Factor the denominator and simplify:
   $\frac{x+2}{-x^3-5x^2-6x}\rightarrow\frac{x+2}{-x\left(x+3\right)\left(x+2\right)}\rightarrow\frac{x+2}{-x\left(x+3\right)\left(x+2\right)}\rightarrow\frac{-1}{x^2+3x}$.

8. **D.** Multiplying the first fraction by $\frac{7}{7}$ and the second fraction by $\frac{m}{m}$, we get $\frac{4}{m}\left(\frac{7}{7}\right)-\frac{m}{7}\left(\frac{m}{m}\right)=\frac{28-m^2}{7m}$.

9. **C.** To make things easier, we can rewrite $\frac{3x}{4-x}$ as $\frac{3x}{-x+4}$. We want the two fractions have a common denominator, so we multiply $\frac{3x}{-x+4}$ by $\frac{-x-4}{-x-4}$ to have a denominator of $x^2-16$. This equals $\frac{-3x^2-12x}{x^2-16}$. Now with a common denominator, we can add the two fractions together: $\frac{3x}{x^2-16}+\frac{-3x^2-12x}{x^2-16}=\frac{-3x^2-9x}{x^2-16}$.

10. **B.** When a square root is in the denominator, we multiply the top and bottom by the same expression with the sign for the square root flipped. In this problem, we would multiply $\frac{5}{7-\sqrt{8}}$ by $\frac{5}{7+\sqrt{8}}$, which equals
    $\frac{5\left(7+\sqrt{8}\right)}{\left(7-\sqrt{8}\right)\left(7+\sqrt{8}\right)}=\frac{35+5\sqrt{8}}{49-8}\rightarrow\frac{35+5\sqrt{8}}{41}$.