FUNCTIONS

ACT Math: Problem Set

1. Given $f(x)=2x^2-3x+8.$ what is the value of $f(-5)$?

A. 43
B. −27
C. 3
D. 13
E. 73

2. Given the function $m(t)=4t^2-7,$ what is $m(-3)$?

A. −19
B. −31
C. −43
D. 17
E. 29

3. For the function $v(n)=3n^2-4n,$ what is the value of $v(-6)$?

A. −624
B. −84
C. −6
D. 84
E. 132

4. A function $f(x)$ is defined as $f(x)=-9x^2;$ what is $f(-4)$?

A. 144
B. −144
C. 72
D. −72
E. 7

5. What is the value of $h\left(\frac{1}{4}\right)$ when $h(x)=-16x^2+32x-9$?

A. −2
B. −5
C. −9
D. 0
E. −8

6. For the function $h(x)=3x^2-5x,$ what is the value of $h(-4)$?

A. 28
B. 68
C. −28
D. 44
E. −172

7. What is the value of $g(-3)$ if $g(x)=3x^2-5x+11$?

A. −1
B. 53
C. 23
D. 44
E. −55

8. Given $f(x)=2x+3$ and $g(x)=x^2-1,$ which of the following is an expression for $f(g(x))$?

A. $4x^2+12x+8$
B. $4x^2+12x+9$
C. x^2+2x+2
D. $2x^2+1$
E. $2x^2+5$

9. The function $f(x)=x^3+3x-2.$ What is $f(x+h)$?

A. $x^3+h^3+3x+3h-2$
B. $h^3+3h^2x+3hx^2+3h+x^3+3x-2$
C. $h^3-3h^2x+3hx^2+3h-x^3+3x-2$
D. $h^3+3h^2x-3hx^2+3h+x^3-3x-2$
E. $h^3+3h^2x+3h^2x^2+3h+x^3+3x-2$

10. $f(x)=\begin{cases}|3x|+1, if\ x>\text{-}4\\ 3x^3+1, if\ x>\text{-}4\end{cases}$

What is the value of $f(-3)$?

A. −80
B. −82
C. 80
D. 82
E. 193

11. There are 2 functions $f(x)$ and $g(x)$ such that $f(x)=\frac{4-x}{7+x}$ and $g(x)=2x^2-3x+1.$ What is $f(g(3))$?

A. $\frac{-6}{17}$
B. $\frac{6}{17}$
C. $\frac{14}{17}$
D. $\frac{14}{3}$
E. $\frac{17}{5}$

12. If $f(x)=\frac{1}{x^2+5}$, what is $f(f(2))$?

A. $\frac{1}{9}$

B. $\frac{1}{86}$

C. $\frac{81}{406}$

D. $\frac{1}{76}$

E. $\frac{1}{21}$

13. Tables of values for the functions $f(x)$ and $g(x)$ are shown below. What is $f(g(7))$?

x	$f(x)$
-4	7
-1	-2
7	3
4	9

x	$g(x)$
-2	-1
3	8
5	2
7	4

A. 9
B. 7
C. 8
D. 4
E. 3

14. If $f(x)=2x+3$ and $g(x)=3x^2-1$, which of the following is the expression for $g(f(x))$?

A. $4x^2+12x+8$

B. $12x^2+36x+26$

C. $12x^2-36x+26$

D. $6x^2+36x-26$

E. $12x^2+18x+26$

15. Of the 5 functions below, each denoted by $h(x)$ and each involving a real number $a\geq 3$, which function yields the smallest value of $f(h(x))$, if $f(x)=\frac{1}{x}, x>1$?

A. $h(x)=ax$

B. $h(x)=\frac{a}{x}$

C. $h(x)=\frac{x}{a}$

D. $h(x)=x+a$

E. $h(x)=x^a$

16. We have 2 functions $f(x)=\sqrt[3]{x}$ and $g(x)=3x-d$. If $f(g(27))=4$, what is the value of d?

A. 13
B. 14
C. 15
D. 16
E. 17

17. If $P(x)=-x^4$, what is $P(P(x))$?

A. $-x^{16}$

B. x^8

C. x^{16}

D. x^{-8}

E. $-x^8$

18. The operation ℧ is defined as "add five to the square of the number on the left of ℧ and subtract the result from the number on the right. What is the value of 3 ℧(4℧5)?

A. 30
B. -2
C. 2
D. -30
E. -100

19. Given the operation $x \mathfrak{H} y = xy^2 + 2y$, what is $(1\mathfrak{H}2)\mathfrak{H}3$?

A. 1200
B. −78
C. 78
D. 66
E. −1200

20. The function $f(x,y) = 2x + 9y$. What is $f(x,y)$ when $y = 4$ and $x = y^{\frac{3}{2}}$?

A. 52
B. 34
C. 80
D. 26
E. 164

21. Given $f(x) = \sqrt[4]{2x+1}$, which of the following expressions is equal to $f^{-1}(x)$ for all real numbers x?

A. $\dfrac{1}{\sqrt[4]{2x+1}}$
B. $\dfrac{x^4+1}{2}$
C. $\dfrac{x^4-1}{2}$
D. $(2x+1)^{\frac{-1}{4}}$
E. $\sqrt[4]{2x+1}$

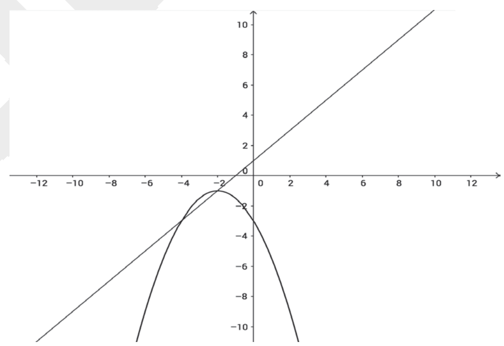
22. For each positive integer q, let $q\nabla$ be the product of all positive odd numbers less than or equal to q. For example, $9\nabla = (9)(7)(5)(3)(1) = 945$ and $10\nabla = (9)(7)(5)(3)(1) \rightarrow 945$. What is $\dfrac{13\nabla}{4\nabla}$?

A. 135135
B. 945
C. 45045
D. 154440
E. 10395

23. Which if the following pairs of functions, $f(x)$ and $g(x)$, form the composite function $g(f(x)) = \sqrt{4x^3 - 9}$?

	$f(x)$	$g(x)$
A.	$4x$	$\sqrt{x^3-9}$
B.	x^3+9	$\sqrt{4x}$
C.	x^3	$\sqrt{4x-9}$
D.	$\sqrt{x^3-9}$	$4x$
E.	x^3+9	$\sqrt{4x-9}$

24. The graph of the functions $y = f(x) = x + 1$ and $y = g(x) = -\dfrac{x^2}{2} - 2x - 3$ are shown in the standard (x,y) coordinate plane below. Which if the following is NOT true?



A. $f(-4) = g(-4)$
B. $g(-2) = f(-2)$
C. $f(g(2)) = -8$
D. $-|f(x)| = f(x)$
E. $-|g(x)| = g(x)$

ANSWER KEY

1. E **2. E** **3. E** **4. B** **5. A** **6. B** **7. B** **8. D** **9. B** **10. D** **11. A** **12. C** **13. A** **14. B**
15. E **16. E** **17. A** **18. D** **19. C** **20. A** **21. C** **22. C** **23. C** **24. D**

ANSWER EXPLANATIONS

1. **E.** Plugging in $x=-5$, we get $f(-5)=2(-5)^2-3(-5)+8$. Simplifying this gives us $f(-5)=2(25)-(-15)+8\rightarrow 50+15+8=73$. So, $f(-5)=73$.

2. **E.** Plugging in $t=-3$, we get $m(-3)=4(-3)^2-7$. Simplifying this gives us $m(-3)=4(9)-7\rightarrow 36-7=29$. So, $m(-3)=29$.

3. **E.** Plugging in $n=-6$, we get $v(-6)=3(-6)^2-4(-6)$. Simplifying this gives us $v(-6)=3(36)-(-24)\rightarrow 108+24=132$. So, $v(-6)=132$.

4. **B.** Plugging in $x=-4$, we get $f(-4)=-9(-4)^2$. Simplifying this gives us $f(-4)=-9(16)\rightarrow -144$. So, $f(-4)=-144$.

5. **A.** Plugging in $x=1/4$, we get $h\left(\frac{1}{4}\right)=-16\left(\frac{1}{4}\right)^2+32\left(\frac{1}{4}\right)-9$. Simplifying this gives us

$h\left(\frac{1}{4}\right)=-16\left(\frac{1}{16}\right)+32\left(\frac{1}{4}\right)-9\rightarrow -1+8-9\rightarrow -2$. So, $h\left(\frac{1}{4}\right)=-2$.

6. **B.** Plugging in $x=-4$, we get $h(-4)=3(-4)^2-5(-4)$. Simplifying this gives us $h(-4)=3(16)-(-20)\rightarrow 48+20\rightarrow 68$. So, $h(-4)=68$.

7. **B.** Plugging in $x=-3$, we get $g(-3)=3(-3)^2-5(-3)+11$. Simplifying this gives us $g(-3)=3(9)-(-15)+11\rightarrow 27+15+11\rightarrow 53$. So, $g(-3)=53$.

8. **D.** Plugging in $g(x)=x^2-1$ into the nested function $f(g(x))$, we get $f(g(x))=2(x^2-1)+3$. (Instead of plugging in a number, we are substituting in the entire expression x^2-1 to the x value in $f(x)=2x+3$). Simplifying this gives us $f(g(x))=2x^2-2+3\rightarrow 2x^2+1$. So, $f(g(x))=2x^2+1$.

9. **B.** Plugging in $x+h$ for x in $f(x)$, we have $f(x+h)=(x+h)^3+3(x+h)-2\rightarrow x^3+3x^2h+3xh^2+h^3+3x+3h-2$. We can rearrange this to be $h^3+3h^2x+3hx^2+3h+x^3+3x-2$.

10. **D.** Since $-3>-4$, we use the second expression to evaluate $f(-3)$. So, $f(-3)=\left|3(-3)^3\right|+1\rightarrow \left|3(-27)\right|+1\rightarrow \left|-81\right|+1\rightarrow 81+1=82$.

11. **A.** We first find $g(3)$ and then plug in that value to find $f(g(3))$. So, solving for $g(3)$, we plug in $x=3$ for $g(x)$ to get $g(3)=2(3)^3-3(3)+1\rightarrow 2(9)-9+1\rightarrow 10$. Now, to find $f(g(3))$, we plug in $x=g(3)=10$ for $f(x)$. This gives us $f(10)=\frac{4-10}{7+10}\rightarrow -\frac{6}{17}$.

12. C. We first find $f(2)$ and then plug in that value to find $f(f(2))$. So, solving for $f(2)$, we plug in $x=2$ for $f(x)$ to get $f(2)=\frac{1}{x^2+5}\to\frac{1}{2^2+5}\to\frac{1}{9}$. Now, to find $f(f(2))$, we plug in $x=f(2)=\frac{1}{9}$ for $f(x)$. This gives us $f\left(\frac{1}{9}\right)=\frac{1}{\left(\frac{1}{9}\right)^2+5}\to\frac{1}{\frac{1}{81}+5}\to\frac{1}{\frac{1}{81}+\frac{405}{81}}\to\frac{1}{\frac{406}{81}}\to\frac{81}{406}$.

13. A. We first find $g(7)$ and then plug in that value to find $f(g(7))$. Looking at the values given in the table for $g(x)$, we see that $g(7)=4$. Now, to find $f(g(7))$, we plug in $x=g(7)=4$ for $f(x)$ and see by the first table that $f(4)=9$.

14. B. To find $g(f(x))$, we replace the x 's in $g(x)$ with $f(x)=2x+3$. This gives us $g(f(x))=3(2x+3)^2-1$. Using FOIL to expand the polynomial, we get $3(4x^2+12x+9)-1$. Distributing the 3, we get $12x^2+36x+27-1=12x^2+36x+26$.

15. E. Since $f(x)=\frac{1}{x}$ for a positive x, the larger x is, the smaller $f(x)$ is. So, we are looking for the largest $h(a)$ value from our answer choices to yield the smallest value of $f(h(x))$. Since $a\geq 3$, x^a would be the greatest value because the exponent increases x most rapidly.

16. E. Plugging in $x=27$ for $g(x)=3x-d$, we get $g(27)=3(27)-d\to 81-d$. Then, plugging in $x=81-d$ for $f(x)=\sqrt[3]{x}$, we get $f(g(27))=f(81-d)\to\sqrt[3]{81-d}$ we are given that this value is equal to 4. So, we now want to solve the equation $4=\sqrt[3]{81-d}$ for d. Cubing both sides, we get $64=81-d$. Subtracting 81 on both sides, we get $-17=-d$. So, $d=17$.

17. A. Plugging in $P(x)=-x^4$ for x in $P(x)$, we get $P(P(x))=-(-x^4)^4\to-(x^{16})\to-x^{16}$.

18. D. We first solve for the value inside the parentheses. $(4\mho 5)$ means $5-(4^2+5)=5-(16+5)\to 5-21\to-16$. Taking this value and plugging it back in we have $3\mho(-16)$ or $-16-(3^2+5)=-16-(9+5)\to-16-14\to-30$.

19. C. We first solve for the value inside the parentheses. $(1\mathfrak{H}2)$ means we plug in $x=1$ and $y=2$ for xy^2+2y. This gives us $(1(2^2))+2(2)=8$. Then, we plug in $1\mathfrak{H}2=8$ to solve for $(1\mathfrak{H}2)\mathfrak{H}3=8\mathfrak{H}3$. This means we plug $x=8$ and $y=3$ in for xy^2+2y. This gives us $8(3)^2+2(3)\to 8(9)+6\to 72+6=78$.

20. A. Plugging in $y=4$ and $x=y^{\frac{3}{2}}\to 4^{\frac{3}{2}}=8$ we get $f(8,4)=2(4)+9(4)\to 16+36=52$.

21. C. To find $f^{-1}(x)$ we switch the x and y values in the function $y=\sqrt[4]{2x+1}$ and solve for y. This gives us $x=\sqrt[4]{2y+1}$. Taking both sides to the power of 4, we get $x^4=2y+1$. Subtracting 1 and dividing both sides by 2, we get $\frac{x^4-1}{2}=y$.

22. C. $\frac{13\nabla}{4\nabla} = \frac{(13)(11)(9)(7)(5)(3)(1)}{(3)(1)}$. Canceling out common factors in the numerator and denominator, we get

$$\frac{(13)(11)(9)(7)(5)(\cancel{3})(\cancel{1})}{(\cancel{3})(\cancel{1})} = (13)(11)(9)(7)(5) \to 45045.$$

23. C. The only tricky thing about this is that we are told to find $g\left(f\left(x\right)\right)$, which is not the same as $f\left(g\left(x\right)\right)$. That rules out answer choice D, because we want the square root to encapsulate the entire expression, which means that the square root should be in the other function, $g(x)$. Then looking at A, B, C, and E, we see that only D works, correctly substituting x with x^3: $\sqrt{4(x^3)-9}$.

24. D. The graph of $f(x)$ crosses the x-axis, which makes all the difference. If it were to exist only below the x axis as $g(x)$ does, $-\left|f(x)\right| = f(x)$ would be true. It we were to graph the absolute value of $f(x)$, we would get a graph that looked like a "v", since all the points must be positive. If we were to reflect this graph over the x-axis the graph would still be shaped like a "v", but it would be upside down.