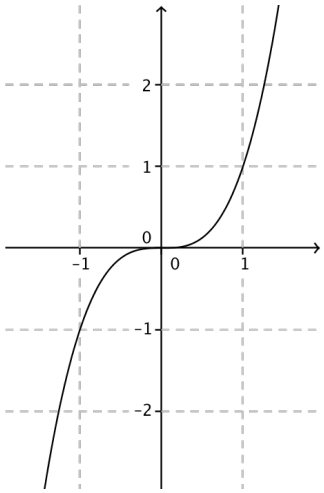
The Best ACT Prep Course Ever

TRANSLATIONS AND REFLECTIONS

ACT Math: Problem Set

1. The graph $y = x^3$ is shown below. Which of the equations below describe this parabola shifted 4 units to the left and 2 units up?



A. $y = (x+2)^2 - 4$

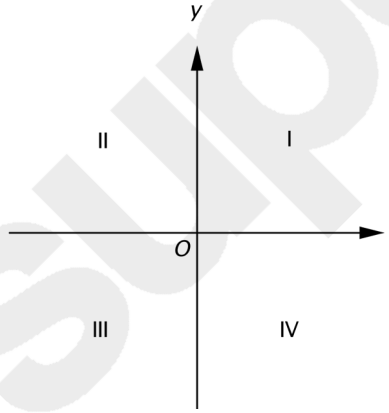
B. $y = (x-2)^2 - 4$

C. $y = (x+4)^2 - 2$

D. $y = (x+4)^2 + 2$

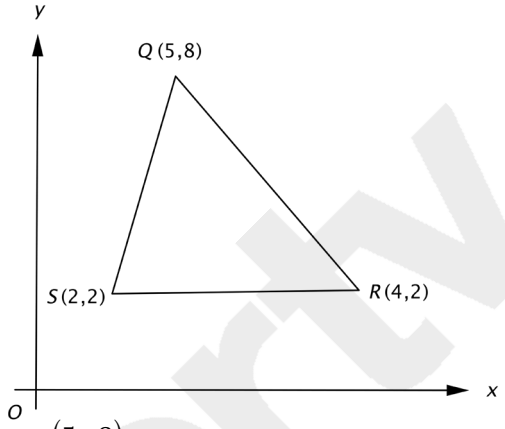
E. $y = (x-4)^2 + 2$

2. In the standard (x, y) coordinate plane shown below, a rectangle has points $(-2,1), (-2,4), (3,1)$, and $(3,4)$. The rectangle is shifted 5 units to the left and 3 units down. Which quadrants does the rectangle exist in?



A. I and II
B. II and III
C. I and IV
D. III
E. All quadrants

3. The triangle below is reflected over $y = 1$. What are the coordinates for point Q after the reflection?



A. $(5,-8)$

B. $(-5,-8)$

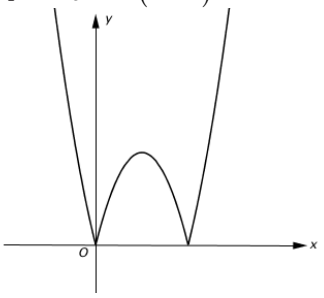
C. $(-5,8)$

D. $(5,-6)$

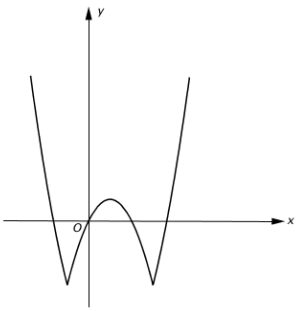
E. $(5,-7)$

4. The graph in the standard (x, y) coordinate plane below is the graph of $y = f(x)$. Which of the following graphs is the graph of $y = f(x+1) - 3$?

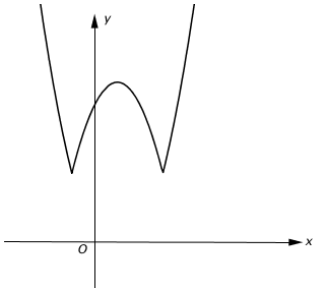
A.



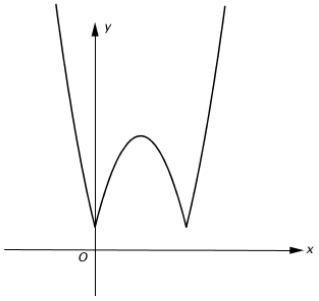
B.



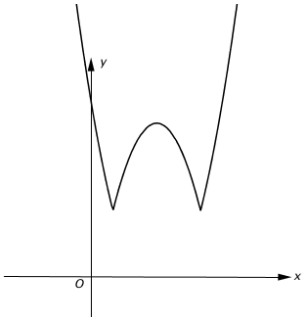
C.



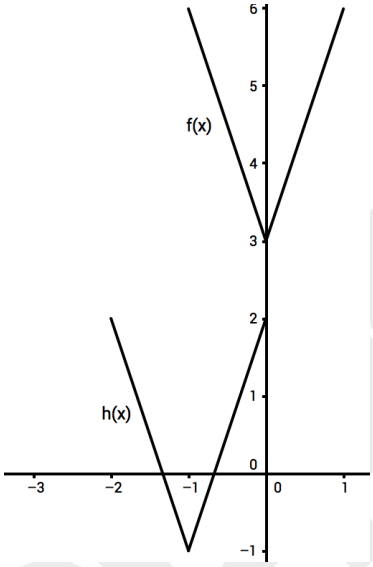
D.



E.



5. Which of the following expressions represents the graph of $h(x)$ in terms of $f(x)$?



A. $|x+1|-4$

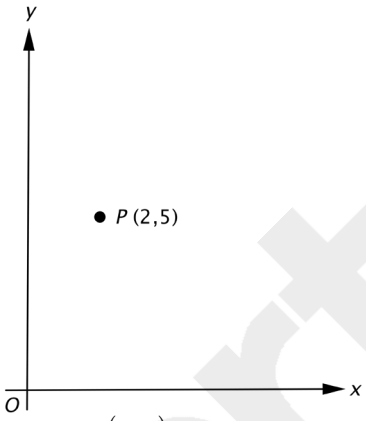
B. $|x-4|+1$

C. $|x-1|+4$

D. $|x+4|-1$

E. $|x-4|+4$

6. The point P is shown below. What are the coordinates of the point P after it is reflected over the line $x=3$?



A. $(1,2)$

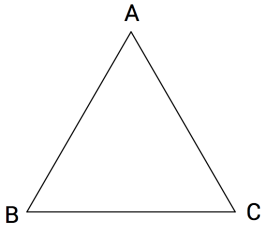
B. $(2,1)$

C. $(4,2)$

D. $(4,5)$

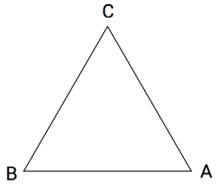
E. $(5,4)$

7.

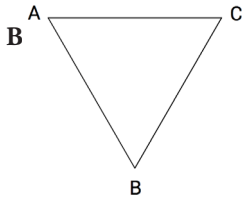


Triangle ABC is pictured above. Which of the following represents the orientation of the triangle after being reflected over $\overline{AB}$ and then rotated $90°$ clockwise around point B?

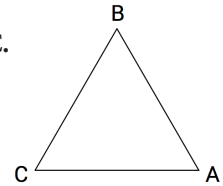
A



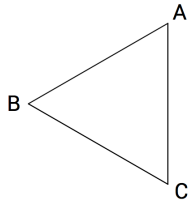
B



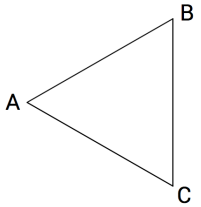
C.



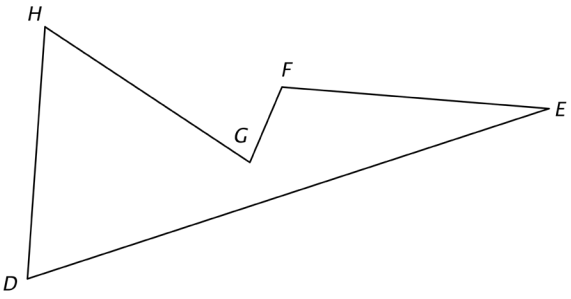
D.



E.



8. The figure below has side lengths of $3,3,2,3,$ and 7. Let $D'H'G'F'E'$ represent the shape after being reflected over $\overline{DE}$. What is the perimeter of the shape $DHGFEF'G'H'$?

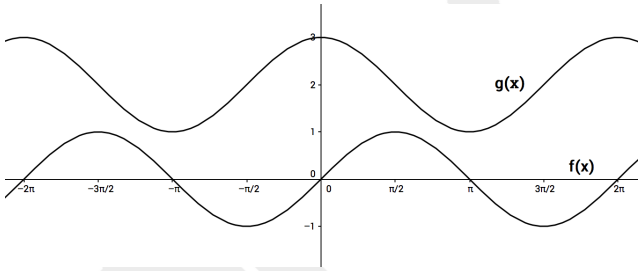


A. 11
B. 18
C. 22
D. 30
E. 36

9. A point at $(-7,3)$ in the standard (x,y) coordinate plane is shifted right 3 units and down 7 units. What are the coordinates of the point?

A. $(-10,10)$
B. $(0,0)$
C. $(-10,-10)$
D. $(-4,10)$
E. $(-4,-4)$

10. The graphs of $f(x)=\sin x$ and $g(x)=\sin\left(x+\frac{\pi}{2}\right)+2$ are shown in the standard (x,y) coordinate plane below. After one of the following pairs of transformations is applied to the graph of $f(x)$, the image of the graph of $f(x)$ is the graph of $g(x)$. Which is it?



A. Shift $f(x)$ 2 units left and $\frac{\pi}{2}$ units down.
B. Shift $f(x)$ 2 units left and $\frac{\pi}{2}$ units up.
C. Shift $f(x)$ 2 units right and $\frac{\pi}{2}$ units down.
D. Shift $f(x)$ $\frac{\pi}{2}$ units right and 2 units up.
E. Shift $f(x)$ $\frac{\pi}{2}$ units left and 2 units up.

11. Rectangle $ABCD$ has vertices in the standard (x,y) coordinate plane at $A(-2,-4)$, $B(-2,6)$, $C(1,6)$, and $D(1,-4)$. A translation of rectangle $ABCD$ is a second rectangle $A'B'C'D'$ with vertices $A'(5,1)$, $B'(x,y)$, $C'(8,11)$, $D'(8,1)$. What are the coordinates of B'?

A. $(3,10)$
B. $(4,10)$
C. $(5,4)$
D. $(5,0)$
E. $(5,11)$

12. In the standard (x,y) coordinate plane, $P(-5,-3)$ will be reflected over the y - axis. What will be the coordinates of the image of P ?

A. $(-5,3)$

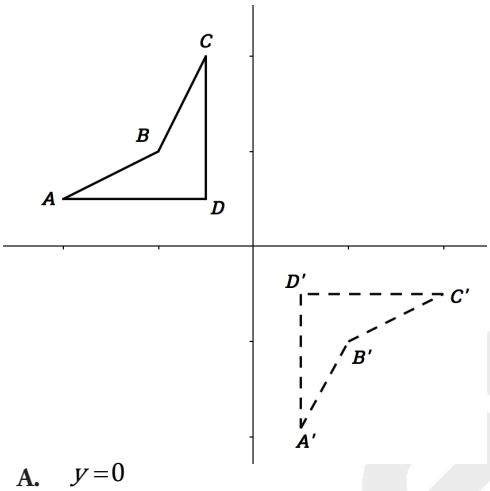
B. $(-3,5)$

C. $(3,-5)$

D. $(3,5)$

E. $(5,-3)$

13. Figure $ABCD$, shown in the standard (x,y) coordinate plane below, has been reflected across a line to figure $A'B'C'D'$. Which of the following lines reflection would best describe this transformation?



A. $y=0$

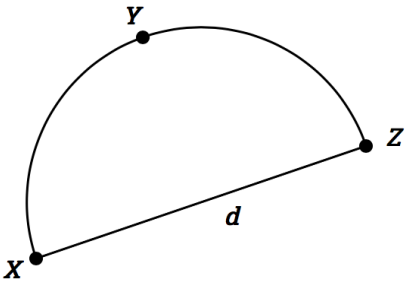
B. $x=0$

C. $x=y$

D. $y=\frac{1}{2}$

E. $y=-x$

14. The semicircle XYZ that is shown below has a diameter of d inches. Let $\overset{\frown}{X'Y'Z}$ be the image of $\overset{\frown}{XYZ}$ reflected across $\overline{XZ}$. Which of the following is an expression for the perimeter, in inches, of the shape formed?



A. $2\left(\overline{XY}+\overline{YZ}\right)$

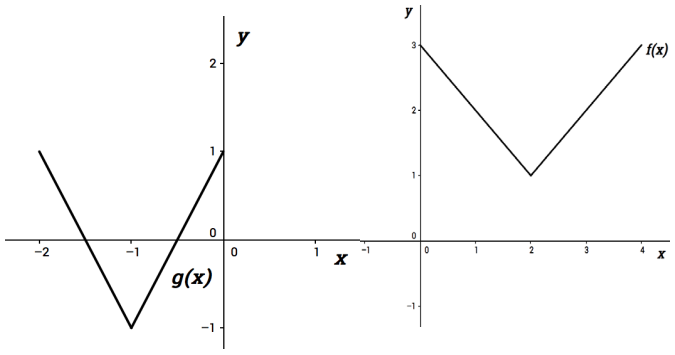
B. $4\left(\overline{XY}+\overline{YZ}\right)$

C. $\pi\overline{XZ}$

D. $2\pi\overline{XZ}$

E. $4\pi\overline{XZ}$

15. The graph of $f(x)$ and $g(x)$ are shown in the standard (x,y) coordinate planes below. Which one of the following expressions represents $g(x)$ in terms of $f(x)$?



A. $2f(x+3)+2$

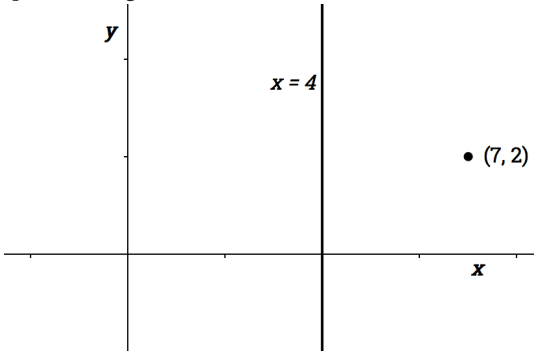
B. $2f(x+3)-2$

C. $2f(x-3)+2$

D. $f(x-3)-2$

E. $f(x+3)-2$

16. The point $(7,2)$ and the line $x = 4$ are graphed in the standard (x,y) coordinate plane below. After the point has been reflected across the line, what are the coordinates of the point's image?



A. $(1,2)$
B. $(-7,2)$
C. $(-1,2)$
D. $(1,7)$
E. $(1,-2)$

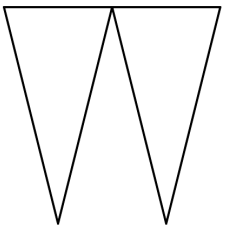
17. In the standard (x,y) coordinate plane, a triangle has vertices $(-1,5),(-1,-2)$, and $(2,5)$. What will be the coordinates of the vertices after the triangle is shifted down 3 units?

A. $(-1,2),(-1,1),(2,8)$
B. $(2,5),(-1,1),(2,8)$
C. $(-1,2),(-1,-5),(2,2)$
D. $(-1,8),(1,-1),(5,-2)$
E. $(2,5),(0,-1),(2,2)$

18. A triangle, $\triangle DEF$, is reflected across the x-axis to have the image $\triangle D'E'F'$ in the standard (x,y) coordinate plane; thus, D reflects to D'. The coordinate of point D' are $(a,-b)$. What are the coordinates of point D'?

A. (a,b)
B. $(-b,a)$
C. $(-a,b)$
D. $(b,-a)$
E. Cannot be determined from the given information.

19. For an assignment on symmetry, Joey created the shape displayed below. His teacher commented on the symmetry when evaluating this assignment. Which of the following is a true comment about the symmetry of this pattern?

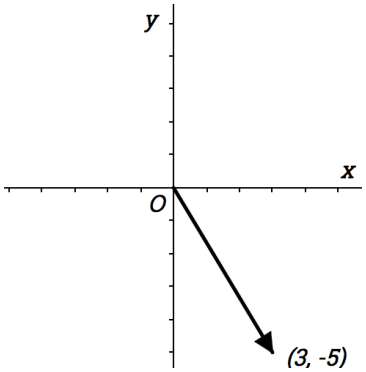


A. The pattern has both a horizontal line and a vertical line of symmetry.
B. The pattern only has a horizontal line of symmetry.
C. The pattern only has a vertical line of symmetry.
D. The pattern has a rotational symmetry of $180°$
E. The pattern has a rotational symmetry of $90°$

20. Which of the following letters of the alphabet has a rotational symmetry of less than $360°$?

A. L
B. W
C. U
D. X
E. N

21. A vector from the origin to the terminal point $(3,-5)$ is shown in the standard (x,y) coordinate plane below. The vector will be rotated $180°$ about the origin, resulting in a new vector. What will be the coordinates of the terminal point of the new vector?



A. $(5,-3)$
B. $(-3,5)$
C. $(-3,-5)$
D. $(3,5)$
E. $(-5,-3)$

22. The graph $\frac{x}{y}=5$ is reflected across the y axis in the in the standard (x,y) coordinate plane. Which of the following is an equation of the reflection?

A. $xy=5$

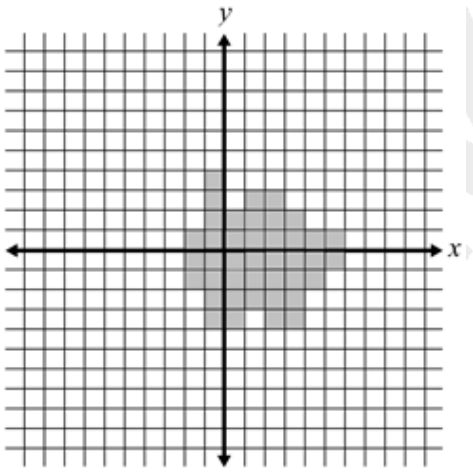
B. $\frac{y}{x}=-5$

C. $\frac{x}{|y|}=5$

D. $-x=5y$

E. $\frac{|x|}{y}=5$

23. Grid lines are shown at 1-unit intervals in the standard (x,y) coordinate plane below. Some of the 1 by 1 square are shaded in the grid. Which is the least number of 1 by 1 square that must be unshaded so that the total shaded region will be symmetric about the x-axis?



A. $xy=5$

B. $\frac{y}{x}=-5$

C. $\frac{x}{|y|}=5$

D. $-x=5y$

E. $\frac{|x|}{y}=5$

ANSWERS

1. D 2. B 3. D 4. A 5. A 6. D 7. D 8. C 9. E 10. E 11. E 12. A 13. C 14 .C
15. B 16. A 17. C 18. A 19. C 20. D 21. B 22. D 23. B

ANSWER EXPLANATIONS

1. D. The formula for a parabola is $y=(x-h)^2+k$. Since h undergoes a change of -4, and k undergoes a change of $+2$, the final equation is $y=(x+4)^2+2$.

2. B. The right side of the rectangle lies on the line $x=3$. After the rectangle has been translated 5 units to the left, the right side of the rectangle lies on $x=-2$. Now the rectangle is in quadrant 2. The top side of the rectangle lies on the line $y=4$ and the bottom side on the line $y=1$. After these lines has been translated 3 units down, they line on the lines $y=1$ and $y=-2$, respectively. Thus, the rectangle exits in quadrants II and III.

3. D. When the triangle is reflected over the line $y=1$, line SR lies on the horizontal axis. Because the height of the triangle is 6 units, Q lies on the line $y=-6$. The *x* coordinate of Q does not change because the triangle was reflected over a horizontal line.

4. A. Adding to x shifts the graph to the left, for example compare $y=x^2$ and $y=(x+5)^2$. Subtracting from the entire function shifts the graph downward. The only graph that does both is A.

5. A. The formula for the absolute value graph is $y=|x-h|+k$. If we look at the graph of $f(x)$, we can see that the vertex has shifted down 4 and to the left 1. This means, for the graph of $h(x)$, the value of $h=1$ and $k=-4$. The correct answer is A.

6. D. First, since the point is being reflected over a vertical line, the y-axis of point *P* will not change. Point *P* lies on the line $x=2$. Because it is one unit to the left of the line of reflection, $x=3$, the coordinates of *P* after reflection will be one unit away to the right, on line $x=4$.

7. D. When the triangle is reflected over $\overline{AB}$, the orientation of line $\overline{AB}$ remains the same, but point C is translated across the line, as in choice C. When the triangle is rotated $90°$ around point B, line $\overline{AC}$ rotates from its horizontal position to a vertical line.

8. C. When the shape is reflected over the line $\overline{DE}$, the reflecting line can be ignored for the calculation of the perimeter. Therefore, we can add the numbers $3,3,2,$ and 3 to calculate the perimeter of the top part of the shape, and again add the numbers $3,3,2,$ and 3 to find the perimeter of the bottom part of the shape. The answer is 22.

9. E. The point lies on the line $x=-7$. Because the point is shifted 3 units right, the point now lies on the line $x=-4$. The point also lies on the line $y=3$. When this point is shifted 7 units down, it will lie on the line $y=-4$. Therefore, the new point lies on $(-4,-4)$.

10. E. The formula for this equation is $y=\sin(x-h)+k$. The h pertains to horizontal shifting and k to vertical shifting. Since h for this equation is $-\frac{\pi}{2}$, the graph of $f(x)$ has shifted to the left $\frac{\pi}{2}$ units. Since k for this equation is $+2$, the graph of $f(x)$ has shifted up 2 units.

11. E. The simplest way to do this problem is to focus on the coordinates of the rectangle after the reflection. Since we can find the coordinate of *B'* with *A', C',* and *D',* there is no need to figure out the line of reflection. Because the shape is a rectangle, each x or y value will occur twice in the rectangle coordinates. For x values, since both *C'* and *D'* are situated at point 8, we know that point *B'* is situated at point 5. For y values, since both *A'* and *D'* are situated at point 11, we know that *B'* is situated at point

12. A. Because the point P is reflected over the y-axis, only the y coordinate of point P will change. P lies on the horizontal line $y=-3$, which is 3 units down from the y-axis. Therefore, when the point is reflected, it will be 3 units up from the y-axis. The point is at $(-5,3)$.

13. C. Each point and its reflected opposite must be equidistant from the reflecting line. If we look at D and D', we can see they are equidistant from the line $y=x$.

14. C. When the semicircle is reflected over $\overline{XZ}$, the new shape is a circle. The formula for the perimeter of any circle is πd.

15. B. The graph of $g(x)$ has shifted from $f(x)$ 2 units down and 3 units left. Since the equation for the graph is $f(x)=(x-h)+k$, h must be -3 and k must be $+2$.

16. A. Because the point is being reflected over a vertical line, only the x coordinate will change. The current point and the reflected point must be equidistant to the reflecting line. The point is 3 units right of the reflecting line. Therefore, the reflected point will be 3 units to the left of the reflecting line, at $y=1$.

17. C. Since we are only concerned with vertical shifting, the x- values for all numbers will remain the same. With this, we can eliminate all but A and C. After this, we subtract 3 from the y coordinates of all points $(5,2,5)$ and find the correct answer.

18. A. Because the point is being reflected over a horizontal line, only the y coordinate will change. Thus a will remain the same, but the sign of the y coordinate will change.

19. C. The line of symmetry can be viewed as the line where we could fold and see each point overlap. The only such line in this problem is the vertical line of symmetry.

20. D. No letters besides X have rotational symmetry until they have turned $360°$. However, the letter X achieves rotational symmetry at $180°$.

21. B. The vector is rotated $180°$ about the origin, which mean that the signs of the x and y values are switched. So, the new vector has terminal point at $(-3,5)$.

22. D. Normally when reflecting over one of the axis, it matters where the negative sign is placed, but in this equation because there is only one monomial, all you need to do is add a negative sign to either side of the equation, (which will reflect the graph over both axis, actually). It's good form to put it next to the x term like you would if the placement matter and you wanted the reflection over the y axis, but for this equation it doesn't really matter. $-\frac{x}{y}=5$ is not given as an answer, but if you multiply both sides by y, you get $-x=5y$, which is given.

23. B. To find the least number of squares that must be unshaded, you must go through all the shaded units on one side of the x axis (above or below). If the corresponding 1 by 1 unit on the other side of the x axis (where a unit is said to correspond if they are mirror images of each other across the x axis) is not shaded, then the unit that is shaded must be unshaded. It you do this, you see that you must un-shade 7 units.