- 1. What is the matrix product  $\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ 3x \\ 4x \end{bmatrix}$ ?
  - A.  $\lceil 11x \rceil$
  - B.  $\lceil 12x \rceil$
  - C.  $\lceil 6x \rceil$
  - **D.** 32x
  - E.  $\lceil 8x \rceil$
- 2. For which values of a and b is the following matrix equation true?

$$\begin{bmatrix} 1 & 4 \\ 2b & \frac{1}{3}b \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 6 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ a & a \end{bmatrix}$$

- **A.**  $a = \frac{-54}{5}$ ,  $b = \frac{42}{5}$
- **B.**  $a = \frac{54}{5}$ ,  $b = \frac{-42}{5}$
- C.  $a = \frac{54}{r}$ ,  $b = \frac{42}{r}$
- **D.**  $a = \frac{-54}{5}$ ,  $b = \frac{-42}{5}$
- E.  $a = \frac{54}{4}$ ,  $b = \frac{42}{5}$
- 3. Matrix  $P = \begin{bmatrix} -3 & 6 \\ -2 & 11 \end{bmatrix}$  and  $Q = \begin{bmatrix} 1 & 4 \\ 7 & -5 \end{bmatrix}$ . What is P 2Q?

- 4. The determinant of a matrix  $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$  is xw yz. If the determinant of  $\begin{bmatrix} 2a & 13 \\ a & a \end{bmatrix}$  is 24, which of the following
  - **A.** 4
  - **B.** 24
  - **C.** 8
  - **D.** -8
  - **E.**  $\frac{3}{2}$
- The 3×3 matrix  $\begin{bmatrix} -3 & 6 & -5 \\ 9 & 2 & -2 \\ 4 & 3 & 1 \end{bmatrix}$  is multiplied by a scalar n.

The resulting matrix is  $\begin{bmatrix} 9 & -18 & 15 \\ -27 & a & 6 \\ -12 & -9 & -3 \end{bmatrix}$ . What is *a*?

- **A.** 2
- **B.** -6
- **C.** -3
- **D.** 3
- E. 6
- The determinant of any  $2 \times 2$  matrix  $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$  is xw - yz. If the determinant of  $\begin{bmatrix} (x+4) & 4 \\ 9 & (x-3) \end{bmatrix}$  is 8. What are all the possible values of x:
  - A. -4 and 3
  - **B.** 8 and −7
  - C. -8 and 7
  - $\mathbf{D.}$  4 and -3
  - E. 7 and 8
- 7.  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} b & c \\ d & a \end{bmatrix} = ?$ 

  - A.  $\begin{bmatrix} ab & bc \\ cd & da \end{bmatrix}$ B.  $\begin{bmatrix} ab+bd & ac+ba \\ cb+d^2 & c^2+da \end{bmatrix}$ C.  $\begin{bmatrix} a+b & b+c \\ c+d & d+a \end{bmatrix}$

  - $\mathbf{D.} \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$
  - E.  $\begin{bmatrix} ab+c^2 & b^2+cd \\ ad+ca & bd+da \end{bmatrix}$

**8.** Which of the following matrices for  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  makes the following expression true?\*\*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ 23 & 37 \end{bmatrix}$$

- A.  $\begin{bmatrix} 7 & \frac{13}{3} \\ \frac{23}{5} & \frac{37}{7} \end{bmatrix}$
- **B.**  $\begin{bmatrix} 6 & 10 \\ 18 & 30 \end{bmatrix}$
- C.  $\begin{bmatrix} 7 & 0 \\ 1 & 4 \end{bmatrix}$
- $\mathbf{D.} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\mathbf{E.} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$
- \*\* denotes problem may be more challenging than typical ACT problems
- 9. If  $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad bc$ , then  $\det \begin{bmatrix} -c & -a \\ d & b \end{bmatrix} = ?$ 
  - A. -ad-bc
  - **B.** bc+ad
  - C. ad-cb
  - **D.** -cb-ad
  - **E.** -ad + cb
- 10. Given the matrix equation shown below, what is  $\frac{b}{a-b}$ ?

(Note: Whenever n is a positive integer, the notation of n! represents the product of the integers from n to 1. For example, 3!=3\*2\*1.)

$$\begin{bmatrix} 4! \\ 2! \end{bmatrix} + \begin{bmatrix} 2! \\ 3! \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

- **A.**  $\frac{4}{9}$
- **B.**  $\frac{13}{9}$
- C.  $\frac{\epsilon}{2}$
- **D.** 5
- E.  $\frac{1}{7}$

11. 
$$\begin{bmatrix} a & a \\ b & b \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} \frac{1}{a+b} & \frac{1}{a+b} \\ \frac{1}{b+c} & \frac{1}{b+d} \end{bmatrix}$$

- A.  $\begin{bmatrix} \frac{2a}{a+b} & 1 \\ 1 & 1 \end{bmatrix}$
- **B.**  $\begin{bmatrix} 2a + \frac{1}{a+b} & a+b + \frac{1}{a+b} \\ b+c + \frac{1}{b+c} & b+d + \frac{1}{b+d} \end{bmatrix}$
- C.  $\begin{bmatrix} 2a+ab & 2a+2b \\ 2b+2c & 2b+2d \end{bmatrix}$
- **D.**  $\begin{bmatrix} aa & ab \\ a+b & a+b \\ bb & cd \\ b+c & b+d \end{bmatrix}$
- E.  $\begin{bmatrix} \frac{1}{3a+b} & \frac{1}{a+2b} \\ \frac{1}{2b+2c} & \frac{1}{2b+2d} \end{bmatrix}$
- 12. Emmy owns 2 juice bars (X and Y) and stocks 3 flavors of juice (A, B, and C). The matrices below show the numbers of each flavor of juice in each shop, and the cost for each flavor. Using the values given below, what is the difference between the value of juice inventories for the two shops?

$$\begin{array}{ccccc}
A & B & C & & \text{Cost} \\
X \begin{bmatrix} 25 & 50 & 20 \\ Y \begin{bmatrix} 50 & 100 & 25 \end{bmatrix} & & A \begin{bmatrix} $5 \\ $10 \\ C \end{bmatrix} & \$15
\end{array}$$

- **A.** \$900
- **B.** \$700
- C. \$2850
- **D.** \$2550
- E. \$1275

13. The number of students who practice an art at a certain conservatory can be shown by the following matrix:

> choir painting pottery 100 60

The head of the conservatory estimates the ratio of the number of art awards that will be earned to the number of students participating with the following matrix:

band choir painting .1 pottery

Given this, which is the best estimate for the number of art awards that will be earned for the year?

- 73
- 74 В.
- 75 C.
- 76
- E. 77

14. The determinant of any  $2 \times 2$  matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is ad - bc.

The determinant of  $\begin{bmatrix} 3 & (x-1) \\ (x+4) & 2 \end{bmatrix}$  is equal to 0.

What are all possible values of x?

- A.  $-\frac{\sqrt{17}-3}{3}$  and  $\frac{\sqrt{17}+3}{3}$
- **B.** 5 and −2
- C. -5 and 2
- D.  $\frac{\sqrt{17}-3}{3}$  and  $-\frac{\sqrt{17}+3}{3}$

E. none of the above

15. What value of x satisfies the matrix equation below? (Assume x is a scalar.)

$$x \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 17 & 4 \end{bmatrix}$$

- **C**. 3
- E. 4

- 16.  $4\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} 2\begin{bmatrix} 3 & -3 \\ -1 & 2 \end{bmatrix} = ?$ 
  - $\mathbf{A.} \begin{bmatrix} 0 & 4 \\ 16 & -8 \end{bmatrix}$

  - **D.** -22
  - E.  $\begin{bmatrix} -6 & 10 \\ 18 & -12 \end{bmatrix}$
- 17. Graph theory is often used to represent connections between different points. Three satellites in space communicate with one another, as represented by the drawing



For example, the arrows indicate that satellite A communicates with satellite B and C, but satellite C only communicates with satellite B. The same relationships are demonstrated in the matrix, where, because satellite A communicates with satellite B, there is a 1 in the A row and B column, but because satellite A does not communicate with itself, there is a 0 in A row and A column.

$$\begin{array}{cccc}
A & B & C \\
A & 0 & 1 & 1 \\
B & 0 & 0 & 1 \\
C & ? & ? & ?
\end{array}$$

Which of the following is the third row of the matrix?

- **A.** 1 1 1
- B. 1 1 0
- C. 1 0 0
- D. 0 1 0
- E. 0 0 0

**18.** For what (x, y) pair is the matrix equation below true?

$$\begin{bmatrix} y & 3x \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -3 & 3 \\ 4 & \frac{x}{2} \end{bmatrix} = \begin{bmatrix} 21 & 9 \\ -4 & -1 \end{bmatrix}$$

- **A.** (4,24)
- **B.** (24,4)
- C. (1,2)
- **D.** (2,1)
- $\mathbf{E} \cdot \left( -\frac{3}{4}, -7 \right)$
- 19. The  $2\times 4$  matrix  $\begin{bmatrix} 1 & 2 & 8 & 4 \\ 2 & 5 & 6 & 1 \end{bmatrix}$  represents quadrilateral

ABCD, with vertices A(1,2), B(2,5), C(8,6), D(4,1) in the standard coordinate plane. After the quadrilateral was reflected over the x-axis, the matrix representing the translated triangle is  $\begin{bmatrix} 1 & 2 & 8 & 4 \\ m & -5 & 3m & -1 \end{bmatrix}$ . What is the value of m?

- **A.** -2
- **B.** 2
- **C.** 3
- **D.** -1
- **E.** −3

**20.** 
$$3\begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 2 & 1 \\ -3 & 8 \end{bmatrix} = ?$$

- A.  $\begin{bmatrix} -1 & -6 \\ 2 & 7 \end{bmatrix}$
- **B.**  $\begin{bmatrix} -3 & -18 \\ 6 & 21 \end{bmatrix}$
- C.  $\begin{bmatrix} 0 & 6 & -12 \\ 12 & 15 & 24 \end{bmatrix}$
- E.  $\begin{bmatrix} -12 & 45 \\ -4 & 84 \end{bmatrix}$

## **ANSWER KEY**

7. E 8. C 1. A 2. C 3. A 4. B 5. C 6. D 9. C 10. C 11. C 12. B 13. C 14. A 19. C 20. A 15. C 16. C 17. E 18. C

## **ANSWER EXPLANATIONS**

- 1. A. Here we have a  $1\times 3$  matrix and a  $3\times 1$  matrix respectively. To multiply, we must ensure the middle two numbers of the matrix dimensions match (they do: 3 and 3). Our solution will be a matrix made up of the first and last digits of the dimensions  $(1\times1)$ . To multiply matrices, multiply the rows times the columns: Row 1 of the first matrix times column 1 of the second. Add the product of the first item in the row times the first item in the column  $(0 \times x)$ , the product of the second item in the row times the second item in the column  $(1 \times 3x)$  and the product of the third item in the row times the third item in the column  $(2\times 4x)$ . Simplify this to 0x+3x+8x=11x. Write your answer in matrix form: [11x].
- 2. C. This problem essentially is four different equations written in one matrix based form. When we add matrices, we look only at the positions—same position means we add together those elements, i.e. in the matrix problem here look only at the positions—same position means we ded to be a same position—so they form the following relationship—a-b=c. The same is true for all other "same position" letters in the above matrix. Let's now take the matrix at hand and reorganize it into a series of equations based on elements in the same position:  $\begin{bmatrix} 1 & 4 \\ 2b & \frac{1}{3}b \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 6 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ a & a \end{bmatrix}$

$$1-3=-2$$
  
 $4--2=6$ 

Those first two only state the obvious—not helping us much—but at least you can use these to verify that you're setting the problem up right. The last two will help more:

$$2b-6=a$$
 and  $\frac{1}{3}b-(-8)=a$ 

Now we have a system of two equations with two unknowns. We can solve by elimination or substitution. Here we will use substitution, substituting for a: First, create a positive from the double negative to get  $2b-6=\frac{1}{2}b+8$ . Solving

for b gives us  $b = \frac{42}{5}$ . Finally, plug in this fraction using the first (or second) equation to find a.  $2\left(\frac{42}{5}\right) - 6 = a$  so  $\frac{84}{5} - \frac{30}{5} = \frac{54}{5} = a$ .

3. E. To understand the basics of how to subtract two matrices, see explanation at question 2. 20 implies that we must first use scalar multiplication before performing the subtraction. To do so, multiply the scalar (2) by each item in the matrix. Let's solve for 2Q:

$$2\begin{bmatrix} 1 & 4 \\ 7 & -5 \end{bmatrix} = \begin{bmatrix} 2*1 & 2*4 \\ 2*7 & 2*-5 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 14 & -10 \end{bmatrix}$$

Now we can set up P-2Q, subtracting numbers in the same positions:

$$\begin{bmatrix} -3 & 6 \\ -2 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 8 \\ 14 & -10 \end{bmatrix} = \begin{bmatrix} -3-2 & 6-8 \\ -2-14 & 11--10 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ -16 & 21 \end{bmatrix}$$

C. For this problem just apply the formula given and you get:

$$2a*a-13*a=24$$
  
 $2a^2-13a=24$   
 $2a^2-13a-24=0$ 

To solve, use the quadratic formula or factoring. I'll use the quadratic formula (you should have this memorized). I suggest using your calculator for the multiplication and roots.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+13 \pm \sqrt{13^2 - 4(2)(-24)}}{2(2)} = \frac{13 \pm \sqrt{169 + 192}}{4} = \frac{13 \pm \sqrt{361}}{4} = \frac{13 \pm 19}{4}$$

Now we can split into two solutions.  $\frac{13-19}{4} = \frac{-6}{4} \rightarrow -1.5$  OR  $\frac{13+19}{4} = \frac{32}{4} \rightarrow 8$ 

8 is the only answer available, so C is correct. You could also "backsolve" this problem, using the answers, or program your calculator to do the quadratic equation (for limitations on calculator programs see the ACT website: http://www.actstudent.org/faq/cas functionality.html

**5. B.** A scalar is a single number that then is multiplied by each individual value in a matrix. To multiply the given matrix by *n*:

$$n \begin{bmatrix} -3 & 6 & -5 \\ 9 & 2 & -2 \\ 4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3n & 6n & -5n \\ 9n & 2n & -2n \\ 4n & 3n & 1n \end{bmatrix}$$

Now we can set this equal to the result:

$$\begin{bmatrix} -3n & 6n & -5n \\ 9n & 2n & -2n \\ 4n & 3n & 1n \end{bmatrix} = \begin{bmatrix} 9 & -18 & 15 \\ -27 & a & 6 \\ -12 & -9 & -3 \end{bmatrix} 9$$

Playing "match" we can match up each value in each position and form up to different equations. We actually will only need two, though. We need to find a <u>not</u> n—but we'll need n to find a. To find n, we have many choices, but let's try for the first row / column value:

$$-3n = 9$$
  
 $n = -3$ 

Now don't go looking for that as the answer choice! Plug in that value into another equation—the one that involves a: 2n = a. Again we found this equation by matching up values in the same positions in the equivalent matrices. We then simplify:

$$2(-3)=a=-6$$

**6.** E. Here apply the formula given, set equal to 8, then FOIL and simplify:

$$(x+4)(x-3)-(4)(9)=8$$

$$x^{2}+4x-3x-12-36=8$$

$$x^{2}+x-48=8$$

$$x^{2}+x-56=0$$

Now we can apply the quadratic equation or factor. Remember you can program your calculator to do the quadratic equation as long as it's under 25 logical lines of code. Here I'll factor to: (x+8)(x-7)=0, which gives the answers x=7 or x=-8.

- 7. B. This problem essentially asks the definition of how to multiply matrices. If you know what you're doing, this should be easy. (A) is wrong—you do not multiply in the same way you add and subtract matrices—you cannot just multiply each item in the same position together. Who knows why this isn't true but it's just the definition of matrix multiplication. (B) is correct—it adds the products of rows times columns. (C) adds the two matrices (D) multiplies the first matrix by a scalar of 2 and (E) mixes up rows and columns—it's rows times columns not columns times rows. If you missed this, go back and review the basics of matrix multiplication.
- **8.** For this problem you must multiply the matrices. Let's first just focus on the setup to the left of the equals sign and simplify that into a single matrix. If you don't know how to do this, review the example problems earlier in the chapter. Take rows times columns and add the products:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} a(1) + b(5) & a(3) + b(7) \\ c(1) + d(5) & c(3) + d(7) \end{bmatrix} = \begin{bmatrix} a + 5b & 3a + 7b \\ c + 5d & 3c + 7d \end{bmatrix}$$

Now let's take what we have and put it with what the original problem was equal to, and match each corresponding position to make four equations:

6 CHAPTER 5

$$\begin{bmatrix} a+5b & 3a+7b \\ c+5d & 3c+7d \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ 23 & 37 \end{bmatrix}$$
$$a+5b=7$$
$$c+5d=23$$
$$3a+7d=13$$
$$3c+7d=37$$

Now we could do the ridiculous task of solving each of these out—but it's a better idea to backsolve a bit—as most of the matrices are VERY different answers. We'll use the first of these equations ONLY—and check each one. Let's try (A): a=7 and  $b=\frac{13}{3}$ —no way will that give us the integer value 7. It's out. Let's try (B) a=6 and b=10.  $6+50 \ne 7$ . Let's try (C) 7+5(0)=7, but we can't assume this is right yet. (D) yields  $0+5\ne 7$ . (E) works too though—2+5=7. Now we've narrowed. Let's try another—this time let's use the last equation 3c+7d=37: for (E) c=3 and d=4, so 3(3)+7(4)=9+28=37—yes. But for (C), c=1 and d=4, so 3(1)+7(4)=3+28=31—not 37.

\*Note that this problem tests skills for which you are responsible on the ACT but is more difficult than most ACT problems.

- 9. C. This is a very straightforward problem: finding the determinant, made even more simple by giving the definition of a determinant in the question. The trick will be in the format of the answers, as the answers will be rearranged to trick you. If approaching the problem traditionally,  $\det \begin{bmatrix} -c & -a \\ d & b \end{bmatrix} = (-c)(b) (-a)(d)$  simplified becomes -cb + ad. While that is not an answer, answer (C), ad cb, is the same by simply rearranging the order of the terms.
- 10. A. This requires multiple steps to solve. First apply the factorial (the !), and then sum the matrices properly, summing the corresponding entries until you find the final values for a and b, which are 26 and 8 respectively. Finally, plug those values in correctly to the expression desired. The result is  $\frac{8}{18}$ , which simplifies into  $\frac{4}{9}$ , answer (A).
- 11. **B.** This question looks more complicated than it really is. You might assume that you will have to simplify the sum, but that's not even necessary. If you take the sum correctly, summing the corresponding entries, B is the obvious answer. If you know how to properly add matrices, then really all this question is testing is your fraction addition skills.
- 12. B. To solve this, you have to find the inventory for each shop, and then subtract to find the difference (it doesn't matter which you subtract from, as the difference is the absolute value). To find the total inventory for each shop, you have to multiply the matrices. (In general, when given word problems with matrices whose inner dimensions match, you usually end up multiplying them for some reason or other). Think of the matrices as tables that organize this information. If you have 25 of Juice A in Shop X, and according to the cost matrix Juice A costs \$5, it makes sense that you have \$125 worth of Juice A in Shop X (\$25\*\$5). Since you have three juices in each shop, you must sum the totals from each juice. Ultimately, it's just as if you were multiplying the matrices, and the resultant matrix tells you the value of the inventory in each shop.

(Shop X flavor) \* (Flavor X value) = Shop X value

Then, \$1625 - \$925 = \$700, which is answer (B).

**13. D.** For this question, again you multiply the matrices, necessary in this problem to find out how many students per department receive an award. To find how many total are awarded across all the departments, sum the elements in that final matrix. You are multiplying:

band choir painting pottery
$$\begin{bmatrix} 80 & 60 & 60 & 100 \end{bmatrix} * band choir painting pottery$$

$$\begin{bmatrix} 1.2 & 2.4 & 6 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1.2 & 2.4 & 6 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1.2 & 2.4 & 6 & 30 \end{bmatrix}$$

school \* art department \* dept. \* award = school \* students awarded per dept.

16+24+6+30=76, which is answer (D).

14. C. With the formula given, all you need to do is substitute the values given with the corresponding variables:

$$\begin{bmatrix} 3 & (x-1) \\ (x+4) & 2 \end{bmatrix} = 0$$

Following the formula we get 6 - (x+4)(x-1) = 0 or  $6 - (x^2+3x-4) = 0$ . After distributing the subtraction sign and simplifying we get  $6 - x^2 - 3x - (-4) = 6 - x^2 - 3x + 4 = -x^2 - 3x + 10 = 0$ . After factoring we get (-x-5)(x-2) = 0 so, x = -5 and 2.

15. C. Answer (C) correctly solves for x. The trick with problems like this is that you don't have to solve the entire matrix. Selecting the top left element, you multiply the matrix by x and get the following:

$$\begin{bmatrix} 3x & 2x \\ 4x & 1x \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 17 & 4 \end{bmatrix}$$

But since we are only going to focus on one address, the top left, all that matters is: 3x + 2 = 11. Solve this and x = 3, which is answer (C).

- **16.** E. Answer (E) is correct because per the rules of scalar multiplication, it multiplies all the elements in the matrix by the scalar, and then it subtracts the elements with the corresponding ones on the second matrix.
- 17. **D.** Satellite C receives messages from both satellite A and B, but it only sends to satellite B, as seen by the double arrow line between C and B, so the correct answer is 0 1 0.
- 18. D. In order to solve this problem, you have to pick the right place to start. Although convention dictates that you multiply matrices starting with the top left element, and go left to right, top to bottom, the order doesn't really matter. In this case, you must first find x, which is possible by creating the equation for the bottom right element in the final matrix. In that case, you get the equation  $0(3)+(-1)(\frac{x}{2})=-1$ , and when solved tells you that x=2. From there you already know your answer, (D), since no other answer has 2 for x.
- 19. A. In this question the matrices merely serve as a way to organize information, in this case coordinate points. Reflecting a polygon over the x axis means that the coordinates stay the same while the y coordinates become negative. Since the top row of the matrix represents the x coordinates, those stay the same, while the bottom row elements, which represents the y coordinates, all become negative. Thus, y, the left most bottom element, negated becomes y. Looking at the translated matrix, y takes the place of where y should be, so y is clearly y.
- **20. B.** This problem combines scalar and matrix multiplication. Scalar multiplication is commutative, but matrix multiplication is not. The scalar factor, **3**, can be applied either before or after the two matrices are multiplied correctly (row by column, adding the products), but if the numbers get too big, it's better to wait until after.

$$3\begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 2 & 1 \\ -3 & 8 \end{bmatrix} = 3\begin{bmatrix} (2)(-2) + (0)(2) + (-1)(-3) & (2)(1) + (0)(1) + (-1)(8) \\ (3)(-2) + (4)(2) + (0)(3) & (3)(1) + (4)(1) + (0)(8) \end{bmatrix} = 3\begin{bmatrix} -1 & -6 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -3 & -18 \\ 6 & 21 \end{bmatrix}$$

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