THE BEST ACT PREP COURSE EVER

QUADRATIC EQUATIONS, PARABOLAS, AND POLYNOMIALS

ACT Math: Problem Set

1. The sum of $\left(-3x^2+4x-8\right)$ and which of the following polynomials is $\left(2x^2-7x+10\right)$?

A. $5x^2-11x+18$

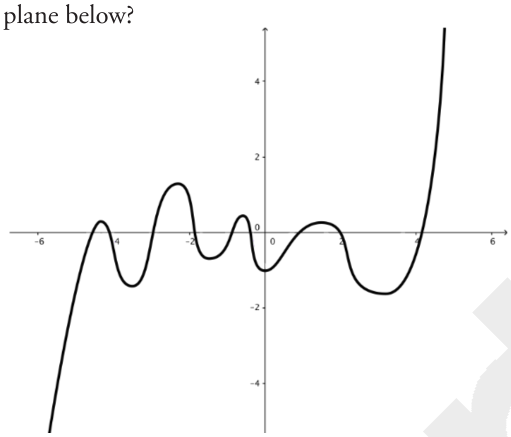
B. $-5x^2+11x-18$

C. $5x^2+18$

D. $5x^2-18$

E. $5x^2-11x$

2. What is the minimum degree possible for the polynomial function whose graph is shown in the standard (x,y) plane below?



A. 5
B. 6
C. 7
D. 8
E. 9

3. What is the solution set of the equation $-2x^2+7=0$?

A. $\left\{-\sqrt{\frac{7}{2}},\sqrt{\frac{7}{2}}\right\}$

B. $\left\{-\sqrt{3},\sqrt{3}\right\}$

C. $\left\{-\frac{4}{2},\frac{4}{2}\right\}$

D. $\left\{-3,3\right\}$

E. $\left\{-\sqrt{5},\sqrt{5}\right\}$

4. The graph of $y=-3x^2+5$ passes through the point $(3,4a)$ in the standard (x,y) coordinate plane. What is the value of a?

A. 32
B. -22
C. 8
D. -5.5
E. -8

5. For what nonzero whole number k does the quadratic equation $x^2+4kx+k^3$ have exactly 1 real solution for x?

A. 2
B. 4
C. 8
D. 16
E. 1

6. Which of the following is the set of real solutions for the equation $5x+12=2(4x+6)$?

A. The empty set

B. The set of all real numbers

C. $\{0,5\}$

D. $\left\{\frac{5}{8}\right\}$

E. $\{0\}$

7. In the standard coordinate plane, what is the vertex of the parabola with the equation $y=-4(x+7)^2+2$?

A. $(-7,-2)$

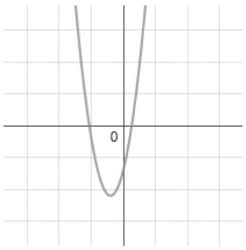
B. $(7,2)$

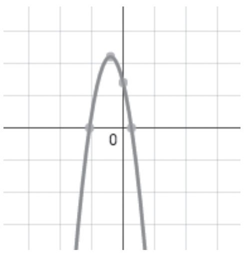
C. $(7,-2)$

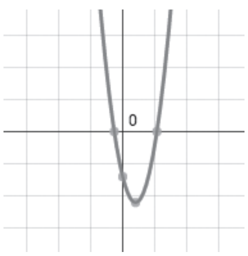
D. $(-7,2)$

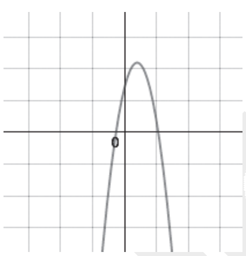
E. $(-14,2)$

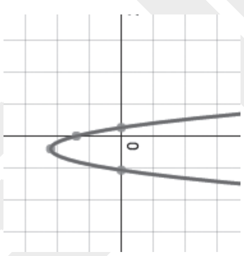
8. The graph of the parabola with the equation $y = -x^2 - 4x + 7$ is shown in the standard (x, y) coordinate plane below. Which of the following graphs is the graph of the given equation rotated $90°$ counterclockwise about the origin?

A. 

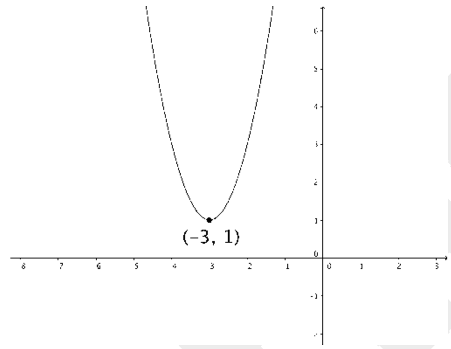
B. 

C. 

D. 

E. 

9. The graph of which of the following equations is the parabola shown in the standard (x, y) coordinate plane below?



A. $y - 1 = (x + 3)^2$

B. $y - 1 = 2(x + 3)^2$

C. $y + 1 = 2(x - 3)^2$

D. $y - 1 = \frac{1}{2}(x + 3)^2$

E. $y + 1 = \frac{1}{2}(x - 3)^2$

10. Using the quadratic formula, what are the two roots for the equation $7x^2 - 3x = 17$?

A. $\frac{3 \pm \sqrt{485}}{14}$

B. $\frac{3 \pm \sqrt{-467}}{14}$

C. $\frac{5}{7}$ and 3

D. $-\frac{5}{7}$ and -3

E. 7

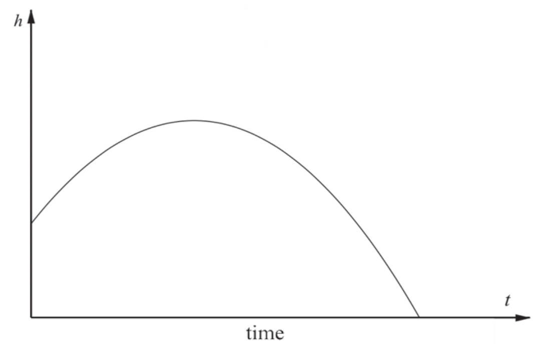
11. For what integer k are both solutions of the equation $x^2 + kx + 19 = 0$ negative integers?

A. 20
B. 19
C. 1
D. −19
E. −20

12. The solution set for x of the equation $x^2 + mx - 4 = 0$ is $\{-4,1\}$. What does m equal?

A. 1
B. 4
C. −4
D. 3
E. −3

13. The graph of the equation $-at^2 + bt + c = 0$, which describes how the height, h, of an object that is thrown directly upward, changes over time, t, is shown below.



If you alter only this equations a term, the leading coefficient,
the alteration has an effect on which of the following?

I. The t-intercept
II. The h-intercept
III. The maximum value of h

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

14. Which of the following equations shows a correct use of the quadratic formula to solve $2x^2 + 8x - 1 = 0$?

A. $x = \dfrac{8 \pm \sqrt{64 - 4(2)(-1)}}{2(2)}$

B. $x = \dfrac{-8 \pm \sqrt{64 - 4(2)(1)}}{2(2)}$

C. $x = \dfrac{-8 \pm \sqrt{64 - 4(2)(-1)}}{2(2)}$

D. $x = \dfrac{8 \pm \sqrt{64 - 4(2)(-1)}}{2}$

E. $x = \dfrac{-8 \pm \sqrt{64 - 4(2)(1)}}{2}$

15. In the equation $x^2 + 2mx - \left(\frac{1}{2}\right)n = 0$, m and n are integers. The *only* possible value for x is −2. What is the value of n?

A. −2
B. −8
C. 2
D. 8
E. 4

16. $7w^3 + 65w - w^3 - 20 - 35w + 2$ is equivalent to:

A. $8w^3 + 30w - 22$
B. $6w^3 + 30w - 22$
C. $6w^3 + 30w - 18$
D. $18w^3$
E. $18w$

17. What polynomial must be added to $2x^3 - 4x - 6$ so that the sum is $-x^3 - 4$?

A. $4x - x^3 + 2$
B. $4x - 3x^3 + 2$
C. $-x^3 - 4x - 2$
D. $-3x^3 - 4x - 2$
E. $-3x^3 - 4x + 2$

18. The height about the ground, h, of an object t seconds after being thrown from the top of a building is given by the equation $h = -3t^2 + 15t + 18$. An equivalent factored form of this equation shows that the object:

A. Starts at a point 6 units off the ground
B. Reaches the ground in 6 seconds
C. Reaches the ground in 1 second
D. Reaches a maximum in 18 seconds
E. Reaches a maximum in 1 second

19. Which of the following expressions is equivalent to $(3x^3 + 5) - (2x^2 - 6x + 7) + (7x - 5) - (5x^2 + 3x + 3x + 2x)$?

A. $3x^3 - 7x^2 + 5x - 7$
B. $3x^3 - 10x^2 + 9x + 7$
C. $-7x^2 + 5x - 7$
D. $3x^3 - 7$
E. $3x^3 + 5x - 7$

20. $(x+4y-2z)-(-3x+2y+5z)$ is equivalent to:

A. $-2x+8y+3z$
B. $-3x+2y-7z$
C. $4x+2y-7z$
D. $-3x+8y+3z$
E. $4x+2y+3z$

21. If $f(x)=4x^3-64x$, which of the following correctly describes the zeroes of the polynomial? (Zeroes are the values where $f(x)=0$)

A. 2 different rational zeroes
B. No real zeroes
C. Only 1 rational zero
D. 3 different rational zeroes
E. 1 number is a double zero

22. One of the roots of $4x^3-18x^2+32x-24=0$ is 2. What are the other roots?

A. $\frac{5}{2}\pm i\sqrt{23}$
B. $\frac{5}{4}\pm\sqrt{23}$
C. $\frac{5}{4}\pm\frac{i\sqrt{23}}{4}$
D. $\frac{5}{2}\pm\frac{i\sqrt{23}}{2}$
E. $5\pm i\sqrt{23}$

23. What is the value of c if $x+1$ is a factor of $x^3+2x^2-cx-20$?

A. 19
B. 18
C. 17
D. 16
E. 15

24. What is the equivalent of $(n+4)^3$?

A. n^3+64
B. $n^3+6n^2+24n+32$
C. $n^3+12n^2+48n+64$
D. $n^3+12n^2+48n+32$
E. $n^3+24n^2+48n+64$

25. Consider the equation $y=-(x+2)^2-4$, where x and y are both real numbers. The table below gives the values of for selected values of x.

x	y
−11	−85
−9	−53
−7	−29
−5	−13
−3	−5
−1	−5
1	−13

For the equation above, which of the following values of x gives the greatest value of y?

A. −8
B. −6
C. −4
D. −2
E. 0

26. Which of the following values is a zero of $f(x)=3x^4+8x^3+4x^2$?

A. $\frac{2}{3}$
B. $\frac{3}{2}$
C. -2
D. 3
E. -3

27. Which of the following expressions is equivalent to $4x^4-8x^2-24$?

A. $(x^2+1)(x^2-3)$
B. $4(x+1)(x-3)$
C. $4(x+8)(x-3)$
D. $4(x^2+3)(x^2-8)$
E. $4(x^2+1)(x^2-3)$

28. Which of the following is NOT a factor of a^7-81a^3?

A. a
B. a^2
C. $a+3$
D. $a-3$
E. a^2+3

29. The function $f(x)$ is a cubic polynomial that has the value of 0 when x is $0, -3,$ and 4. If $f(1) = -6$, which of the following is an expression for $f(x)$?

A. $x(x-3)(x+4)$

B. $x(x+3)(x-4)$

C. $2x(x+3)(x-4)$

D. $\frac{x}{2}(x+3)(x-4)$

E. $x^2(x-3)(x+4)$

30. $f(x)$ is a quartic (fourth order) polynomial that has zeroes at $x = 2, 6, -4, -9.$ If $f(3) = 63$, which of the following is an expression for $f(x)$?

A. $(x-2)(x-6)(x+4)(x+9)$

B. $\frac{1}{4}(x-2)(x-6)(x+4)(x+9)$

C. $-\frac{1}{4}(x-2)(x-6)(x+4)(x+9)$

D. $\frac{1}{4}(x+2)(x-6)(x+4)(x+9)$

E. $-\frac{1}{4}(x+2)(x+6)(x-4)(x-9)$

ANSWER KEY

1. A	**2. E**	**3. A**	**4. D**	**5. B**	**6. E**	**7. D**	**8. D**	**9. A**	**10. A**	**11. A**	**12. D**	**13. E**	**14. C**
15. B	**16. C**	**17. B**	**18. B**	**19. A**	**20. C**	**21. D**	**22. C**	**23. A**	**24. C**	**25. D**	**26. C**	**27. E**	**28. E**
29. D	**30. C**												

ANSWER EXPLANATIONS

1. **A.** We wish to solve the equation $-3x^2+4x-8+Y=2x^2-7x+10$ for the polynomial Y. So, subtracting $-3x^2+4x-8$ on both sides, we get $Y=2x^2-7x+10-\left(-3x^2+4x-8\right)$. Distributing the negative sign, we get $Y=2x^2-7x+10+3x^2-4x+8$. Now, combining like terms, we get $Y=5x^2-11x+18$.

2. **E**. For a polynomial with n turning points (whenever the slope of the graph changes signs, the minimum degree of the polynomial is $n+1$. The graph has 8 turning points, so the minimum degree of the polynomial is $8+1=9$.

3. **A.** Using the quadratic formula with $a=-2$, $b=0$, and $c=7$, we have $x=\dfrac{0\pm\sqrt{-4(-2)7}}{2(-2)}=\pm\dfrac{\sqrt{56}}{-4}=\pm\dfrac{2\sqrt{14}}{4}=\pm\dfrac{\sqrt{14}}{2}$ $=\pm\dfrac{\sqrt{14}}{\sqrt{4}}=\pm\sqrt{\dfrac{14}{4}}=\pm\sqrt{\dfrac{7}{2}}$.

4. **D.** Plugging in $x=3$, we get $y=-3(3)^2+5=-3(9)+5=-27+5=-22$. So, we can equate $4a=-22\rightarrow a=-\dfrac{22}{4}=-5.5$

5. **B.** If the polynomial only has one solution, it means that it is a perfect square that can be factored into $(x+a)(x+a)$. So, we set $x^2+4kx+k^3=(x+a)(x+a)=x^2+2ax+a^2$. This means that $2ax=4kx$ and $a^2=k^3$. Simplifying the first equation, we get $a=2k$. Plugging in this value for a in $a^2=k^3$, we get $(2k)^2=k^3\rightarrow 4k^2=k^3\rightarrow k=4$.

6. **E.** Distributing the 2 on the right hand side of the equation, we get $5x+12=8x+12$. Subtracting 12 on both sides, we get $5x=8x$. This is only true if $x=0$.

7. **D.** The equation of a parabola is in the form $y=a(x-h)^2+k$ where (h,k) is the vertex of the parabola. So, the parabola with equation $y=-4(x+7)^2+2$ has vertex $(-7,2)$.

8. **D.** Although we could attempt to figure out what the original parabola looked like and then try to match specific points to a graph, since we know that the original parabola was downward facing $(-x^2)$, we know that rotating the graph 90° would produce a parabola that opens to the right. Only one answer choice has a rightwards-opening parabola.

9. **A.** The parabola shown has a vertex at $(-3,1)$, so when $x=-3$ and $y=1$, the equation must be true. We see that when we plug these values into answer choice A, we get $y-1=2(x+3)^2\rightarrow 1-1=2(-3+3)^2\rightarrow 0=0$. The vertex of answer choices A, B, and D are the same, but the parabola in the figure also passes through $(-2,2)$, which is only true of answer choice A.

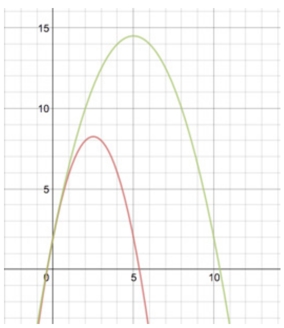
10. **A.** We first subtract 17 on both sides to bring everything to the left side of the equation. We get $7x^2-3x-17=0$. Now, we plug in $a=7$, $b=-3$, and $c=-17$ into the quadratic equation to get
$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(7)(-17)}}{2(7)}=\frac{3\pm\sqrt{9+476}}{14}=\frac{3\pm\sqrt{485}}{14}.$$

11. A. We plug in the values $a=1$, $b=k$ and into the quadratic equation to get the value of x. We get

$x=\frac{-k\pm\sqrt{k^2-4(1)(19)}}{2(1)}=\frac{-k\pm\sqrt{k^2-76}}{2}$. If both solutions for x are negative, then we know that $\frac{-k+\sqrt{k^2-76}}{2}$ is negative and $\frac{-k-\sqrt{k^2-76}}{2}$. This means that k is positive because if k were negative, the negative sign in front of it will cancel it out to make x a positive value. We also know that in order for $\frac{-k-\sqrt{k^2-76}}{2}$ to be an integer, $\sqrt{k^2-76}$ must be an integer. If we plug in the positive answer choices $20, 19$ and 1 for k, we see that only $k=20$ gives us an integer solution. $\sqrt{20^2-76}=18$ while $\sqrt{19^2-76}=16.88$ and $\sqrt{1^2-76}=$ undefined.

12. D. The question says that -4 and 1 are solutions to a quadratic equation, so it's best to work backwards. If those are solutions, then $(x-1)(x+4)=0$. FOIL to get $x^2+3x-4=0$. Comparing the equation given with the one we found, we see that $m=3$.

13. E. The best way to solve this is to graph our own made up, arbitrary examples on a calculator and see for ourselves. Try graphing $-x^2+5x+2$ and $-\frac{1}{2}x^2+5x+2$.



From the graphs we can see that the x-intercept (t in the question) and the vertex have moved, but the y-intercept (h in the question) hasn't. If we don't have a graphing calculator or don't have time to graph them, remember that the leading coefficient of a parabola can tell us only a couple things about the parabola: it's sign indicates what direction the parabola is facing, it magnitude tells how 'fat' or 'skinny' the parabola is, and it's used to determine the x-value of the vertex, whose formula is $\frac{-b}{2a}$. Thus, altering the a term would potentially change both the x and y value of the vertex, and the y value is the maximum height. However, the only way to change the y-intercept would be to change the *c* value, so it makes sense that the y-intercept would stay the same, as c is not affected when changing the a value.

14. C. The quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, so plugging in 2 for a, 8 for b, and -1 for c, we get $\frac{-8\pm\sqrt{64-4(2)(-1)}}{2(2)}$, answer (C).

15. B. We cannot solve this by plugging in -2 and trying to factor. Instead, work backwards. Think about what the question means by "the *only* possible value for x is -2". Putting together the fact that it must be a polynomial of degree two, which means that if factored there would be two expressions with x, and that both must show that $x=-2$, it makes sense to assume that $(x+2)(x+2)=0$. Simplify to get x^2+4x+4. Compare this with the expression given, and we realize that $2mx=4$ and $-\frac{1}{2}n=4$. Solving for n in the second equation gives us -8.

16. C. Add like terms.

$$7w^3+65w-w^3-20-35w+2$$
$$=7w^3-w^3+65w-35w-20+2$$
$$=6w^3+30w-18$$

17. B. if $2x^3-4x-2$ plus some random expression equals $-x^3-4$, then subtract the first from the second.

$$\left(-x^3-4\right)-\left(2x^3-4x-2\right)$$
$$=-x^3-2x^3-\left(-4x\right)-4-\left(-2\right)$$

$=-3x^3+4x-2$, which is answer (B).

18. B. Looking at the coefficients, realize that we can pull out -3, which gives us $h\left(t\right)=-3\left(t^2-5t-6\right)$, which is much easier to factor. $h\left(t\right)=-3\left(t^2-5t-6\right)$ factored becomes $h\left(t\right)=-3\left(t-6\right)\left(t+1\right)$. Since the graph relates height and time, we know that $h=0$ at $t=6$ seconds and $t=-1$ seconds. There is no such thing as negative time, so we know that the ball is dropped from a height of 18 feet (our y-intercept found from the original form) and hits the ground at $t=6$. Looking at our answer choices, hitting the ground after 6 seconds is the only correct statement.

19. A. Distributing out the negative signs, we write the expression as $3x^3+5-2x^2+6x-7+7x-5-5x^2-3x-3x-2x$. Adding like terms, we get $3x^3-2x^2-5x^2+6x+7x-3x-3x-2x+5-7-5=3x^3-7x^2+5x-7$.

20. C. Distributing out the negative sign, we get $x+4y-2z+3x-2y-5z$. Adding like terms, we get $x+3x+4y-2y-2z-5z=4x+2y-7z$.

21. D. Factoring out $4x$, we get $f\left(x\right)=4x\left(x^2-16\right)$. x^2-16 is a difference of squares, so we can rewrite the equation as $f\left(x\right)=4x\left(x+4\right)\left(x-4\right)$. This gives us three different rational zeros. Namely, $0,4,$ and -4.

22. C. We can first factor out 2 from the equation because every term in the equation is divisible by 2. We get $2\left(2x^3-9x^2+16x-12\right)=0$. Knowing that 2 is a root, we know that when $x=2$, the equation equals zero. Thus, it must have the factor $\left(x-2\right)$. Using long division to factor out $\left(x-2\right)$, we get $2\left(x-2\right)\left(2x^2-5x+6\right)=0$. Now, we find the remaining two roots by using the quadratic formula on $2x^2-5x+6$. Plugging in $a=2$, $b=-5$, and $c=6$, we get

$$x=\frac{-\left(-5\right)\pm\sqrt{\left(-5\right)^2-4\left(2\right)\left(6\right)}}{2\left(2\right)}=\frac{5\pm\sqrt{25-48}}{4}=\frac{5}{4}\pm\frac{\sqrt{-23}}{4}=\frac{5}{4}\pm\frac{i\sqrt{23}}{4}.$$

23. A. Using long division to factor out $\left(x+1\right)$ from $x^3+2x^2-cx-20$, we get $\left(x+1\right)\left(x^2+x-\left(c+1\right)\right)$ where the constant term is equal to 20. $c+1=20$ so $c=19$.

24. C. Foiling out $\left(n+4\right)^3$, we get $\left(n+4\right)^3=\left(n+4\right)\left(n^2+8n+16\right)=\left(n^3+4n^2+8n^2+32n+16n+64\right)$
$=n^3+12n^2+48n+64$

25. **D.** Without even looking at the table, we see that the equation is in vertex form. From the equation we see that it is a downward facing parabola (because the leading term is negative) with a vertex at $(-2,-4)$. Because the parabola is facing down, we know the vertex has the greatest y- value, so the answer is -2. Alternatively, looking at the table we see that the y-values increase while $x<-3$, and decrease when $x>-1$, so the greatest point must be in between those two numbers, and -2 is the only answer that fulfills that condition.

26. **C.** First factor out an x^2: $f(x)=3x^4+8x^3+4x^2 \rightarrow f(x)=x^2(3x^2+8x+4)$. Now we can factor by reverse foiling: $f(x)=x^2(3x+2)(x+2)$. Our zeros are: 0, $\frac{-2}{3}$, and -2, and -2 is the only correct answer given.

27. **E.** Notice that we can pull out a constant of 4: $4x^4-8x^2-24 \rightarrow 4(x^4-2x^2-6)$. We don't know how to factor polynomials to the fourth degree easily, but we can substitute. If we let $w=x^2$, our expression becomes $4(w^2-2w-6)$, which factors easily into $4(w+1)(w+3)$. When we plug back in x^2 for w, we get $4(x^2+1)(x^2-3)$.

28. **E.** Factor out a^3 and then look at the problem as the difference between squares.
$a^7-81a^3 \rightarrow a^3(a^4-3^4) \rightarrow a^3\left((a^2)^2-(3^2)^2\right) \rightarrow a^3(a^2-3^2)(a^2+3^2) \rightarrow a^3(a-3)(a+3)(a^2+9)$. Thus, $a, a^2, a^3, (a+3), (a-3), (a^2-9), \text{and} (a^2+9)$ are all factors, but a^2+3 is not.

29. **D.** If our zeros are at 0, -3, and 4, then we can say that $f(x)=x(x+3)(x-4)$. When we plug in 1 to test, $f(1)=1(1+3)(1-4)=-12 \neq -6$. In order to satisfy the condition that says that $f(1)=-6$, we look at what our $f(1)$ currently equals and adapt the equation accordingly. In order to get $f(1)=-6$, we must divide our current $f(1)$ by 2, so our equation becomes $\frac{x}{2}(x+3)(x-4)$.

30. Since $2, 6, -4,$ and -9 are zeros for the polynomial, we know that it can be written in the form $(x-2)(x-6)(x+4)(x+9)k=0$ for some constant k. We are given that $f(3)=63$, so plugging in 3, we have $(3-2)(3-6)(3+4)(3+9)k=63 \rightarrow 1(-3)(7)(12)k=63 \rightarrow -252k=63 \rightarrow k=-\frac{63}{252}=-\frac{1}{4}$. So, the polynomial is $-\frac{1}{4}(x-2)(x-6)(x+4)(x+9)$.