4   MANIPULATIONS

SKILLS TO KNOW

- How to solve an equation in terms of any variable
- How to rework an equation or expression into another form or expression

To save time, DO THESE PROBLEMS ALGEBRAICALLY!

Some teachers will tell you to "plug in numbers" for these problems. That way does work, but it is time consuming. We'll show you how to do that (in a pinch), but if you have trouble with timing, pay attention to the algebraic methods below.

The idea behind manipulation is that you need to make what you have look like what you need. Sometimes these problems can look scary—but don't be afraid. Instead remember your two S's and two F's:

- **S**implify
- **S**ubstitute
- **F**actor
- **F**OIL

As long as you apply these ideas, you'll find the answers.

Don't be overwhelmed! Most of the time there are variables in the answer choices. You're not being asked to solve for the impossible, but simply to rearrange things.

SIMPLIFY

Which of the following is the equation $9(x+y)=8$, solved for x?

A. $\dfrac{8-9x}{9}$ **B.** $\dfrac{8+9y}{9}$ **C.** $\dfrac{8-9y}{9}$ **D.** $\dfrac{8+y}{9}$ **E.** $\dfrac{8-y}{9}$

Here, we want x alone and the everything else (the numbers and the y) together—so don't distribute! Divide by 9 first and you'll save time. To speed up this process, think about which step will help you get numbers together and variables together the fastest. In other words, **combine like terms:**

$$9(x+y)=8$$
$$x+y=\frac{8}{9}$$
$$x=\frac{8}{9}-y$$

Now we have x alone, but if we look at the answers, nothing matches. What doesn't match? All the answers are single fractions. Thus we need to turn our answer into a single fraction:

$$x = \frac{8}{9} - \frac{9y}{9} = \frac{8-9y}{9}$$

Answer: **C.**

SUBSTITUTE

Often, you'll see a pattern in manipulations problems: an **EQUATION** (or two) then an **EXPRESSION** you need to solve for. The key here is that you will always **ISOLATE** some variable in the equation that you will then **SUBSTITUTE** into the expression. The equation part has an equals sign—the expression is what you are solving for. If you can break these problems into parts, seeing the equation half (usually first) and the expression you want (usually second) this strategy becomes easier to apply.

> If $n-3=a$ and $n+8=b$, what is the value of $a-b$?
>
> **A.** –11   **B.** 5   **C.** 11   **D.** $2n-5$ **E.** $2n+5$

Here our equations ($n-3=a$ and $n+8=b$) already have a and b isolated, so we can substitute straight into the expression $a-b$:

$$a-b = (n-3) - (n+8)$$
$$n-3-n-8 = -3-8$$
$$\therefore -11$$

Answer: **A.**

TIP: TRIAL AND ERROR IS OK! Don't give up! If you try substituting in for one variable and don't get anything similar to the answer choices, try substituting for the other.

MAKE UP A NUMBER

If you can't make progress algebraically, make up a number. Remember, this technique works when there are **variables** in the **answers**, but it is typically slower than working algebraically.

> When $\frac{a}{b} + c = d$, and $b \neq 0$, $b = ?$
>
> **A.** $\frac{a}{d} - \frac{a}{c}$ **B.** $\frac{a+c}{d}$ **C.** $\frac{d}{a+c}$ **D.** $\frac{a}{d-c}$ **E.** $\frac{a}{d+c}$

TIP: Use distinct numbers so you don't get confused!

$$Let\ a=2;\ b=?;\ c=3;\ d=7$$

$$\frac{a}{b}+c=d$$

$$\frac{2}{b}+3=7$$

$$\frac{2}{b}=4$$

$$b=\frac{1}{2}$$

A. $\dfrac{a}{d}-\dfrac{a}{c}=\dfrac{2}{7}-\dfrac{2}{3}=\dfrac{6-14}{21}=\dfrac{-8}{21}\neq\dfrac{1}{2}$

B. $\dfrac{a+c}{d}=\dfrac{2+3}{7}=\dfrac{5}{7}\neq\dfrac{1}{2}$

C. $\dfrac{d}{a+c}=\dfrac{7}{2+3}=\dfrac{7}{5}\neq\dfrac{1}{2}$

D. $\dfrac{a}{d-c}=\dfrac{2}{7-3}=\dfrac{2}{4}=\dfrac{1}{2}$ THIS ONE

E. $\dfrac{a}{d+c}=\dfrac{2}{7+3}=\dfrac{2}{10}=\dfrac{1}{5}\neq\dfrac{1}{2}$

Answer: **D.**

WARNING: When you make up numbers, you must plug in to EVERY lettered answer to be sure that you have the right answer. Occasionally two answers (or more) will work with this method—if you get a duplicate yes, you must start over with new numbers. The only time not to check every answer would be if you have issues finishing on time.