Polynomial Factors and Graphs Answer Key

1. **D.** The function can be written as: $P(t)=2t(t^2-16)=2t(t-4)(t+4)$. Solving for the zeros by setting each factor to zero, we get $2t=0, t=0; t-4=0, t=4,; t+4=-, t=-4; t=\{-4,0,4\}$. Choice A is incorrect and results from taking the coefficient of the factor $2t$. Choice B proposes incorrect zeros. Choice C is incorrect and results from setting the polynomial equal to zero.

2. **B.** Based on the end behavior of the graph, we can tell that the function has an odd degree. Because one of the zeros of the graph occurs at x=0, we will know that x is a factor of the entire polynomial, so there will be no constant term. We can see that the other 2 zeros are both negative. Of the answer choices above, only choice B has two negative zeros and one 0 zero. $f(x)=x(x^2+5x+6)=x(x+3)(x+2); x=\{0,-3,-2\}$. Choice A and C are incorrect because they are polynomials of degree 2, thus they can only have a maximum of two zeros. Choice D is incorrect because when factored out, it produces zeros of 0, 3, and 2. Looking at the graph, it is clear that there are two negative roots and a 0 root.

3. **B.** From the given roots, we can determine that the polynomial has the factors $(x+2),(x-8)$, and x. Multiplying these three factors gives us the function and gives us x^3-6x^2+16x. Choice A, C and D are incorrect because none of them produce zeros of $-2, 8$, and 0.

4. **C.** To find the zeros, or roots, set each factor equal to zero. $(x+9)=0;\ x=-9; (2x-8)=0; 2x=8;\ x=4; (8x+2)=0; 8x=-2;\ x=-\frac{2}{8}=-\frac{1}{4}; x=\left\{-9,4,-\frac{1}{4}\right\}$ Choice A, B, and D are incorrect because they all misplace a negative sign.

5. **D**. Set each factor equal to zero.
$3=0$ is never true, so no value for x will make this true. Thus, this factor does not have a corresponding zero. $(x+18)=0, x=-18$ is a zero. $(3x-27)=0, 3x=27, x=-9$ is a zero. Choice A and B are incorrect because 3 is not a zero. Choice B is also incorrect because $\frac{1}{3}$ is not a zero. Choice C is incorrect because the root 9 should not be negative.

6. **A**. If (2x+3) is a factor of $6x^2+ax-27$, then we know that $6x^2+ax-27=(2x+3)(\ldots)$ multiplied by some other factor (…). Since we see the coefficient 6 before x^2 and the constant -27, we can guess that the other factor must be $3x-9$. Multiplying the two factors out, we get $(3x-9)(2x+3)=6x+9x-18x-27=6x^2-9x-27$. Thus, a must equal -9. Choices B, C, and D are incorrect and potentially result from algebra errors from multiplying the two factors out.

7. **C**. We plug in $(x+a)$ into the function and expand to get: $f(x+a)=3(x+a)^2-5=3(x^2+2ax+a^2)-5=3x^2+6ax+(3a^2-5)$. Now we can set $6ax=24x$ to get $a=4$. We can double check by setting $3a^2-5=43$ to also get $a=4$. Thus, $a=4$. Choices A, B, and D are incorrect and potentially result from algebra errors.

8. **B**. A root of a polynomial is a value for the variable that makes the entire polynomial equal to 0. Only choice B satisfies this; if we try plugging $-2,-4,$ or 6 into the choices, only choice B will give a 0 with x=6. Choice A is incorrect because it gives a root of $-6.$ Choice C is incorrect because it gives a root of 2. Choice D is incorrect because it gives a root of 4.

9. **C**. The four-term polynomial expression can be factored complete, by grouping, as follows:
$\left(x^3-3x^2\right)+\left(4x-12\right)=0;\ x^2\left(x-3\right)+4\left(x-3\right)=0;\ \left(x^2+4\right)\left(x-3\right)=0$
By the zero-product property, set each factor of the polynomial equal to 0 and solve each resulting equation for x. This gives $x=3$ or $x=\pm 2i$, respectively. Because the question asks for the real value of x that satisfies the equation, the correct answer is 3. Choices A, B, and D are incorrect because when plugged into the equation, none of them equals 0.

10. **D**. The equation $f\left(x\right)=k$ gives the solutions to the system of equations $fy=\left(x\right)=x^3+x^2-x-\frac{13}{4}$ and $y=k$. A real solution of a system of two equations corresponds to a point of intersection of the graphs of the two equations in the xy-plane. The graph of y=k is a horizontal line that contains the point (0,k). Thus, the line with the equation $y=-3$ is a horizontal line that intersects the graph of the cubic equation three times, and it follows that the equation $f\left(x\right)=-3$ has three real solutions. Choice A is incorrect because it only intersects the graph once when a horizontal line $y=-2$ is drawn. Choices B and C are incorrect for the same reasoning.