MANIPULATIONS

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- How to solve an equation in terms of any variable
- How to rework an equation into another form or expression

To save time, DO THESE PROBLEMS ALGEBRAICALLY!
Some teachers will tell you to "plug in numbers" for these problems. That way does work, but it is time consuming. We'll show you how to do that (in a pinch) below.

The idea behind manipulation is that you need to make what you have look like what you need. Sometimes these problems can look scary—but don't be afraid. Instead remember your two S's and two F's.

- Simplify
- Substitute
- Factor (see "Factoring and Foiling")
- FOIL (see "Factoring and Foiling")

As long as you apply these ideas, you'll find the answers.
Don't be overwhelmed! Much of the time there are variables in the answer choices. You're not being asked to solve for the impossible, but simply to rearrange things.

SIMPLIFY

These are the easiest.

Which of the following is the equation $9(x+y)=8$, solved for x?

A. $\frac{8-9x}{9}$ **B.** $\frac{8+9y}{9}$ **C.** $\frac{8-9y}{9}$ **D.** $\frac{8+y}{9}$ **E.** $\frac{8-y}{9}$

Here we want x alone and the everything else (the numbers and the y) together—so don't distribute! Divide by 9 first and you'll save time. To speed up this process, think about which step will help you get numbers together and variables together the fastest.

$$9(x+y)=8$$

$$x+y=\frac{8}{9}$$

$$x=\frac{8}{9}-y$$

Now we have x alone, but if we look at the answers, nothing matches. What doesn't match? All the answers are single fractions. Thus we need to turn our answer into a single fraction:

$$x=\frac{8}{9}-\frac{9y}{9}=\frac{8-9y}{9}$$

Answer: **C.**

SUBSTITUTE

Often, you'll see a pattern in manipulations problems: an EQUATION (or two) then an EXPRESSION you need to solve for. The key here is that you will always ISOLATE some variable in the equation that you will then SUBSTITUTE into the expression. The equation part has an equals sign—the expression is what you are solving for. If you can break these problems into parts, seeing the equation half (usually first) and the expression you want (usually second) this strategy becomes easier to apply.

If $n-3=a$ and $n+8=b$, what is the value of $a-b$?

A. -11 **B.** 5 **C.** 11 **D.** $2n-5$ **E.** $2n+5$

Here our equations ($n-3=a$ and $n+8=b$) already have a and b isolated, and as such, we can substitute straight into the expression $a-b$:

$$a-b=(n-3)-(n+8)$$

$$n-3-n-8=-3-8$$

$$\therefore -11$$

Answer: **A.**

Don't give up! If you try substituting in for one variable and don't get anything similar to the answer choices, try substituting for the other. Oftentimes if it doesn't work you may have made a careless error.

MAKE UP A NUMBER

If you can't make progress algebraically, make up a number. Remember, this technique works when there are **variables** in the **answers**, but it is typically slower than working algebraically.

When $\frac{a}{b}+c=d$, and $b\neq 0$, $b=?$

A. $\frac{a}{d}-\frac{a}{c}$ **B.** $\frac{a+c}{d}$ **C.** $\frac{d}{a+c}$ **D.** $\frac{a}{d-c}$ **E.** $\frac{a}{d+c}$

TIP: Use distinct numbers so you don't get confused!

$Let\ a=2; b=?; c=3; d=7$

$$\frac{a}{b}+c=d$$

$$\frac{2}{b}+3=7$$

$$\frac{2}{b}=4$$

$$b=\frac{1}{2}$$

A. $\frac{a}{d}-\frac{a}{c}=\frac{2}{7}-\frac{2}{3}=\frac{6-14}{21}=\frac{-8}{21}\neq\frac{1}{2}$

B. $\frac{a+c}{d}=\frac{2+3}{7}=\frac{5}{7}\neq\frac{1}{2}$

C. $\frac{d}{a+c}=\frac{7}{2+3}=\frac{7}{5}\neq\frac{1}{2}$

D. $\frac{a}{d-c}=\frac{2}{7-3}=\frac{2}{4}=\frac{1}{2}$ THIS ONE

E. $\frac{a}{d+c}=\frac{2}{7+3}=\frac{2}{10}=\frac{1}{5}\neq\frac{1}{2}$

Answer: **D.**

WARNING: When you make up numbers, you must plug in to EVERY lettered answer to be sure that you have the right answer. Occasionally two answers (or more) will work with this method—if you get a duplicate yes, you must start over with new numbers. The only time not to check every answer would be if you have issues finishing on time.

1. If $y \neq 0$, when $\frac{x^2}{y} = 5$, $25y^2 - x^4 = ?$

A. -25
B. -24
C. 0
D. 24
E. 25

2. If $x, y,$ and z are nonzero real numbers and $2xy - z = yz$, which of the following equations for x must always be true?

A. $x = zy + y + 2$
B. $x = 2yz - y$
C. $x = \sqrt{yz} - y$
D. $x = \frac{yz - z}{2y}$
E. $x = \frac{yz + z}{2y}$

3. For the equation $4x - 3a = -b$, which of the following expressions gives x in terms of a and b?

A. $\frac{3a - 4}{b}$
B. $\frac{3a - b}{4}$
C. $\frac{3a + 4}{4}$
D. $\frac{-b - 3a}{4}$
E. $3a - b - 4$

4. Which of the following is $3(m + n)^2 = 13$ solved for n?

A. $\pm\sqrt{\frac{13}{3}} - m$
B. $\pm\sqrt{\frac{13}{3}} + m$
C. $\pm\sqrt{\frac{3}{13}} + m$
D. $\pm\sqrt{\frac{13}{3} - m}$
E. $\pm\sqrt{\frac{3}{13}} - m$

5. If $x = 6z$ and $y = 15z$, which of the following is the relationship between x and y for each nonzero value of z?

A. $x = 2x - 1$
B. $y = 5y$
C. $x = y$
D. $x = \frac{2}{5}y$
E. $y = \frac{2}{5}x$

6. If $\sqrt{\left(14 - \sqrt{x}\right)} = 3 - \sqrt{5}$, then $x = ?$

A. 0
B. 20
C. 25
D. 30
E. 180

7. For all nonzero $a, b,$ and c such that $2a = \frac{b}{c}$, which of the following *must* be equivalent to ab?

A. $\frac{c}{2a}$
B. $2ac^2$
C. $\frac{b^2}{2c}$
D. $\frac{a^2}{2c}$
E. $\frac{2c}{b}$

8. The relation between enthalpy and energy is $H = E + 8.31nT$, where H is the change in enthalpy, E is the change in energy, n is the change in moles, and T is the change in temperature. Which of the following expressions gives n in nonzero terms of H, E, and T?

A. $\frac{H-E}{8.31}$

B. $\frac{8.31T}{H-E}$

C. $\frac{H-E}{8.31T}$

D. $8.31E(H-T)$

E. $8.31T\left(H-E\right)$

9. Which of the following is *not* true for all $\left(m,n\right)$ that satisfy the equation $\frac{n}{2}=\frac{m}{3}$ for $n, m \neq 0$?

A. $\frac{m}{2}=\frac{n}{2}$

B. $3n = 2m$

C. $m=\frac{3}{2}n$

D. $n+m=\frac{5}{2}m$

E. $n \neq m$

10. If $x-y \neq 0$ and $\frac{3y+5x}{x-y}=\frac{5}{7}$, then $\frac{y}{x}=?$

A. $\frac{-15}{13}$

B. $\frac{15}{8}$

C. $\frac{5}{7}$

D. $\frac{12}{43}$

E. 3

11. If $\frac{2y-3x}{4y-5x}=\frac{3}{10}$, then $\frac{y}{x}=?$

A. $\frac{15}{8}$

B. $\frac{16}{30}$

C. $\frac{3}{10}$

D. $\frac{3}{16}$

E. 3

12. When $y=-x^3$, which of the following expressions is equal to $\frac{1}{y}$?

A. $\frac{1}{-x^3}$

B. x^{-3}

C. $\frac{1}{-x^{-3}}$

D. $\frac{1}{-x}$

E. $-x^3$

13. Given that $A,B,C,$ and D are all positive real numbers satisfying $A^2=\frac{1}{2}B^2$, $C=D$, and $B=\sqrt{D}$, which of the following equations is NOT necessarily true?

A. $B^2=C$

B. $A=\sqrt{\frac{1}{2}D}$

C. $A=\frac{1}{2}D$

D. $A^2=\frac{1}{2}C$

E. $A=\frac{\sqrt{2}}{2}B$

14. For all real numbers x and y such that x is the quotient of y divided by 5, which of the following represents the difference of y and 5 in terms of x?

A. $x-5$

B. $\frac{x}{5}-5$

C. $5x-5$

D. $5(x-5)$

E. $\frac{x-5}{5}$

15. The area of a rectangle is A square units. The length is l units, and the width $5n+3$ units longer than l. What is n in terms of A and l?

A. $n=\frac{A-l^2-3l}{5l}$

B. $n=\frac{l^2+3l-A}{5l}$

C. $n=\frac{A-3l}{5l}$

D. $n=\frac{A-l-3}{5l}$

E. $n=\frac{A-3l}{5}$

ANSWER KEY

1. C **2. E** **3. B** **4. A** **5. D** **6. E** **7. C** **8. C** **9. A** **10. A** **11. A** **12. A** **13. C** **14. C** **15. A**

ANSWER EXPLANATIONS

1. **C.** We wish to write one variable in the terms of the other. For this problem, it is easier to write y in terms of x. We are given $\frac{x^2}{y}=5$, so multiplying y on both sides and dividing by 5 on both sides gives us $\frac{x^2}{5}=y$. Now we can substitute in $y=\frac{x^2}{5}$ into the equation $25y^2-x^4$. This is equal to $25\left(\frac{x^2}{5}\right)^2-x^4=25\left(\frac{x^4}{25}\right)-x^4 \rightarrow x^4-x^4=0$.

2. **E.** We wish to write x in terms of y and z, so our goal is to move the equation around so that x is on a side by itself. We do this by first adding z to both sides of the equation, giving us $2xy=yz+z$. Then, we divide both sides by $2y$, giving us $x=\frac{yz+z}{2y}$.

3. **B.** We wish to write x in terms of a and b, so our goal is to move the equation around so that x is on a side by itself. We do this by first adding $3a$ to both sides of the equation, giving us $4x=3a-b$. Then, we divide both sides by 4, giving us $x=\frac{3a-b}{4}$.

4. **A.** We wish to write n in terms of m, so our goal is to move the equation around so that n is on a side by itself. We do this by first dividing both sides by 3, giving us $(m+n)^2=\frac{13}{3}$. Then, taking the square root of both sides gives us $m+n=\pm\sqrt{\frac{13}{3}}$. Finally, subtracting m on both sides gives us $n=\pm\sqrt{\frac{13}{3}}-m$.

5. **D.** To solve this, we want to first write z in terms of y so we can plug in that expression into the variable z to evaluate x in terms of y. So, our first step is to write z in terms of y by dividing both sides of the second equation by 15. This gives us $\frac{y}{15}=z$. Now, we plug in $\frac{y}{15}$ for the z value in $x=6z$ to get $x=6\left(\frac{y}{15}\right)\rightarrow\frac{2y}{5}\rightarrow\frac{2}{5}y$.

6. **E.** We want to isolate x, so we first square both sides of the equation. This gives us $14-\sqrt{x}=\left(3-\sqrt{5}\right)^2$. FOILing out the right side of the equation, we get $14-\sqrt{x}=9-6\sqrt{5}+5$. Subtracting 14 on both sides, we get $-\sqrt{x}=-6\sqrt{5}$. Squaring both sides gives us $x=6^2*5=36*5=180$.

7. **C.** Dividing $2a=\frac{b}{c}$ by 2 on both sides, we get $a=\frac{b}{2c}$. To solve for the value of ab, we can plug in $a=\frac{b}{2c}$ into ab to get $ab=\frac{b}{2c}(b)\rightarrow\frac{b^2}{2c}$.

8. **C.** Subtracting E from both sides of the equation, we get $H-E=8.31nT$. Dividing both sides of the equation by $8.31T$, we get $\frac{H-E}{8.31T}=n$.

9. A. Multiplying the equation, we get $3n = 2m$, which eliminates answer choice (B). Dividing both sides of $3n = 2m$ by 2 gives us $\frac{3n}{2} = m$, which eliminates answer choice (C). Dividing both sides of $3n = 2m$ by 3 gives us $\frac{2m}{3} = n$, and adding m to both sides to this equation gives us $\frac{2m}{3} + m = n + m \rightarrow \frac{2m}{3} + \frac{3m}{3} = n + m \rightarrow \frac{5m}{3} = n + m$, which eliminates answer choice (D). For all values of n and m not equal to zero, answer choice (E) is true. Lastly, Answer choice (A) is false for all values of n and m except when n and m both equal 0, but the problem states that $n, m \neq 0$. So, the only answer that is false is (A).

10. A. Cross-multiplying the equation, we get $(3y + 5x)7 = (x - y)5$. Distributing the 7 and 5, we get $21y + 35x = 5x - 5y$. Now, we subtract $5x$ from both sides to get $21y + 30x = -5y$. Then, we subtract $21y$ from both sides to get $30x = -26y$. To find the value of we can divide both sides by x to get $30 = \frac{-26y}{x}$. Then divide both sides by -26, which is $-\frac{30}{26} = \frac{y}{x}$. $-\frac{30}{26}$ simplifies to $-\frac{15}{13}$.

11. A. Cross-multiplying the equation, we get $10(2y - 3x) = 3(4y - 5x)$. Distributing out the x and 3, we get $20y - 30x = 12y - 15x$. Combining like terms, we get $8y = 15x$. Dividing both sides by 8, we get $y = \frac{15x}{8}$. Then finally, to find the value of $\frac{y}{x}$, we divide both sides of the equation by x to get $\frac{y}{x} = \frac{15}{8}$.

12. A. Since $y = -x^3$, $\frac{1}{y} = \frac{1}{-x^3}$ by plugging in $y = -x^3$ in the denominator.

13. C. Since $B = \sqrt{D}$ and $C = D$, then by substitution, $B = \sqrt{C} \rightarrow B^2 = C$ so answer choice (A) is true. Since $A^2 = \frac{1}{2}B^2$ and $B = \sqrt{D}$, by substitution, $A^2 = \frac{1}{2}\sqrt{D}^2 \rightarrow A^2 = \frac{1}{2}D \rightarrow A = \sqrt{\frac{1}{2}D}$ so answer choice (B) is true. Since answer choice (B) is always true, answer choice (C) is not always true unless $A = 1$ so the correct answer is (C). Verifying that the rest of the answer choices are always true, we see that from answer choice (B) we know that $A = \sqrt{\frac{1}{2}D}$ and $D = C$, so $A = \sqrt{\frac{1}{2}C} \rightarrow A^2 = \frac{1}{2}C$ and answer choice (D) is true. Lastly, since $A^2 = \frac{1}{2}B^2$, taking the square root on both sides, answer choice (E) is true: $A = \frac{B}{\sqrt{(2)}} = \frac{\sqrt{2}}{2}B$.

14. C. x is the quotient of y divided by 5, which means $x = \frac{y}{5} \rightarrow 5x = y$. The difference between y and 5, using our new expression for y, is $5x - 5$ (or $5 - 5x$, but only $5x - 5$ is an answer provided).

15. A. The width of the rectangle is $l + 5n + 3$. The area of the rectangle, A, is $l(l + 5n + 3)$. Distributing gives us $l^2 + 5nl + 3l = A$. We isolate the term containing n : $5nl = A - l^2 - 3l$. Finally, isolate n : $n = \frac{A - l^2 - 3l}{5l}$.