1. A 1st grade teacher has 30 students. She needs to choose 3 students for 3 different leadership positions: class president, class secretary, and class treasurer. If she picks three students at random to fill the positions, which of the following gives the number of different possible results for choosing these 3 class officers?

A. $30 \cdot 29 \cdot 28$
B. 30^3
C. $29 \cdot 28 \cdot 27$
D. $3 \cdot 2 \cdot 1$
E. 3^3

2. Six friends are standing in line to buy movie tickets. In how many different ways can they stand in line?

A. 1
B. 21
C. 120
D. 720
E. 360

3. Dwayne and 3 of his friends go on a carnival ride that can seat 4 people from front to back. They only go on the ride once, but Dwayne is curious in how many different ways he and his friends could sit on the ride. How many different ways can they sit on the ride?

A. 10
B. 6
C. 40
D. 1
E. 24

4. If 5 letters are taken from the word COMPUTER without repeating a letter, how many different orderings of these letters are possible?

A. 120
B. 6875
C. 6720
D. 3125
E. 25

5. California license plates consist of 1 number (1 to 9) followed by 3 letters (from the 26-letter alphabet) followed by 3 more numbers (0 to 9). How many possible license plates are there?

A. $9 \cdot 10^3 \cdot 26^3$
B. $10^4 \cdot 26^3$
C. $10 \cdot 9 \cdot 8 \cdot 7 \cdot 26 \cdot 25 \cdot 24 \cdot 23$
D. $9 \cdot 9 \cdot 8 \cdot 7 \cdot 26 \cdot 25 \cdot 24 \cdot 23$
E. $9^4 \cdot 26^3$

6. The School Store has in-stock 15 packages of blue pens, 12 packages of black pens, and 20 of red pens. Ramsey will select 3 packages of pens to purchase. How many different selections of 3 pen packages are possible?

A. 3
B. 6
C. 9
D. 10
E. 27

7. In a polygon, diagonals can be drawn between every vertex of the polygon and every other vertex of the polygon, excluding the two adjacent vertices. A regular polygon has equal side lengths and equal angle measures. How many distinct diagonals can be drawn in a 6-sided regular polygon?

A. 6
B. 9
C. 12
D. 18
E. 36

8. Sabrina has been instructed to make up three linear equations and then graph them on the coordinate plane. The equations do not necessarily need to form distinct lines. If she graphs any such three linear equations, what are all the possible numbers of distinct regions that these lines could divide the plane into?

A. 2,3,4,6,7
B. 2,4,5,6,7
C. 3,4,5,6,7
D. 1,2,4,6,7
E. 3,4,5,6,7

9. Given 8 points, no 3 of which lie on the same straight line, what is the maximum number of straight lines that can be drawn through pairs of the 8 points.

A. 16
B. 28
C. 32
D. 56
E. 64

10. Asa has 4 pairs of shoes, 5 pairs of pants, and 4 shirts which can be worn in any combination. He needs to choose an outfit to wear to his friend's birthday party. How many different combinations consisting of one item in each category are possible?

A. 9
B. 13
C. 20
D. 40
E. 80

11. Buildings A, B, C, and D all lie on the same city street though not necessarily in that order. The street runs straight from east to west. If Building B is 5 blocks east of building D, building A is 8 blocks west of building C, and building B is 5 blocks west of building C, what is the relationship between building A and D?

A. Building A is 3 blocks east of Building D
B. Building D is 3 blocks east of Building A
C. Building A is 2 blocks east of Building D
D. Building D is 2 blocks east of Building C
E. Cannot be determined

12. In a poetry contest, a group of 6 poets stand in a circle. A designated poet is chosen to speak first. He or she then must call out the name of another poet to speak, but that poet cannot be to her immediate right or left, and cannot be the last poet who spoke. Given this arrangement, what is the earliest speaking turn that the first poet could possibly speak again?

A. 3^{rd}
B. 4^{th}
C. 5^{th}
D. 6^{th}
E. 7^{th}

13. At a school, one junior and one senior are named student of the month. If there are 40 juniors and 50 seniors, how many different 2-person teams of 1 junior and 1 senior are possible?

A. 10
B. 90
C. 200
D. 1000
E. 2000

14. A programmable lock allows users to make an alpha numeric pin consisting of one upper case letter in the first position (A through Z) followed by two digits (each digit can be any digit zero through 9). How many different such pins can be made?

A. $9 \cdot 2 \cdot 26^3$
B. $10^2 \cdot 26$
C. $26 \cdot 25 \cdot 24$
D. $9 \cdot 8 \cdot 26$
E. $26 \cdot 10$

15. On a camping trip, the Lee family rents a paddle boat that seats two people. The Lee family has two adults and four children, one of whom is an infant so cannot go on the paddle boat. At least one adult must remain on shore to care for the infant. Given these restrictions, how many different pairs of two family members could ride together on the paddle boat?

A. 6
B. 9
C. 10
D. 12
E. 14

16. At a rice bowl shop, Hanna can choose her own rice, protein and vegetable. For rice she can choose from 3 styles of rice, for protein, she can choose tofu, chicken, or beef, and she can choose one of 4 different vegetables. How many possibilities are there for Hanna's three choices for her rice bowl?

A. 10
B. 12
C. 36
D. $3^2!4!$
E. $3!3!4!$

18. Serial codes on a line of toys consist of 3 digits from the 10 digits, 0 through 9, 4 letters taken from the 26 letters, A through Z, followed by another 2 digits from the 5 digits, 0 through 4. Which of the following expressions gives the number of distinct serial codes that are possible given that repetition of both letters and digits is allowed?

A. $10^5 26^4$
B. $10^3 26^4 5^2$
C. $(10\cdot 3)(26\cdot 4)(5\cdot 2)$
D. $3^{10} 4^{26} 2^5$
E. $10!^3\, 26!^4\, 5!^2$

19. Which of the following expressions gives the number of permutations of 27 objects taken 6 at a time?

A. $(27)(6)$
B. $(27-6)!$
C. $\frac{27!}{6!}$
D. $\frac{27!}{(27-6)!}$
E. $\frac{27!}{(6!)(27-6)!}$

20. Students at a university are assigned identification codes consisting of letters and numbers. The codes consist of 3 digits out of the 10 possible digits, and 3 out of the 26 possible letters. No code will repeat digits or letters. How many codes are possible?

A. $10^3 \cdot 26^3$
B. $9\cdot 8\cdot 7\cdot 26\cdot 25\cdot 24$
C. $10\cdot 9\cdot 8\cdot 26\cdot 25\cdot 24$
D. $10\cdot 9\cdot 8\cdot 25\cdot 24\cdot 23$
E. $3\cdot 10\cdot 3\cdot 26$

21. Amy is making her schedule for next semester's classes. She needs to take six specific classes, but in no particular order. How many different possible schedules can Amy make if she takes those six classes, assuming she has six academic periods each day?

A. 36
B. 120
C. 360
D. 720
E. 46,656

22. Which of the following expressions gives the number of distinct permutations of the letters in QUADRATIC?

A. $9!$
B. $8!(2)$
C. $\frac{8!}{2}$
D. $\frac{9!}{2}$
E. $(9)(8)(7)(6)(5)(3)(2)(1)$

23. The factorial of a number, notated $n!$, is the product of all positive integers less than or equal to n. For example, $4! = 4\cdot 3\cdot 2\cdot 1 = 24$. What is $\frac{7!}{4!2!}$?

A. 15
B. 70
C. 105
D. 210
E. 420

24. If n is an integer greater than 3, what expression is equivalent to $\frac{(n+4)!(n-2)!}{((n+3)!)^2}$?

A. $\frac{n+3}{(n+4)(n+2)(n+1)(n)(n-1)}$
B. $\frac{n+4}{(n+3)(n+1)(n)(n-1)}$
C. $\frac{n+4}{(n+3)(n+2)(n+1)(n-1)}$
D. $\frac{n+3}{(n+3)(n+2)(n+1)(n)}$
E. $\frac{n+4}{(n+3)(n+2)(n+1)(n)(n-1)}$

25. If $\frac{(x+1)!}{(x-1)!} = 42$ for positive x, then $x! = ?$

A. 6
B. 7
C. 5040
D. 720
E. 26

ANSWERS

1. A **2. D** **3. E** **4. C** **5. A** **6. D** **7. B** **8. A** **9. B** **10. E** **11. C** **12. B** **13. E** **14. B**
15. B **16. A** **17. C** **18. B** **19. D** **20. C** **21. D** **22. D**

ANSWER EXPLANATIONS

1. **A.** To solve arrangements problems we first must determine whether order matters. In this problem, order does matter: assigning Amy to be president instead of secretary is a different outcome. As such, we can think of this problem as a "tree-diagram" style problem. Drawing out a tree is pretty time consuming, so we use the "collapsed" method based on the tree diagram. For more on this, see the explanation at the beginning of this chapter. Make a slot for each choice we have to make as so:

______	______	______
president	secretary	treasurer

   Now we, how many people do we have to choose from for this position? For the first position we have 30 people, but then one person will not be eligible for the second position (the person chosen for the first). Now fill in "30" for that first slot (president). For the second blank, ask again, how many people can we choose from? Now we have 29 people—so we write that in. For the third blank, two people are spoken for already, so we have 28 people to choose from. Now we multiply the three numbers together. We're left with: $30 \cdot 29 \cdot 28$. Another way to solve this problem is to recognize this is a permutation (an arrangement in which order matters with no repeated terms and with no special conditions). Permutations can be solved using the permutation function on our calculator: ${}_nP_r$—on the TI-84 plus this is found in the MATH menu under PROB. Enter ${}_{30}P_3$ (30 then key ${}_nP_r$ then 3) and we'll get the answer.

2. **D.** The method for solving this problem is identical to problem #1, but now with six blanks. As order matters, this is a permutation: We draw out six blanks: __ __ __ __ __ __ —then fill them in with numbers of how many students we have to choose from to fill each blank, and then we multiply together. With each step we have one less person to choose from:

   $$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

   We can alternatively use the ${}_nP_r$ function on a calculator ($n = 6, r = 6$).

3. **E.** This question is a bit of a trick question—Dwayne and THREE OF HIS FRIENDS is actually FOUR people—Dwayne plus the other three. So we're taking 4 people 4 at a time in a permutation. Method is the same as in questions 1 & 2—so $4 \cdot 3 \cdot 2 \cdot 1 = 24$.

4. **C.** Here, order matters (how many different ORDERINGS). As such this is a permutation—there are 8 distinct letters in COMPUTER and we're choosing five at a time, or ${}_8P_5$. Imagine five slots for each letter we are choosing, and we would have $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$. Remember the letters cannot be used twice—so we subtract one option as we go. This is a permutation—see questions 1 & 2 explanations or the chapter start for more on solving these.

5. **A.** Here order matters, but because of the special conditions (letters and numbers) as well as the fact that numbers CAN repeat, we cannot use the permutations button on our calculator any more to find the answer. Instead, set our blanks up:

____	____	____	____	____	____	____
#	Letter	Letter	Letter	#	#	#

   Now for the first blank we have 1 to 9—that's NINE numbers (1, 2, 3, 4, 5, 6, 7, 8, 9). So we write a "9" in the first blank. In the next three blanks, we have 26 three times. In the last three blanks, we actually have TEN numbers—as this part calls for numbers ZERO through nine, not 1-9—if we're not sure, we can write them all out (1-9 is nine plus zero). "10" choices—or:

   $$9 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$$

   Now the answer choices are in exponent form—so we group our like terms together:

   $$9 \cdot 10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26$$

   That's one nine times three 10's or 10^4 times three 26's or $26^3 = 9 \cdot 10^3 \cdot 26^3$. Be sure to READ CAREFULLY!

6. **D.** In this case, order doesn't matter. Picking two packs of red pens first and a black package last is the same as picking black first, and two reds second—one either buys the packs or doesn't. As such, and given the fact that we CAN repeat terms (i.e. we could buy 3 packs of all black pens—once we pick black it's not "out"), we cannot use traditional permutation or combination equations to solve this problem. The store, furthermore, has plenty of pens in stock—i.e. if we're only buying 3 packs, there are more than enough of EVERY type of pen—so we don't need to worry about that as a limiting factor. Because the numbers are small, this is easy enough to just write out—but try to be systematic (organized). Here, X is used for black, R for red and B for blue:

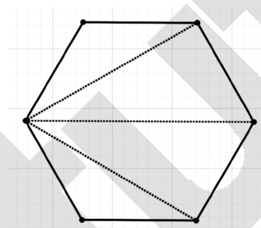
THREE OF A KIND: All Black, All Red, All Blue (3 options)

TWO OF A KIND: XXR, XXB, RRX, RRB, BBX, BBR (6 options)

ONE OF EACH: XRB (1 option)

Altogether, that's 10 options.

7. **B.** This is another arrangements problem, but in this problem, order doesn't matter—the diagonal from vertex A to vertex C is the same as a diagonal from vertex C to vertex A. As such it's not a permutation, but more of a combination. Likewise, we have a special condition: we don't count the lines between a point and the adjacent points. So we can't just use the combination function in our calculator, but we do have to divide out the "repeats" at the end of the problem (which we do WHENEVER order doesn't matter). First, figure out how many diagonals can be drawn from any given vertex. With 6 vertices, we'll have 5 OTHER vertices besides the one we're drawing from. Now subtract TWO from the five, as we don't draw a diagonal to those two according to the problem. That leaves us with THREE diagonals per vertex. See the picture below:



Every vertex is connected to three diagonals, so $6 \cdot 3 = 18$. But we're not done yet! 18 includes REPEATS—i.e. we've counted the diagonal from vertex A to vertex C AND a diagonal from vertex C to vertex A, etc. As such, we divide 18 by 2 to get the answer: 9. One can also memorize the formula for number of diagonals in a polygon with n sides: $\frac{n(n-3)}{2}$.

8. **A.** The best way to solve something like this is to draw it out. To divide the plane by 2, the 3 linear equations will be the same. To divide the plane by 3, there will be 3 lines that are the same and another line parallel to them. To divide the plane by 4, we can draw 3 different parallel lines. To divide the plane by 6, we can draw 2 different parallel lines and 1 line perpendicular to those lines (so they will intersect). To divide the plane by 7, we can form a triangle with the 3 lines, so each line has 2 intersections.

9. **B.** Order doesn't matter on this one. Essentially we are taking two points at a time. If we labeled our points A-H and made a list of the combinations it would look something like:

AB, AC, AD, AE, AF, AG, AH…

This is not a permutation, but we'll start off by figuring out the number of arrangements if order DID matter, and then divide out the repeated terms. If there are eight points, then there would be 7 in the list for pairs starting with "A"—If order matters, there would be 7 in the list for each of the other points as well (BA, BC, and so on), so that gives us 56. But right now, we're counting BA and AB as different lines—that's not ok. Because there are two terms, there are two ways to arrange any given pairing (i.e. if I know point A and point B are involved, there are 2 ways to arrange them). As such we need to divide by 2 to divide out all the repeated terms: $56 \div 2 = 28$.

10. **E.** This is similar to a permutation—order "matters" as each category we are choosing is distinct. Make a slot for each choice we have to make as so:

$$\underset{\text{shoes}}{\underline{\qquad}} \times \underset{\text{pants}}{\underline{\qquad}} \times \underset{\text{shirts}}{\underline{\qquad}}$$

Then fill in the blanks with the number of choices we have: $4,\ 5,\ 4,$ and multiply together: $4\cdot5\cdot4=80$. Because order "matters" or each slot is distinct, no need to worry about repeats. We're done! If this confusing, we might start listing out what is possible. Shoes A would have five pant options—each of those pant options 4 shirt options, as shown below, etc. As we can see for shoes A alone we'll have 5×4 or 20 options—for shoes B, C, and D we'll also have 20 options each…that's why we multiply in the work above!

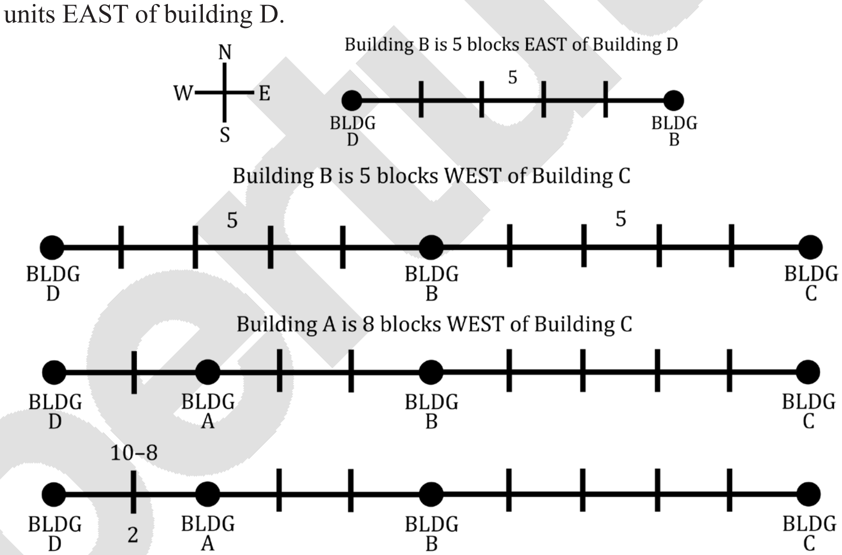
Shoes A:

Pants 1
- Shirt A
- Shirt B
- Shirt C
- Shirt D

Pants 2
- Shirt A
- Shirt B
- Shirt C
- Shirt D

Pants 3—4 options
Pants 4—4 options
Pants 5—4 options

11. **C.** For this problem, we draw a picture, taking one command at a time. We draw the 1st and 3rd commands last—as the middle command has two new letters, so we might place them in the wrong order if we do this command first. We solve this using a number line, as shown below. Once we see the distance from D to C is 10 units, we can subtract the 8 units to get that A is 2 units EAST of building D.



12. **B.** Problems like this can seem confusing—but the best way to solve them is to draw out as many possibilities as we can, be systematic by always being aware of when we have more than one option, and know that we should aim for the smallest number first. Let's name our poets Amy, Betsy, Candy, Dina, Emily, Farrah – and imagine they are in that order around the circle (so Farrah is next to Amy).

FIRST: Let's try to do this on the 3rd turn.
1st Turn—Amy speaks—then can choose CANDY, DINA, or EMILY—let's stick with CANDY for now. If she doesn't work we'll potentially have to consider the others.
2nd Turn—Candy speaks
3rd Turn—Amy can't speak because Candy can only pass to someone who is not the LAST person who spoke—which would be Amy. So 3rd turn doesn't work—and this would be true no matter which lady spoke first.

SECOND: Let's try for the 4th turn.
1st Turn—Amy speaks—then can choose CANDY, DINA, or EMILY—let's stick with CANDY for now. If this doesn't work we'll have to check the other ladies, too.

2nd Turn—Candy speaks—now Candy can't pass to neighbors (Betsy, Dina), or Amy (last speaker)—she can pass to EMILY or FARRAH.
3rd Turn—two options—EMILY or FARRAH—if FARRAH speaks, she's next to Amy, so Amy can't be 4th. Let's try EMILY—if Emily speaks, she can't pass to Candy (last speaker), or Farrah or Dina (neighbors)—AHA! If Emily speaks she CAN pass to Amy on the 4th turn—so Amy can be on the 4th turn.

Obviously, there are other combinations—we could have tried DINA 2nd, and then would have ended up with BETSY or FARRAH 3rd—making it impossible for AMY to be 4th—that's why we need to be aware of each choice we have made between multiple options so we can track back to it and look at other possibilities.

13. E. For this problem, we ask ourselves how many juniors we have to choose from (40) and then how many seniors (50) —simply multiply these numbers together. If we were to listed them out we would do so as Amy Jr.—Allan Sr., Amy Jr.—Brian Sr., Amy Jr.—Carl Sr.…Amy would have 50 seniors she could be paired with, and so on for each of the 40 juniors. This is like a permutation in that there is NO overlap between juniors and seniors—i.e. we have 40 different people who are juniors to choose from and none of them can also be seniors—as such order matters. If you put (B) you likely added the numbers together—multiply, don't add. If you put (C) you likely tried to multiply in your head and messed up the decimal place—use your calculator if you need it. If you put (D) you likely thought you had to divide out repeats. No need to divide out any repeats—Amy the junior and John the senior can't be created in reverse order (because John is not a junior).

14. B. In a combo lock, order matters. As such this is a permutation—we first ask "how many options do we have to choose from" for each element, then multiply the number of options together. Remember that there are 26 letters, and 10 digits for each of two digit positions (0-9 is TEN digits—1-9 is nine digits and zero makes 10). $26\times10\times10=10^2\cdot26$

15. B. Occasionally an arrangements problem has so many restrictions, it's best to work out the logic and list out all the options rather than fully rely on formulas. Here, we've got a family that is going to divide out into groups of two—and only one parent at a time can be on the boat. Now we could also have kids alone on the boat. As far as we can tell, order DOES NOT matter—you're either on the boat together or not. We don't care who sits in what seat. Let's start with the condition that mom is on the boat (and dad is on shore with the baby), then dad is on the boat, then kids only. Let's label the kids A,B, C and mom and dad M, D.

If mom is on the boat: MA, MB, MC—3 options
If dad is on the boat: DA, DB, DC—3 options
If no parents are on the boat: (here I can either do ${}_3C_2$ on my calculator, $3\cdot2$ then divide by 2 to eliminate repeats, or I can list out) AB, BC, CA (if I list BA, CB or AC those are repeats)—also 3 options
As we can see—we have 9 total options

If you chose (A) you may have added all the members together or missed some options, if you chose (C) you may have added the pair MD (Mom and Dad), which would be impossible (abandoned baby!)—or done ${}_5C_2$ —but this is not a simple combination; it has restrictions (D) would double count the kid pairs, and (E) would be a permutation (if order mattered) of 5 people taken two at a time with no restrictions; again we have restrictions so you can't just multiply $5\cdot4$, and order doesn't matter so you must eliminate repeats.

16. C. To find the number of possible combinations of rice, proteins, and vegetables, we multiply the number of items in each category together. There are 3 kinds of rice, 3 kinds of protein, and 4 kinds of vegetable, so our answer is $(3)(4)(4)=36$.

17. B. The number of different possibilities is equal to the number of possible numbers or letters that can occupy each position in the code multiplied together. For example, in a 3 digit code using the digits 0 through 9 , we have 3 positions with 10 different possible digits in each position, so the number of combinations is $(10)(10)(10)=10^3$. We use exponents to make the numbers easier to handle. The exponential expression will be the possible digits/letters to the power of the number of positions. For this problem, we have 3 positions using 10 digits, 4 positions using 26 letters, and 2 digits using 5 digits. Thus, the total number of combinations is $10^3 26^4 5^2$.

18. D. The number of different permutations will be equal to the number of members available to fill the first spot, times the number of members available to fill the second spot, and so on. Since we do not reuse objects (e.g. the object in the first position can't also be in the third position) every spot has one fewer possible objects to fill it than the one before it. Thus, the number of different permutations is equal to $(27)(26)(25)(24)(23)(22)$. A shorthand way to express this kind of answer is as the factorial of the members of a set divided by the factorial of the members that won't be used at the end. We don't use any of the 27 objects except for the 6 that we do use, so the number of objects we don't use is $(27-6)$. Our answer is $\frac{27!}{(27-6)!}$.

19. C. The first letter in the identification code can be chosen out of any of the 10 digits, so there are 10 possible outcomes for the first letter. The second letter cannot repeat the first letter, so there are 9 possible outcomes for the second letter. The third letter cannot repeat the first or second letters, so there are 8 possible outcomes for the third letter. Likewise, there are 26 numbers to choose from for the first digit, 25 for the second, and 24 for the third. The total number of possible permutations is the product of these possible outcomes. So, our answer is $10\cdot 9\cdot 8\cdot 26\cdot 25\cdot 24.$

20. D. When creating her schedule, Amy has six time slots to fill six classes. For the first time slot, she can place any class. For the second time slot, she has 5 remaining classes to choose from, for the third time slot she has 4 remaining classes to choose from, and so on until all time slots are filled. The total number of possible permutations of classes Amy can schedule is calculated by $6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$ or $6! = 720.$

21. D. There are 8 individual letters used, with one, the 'A', occurring twice to make 9 total letters in the set. If we imagine that each of the 'A's are different, then the number of possible permutations is $9!$. However, since in each of the permutations the 'A's can switch positions without changing the permutation, there are actually only half the number of permutations. Thus, we have $\frac{9!}{2}$ permutations.

22. C. Using the definition given in the problem, $7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 5054$ and $4! = 4\cdot 3\cdot 2\cdot 1 = 24$ and $2! = 2\cdot 1 = 2$. So $\frac{7!}{4!2!} = \frac{5054}{24\cdot 2} = 105.$ Another way to approach the problem is to write out $\frac{7!}{4!2!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(4\cdot 3\cdot 2\cdot 1)(2\cdot 1)}$. Then, canceling out equivalent numbers in the numerator and denominator, we get $\frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(4\cdot 3\cdot 2\cdot 1)(2\cdot 1)} = \frac{7\cdot 6\cdot 5}{2\cdot 1}$ which is a lot easier to calculate. $\frac{7\cdot 6\cdot 5}{2\cdot 1} = 7\cdot 3\cdot 5 = 105$.

23. E. $\frac{(n+4)!(n-2)!}{(n+3)^2!} = \frac{(n+4)!(n-2)!}{(n+3)!(n+3)!}$. Since we know:

$(n+4)! = (n+4)(n+3)!$ and $(n+3)! = (n+3)(n+2)(n+1)(n)(n-1)(n-2)!,$ we can rewrite the expression as:

$$\frac{(n+4)(n+3)!(n-2)!}{(n+3)!(n+3)(n+2)(n+1)(n)(n-1)(n-2)!}$$

Now we can cancel out equal terms from the numerator and denominator to get:

$$\frac{(n+4)(n+3)!(n-2)!}{(n+3)!(n+3)(n+2)(n+1)(n)(n-1)(n-2)!} = \frac{(n+4)}{(n+3)(n+2)(n+1)(n)(n-1)}$$

24. D. We can write $(x+1)!$ as $(x+1)(x)(x-1)!.$ So, we can rewrite the equation as $\frac{(x+1)(x)(x-1)!}{(x-1)!} = 42.$ Canceling out like terms from the numerator and denominator, we get $(x+1)(x) = 42 \rightarrow x^2 + x = 42.$ Subtracting 42 on both sides, we get $x^2 + x - 42 = 0.$ Now, we factor this to find the zeros to get $(x+7)(x-6) = 0$. So, $x = -7$ or 6. Since x is positive, $x = 6.$ So, $x! = 6(5)(4)(3)(2)(1) = 720.$