LOGARITHMS

ACT Math: Problem Set

1. If x, y, z are positive real numbers, which of the following expressions is equal to $3\log_2 x - \log_4 y + \frac{1}{2}\log_2 z$?

A. $\log_2 \frac{x^2\sqrt{z}}{2y}$

B. $\log_2 \frac{x^3 z}{2} - \log_4 y$

C. $\frac{3}{2}\log_2(x+z) - \log_4 y$

D. $\log_2 x^3\sqrt{z} - \log_4 y$

E. $\log_2\left(x^3 + \sqrt{z}\right) - \log_4 y$

2. If $\log_4 3 = a$ and $\log_4 5 = b$, which of the following is equal to 8?

A. 4^{a+b}

B. $4^a + 4^b$

C. 16^{a+b}

D. ab

E. $a+b$

3. If $\log_a x = n$ and $\log_a y = m$ then $\log_a\left(\frac{x}{y}\right)^3 = ?$

A. $3(n-m)$

B. $3(n+m)$

C. $n-m$

D. $3(m-n)$

E. $\frac{n}{m}$

4. If $3^{x-1} = 3y$, what is 3^{x+1} in terms of y?

A. $27y$

B. $3y$

C. $3y+2$

D. $(3y)^2$

E. $9y$

5. If $2^{a+2} = 4b$, which of the following is an expression for b^2 in terms of a?

A. $\frac{1}{2^{2a}}$

B. 4^a

C. 2^{a+1}

D. 2^{a+2}

E. 2^{2a}

6. If $2^n = 53$, then which of the following must be true?

A. $2 < n < 3$

B. $3 < n < 4$

C. $4 < n < 5$

D. $5 < n < 6$

E. $6 < n$

7. Which of the following is a value of x that satisfies $\log_x 27 = 3$?

A. 3

B. 6

C. 9

D. 24

E. 27

8. If $16 \cdot 2^{x-4} = 4^{y+3}$ and $y = 4$, what is the value of x ?

A. $\frac{1}{2}$

B. $\frac{15}{2}$

C. 7

D. 14

E. $\frac{34}{5}$

9. If $\log_x 625 = 4$, then $x = ?$

A. 5

B. 25

C. $\frac{625}{4}$

D. $\frac{625}{\log 4}$

E. 625^4

10. In the realm of real numbers, what is the solution of the equation $9^{2x-1}=3^{1+x}$?

A. 0

B. $\frac{2}{3}$

C. -1

D. 2

E. 1

11. What is x if $\log_6 x=2$?

A. 3

B. $\sqrt{6}$

C. $\sqrt[6]{2}$

D. 36

E. 12

12. For all $x>0$, which of the following expressions is equivalent to $\log\left[\left(\frac{3}{x}\right)^{\frac{1}{3}}\right]$?

A. $\log\frac{1}{x}$

B. $\log 1-\log\frac{x}{3}$

C. $\frac{1}{3}\left[(\log 3)+(\log x)\right]$

D. $\frac{1}{3}(\log 3-\log x)$

E. $\log 3-\frac{1}{3}\log x$

13. What is the value of $\log_4 64$?

A. 2

B. 3

C. 60

D. 4

E. 16

14. What value of x satisfies the following equation $\log_{16} x=\frac{-3}{4}$?

A. $\frac{-16}{3}$

B. -4

C. $\frac{1}{8}$

D. $\frac{1}{4}$

E. 4

15. If a is a positive number such that $\log_a\left(\frac{1}{125}\right)=-3$, then $a=$?

A. 5

B. 25

C. 128

D. $\frac{1}{5}$

E. $\frac{1}{25}$

16. What is the set of all values of a that satisfy the equation $\left(y^2\right)^{a^2+10a+25}=1$ if $y\neq 1$?

A. $\{0\}$

B. $\{5\}$

C. $\{-10\}$

D. $\{-5\}$

E. $\{-5,5\}$

17. What is the real value of a in the equation $\log_3 54-\log_3 6=\log_6 a$?

A. 3

B. 12

C. $\frac{1}{3}$

D. 36

E. $\frac{8}{3}$

ANSWERS

1. D 2. B 3. A 4. A 5. E 6. D 7. A 8. D 9. A 10. E 11. D 12. D 13. B 14. C 15. A 16. D 17. D

ANSWER EXPLANATIONS

1. **D.** Since $a\log_b x = \log_b x^a$, we can rewrite $3\log_2 x$ as $\log_2 x^3$ and $\frac{1}{2}\log_2 z$ as $\log_2 \sqrt{z}$. Our equation can now be written as $\log_2 x^3 - \log_4 y + \log_2 \sqrt{z}$. Combining the two terms with log base 2, we use the property $\log_a x + \log_a y = \log_a xy$ to rewrite the expression: $\log_2 x^3 - \log_4 y + \log_2 \sqrt{z} \to \log_2 x^3 + \log_2 \sqrt{z} - \log_4 y \to \log_2 x^3\sqrt{z} - \log_4 y$.

2. **B.** By the definition of a logarithm, $y = b^x$ is equivalent to $log_b(y) = x$. Thus, $log_4(3) = a$ is equivalent to $4^a = 3$, and $log_4(5) = x$ is equivalent to $4^b = 5$. We can then add the two equations.

$$\begin{array}{rl} & 4^a = 3 \\ + & 4^b = 5 \\ \hline & 4^a + 4^b = 3 + 5 \end{array}$$

Thus, $8 = 4^a + 4^b$.

3. **A.** Since $a\log_b x = \log_b x^a$, we can write $\log_a\left(\frac{x}{y}\right)^3$ as $3\log_a\left(\frac{x}{y}\right)$. Since $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$, we can write $3\log_a\left(\frac{x}{y}\right)$ as $3(\log_a x - \log_a y)$. Now, substituting in $\log_a x = n$ and $\log_a y = m$, we get $3(\log_a x - \log_a y) = 3(n-m)$.

4. **A.** $3^{x+1} = 3^{x-1}(3^2)$ so substituting $3y$ for 3^{x-1}, we get $3^{x+1} = 3y(3^2) = 27y$.

5. **E.** We can write 2^{a+2} as $2^a 2^2$ which simplified becomes $2^a(4)$. Dividing both sides of the equation by 4, we get $2^a = b$. Now, squaring both sides, we get $2^{2a} = b^2$.

6. **D.** Looking at the powers of 2, we know that $2^5 = 32$ and $2^6 = 64$. Since $2^5 = 32 < 2^n = 53 < 2^6 = 64$, $5 < n < 6$.

7. **A.** Raising x to the values on both sides of the equation, we get $x^{(\log_x 27)} = x^3 \to 27 = x^3$. Taking the cube root of both sides, we get $3 = x$.

8. **D.** Plugging in $y = 4$, we get $16(2^{x-4}) = 4^{4+3}$. Since $16 = 2^4$, and $4 = 2^2$, we rewrite this as $2^4(2^{x-4}) = (2^2)^7$. This is equal to $2^x = 2^{14} \to x = 14$.

9. **A.** By the definition of a logarithm, $\log_x 625 = 4$ is equivalent to $x^4 = 625$. Taking the 4th root of both sides, we get $x = \sqrt[4]{625} \to x = 5$.

10. **E.** Since $9 = 3^2$, we can write $(3^2)^{2x-1} = 3^{1+x}$. This is equal to $3^{4x-2} = 3^{1+x}$. So, $4x - 2 = 1 + x$. Adding 2 and subtracting x to both sides, we get $3x = 3 \to x = 1$.

11. D. By the definition of a logarithm, $\log_6 x = 2$ is equivalent to $6^2 = x$, so $x = 36$.

12. D. Since $a\log_b x = \log_b x^a$, we can write $\log\left[\left(\frac{3}{x}\right)^{\frac{1}{3}}\right] = \frac{1}{3}\log\left(\frac{3}{x}\right)$. Since $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$, we can write $\frac{1}{3}\log\left(\frac{3}{x}\right) = \frac{1}{3}(\log 3 - \log x)$.

13. B. We want to find the value that $\log_4 64$ is equal to, which we will call x. By the definition of a logarithm, $\log_4 64 = x$ is equivalent to $4^x = 64$. Since we know that $4^3 = 64$, we know $x = 3$.

14. C. Because we understand what a logarithm represents, we know that $\log_{16} x = \frac{-3}{4}$ is equivalent to $x = 16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{2^3} = \frac{1}{8}$.

15. A. Again, because we know the definition of a logarithm, we know that $\log_a\left(\frac{1}{125}\right) = -3$ is equivalent to $a^{-3} = \frac{1}{125}$. This implies that $\frac{1}{a^3} = \frac{1}{125} \to a^3 = 125 \to a = 5$.

16. D. If $y \neq 1$, then the only way the equation is true is if the exponent equals 0, because $y^0 = 1$. Thus we know that $2\left(a^2 + 10a + 25\right) = 0$. Factoring, we get $2\left(a+5\right)\left(a+5\right) = 0$, which means $y = -5$ only.

17. D. Since $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$, we can write $\log_3 54 - \log_3 6$ as $\log_3\left(\frac{54}{6}\right) = \log_3 9$. So, $\log_3 9 = \log_6 a$. Raising 3 to the values on both sides of the equation gives us $3^{\log_3 9} = 3^{\log_6 a} \to 9 = 3^{\log_6 a}$. Since $9 = 3^2$, we have $3^2 = 3^{\log_6 a} \to 2 = \log_6 a$. Raising 6 to the values on both sides of this equation, we get $6^2 = 6^{\log_6 a} \to 6^2 = a$. So, $a = 36$.