CHAPTER

20 TRIGONOMETRY

SKILLS TO KNOW

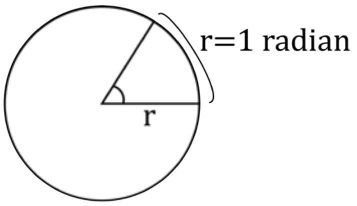
- Radian and Degree Measure
- Unit Circle
- ASTC (All Students Take Calculus) positive and negative values of trigonometric functions on a graph
- Word problems involving Sine/Cosine/Angles
- Reference angles
- Trig Identities

Note: This chapter is a continuation of skills covered in SOHCAHTOA (right triangle trigonometry). Trig concepts are additionally covered in Trig Graphs and Laws of Sines and Cosines. Trig questions (including SOHCAHTOA) typically occur 1-5 times on any given ACT© exam. They often take longer to drill and understand than many other areas of the exam and tend to be varied in form, hence the length of this chapter and the multiple chapters on trigonometry topics. I recommend that ALL students feel proficient in SOHCAHTOA, as it occurs on many exams and tends to be easier, but this chapter is best suited for those aiming for top scores (32+).

RADIAN AND DEGREE MEASURE

Just as we can measure short lengths in inches or centimeters, angles in math can be measured in two ways: using radians or using degrees.

Most of us are familiar with measuring angles in degrees. We know that a straight angle is $180°$, or that there are $360°$ in a circle. Sometimes, however, it makes sense to use radians. Radians, like the area of a circle and the circumference of a circle, are typically described in terms of π (though they don't need to have a π in them to be in radians). You can see below how an angle of one radian has an arc length equal to the radius. Thus, a radian measure is a calculation of how many "radius lengths" along you are on a circle at a given angle.



But that can be confusing; it's easier to just remember how to convert between degrees and radians:

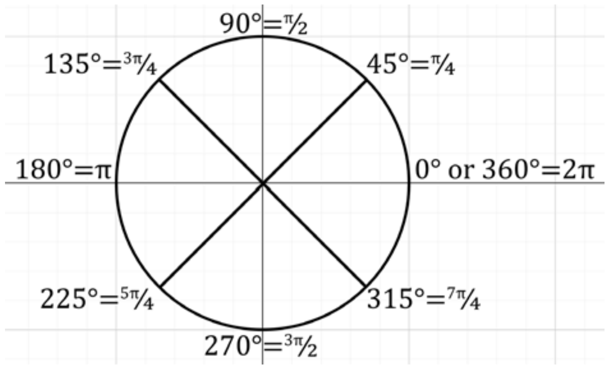
There are π radians in 180 degrees, or 2π radians in 360 degrees:

$$\pi = 180°$$

$$2\pi = 360°$$

We can delineate radian measures, along with their corresponding degree measures, drawing angles centered at $(0,0)$ on a coordinate plane.

Angles in Quadrant I are between 0 and $\frac{\pi}{2}$, in Quadrant II they are between $\frac{\pi}{2}$ and π, in Quadrant III, between π and $\frac{3\pi}{2}$, and in Quadrant IV, between $\frac{3\pi}{2}$ and 2π.



Converting between Radians and Degrees

In the chapter on Units, Rates, and Ratios, in the first book of this series, we learned how to convert between two equivalent measures using **dimensional analysis**. We use that same tactic to convert between radians and degrees.

Remember $\pi = 180°$, so our conversion factors are $\frac{\pi \text{ radians}}{180 \text{ degrees}}$ and $\frac{180 \text{ degrees}}{\pi \text{ radians}}$.

First, figure out what you **HAVE**. If you have **DEGREES**, multiply by the conversion factor with degrees **on the bottom so they cancel,** and **put what you need (radians) on top**. Here is how we set up that problem. Put the angle measure you know in the first blank.

$$\_\_\_\_\_\cancel{\text{degrees}} \times \frac{\pi \text{ radians}}{180\ \cancel{\text{degrees}}} = \_\_\_\_\_\text{radians}$$

If you have RADIANS, multiply by the conversion factor with radians on the bottom so they cancel, and put what you need (degrees) on top. Here is how we set up that problem. Put the angle measure you know in the first blank.

$$\_\_\_\_\_\cancel{\text{radians}} \times \frac{180 \text{ degrees}}{\pi\ \cancel{\text{radians}}} = \_\_\_\_\_\text{degrees}$$

What is the measure, in degrees, of an angle of $-\frac{4}{5}\pi$ radians?

To solve, we set up our conversion as denoted above:

$$-\frac{4}{5}\cancel{\pi}\ \cancel{\text{radians}} \times \frac{180 \text{ degrees}}{\cancel{\pi}\ \cancel{\text{radians}}} = -\frac{4}{5} \times 180 \text{ degrees}$$

Because the radians and π's cancel, we solve:

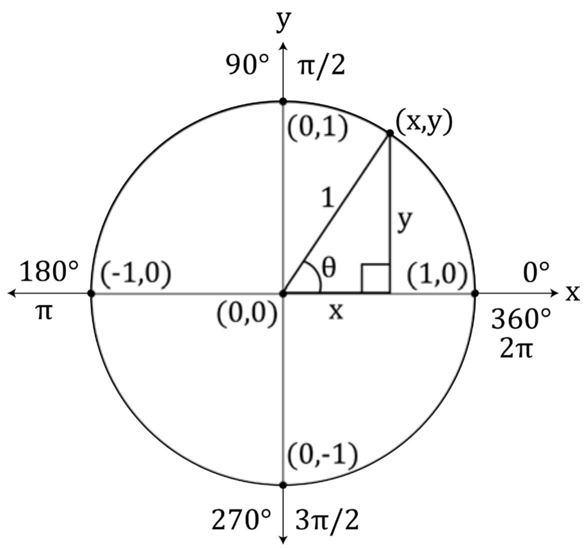
$$-\frac{4}{5}\times 180 = -36 \text{ degrees}$$

Answer: $-36°$.

UNIT CIRCLE

The Unit Circle is a tool we use in trigonometry to solve and understand trigonometric problems. It is defined by the equation:

$$x^2 + y^2 = 1$$

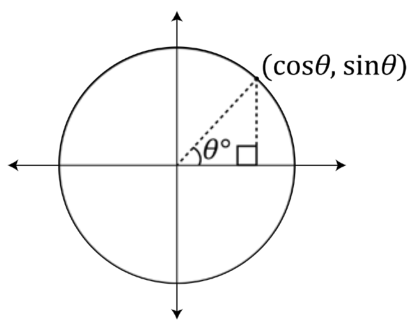


If you know the base form of a circle equation, or rather how that equation is derived, you realize that this is like the Pythagorean theorem: the x length and the y lengths are the sides of a little triangle we can draw that has a hypotenuse equal to the radius, 1 (1 for "unit" circle).

But here's the catch. When we find $\sin$ and $\cos$ values on this little triangle, the hypotenuse is 1.

Take the central angle in the picture is θ. $\sin\theta = \frac{o}{h} = \frac{y}{1}$ right? So $\sin\theta = y$. $\cos\theta$ would equal $\frac{a}{h}$ or $\frac{x}{1}$ right? So $\cos\theta = x$. Because the radius is 1, we thus know in any unit circle:

$$\boldsymbol{x = \cos\theta} \text{ and } \boldsymbol{y = \sin\theta}$$



Thus, because $\boldsymbol{x^2 + y^2 = 1}$, $\boldsymbol{\sin^2\theta + \cos^2\theta = 1}$. We call this formula the **Pythagorean Identity**.

Now that we realize this fact, we can use a unit circle to quickly jump between the $\sin$ of an angle and a $\cos$ of an angle by plugging into the Pythagorean identity ($\sin^2 x + \cos^2 x = 1$), or by using memorized values we visualize on the unit circle which adhere to this pattern.

Many students memorize the unit circle values such as $\frac{\pi}{3}$, $\frac{\pi}{2}$, or $\frac{\pi}{4}$ for their trig coursework. For the ACT, this level of memorization is likely overkill. However, you should know a few things that relate to the idea of a unit circle, and it's good to generally know what it is.

If $1 - sin^2 x = \frac{5}{12}$, what is $cos^2 x$?

Using the Pythagorean Identity, we can very quickly solve this problem. Because I know $\sin^2\theta + \cos^2\theta = 1$, I can plug in this expression where I see the 1 and solve:

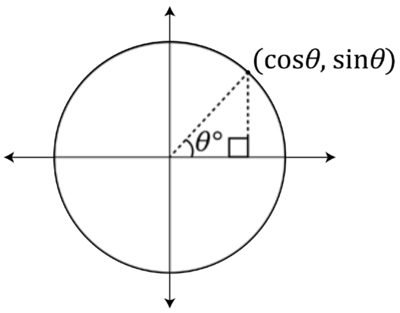
$$1 - sin^2 x = \frac{5}{12}$$
$$\left(\sin^2\theta + \cos^2\theta\right) - \sin^2 x = \frac{5}{12}$$
$$cos^2\theta = \frac{5}{12}$$

Answer: $\frac{5}{12}$.

ASTC: ALL STUDENTS TAKE CALCULUS, ALL STUDENTS TAKE CLASSES, ADD SUGAR TO COFFEE, ALL SILVER TEA CUPS

Whatever phrase you use to remember it, ASTC is a way to remember when trig functions are positive or negative, based on the measure of the angle or the range of that measure.

To use this technique, first imagine a radius drawn on a unit circle.



Imagine I know that my x value is $\frac{3}{5}$, or in other words, $\cos\theta = \frac{3}{5}$. I can solve:

$$\left(\frac{3}{5}\right)^2 + y^2 = 1$$
$$\frac{9}{25} + y^2 = 1$$
$$y^2 = \frac{16}{25}$$
$$y = \frac{4}{5}$$

But if I do, I always assume my trig value is positive. That works in this instance, but what if we were in Quadrant III? When we use the formula to solve for the trig identity we don't know, we can't use this formula alone to determine the correct sign. Because this formula squares each term, any negative signs vanish! Nor can I use the simple "triangle" method to track signs (we'll get to that soon)– triangle sides are always positive, never negative. We must thus track our signs in some other fashion. That's where ASTC comes in.

A=ALL

In the first quadrant ($0°$ to $90°$ or 0 to $\frac{\pi}{2}$), $\underline{\boldsymbol{A}}$ll trigonometric ratios are positive.

S=SINE

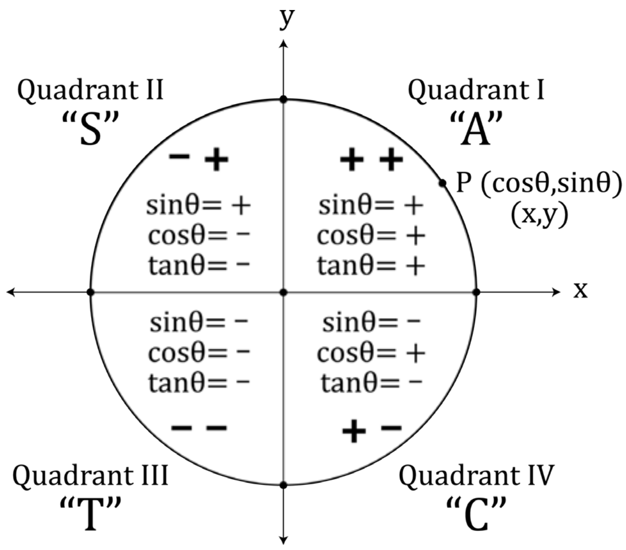
In the second quadrant ($90°$ to $180°$ or $\frac{\pi}{2}$ to π), $\underline{\boldsymbol{S}}\text{in}$ is positive, and $\cos$ and $\tan$ are negative. (By extension, $\csc$ is also positive, and $\sec$ and $\cot$ are negative.)

T=TANGENT

In the third quadrant ($180°$ to $270°$ or π to $\frac{3\pi}{2}$), $\underline{\boldsymbol{T}}\text{an}$ is positive, and $\sin$ and $\cos$ are negative. (By extension, $\cot$ is also positive, and $\csc$ and $\sec$ are negative.)

C=COSINE

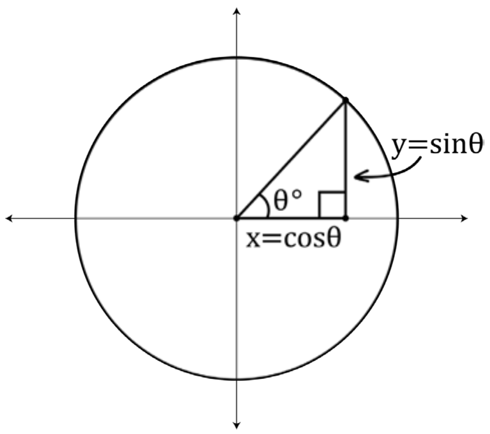
In the fourth quadrant (in degrees, $270°$ to $360°$ or in radians, $\frac{3\pi}{2}$ to 2π), $\underline{\boldsymbol{C}}\text{os}$ is positive, and $\tan$ and $\sin$ are negative. (By extension, $\sec$ is positive, and $\csc$ and $\cot$ are negative.)



The idea is, if you measure an angle starting at the horizontal line between Quadrant I and IV as the base of the angle and draw a radius segment on the unit circle to form the other side of that angle in one of the four quadrants, the quadrant in which that radius (terminal side of the angle) falls will have trig function values according to this pattern.

An Alternative to ASTC: Think About X and Y

Remember on the unit circle, x corresponds to $\cos$ and y corresponds to $\sin$. Thus when x is positive on our quadrant, so is our cosine. When y is positive on the coordinate plane, so is our sine. Just as Quadrant I indicates positive values for both x and y, so it determines that angles falling in that Quadrant I range, when mapped onto the unit circle, also will have positive $\sin$ and $\cos$, and by extension, $\tan$ (remember tangent equals $\frac{\sin}{\cos}$).



We can thus alternatively use the idea of the signs of x and y that determine the terminal side of our angle to derive the correct sign for a trig function of any angle mapped onto the unit circle. All of this may confuse you, but what matters most is that you can apply this knowledge, so let's get into the problem type this information will help you solve.

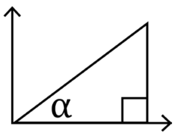
DERIVING THE VALUE OF SINE, COSINE, OR THETA (AND COT, SEC, CSC) FROM ONE ANOTHER

Now that we know the basic idea of a unit circle, the Pythagorean identity, and ASTC, we can use these tools, along with simple SOHCAHTOA, to solve for any trig function of an angle when given a different trig function.

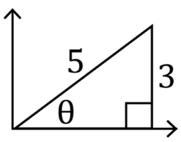
Oftentimes, you will be given the value of one trigonometric function (say $\sin\theta$) and be asked to find another, like $\cos\theta$. Here's how to approach these:

If $\sin\alpha = \frac{3}{5}$ and α is between $\frac{\pi}{2}$ and π, what is $\cos\alpha$?

1. First, sketch a little right triangle.



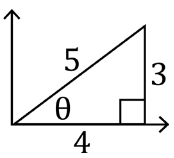
2. Next, label an angle (not the right angle). Make up side lengths that adhere to the SOHCAHTOA ratio you know. The easiest way is to use the fraction's numerator and denominator. For example, if you have $\sin x = \frac{4}{5}$ make the opposite side 4 and the hypotenuse 5 ($\sin$ equals opposite over hypotenuse). We assume $\frac{3}{5} = \frac{opp}{hyp}$ and assign 3 to the opposite side and 5 to the hypotenuse opposite our reference angle. Now label the triangle:



3. Now, we solve for the missing side using the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 5^2$$
$$9 + b^2 = 25$$
$$b^2 = 16$$
$$b = 4$$

Write down this value for our missing side:



4. Now we use these sides to find the value of $\cos$, $\sin$, $\tan$ or other trig function you need of this same angle. Here, it's a simple matter to find the cosine. Just divide the adjacent side by the hypotenuse:

$$\cos\theta = \frac{4}{5}$$

But we're not done yet! This is the cos of the angle theta we made up in our little triangle, a triangle that has all positive side lengths and trig values. We need the angle in between $\frac{\pi}{2}$ and π that holds this ratio but may have a different sign.

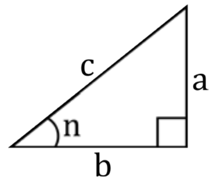
5. Adjust the sign according to ASTC if necessary. Because the $\cos\alpha$ is in the 2nd quadrant (between $\frac{\pi}{2}$ and π), we know that quadrant is "S" meaning ONLY $\sin$ is positive. Thus, cosine is negative, so $\cos\alpha = -\frac{4}{5}$.

Answer: $-\frac{4}{5}$.

If $\tan n = \frac{a}{b}$ and n is in the first quadrant, what is $\csc n$?

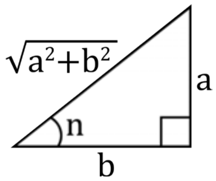
Here we don't have numbers but letters. Still the approach is the same.

1. Draw a right triangle.
2. Label according to what we know:



Angle n has an opposite side a and an adjacent side b.

3. Solve using the Pythagorean theorem. Since $a^2 + b^2 = c^2$, the third side, c, equals $\sqrt{a^2+b^2}$.



4. Find what you need, $\csc n$. Because $\csc$ is equal to $\frac{1}{\sin}$, we want $\frac{h}{o}$ instead of $\frac{o}{h}$. Our h is $\sqrt{a^2+b^2}$. And our o is a. That's $\frac{\sqrt{a^2+b^2}}{a}$.
5. Adjust the sign using ASTC if necessary. Our value is first quadrant, where $\sin$ and thus $\csc$ are positive. So, no adjustment is necessary.

Answer: $\frac{\sqrt{\mathrm{a}^2+\mathrm{b}^2}}{a}$.

REFERENCE ANGLES

Some students find trig values involving angles greater than $90°$ confusing. How do you do "SOHCAHTOA" if you can't create a little right triangle for reference that includes the angle you're analyzing? The answer lies in reference angles.

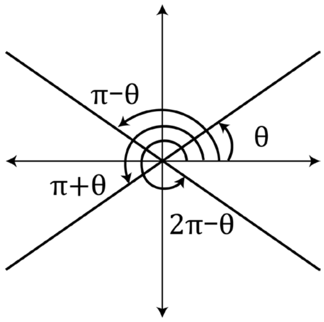
In each of the last two problems, we used a reference angle (the angle we "made up" in our little right triangle) to first find that angle's cosine and then to find the actual cosine of the angle in question. Here, we'll explain that concept in greater depth.

The reference angle is the angle that falls between 0 and $90°$ that has the magnitudes (absolute values) of sine, cosine, and tangent that we are looking for in the other quadrants. It has positive sine, cosine, and tangent. It's the angle we focus on when we use the triangle method. When we combine the information, we know about the reference angle and our ASTC, we can calculate the $\sin$, $\cos$, etc. of our larger angle.

With problems like the one previously, when we jump from $\sin$ to $\cos$ or $\cot$ to $\sec$, we don't really have to think much of the reference angle's actual measure even though we're using it. We can simply use the triangle method and ASTC. But on problems in which we are given an angle measure that is negative or above 90 degrees, we need to understand more clearly what our reference angle is and how to calculate it.

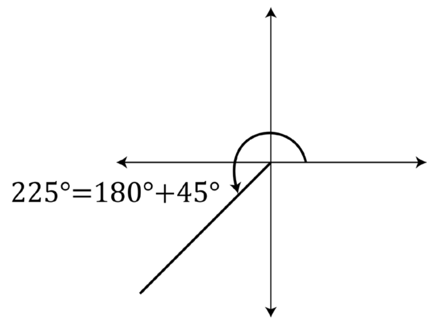
Here, θ acts as a reference angle for any angle with terminal sides that match the following conditions:

Any angle with terminal side in Quadrant II at $\pi-\theta$ or $180°-\theta$.
Any angle with terminal side in Quadrant III as $\pi+\theta$ or $180°+\theta$.
Any angle with terminal side in Quadrant IV as $2\pi-\theta$ or $360°-\theta$.



To make this easier, **the reference angle is ALWAYS the acute angle formed by the horizontal x-axis and the terminal side of the angle we're evaluating.** Knowing this, we can now calculate trig values of angles that are greater than 90 degrees.

What is the cosine of an angle that measures 225 degrees?

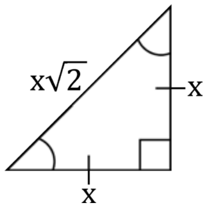


First, we need to calculate the reference angle. I know that 225 degrees is 45 more degrees than 180 degrees, so the acute angle I can draw using the terminal side of this angle and the horizontal x-axis is 45 degrees.

This is the 3rd quadrant, so following to the 3rd letter in AS**T**C. T (tangent) is the only positive value. I know cosine must be negative.

Now I simply need the cosine of 45 degrees. I can use my calculator, the value I've memorized from the unit circle, or my knowledge of $45-45-90$ triangles to answer this.

Let's sketch a quick $45-45-90$ triangle.



We have x , x , $x\sqrt{2}$. Our adjacent side is x and our hypotenuse $x\sqrt{2}$, so $\dfrac{x}{x\sqrt{2}}=\dfrac{1}{\sqrt{2}}$.

Rationalizing the denominator, we get:

$$\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

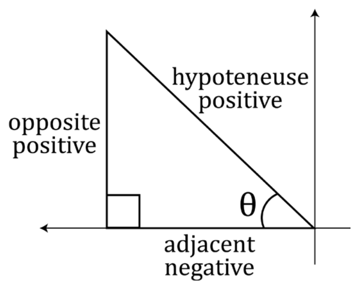
We apply the sign from ASTC to get $-\dfrac{\sqrt{2}}{\sqrt{2}}$.

Answer: $-\dfrac{\sqrt{2}}{\sqrt{2}}$.

One more way to solve

One final way we can solve these sorts of problems is drawing the little triangle right on the coordinate plane itself, using the "reference" angle as the acute angle in our right triangle.

In this method, we now can accommodate for the differing signs as we have negative and positive values on the coordinate plane. In essence, we don't have to think about ASTC if we pay attention to the positive and negative values on the axis.



Here, you can see that angle is theta (θ).

When we take the $\cos\theta$ (CAH) here, for example, we divide the adjacent side, which is negative, by the hypotenuse, which is positive, so we get a negative value.

TRIG IDENTITIES & FORMULAS

(Note: formulas defining $\cot$, $\csc$, and $\sec$ are in the chapter SOHCAHTOA on page 298.)

TRIGONOMETRY IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\csc^2\theta - \cot^2\theta = 1$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$

The first two of these, the equation for tangent and the Pythagorean identity, we've already discussed. It is vital that you memorize them. The others are not as important but still may be helpful on some ACT® problems. In general, if you need to know a trig identity or formula besides the first two, it will be given to you as part of the problem you're solving. As with the Law of Sines and Law of Cosines (covered in a separate chapter), however, there is a small chance memorizing the last four above may help you on the exam. Doing so I would only recommend to those aiming for a perfect math score. However, you will need to know how to manipulate expressions using these formulas and identities. Note these problems are pretty rare, occurring on maybe 10-20% of exams.

CALCULATOR TIP: If you think you need one of these, make up numbers and use your calculator to back solve the answer.

Whenever $\frac{\sin(2x)}{2\cos x}$ is defined it is equivalent to which of the following?

Note: $\sin(2\theta) = 2\sin\theta\cos\theta$

A. 1 **B.** $\sin x$ **C.** $\cos x$ **D.** $\tan x$ **E.** $\csc x$

Method 1: Plug in using the given equation.

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Thus, we know $\sin(2x) = 2\sin x\cos x$.

Then we "group substitute" (i.e. substitute in for a whole big, ugly piece rather than just a single variable) into the value of $\sin(2x)$, in our given expression, knowing that it will equal $2\sin x\cos x$.

$$\frac{\sin(2x)}{2cosx} = \frac{2\sin x\cos x}{2\cos x}$$

Now we cancel the two instances of $\cos x$:

$$\frac{2\sin x\cancel{\cos x}}{2\cancel{\cos x}} = \sin x$$

Method 2: Use your calculator and make up numbers.

What do we do if we don't have the formula or it's not given by ACT? Make up numbers and use our calculator.

Here, I can make up $\frac{\sin(2x)}{2\cos x}=\frac{\sin(28)}{2(\cos 14)}\cong .24$

I plug into my calculator and then I start back-solving:

A. 1**—No!**
B. $\sin(14)\cong .24$**—Correct!**

Depending on the amount of time I have, I may stop or may double check each additional answer choice.

Answer: **B**.

ADDITIONAL TIP: If you're still stuck, you can also use o's, a s, h s in place of $\sin$, $\cos$, $\tan$, etc. and sometimes simplify using those variables.

The expression $\csc^2 x-\cot^2 x+2$ is equal to:

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

Here, I might forget my more complex trig identities. What I can do instead is turn all into o's and h's.

$$\csc=\frac{1}{\sin}=\frac{h}{o}$$

$$\cot=\frac{1}{\tan}=\frac{a}{o}$$

We thus have:

$$\left(\frac{h}{o}\right)^2-\left(\frac{a}{o}\right)^2+2$$

This simplifies to:

$$\frac{h^2-a^2}{o^2}+2$$

Now let's think. I know that everything with a variable must collapse or disappear because all my answers are integers. So, I know $\frac{h^2-a^2}{o^2}$ will either have to equal 0 or an integer. 0 doesn't make sense. h is the hypotenuse, a greater value than the triangle leg, a. Hypotenuse, leg, two values squared…wait! That's it. This looks like the Pythagorean theorem! $a^2+o^2=h^2$ so $h^2-a^2=o^2$! Thus $\frac{h^2-a^2}{o^2}=\frac{o^2}{o^2}=1$.

Now we add $1+2$ and get 3, answer C.

Answer: **C**.

SOLVING PROBLEMS WITH CSC, COT, AND SEC

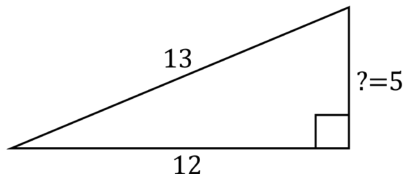
These problems occur rarely on the ACT®, but when they do, many students miss them. For that reason, this chapter includes a good amount of practice on these. Again, though, if you're not aiming for 32+ on this test, spend your time elsewhere.

When μ is between π and $\frac{2\pi}{3}$, and $\csc\mu = \frac{13}{12}$, what is the value of $\cot\mu - \sec\mu$?

Start by remembering what each of these trig functions means:

$$csc\theta = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite} \qquad sec\theta = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent} \qquad csc\theta = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$$

Now I use the triangle method, and from $\csc\mu = \frac{13}{12}$, sketch the following:



13 is the hypotenuse
12 is the opposite
By memorizing the $5-12-13$ Pythagorean triple, I know the other side (adjacent) must be 5.

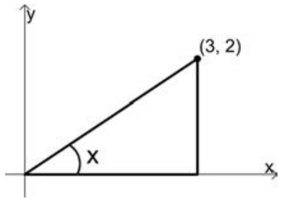
Now, $\cot$ equals adjacent/opposite: $\frac{5}{12}$.

And $\sec$ equals hypotenuse over adjacent, $\frac{13}{5}$.

Then I subtract $\frac{5}{12} - \frac{13}{5}$ using my TI-84 calculator, then I hit "MATH" ("FRAC" is highlighted once I do) and "ENTER" to turn my decimal into a fraction.

Answer: $-\frac{131}{60}$.

1. In the figure below, what is $\cos(x)$?



A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{3}{\sqrt{13}}$

D. $\frac{2}{\sqrt{13}}$

E. $\frac{4}{9}$

2. Roman is standing 55 feet from a cell-phone tower, on level ground. He can see the tip of the tower at an angle of inclination of $38°$. How many feet is the tip of the cell-phone tower from his eyes?

A. $\frac{55}{\sin 38°}$

B. $\frac{55}{\sin 142°}$

C. $\frac{55}{\sin 52°}$

D. $\frac{\sin 52°}{55}$

E. $\frac{\sin 38°}{55}$

3. If $\sin(A)=\frac{10}{26}$, which of the following values could $\cos A$ equal?

A. $\frac{24}{26}$

B. $\frac{16}{26}$

C. $\frac{26}{24}$

D. $\frac{10}{24}$

E. 24

4. If $\tan(x)=\frac{8}{15}$ and $\cos(x)=-\frac{15}{17}$, then $\sin(x)=?$

A. $\frac{8}{17}$

B. $-\frac{7}{17}$

C. $\frac{8}{15}$

D. $-\frac{8}{17}$

E. $-\frac{7}{15}$

5. If $90° < \theta < 180°$ and $\sin\theta=\frac{12}{20}$ then $\cos\theta=?$

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{4}{5}$

D. $-\frac{5}{4}$

E. $-\frac{5}{3}$

6. For right triangle $\triangle XYZ$, $\sin\angle X=\frac{5}{6}$. If $\angle Z$ is a right angle, what is $\tan\angle Y$?

A. $\frac{\sqrt{11}}{5}$

B. $\frac{5}{\sqrt{11}}$

C. $\frac{6}{5}$

D. $\frac{11}{5}$

E. $\frac{\sqrt{61}}{5}$

7. If $\sin\theta = 0.6$, what is $\cot\theta$?

A. $\frac{6}{\sqrt{164}}$

B. $\frac{3}{5}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

E. $\frac{4}{5}$

8. If $\sin\beta = \frac{4}{7}$, what is $\cos\beta$?

A. $\frac{\sqrt{33}}{4}$

B. $\frac{\sqrt{33}}{7}$

C. $\frac{4}{\sqrt{33}}$

D. $\frac{7}{\sqrt{33}}$

E. $\frac{\sqrt{23}}{7}$

9. If $\frac{\pi}{2} < \alpha < \pi$ and $\csc\theta = \frac{29}{21}$, what is $\cos\theta$?

A. $\frac{-20}{29}$

B. $\frac{20}{29}$

C. $\frac{29}{20}$

D. $\frac{21}{\sqrt{1282}}$

E. $\frac{-21}{29}$

10. Given that $\tan\theta = \frac{9}{40}$, what are all possible values of $\sec\theta$?

A. $\frac{41}{9}$

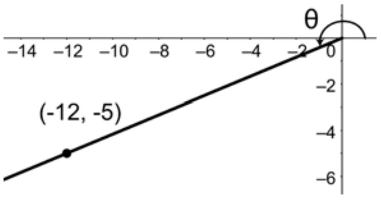
B. $\frac{41}{40}$

C. $\frac{41}{40}$ & $\frac{-41}{40}$

D. $\frac{41}{9}$ & $\frac{-41}{9}$

E. $\frac{39}{40}$

11. In the standard (x, y) coordinate place below, an angle is shown whose vertex is the origin. One side of this angle with measure θ passes through $(-12, -5)$ and the other side includes the positive x-axis. What is the sine of θ?



A. $-\frac{5}{13}$

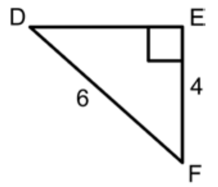
B. $\frac{5}{13}$

C. $\frac{5}{12}$

D. $\frac{12}{13}$

E. $-\frac{12}{13}$

12. What is the sine of angle F in right triangle ΔDEF below?



A. $\frac{2}{3}$

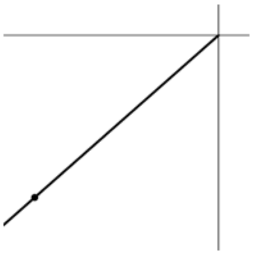
B. $\frac{3}{2}$

C. $\frac{3}{\sqrt{5}}$

D. $\frac{\sqrt{5}}{2}$

E. $\frac{\sqrt{5}}{3}$

13. An angle with measure γ such that $\cos\gamma = \frac{-15}{17}$ is in standard position with its terminal side extending into Quadrant III as shown in the standard (x, y) coordinate plane below. What is the value of $\sin\gamma$?



A. $-\frac{17}{8}$

B. $\frac{15}{17}$

C. $-\frac{15}{17}$

D. $\frac{8}{17}$

E. $-\frac{8}{17}$

14. What are the values of θ between 0 and 2π such that $\cot\theta = -1$?

A. $\frac{\pi}{4}$ and $\frac{7\pi}{4}$

B. $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

C. $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

D. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

E. $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

15. For an angle with measure ϕ in a right triangle such that $\sin\phi = \frac{13}{85}$ and $\sec\phi = \frac{85}{84}$ $\tan\phi$?

A. $\frac{13}{84}$

B. $\frac{84}{13}$

C. $\frac{13}{\sqrt{6887}}$

D. $\frac{\sqrt{6887}}{13}$

E. $\frac{13}{\sqrt{7225}}$

16. If $\sin^2\theta = \frac{9}{16}$, then $\cos^2\theta = ?$

A. $\frac{16}{25}$

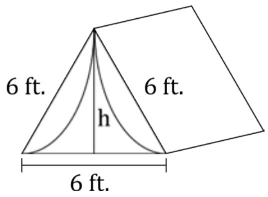
B. $\frac{3}{4}$

C. 1

D. $\frac{1}{4}$

E. $\frac{7}{16}$

17. A man builds a tent (as shown in the figure below) in the shape of a triangular prism whose triangular front silhouette has three sides of length 6 feet. If the man would like to calculate the height of the tent, which of the following would be LEAST directly applicable?



A. The Law of Sines: For any ΔABC, where a is the length of the side opposite $\angle A$, b is the length of the side opposite $\angle B$, and c is the length of the side opposite $\angle C$, $\frac{\sin(\angle A)}{a}=\frac{\sin(\angle B)}{b}=\frac{\sin(\angle C)}{c}$.
B. The Pythagorean Theorem.
C. The ratios for the side lengths of $45-45-90$ triangle.
D. The ratios for the side lengths of a $30-60-90$ triangle.
E. The formula for the altitude of an equilateral triangle of side length x, where h is the altitude: $h=\frac{\sqrt{3}}{2}x$

18. If $\cot C=\frac{t}{p}$ and C is in the first quadrant of the unit circle, what is $\sec C$?

A. $\frac{\sqrt{p^2+t^2}}{t}$
B. $\frac{\sqrt{p^2-t^2}}{t}$
C. $\frac{p^2+t^2}{t}$
D. $\frac{p}{\sqrt{p^2+t^2}}$
E. $\frac{\sqrt{p^2+t^2}}{p}$

19. If a is in the first quadrant of the unit circle and $\cos a=\frac{5}{13}$, what is $\csc a$?

A. $\frac{13}{12}$
B. $\frac{12}{13}$
C. $\frac{5}{12}$
D. $\frac{5}{\sqrt{194}}$
E. $\frac{13}{\sqrt{194}}$

20. The expression $\sec^2\theta-\tan^2\theta+5$ is equal to?

A. 5
B. 6
C. 4
D. 9
E. 3

21. The expression $8\cos^2\theta-4$ is equivalent to? (Note: $\cos 2\theta=\cos^2\theta-\sin^2\theta$)

A. $4\cos^2\theta$
B. $4\cos 2\theta$
C. 4
D. 0
E. $8\cos 2\theta$

22. Which of the following is equivalent to the function $h(x)=\csc x\tan x$? (Note: $\csc x=\frac{1}{\sin x}$)

A. $\cot x$
B. $\cos x$
C. $\sin x$
D. $\csc x$
E. $\sec x$

23. For all values of θ where $\sin\theta$ and $\cos\theta$ are positive, which of the following is equal to $\frac{1}{8}\csc\theta\geq\sec\theta$?

A. $\cot\theta\leq\frac{1}{8}$
B. $\tan\theta\geq 8$
C. $\cot\theta\geq 8$
D. $\sin^2\theta\cos\theta\geq 8$
E. $\cot\theta\leq-\frac{1}{8}$

24. For all θ such that $\sin\theta \,\&\, \cos\theta \neq 0$, which of the following is equivalent to $\dfrac{\cot^2\theta\sec\theta}{\csc^2\theta}$?

A. $\dfrac{\sin^4\theta}{\cos^3\theta}$

B. $\dfrac{\cos\theta}{\sin^4\theta}$

C. $\cos\theta$

D. $\cos^2\theta$

E. $\dfrac{1}{\cos^3\theta}$

25. For all β such that $0° < \beta < 90°$ the expression $\dfrac{\sqrt{\sec^2\beta - 1}}{\tan\beta} - \dfrac{\sqrt{\csc^2\beta - 1}}{\cot\beta}$ is equal to?

A. $\cot^2\beta - \tan^2\beta$

B. $\csc\beta - \sec\beta$

C. $\tan\beta - \cot\beta$

D. 2

E. 0

26. If x is in the first quadrant of the unit circle and $\cos x = \dfrac{5}{7}$, which of the following is equal to $\sin x \cot x$?

A. $\dfrac{24}{35}$

B. $\dfrac{5}{7}$

C. $\dfrac{\sqrt{24}}{7}$

D. $\dfrac{5\sqrt{24}}{49}$

E. $\dfrac{\sqrt{24}}{5}$

27. When θ is between 0 and π and $\cot\theta = \dfrac{16}{5}$, what is the value of $\csc\theta - \sec\theta$?

A. $\dfrac{11}{80}$

B. $\dfrac{-11}{80}$

C. $\dfrac{-11(281)}{80}$

D. $\dfrac{11\sqrt{281}}{80}$

E. $\dfrac{-11\sqrt{281}}{80}$

28. When $\sin x \neq 0$ and $\cos x \neq 0$, the expression $\csc^2 x - \cot^2 x$ is equal to?

A. 1

B. 0

C. 2

D. $\cos^2 x$

E. $\tan^2 x$

29. Which of the following is equivalent to $\dfrac{\sec^2\alpha - 1}{\sin^2\alpha}$?

A. $\sec^2\alpha$

B. $\cos^2\alpha$

C. $\dfrac{\cos^2\alpha}{\sin^4\alpha}$

D. 1

E. $\tan^2\alpha$

30. In trigonometry, an angle of $\dfrac{-11\pi}{4}$ has the same tangent as which of the following angle measures in degrees?

A. $-225°$

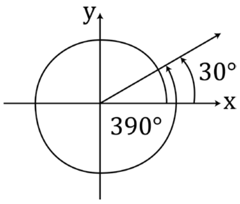
B. $-45°$

C. $135°$

D. $45°$

E. $225°$

31. Two angles are coterminal if they have the same initial and terminal sides. The angles shown below with measures $30°$ and $390°$ are coterminal, for example.



An angle with measure $45°$ is coterminal with a second angle. Which of the following measures could be that of the second angle?

A. $225°$
B. $270°$
C. $315°$
D. $360°$
E. $405°$

32. What are the values of θ, between 2π and 4π, when $\tan\theta = 1$?

A. $\frac{9\pi}{4}$ and $\frac{11\pi}{4}$ only
B. $\frac{9\pi}{4}$ and $\frac{13\pi}{4}$ only
C. $\frac{11\pi}{4}$ and $\frac{13\pi}{4}$ only
D. $\frac{11\pi}{4}$ and $\frac{15\pi}{4}$ only
E. $\frac{13\pi}{4}$ and $\frac{15\pi}{4}$ only

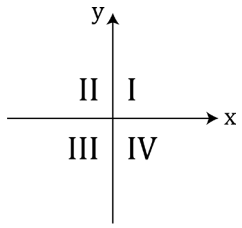
33. If $\cos\theta = -\frac{1}{4}$, what is the value of $2\cos 2\theta$? (Note: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$)

A. $-\frac{9}{8}$
B. $\frac{1}{8}$
C. $-\frac{14}{8}$
D. $-\frac{7}{8}$
E. $\frac{1}{16}$

34. Tyler is lying on his back on a picnic blanket on the ground twenty-two feet away from the point where a tree touches the ground. If the treehouse is sixteen feet high, what is the measure of the angle of elevation of the Tyler's line of sight?

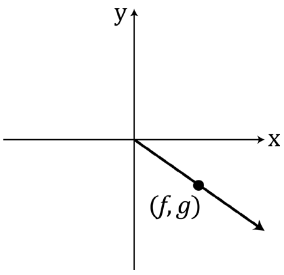
A. $\arcsin\frac{8}{11}$
B. $\arccos\frac{11}{8}$
C. $\arctan\frac{8}{11}$
D. $\text{arccot}\frac{8}{11}$
E. $\text{arccsc}\frac{11}{8}$

35. The vertex of $\angle A$ is the origin of the standard (x, y) coordinate plane shown below. One ray of $\angle A$ is the positive x axis. The other ray, AB, is positioned so that $\sin\angle A < 0$ and $\cos\angle A < 0$. In which quadrant, if it can be determined, is point B?



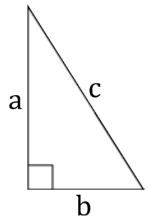
A. Quadrant IV
B. Quadrant III
C. Quadrant II
D. Quadrant I
E. Cannot be determined from the given information

36. In the standard (x,y) coordinate plane below, θ is the radian measure of an angle in standard position with the point (f,g) on the terminal side, where (f,g) falls in Quadrant IV of the coordinate plane. Which of the following points is on the terminal side of the angle in standard position having radian measure $\pi+\theta$?



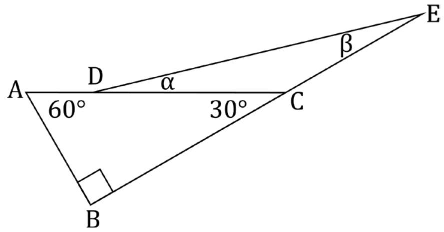
A. $(-f,g)$
B. $(f,-g)$
C. $(-f,-g)$
D. (g,f)
E. $(-g,-f)$

37. In the figure below, ΔABC is a right triangle with all three sides of different lengths. What is the value of $\cos^2 A+\cos^2 B$?



A. $\frac{1+\sqrt{3}}{2}$
B. $\sqrt{2}$
C. $\frac{1}{2}$
D. 1
E. Not enough information

38. The angle measures of ΔABC are shown below. Point D of ΔDEC lies on $\overline{AC}$, and point C lies on $\overline{BE}$. What is the value of $\sin(\alpha+\beta)$?



A. $\frac{\sqrt{2}}{2}$
B. $\frac{\sqrt{3}}{2}$
C. $\frac{1}{2}$
D. 1
E. 0

39. In ΔDEF, the measure of $\angle D$ is 90°, the measure of $\angle E$ is θ, $\overline{DF}$ is 24 units, and $\tan\theta=\frac{6}{5}$. What is the area of ΔDEF in square units?

A. 200
B. 240
C. $8\sqrt{30}$
D. 480
E. $24\sqrt{30}$

40. The measure of the sum of interior angles of a regular polygon with n sides is $[(n-2)180]$ degrees. What is the measure of the sum of the interior angles of a regular polygon with n sides in radians?

A. $(n-2)2\pi$
B. $\frac{(n-2)}{\pi}$
C. $(n-2)\pi$
D. $\frac{n-2}{2\pi}$
E. $\frac{(n-2)\pi}{2}$

41. What is the measure, in degrees, of an angle of $-\frac{2\pi}{9}$ radians?

A. 20°
B. -20°
C. -40°
D. -80°
E. 90°

42. If the value to the nearest thousandth of $\cos\theta$ is $-.734$, which of the following could be true about θ?

A. $\frac{\pi}{2}<\theta<\frac{2\pi}{3}$
B. $\frac{5\pi}{6}<\theta<\pi$
C. $\frac{7\pi}{6}<\theta<\frac{4\pi}{3}$
D. $\frac{\pi}{6}<\theta<\frac{\pi}{3}$
E. $\frac{5\pi}{3}<\theta<\frac{11\pi}{6}$

43. Angle A measures $\frac{10\pi}{3}$ from its initial to its terminal side. Angle B has the same initial and terminal side as Angle A. Which of the following measures could be that of Angle B?

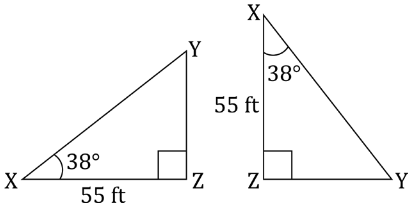
A. 54°
B. 60°
C. 108°
D. 120°
E. 240°

ANSWER KEY

1. C	**2. C**	**3. A**	**4. D**	**5. C**	**6. A**	**7. D**	**8. B**	**9. A**	**10. C**	**11. A**	**12. E**	**13. E**	**14. C**
15. A	**16. E**	**17. C**	**18. A**	**19. A**	**20. B**	**21. B**	**22. E**	**23. C**	**24. C**	**25. E**	**26. B**	**27. D**	**28. A**
29. A	**30. E**	**31. E**	**32. B**	**33. C**	**34. C**	**35. A**	**36. C**	**37. D**	**38. C**	**39. B**	**40. C**	**41. C**	**42. C**
43. E													

ANSWER EXPLANATIONS

1. **C.** If we draw a triangle with a side on the x-axis, a side on the line $x=3$, and the hypotenuse going from the origin to point $(3,2)$, we get a right triangle with $2=$ the side opposite x and $3=$ the side adjacent angle x. Since $\cos(x)=\frac{adjacent}{hypotenuse}$, we need to find the length of the hypotenuse first by using the Pythagorean theorem $a^2+b^2=c^2$. Plugging in $a=3$ and $b=2$ and solving for c we get $3^2+2^2=c^2 \to 9+4=c^2 \to 13=c^2 \to c=\sqrt{13}$. Now, we can plug in $adjacent=3$ and $hypotenuse=\sqrt{13}$ to get $\cos(x)=\frac{3}{\sqrt{13}}$.
2. **C.** We can draw a triangle ΔXYZ using the points from his eyes (X) to the tip of the tower (Y) to the point on the tower that matches his eye-level (Z). The angle at Z is equal to 90, because the tower is perpendicular to the ground and $\overline{XZ}$ is parallel to the ground, so $\angle Y=52°$. It would be easier to use cosine, but because none of the answer choices include a cosine, we must use sine. In order to use sine, we must look at the triangle in another way. If we rotate the triangle by $90°$ and look at $\angle Y$, , by SOHCAHTOA, $\sin\angle X=\sin 52°=\frac{opposite}{hypotenuse}=\frac{55}{\overline{XY}}$, so $\overline{XY}$, the distance we want to find, is equal to $\frac{55}{\sin 52}$.



3. **A.** Since we know that $\sin(A)=\frac{10}{26}$, we know by SOHCAHTOA that the opposite and hypotenuse sides are 10 and 26, respectively. We can solve for the adjacent side, a, by using the Pythagorean Theorem $a^2+b^2=c^2$. $a^2+10^2=26^2 \to a^2+100=676 \to a^2=576 \to a=24$. Then, $\cos(A)=\frac{adjacent}{hypotenuse}=\frac{24}{26}$. Notice it can save time to know the $5-12-13$ right triangle.
4. **D.** By SOHCAHTOA, we know that $\tan(x)=\frac{opposite}{adjacent}$ and $\cos(x)=\frac{adjacent}{hypotenuse}$ so the opposite, adjacent, and hypotenuse sides are $-8,-15,$ and 17 respectively. We know that the opposite and adjacent sides are negative because $\cos(x)$, which involves the adjacent side and the hypotenuse, is negative, which means that either but not both the adjacent side and the hypotenuse must be negative, but $\tan(x)$, which involve the adjacent and hypotenuse side, is positive, which means that both the adjacent and opposite side must be negative and cancel each other out). So, $\sin(x)=\frac{opposite}{hypotenuse}=\frac{-8}{17}$.
5. **C.** By SOHCAHTOA, we know that $\sin(x)=\frac{opposite}{hypotenuse}$. So, the opposite and hypotenuse sides are 12 and 20 respectively. We can solve for the adjacent side, a, by using the Pythagorean Theorem ($a^2+b^2=c^2$) $a^2+12^2=20^2 \to a^2+144=400 \to a^2=256 \to a=16$. Then $\cos(x)=\frac{adjacent}{hypotenuse}=\frac{16}{20}$, which simplifies to $\frac{4}{5}$. However, since $90°<\theta<180°$, it is in the 2nd quadrant, which means the adjacent side is negative. So, $\cos(x)=-\frac{4}{5}$.

6. **A.** If $\angle Z$ is the right angle, then the hypotenuse is opposite of $\angle Z$. Since we know $\sin\angle X=\frac{5}{6}$, according to SOHCAHTOA, the hypotenuse is equal to 6 and the side opposite of $\angle X$ is equal to 5. We now use the Pythagorean Theorem $a^2+b^2=c^2$ to find the adjacent side. We plug in $a=5$ and $c=6$ to get $5^2+b^2=6^2 \to 25+b^2=36 \to b^2=11 \to b=\sqrt{11}$. So, the adjacent side of $\angle X$ is $\sqrt{11}$. Now, if we look at the graph from a different perspective, taking $\angle Y$ as our featured angle but keeping the same lengths on either side of it, the adjacent side becomes 5 and the opposite side becomes $\sqrt{11}$. So, $\tan\angle Y=\frac{\text{opposite}}{\text{adjacent}}=\frac{\sqrt{11}}{5}$.

7. **D.** If $\sin\theta=0.6=\frac{6}{10}$ and by SOHCAHTOA we know that $\sin\theta=\frac{opposite}{hypotenuse}$, so we can assign the opposite side to be equal to 6 and the hypotenuse to be equal to 10. Then, using the Pythagorean Theorem $a^2+b^2=c^2$, we plug in $a=6$ and $c=10$ to solve for the adjacent side b. We have $6^2+b^2=10^2 \to 36+b^2=100 \to b^2=64 \to b=8$. So, the adjacent side is equal to 8. By SOHCAHTOA, we know that $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{\frac{opposite}{adjacent}}=\frac{adjacent}{opposite}=\frac{8}{6}=\frac{4}{3}$. It is helpful to know the $3-4-5$ right triangle in this problem because $6-8-10$ is a just multiple.

8. **B.** If $\sin\beta=\frac{4}{7}$, by SOHCAHTOA, we know that the opposite side is equal to 4 and the hypotenuse is equal to 7. Then, using the Pythagorean Theorem $a^2+b^2=c^2$, we plug in $a=4$ and $c=7$ to solve for the adjacent side b. We have $4^2+b^2=7^2 \to 16+b^2=49 \to b^2=33 \to b=\sqrt{33}.$ So, the adjacent side is equal to $\sqrt{33}$. By SOHCAHTOA, we know that $\cos\theta=\frac{adjacent}{hypotenuse}=\frac{\sqrt{33}}{7}$.

9. **A.** Since $\frac{\pi}{2}<\alpha<\pi$, the triangle is in the second quadrant, which means it is on the negative side of the x-axis. This means the adjacent side is negative. Since $\csc\theta=\frac{29}{21}$, we know by SOHCAHTOA that $\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{opposite}{hypotenuse}}=\frac{hypotenuse}{opposite}$.

So, the hypotenuse is 29 and the opposite side is 21. We use the Pythagorean Theorem $a^2+b^2=c^2$ to solve for the adjacent side b, which will be negative. Plugging in $a=21$ and $c=29$, we get $21^2+b^2=29^2 \to b^2=400 \to b=20$. So, the adjacent side is -20. We can now solve $\cos\theta=\frac{adjacent}{hypotenuse}=-\frac{20}{29}$.

10. **C.** Given $\tan\theta=\frac{9}{40}$, by SOHCAHTOA, we know that $\tan\theta=\frac{opposite}{adjacent}$, so the opposite side is 9 and the adjacent side is 40 in the first quadrant, or -9 and -40 if θ is in the third quadrant, since tangent is only positive for an angle in the first and third quadrant. Using the Pythagorean Theorem and plugging in $a=\pm 9$ and $b=\pm 40$ (because we are squaring the side lengths, it doesn't matter if they are positive or negative) then solving for the hypotenuse c, we get $(\pm 9)^2+\left(\pm 40\right)^2=c^2 \to 1681=c^2 \to c=41$. (Although the sides might be negative, the hypotenuse is always positive, both mathematically and on graphs) Then, the possible values of $\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{adjacent}{hypotenuse}}=\frac{hypotenuse}{adjacent}$ are $\frac{41}{40}$ or $\frac{41}{-40}$.

11. **A.** The sine of this angle is the same as the sine of its reference angle (the angle the segment makes with the closest side of the x-axis, which in this case is the negative x-axis). It we draw a triangle with vertices at $\left(0,0\right)$, $\left(0,-12\right)$, and $\left(-12,-5\right)$, we can use the Pythagorean theorem to find the length of the hypotenuse, which is 13. Using the triangle, we see that the sine of the reference angle is $\frac{5}{13}$. However, because the angle is, in reality, not the same as the reference angle because lies in the third quadrant, we know it is negative (because sine is negative in the third quadrant). Thus, our answer is $-\frac{5}{13}$.

12. **E.** First we need to find $\overline{DE}$ by using the Pythagorean theorem: $\overline{DE}^2+4^2=6^2 \to \overline{DE}=\sqrt{20}=2\sqrt{5}$. By SOCAHTOA, $\sin\angle F=\frac{opposite}{adjacent}=\frac{\overline{DE}}{6}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$.

13. E. In Quadrant III, sine is negative. We can find the length of the opposite side using the Pythagorean theorem and the lengths of the adjacent side and hypotenuse we are given from the cosine. $a^2+b^2=c^2 \rightarrow a^2+(-15)^2=17^2 \rightarrow a^2+225=289 \rightarrow a^2=64 \rightarrow a=8$. However, since we are in Quadrant III, sine must be negative, so $a=-8$. Now we plug that value in with the value of the hypotenuse to find the sine: $\sin\gamma=\frac{-8}{17}$.

14. C. $\cot\theta=\frac{1}{\tan\theta}$. If we want the cotangent to be equal to -1, then the tangent must be equal to -1. $\tan\theta=\frac{sin\theta}{\cos\theta}$, so it will only equal -1 when sine and cosine have the same magnitude but opposite signs, which we can express as $\sin\theta=-\cos\theta$. The Quadrants where sine and cosine have opposite signs are Quadrants II and IV. Sine and cosine have equal magnitude when both sides of a right triangle are equal, (this is because $\sin\theta=\cos\theta \rightarrow \frac{opposite}{hypotenuse}=\frac{adjacent}{hypotenuse} \rightarrow opposite=adjacent$). This makes it an isosceles triangle, and a right-isosceles triangle has angles of $45-45-90$. In terms of radians, the $45°$ angle is $\frac{\pi}{4}$. However, this is only the reference angle. We are looking for the angles that fall within Quadrants II and IV. The angle in Quadrant II will be $\pi-\frac{\pi}{4}=\frac{3\pi}{4}$. The angle in Quadrant IV will be $2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$. Thus, our answer is C.

15. A. If sine is $\frac{13}{85}$, we can define the measure of the opposite side as 13 and the measure of the hypotenuse as 85. Since the secant is $\frac{85}{84}$, we can define the adjacent side as 84, since $\sec\theta=\frac{hypotenuse}{adjacent}$. We have everything we need to find the tangent now. $\tan\phi=\frac{opposite}{adjacent}=\frac{13}{84}$.

16. E. The fastest way to solve this is with the Pythagorean identity: $sin^2\theta+cos^2\theta=1$. We simply plug into this using our given value $\sin^2\theta=\frac{9}{16}$: $\frac{9}{16}+\cos^2\theta=1,\ \cos^2\theta=1-\frac{9}{16},\ \cos^2\theta=\frac{7}{16}$. Alternatively, we can take the root of our given value, $\sin^2\theta=\frac{9}{16}$, to find $\sin\theta=\frac{3}{4}$. Now by SOHCAHTOA we can draw a little triangle and assign opposite and hypotenuse sides as 3 and 4, respectively. We can solve for the adjacent side, a, by using the Pythagorean Theorem: $a^2+b^2=c^2$: $a^2+3^2=4^2 \rightarrow a^2+9=16 \rightarrow a^2=7 \rightarrow a=\sqrt{7}$. Then, $\cos^2\theta=\left(\frac{opposite}{hypotenuse}\right)^2=\left(\frac{\sqrt{7}}{4}\right)^2=\frac{7}{16}$.

17. C. Since the front silhouette of the tent is an equilateral triangle, the triangle is split down the middle to make two $30-60-90$ triangles. There are no $45-45-90$ triangles, so the answer choice C is the least relevant to calculating the height of the tent. Answer choice A could be applied to solve for the height by plugging in $\frac{\sin 90°}{6}=\frac{\sin 60°}{h}$ and solving for h. Answer choice B can be applied to solve for the height by plugging in $h^2+3^2=6^2$ and solving for h. Answer choice D can be applied to find the height by using the ratio $x-x\sqrt{3}-2x$ and plugging in $x=3$ to solve for $h=x\sqrt{3}$. Answer choice E could be applied to solve for the height by plugging in $x=6$ to get $h=\frac{\sqrt{3}}{2}(6)$.

18. A. If C is in the first quadrant, that means all its sides are positive. We know that $\cot C=\frac{1}{\tan C}=\frac{adjacent}{opposite}=\frac{t}{p}$. So, $t=$ adjacent side and $p=$ opposite side. We can then use the Pythagorean theorem $t^2+p^2=c^2$ and take the square root of both sides to write the hypotenuse side as $\sqrt{p^2+t^2}$. We then find $\sec C=\frac{1}{\cos C}=\frac{1}{\frac{adjacent}{hypotenuse}}=\frac{hypotenuse}{adjacent}=\frac{\sqrt{p^2+t^2}}{t}$.

19. A. Since a is in the first quadrant of the unit circle, all of its sides are positive. Using SOHCAHTOA, we know that $\cos a=\frac{adjacent}{hypotenuse}=\frac{5}{13}$, so the adjacent side is equal to 5 and the hypotenuse is 13. Using the Pythagorean Theorem $a^2+b^2=c^2$, we can plug in $a=5$ and $c=13$ to solve for the opposite side, b. Thus, we have $5^2+b^2=13^2 \to 25+b^2=169 \to b^2=144 \to b=12$. We now know that $\csc a=\frac{1}{\sin a}=\frac{1}{\frac{opposite}{hypotenuse}}=\frac{hypotenuse}{opposite}=\frac{13}{12}$. Note that this problem could be solved much faster if we remember that 5, 12, and 13 are a Pythagorean triple.

20. B. This problem is easiest if you remember the identity: $\tan^2\theta=\sec^2\theta-1$. You can rearrange this to get $1=\sec^2\theta-\tan^2\theta$. We are told to find the value of the expression $\sec^2\theta-\tan^2\theta+5$, so we substitute $\sec^2\theta-\tan^2\theta$ as 1 and get $1+5=6$. If you can't remember the identity or how to derive it, you have two options: make up any angle value for theta and plug this into your calculator (probably best/fastest) or make up o's, a's and h's in place of $\tan$ and $\sec$ and solve down with those letters. Here let's try the first method, using $37°$ for θ. I get out my calculator, remembering that $\sec$ is $\frac{1}{\cos}$, and plug in $\frac{1}{\cos^2(37°)}-\tan^2(37°)$ to get 1. Then I add 1 to 5 and get 6.

21. B. Although our answer is in terms of cosine, our expression in its current form cannot be manipulated to one of the answers provided. Thus, we'll need to either make up a number for theta and back solve using our calculator OR factor and substitute. Let's try the 2nd method (I give an example of the 1st method earlier in the chapter). We can factor 4 out of our expression to get $4(2\cos^2\theta-1)$.Whenever I see a 1 and I know I need to substitute, I try group subbing in the entire left hand side of the Pythagorean identity for 1: $\sin^2 x+\cos^2 x=1$. Here we have θ, not x, so I sub in: $4\left(2\cos^2\theta-(\sin^2\theta+\cos^2\theta)\right)=4\left(2\cos^2\theta-\sin^2\theta-\cos^2\theta\right)$. Now I can simplify this to $4\left(\cos^2\theta-sin^2\theta\right)$. At this point, I regroup and look at my given information and see that the elements I have in parenthesis are equal to $\cos^2\theta$. Now I substitute in again and get $4(\cos 2\theta)$.

22. E. We can rewrite $\csc x=\frac{1}{\sin x}$ and $\tan x=\frac{\sin x}{\cos x}$. So, $\csc x\tan x=\frac{1}{\sin x}\left(\frac{\sin x}{\cos x}\right)=\frac{1}{\cos x}=\sec x$.

23. C. We can rewrite $\csc\theta$ as $\frac{1}{\sin\theta}$ and $\sec\theta$ as $\frac{1}{\cos\theta}$. The inequality $\frac{1}{8}\csc\theta\geq\sec\theta$ can then be written as $\frac{1}{8}\left(\frac{1}{\sin\theta}\right)\geq\frac{1}{\cos\theta}$. Multiplying $8\cos\theta$ to both sides we get $\frac{\cos\theta}{\sin\theta}\geq 8$. Since $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{\frac{\sin\theta}{\cos\theta}}=\frac{\cos\theta}{\sin\theta}$, we can rewrite the inequality by substituting $\cot\theta$ in for $\frac{\cos\theta}{\sin\theta}$. We get $\cot\theta\geq 8$.

24. C. We can rewrite $\cot^2\theta$ as $\frac{1}{\tan^2\theta}=\frac{1}{\frac{\sin^2\theta}{\cos^2\theta}}=\frac{\cos^2\theta}{\sin^2\theta}$, $\sec\theta$ as $\frac{1}{\cos\theta}$, and $\csc^2\theta$ as $\frac{1}{\sin^2\theta}$. Substituting these expression into the equation $\frac{\cot^2\theta\sec\theta}{\csc^2\theta}$, we get $\frac{\frac{\cos^2\theta}{\sin^2\theta}\left(\frac{1}{\cos\theta}\right)}{\frac{1}{\sin^2\theta}}=\frac{\cos^2\theta}{\sin^2\theta}\left(\frac{1}{\cos\theta}\right)(\sin^2\theta)=\frac{\cos^2\theta}{\sin^2\theta}\left(\frac{\sin^2\theta}{\cos\theta}\right)=\cos\theta$.

25. E. $0°<\beta<90°$ means that all of the values are positive. Using the trig identity $\sin^2\beta+\cos^2\beta=1$, we can divide both sides of the equation by $\cos^2\beta$ to get $\frac{\sin^2\beta}{\cos^2\beta}+1=\frac{1}{\cos^2\beta}\to\tan^2\beta+1=\sec^2\beta\to\tan^2\beta=\sec^2\beta-1$ On the other hand, taking $\sin^2\beta+\cos^2\beta=1$ and dividing both sides by $\sin^2\beta$, we get $1+\frac{\cos^2\beta}{\sin^2\beta}=\frac{1}{\sin^2\beta}\to 1+\cot^2\beta=\csc^2\beta\to$ $\cot^2\beta=\csc^2\beta-1$. We can now substitute $\tan^2\beta=\sec^2\beta-1$ and $\cot^2\beta=\csc^2\beta-1$ into the expression $\frac{\sqrt{\sec^2\beta-1}}{\tan\beta}-\frac{\sqrt{\csc^2\beta-1}}{\cot\beta}$ to get $\frac{\sqrt{\tan^2\beta}}{\tan\beta}-\frac{\sqrt{\cot^2\beta}}{\cot\beta}=\frac{\tan\beta}{\tan\beta}-\frac{\cot\beta}{\cot\beta}=1-1=0$.

26. B. If x is in the first quadrant, this means that all sides of the triangle formed by x are positive. $\sin x \cot x = \sin x\left(\frac{1}{\tan x}\right) = \sin x\left(\frac{1}{\frac{\sin x}{\cos x}}\right) = \sin x\left(\frac{\cos x}{\sin x}\right) = \cos x$. We are given that $\cos x = \frac{5}{7}$.

27. D. Because θ is between 0 and π, it must be in the first or second quadrant, and because we are dealing with cotangent, which is only positive in the first and third quadrant, we know that for this problem we are working only in the first quadrant. cosecant, secant, and cotangent are reciprocals of sine, cosine, and tangent, respectively, so $\cot\theta = \frac{adjacent}{opposite}$, $\csc\theta = \frac{hypotenuse}{opposite}$, and $\sec\theta = \frac{hypotenuse}{adjacent}$. We are given that $\cot\theta = \frac{16}{5}$, so our adjacent side must be 16 and our opposite side must be 5. Using the Pythagorean theorem, we solve to find our hypotenuse, which we find is $\sqrt{281}$. Now that we know our three side lengths, we can determine that $\csc\theta = \frac{\sqrt{281}}{5}$ and $\sec\theta = \frac{\sqrt{281}}{16}$. Subtracting the two, $\frac{\sqrt{281}}{5} - \frac{\sqrt{281}}{16} = \frac{16\sqrt{281} - 5\sqrt{281}}{5(16)} = \frac{11\sqrt{281}}{80}$.

28. A. Using the trigonometric identity $\sin^2 x + \cos^2 x = 1$, we can divide both sides by $\sin^2 x$ to get: $\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \rightarrow 1 + \cot^2 x = \csc^2 x$. Subtracting $\cot^2 x$ on both sides, we get $1 = \csc^2 x - \cot^2 x$.

29. A. Using the trigonometric identity $\sin^2\alpha + \cos^2\alpha = 1$, we can divide both sides by $\cos^2\alpha$ to get $\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \rightarrow$ $\tan^2\alpha + 1 = \sec^2\alpha$. Subtracting 1 from both sides of the equation, we get $\sec^2\alpha - 1 = \tan^2\alpha$. Plugging this into the expression $\frac{\sec^2\alpha - 1}{\sin^2\alpha}$, we get $\frac{\tan^2\alpha}{\sin^2\alpha}$. Simplifying this, we get $\frac{\sin^2\alpha}{\cos^2\alpha}\left(\frac{1}{\sin^2\alpha}\right) = \frac{1}{\cos^2\alpha} = \sec^2\alpha$.

30. E. Because our answer is in terms of degrees, it's best to convert $\frac{-11\pi}{4}$ from radians to degrees first, by multiplying by $\frac{180}{\pi}$: $\frac{-11\pi}{4} \times \frac{180}{\pi} = -495°$. We don't know the tangent of $-495°$, but we at least know that $-495°$ has the same tangent as an angle it is coterminal with. $-495°$ is coterminal with every angle that is a full $360n°$ before or after it, where n is any integer, such as $-855°, -135°$, and $225°$. Luckily $225°$ is one of our answers, so we can end there. But if we had not been so fortunate, we would have had to look at the reference angle of $225°$, which is $45°$, and find another answer whose reference angle is $45°$ and has a positive tangent, as we know $225°$ does because we know the unit circle.

31. E. An angle in a circle that is greater than $360°$ is congruent, or coterminal, to an angle with the same measure plus or minus $360°$, or any multiple thereof (e.g. $370°$ is coterminal with $10°$ and $750°$ is coterminal with $30°$). So to find an angle that is coterminal with $45°$, we can try adding $360°$. This gives us $405°$, which is one of the answers provided.

32. B. Because of the unit circle, we know that $\tan\frac{\pi}{4} = 1$ and $\tan\frac{5\pi}{4} = 1$. But because we want our angle to be between 2π and 4π, we must find the coterminal angles. $\frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$ and $\frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}$, so our answer is $\frac{9\pi}{4}$ and $\frac{13\pi}{4}$.

33. C. If $\cos\theta = -\frac{1}{4}$, then $\cos^2\theta = \frac{1}{16}$. Because we are given an identity, we can set the two equal and manipulate the expression to get our answer: $\cos^2\theta = \frac{1}{16} = \frac{1 + \cos 2\theta}{2}$. Multiply both sides by 2: $\frac{1}{8} = 1 + \cos 2\theta$. Subtract 1 from both sides: $\frac{1}{8} - 1 = \cos 2\theta = -\frac{7}{8}$. Since we're looking for $2\cos 2\theta$, we can multiply this by 2 to get $-\frac{14}{8}$.

34. C. If we draw a diagram based on what we are told, we are basically given a right triangle, where the distance between Tyler and the tree house, 22, is the adjacent side, and the height of the tree house, 16, is the opposite side. We could find the third side using the Pythagorean Theorem, but since all of the answer choices actually use multiples of the side lengths already given $\left(\frac{22}{2} = 11 \text{ and } \frac{16}{2} = 8\right)$, our answer must be the trigonometric function that only uses the opposite and adjacent sides, either $\arctan$ or arccot. $\arctan\frac{opposite}{adjacent} = \theta$, so our answer is $\arctan\frac{16}{22} = \arctan\frac{8}{11}$.

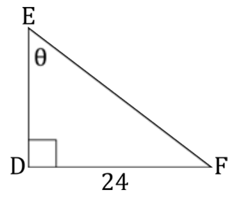
35. A. Per the unit circle, since $\sin\angle A<0$, then point B must be below the x-axis, and since $\cos\angle A<0$, then point B must also be to the right of the x-axis. Quadrant IV is the only quadrant that is below the x-axis and to the right of the y-axis.

36. C. In this problem f and g are the x and y components of a point on a line. For simplicity's sake, imagine that it is the endpoint of a radius of the unit circle. The point we want to find is on the line whose angle measures $\pi+\theta$ radians, in other words, the line directly opposite of the original line on the unit circle (a $180°$ degree rotation from the original position to the new one). The new angle will, therefore, be in quadrant II, and this will change both the sign of the x and the sign of y components. Thus, our answer is $\left(-f,-g\right)$.

37. D. Using basic SOHCAHTOA we know that $\cos A=\frac{b}{c}$ and $\cos B=\frac{a}{c}$, so $\cos^2 A+\cos^2 B$ becomes $\left(\frac{b}{c}\right)^2+\left(\frac{a}{c}\right)^2$. Distribute the exponent and combine the fractions to get $\frac{b^2+a^2}{c^2}$. By the Pythagorean theorem we know that $a^2+b^2=c^2$, so we can substitute c^2 for b^2+a^2, giving is $\frac{c^2}{c^2}$, which is just 1.

38. C. Although there is a formula to find $\sin\left(\alpha+\beta\right)$ but using the sines and cosines of α and β, that actually doesn't matter here. This is actually more of a triangle question. Assuming we don't use the exterior angle theorem as a shortcut, we could find the measure of $\alpha+\beta$ by taking the following steps. $\overline{BE}$ is a straight line, which means $\angle BCA$ and $\angle DCE$ are supplementary pairs. Thus, we can solve for $\angle DCE$: $\angle BCA+\angle DCE=180°\rightarrow 30°+\angle DCE=180°\rightarrow \angle DCE=150°$. Now, looking at $\angle DEC$, we know by the triangle sum theorem that $\angle DCE+\alpha+\beta=180°$. If we subtract $\angle DCE$, which we just found, from both sides, we get $\alpha+\beta=30°$. $\sin 30°=\frac{1}{2}$.

39. B. We have a right triangle with one angle and one side given. We know that $\tan\theta=\tan\angle E=\frac{6}{5}=\frac{opposite}{adjacent}$. If we sketch the triangle, we see that $\overline{DF}$ is our "opposite" side, so we can plug that in to solve for the "adjacent" side, $\overline{DE}$. $\frac{6}{5}=\frac{24}{\overline{DE}}$, so $\overline{DE}=20$. Because this is a right triangle, the side lengths are the base and height of the triangle, so we can use them to find the area. $A=\frac{1}{2}bh=\frac{1}{2}\left(24\right)\left(20\right)=240$.



40. C. To convert from degrees to radians, simply multiply the degrees by the conversion ratio $\frac{\pi}{180°}$: $\left(n-2\right)180°\times\frac{\pi}{180°}=\left(n-2\right)\pi$.

41. C. Convert by multiplying by the conversion ratio $\frac{180°}{\pi}$: $-\frac{2\pi}{9}\times\frac{180°}{\pi}=-40$.

42. C. Cosine is negative in the second and third quadrant, so we know that $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$ at the very least. We know that the cosine of angles with a reference angle of $\frac{\pi}{6}$ is $\pm\frac{1}{2}$, and that the sine of angles with a reference angle of $\frac{\pi}{3}$ is $\pm\frac{\sqrt{3}}{2}\approx .866$, so our mystery angle must be between a pair like that (A trick to know what the reference angle is to ignore the numerator and just look at the denominator) In the second quadrant, the angle with a reference angle of $\frac{\pi}{3}$ is $\frac{2\pi}{3}$, which the angle with a reference angle of $\frac{\pi}{6}$ is $\frac{5\pi}{6}$, so a possible answer is $\frac{2\pi}{3}<\theta<\frac{5\pi}{6}$. That answer is not provided but using the same logic in the third quadrant, we get $\frac{7\pi}{6}<\theta<\frac{4\pi}{3}$, which is provided.

43. E. First we convert $\frac{10\pi}{3}$ to degrees: $\frac{10\pi}{3}\times\frac{180°}{\pi}=600°$. Angles that share the same initial and terminal sides are coterminal, and coterminal angles are all $\pm 2\pi$ (or $\pm 360°$) of each other. While $600°$ is not one of the answers, $600°-360°=240°$ is a given answer.