AVERAGES

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- Mean (Average), Median, Mode, and Range
- Stem-and-leaf plot
- Set transformations (adding members, multiplying/dividing members, increasing/decreasing members of a set) and how they affect mean, median, and mode
- Reading tables that involve these measures of central tendency

MEAN, MEDIAN, MODE, RANGE

Mean (Average)

The words MEAN and the AVERAGE are the same thing: the average value of a set of numbers, found by adding all of the numbers in a set and then dividing by the number of items in the set!

Average of n items in the set $\{x_1, x_2, \ldots x_n\} = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$ or $AVERAGE = \dfrac{SUM\ of\ ALL\ ITEMS}{NUMBER\ OF\ ITEMS}$

The average of $6,\ 9,\ 15 = \dfrac{6+9+15}{3} = 10$

Median

If you line up a set of numbers in **numerical** (chronological) order, and you find the number physically in the middle of this list, that number is called the MEDIAN. If there are an even number of items in a list, then average the two middle values.

The median of $2, 6, 9,\ 15,\ 17$ is 9, because 9 is physically in the middle of the list.

The median of $2,\ 6,\ 9,\ 10,\ 15,\ 17 = 9.5$, because the average of $9\ \&\ 10$ is 9.5, and $9\ \&\ 10$ are physically in the middle of the list.

Mode

The mode occurs most often in a set. I like to think that **MO**de and **MO**st both start with "**MO**" to remember this one.

If you have more than one number that occurs the most (i.e. if three numbers occur five times each), **you can have MULTIPLE modes, unlike median and mean.**

Mode of $\{x_a, x_b, x_b\} = x_b$

Mode of the set $\{5,6,6,6,7,7,7\} = 6\ \boldsymbol{AND}\ 7$

Range

The range of a set of values is the difference between the greatest value and the least value.

> The range of this set: $2,6,9,10,15,17$ is found by subtracting the least number from the greatest one:
>
> $17-2=15$

Average/Mean Word Problems

On these problems, usually the normal equation for mean comes into play, but more often, your problem won't be as simple as this one:

> In Los Angeles, the daily high temperatures in degrees Fahrenheit (F) over one week of June were $72°, 72°, 75°, 82°, 88°, 97°,$ and $104°$.
> To the nearest degree Fahrenheit, what was the mean daily high temperature for that week?
>
> **A.** 72° **B.** 74° **C.** 82° **D.** 84° **E.** 85°

All we have to do here is add up all the numbers and divide by the number of temperatures—that's easy:

$$\frac{72+72+75+82+88+97+104}{7}=\frac{590}{7}=84.285\ldots$$

Answer: **D**

What's not so easy are problems that actually give you the sum, the number of terms, and maybe a few numbers that contribute to the sum. These kind of "average" problems are much more common.

METHOD 1: USE ALGEBRA

One way to solve them is to plug into your original average equation and then roll up your Algebra sleeves!

> Mario has taken 4 of 6 equally weighted tests in his Biology class and has an average score of exactly 87 points. What must he score on the 5th and 6th test, on average to bring his average up to 90 points?
>
> **A.** 96 **B.** 100 **C.** 93 **D.** 99 **E.** 94

We'll have to plug into the equation and solve for what we don't know. Let's assess what we know. We will build out multiple copies of the equation. Just write down the formula and plug in what you know. Repeat if necessary.

He's averaged 87 points (average) on 4 tests (number of items). Let's plug that into our equation:

$$AVERAGE=\frac{SUM\ of\ ALL\ ITEMS}{NUMBER\ OF\ ITEMS}$$

$$87=\frac{SUM\ of\ FIRST\ FOUR\ TESTS}{4}$$

As you can see, I use plain words to represent what goes where—these are easier to understand than variables and the technique helps me more clearly set up my equation. Ok, so I can't solve for the individual test score, but I CAN solve for the sum of the first four test scores.

$$87(4)=\text{SUM of FIRST FOUR TESTS}=348$$

Now let's go back to what we need. We need to know the average of the last two tests. We also know he's got 2 tests left (out of a total of 6 tests) to bring his grade up to a 90 (desired average). Let's start by creating a new equation incorporating in the information we know, again using our formula, but this time for the final grade equation, and filling in what we don't know with English. When in doubt, write it all out:

$$90\left(\textit{Desired Average}\right)=\frac{348\left(\textit{Sum of First Four Tests}\right)+\_\_+\_\_\_\left(\textit{Sum of Last Two Tests}\right)}{6\left(\textit{Number of Tests}\right)}$$

Now I am going to think about the last two test scores—those two blanks. To make this easy, let's assume they are identical and equal to n—remember if you can't solve for a precise value you can make an assumption like this. Also, if they are identical, then n would also be their average. (If two values are equal, they are also their own average).

$$90=\frac{348+2n}{6}$$

$$90(6)=348+2n$$

$$540=348+2n$$

$$192=2n$$

$$n=96$$

He needs an average of 96, because two tests of score 96 would get him the points he needs!

Answer: **A**

METHOD 2: NO ALGEBRA METHOD

Another way to solve many of these problems involves no algebra at all.

The first step I take with this method is to start with some assumptions that could be true. First, I know his average on 4 tests is 87 points. I don't know what he got on each test, but I know that one possibility would be that he actually scored 87 on every single test. That's my first assumption. I write out blanks for each number I "know" and the two that I don't know and fill them in as so:

$$\underline{87}\quad \underline{87}\quad \underline{87}\quad \underline{87}\quad \_\quad \_$$

My goal is to actually make this list look like six 90's in a row-- if he averaged 90 it would be as if he scored 90 on each test. With that in mind, now I add to each score that already exists "3" points—that is what he needs to "catch up" to 90 points for each of those tests, i.e. I'm three points short on each of those four tests, so he needs three more points for each time he got an 87. Then for the last two tests, he will need the equivalent of 90 points for each. If I add the "catch up" three points for the first four tests, plus 90 points from the last two, that is the total amount of points he will need to have averaged 90:

	87	87	87	87	_	_	This row represents the points he has so far on four tests.
+	3	3	3	3	90	90	This row represents the number of points he still needs.
	90	90	90	90	90	90	This is what I actually want his grade card to look like.

So now I add up all the numbers in that 2nd horizontal row:

$$3+3+3+3+90+90=192$$

I also know he needs to score 192 points in two test sittings – so the number of points, on average, per test he needs to score is 96 points per test (192 divided by 2):

$$192\div 2=96$$

I can also think of this as redistributing the "catch up" points from the four 3's – I need 12 more points and I can distribute 6 of these on the first additional 90 point test and 6 on the 2nd to get 96 points per test remaining. Make sense?

Answer: **A**

Median Problems

What is the median of the following set?

42, 33, 85, 60, 15, 29

A. 33 **B.** 37.5 **C.** 44 **D.** 60 **E.** 72.5

The first rule of medians: <u>put everything in order!</u>

42,33,85,60,15,29 becomes 15,29,33,42,60,85

Now cross off equal numbers of numbers on the left and right to find the center values:

$$\cancel{15},\cancel{29},33,42,\cancel{60},\cancel{85}$$

Because we have an even number of items in our set, we can now find the average of 33 and 42:

$$\frac{33+42}{2}=\frac{75}{2}=37.5$$

Answer: **C**

Mode Problems

What is the mode of the following set?

0, 1, 3, 4, 8, 9, 9, 1, 2, 6, 3, 1, 5

A. 1 **B.** 3 **C.** 3.5 **D.** 4 **E.** 9

0, 1, 3, 4, 8, 9, 9, 1, 2, 6, 3, 1, 5

To identify the mode, we also want to rearrange numbers in order so we can be sure which occur most often—if the mode is very apparent though we can just count in place.

0, 1, 1, 1, 2, 3, 3, 4, 5, 6, 8, 9, 9

Here 1 occurs three times. It is the mode.

Answer: **A**

STEM-AND-LEAF PLOT

Tammy surveyed the ages of 30 adults at her most recent family reunion using the stem-and-leaf plot below. What is the median age of these thirty adults surveyed?

Stem	Leaf
2	1,2,8,9,9
3	1,2,2,2,3
4	1,1,1,2,6,7
5	1,2,5,7,7,7,8,8,8
6	4,5,7,9,9

Key:

Stem	Leaf
2	1

Represents "21"

Now I know what you're thinking—what is a stem and leaf plot!?! Look at the KEY! The ACT is really nice about giving you clues to obscure ways of displaying information. The 1,2,8,9,9 in the first row stands for 21,22,28,29,29—the stem is the tens place, and the leaf is the ones place.

SPEED TIP: If you have an exceedingly long list of numbers in which to find a median and know the number of items in a list, you can count "up" or "down" to find the median. For example, if there are 20 numbers in a set, the median is the average of the 10th and 11th numbers in the set. Rather than count off 18 numbers (9 from the left, 9 from the right), just count in 9 numbers to get to the 10th and 11th terms and save some time.

Here I know there are 30 adults, so I know the median is the average of the 15th and 16th number. I count upwards 14, and circle the next two terms (15 & 16) then find their average.

Stem	Leaf	
2	~~1~~,~~2~~,~~8~~,~~9~~,~~9~~	
3	~~1~~,~~2~~,~~2~~,~~2~~,~~3~~	
4	~~1~~,~~1~~,~~1~~,~~2~~, $\boxed{6, 7}$	← 15th and 16th number
5	1,2,5,7,7,7,8,8,8	
6	4,5,7,9,9	

46 and 47 are the 15th and 16th items in the list. Their average is 46.5.

Answer: 46.5

SET TRANSFORMATIONS

Each element in a data set is divided by 3, and each resulting quotient is then increased by 8. If the median of the final data set is 21, what is the median of the original data set?

With set transformations, think about what happens to a set of numbers—remember a median is often an actual number in a set when there are an odd number of items, and if we divide, multiply, add or subtract to the whole set, what we do to the median is the same as what we do to each number. Now we might conclude if there is no "cannot be determined" than what is true for sets that include a median must be true for sets that do not. As such we can do to the median what we do to the set and imagine that it will hold its position.

What we essentially have in a number set is a giant inequality.

$$a < b < c < d < \boldsymbol{n} < f < g < h < k$$

As long as we're dealing with the median, the relationship will hold because even if we had to flip the sign, it's still in the middle. We can multiply, divide, add or subtract to all the elements, and n will remain in the center position. The key to these problems is thinking things through: use logic, make up numbers, and seek to understand what is happening.

As such, we can simply do unto the median as to the rest of the set:

Divide it by 3, add 8, get the new data set version, 21:

$$\frac{n}{3} + 8 = 21$$

$$\frac{n}{3} = 13$$

$$n = 39$$

Answer: 39

READING TABLES

The biggest problem students often have with average problems is the fact that oftentimes, they have to pay attention to information presented in unexpected ways. Be careful, watch your details, and think it through!

For each of 3 years, the table below gives the number of auctions held at an auction house, the number of items sold, and the auction house's income.

Year	Auctions	Items sold	Income
2007	45	405	\$1,206,903
2008	38	266	\$682,024
2009	42	504	\$1,568,448

1. To the nearest dollar, what is the average income the auction house made on an item sold in 2007?
2. If, on average, an item between 2007-2009 cost the auction house \$1400 to acquire, process, and sell at auction, disregarding any other expenses, what was the average amount of profit per auction over the three-year period?

Here we want the average income per item, NOT the average overall income. DO NOT add up the income column and divide by 3! Divide just the 2007 income by the number of items sold to get the average per item.

$$\frac{2007\ \textit{income}}{\#\ \textit{of items sold in}\ 2007} = \text{average selling price per item, so } \frac{\$1{,}206{,}903}{405} = \$2980$$

Now we have to do more work. First let's figure out the average sale price per item, then we'll figure out the profit per item, then the total profits, then divide by the total number of auctions:

Basically, we need total profit.

Remember, $\frac{SUM}{\#\,of\ items} = average$

$$\frac{1206903+682024+1568448}{405+266+504} = 2942.45$$

That's what they sell items for. Now to get the profit per item, we subtract $\$1400$:

$$2942.45 - 1400 = 1542.45$$

Now we multiply this times the number of items to get the sum of the profits ($\frac{SUM}{\#\,of\ items} = average$)

$$\frac{SUM\ of\ profits}{405+266+504} = 1542.45\left(average\ profit\ per\ item\right)$$

$$Sum\,of\ Profits = \$1,812,375$$

Now we take this number, and divide by the number of AUCTIONS to get the profit PER AUCTION (remember **<u>PER MEANS DIVIDE</u>**!! If you have the word "per" translate that into a division bar!)

$$\frac{1812375}{45+38+42} = Average\,Profit\ Per\ Auction$$

$$= \$14,499$$

Answer: $\$14,499$

1. In a science class, Bob scored 95 on one test, 89 on another test, and had an average test score of 83 in the class before taking those two tests. If Bob's average test score for the entire class, including all tests, is 85, and each test he takes in the course is weighted equally, how many tests has Bob taken in the class?

A. 3

B. 9

C. 7

D. 14

E. Cannot be determined from the given information

2. Mary's test average after 8 tests is 80. Her score on the 9^{th} test was 71. If all 9 tests are weighted equally, what is Mary's test average for all 9 tests?

A. 71

B. 75.5

C. 77

D. 79

E. 80

3. Theo has taken 6 of 7 equally weighted tests in his Chemistry class and has an average score of exactly 81 points. What must he score on the 7^{th} test to bring his average up to 83 points?

A. 95

B. 93

C. 85

D. 83

E. 82

4. The mean of 6 numbers is 27. The smallest of the 6 numbers is 12. What is the mean of the other 5 numbers?

A. 27

B. 30

C. $30\frac{3}{4}$

D. $41\frac{2}{3}$

E. 42

5. The 7 positive integers a, a, a, a, a, b, c have an average of a. Which of the following equations must be true?

A. $b = c$

B. $b + c = a$

C. $b + c = 2a$

D. $c = -b$

E. $b + c = 0$

6. Each element in a data set is increased by 3 then divided by 7. If μ is the mean of the final data set, what is the mean of the original set?

A. $\frac{\mu}{7} + 3$

B. $\frac{\mu + 3}{7}$

C. $\frac{\mu - 3}{7}$

D. $7\mu - 3$

E. $7\mu + 3$

7. The average of a set of 8 numbers is 13. When a 9^{th} number is added to the set, the average increases to 16. What is the 9^{th} number?

A. 16

B. 20

C. 29

D. 32

E. 40

8. A music concert was rated on a 5 point scale by the audience. 10% gave a 1, 18% gave a 2, 33% gave a 3, 20% gave a 4, and 19% gave a 5. To the nearest tenth, what is the average rating given by the audience?

A. 2.5

B. 2.8

C. 3.2

D. 3.4

E. 4.6

9. In a town of 600 people, 250 males have an average age of 43, and 350 females have an average age of 38. To the nearest whole number, what is the average age of the entire town?

A. 38

B. 39

C. 40

D. 41

E. 42

10. Each of 16 students took a test and received a whole number of points. The median of the scores was 78, and 25% of the students scored 74 or below. No student received a score of 78. How many students scored 75, 76, or 77?

A. 2

B. 3

C. 4

D. 5

E. 8

11. Nick has 30 collectible coins. He paid $\$36.50$ for each coin 2 years ago. The coins are currently valued at $\$38.15$. To the nearest cent, how much *more* must the average value per coin rise for the combined value of these 30 coins to be $\$350.00$ more than Nick paid for them?

A. \$8

B. \$8.50

C. \$10.02

D. \$11.67

E. \$300.50

12. A data set has 20 elements. A second data set of 20 elements is obtained by adding 7 to each element of the first set. A third data set of 20 elements is obtained by multiplying each element of the second data set by 3. The median of the third data set is 63. What is the median of the first data set?

A. 7

B. 14

C. 21

D. 196

E. 210

13. What is the median of the following test scores?

$$\{56,93,64,78,92,83,88,40,61\}$$

A. 64

B. 73

C. 78

D. 83

E. 92

14. Each number on a list of 9 numbers is multiplied by 13 to produce a 2nd list of 9 numbers. Each of the 9 numbers on the second list is increased by 4 to produce a 3rd list of 9 numbers. The median of the 3rd list is x. What is the median of the first list?

A. $\dfrac{x-4}{13}$

B. $\dfrac{x}{13}$

C. $x-4$

D. $\dfrac{x}{13}-4$

E. $13(x+4)$

15. List *B* contains all the elements of List *A* including integers x, y, and z, where $x \ge 43, y = z, y \le 18$. What is the median of List *B*?

List *A*: $\{12,17,18,30,30,36,42,43,48,48,51\}$

A. 27

B. 30

C. 33

D. 36

E. 39

16. The following table shows the total number of cabinets built in a workshop over 50 consecutive days. What is the average number of cabinets built per day, rounded to the nearest tenth?

Number of cabinets built in a day (Output)	Number of days with this output
0	5
1	11
2	10
3	18
4	6

A. 2

B. 2.1

C. 2.2

D. 3

E. 3.3

Use the following information to answer questions 17-19.

Mr. Rivera gives his 20 students a progress report once a month that gives a student their scores on tests, quizzes, and homework, and the average score of all 20 students. The following is a progress report for Isaac Shemtov.

Student: Isaac Shemtov			
Task	Possible Points	Student score	Class average
Homework #1	100	89	93
Homework #2	100	94	90
Quiz #1	100	75	80
Quiz #2	100	87	85
Quiz #3	100	100	95
Quiz #4	100	89	78
Test #1	100	85	83
Test #2	100	93	85
Test #3	100	95	90

17. Isaac is about to take Test #4. What score does he need to get an average test score of 93?

A. 91

B. 93

C. 95

D. 97

E. 99

18. In Mr. Rivera's class, a homework assignment that has not been turned in receives a score of 0. Of the 20 students in his class, what is the maximum number of students who could have not turned in homework #2?

A. 0

B. 1

C. 2

D. 3

E. Cannot be determined from the given information

19. What is Isaac's average quiz score to the nearest point?

A. 86

B. 87

C. 88

D. 89

E. 90

20. The 6 positive integers $a, a, b, b, c,$ and d have an average of c. What is the value of $a+b$?

A. $\frac{5c-d}{2}$

B. $5c+d$

C. $\frac{5c+d}{2}$

D. $3c+d$

E. Cannot determine from the given information.

21. In a biology course, a student scored 94 on one test, 100 on another test, and 90 on each of the other tests. The students final test average was 91.75. How many tests did the student take?

A. 6

B. 7

C. 8

D. 9

E. 10

22. What is the median of the following set?
$-5,8,3,17,7,12,100,7,0,20$

A. 7

B. 7.5

C. 8

D. 9.5

E. 16.9

23. The median of a set of data containing 12 items, all of different values, was found. Three data items, a,b, and c, were added to the set. a was greater than the original median, but less than all of the original data greater than the median. b was greater than all of the original data. c was equal to the original median. What is the new median of the set?

A. a

B. b

C. c

D. The average of a and c

E. Cannot be determined from the given information.

24. What is the mode of the following data?
$-3,-1,0,3,5,5,8,9,10$

A. 0

B. 4

C. 5

D. 5.7

E. 6

25. Mr. Harker organized his students' test grades into a stem and leaf plot in order to see the frequency of different letter grades. What is the range of the set of test grades?

Stem	Leaf
6	7 9
7	2 3 6
8	0 0 2
9	1 4 5 9

A. 2

B. 9

C. 32

D. 40

E. 166

26. To decrease the mean of 5 numbers by 2, by how much would the sum of the 5 numbers have to decrease?

A. 2

B. 5

C. 7

D. 10

E. 17

27. A data set has 4 members. The mode of the data set is both 6 and 8. What are the mean and median of the data set, respectively?

A. 2,7

B. 6,8

C. 7,7

D. 24,6

E. 24,7

28. What is the mode of the data given below?
12, 26, 13, 7, 20, 35, 38, 13

A. 12.5

B. 13

C. 16.5

D. 20.5

E. 31

29. The stem and leaf plot below shows the number of walk-ins to The Doctor's Office during a 30-day observation period. What is the median number of daily walk-ins?

Stem	Leaf
2	2 3 3 4 7 9
3	1 1 4 4 5 5 9
4	0 0 2 3 7 7 7 8
5	0 3 4 6 6
6	1 3 7 8

Key: 2 | 2 = 22

A. 41

B. 42

C. 43

D. 44

E. 47

30. Mr. Eames was hired for a new job and was requested to keep a log of all his working hours. He recorded the number of hours he worked each day in a table, as shown below. What was the mean number of hours he spent per day?

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Hours Worked	7	4	6	4	9

A. 4

B. 6

C. 7

D. 9

E. 13

31. Which of the following statements is correct concerning the data set below?

36, 47, 47, 47, 59, 72, 89, 89, 98

A. Its mean is 67

B. Its median is approximately 63.8

C. Its range is 98

D. Its mode is 47

E. Its median is not a member of the data set

32. The signs of a and d are positive, but the signs of b and c are negative. If it can be determined, what are the signs of the mean and median of the four numbers, respectively?

A. Both positive

B. Both negative

C. Both neither (zero)

D. The mean is neither, the median is negative

E. Cannot be determined from the given information

33. The 8 consecutive integers below add up to 332.

$$\begin{gathered} x-3 \\ x-2 \\ x-1 \\ x \\ x+1 \\ x+2 \\ x+3 \\ x+4 \end{gathered}$$

What is the value of $x+2$?

A. 40

B. 41

C. 42

D. 43

E. 44

34. A new show was in town for the past 7 days. The average attendance of the slowest and busiest night was 224 people. The average attendance for the other 5 days was 311. How many people attended the show over the course of 7 days?

A. 1704

B. 1405

C. 2093

D. 2003

E. 1967

ANSWER KEY

1. B **2. D** **3. A** **4. B** **5. C** **6. D** **7. E** **8. C** **9. C** **10. C** **11. C** **12. B** **13. C** **14. A**
15. C **16. C** **17. E** **18. C** **19. C** **20. A** **21. C** **22. B** **23. A** **24. C** **25. C** **26. D** **27. C** **28. B**
29. A **30. B** **31. D** **32. E** **33. D** **34. D**

ANSWER EXPLANATIONS

1. **B.** If the average of Bob's test scores for the entire class is 85, that means the sum of his test scores can be represented as $85x$, where $x =$ the number of tests he took in total. We know two of his test scores are 95 and 89, and the average of the test scores before those two tests was 83. So, the sum of his test scores can also be represented by $83(x-2)+95+89$. Setting the two equations expressions to each other, we get $85x = 83(x-2)+95+89$. Distributing the 83 on the right hand side, we get $85x = 83x - 166 + 95 + 89$. Combining like terms, we get $2x = 18$. So, $x = 9$.

2. **D.** Mary's average after 8 tests was 80, so the sum of those 8 test scores is $80*8 = 640$. Now, adding the 9^{th} test score, we have $640 + 71 = 711$. So, the average of her 9 test scores is the sum of the 9 test scores divided by 9. That is, $\frac{711}{9} = 79$.

3. **A.** Theo's average after 6 tests was 81, so the sum of those 6 test scores is $81*6 = 486$. If he wants the average of 7 tests to be 83, then the sum of the 7 tests should be $83*7 = 581$. Since we already know the sum of her 6 test scores, the 7^{th} test score can be calculated as $581 - 486 = 95$.

4. **B.** If the mean of 6 numbers is 27, then the sum of the 6 numbers is $27*6 = 162$. We know that the smallest number is 12, so if we take away that number, we know that the sum of the remaining 5 numbers is $162 - 12 = 150$. This means that the average of those 5 numbers is $\frac{150}{5} = 30$.

5. **C.** We can use the formula for finding the average in order to solve this problem. First, the average, a , is equal to $\frac{a+a+a+a+a+b+c}{7}$. Through algebraic calculation, we can find the $7a = 5a + b + c$. Therefore, $2a = b + c$.

6. **D.** If every element in a data set is increased by 3 and then divided by 7, then the average would also be increased by 3 and divided by 7. So, if μ is the mean of the final data set, then $\mu = \frac{x+3}{7}$ is true for the original mean $= x$. Solving for x now, we get $7\mu = x + 3$ so $x = 7\mu - 3$.

7. **E.** If the average of 8 numbers is 13, then the sum of those 8 numbers is $8*13 = 104$. When a 9^{th} number is included, the average changes to 16, so the sum of those 9 numbers is $9(16) = 144$. The 9^{th} number can then be calculated as $144 - 104 = 40$.

8. **C.** In this case—to make the math easier—we can view each percent of the audience as one count (one person). If 10% of the audience gave the concert a 1, then the concert has $10*1 = 10$ points from the 10%. Multiplying out the other percentages and points, we have $18*2 = 36$, $33*3 = 99$, $20*4 = 80$, and $19*5 = 95$. Adding these points together, we get the total number of points accumulated by the ratings. We get $10 + 36 + 99 + 80 + 95 = 320$. The average can then be calculated by dividing this sum by the total of 100 counts. $\frac{320}{100} = 3.2$.

9. **C.** Since the average of 250 males is 43, then the sum of their ages is $250*43=10750$. Likewise, the average of 350 females is 38, so the sum of their ages is $350*38=13300$. Adding these two sums together we get $10750+13300=24050$. To find the average of the total 600 people, we divide this sum by 600 to get $\frac{24050}{600}=40.08$. Rounding this to the nearest whole number, we get the average age $=40$.

10. **C.** If 25% of the students scored 74 or below, then 4 students scored 74 or below. If the median score was 78, but no one scored a 78, then the 8^{th} and 9^{th} numbers average out to be 78. This means that the 5^{th}, 6^{th}, 7^{th}, and 8^{th} numbers are either 75, 76, or 77. This means 4 students scored 75, 76, or 77.

11. **C.** $\$350.00$ more than what Nick paid for the coins is $350+36.5(30)=1445$. This means that each coin must be worth $\frac{1445}{30}=48.17$. This is $48.17-38.15=10.02$ more than the coins' current value.

12. **B.** As we learned in the lesson, operations on a set of elements don't change the order of the set, so the median for a previous set of numbers can simply be found by applying those operations to the newest median in reverse order. In this case, we can divide 63 by 3 and then subtract by 7. $63\div 3=21$. $21-7=14$. The median of the first set of elements is 14.

    If we didn't know that, we can also solve this questions algebraically. $x_1, x_2, x_3 \ldots x_{20}$ denotes the 20 elements in the first data set, then the second data set is $x_1+7, x_2+7 \ldots x_{20}+7$ and the third data set is $3(x_1+7), 3(x_2+7) \ldots 3(x_{20}+7)$. The median of the third data set is 63, which means the average of the two middle terms is 63. $\frac{3(x_{10}+7)+3(x_{11}+7)}{2}=63$. We want to manipulate this equation to find the median of the first data set, which is $\frac{x_{10}+x_{11}}{2}$. Distributing the $3's$ in the numerator and simplifying, we get $\frac{3x_{10}+3x_{11}+42}{2}=63$. Multiplying by 2 on both sides, we get $3x_{10}+3x_{11}+42=126$. Subtracting 42 on both sides, we get $3x_{10}+3x_{11}=84$. Dividing by 6 on both sides, we get $\frac{x_{10}+x_{11}}{2}=14$.

13. **C.** Putting the scores in numerical order, we get $\{40,56,61,64,78,83,88,92,93\}$. Now, cancelling out the largest and smallest numbers one by one, we arrive at the median. $\{40,56,61,64,78,83,88,92,93\}$. So, the median is 78.

14. **A.** This question can be solved the same way question 12 was solved. Because we know that the operations don't change the order of the elements in the set, the old median can be found by applying the operations to the new medians in reverse order. For this problem, we take x and subtract 4 to get the median of the second list: $x-4$. Then, we divide it all by 13 to get the median of the first list: $\frac{x-4}{13}$.

15. **C.** Integer x comes somewhere after 43 in list B, and both y and z come before 18. We do not need to know their specific locations to find the median. Let us arbitrarily place the integers into the list as such: $\{12,y,z,17,18,30,30,36,42,43,48,48,x,51\}$. It does not matter where we place x, y, and z as long as they satisfy their inequalities. Cancelling out the largest and smallest numbers one by one, we are left with $\{30,36\}$. Since we have two numbers, we take the average between them and get the median: 33.

16. C. The average number of cabinets built per day is equal to the total number of cabinets built over the time period divided by the number of days. We multiply each of the possible number of cabinets built in a day by the number of days when that output was reached and add together each of our results to get the total number of cabinets built: $5*0+11*1+10*2+18*3+6*4=109$. We divide by the number of days: $\frac{109}{50}=2.18$, which rounds to **2.2.**

17. E. Isaac's average test score after he takes another test will be $\frac{85+93+95+n}{4}$ where n is his score on the newest test. We can find out what n must be to get an average of 93 by setting the equation equal to 93: $\frac{85+93+95+n}{4}=93$. This becomes $85+93+95+n=372.$ Isolating n, $n=99.$

18. C. We can find the maximum number of students who could have not turned in homework $\#2$ by seeing how many students with the maximum score it would take to reach or exceed the class average. This is $\frac{n(0)+x(100)}{20}\geq 90$ where are the number of students who did not turn in the homework, and x is the number who did and received $100.$ Together, $x+n=20$ Solving this gives us $x\geq 18.$ Since has a minimum value of $18,$ has a maximum value of $2.$

19. C. Isaac's average quiz score is $\frac{75+87+100+89}{4}=\frac{351}{4}=87.75$, which rounds to $88.$

20. A. The average of the 6 integers is $\frac{a+a+b+b+c+d}{6}=c.$ This becomes $a+a+b+b+c+d=6c.$ We simplify to $2(a+b)+c+d=6c.$ Isolate $(a+b)$: $2(a+b)=5c-d.$ $a+b=\frac{5c-d}{2}.$

21. C. Let be the number of tests the student scored 90 on. The average score of all the tests will be $\frac{91+100+n(90)}{n+2}.$ Setting this equal to 91.75 and restructuring the equation gives us $94+100+n(90)=91.75(n+2).$ Simplify: $90n+194=91.75n+183.5$. Simplifying further, $1.75n=10.5$. Then, $n=6$. Since the student took 2 tests besides the ones they scored 90 on, the total number of test is $n+2=8$.

22. B. Reorganizing the set in ascending order gives us $-5, 0, 3, 7, 7, 8, 12, 17, 20, 100.$ Crossing off the lowest and highest number one by one leaves us with $7,8.$ Since we have two numbers left, we average them to get $7.5.$

23. A. Let's sketch the set in a very rudimentary manner: $\circ\circ\circ\circ\circ\circ \quad \circ\circ\circ\circ\circ\circ$. There is a space between the two sets of 6 shapes that make up the lower and higher halves of the set. Since there is an even number of items in the set, the median will be the average of the two shapes in the center (the shapes adjacent to the gap). We will place c there since it is equal to this original median: $\circ\circ\circ\circ\circ\circ\ c\ \circ\circ\circ\circ\circ\circ$. Next, place , which is greater than (the original median), but less than the items greater than it: $\circ\circ\circ\circ\circ\circ\ ca\ \circ\circ\circ\circ\circ\circ$. Finally, add b, which is greater than all of the original data: $\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\ \cancel{c}\ a\ \cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{\circ}\cancel{b}.$ Crossing off the highest and lowest items one by one gives us $a.$ The median is $a.$

24. C. The mode is the data value that appears most often. In this case, 5 occurs twice, and all other values occur once, making 5 the mode.

25. C. The stem in a stem and leaf plot is typically the largest place digit in common with the rest of the data set. Thus, if a number which has a 3 in the stem column and a 2 in the leaf column represents a $32.$ The range of a set of data is the difference between the greatest and smallest number. Therefore, the range for this stem plot would be $99-67=32.$

26. D. The best way to do this problem would be to pick the simplest numbers and plug them into the problem. Let us use $1,2,3,4,$ and $5.$ The mean of this set would be $\frac{15}{3}$, or $3.$ If we decrease this mean by $2,$ we get the mean of $1.$ This means that the new sum, or *x*, divided by the number of numbers, $5,$ is now equal to $1.$ Through algebraic calculation, we know that the new sum, *x,* would have to be $5.$ From the original sum of 15 to the new sum of $5,$ there would have to be a decrease of $10.$

27. C. In a data set of four members, 2 of the members must be 6 and the other 2 of the members must be 8 in order to have a mode of both 6 and $8.$ If there were only 1 member each of 6 and $8,$ and the 2 remaining members were different numbers, then each member would occur exactly once and there would be no mode. Therefore, the data set is $6,6,8,8.$ The mean of the set is $\frac{28}{4}$, or $7.$ The median of the set, since there are two remaining numbers, is the average of 6 and $8,$ or $7.$

28. B. The mode of a set if the number that occurs the most. In this set the number 13 occurs twice, and all other numbers occur only once. The mode is $13.$

29. A. The median is the number in the middle of an ordered set. Thanks to the stem and leaf plot, the data set is already in order. All that is left to do is to begin crossing out numbers from the beginning, $22,23,23,$ and then cross out the same amount of numbers from the end, $68,67,63,$ until we reach 40 and 42, which we average to get the median: $41.$

30. B. The mean is the statistical term for the average. To find the mean in a data set, we add all the numbers, then divide by the total number of members in the set. In this case, $\frac{4+4+6+7+9}{5}=6.$

31. D. One value appears more often than any other values in the data set: $47.$ This makes it the mode.

32. E. A common mistake would be to assume that two negative and two positive numbers always cancel each other out by summing to zero, when that is not always the case. We know that when summing positive and negative numbers, the answer could be positive *or* negative. Diving by the total number of numbers to get the mean will not change the sign of that sum. Thus, the mean's sign cannot be determined. The median, because we are dealing with an even count of numbers (4 numbers), is the mean of the two center numbers in the set. This mean is again the average of a combination of positive and negative numbers, so its sign cannot be determined either.

33. D. An easy way to add the 8 consecutive integers is to add up the x values and constant values separately, and only then add those two sums together. Adding up the x values, we see that there are 8 $x's$, so their sum is $8x.$ Then we add the constants: $-3-2-1+0+1+2+3+4.$ $-3-2-1$ cancels out with $1+2+3,$ so the sum of the constants is equal to $4.$ Now, we can add the values and constant values together to get as the total sum. This, we are given, is equal to $332.$ So solving this equation, we get $8x+4=332 \rightarrow 8x=328 \rightarrow x=41.$ We are asked to find the value of $x+2$ which is $41+2=43.$

34. D. The average of the slowest and busiest nights was 224 which means that the total attendance in those two days was $2*224=448.$ The average of the other 5 days was 311 which means that the total attendance for those 5 days was $5*311=1555.$ Then, the total attendance for the week is $448+1555=2003.$