1. If a is a positive integer, then the sum of $9a$ and $4a$ is always divisible by which of the following?

A. 13
B. 9
C. 4
D. 5
E. 36

2. Which of the following could be the last 3 digits of an integer that is a perfect square?

A. 232
B. 753
C. 689
D. 597
E. 168

3. What percent (to the nearest tenth) of numbers from 10 to 99 have a tens digit that is 2 times as large as the units digit?

A. 4.1%
B. 4.2%
C. 4.3%
D. 4.4%
E. 4.7%

4. For every 2-digit integer number, a, with tens digit x and units digit y, let b be the 2-digit number reversing the digits of a. Which expression is equivalent to $a+b$?

A. $2xy$
B. $10(x+y)$
C. $10x+10y$
D. $11(x+y)$
E. $x+y$

5. Let a be a positive, odd integer. The expression ab^2 is a positive, odd integer when b is part of what set of integers?

A. Positive Even Integers
B. Negative Even Integers
C. Positive Odd Integers
D. All Integers
E. All Odd Integers

6. x and y are consecutive integers such that $x>y$. All of the following are true except?

A. $x-y$ is odd
B. $x-y$ is even
C. $x+y$ is odd
D. $y-x<0$
E. x^2+y^2 is odd

7. Which set gives the 3 largest prime numbers that are less than 100?

A. $\{37,47,59\}$
B. $\{83,89,97\}$
C. $\{59,61,67\}$
D. $\{97,98,99\}$
E. $\{95,97,99\}$

8. The sum of any two prime numbers except the number 2 must be which of the following?

A. Prime Number
B. Perfect Square
C. Even Number
D. Odd Number
E. A multiple of 3

9. How many positive prime factors does the number 60 have?

A. 1
B. 2
C. 3
D. 4
E. 5

10. For all positive integers k where $k>2$, what is the correct ordering of $k^{k!}$, $(k+1)!^k$, $k!^{k!}$?

A. $k!^{k!} \ge k^{k!} \ge (k+1)!^k$
B. $k!^{k!} \ge (k+1)!^k \ge k^{k!}$
C. $k^{k!} \ge (k+1)!^k \ge k!^{k!}$
D. $(k+1)!^k \ge k^{k!} \ge k!^{k!}$
E. $(k+1)!^k \ge k!^{k!} \ge k^{k!}$

11. On the real number line, -0.1395 is between $\frac{a}{1000}$ and $\frac{a+1}{1000}$ for some integer a. What is a?

A. -138
B. -139
C. -140
D. -141
E. -142

12. The variables x, y, and z are all integers and $x+y+z=200$. If $45<x<75$ and $3<y<18$ what is the maximum possible value of z?

A. 107
B. 152
C. 151
D. 150
E. 149

13. What is the correct order from greatest to smallest for the following values: $6\frac{4}{9}$, $\frac{44}{7}$, 6.374?

A. $\frac{44}{7}$, $6\frac{4}{9}$, 6.374

B. $6\frac{4}{9}$, 6.374, $\frac{44}{7}$

C. $6\frac{4}{9}$, $\frac{44}{7}$, 6.374

D. 6.374, $\frac{44}{7}$, $6\frac{4}{9}$

E. 6.374, $6\frac{4}{9}$, $\frac{44}{7}$

14. What is the correct order from least to greatest of the following values: $\sqrt{5}$, 2.1, $\frac{9}{4}$, $2\frac{2}{3}$?

A. $\sqrt{5}$, $\frac{9}{4}$, $2\frac{2}{3}$, 2.1

B. $\sqrt{5}$, $\frac{9}{4}$, 2.1, $2\frac{2}{3}$

C. 2.1, $\sqrt{5}$, $\frac{9}{4}$, $2\frac{2}{3}$

D. 2.1, $2\frac{2}{3}$, $\sqrt{5}$, $\frac{9}{4}$

E. $\frac{9}{4}$, 2.1, $2\frac{2}{3}$, $\sqrt{5}$

15. x, y, and z are positive real numbers such that $3x=4z$ and $\frac{1}{3}x=\frac{1}{5}y$. Which of the following inequalities is true?

A. $y>z>x$
B. $x>y>z$
C. $z>x>y$
D. $x>z>y$
E. $y>x>z$

16. For every pair of integers n and m, to which of the following sets must $\frac{m}{n}$ belong?

I. The natural numbers
II. The integers
III. The rational numbers
IV. The real numbers
V. The complex numbers

A. I, II, and III only
B. II, III, and IV only
C. III and IV only
D. III, IV, and V only
E. IV and V only

17. What is the lowest possible value for the product of two integers that differ by ten?

A. -36
B. -16
C. -25
D. -24
E. -4

18. x is a factor of 12 and y is a factor of 39. Which of the following cannot be the value of xy?

A. 18
B. 78
C. 72
D. 36
E. 468

19. There are 3 integers A, B, and C. A is a prime factor of 18. B is a factor of 36 such that $6 < B < 12$. C is a perfect cube such that $10 < C < 30$. What is one possible value of $\frac{AB}{C}$?

A. 3
B. 1
C. $\frac{4}{9}$
D. $\frac{8}{9}$
E. $\frac{4}{3}$

20. Two numbers are reciprocals if their product is equal to 1. If a and b are reciprocals and $a < -1$, then b must be:

A. Greater than 1
B. Between 0 and 1
C. Equal to 0
D. Between 0 and -1
E. Less than -1

21. If m and n are real numbers such that $6 \le m \le 18$, and $-3 \le n \le 9$, then the minimum value for $\frac{m}{n}$ is:

A. -9
B. -6
C. -2
D. 3
E. 2

22. If A, B, and C are real numbers, and if $ABC = -1$, which of the following conditions must be true?

A. $AB = \frac{1}{C}$
B. A, B, and C must all be negative
C. Either A, B, or C must equal -1
D. Either A, B, or C is between -1 and 0
E. $BC = -\frac{1}{A}$

23. a is a positive real number and $\frac{9a^2}{16a}$ is a rational number. Which statement about a must be true?

A. a is rational
B. a is irrational
C. $a = \frac{16}{9}$
D. $a = \frac{4}{3}$
E. $a = \frac{3}{4}$

24. Which of the following is irrational?

A. $\sqrt{\frac{4}{9}}$
B. $\sqrt{\frac{16}{25}}$
C. $\sqrt{\frac{100}{36}}$
D. $\sqrt{\frac{3}{4}}$
E. $\sqrt{\frac{64}{81}}$

25. Which of the following statements is false about rational and/or irrational numbers?

A. The sum of any 2 irrational numbers is irrational
B. The product of any 2 irrational numbers is always rational
C. The quotient of any 2 rational numbers is always rational
D. The product of a rational and irrational number is irrational
E. The quotient of any 2 irrational numbers could be irrational

ANSWER KEY

1. A **2. C** **3. D** **4. D** **5. E** **6. B** **7. B** **8. C** **9. C** **10. A** **11. C** **12. D** **13. B** **14. C**
15. E **16. D** **17. C** **18. C** **19. B** **20. D** **21. B** **22. E** **23. A** **24. D** **25. B**

ANSWER EXPLANATIONS

1. **A.** $9a+4a=13a$. So, $13a$ is always divisible by 13 and a.

2. **C.** The squares of the first 10 digits are $1^2=1, 2^2=4, 3^2=9, 4^2=16, 5^2=25, 6^2=36, 7^2=49, 8^2=64, 7^2=49,$ $8^2=64, 9^2=81,$ and $10^2=100$. These only end with the digits $1,4,9,5,6$ and 0. In the answer choices, only answer choice (C) ends with a 9. So, only answer choice (C) could be the last 3 digits of a square number.

3. **D.** There are a total of $99-10+1=90$ numbers from 10 to 99. Of these 90 numbers, the numbers that have tens digits that are twice as large as the units digits include $21, 42, 63$ and 84. So, 4 out of 90 numbers $=\frac{4}{90}(100\%)\rightarrow 4.4\%$ of the numbers from 10 to 99 satisfy the description.

4. **D.** The 2-digit number a can be written as $10x+y$. When we reverse x and y we get $b=10y+x$. So, $a+b=10x+y+10y+x\rightarrow 11x+11y\rightarrow 11(x+y)$.

5. **E.** The square of an odd number is odd. So, in order for ab^2 to be odd, b^2 must be odd, which indicates b is odd. It does not matter if b is positive or negative because all integers are positive when squared. So, b belongs to the set of all odd integers.

6. **B.** If x and y are consecutive integers with $x>y$, that means $x=y+1$. So, evaluating the expression $x-y$, we can substitute in $x=y+1$ for x to get $y+1-1=y$. Since 1 is odd, answer choice (A) is true. The statement "$x-y$ is even" in answer (B) is the only false statement. Answer choice (C) is true because $x+y=y+1+y=2y+1$ is an even number plus an odd number = odd. Answer choice (D) is true because $y-x=y-(y+1)\rightarrow -1<0$. Answer choice (E) is true because $x^2+y^2=(y+1)^2+y^2=y^2+2y+1+y^2=2y^2+2y+1=2\left(y^2+y\right)+1$ is an even number plus an odd number = odd.

7. **B.** To determine if a number is prime, we can divide the number by all integers less than its square root, and if none of the integers divide the number, then it is prime. So, we start counting down from 99 until we find three numbers that are prime. To make the process faster, we only need to evaluate the numbers given to us in the answer choices. We can eliminate all even numbers because they are divisible by 2. We can also eliminate all numbers that end with 5 because they are divisible by 5. We have 99 is divisible by 9, so it is not prime. 97 is difficult to tell, so we take $=\sqrt{99}=9.8$ so we test $\frac{97}{9}=10.77, \frac{97}{8}=12.125, \frac{97}{7}=13.86$ etc. and we find that 97 is not divisible by any integer less than 9, so it is prime. Now, we can already eliminate all of the answer choices except for answer choice (B) because $97>67>59$. So, we know the answer is (B). To double check, we see if 89 and 83 are prime. $\sqrt{89}=9.43$ and $\frac{89}{9}=9.89, \frac{89}{8}=11.125, \frac{89}{7}=12.714$ etc. and we see that 89 is not divisible by any integer less than 9. $\sqrt{83}=9.11$ and $\frac{83}{9}=9.22, \frac{83}{8}=10.375, \frac{83}{7}=11.857$ etc. and we see that 83. is not divisible by any integer less than 9 either.

8. **C.** 2 is the only even prime number, so if summing all the prime numbers other than two, we are summing odd numbers only. The sum of any two odd numbers is always an even number.

9. **C.** We find the prime factorization of 60. $60=6(10)=3(2)(5)(2)=2^2(3)(5)$. So, 60 has three prime factors: 2, 3, and 5.

10. A. For all positive integers k we know that $(k+1)! > k! > k$ and numbers with larger exponents increase at a faster rate. So, $(k+1)!^k$ has the smallest value because it is raised to the smallest power. Next, we compare $k^{k!}$ and $k!^{k!}$, and we can see that $k^{k!} < k!^{k!}$ because they are raised to the same power but $k < k!$. So, the correct order is $k!^{k!} \geq k^{k!} \geq \left(k+1\right)!^k$.

11. C. The problem tells us that $\frac{a}{1000} < -0.1395 < \frac{a+1}{1000}$. So, multiplying both sides by 1000, we get $a < -139.5 < a+1$. We can tell that the number -139.5 is in between the numbers -140 and -139. So, $a = -140$ and $a+1 = -139$.

12. D. Since $x + y + z = 200$, to find the maximum possible value of z we want to find the smallest possible values of x and y Since $45 < x < 75$ and $3 < y < 18$, the smallest integer values for x and y are 46 and 4. So, plugging in $x = 46$ and $y = 4$, we have $46 + 4 + z = 200$. Subtracting $(46+4)$ on both sides, we get $z = 150$.

13. B. Plugging these values into the calculator, we get $6\frac{4}{9} = 6.444$, $\frac{44}{7} = 6.286$, and $6.374 = 6.374$. So, putting these in order from greatest to smallest, we have $6.444 > 6.374 > 6.286$. Putting these values in their original forms, we get $6\frac{4}{9} > 6.374 > \frac{44}{7}$.

14. C. Plugging these values into the calculator, we get $\sqrt{5} = 2.236$, $2.1 = 2.1$, $\frac{9}{4} = 2.25$, and $2\frac{2}{3} = 2.667$. So, putting these in order from least to greatest, we have $2.1 < 2.236 < 2.25 < 2.667$. Putting these values in their original forms, we get $2.1 < \sqrt{5} < \frac{9}{4} < 2\frac{2}{3}$.

15. E. Multiplying both sides of $\frac{1}{3}x = \frac{1}{5}y$ by 15 , we get $5x = 3y$. So, we know that $y > x$ because it has a smaller coefficient. Similarly, we know $3x = 4z$ means that $x > z$ because x has the smaller coefficient. So, we can write $y > x > z$.

16. D. Recognize that every element in the list "falls under" the item after it. For example, all natural numbers are integers, all integers are rational numbers, and all real numbers are complex numbers, although none of the reverse are true. Thus, we only need to prove the most specific case, and the rest will follow. $\frac{m}{n}$ does not have to be a natural number, since one could be positive and one negative, making the term negative, not a natural number. $\frac{m}{n}$ doesn't have to be an integer, since it could fall between the integers, for example, as $\frac{3}{2}$. However, $\frac{m}{n}$ has to be a rational number, since a rational number can be expressed as a fraction of rational numbers, and m and n are both rational (since they are integers.) So III is true, and as we pointed out before, all the rest follow, so III, IV, and V are true.

17. C. If we take two negative numbers, then their product will be positive. If we take two positive numbers, their product will also be positive. So, we are looking for one positive and one negative integer that differ by 10. This narrows the options down to 9 pairs of numbers: $\left(-1,9\right)$, $\left(-2,8\right)$, $\left(-3,7\right)$, $\left(-4,6\right)$, $\left(-5,5\right)$, $\left(-6,4\right)$, $\left(-7,3\right)$, $\left(-8,2\right)$, $\left(-9,1\right)$. The pair of integers that yields the largest negative value is $(-5,5) \rightarrow (-5)5 = -25$.

18. **C.** x is a factor of 12 means that $x=\frac{12}{a}$ for some integer a. Likewise, y is a factor of 39 means that $y=\frac{39}{b}$ for some integer b. This means that $xy=\frac{12}{a}\left(\frac{39}{b}\right)\rightarrow\frac{468}{ab}\rightarrow\frac{2^2 3^3 13}{ab}$. All of our answer choices are a combination of the factors in the prime factorization of $468=2^2 3^3 13$ except for 72 whose prime factorization is $72=3^3 2$. The factor 3^3 make it impossible for $\frac{2^2 3^3 13}{ab}=3^3 2$. Alternatively, if using variables feels like a nuisance, just list the prime factors of 12 and 29 using a factor tree or not, and go through the answers and see if they can be found with those factors.

19. **B.** $18=9(2)=3^2(2)$ so the prime factors of 18 are 2 and 3, which means that A must be either 2 or 3. The factors of 36 are $1, 2, 3, 4, 6, 9, 12, 18, 36,$ and the only factor that satisfies the condition $6<B<12$ is 9, so $B=9$. The only perfect cube between 10 and 30 is $27=3^3$, so $C=27$. Plugging in $A=2$, $B=9$, and $C=27$ to the expression $\frac{AB}{C}$, we get $\frac{2(9)}{27}=\frac{18}{27}\rightarrow\frac{2}{3}$. Plugging $A=3$ $B=9$ and $C=27$ we get $\frac{3(9)}{27}=\frac{27}{27}\rightarrow 1$. $\frac{2}{3}$ is not one of the answer options, but 1 is.

20. **D.** Because $a<-1$, we know that a is a negative number. In order for ab to equal 1, b must be $\frac{1}{a}$. Since a is negative, b must be negative, and since $a<-1$, $\frac{1}{a}$ must be less than -1. Combining these two conditions, we see that b is between -1 and 0.

21. **B.** To get the minimum value for $\frac{m}{n}$, we would usually want to get each variable's respective minimum. However, because we can chose from negative numbers, $\frac{m}{n}$ would be smallest when the lowest negative number and the highest positive number is chosen from our pool of potential numbers. Thus, if we let $m=18$ and $n=-3$, $\frac{m}{n}=-6$. Had we taken the smallest numbers of each set, letting $m=6$ and $n=-3$, we would have gotten -2, which is greater than -6.

22. **E.** If $ABC=-1$, by dividing by A on both sides, we get $BC=-\frac{1}{A}$.

23. **A.** If $\frac{9a^2}{16a}$ is a rational number, that means that it could be expressed as a fraction with integers in the numerator and denominator. Simplifying the fraction to get $\frac{9a^2}{16a}=\frac{9a}{16}$ we know that a must be rational in order for $\frac{9a}{16}$ to be rational.

24. **D.** Every answer choice except for answer choice D can be simplified to be a rational number because they are fractions with perfect squares in the numerator and denominator.

25. **B.** Irrational numbers cannot be expressed as a fraction. Multiplying any 2 irrational numbers by each other will not make them rational. For example, π is irrational, and multiplying it by π produces π^2, which is also irrational. Thus, (B) is false.