BASIC ALGEBRA

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- How to solve single variable equations
- How to solve two-variable equations with given values
- How to plug in to equations when given values for variables
- How to simplify expressions & equations (PEMDAS, The Distributive Property, etc.)

SOLVING SINGLE VARIABLE EQUATIONS

Oftentimes on the ACT, you'll need to know how to solve a basic algebraic equation. Sometimes you'll confront "Single Variable Equations" (i.e. $3x = 45 - x$ is a single variable equation). At other times, you'll see "Two Variable Equations" (i.e. $2x + 3y = 45$ has two variables, x and y), and be given the exact value for one of those variables. Either way, the path to the solution is similar: Your job is to isolate the variable and solve for it while combining like terms and following the order of operations. Most of the time when students miss these questions, they're making careless errors or they're thrown by an awkward word or two or by a setup they don't recognize. When in doubt, **substitute** in any variables, **simplify** anything you have and reassess the situation! These two "S's" can get you through many seemingly challenging problems that are much easier than they look!

When $3x + 5y = 45$ and $y = 6$ what is the value of x ?

To solve, simply plug in the value of y to get a single variable equation:

$$3x + 5(6) = 45$$
$$3x + 30 = 45$$
$$3x = 15$$
$$x = 5$$

If you're having trouble with that problem, you probably need to do some serious work on your algebra skills—this book may not be enough. Get a hold of a basic algebra textbook and start reviewing! Get online and Google "Free Algebra 1 worksheets." Kuta Software, as well as other websites, have tons of free "solving equations" exercises that could help you drill. Then come back here!

SOLVING TWO VARIABLE EQUATIONS/USING GIVEN VALUES

Let's look at another problem:

When $a = 6$ is the solution to the equation $3b - 1 = a + 10b$, what must b equal?

This problem isn't hard, it's just worded awkwardly. **WHEN IN DOUBT, plug in what you have:** $a = 6$

$$3b - 1 = a + 10b$$

Substitute

$$3b - 1 = 6 + 10b$$

Combine like terms

$$-1 = 6 + 7b$$
$$-7 = 7b$$
$$-1 = b$$

USING SIMPLIFICATION AND SUBSTITUTION TO SOLVE

Here's one more:

If $5+2p=21$, then $\frac{4}{3}p+\sqrt[3]{p}=?$

If you look at the $\frac{4}{3}p+\sqrt[3]{p}=?$ part of the problem, you may freak out. But don't! This is easy—find p first, then finding $\frac{4}{3}p+\sqrt[3]{p}$ will simply require you to "plug in" the value of p you found into the expression.

Sometimes we call this setup **EQUATION—EXPRESSION**. Here's how it works: You're given an equation. Then the test asks to evaluate some "expression" (some group of variables, monomials, polynomials whatever) that is impossible to "solve" because there's a question mark on the other side of the equals sign. For these problems, always **simplify—**that first magical "S"— and then **substitute.**

$$5+2p=21$$
$$2p=16$$
$$p=8$$

Great! Now we substitute 8 into the expression the expression we're trying to solve:

$$\frac{4}{3}p+\sqrt[3]{p} \qquad \text{Substitute}$$
$$\frac{4}{3}(8)+\sqrt[3]{8}$$
$$\frac{32}{3}+2$$

At this point, check the answers. Are they improper fractions? Mixed? Decimals? Let's say the answers are improper.

$$\frac{32}{3}+2\left(\frac{3}{3}\right)$$
$$\frac{32}{3}+\frac{6}{3}$$
$$\frac{38}{3}$$

If this problem still makes you panic, you can **use your calculator** if you can't remember cube roots or if you want to add fractions more easily. If you have a TI-83 or TI-84, you can hit the MATH key and then press "1" for "$\triangleright$ Frac" to turn any decimal solution into fraction form.

Once you get the hang of these problems, they'll likely be easy—just watch out for common mistakes and careless errors!

Remember the following tips:

1. Always distribute across items added or subtracted in parenthesis—don't distribute across multiplied or divided items:

OKAY: $3(x+7)=3x+21$

NOT OKAY: $3\left(7\left(\frac{x}{y}\right)\right)\neq 21\left(\frac{3x}{y}\right)$

Notice how the 2nd example "distributes" the 3 to the fraction being multiplied? Don't do that. It's wrong.

2. Keep track of your negative signs! Always distribute them across added and subtracted elements when there is a negative in front of the parenthesis!

YES: $5-(x+2)=5-x-2$

Notice how the negative sign carries over the "$(x+2)$" to become "$-x-2$" multiplies times the 2?
Oftentimes students know this rule but somehow forget to apply it. If that's you, get in the habit of double checking yourself as you crunch numbers in algebra problems. Sure, this problem above is easy—but when your paper has 12 different terms to combine, keeping track of everything and remembering what you're multiplying can be a challenge.

3. A negative sign is not the same as a subtraction sign.
Sometimes we'll have students (incorrectly) write things like:

$$3x\bullet-7=5$$
$$3x=12$$
$$x=4$$

Oftentimes, they know that the -7 was being multiplied at some point, but because of their sloppy work—or because they use a dot instead of parenthesis—they end up off track and start thinking that they were subtracting the 7 instead of multiplying something by it. If you do this—be extra cautious! Put your elements in parenthesis when necessary.

4. Write neatly and organized enough to keep track of your work—and above all WRITE!
So often students miss these types of questions because they're not writing down much work. You don't need to do all this in your head—and when you do so—you're far more likely to make mistakes and careless errors. Thinking is often not much faster than writing when working through algebra problems. Use the pencil! It is your FRIEND!

1. Which of the following is a simplified form of the expression $-9(3-4y)+7+5y$?

A. $-y+10$
B. $41y-20$
C. $41y-34$
D. $-20y+41$
E. $-20y+10$

2. If $x=9$, $y=\frac{3}{4}$, and $w=-10$, what is the value of $\frac{2x^2y+w}{12yw-5x}$?

A. $-\frac{202}{270}$
B. $\frac{223}{270}$
C. $\frac{111.5}{270}$
D. 82
E. $-\frac{223}{270}$

3. If $x=20$, then which of the following is equal to 6065?

A. $15x+5$
B. $300x+15$
C. $15x^2+3x+5$
D. $15x^3+3x+5$
E. $15x^4+3x+5$

4. What is the value of $7\cdot 4^{2y+x}$ when $x=-3$ and $y=3$?

A. .1093
B. 84
C. 448
D. 1372
E. 21,952

5. If $k-4=q$ and $k-7=p$, what is the value of $p-2q$?

A. 3
B. -3
C. $1-k$
D. $k-1$
E. $3k-11$

6. What value of x makes $\frac{2}{3}(x-3)+2x=21$ true?

A. $8\frac{3}{8}$
B. $8\frac{5}{8}$
C. $10\frac{2}{3}$
D. 18
E. 69

7. If $6+7x=27$, then $2x=$?

A. 6
B. 3
C. 12
D. 9
E. 15

8. What is the solution to the equation $4(3x-3)=4x+3$?

A. 0
B. -15
C. $\frac{15}{8}$
D. 7
E. 8

9. If $5x-7=3x+8$, then $x=$?

A. 4
B. $\frac{7}{2}$
C. 7
D. $-\frac{15}{2}$
E. $\frac{15}{2}$

10. When $4x+8y=24$ and $y=5$, what's the value of x?

A. -16
B. -4
C. -1
D. $\frac{1}{4}$
E. 8

11. If $7+3m=30$, then $\frac{9}{2}m=$?

A. 1.70
B. 7.67
C. 13.5
D. 34.5
E. 69

12. When $b=-2$ is a solution to the equation $2b-7=ab+10$, what must a equal?

A. $\frac{21}{2}$
B. 2
C. $\frac{1}{2}$
D. -2
E. -10

13. If $4x-5(x+2)=3$, what is the value of x^2-2x?

A. -221
B. -195
C. -143
D. 143
E. 195

14. What value of z will satisfy the equation $0.2(z-1230)=-z$?

A. 102
B. 205
C. 246
D. 308
E. 1025

15. What is the solution to the equation below?
$5(w+12)-7(2-3w)=9(w+4)-13$

A. $-\frac{9}{2}$
B. $-\frac{23}{17}$
C. $\frac{9}{12}$
D. $\frac{23}{17}$
E. $\frac{9}{2}$

16. If $\frac{x}{6}-\frac{x}{9}=\frac{2}{3}$, then $x=$?

A. $\frac{1}{15}$
B. $\frac{1}{2}$
C. 3
D. 6
E. 12

17. When $\frac{1}{2}x+\frac{1}{5}x=2$, what is the value of x?

A. $\frac{1}{7}$
B. $\frac{20}{7}$
C. $\frac{10}{7}$
D. 7
E. 20

18. How many ordered pairs (x,y) of real numbers satisfy the equation $3x+7y=63$?

A. 0
B. 1
C. 2
D. 3
E. Infinitely many

19. Which of the following is an equivalent form of $x-x-x+x(x+x+x)$?

A. $7x$
B. $2x^2$
C. $3x$
D. x^2+x
E. $3x^2-x$

ANSWER KEY

1. B **2. E** **3. C** **4. C** **5. C** **6. B** **7. A** **8. C** **9. E** **10. B** **11. D** **12. A** **13. E** **14. B**
15. B **16. E** **17. B** **18. E** **19. E**

ANSWER EXPLANATIONS

1. **B.** Distributing the $-9,$ we get $-27+36y+7+5y=-20+41y=41y-20.$

2. **E.** Plugging in all the values for the correct variables, we get

$$\frac{2x^2y+w}{12yw-5x}=\frac{2(9)^2\left(\frac{3}{4}\right)+(-10)}{12\left(\frac{3}{4}\right)(-10)-5(9)}=\frac{2(81)\left(\frac{3}{4}\right)-10}{-12\left(\frac{3}{4}\right)(10)-45}=\frac{\left(\frac{243}{2}-\frac{20}{2}\right)}{-9(10)-45}=\frac{\frac{223}{2}}{-135}=\frac{223}{-270}.$$

3. **C.** We must plug in $x=20$ into each answer option until we get an expression that is equal to $6065.$ Plugging in $x=20$ for the equation in answer choice (A) gives us $15(20)^2 \; 3(20) \; 5 \quad 15(400) \quad 3(20) \quad 5 \quad 6000 \quad 60 \quad 5 \quad 6065.$

4. **C.** Plugging in $x=-3$ and $y=3$, we get $7\cdot 4^{2(3)+(-3)}=7\cdot 4^{6-3}=7\cdot 4^3=7\cdot 64=448.$

5. **C.** Plugging in $k-4=q$ and $k-7=p$ into the expression $p-2q,$ we get $(k-7)-2(k-4)$. Distributing the -2 (don't forget the negative!) gives us $k-7-2k+8.$ Combining like terms, we get $-k+1$ which is $1-k$.

6. **B.** Distributing the $\frac{2}{3},$ we get $\frac{2}{3}x-\frac{2}{3}(3)+2x=21.$ This simplifies into $\frac{2}{3}x+2x-2=21.$ Combining like terms, we get $\frac{8}{3}x=23.$ Now, to find the value of $x,$ we multiply both sides by the reciprocal of $\frac{8}{3},$ which is $\frac{3}{8}.$
So, $\left(\frac{3}{8}\right)\left(\frac{8}{3}\right)x=\left(\frac{3}{8}\right)23$. This simplifies into $x=\frac{69}{8}=8\frac{5}{8}.$

7. **A.** First solve for $x,$ then worry about $2x$ later. Subtracting 6 on both sides of the equation, we get $7x=21$. Dividing both sides by 7, we find the value of $x=3.$ Now, to find the value of $2x$, we multiply by 2 to get $2x=6.$

8. **C.** Distributing the 4 on the left side, we get $12x-12=4x+3$. Adding 12 on both sides, we get $12x=4x+15$. Now, subtracting $4x$ on both sides gives us $8x=15$. Finally, dividing both sides by 8, we get $x=\frac{15}{8}.$

9. **E.** Adding 7 to both sides of the equation, we get $5x=3x+15$. Subtracting $3x$ from both sides gives us $2x=15.$ Finally, dividing both sides by 2, we get $x=\frac{15}{2}.$

10. **B.** Substituting in $y=5$ into $4x+8y=24,$ we get $4x+8(5)=24.$ Simplifying this gives us $4x+40=24.$ Subtracting 40 on both sides gives us $4x=-16$, and dividing both sides by 4 gives us $x=-4.$

11. **D.** Subtracting 7 on both sides of the equation gives us $3m=23$. Dividing both sides by 3 gives us $m=\frac{23}{3}.$ Now, to find the value of $\frac{9}{2}m,$ we multiply the value of m by $\frac{9}{2}$, which is $\frac{9}{2}\left(\frac{23}{3}\right)=\frac{207}{6}=\frac{69}{2}=34.5$.

12. **A.** Plugging in $b=-2$ into the equation $2b-7=ab+10,$ we get $2(-2)-7=a(-2)+10$. This simplifies to $-4-7=-2a+10\,;-11=-2a+10\,;-21=-2a\,;a=\frac{-21}{-2}=\frac{21}{2}.$

13. E. Distributing out the -5 on the left side of the equation gives us $4x-5x-10=3$. Combining like terms, we get $-x=13$, so $x=-13$. Plugging in $x=-13$ to the expression x^2-2x, we get $(-13)^2-2(-13)=169+26=195$.

14. B. Dividing both sides by 0.2 we get $z-1230=-\frac{z}{0.2}$. Recognizing that $-\frac{1}{0.2}$ is equivalent to $-\frac{5}{1}$, (use your calculator if necessary) we can rewrite the equation as $z-1230=-5z$. Adding $5z$ on both sides gives us $6z-1230=0$. Adding 1230 on both sides gives us $6z=1230$. Dividng both sides by 6, we get $z=\frac{1230}{6}=205$.

15. B. Distributing out the constants in the equation, we get $5w+60-14+21w=9w+36-13$. Combining like terms, we get $5w+21w-9w=-60+14+36-13$ which simplifies to $17w=-23$. Dividing both sides by 17 gives us $w=-\frac{23}{17}$.

16. E. Rewriting $\frac{x}{6}-\frac{x}{9}$ with the common denominator $=18$, we get $\frac{x}{6}\left(\frac{3}{3}\right)-\frac{x}{9}\left(\frac{2}{2}\right)=\frac{3x}{18}-\frac{2x}{18}-\frac{3x-2x}{18}=\frac{x}{18}$. So now we have $\frac{x}{18}=\frac{2}{3}$. Cross multiplying this equation gives us $3x=36$. Dividing both sides by 3 gives us $x=12$.

17. B. We find the least common denominator of the fractions, which is the least common multiple of 2 and 5, to be 10. So we convert our fractions into $\frac{5}{10}x$ and $\frac{2}{10}x$. Adding them together, we get $\frac{7}{10}x=2$. Multiply by the reciprocal of the fraction to get $x=2\left(\frac{10}{7}\right)=\frac{20}{7}$.

18. E. Since the real numbers are infinite, there can be an infinite number of ordered pairs that satisfy the equation. For any value of x, we can always find a value of y in the real numbers that makes the equation true. As further proof we know that, graphically, this is a line, which infinitely extends in both directions with no discontinuities.

19. E. Use PEMDAS. Simplify what is in the parenthesis to get $x-x-x+x(3x)$; then, as there are no exponents, multiply to get $x-x-x+3x^2$. Finally, sum all like terms to get $3x^2-x$.