1. How many different integer values of d satisfy this inequality $\frac{2}{15} < \frac{4}{d} < \frac{6}{13}$?

A. 20
B. 22
C. 29
D. 18
E. 21

2. For which values of n is $n > n^2$?

A. $n < -1$
B. $n > 1$
C. $0 < n < 1$
D. $-1 < n < 0$
E. No real numbers

3. Which of the following inequalities is representative of the statement below? 35 is greater than $\frac{1}{3}$ the difference between 10 and the reciprocal of a number p. (Two numbers are reciprocals if their product equals 1).

A. $35 > \frac{1}{3}\left(10 - \frac{1}{p}\right)$

B. $35 + \frac{1}{3} > 10 - \frac{1}{p}$

C. $35 > \frac{10}{3} - \frac{1}{p}$

D. $35 > \frac{1}{3} - 10\left(\frac{1}{p}\right)$

E. $35 > \frac{1}{3}\left(\frac{1}{10 - p}\right)$

4. R is a negative integer and $1 > t > 0$. Which of the following must be true about t^r?

A. $t^r \le 1$
B. $t^r < 1$
C. $t^r > 1$
D. $t^r \ge 1$
E. $t^r = 1$

5. A and b are two numbers such that $0 < b < a$. All of the following statements are true except:

A. $a + 1 > b + 1$
B. $a^2 > b^2$
C. $\frac{1}{b} > \frac{1}{a}$
D. $\frac{1}{a} > \frac{1}{b}$
E. $-b > -a$

6. If $t < 0$ and $w > 0$, which of the following statements is negative for all values of t and w?

A. $t^2 + w$
B. tw^2
C. w^2t^2
C. $t + w$
E. $w - t$

7. Which of the following gives the solution set for the system of inequalities below?

$$n \ge 10$$
$$9 - 3n \le 0$$

A. $n \ge 3$
B. $n \ge 10$
C. $10 \ge n \ge 3$
D. $n \le 3$ or $n \ge 10$
E. $n \le 3$

8. If $7x^{23}y^{54} < 0$, which of the following must be true?

A. $x < 0$
B. $y > 0$
C. $x > y$
D. $y < 0$
E. $x < y$

9. The variables x, y, z are defined by the following statements.

1. y is 8 more than x
2. z is at least 4 less than y

Which of the following inequalities expresses the relationship between x and z?

A. $x \ge z - 4$
B. $x \ge -z - 4$
C. $x \le z - 4$
D. $x \le -z - 4$
E. $x \ge z + 12$

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10. If g is a variable between 0 and 1, not inclusive, which of the following is the largest?

A. g^2
B. g^3
C. g^4
D. g^5
E. $\sqrt[3]{g}$

11. For positive real numbers x, y, z such that $\frac{1}{2}x = y\sqrt{5} = \frac{z\sqrt{5}}{7}$, what is the correct ordering of x, y, z?

A. $z > x > y$
B. $y > x > z$
C. $y > z > x$
D. $x > z > y$
E. $z > y > x$

12. If p, q, r are real numbers such that $p^6q^9r^{10} > 0$, which of the following must be greater than 0?

A. pr
B. pq
C. rq
D. p^2q
E. pq^2

13. When $-7 \le a \le -8$ and $2 \le b \le 3$, what is the largest value of $a - b$?

A. -10
B. -9
C. -11
D. -6
E. -5

14. The Congress of a certain country has 267 members. If more than $\frac{2}{3}$ of the members of the Congress must vote yes on a certain bill to pass it, which of the following expressions shows the necessary number of yes votes, x, for a bill to pass?

A. $x = 178$
B. $x > 178$
C. $x < 179$
D. $x < 89$
E. $x < 177$

15. A line contains the points $P, Q, R,$ and S. Point R is between P and S. Point Q is between Point R and Point S. Which of the following inequalities must be true?

A. $PQ > PR$
B. $PR > QR$
C. $PR < RS$
D. $RQ > RS$
E. $QS > RS$

16. Whenever $(1-x)(x-5) > 0$, which of the following expressions *always* has a negative value?

A. $-x+5$
B. $x-1$
C. $-2x+5$
D. x^2-6x+2
E. $(x-3)(x-1)$

17. If $a \ne b$, what are the real values of a that make the following inequality true?

$$\frac{ab-b}{2b-4b} < 0$$

A. $a = 1$
B. $a < 1$
C. $a > 1$
D. all positive real numbers
E. all negative real numbers

18. If j and k can be any integers such that $4j - 2k = 12$ and $k < 8$, which of the following is the solution set for j?

A. $j < 7$
B. $j > 0$
C. $j > 7$
D. $j < 0$
E. $j > 1$

19. For real numbers a and b such that $0 < ab < a < b$, which of the must be true?

A. $a > 1$ and $b > 1$
B. $a < 1$ and $b < 1$
C. $a < 1$ and $b > 1$
D. $ab > a + b$
E. $ab > a - b$

supertutortv

20. Whenever $\frac{a}{b} < \frac{a}{c} < \frac{a}{d} < 1$ is true for some positive integers a, b, c, and d, which of the following, if it can be determined, must be the order of $\frac{b}{a}$, $\frac{c}{a}$, and $\frac{d}{a}$?

A. $1 < \frac{d}{a} < \frac{c}{a} < \frac{b}{a}$

B. $\frac{d}{a} < \frac{c}{a} < \frac{b}{a} < 1$

C. $1 < \frac{b}{a} < \frac{c}{a} < \frac{d}{a}$

D. $\frac{b}{a} < \frac{c}{a} < \frac{d}{a} < 1$

E. $1 < \frac{c}{a} < \frac{d}{a} < \frac{b}{a}$

21. Given that $a > b$ and $(a+b) < (a^2 - b^2)$, then a and b must be:

A. Greater than 1
B. Positive
C. Negative
D. Between 0 and 1
E. Between 0 and -1

22. Which of the following is the solution statement for the inequality $14x - 3 \leq 9x + 4$?

A. $x \leq \frac{7}{5}$

B. $x \leq \frac{5}{7}$

C. $x \leq \frac{1}{5}$

D. $x \leq \frac{1}{3}$

E. $x \leq \frac{7}{3}$

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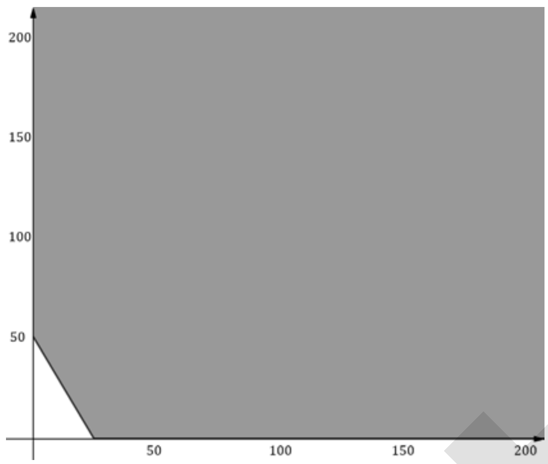
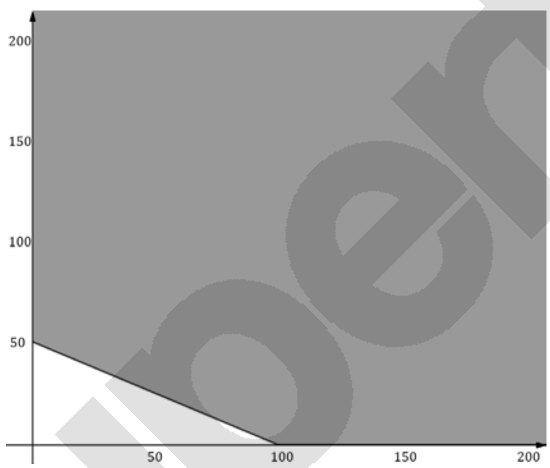
23. Tickets for the Annual Poetry Slam at Van Buren High School are $\$8$ for adults and $\$4$ for students. In order to make a profit, at least $\$200$ must be collected from ticket sales for the show. One of the following graphs in the standard (x,y) coordinate plane, where x is the number of adult tickets sold and y is the number of student tickets sold represents all possible combinations of ticket sales necessary to make a profit. Which graph is it?

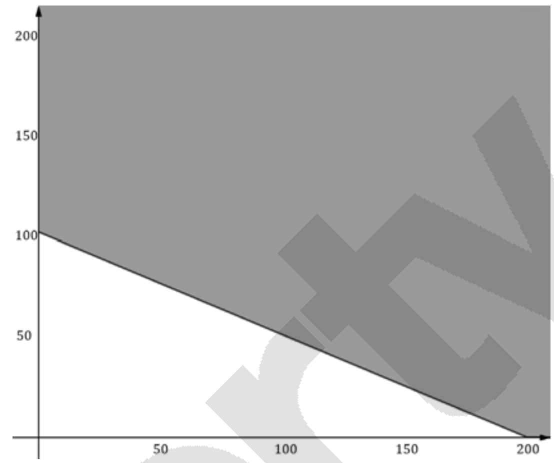
A.



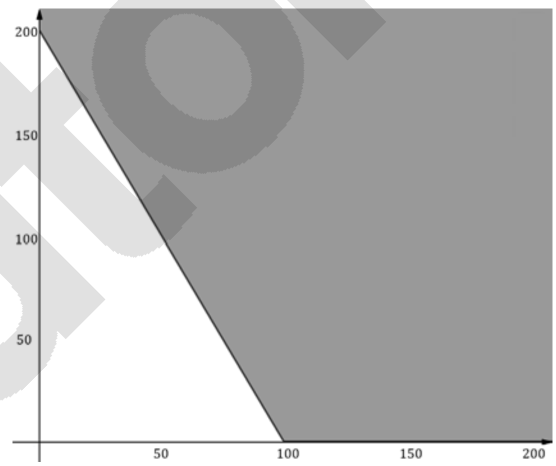
B.



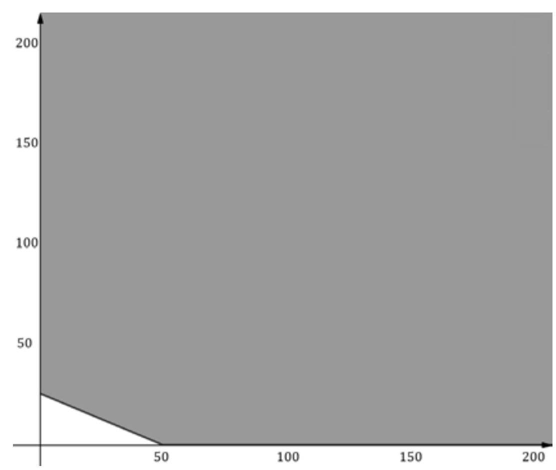
C.



D.

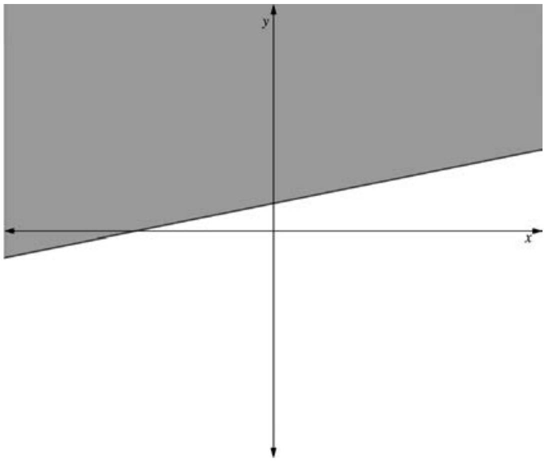


E.

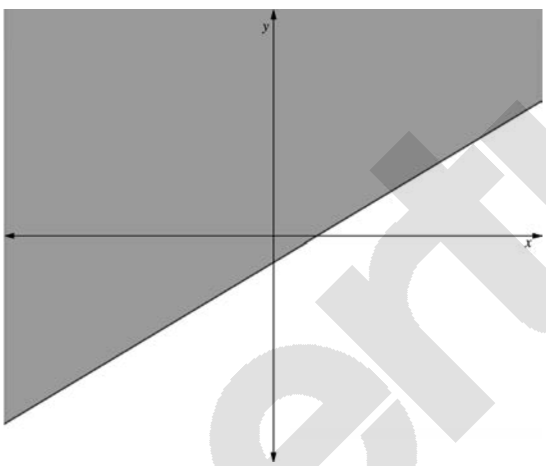


24. One of the following graphs in the standard (x, y) coordinate plane is the graph of $y > ax + b$ for some positive a and positive b. Which graph?

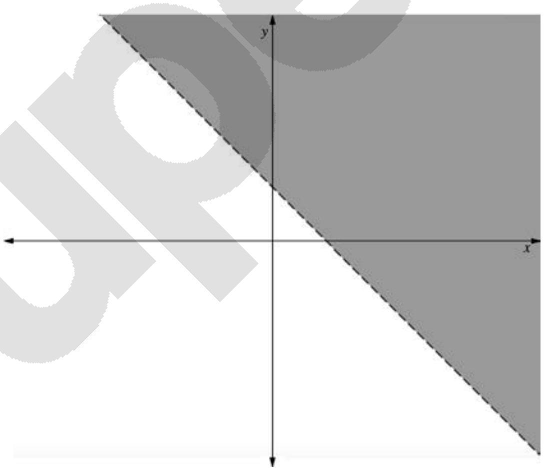
A.



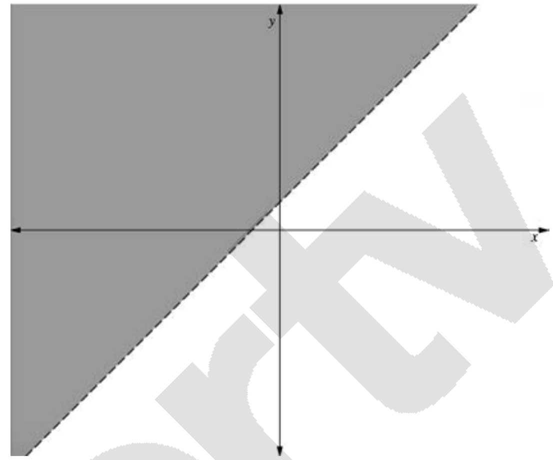
B.



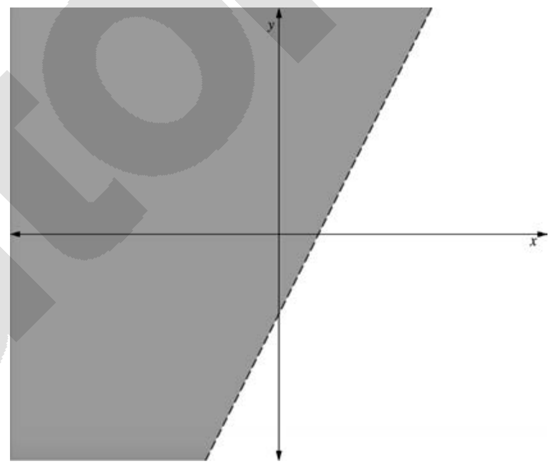
C.



D.



E.



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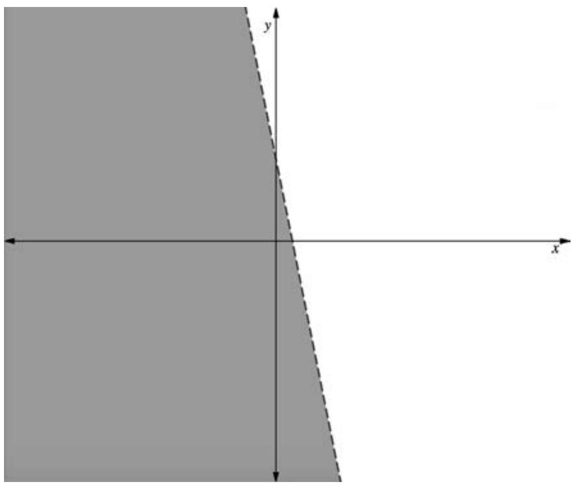
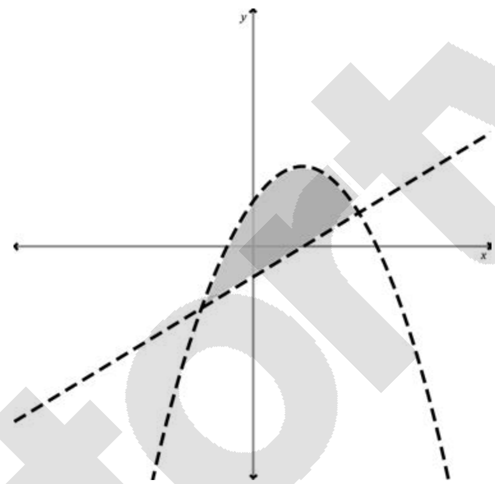
25. One of the following inequalities, where both constants a and b are positive real numbers, is graphed in the standard (x,y) coordinate plane below. Which inequality is it?



A. $y \geq ax + b$
B. $y > -ax + b$
C. $y < ax - b$
D. $y < -ax + b$
E. $y \leq ax + b$

26. The shaded region in the standard (x,y) coordinate plane below is bounded by a parabola and a line. The shaded region and its boundary is the solution set of which of the following systems of inequalities?



A. $\begin{cases} y \leq \left(\dfrac{x}{2} - 2\right)^2 + 6 \\ y \geq \dfrac{x}{2} - 2 \end{cases}$

B. $\begin{cases} y > \left(\dfrac{x}{2} - 2\right)^2 + 6 \\ y < \dfrac{x}{2} - 2 \end{cases}$

C. $\begin{cases} y \leq \left(\dfrac{x}{2} - 2\right)^2 + 6 \\ y < \dfrac{x}{2} - 2 \end{cases}$

D. $\begin{cases} y < \left(\dfrac{x}{2} - 2\right)^2 + 6 \\ y > \dfrac{x}{2} - 2 \end{cases}$

E. $\begin{cases} y > \left(\dfrac{x}{2} - 2\right)^2 + 6 \\ y > \dfrac{x}{2} - 2 \end{cases}$

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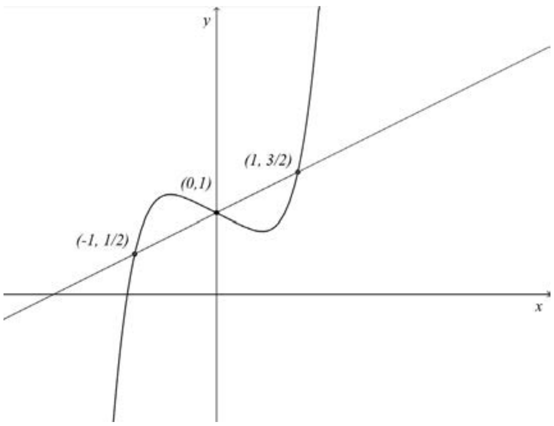
27. The graph of the equations $y_1 = x^5 - \frac{x}{2} + 1$ and $y_2 = \frac{x}{2} + 1$ are shown in the standard (x, y) coordinate plane below. What real values of x, if any, satisfy the inequality $x^5 - \frac{x}{2} + 1 > \frac{x}{2} + 1$?



A. $-1 < x < 1$
B. $-1 < x < 0$ and $x > 1$
C. $-1 < x < 0$
D. $x < -1$ and $0 < x < 1$
E. $0 < x < 1$

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28. Which of the graphs below best represents the system of inequalities below?

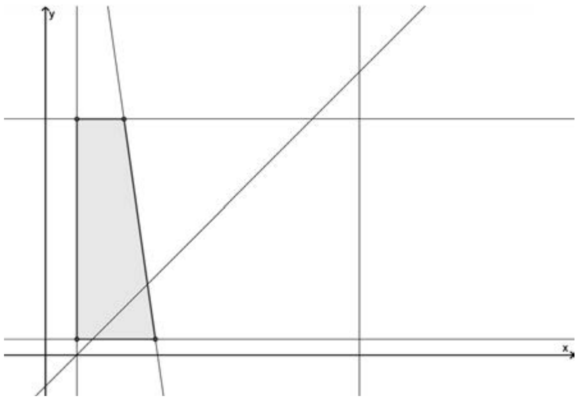
1. $2 \le x \le 20$

2. $1 \le y \le 10$

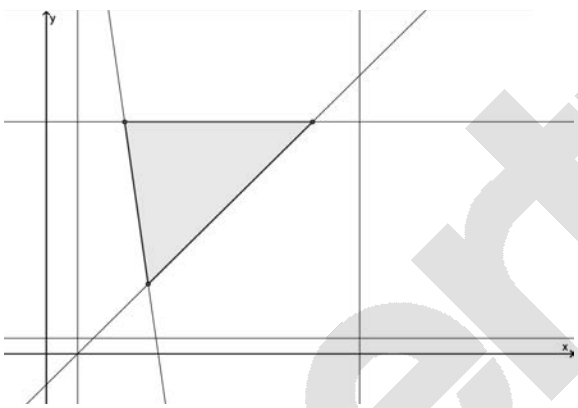
3. $y \le x - 2$

4. $y \ge -7x + 50$

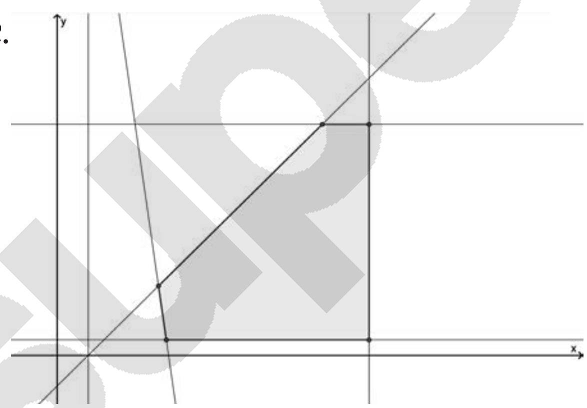
A.



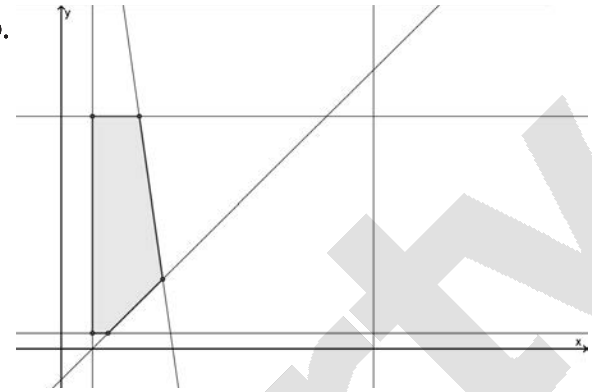
B.



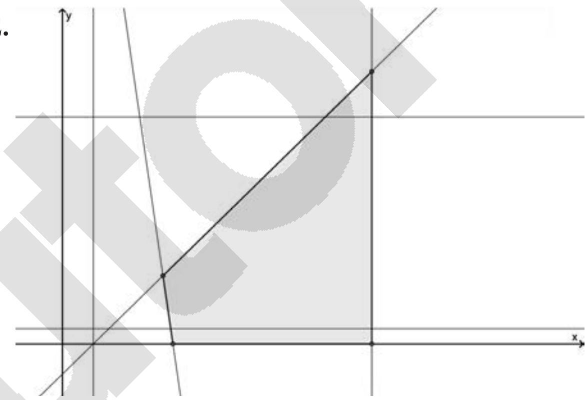
C.



D.



E.



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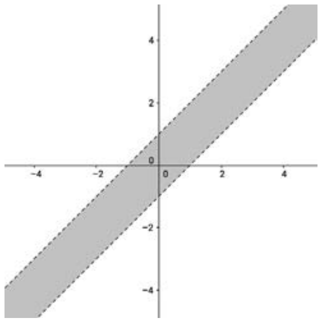
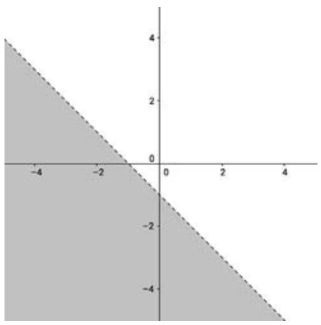
29. Which of the following graphs in the standard (x, y) coordinate plane represents the solution set of the inequality $|x-y|<1$?

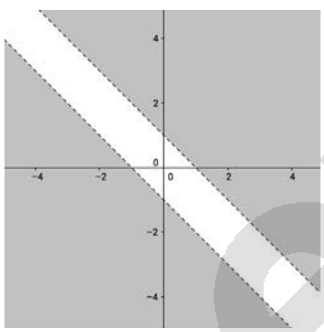
A.



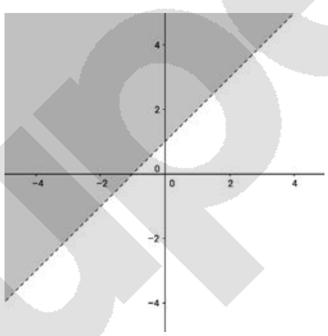
B.



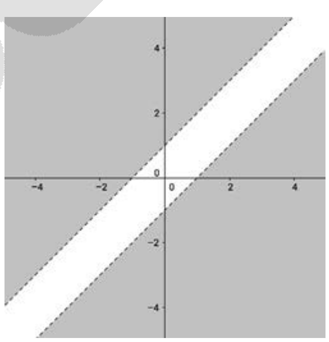
C.



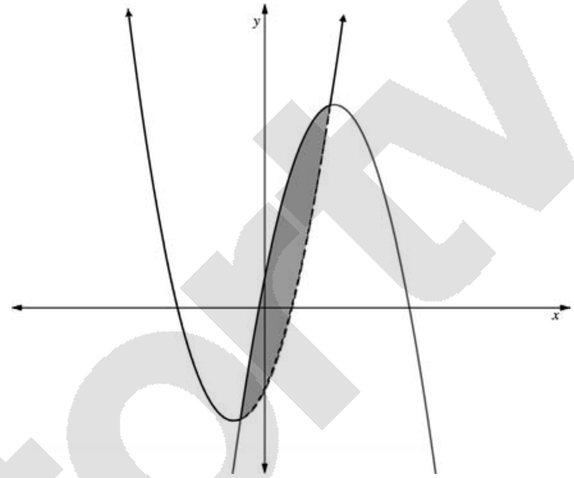
D.



E.



30. Which of the following systems of inequalities is represented by the shaded region of the graph below?



A. $y > x^2+2x-3$ and $y < -x^2+5+1$

B. $y > x^2+2x-3$ and $y \le -x^2+5+1$

C. $y \le x^2+2x-3$ and $y < -x^2+5+1$

D. $y > x^2+2x-3$ and $y > -x^2+5+1$

E. $y \ge x^2+2x-3$ and $y \le -x^2+5+1$

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ANSWERS

1. E	**2. C**	**3. A**	**4. C**	**5. D**	**6. B**	**7. B**	**8. A**	**9. C**	**10. E**	**11. A**	**12. D**	**13. B**	**14. B**
15. A	**16. D**	**17. C**	**18. A**	**19. B**	**20. A**	**21. E**	**22. A**	**23. A**	**24. D**	**25. D**	**26. D**	**27. B**	**28. C**
29. A	**30. B**												

ANSWER EXPLANATIONS

1. **E.** Multiplying the inequality by d, we get $\frac{2}{15}d<4<\frac{6}{13}d$. Now splitting the inequality into two separate ones, we have $\frac{2}{15}d<4$ and $4<\frac{6}{13}d$. Dividing $\frac{2}{15}d<4$ by $\frac{2}{15}$ we get $d<4\left(\frac{15}{2}\right)\to d$. Dividing $4<\frac{6}{13}d$ by $\frac{6}{13}$, we get $\frac{26}{3}<d$. Now, combining the two inequalities, we have $\frac{26}{3}<d<30$. $\frac{26}{3}\approx 8.67$. So, the integer values that would satisfy the inequality are $9\le d\le 29$. This gives us $29-9+1=21$ possible integers. Notice that we must add 1 after subtracting the boundaries because we include both boundaries in our count.

2. **C.** The square of a fraction less than 1 is larger than the fraction itself because when the denominator is larger than the numerator, the denominator increases more than the numerator when squared. This makes the resulting fraction smaller.

3. **A.** The reciprocal of a number p is written $\frac{1}{p}$. The difference between 10 and $\frac{1}{p}$ is $10-\frac{1}{p}$. $\frac{1}{3}$ of that difference is $\frac{1}{3}\left(10-\frac{1}{p}\right)$. Finally, the inequality states that 35 is greater than the expression we've written, so $35>\frac{1}{3}\left(10-\frac{1}{p}\right)$.

4. **C.** Since r is a negative integer, we can write t^y as t^{-x} for some positive x and since t is a fraction, we can rewrite it again as $\left(\frac{1}{y}\right)^{-x}$ for some $y>1$. This is equivalent to $\frac{1}{\left(\frac{1}{y}\right)^{-x}}=\frac{1}{\frac{1^x}{y^x}}\to\frac{y^x}{1^x}\to y^x$ for positive x and y. This makes $y^x>1$.

5. **D.** Since a and b are positive numbers, $\frac{1}{a}<\frac{1}{b}$ because the larger the denominator, the smaller the value of the fraction. So, since $a>b$, $\frac{1}{a}<\frac{1}{b}$. This makes answer choice (D) the only false statement.

6. **B.** w^2 is positive and t is negative. tw^2 is a negative number multiplied by a positive number, so it will always result in a negative value.

7. **B.** Adding $3n$ to both sides of the second equation, we get $9\le 3n$ or $3n\ge 9$. Dividing both sides by 3, we get $n\ge 3$. So, our two inequalities are $n\ge 3$ and $n\ge 10$. The set of solutions that satisfies these inequalities is $n\ge 10$ because if n is greater than 10, it will automatically be greater than 3.

8. **A.** Any number to an even power is positive, so y^{54} is positive. If $7x^{23}y^{54}$ is given to be negative, we know that x must be negative because it is the only possible term that could make the product negative. So, $x<0$.

9. **C.** Statement #1 can be written as $y=x+8$. Statement #2 can be written as $z\ge y-4$. Plugging in $y=x+8$ into $z\ge y-4$ we get $z\ge x+8-4$ or $z\ge x+4$ Subtracting 4 on both sides, we get $z-4\ge x$ or $x\le z-4$.

10. **E.** g is a positive fraction, so the higher the power it is raised to, the smaller its value, and the higher the root taken, the larger its value. So, $\sqrt[3]{g}$ would yield the largest value.

11. **A.** Because the expressions set equal, the variable with the smallest coefficient is the largest. Rewriting $\frac{z\sqrt{5}}{7}$ as $\frac{z\sqrt{5}}{\sqrt{49}}$, It is easy to see that $\frac{\sqrt{5}}{\sqrt{49}}<\frac{1}{2}$. Putting the coefficients in the correct order, we have $\frac{\sqrt{5}}{\sqrt{49}}<\frac{1}{2}<\sqrt{5}$. This means the order of the variables is $z>y>x$.

12. **D.** Since $p^6q^9r^{10}>0$ we know that $q>0$ since it is the only variable risen to an odd power. Thus, the expression p^2q is guaranteed to be greater than zero because we know $q>0$ and a square number such as p^2 is always positive.

13. **B.** The largest value of $a-b$ would be the difference between the largest possible value of a and the smallest possible value of $b: -7-2=-9$.

14. **B.** The number of yes votes, x, must be greater than $\frac{2}{3}$ of 267. This can be mathematically written as $x>\frac{2}{3}(267)$ or $x>178$.

15. **A.** Drawing the points on a number line according to the description given, we have P R Q S. We can see that $PQ>PR$ because $PQ=PR+RQ$.

16. **D.** First we need to find the set of numbers for which the inequality is true. We can do this by setting up a number line. We know that the expression equals zero at $x=1$ and $x=5$, so we can split up the number line into three main regions to test. If we plug in numbers less than 1 or numbers greater than 5 into the expression for x, the expression is negative, so our inequality is not satisfied. If $1<x<5$, the expression is positive, so the inequality is satisfied, and we now know which values of x we are looking at. Next, we need to find the expression that is negative for all these values of x. The only expression given that is negative while $1<x<5$ is x^2-6x+2. You could find this out by sketching the expressions given or plugging in numbers to test, as we did with initial expression.

17. **C.** Factor and simplify: $\frac{ab-b}{2b-4b}<0\rightarrow\frac{b(a-1)}{-2b}<0\rightarrow\frac{(a-1)}{-2}<0$. Since the denominator is a positive constant, the numerator must always be positive in order for the fraction to be less than zero (negative). Thus, $a-1>0\rightarrow a>1$.

18. **A.** Let's use substitution to solve this problem. First divide both sides of the expression $4j-2k=12$ by 2 and isolate k to get $k=-6+2j$. We know that $k<8$, so we can set the equivalent expression to be less than 8 as well to get $-6+2j<8$. Solving for j leaves us with $j<7$.

19. **B.** The ONLY way for the product to be less than *both* its factors is for both its factors to be fractions themselves. To test this theory out, try picking your own arbitrary numbers. If $a=2$ and $b=6$, the product ab would be 12, so that doesn't work. If $a=\frac{1}{2}$ and $b=6$, $ab=3$, which is less than b but greater than a, so that doesn't work either. Now let $a=\frac{1}{2}$ and $b=\frac{1}{6}$. Their product is $\frac{1}{18}$, which *is* less than both a and b. Thus, it is given that $a>0$ and $b>0$ and in order for a and b, to be fractions, $a<1$ and $b<1$.

20. **A.** This question looks overwhelming because of the fractions and variables but it's not that bad. We can assign arbitrary numbers to our variables, which will make it easier. Because the numbers are less than 1 and positive they must be fractions, which means the numerator a must be less than all of the three other variables. Because the fractions all share a numerator, so $b<c<d$ to make the inequality given to us true. Thus, let $a=2$, $b=3$, $c=4$, and $d=5$. When we take the reciprocals we get $\frac{b}{a}=\frac{5}{2},\frac{c}{a}=\frac{4}{2}$, and $\frac{d}{a}=\frac{3}{2}$. We know that $1<\frac{3}{2}<\frac{4}{3}<\frac{5}{2}$, so $1<\frac{d}{a}<\frac{c}{a}<\frac{b}{a}$.

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21. E. This problem is best done by backsolving. Come up with arbitrary pairs of numbers and test each of the answers given. A tip is to start with the bottom answers first, since the test will expect you to start at the top and go one by one. Try letting $a=\frac{1}{2}$ and $b=\frac{1}{3}$—that doesn't work because $\frac{5}{6}<\frac{5}{36}$ is false. Now try $a=-\frac{1}{2}$ and $b=\frac{1}{3}$. This works because $-\frac{5}{6}<-\frac{5}{36}$. Just to be safe, we should check that the inequality doesn't work for all negative numbers, as answer (C) states. If we let $a=-2$ and $b=-5$, we get $-7<-21$, which is false, so our answer is (E).

22. A. The only thing slightly tricky about this problem is the diction, but the question just asks you to solve the inequality, so combine like terms and simplify: $14x-3\le 9x+4 \rightarrow 5x\le 7 \rightarrow x\le \frac{7}{5}$.

23. A. The amount of money raised is equal to $8x+4y$, where x is the number of adult tickets sold and y is the number of student tickets sold. This sum must be at least $\$200$ to make a profit, so $8x+4y\ge 200$ is our equal. If we wanted change that into a more recognizable form, our equation would be $y\ge 50-2x$, so we look for a graph with a y-intercept of 50, and an x-intercept of 25 (per this equation) and only answer (A) has that. We do not need to test regions because all of the graphs shade on the same side, but if they didn't we would need to determine which side was shaded as well.

24. D. Only (A) and (D) have positive slopes and positive y-intercepts, which are what a and b represent respectively. Because the inequality is "greater than," not "greater than or equal to" we want the graph a dashed, not solid line, which leaves us with graph (D).

25. D. The graph has a negative slope and a dotted line, so we know that the a term, which was said to be positive, must have a negative sign in front of it, and the inequality sign must be either "greater than" or "less than," not "equal to." At this point we are deciding between (B) and (D). Now we must test a point to see what direction the sign is facing, plugging in that point's x and y coordinates to see if the inequality is true. In general, if you can, use the origin to test. $0>a(0)+b$ is false, since we are told (and we see on the graph) that b is positive, but $0<a(0)+b$ is true, so our answer is (D).

26. D. The question is testing your understanding of both inequalities and graphs. This graph has dashed lines, not solid, so our signs must be either $<$ or $>$, not $\le$ or $\ge$, narrowing our answer choices down to (B), (D), or (E). Now we must test points, and the easiest point to test is the origin, which is part of the shaded area. Testing answer B: $0>\left(\frac{0}{2}-2\right)^2+6$ is false, so answer (B) is incorrect. Now we test answer (D): $0<(0-2)^2+6$ is true and $0<\frac{0}{2}-2$ is true, so (D) is our answer.

27. B. Really what this question is asking is: when does $y_1>y_2$, since their y values are being directly compared. When you look at the graph, you see that the graph y_1 is above y_2 when $-1<x<0$ and $x>1$.

28. C. The first two inequalities tell you that the solution set is within a certain range of x and y values. What you must look for in the shaded region is within this range (which forms a rectangle) and whose boundaries are the third and fourth inequalities. We can identify which line is which in the graph easily because the third inequality has a positive slope but the fourth has a negative slope. This eliminates (A), because the inequality $y\le x-2$ is not evaluated, not even incorrectly (you can see that the region is shaded both above and below the line $y=x-2$, which is impossible in graphing an inequality), and (E) because the shaded region in (E) goes outside the box. Now we must test points to determine which of the remaining graphs, (B), (C), or (D), is shaded correctly. When we solve $-7x+50=x-2$, we find that the lines intersect at the point $(6.5, 4.5)$. Thus, it can safely be assumed that the region in answer (B) contains the point $(6.5, 8)$. Testing that point in the third inequality given, $y\le x-2$, we find that $8\le 6.5-2$ is false, so the graph must be shaded to the *right* of this positive sloping line, not the left. Now that we know this, we can also eliminate answer choice (D), since that is also shaded to the left. This leaves (C) as our answer.

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29. **A.** $|x-y|<1$, which means $-1<x-y$ and $x-y<1$, so the *boundary* lines of our inequality are $y=x-1$ and $y=x+1$. Because of this we know that our answer is either (A) or (E), as those are the only two choices that include both of those lines. To know whether we shade inside or outside, we can test the origin: $-1>0$ and $0<1$ are true, so we do shade between the lines, making answer (A) correct.

30. **B.** Looking at the graph we see that the upward facing parabola is dotted as it bounded the region and the downward facing parabola is solid. This means that in the system of inequalities the expression with a positive leading term is part of a $\leq$ or $\geq$ equation, but the expression with a negative leading term is part of a $<$ or $>$ equation (it doesn't actually matter *what* the expressions are, because we can easily tell them apart by their sign). This eliminates all answer choices except (C), which must be our answer. Note that if it didn't, we would have had to test out the inequalities with points, and in this case we could have used the origin.

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