INEQUALITIES: CORE

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- Flip the sign when multiplying/dividing by a negative
- How to graph inequalities on a number line

THE BASICS

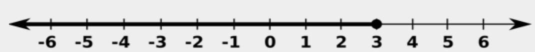
Inequalities are **just like equations**—with one big exception. If you're multiplying or dividing both sides of the inequality by a negative number, you must "flip" the sign to the other direction:

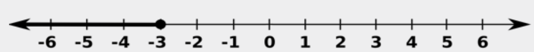
$-3x > 9$	Divide by negative three
$\frac{-3x}{-3} < \frac{9}{-3}$	Flip the sign!
$x < -3$	Simplify

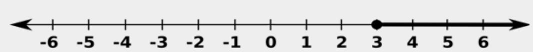
INEQUALITIES ON A NUMBER LINE

You'll need to know how to graph a basic inequality on a number line and properly shade. Remember open circles refer to less than / greater than and closed circles refer to less than or equal to / greater than or equal to.

Which of the following number line graphs shows the solution set of $-6(x+2)+1 \geq 4-x$?

A. 

B. 

C. 

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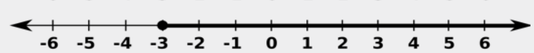
E. 

$-6(x+2)+1 \geq 4-x$	To simplify, distribute the multiplier, -6
$-6x-12+1 \geq 4-x$	Group like terms together
$-6x+x \geq 4+12-1$	Simplify
$-5x \geq 15$	Divide by negative five
$\frac{-5x}{-5} \leq \frac{15}{-5}$	Flip the sign!
$x < -3$	Simplify

Answer: **B.**

1. Which of the following is a solution for the inequality $\frac{9}{5}a+3>\frac{3}{4}a-7$?

A. $a>-\frac{5}{3}$

B. $a>-\frac{210}{20}$

C. $a>-\frac{200}{21}$

D. $a<-\frac{200}{21}$

E. $a<-\frac{210}{20}$

2. Which of the following is equal to the inequality $5n-21<13+2n$?

A. $n<\frac{34}{3}$

B. $n<-\frac{8}{3}$

C. $n>-\frac{8}{3}$

D. $n>\frac{34}{3}$

E. $n<\frac{8}{3}$

3. The inequality $8(y-4)>7(y+2)$ is equivalent to which of the following?

A. $y<46$

B. $y>46$

C. $y>6$

D. $y>18$

E. $y<34$

4. Which of the following is equal to $\frac{4}{2-x}-8>0$?

A. $x>2$

B. $\frac{3}{2}<x<2$

C. $x<\frac{3}{2}$

D. $x>\frac{3}{2}$

E. $x>\frac{3}{2}$ or $x>2$

5. Which of the following is equivalent to $\left(|a|-7\right)^3\geq 8$?

A. $a\geq 7$ or $a\leq -7$

B. $a\geq 8$ or $a\leq -8$

C. $a\geq 15$ or $a\leq -15$

D. $a\geq 3$ or $a\leq -3$

E. $a\geq 9$ or $a\leq -9$

6. For what values of n is $\frac{1}{4}n-9>\frac{5}{2}n$?

A. $n>-4$

B. $n<-4$

C. $n<4$

D. $n<36$

E. $n>46$

7. What is the smallest integer value x that satisfies the inequality $\frac{x}{20}>\frac{13}{23}$?

A. 10

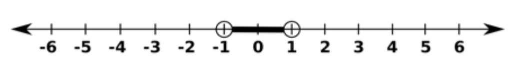
B. 11

C. 12

D. 13

E. 14

8. Which of the following graphs shows the solution set for the inequality $7x+2>9$?

A.


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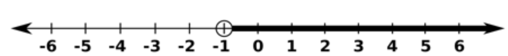

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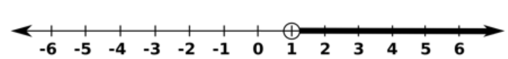

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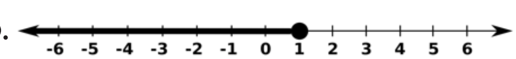

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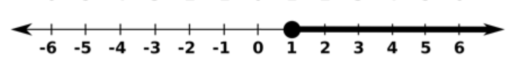
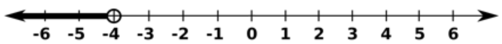
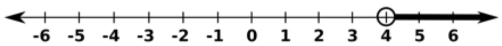
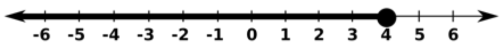
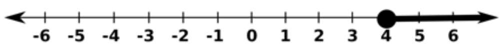


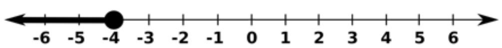
9. Which of the following is the graph of the solution set for the inequality $11-\frac{x}{2}\le 9$?

A. 

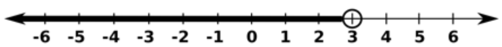
B. 

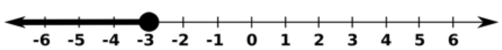
C. 

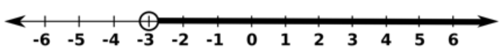
D. 

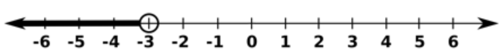
E. 

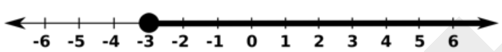
10. When 3 times x is increased by 13, the result is less than 4. Which of the following is a graph of the real numbers x that satisfy this relationship?

A. 

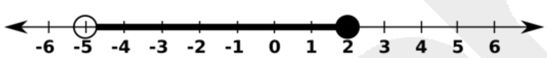
B. 

C. 

D. 

E. 

11. Which of the following inequalities represents the graph shown below on the real number line?



A. $-5\le x\le 2$
B. $-5<x<2$
C. $-5<x\le 2$
D. $x<-5$ and $x\ge 2$
E. $x\le -5$ and $x>2$

12. The number lined graphed below is the graph of which of the following inequalities?

A. $x\ge 0$ and $x\le 4$
B. $x>4$ and $x<0$
C. $x>0$ or $x<4$
D. $x<0$ or $x>4$
E. $x\le 0$ or $x\ge 4$

13. Which of the following graphs illustrates the solution set for the system of inequalities $3x-25\ge -15$ and $-4x+10<-10$?

A. 

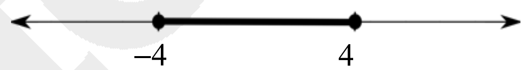
B. 

C. 

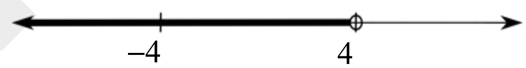
D. 

E. 

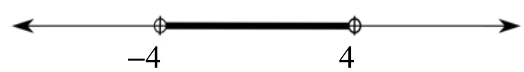
14. Which of the following number line graphs shows the solution set for x of $x^2<16$?

A. 

B. 

C. 

D. 

E. 

15. Given real numbers $a, b, c, d,$ and e such that $b<a$, $e<d$, $a<c$, and $d<b$, which of these numbers is the least?

A. a
B. b
C. c
D. d
E. e

ANSWER KEY

1. C **2. A** **3. B** **4. B** **5. E** **6. B** **7. C** **8. C** **9. D** **10. D** **11. C** **12. E** **13. D** **14. E** **15. E**

ANSWER EXPLANATIONS

1. **C.** Adding 7 on both sides, we get $\frac{9}{5}a+10>\frac{3}{4}a.$ Subtracting $\frac{9}{5}a$ on both sides, we get $10>\frac{3}{4}a-\frac{9}{5}a \to 10>\frac{3}{4}\left(\frac{5}{5}\right)a-\frac{9}{5}\left(\frac{4}{4}\right)a \to 10>\frac{15}{20}a-\frac{36}{20}a \to 10>-\frac{21}{20}a.$ Now, dividing both sides by $-\frac{21}{20},$ we get $10\left(-\frac{20}{21}\right)<a \to -\frac{200}{21}<a$ or $a>-\frac{200}{21}.$

2. **A.** Adding 21 on both sides, we get $5n<34+2n.$ Subtracting $2n$ on both sides, we get $3n<34.$ Dividing by 3 on both sides, we get our answer $n<\frac{34}{3}.$

3. **B.** Distributing the constants on both sides of the inequality, we get $8y-8(4)>7y+7(2) \to 8y-32>7y+14.$ Adding 32 on both sides, we get $8y>7y+46.$ Subtracting both sides by $7y$, we get $y>46.$

4. **B.** Adding 8 on both sides, we get $\frac{4}{2-x}>8.$ The denominator is an expression that could be either negative or positive, depending on the value of $2-x$. Thus, when we multiply by $2-x$ on either sides, we get $4<8(2-x)$ if $x<2$ (which would make the expression positive) and $4>8(2-x)$ if $x>2$ (which would make the expression negative, and this requires the sign to change directions). Distributing the 8 on the right side, we get $4<16-8x$ or $4>16-8x.$ Subtracting 16 on both sides, we get $-12<-8x$ or $-12>-8x$. Lastly, dividing each side by -8 gives us $\frac{3}{2}<x$ when $x<2$, or $\frac{3}{2}>x$ when $x>2$. The second statement gives us no solution because no number can be simultaneously less than $\frac{3}{2}$ and greater than 2. Thus, our answer is when $\frac{3}{2}<x<2.$

5. **E.** Taking the cube root of both sides of the equation, we get $|a|-7\ge 2$. Adding 7 on both sides gives us $|a|\ge 9.$ This means $a\ge 9$ or $a\le -9$.

6. **B.** Subtracting $\frac{1}{4}n$ on both sides, we get $-9>\frac{5}{2}n-\frac{1}{4}n \to -9>\frac{9}{4}n.$ Dividing both sides by $\frac{9}{4}$, we get $-9\left(\frac{4}{9}\right)>n \to -4>n$ or $n<-4.$

7. **C.** Rewriting the fractions with a common denominator, we get $\frac{x}{20}\left(\frac{23}{23}\right)>\frac{13}{23}\left(\frac{20}{20}\right) \to \frac{23x}{460}>\frac{260}{460}$, which simplies to $23x>260$. Dividing both sides by 23, we get $x>11.3$ and the smallest integer value that is greater than 11.3 is 12.

8. **C.** Subtract 2 from both sides and divide by 7 to get $x>1$. x is greater than, not greater than or equal to, so the circle is open or unfilled, since $x\ne 1$.

9. **D.** Subtract 11 from both sides and multiply both sides by -2. Remember to switch the direction of the sign because we are multiplying both sides by a negative number. $11-\frac{x}{2}\le 9 \to -\frac{x}{2}\le -2 \to x\ge 4$. Because $x\ge 4$, x can equal 4, so the circle on 4 is filled.

10. D. Translating our inequality into numbers, we have $3x+13<4$. Simplify: $3x+13<4 \to 3x<-9 \to x<-3$. x is less than but *not* equal to 3, so our bubble is empty.

11. C. Looking at the graph, we see that the thick line starts at -5 and ends at 2, so x can be any value in between the two, and that the bubble is unfilled at -5 but filled at 2, meaning that x *cannot* be -5 but *can* be 2. As an inequality, this looks like $-5<x\leq 2$.

12. E. Looking at the graph, we see that the thick line that is our x spans values that are less than 0 and greater than 4 and that the circles at 0 and 4 are filled, which means that x could potentially be 0 or 4. Than means that our x values are less than or equal to 0 *or* greater than or equal 4: $x\leq 0$ or $x\geq 4$. It is logically impossible for a number to be less than or equal to 0 *and* greater than or equal 4, which is why we say *or*.

13. D. First simplify the inequalities given: $3x-25\geq -15 \to x\geq \frac{10}{3}$ and $-4x+10<-10 \to -4x<-20 \to x>5$. The solution of the system is the intersection of the two inequalities: $x\geq \frac{10}{3} \cap x>5$, which simplifies to $x>5$, because anything that is greater than 5 will always be greater than $\frac{10}{3}$, and the solution set must be greater than 5.

14. E. We cannot just take the square root of either side. When we take the square root, it is the same as when we multiply or divide by a negative: $\sqrt{x^2} < \pm\sqrt{16} \to x<\sqrt{16}$ and/or $x>-\sqrt{16}$. Solving both inequalities we see that $x<4$ and $x>-4$, which can also be written as $-4<x<4$, which is represented by the bottommost graph because that graph shows the set of numbers between but not including -4 and 4.

15. E. We cannot relate the first two inequalities given, but the first and the third combined is $b<a<c$. Because $e<d$ and $d<b$, we get $e<d<b<a<c$. Thus, e is the least element.