SAT 39 Circle Theorems Answers and Explanations

1) A

First, we must find the area of the shaded region by using this formula:

$$Area = \frac{1}{2}r^2\theta$$

Where theta is measured in radians. So, we can use simple algebra to find the angle θ.

$$16 = \frac{1}{2}(4)^2\theta$$
$$16 = \frac{16}{2}\theta$$
$$16 = 8\theta$$
$$2\ radians = \theta$$

Now we know that the radian measure of angle ABC is 2 radians. Since 180 degrees is the same as π radians, then angle CBD would be $(\pi - 2)$

2) C

Notice that an isosceles triangle that is also a right triangle will be a special 45-45-90 triangle. We know that the two side lengths will have the length of x and the hypotenuse will be $x\sqrt{2}$. So we can find the length of XY by finding out the side lengths from the perimeter.

$$6 + 3\sqrt{2} = 2x + x\sqrt{2}$$

The structure of the perimeter shows that x, which also corresponds to the radius of the circle, is 3. Now we can use the following arclength equation

$$s = r\theta$$

Plugging in $r = 3$ and $\theta = \frac{\pi}{2}$, we get $s = \frac{3\pi}{2}$, which is the length of arc XY.

3) D

First, we must find the radius of the outer circle as the radius is also the side length of a 45-45-90 triangle created when dividing the square as shown in the diagram. We find the radius by using algebra from the known area of the circle

$$64\pi = \pi r^2$$
$$64 = r^2$$
$$8 = r$$

If the radius of the circle is 8 feet, then that means the hypotenuse (the square's side length) is $8\sqrt{2}$ due to the special properties of a 45-45-90 triangle. And to find the perimeter we just add up all the sides of the square.

$$4\left(8\sqrt{2}\right) = 32\sqrt{2}\ feet$$

4) C

First, notice that triangle ABC is 3-4-5 triangle. That means that AC has to have a measurement of 3 since we know BC is half the length of BD which has a length of 8 meters. Next, we know that the radius of the circle is 15 meters, meaning the length of CO is 15-3=12 meters

5) C

The formula for arc length is just the product of the measure of the central angle in radians by the radius. In this case, we can use algebra to find out what the radians are.

$$24 = 6\theta$$
$$4 = \theta$$

The measure of the central angle is 4 radians

6) B

First, we must find the length of the radius. We can use algebra to find this out.

$$18 = \frac{1}{2}r^2(1)$$
$$36 = r^2$$
$$6 = r$$

To find chord XZ, we can use the Pythagorean theorem to find out the side length of the congruent triangles. We know that the hypotenuse is 5 feet and since OY is 4 feet in length, this means YW is 2 feet, since the radius equals 6 feet.

So now we use the Pythagorean theorem to find the length of XY and YZ.

$$a^2 + 2^2 = 5^2$$
$$a^2 + 4 = 25$$
$$a = \sqrt{21}$$

Now the total length of XZ will be $2\sqrt{21}$ feet

7) B

First, let's find the radius of this circle by using algebra

$$64\pi = \pi r^2$$
$$64 = r^2$$
$$8 = r$$

Now, we know that arc length is the radius multiplied by the angle in radians. In this case, angle BOC that creates arc BDC is a right angle, meaning that the it has a measure of $\frac{\pi}{2}$ radians.

$$\frac{\pi}{2} \times 8 = 4\pi$$

8) C

First, we know that the radius of the larger circle is 10 feet as the diameter would be 20 making the circumference 20π feet. Notice that the radius of 10 feet is also the length of BC, meaning that the triangle of BEC will have a hypotenuse of 10 feet and the side length EC of 8. We use the Pythagorean theorem to find the length of BE.

$$BE^2 + 8^2 = 10^2$$
$$BE^2 + 64 = 100$$
$$BE^2 = 36$$
$$BE = 6$$

If BE is 6 feet and EA is 4 feet (since it is the mid-point of EC) we can conclude that the area is:

$$A = \frac{1}{2}(6)(4)$$
$$A = 12\ ft^2$$

9) D

The formula for an arc length is the product of the central angle in radians by the radius. If the diameter is 50 feet, the radius must be 25 feet. We multiply this by 6 radians to get a total arc length of 150 feet.

10) B

Instead of finding the area of the shape, we can find the area of the missing piece of the circle and then subtract that from the overall area of the entire circle. If the radius is 6 inches, we know that the area of the entire circle will be as follows.

$$A = \pi(6)^2$$
$$A = 36\pi$$

Next, we must find the area of the missing sector. The first step is to convert 100 degrees into radians.

$$\frac{x}{100} = \frac{2\pi}{360}$$
$$360x = 200\pi$$
$$x = \frac{5\pi}{9}$$

Now we find the area of the sector.

$$A = \frac{1}{2}(6)^2\frac{5\pi}{9}$$
$$A = \frac{90\pi}{9}$$
$$A = 10\pi$$

Subtract the area of the sector from the area of the entire circle to get the area of the shape.

$$36\pi - 10\pi = 26\pi\ in^2$$