CHAPTER

21 LAW OF SINES AND COSINES

SKILLS TO KNOW

- Law of Sines
- Law of Cosines

The ACT® sometimes incorporates questions that require you to understand how to apply the Law of Sines and Law of Cosines. Though historically, the ACT® nearly always published these equations as part of the question itself whenever you needed to apply them, recent tests have diverged from this trend. In other words, ACT® has asked questions solvable with these rules that did not include the formulas for reference. I recommend that students **aiming for a 34+ on the math section <u>memorize</u> these laws (or have a calculator program that peforms them and adheres to ACT's® strict rules).** Students aiming for a 30+ should simply know how to use them if given the equation. Students aiming for below a 30 can skip this chapter until other areas are mastered.

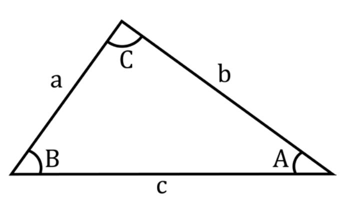
Given the calculator program rule, **you could potentially program a short program into your calculator that performs each of these functions**, as they are similar to the Quadratic Equation (the example of what program is allowed on the ACT®). I.e. there are a few variables and you must know how to plug in to solve for what you need. We offer a free pdf download on programming your calculator that you can download at **supertutortv.com/bookowners**. If you're interested in this option, check out our information there for more tips.

Why do we need these laws?

We use SOHCAHTOA to solve for missing pieces of right triangles, but what about triangles that have no 90 degree angles? That's when the Law of Sines and Cosines come in.

If you have enough pieces of a triangle, namely ASA, SAS, SSA, or SSS, you can solve for every angle and side length in the triangle using these formulas. It can be confusing to know which one to use when, but with practice that part gets easier.

LAW OF SINES

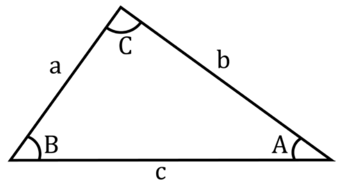


Law of Sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

This rule states that the length of a side over the opposite angle equals another side over that respective opposite angle.

We use the Law of Sines when we know two sides and one of the corresponding angles (**SSA**) or two angles and one of the corresponding sides (**ASA** or **AAS**). Feel free to come up with some silly way to remember all this.

LAW OF COSINES

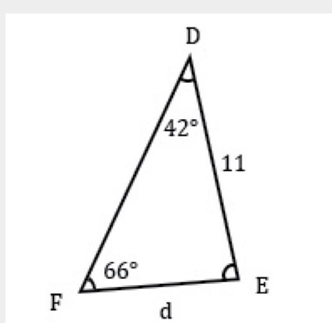


$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

As you can see, Law of Cosines has three "versions"—but in fact they all are essentially the same idea. The side to the left of the equal sign corresponds to the angle we take the cos of on the right. Note that if the ACT® gives you the Law of Cosines for reference, **it will NOT give you all three versions**. Even more confusing, it may reuse letters that are arbitrarily already on your diagram, so don't rely on the "letters" as much as the concept. The biggest mistake students make is that they assign the longest side arbitrarily to C because they mix up this formula with the Pythagorean theorem, and think C must always be the long side. That's incorrect. With the Law of Cosines we know three sides and one angle by the end of the calculation, and again the variable to the left of the equals sign is always representative of the side opposite the angle in question (whether we know the angle or are solving for it.)

We use Law of Cosines when we know the lengths of three sides (**SSS**) and need an angle (the side opposite the angle goes to the left of the equal sign in all versions of the equation above), or we know the length of two sides and the angle that joins them (**SAS**) and need the third side (in this case, the missing side is opposite the "A" angle so that is what goes to the left of the equal sign).

What is the approximate length of d in triangle DEF ?



A. 6 **B.** 7 **C.** 7.4 **D.** 8 **E.** 8.7

We know the angle of F, which is $66°$, the angle of D, which is $42°$, and side $\overline{DE}$, which is 11. This gives us AAS. Anytime we know **two angles** we are always using the Law of Sines not the Law of Cosines.

Using the Law of Sines we set up the proportion:

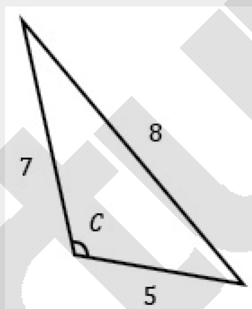
$$\frac{d}{\sin 42°} = \frac{11}{\sin 66°}$$
$$d = 8.057$$

Then we multiply both sides by $\sin 42°$ to solve for d:

$$d = \left(\frac{11}{\sin 66°}\right) * \left(\sin 42°\right)$$

Answer: **D**.

Using the diagram below, find the approximate angle C.



A. 78.2° **B.** 81.8° **C.** 88.1° **D.** 96.2° **E.** 112.7°

Using the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\left(C\right)$ we can plug in the numbers into the equation. Again, I am very careful to place the 8 on the LEFT side of the equation, as it is the side opposite the angle I need. **The corresponding side opposite the angle in question always goes on the left by itself.** This gives us:

$$8^2 = 5^2 + 7^2 - 2\left(5\right)\left(7\right)\cos\left(C\right)$$
$$64 = 25 + 49 - 70\cos\left(C\right)$$
$$-10 = -70\cos\left(C\right)$$

After combining like terms, 64, 25, and 49, by subtracting 25 and 49 from both sides, divide both sides by -70:

$$\frac{1}{7} = \cos\left(C\right)$$

Now it's time to get out your calculator and calculate $\cos^{-1}$ (the inverse of $\cos$) of $\frac{1}{7}$. Be sure your calculator is in degree mode!

We get $C \approx 81.787°$.

Answer: **B**.