EXPONENTS AND RADICALS

ACT Math: Problem Set

1. If $3^4\left(81^2\right)=3^n$, what is n?

A. 6

B. $\frac{1}{6}$

C. $\frac{1}{12}$

D. 12

E. 2

2. For what value(s) of x does $\sqrt[3]{x^2-12x}=4$?

A. 16 and -4

B. 16 and 4

C. -16 and -4

D. -16 and 4

E. 16

3. What real value of k satisfies the equation $25^{2k}=\frac{1}{625^{k-1}}$?

A. 1

B. 2

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 4

4. For all nonzero x and y, $\frac{\left(32x^4y^{-5}\right)\left(-3x^2y^3\right)}{8x^5y^{-6}}=$?

A. $12xy^4$

B. $12xy^2$

C. $-12xy^4$

D. $12xy^{-4}$

E. $-12xy^{-4}$

5. $-\left(-ab\right)^4\left(a^3b^4\right)^5$ is equal to?

A. $a^{19}b^{24}$

B. $-a^{19}b^{24}$

C. $-a^7b^8$

D. $-a^{60}b^{80}$

E. $-a^7b^9$

6. When c and d are nonzero values, $\left(\frac{c^4d}{d^{-3}}\right)\frac{c^{-2}d^{-5}}{cd}$ is equivalent to?

A. $\frac{c}{d^2}$

B. c^5d^2

C. $\frac{c}{d^8}$

D. $\frac{c^5}{d^8}$

E. cd

7. Which of the following is equal to $\frac{1}{100^{100}}-\frac{1}{100^{99}}$?

A. $\frac{99}{100^{100}}$

B. $\frac{99}{100^{99}}$

C. $\frac{-99}{100^{100}}$

D. $\frac{-99}{100^{99}}$

E. $\frac{1}{100^{200}}$

8. Given that $x=3-y$, what is $\left(x+y\right)^5$?

A. 243

B. -243

C. 3

D. 81

E. -81

9. If $-x^t>\left(-x\right)^s$ and x,t,s are all positive integers, which of the following must be true?

A. s is even

B. $t>s$ and s is even

C. $t<s$ and s is even

D. $t<s$ and s is odd

E. $t>s$ and s is odd

10. If $a=\sqrt[3]{b}$ and $c=b^5$, what is c in terms of a?

A. $a^{\frac{5}{3}}$
B. $a^{\frac{3}{5}}$
C. a^3
D. a^{15}
E. a^5

11. If a and b are nonzero numbers, $\sqrt{\frac{b}{a}}-\sqrt{\frac{a}{b}}$ is equal to which of the following?

A. $\sqrt{\frac{b-a}{a-b}}$
B. $\frac{b-a}{\sqrt{ab}}$
C. x and $1s$
D. $\frac{b-a}{ab}$
E. $\sqrt{\frac{b^2-a^2}{ab}}$

12. Which of the following is equivalent to $\sqrt[4]{16b^{12}}$?

A. $4b^3$
B. $2b^2$
C. $2b^3$
D. $4b^2$
E. $16b^3$

13. If $2\sqrt{r}+\sqrt{s}=5\sqrt{s}$, what is r in terms of s?

A. $2s$
B. $2\sqrt{s}$
C. $4\sqrt{s}$
D. $4s$
E. $2s^2$

14. If for all x, $(x^{3a+2})^2=x^3$, then $a=$?

A. -1
B. $\pm\frac{\sqrt{3}-2}{3}$
C. $-\frac{1}{3}$
D. 1
E. $-\frac{1}{6}$

15. For what real number value of n is the equation $\left(x^{1/2}\right)^4\left(x^{-1}\right)^6=x^{n/2}$ true?

A. -24
B. -8
C. -4
D. 16
E. 19

16. If $\frac{x^{a/2}}{x^{b/2}}=x^6$ for all $x\neq 0$, which of the following must be true?

A. $a-b=12$
B. $\frac{a}{b}=6$
C. $a-b=3$
D. $\frac{ab}{4}=6$
E. $a-b=6$

17. $\frac{1}{2}x^2y^2*4x*6xy^3$ is equivalent to:

A. $12x^4y^5$
B. $12x^2y^6$
C. $12x^2y^3$
D. $\frac{21}{2}x^2y^6$
E. $\frac{21}{2}x^6y^5$

18. Which of the following is equivalent to $\left(5x^4\right)^3$?

A. $5x^{12}$
B. $15x^7$
C. $15x^{12}$
D. $125x^7$
E. $125x^{12}$

19. For all real values of x, which of the following expressions is equal to $x^n*x^n*x^n*x^n*x^n$?

A. x^{n^5}
B. $5x^n$
C. $n(x^5)$
D. x^{5n}
E. $(5x)^n$

20. What is the value of $f(x)=16^{1/x}-6$ when $x=4$?

A. 65530
B. -2
C. 2
D. 4
E. -4

21. For any non-value of y, $\left(y^{-2}\right)^{1/5}=$?

A. $\frac{1}{y^{9/5}}$
B. $y^{11/5}$
C. $\frac{1}{y^{2/5}}$
D. $y^{2/5}$
E. $\frac{1}{y^{10}}$

22. For all real x, $\left[\frac{27(x+1)^2}{3}\right]^{1/2}=$?

A. $81(x+1)$
B. $9x^2+18x+9$
C. $x+1$
D. $3(x+1)$
E. $9x^2-18x+9$

23. For all $a\neq 0$, $\frac{\left(3a^{5/2}\right)^2+5\left(a^4\right)^2+4a^8}{3a^2}=$?

A. $3\left(a^3+a^6\right)$
B. a^3+a^6
C. a^3+3a^6
D. $3a^{5/2}+5a^4+4a^6$
E. $3a^{5/2}+\frac{5}{3}a^4+\frac{4}{3}a^6$

24. Which of the following is equivalent to $\left(-2x^3y^7\right)^3$?

A. $8x^9y^{21}$
B. $-8x^9y^{21}$
C. $8x^6y^{10}$
D. $6x^9y^{21}$
E. $-6x^6y^{10}$

25. If x and y are real and $\sqrt{4\frac{x^3}{y}}=2$, then what must be true?

A. x and y must both be positive
B. x and y must both be negative
C. $x^3=y$
D. $x=y^3$
E. x and y must be the same number

26. A certain perfect square has exactly 5 digits (that is, it is an integer between 10,000 and 99,999). What is the maximum number of digits the positive square root of that perfect square could have?

A. 2
B. 3
C. 4
D. 316
E. 317

27. Given $13^{\frac{(x+1)^2}{(x^2-1)}}=1$, $x=$?

A. 1
B. -1
C. 1 and -1
D. 0
E. $\frac{1}{2}$

28. For all positive values of a and b, which of the following expressions is equivalent to $a^2\sqrt[3]{b^7}*b^4\sqrt[6]{a^8}$?

A. $a^3b^5*\sqrt[6]{ab}$
B. $a^3b^6*\sqrt[3]{ab}$
C. $a^6b^3*\sqrt[6]{ab^2}$
D. $a^2b^4*\sqrt[6]{ab}$
E. $a^2b^4*\sqrt[3]{ab}$

29. For how many integers x is the equation $8^{x-1}=2^{x^2-1}$ true?

A. 0
B. 1
C. 2
D. 4
E. 5

30. What is the value of the expression $\sqrt[3]{\frac{3m}{2n+2}}$ if $m=6$ and $n=-4$?

A. $i\sqrt[3]{3}$

B. $-\sqrt{3}$

C. $i\sqrt{3}$

D. $\sqrt[3]{3}$

E. $-\sqrt[3]{3}$

31. The square root of a certain number is approximately 6.324555. The certain number is between what two integers?

A. 2 and 3
B. 6 and 7
C. 25 and 37
D. 35 and 50
E. 63 and 82

32. For nonzero values of x and y, which of the following expressions is equivalent to $-\frac{35x^5y}{7x^3y^4}$?

A. $-28x^2y^{-3}$

B. $-42x^2y^{-3}$

C. $-5x^8y^4$

D. $-5x^2y^{-3}$

E. $-5x^2y^3$

33. What is the positive solution to the equation $14c^2=25$?

A. $\frac{25}{14}$

B. $\sqrt{\frac{14}{25}}$

C. $\sqrt{\frac{25}{14}}$

D. $\frac{30\sqrt{14}}{14}$

E. $\left(\frac{25}{14}\right)^2$

ANSWER KEY

1. D 2. A 3. D 4. C 5. B 6. A 7. C 8. A 9. D 10. D 11. B 12. C 13. D 14. E
15. B 16. A 17. A 18. E 19. D 20. E 21. C 22. D 23. A 24. B 25. C 26. B 27. B 28. B
29. C 30. E 31. D 32. D 33. C

ANSWER EXPLANATIONS

1. D. $81 = 3^4$, so $3^4(81)^2 = 3^4(3^4)^2 \to 3^4(3^8) \to 3^{12}$. So, if $3^4(81)^2 = 3^n$, then $n = 12$.

2. A. We cube both sides to get $\sqrt{x^2 - 12x} = 64$, and then square both sides to get $x^2 - 12x = 4096$. Subtracting 4096 on both sides gives us $x^2 - 12x - 4096 = 0$. We factor to find that $x = -16$ and 4.

3. D. Multiplying 625^{k-1} on both sides, we get $25^{2k}(625)^{k-1} = 1$. Since $625 = 25^2$, we can rewrite this as $25^{2k}(25^2)^{k-1} = 1 \to 25^{2k}(25^{2k-2}) = 1 \to 25^{2k+2k-2} = 1 \to 25^{4k-2} = 1$. The only way for a number other than 1 raised to a power to equal 1 is for the exponent to equal 0, as $25^0 = 1$. Thus, $4k - 2 = 0$, which once solved tells us that $k = \frac{2}{4} \to \frac{1}{2}$.

4. C. Combining the like terms in the numerator, we get $\frac{32(-3)x^4x^2y^{-5}y^3}{8x^5y^{-6}} = \frac{-96x^6y^{-2}}{8x^5y^{-6}}$. Now, canceling out like terms in the numerator and denominator, we get $-12xy^4$.

5. B. Distributing out the exponents, we get $-(a^4b^4)(a^{15}b^{20})$. Combining like terms, we get $-a^{19}b^{24}$.

6. A. Combining like terms in the numerator and denominator, we get $\frac{c^2d^{-4}}{d^{-2}c}$. Canceling out like terms in the numerator and denominator, we get $cd^{-2} = \frac{c}{d^2}$.

7. C. We can rewrite $\frac{1}{100^{99}}$ as $\frac{100^1}{100^{100}}$, which allows us to rewrite the expression as $\frac{1}{100^{100}} - \frac{100}{100^{100}}$. This simplifies into $-\frac{99}{100^{100}}$ which is one of the answers provided.

8. A. Adding y to both sides of the equation $x = 3 - y$, we get $x + y = 3$. Substituting in $x + y = 3$ to $(x+y)^5$, we get $3^5 = 243$.

9. D. We should first acknowledge that $-x^t$ will have a negative value no matter what because the negative sign is not raised to the power of t, so it cannot be canceled out when t is even. Thus, in order for $-x^t > (-x)^s$, the expression $(-x)^s$ must also be negative. Since the negative sign is raised to the power of s in $(-x)^s$, s must be odd in order for $(-x)^s$ to be negative. This eliminates answer choices A, B, and C. Lastly, if $t > s$ then $-x^t$ will have a greater magnitude and therefore be more negative than $(-x)^s$, yielding $-x^t < (-x)^s$. So, $t < s$ and s must be odd.

10. D. We want to write b in terms of a and then write c in terms of a using the given that $c = b^5$. To write b in terms of a cube both sides of the equation $a = \sqrt[3]{b}$, which tells us that $a^3 = b$. Substituting this value of b into $c = b^5$, we get $c = (a^3)^5 = a^{15}$.

11. B. Rewriting the fractions to have a common denominator, we get $\sqrt{\frac{b}{a}}\sqrt{\frac{b}{b}}-\sqrt{\frac{a}{b}}\sqrt{\frac{a}{a}}=\sqrt{\frac{b^2}{ab}}-\sqrt{\frac{a^2}{ab}}\rightarrow\frac{b}{\sqrt{ab}}-\frac{a}{\sqrt{ab}}$.

Combining the numerators, we get $\frac{b-a}{\sqrt{ab}}$.

12. C. We recognize that $16=2^4$. So, we can simplify the expression to be $\sqrt[4]{2^4\left(b^3\right)^4}=\sqrt[4]{\left(2b^3\right)^4}\rightarrow 2b^3$.

13. D. Subtracting $\sqrt{s}$ on both sides, we get $2\sqrt{r}=4\sqrt{s}$. Squaring both sides, we get $4r=16s$. Now, dividing both sides by 4, we get $r=4s$.

14. E. Given $(x^{3a+2})^2=x^3$ distribute the power of 2 to get $x^{6a+4}=x^3$. Because the base is the same on either side, the exponent expressions are equal, which means $6a+4=3$. Solve to find that $a=-\frac{1}{6}$.

15. B. Given a power to a power, multiply the exponents to simplify $\left(x^{1/2}\right)^4\left(x^{-1}\right)^6$ into $(x^{4/2})\left(x^{-6}\right)$, and then add the exponents of the terms with like bases: $(x^{4/2})\left(x^{-6}\right)\rightarrow x^{2-6}\rightarrow x^{-4}$. Stopping there would lead to answer (C), which is incorrect. Going back to the original equation, $x^{-4}=x^{\frac{n}{2}}$, and since the two expressions have the same base their exponents are equal: $-4=\frac{n}{2}$. Solving this tells us that $n=-8$, answer (B).

16. A. We simplify the fraction by subtracting the exponents. $\frac{x^{a/2}}{x^{b/2}}=x^6$ becomes $x^{\frac{a}{2}-\frac{b}{2}}=x^6$, and because the bases are the same, the exponents are equal: $\frac{a}{2}-\frac{b}{2}=6$. This can be rearranged due to their common denominator, $\frac{a-b}{2}=6$, and simplified further becomes $a-b=12$, which is answer (A).

17. A. We combine like terms. $\left(\frac{1}{2}*4*6\right)\left(x^2*x^1*x^1\right)\left(y^2*y^3\right)\rightarrow 12\left(x^{2+1+1}\right)\left(y^{2+3}\right)\rightarrow 12x^6y^5$, answer (A).

18. E. Because the two terms are implicitly (meaning there is no multiplication sign to tell you it's multiplication) multiplied inside the parenthesis, the exponent outside the parenthesis is applied to both terms. $\left(5x^4\right)^3=5^3x^{4\cdot 3}=125x^{12}$.

19. D. We combine same base terms by adding the exponents. $x^n*x^n*x^n*x^n*x^n=x^{n+n+n+n+n}\rightarrow x^{5n}$, answer (D).

20. E. Like any function, we can simply plug in the given value for the variable.
$16^{1/4}-6=\sqrt[4]{16}-6\rightarrow 2-6\rightarrow -4$. We get -4, which is answer (E).

21. C. We simplify the exponents to rewrite the expression: $\left(y^{-2}\right)^{1/5}\rightarrow y^{-2\cdot 1/5}\rightarrow y^{-2/5}\rightarrow\frac{1}{y^{2/5}}$, answer (C).

22. D. Before trying to take the square root of anything, we should simplify whatever possible inside the radical, and then distribute the root.

$$\left[\frac{27\left(x+1\right)^2}{3}\right]^{1/2}=\left[9\left(x+1\right)^2\right]^{1/2}\rightarrow 9^{\frac{1}{2}}\left(x+1\right)^{2\frac{1}{2}}\rightarrow 3\left(x+1\right)^1$$

23. A. We simplify the numerator first.

$$\frac{\left(3a^{5/2}\right)^2+5\left(a^4\right)^2+4a^8}{3a^2}=\frac{3^2\left(a^{\frac{5}{2}2}\right)+5a^8+4a^8}{3a^2}\rightarrow\frac{9a^5+9a^8}{3a^2}$$

Because the denominator is a monomial, we can split up the terms in the numerator, divide by the denominator, and then simplify further.

$$\frac{9a^5+9a^8}{3a^2}\rightarrow\frac{9a^5}{3a^2}+\frac{9a^8}{3a^2}\rightarrow 3a^3+3a^6\rightarrow 4\left(a^3+a^6\right).$$

24. B. Because of the parenthesis, we apply the power of 3 to all terms within the parenthesis.

$$\left(-2x^3y^7\right)^3=(-1)^3 2^3(x^3)^3\left(y^7\right)^3\rightarrow -8x^9y^{21}$$

25. C. We first simplify the problem, although the point is not to solve. $\sqrt{4\frac{x^3}{y}}=2\rightarrow 4\frac{x^3}{y}=4\rightarrow\frac{x^3}{y}=1.$ Looking at this equation, we go through the answers one by one and use process of elimination. Answer (A) and (B) are wrong because they say 'must'. It's true that in order to equal 1, which is positive, x and y must be the same sign, but that also means that they could be either negative or positive, as long as they are the same sign. Answer (C) is correct because it simply rearranges the expression we found, multiplying y to either side. Just to be safe, check answers (D) and (E). Answer (D) confuses the powers, and answer (E) is wrong the same way (A) and (B) are, saying 'must'. x and y *could* be the same number (if they both equaled 1), but they don't have to be.

26. B. This theoretical question is harder than it looks. The question asks for the maximum number of digits a number could have if its square has five digits. If we take the square root of $99{,}999$, the largest five-digit number, we get approximately $316.22618.$ This means that 316^2 is less than $99{,}999$ but 317^2 is greater than $99{,}999.$ If 316 is the maximum number that can yield a square with 5 digits, and 316 has 3 digits, then 3 is the maximum number of digits that a number can have and still have a square of five digits or less, which is answer (B). Answers (A) and (C) might tempt a student who is used to finding the maximum and going either one under, or one over. Answers (D) and (E) are there for the students who stop early and forget that they are looking for the digits, not the actual maximum possible number.

27. B. The only way for a number other than 1 to equal 1 is it is raised to an exponent is if that exponent equals 0. Thus, we can set up the equation: $\frac{\left(x+1\right)^2}{\left(x^2-1\right)}=0.$ If we factor, we see that $\frac{\left(x+1\right)\left(x+1\right)}{\left(x+1\right)\left(x-1\right)}=0$, so $x=-1.$

28. B. Looking at the answer choices we see that the point is not to simplify the expression *all* the way, but rather to combine the terms of different bases under the same radical. First, simplify the radicals by pulling out whole integers: $a^2\sqrt[3]{b^7}\cdot b^4\sqrt[6]{a^8} \to a^2\sqrt[3]{b^3*b^3*b^1}*b^4\sqrt[6]{a^6*a^2} \to a^2b^2\sqrt[3]{b}*b^4a^1\sqrt[6]{a^2}$. Notice that $\sqrt[6]{a^2}$, which is the same as, $a^{\frac{2}{6}}$, can therefore be rewritten as $a^{\frac{1}{3}}$, which is $\sqrt[3]{a}$. Thus, our expression is now: $a^2b^2\sqrt[3]{b}\cdot b^4a^1\sqrt[3]{a}$. Combine like terms to get: $a^3b^6\sqrt[3]{ab}$.

29. C. We want the bases to be the same so that the exponents can be set equal. 8 can be rewritten as 2^3, so our expression becomes $(2^3)^{x-1}=2^{x^2-1}\to 2^{3x-3}=2^{x^2-1}$. Because our bases are the same, we know that our exponents are equal, so we can set up the new equation: $3x-3=x^2-1$. Transfer everything to one side so that the sum of the terms equals zero and factor: $x^2-3x+2=0\to(x-1)(x-2)=0$. There are two solutions to this equation. We can plug 1 and 2 back into the equation if you like to confirm.

30. E. First we plug in the values given and simplify: $\sqrt[3]{\frac{3(6)}{2(-4)+2}}\to\sqrt[3]{\frac{18}{-6}}\to\sqrt[3]{-3}\to\sqrt[3]{-1\cdot 3}\to -1\sqrt[3]{3}$.

31. D. Because the square root of this certain number is greater than 6 but less than 7, the number must be greater than 6^2 and less than 7^2: $6.324555^2=x$, the number we want to find, and $6^2<6.324555^2<7^2\to 36<6.324555^2<49$. If a number is less than 49, it is also less than 50, and if a number is greater than 36 is it also greater than 35, making answer D correct.

32. D. We can cancel out $7x^3y$ from the numerator and denominator of the fraction to get $-\frac{5x^2}{y^3}$. Since $\frac{1}{y^3}=y^{-3}$, the fraction can be rewritten as $-5x^2y^{-3}$.

33. C. Dividing both sides by 14, we get $c^2=\frac{25}{14}$. Now, taking the square root of both sides gives us $\pm\sqrt{\frac{25}{14}}$. The question asks for the positive solution, so it is $\sqrt{\frac{25}{14}}$.