

## Heart of Algebra Overview

### Linear Equation Word Problems

1. At 7:00 AM, Justin had 30 cups of lemonade at his lemonade stand. At 8:00 AM, Justin begins to sell lemonade at a constant rate of 5 cups per hour. At what time will Justin run out of lemonade?
2. At 7:00 AM, Justin had 30 cups of lemonade at his lemonade stand. At 7:00 AM, Justin begins to sell lemonade at a constant rate of 5 cups per hour. At this rate, if  $h$  represents the number of hours passed since 7:00 AM, which of the following functions  $l$  gives the number of cups of lemonade Justin currently has?

A)  $l(h) = 30h - 5$

B)  $l(h) = 30 - 5(h - 1)$

C)  $l(h) = 30 + 5h$

D)  $l(h) = 30 - 5h$

3. At 7:00 AM, Justin had 30 cups of lemonade at his lemonade stand. At 8:00 AM, Justin begins to sell lemonade at a constant rate of 5 cups per hour. At this rate, if  $h$  represents the number of hours passed since 7:00 AM, which of the following functions  $l$  gives the number of cups of lemonade Justin currently has?

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D)  $l(h) = 30 - 5h$

4. At 7:00 AM, Justin had 30 cups of lemonade at his lemonade stand. At 8:00 AM, Justin begins to sell lemonade at a constant rate of 5 cups per hour. How many cups of lemonade will Justin have left at 11:00 AM?

### Linear Function Word Problems

5. Frank is a piano player practicing for a concert. He spends the first 10 minutes of his practice playing slowly, at a rate of 10 notes per minute. He then increases his speed to 30 notes per minute for the rest of his practice. Which of the following represents  $N$ , the number of notes Frank plays if he practices for  $y$  minutes, where  $y > 10$ ?

A)  $N = 100 + 30(y - 10)$

B)  $N = 40y$

C)  $N = 100 + 30y$

D)  $N = 30(10y)$

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6. A chef uses the equation  $f(a) = 30a + 10$  to determine the amount of cupcakes,  $f(a)$ , that he can make with  $a$  bags of flour. If the chef wants to make 100 cupcakes for a birthday party, how many bags of flour should he buy?

### Systems of Equations Word Problems

7. Schubert bought 2 sodas and 3 cotton candies at the fair. The sodas came at a 50% discount. The sum of the original prices of the sodas and the cotton candies before discount was \$6.75, and the total cost of the soda and cotton candies after the discount was \$5.25. What was the price, in dollars, of one soda before the discount?

### Systems of Inequalities Word Problems

8. Every day, Alex reads *David Copperfield* at a rate of 12 pages per hour and *Oliver Twist* at a rate of 20 pages per hour. His goal is to read at least 10 pages in total in no more than half an hour. If  $g$  represents the number of pages of *David Copperfield* he has read and  $f$  represents the number of pages of *Oliver Twist* he has read, which of the following systems of inequalities represents Alex's goal?

A)  $\frac{g}{12} + \frac{f}{20} \leq \frac{1}{2}$   
 $g + f \leq 10$

B)  $\frac{g}{12} + \frac{f}{20} \leq \frac{1}{2}$   
 $g + f \geq 10$

C)  $12g + 20f \leq \frac{1}{2}$   
 $g + f \geq 10$

D)  $\frac{g}{12} + \frac{f}{20} \geq \frac{1}{2}$   
 $g + f \leq 10$

### Solving Linear Equations

9. What is the solution to the equation shown below?

$$\frac{2}{3} \left( \frac{g}{20} - 5 \right) = \frac{1}{2}g + \frac{3}{15}g$$

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10. What is the solution to the equation shown below?

$$-21x = -7(3x + 5)$$

## Solving Systems of Equations

11. What is the solution  $(a, b)$  to the system of equations shown below?

$$3a + \frac{9}{2} = 2b$$

$$-12a + 3b = 3$$

12. How many solutions  $(x, y)$  are there in the following system of equations?

$$4x + 16y = 39$$

$$-\frac{8}{3}(x - y) = 13 - 4x - \frac{8}{3}y$$

- A) One
- B) Infinitely Many Solutions
- C) No Solution
- D) Two

13. In the system of equations shown below,  $m$  and  $n$  are both constants. If the equation has no solution, what is the value of  $n$ , assuming  $m \neq 30$ ?

$$5x + 7y = 10$$

$$15x + ny = m$$

## Interpreting Linear Functions

14. Frank's Pizza Joint released an advertisement program promoting their pizza a few months ago. The number of pizzas sold per week,  $P$ , sold  $d$  months after the start of the advertisement program can be estimated by the equation  $P = 18 + 3d$ . What does the number 18 represent in this equation?

- A) The increase in pizzas sold per week for each month after the start of the program.
- B) The total number of pizzas sold per week at the end of the program.
- C) The number of pizzas sold per week at the start of the program.
- D) The number of pizzas sold per week 6 months after the program started.

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### Graphing Linear Functions



The graph of line  $l$  is shown in the  $xy$ -plane above. Which of the following functions represents a line parallel to  $l$ ?

- A)  $-\frac{3}{2}x + 3 = y$
- B)  $\frac{3}{2}x + 4 = y$
- C)  $-\frac{2}{3}x + 5 = y$
- D)  $-\frac{2}{3}x + 3 = y$

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### Answers:

1. 2:00 PM    2. D    3. B    4. 15 cups    5. A    6. 3    7. \$1.50    8. B    9. -5    10. No  
Solution    11.  $(\frac{1}{2}, 3)$     12. B    13. 21    14. C    15. C

### Explanations:

All questions have video solutions under the **CONTENT** tab of the **Math Videos** section. Look for the appropriate subheading name to find corresponding explanations.

Select written answer explanations below.

1. **2:00 PM.** The question asks for the time when Justin runs out of lemonade, which essentially means that it is asking for when Justin sells all of his lemonade. Since we are given a set number of cups he has to sell and a set rate at which he sells them, we can use dimensional analysis to determine the amount of time it takes as seen below:

$$30 \text{ cups} * \frac{1 \text{ hour}}{5 \text{ cups}} = 6 \text{ hours}$$

Since it takes 6 hours for Justin to sell all of his lemonade, and he starts selling at 8:00 AM, he will run out of lemonade by 2:00 PM

2. **D.** The question asks for a function  $l(h)$  that models the number of cups of lemonade Justin has left  $h$  hours after 7:00 PM. We know that Justin sells lemonade at a rate of 5 cups per hour starting at 7:00 AM, so we can begin to model our equation:

$$l(h) = -5h$$

$h$  is negative since Justin loses lemonade once he sells it. This answer automatically knocks out choices A and C because they both have a positive  $h$ . However, our equation is not done. Justin starts off with 30 cups of lemonade at 7:00 AM (i.e. when he has not yet sold any lemonade and  $h = 0$ ), and we can use this information to finish our equation:

$$l(h) = 30 - 5h$$

This makes choice D correct. Choice B may result from misinterpretation of the information.

3. **B.** This question is very similar to question 2, however, this question states that Justin begins selling lemonade at 8:00 AM instead of 7:00 AM. This means that from 7:00 AM to 8:00 AM, Justin has not sold any cups of lemonade. This forces us to modify our equation:

$$l(h) = 30 - 5h \rightarrow l(h) = 30 - 5(h - 1)$$

Since no lemonade is sold in the first hour from 7:00 AM to 8:00 AM, we must use  $5(h - 1)$  instead of  $5h$ . Thus, choice B is correct.

4. **15 cups.** The question asks for the number of cups Justin has left at 11:00 PM. This is 3 hours after 8:00 PM, the time when Justin begins to sell lemonade. We can multiply the rate at which Justin sells lemonade by the time to get the cups of lemonade he has sold:

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$$3 \text{ hours} * \frac{5 \text{ cups}}{1 \text{ hour}} = 15 \text{ cups}$$

We can then subtract the number of cups sold from the total number of cups Justin starts off with to get the number of cups Justin has left:

$$30 \text{ cups} - 15 \text{ cups} = 15 \text{ cups}$$

Thus, 15 cups is correct.

5. **A.** The question asks for an equation that represents  $N$ , the number of notes Frank plays, based off of  $y$ , the number of minutes he practices. Since it is given that  $y > 10$ , and Frank practices at a rate of 10 notes per minute for the first 10 minutes, we can determine that he has played  $10 * 10 = 100$  notes after 10 minutes. He then continues to play at a rate of 30 notes per minutes. Since  $y > 10$ , the number of minutes he plays after 10 minutes is equal to  $(y - 10)$ . Thus, we can model  $N$  as such:

$$N = 100 + 30(y - 10)$$

Therefore, choice A is correct. Choices B, C, and D all may result from misinterpretation of the question.

6. **3.** The question asks for the number of bags of flour (a.k.a  $a$ ) the chef needs to make 100 cupcakes. Since we are given the equation to determine the number of cupcakes (a.k.a  $f(a)$ ) based off of the number of bags of flour, we can simply plug in 100 to get the answer:

$$f(a) = 30a + 10 \rightarrow 100 = 30a + 10 \rightarrow 90 = 30a \rightarrow 3 = a$$

Thus, 3 is correct.

7. **\$1.50.** The question asks for the cost of one soda before the 50% discount. In order to solve, we can set up a system of equations. The question states that 2 sodas and 3 cotton candies before the soda discount costed \$6.75. We can use this to set up an equation where  $s$  is the cost of one soda and  $c$  the cost of one cotton candy:

$$6.75 = 2s + 3c$$

The question then states that after the 50% discount on soda, the total cost of 2 sodas and 3 cotton candies was \$5.25. We can use this to make another equation with the same variables:

$$5.25 = 2(0.5)2 + 3c$$