

1.  $i$  is a complex number and  $n$  is an integer. Which of the following is not a possible value of  $i^{n+1}$ ?
  - A. 1
  - B.  $i$
  - C. 0
  - D.  $-1$
  - E.  $-i$
2. Maria is finding the zeroes of a polynomial, and the quadratic formula gives  $x = 9 \pm \sqrt{-16c^2}$ . If  $c$  is a non-negative number, what is  $x$  written as a complex number?
  - A.  $9 \pm 4c$
  - B.  $9 \pm 4ci$
  - C.  $9 + 4c$
  - D.  $9 - 4ci$
  - E.  $9 + 4ci$
3. If  $i = \sqrt{-1}$ , then what does  $\left(\frac{1-i}{1+i}\right)\left(\frac{-1}{1-i}\right) = ?$ 
  - A.  $\frac{1+i}{2}$
  - B.  $1-i$
  - C.  $\frac{-1-i}{2}$
  - D.  $\frac{i-1}{2}$
  - E.  $-1+i$
4. For all pairs of nonzero real numbers  $x$  and  $y$ , the product of the complex number  $x - yi$  and which of the following complex numbers is a real number?
  - A.  $x + yi$
  - B.  $x - yi$
  - C.  $xyi$
  - D.  $y - xi$
  - E.  $x + i$
5. The product of two numbers is 41. One of the numbers is the complex number  $5 + 4i$ . What is the other number?
  - A.  $5 - 4i$
  - B.  $5 + 4i$
  - C.  $-5 - 4i$
  - D.  $-5 + 4i$
  - E.  $\frac{41}{5 - 4i}$
6. Which equation given in factored form has the roots  $\frac{1}{4}, \frac{2}{3}, i$ , and  $-i$ ?
  - A.  $(4x-1)(3x-2)(x^2-1)$
  - B.  $(4x+1)(3x-2)(x^2-1)$
  - C.  $(4x-1)(3x-2)(x^2+1)$
  - D.  $(4x-1)(3x+2)(x^2+1)$
  - E.  $(4x-1)(3x+2)(x^2-1)$
7. For the complex number  $i$  such that  $i^2 = -1$ , what is the value of  $i^8 - 2i^2 - 1$ ?
  - A.  $-4$
  - B.  $-2$
  - C. 0
  - D. 2
  - E. 4
8. What is the sum of  $\sqrt{-20}$  and  $\sqrt{-125}$ ?
  - A.  $-7i\sqrt{5}$
  - B.  $7i\sqrt{5}$
  - C.  $-21i\sqrt{5}$
  - D.  $21i\sqrt{5}$
  - E.  $i\sqrt{105}$
9. What is the square of the complex number  $(2i - 4)$ ?
  - A.  $12 - 16i$
  - B.  $20 - 16i$
  - C.  $-20$
  - D. 12
  - E. 20
10. For  $i^2 = -1$ ,  $(3 - i)^2 = ?$ 
  - A. 8
  - B. 10
  - C.  $8 - 6i$
  - D.  $8 + 6i$
  - E.  $10 - 6i$

11. The solution set for the equation  $3^{x^2+3} - 1 = 0$  contains:

- A. Only 1 imaginary numbers
- B. Only 2 imaginary number
- C. 1 imaginary and 1 real number
- D. 1 negative real number and 1 imaginary number
- E. 1 real number, which is 0.

12. For all  $x < 0$ , which of the following expressions is

equivalent to  $\frac{\sqrt{x}}{\sqrt{x}-i}$ ?

- A.  $\frac{x-\sqrt{x}}{x-1}$
- B.  $\frac{x+\sqrt{x}}{x-1}$
- C.  $\frac{-x-\sqrt{x}}{-x+1}$
- D.  $\frac{x+\sqrt{-x}}{x+1}$
- E.  $\frac{(x-\sqrt{x})}{x+1}$

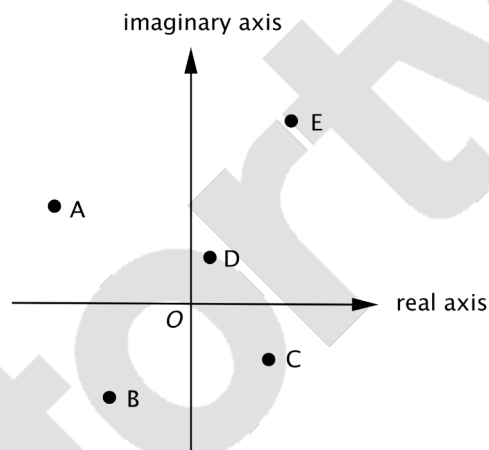
13. Which of the following expressions is equivalent to  $9x^2 + 169$ ?

- A.  $(3x+13)^2$
- B.  $(3x+13i)^2$
- C.  $(3x-13i)^2$
- D.  $(3x-13)(3x+13)$
- E.  $(3x-13i)(3x+13i)$

14. What complex number equals  $(3-4i)(\pi+3i)$ ?

- A.  $(12+3\pi)i + (9-4\pi)$
- B.  $(12-3\pi) + (9-4\pi)i$
- C.  $(12+3\pi) + (9+4\pi)i$
- D.  $(12+3\pi) + (9-4\pi)i$
- E.  $(12-3\pi)i + (9-4\pi)i$

15. The figure below depicts a complex plane with the horizontal axis representing real values and the vertical axis representing imaginary values. The modulus of a complex number  $a+bi$  is  $\sqrt{a^2+b^2}$ . By looking at the points below, which point has the smallest modulus?



- A. A
- B. B
- C. C
- D. D
- E. E

**ANSWER KEY**

1. C   2. B   3. D   4. A   5. A   6. C   7. D   8. B   9. A   10. C   11. B   12. E   13. D   14. D  
15. D

**ANSWER EXPLANATIONS**

1. C. By definition,  $i$  is the complex number equal to  $\sqrt{-1}$ , and when taken to a power, equals one of four possible answers ( $i, -1, -i$ , or  $1$ ). For A,  $i^4 = \sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1} = 1$ . B is wrong because  $i^1 = \sqrt{-1} = i$ . D:  $i^2 = \sqrt{-1}\sqrt{-1} = -1$ . E:  $i^3 = \sqrt{-1}\sqrt{-1}\sqrt{-1} = -1\sqrt{-1} = -i$ . As such, the answer is C. Also, the only number that can equal 0 when taken to any power is zero. Therefore, 0 cannot be the answer.
2. B.  $x = 9 \pm \sqrt{-16c^2} = 9 \pm 4c\sqrt{-1}$ . Since  $i^2 = -1$ , we can rewrite  $\sqrt{-1} = i$ . This gives us  $x = 9 \pm 4ci$ .
3. D. Canceling out  $1-i$  from the numerator and denominator, we get  $-\frac{1}{1+i}$ . To get the  $i$  term out of the denominator, we multiply the fraction by the conjugate on the top and bottom to get  $-\frac{1}{1+i} \left( \frac{1-i}{1-i} \right) = -\frac{1-i}{1+i-i+1} = -\frac{1-i}{2} = \frac{i-1}{2}$ .
4. A. The product of a complex number and its conjugate is a real number because the  $+$  and  $-$  in front of the imaginary terms cancel out when foiled. So,  $(x-yi)(x+yi)$  will be equal to a real number. To verify this, we foil the factors and get  $x^2 - xyi + xyi - y^2i^2 = x^2 - (-1)y^2 = x^2 + y^2$ . This is a real number because  $x$  and  $y$  are real numbers.
5. A. The product of a complex number and its conjugate is a real number because the  $+$  and  $-$  in front of the imaginary terms cancel out when expanded with FOIL. So, the only way  $(5-4i)$  multiplied by something can yield a real number will be when it is multiplied by its conjugate. To verify, we multiply  $(5-4i)(5+4i)$  and get  $25 - 4i(5) + 5(4i) - 16i^2 = 25 - (-1)16 \rightarrow 25 + 16 \rightarrow 41$ .
6. C. If a polynomial has roots equal to  $\frac{1}{4}, \frac{2}{3}, i$ , and  $-i$ , it means that these terms make the polynomial equal to zero when plugged in. So, the following equations must be true:  $x - \frac{1}{4} = 0 \rightarrow 4x - 1 = 0$ ,  $x - \frac{2}{3} = 0 \rightarrow 3x - 2 = 0$ ,  $x - i = 0$ , and  $x + i = 0$ . So, the polynomial can be written as  $(4x-1)(3x-2)(x-i)(x+i)$ . Multiplying the last two factors, using FOIL, we get  $(4x-1)(3x-2)(x^2 - xi + xi - 1) = (4x-1)(3x-2)(x^2 + 1)$ .
7. D. The first term,  $i^8 = (i^2)^4 = (-1)^4 = 1$ . The second term,  $-2i^2 = -2(-1) = 2$ . By substituting these into the equation, we get  $1 + 2 - 1$ , which equals 2.
8. B. We can break down the square roots to  $\sqrt{(-1)(4)(5)}$  and  $\sqrt{(-1)(25)(5)}$ . The  $\sqrt{-1}$ 's become  $i$ , and the perfect squares become their square roots. Thus, we get  $2i\sqrt{5}$  and  $5i\sqrt{5}$ . Their sum is  $7i\sqrt{5}$ .
9. A. Using FOIL, we get  $(2i-4)^2 = 2i*2i - 8i - 8i + 16$ .  $2i*2i$  is equal to  $4*-1 = -4$ , so simplifying, we get  $-4 - 16i + 16$ . We combine the integers to get  $12 - 16i$ .
10. C. Using FOIL, we get  $(3-i)^2 = 3*3 - 3i - 3i - i*i$ . Simplifying gets us  $9 - 6i + i^2$ . We are given that  $i^2 = -1$ , so plugging that in:  $9 - 6i - 1 = 8 - 6i$ .
11. A. In order to satisfy the equation,  $3^{x^2+3}$  must equal 1. An exponential function only equals 1 when its exponent is equal to 0. Solve for  $x^2 + 3 = 0$ . This becomes  $x^2 = -3$ .  $x$  must then be equal to  $i\sqrt{3}$  and  $-i\sqrt{3}$ . There are 2 imaginary numbers in the solution set.

12. **D.** In order to simplify the expression, we multiply the top and bottom of the fraction by the conjugate of the denominator. Since the denominator is  $\sqrt{x} - i$ , its conjugate is  $\sqrt{x} + i$ . Our expression now becomes  $\frac{\sqrt{x}(\sqrt{x} + i)}{(\sqrt{x} - i)(\sqrt{x} + i)}$ .
- Distributing the top and apply FOIL to the bottom gives us  $\frac{x + i\sqrt{x}}{x + i\sqrt{x} - i\sqrt{x} - i^2}$ , which we can simplify to  $\frac{x + \sqrt{-x}}{x + 1}$ .
13. **E.**  $9x^2 + 169$  can be expressed as the product of complex conjugates. The first term is the square of  $3x$ , and the second term is the product of  $13i$  and  $-13i$ . Thus, we can set up our equation as  $(3x + 13i)(3x - 13i)$ . The 'O' and 'I' of FOIL cancel each other out, leaving us with  $9x^2 + 169$ .
14. **D.** Multiplying the expression out using FOIL, we get  $(3 - 4i)(\pi + 3i) = 3\pi - 4i\pi + 9i - (-1)12$ . Now, separating the real and imaginary terms, we get  $3\pi + 12 - 4\pi i + 9i = (12 + 3\pi) + (9 - 4\pi)i$ .
15. **D.** The modulus of the complex number is essentially the distance from the origin to the point. This can be seen since the value of the modulus,  $\sqrt{a^2 + b^2}$ , is the Pythagorean theorem, which is used to find the distance of a point from the origin. Thus, we can tell what the smallest modulus is by seeing which point is the closest to the origin. In this case, it's D.