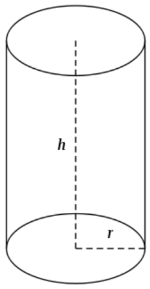
SIMILAR SHAPES AND RATIOS

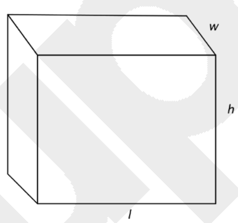
ACT Math: Problem Set

1. The volume, V, of the right circular cylinder below is given by the formula $V=\pi r^2 h$, where r is the radius of the base and h is the height of the cylinder shown below. If r is tripled and h is halved, the cylinder's new volume would be:



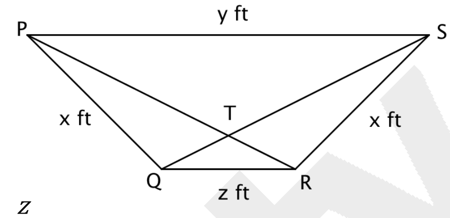
A. $\frac{3}{4}V$
B. $\frac{9}{2}V$
C. $\frac{3}{2}V$
D. $\frac{9}{4}V$
E. $\frac{9}{2}V$

2. The formula for the surface area (S) of a rectangular solid (shown below) is $S=2lw+2lh+2wh$, where l represents the length, w the width, and h the height of the solid. Tripling each of the dimensions (l, w, and h) will increase the surface area to how many times its original size?



A. 3
B. 6
C. 9
D. 12
E. Impossible to determine without knowing the original measurements

3. Isosceles trapezoid $PQRS$ below has side lengths as marked. Its diagonals intersect at T. What is the ratio of the length of $\overline{PR}$ to $\overline{TQ}$?



A. $\frac{z}{y}$

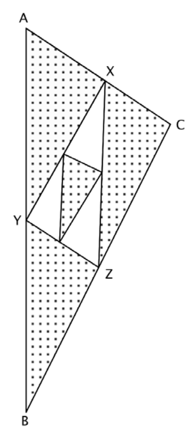
B. $\frac{y}{y+z}$

C. $\frac{y+1}{z}$

D. $\frac{y}{z}$

E. $\frac{y+z}{z}$

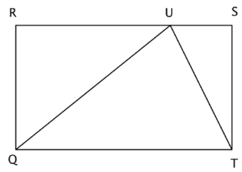
4. In the figure below, $X, Y,$ and Z are the midpoints of the sides of $\triangle ABC$, and $D, E,$ and F are the midpoints of the sides of $\triangle XYZ$. The interiors of $\triangle AXY$, $\triangle CXZ$, $\triangle BYZ$, and $\triangle DEF$ are dotted. What percent of the interior of $\triangle ABC$ is *not* dotted?



A. 18.75%
B. 22%
C. 37.5%
D. 56.25%
E. Impossible to determine from the given information.

5. In the figure below, U lies $2/3$ of the way from R to

S on the rectangle $QRST$. The area of $\triangle QUT$ is what fraction of the area of rectangle $QRST$?



A. $\frac{1}{3}$

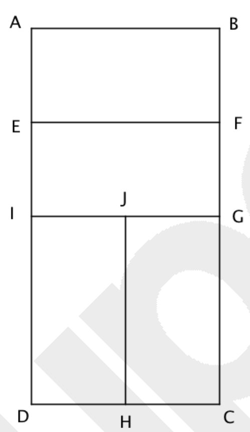
B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

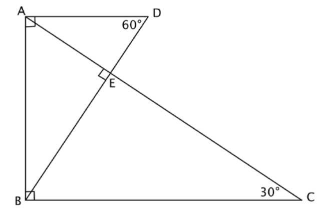
E. $\frac{4}{5}$

6. Rectangle $ABCD$ consists of 4 congruent rectangles as shown in the figure below. Which of the following is the ratio of the length of $\overline{EF}$ to the length of $\overline{AD}$?



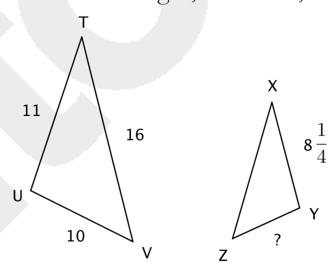
A. $1:4$
B. $1:3$
C. $1:2$
D. $2:3$
E. $3:4$

7. For the triangles in the figure below, which of the following ratios of side lengths is equivalent to the ratio of the perimeter of $\triangle ABD$ to $\triangle ABC$?



A. $AB:BC$
B. $AB:AC$
C. $AD:BC$
D. $AD:AC$
E. $DB:CB$

8. Triangles $\triangle TUV$ and $\triangle XYZ$, shown below, are similar with $\angle T \cong \angle X$ and $\angle U \cong \angle Y$. The given lengths are in meters. What is the length, in meters, of $\overline{YZ}$?



A. $5\frac{5}{32}$

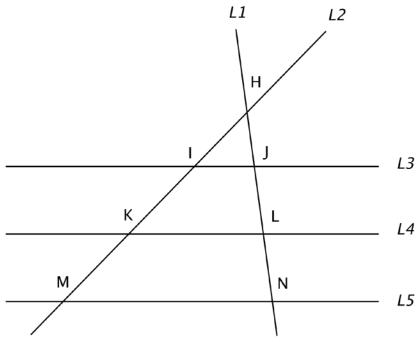
B. $7\frac{1}{4}$

C. $7\frac{1}{2}$

D. $9\frac{3}{40}$

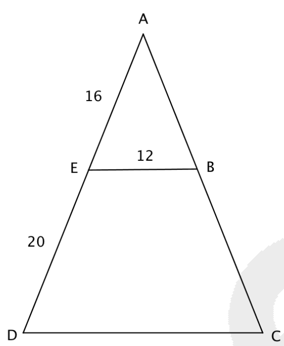
E. $13\frac{1}{5}$

9. Lines $L1$ and $L2$ intersect each other and 3 parallel lines, $L3$, $L4$, and $L5$, as shown below. The ratio of the perimeter of $\triangle HKL$ to $\triangle HMN$ is 5:8. The ratio of IJ to MN is 3:10. What is the ratio of KL to IJ ?



A. $3:16$
B. $12:25$
C. $24:50$
D. $25:12$
E. $16:3$

10. In $\triangle ACD$ below, $\overline{EB} \parallel \overline{DC}$. The lengths are given in feet. What is CD?



A. 6.7
B. 9.6
C. 15
D. 27
E. 60

ANSWERS

1. E **2.** C **3.** E **4.** B **5.** B **6.** C **7.** A **8.** C **9.** D **10.** D

ANSWER EXPLANATIONS

1. **E.** Let the original volume be $V=\pi r^2 h$. The new volume is $\pi(3r)^2\left(\frac{h}{2}\right)$. This becomes $\frac{9}{2}\pi r^2 h=\frac{9}{2}V$.
2. **C.** Tripling the measurement of each dimension turns our formula into $2(3l)(3w)+2(3l)(3h)+2(3w)(3h)=9(2lw)+9(2lh)+9(2lw)=9(2lw+2lh+2wh)=9\cdot$
3. **E.** The ratio of PT to QT is $\frac{y}{z}$. The ratio of TR to QT is $\frac{z}{z}$. So, since $PR=PT+TR$, $\frac{PR}{TQ}=\frac{PT+TR}{TQ}=\frac{y}{z}+\frac{z}{z}=\frac{y+z}{z}$.
1.
4. **A.** The triangles formed by connecting the midpoints of the larger triangles are congruent. Thus, the center triangle is $\frac{1}{4}$ of the total area. Similarly, the 3 undotted triangles within the center triangle are $\frac{3}{4}$ of the center triangle's area. Thus, the undotted triangles are $\frac{3}{4}*\frac{1}{4}=\frac{3}{16}=18.75\%$ of the total triangle's area.
5. **B.** Draw a line straight down from U to the bottom of the rectangle. It's clear that the two triangle's that $\triangle QUT$ has been divided into are half of the respective rectangles that $QRST$ has been divided into. Thus, $\triangle QUT$ is half of the area of rectangle $QRST$.
6. **C.** Since the 2 shorter sides of the bottom rectangles together equal the longer side of the rectangle above them, we can conclude that the shorter side of any of the rectangles are half of the long side. Let s be the length of the short end and l be the length of the long end. $EF=l$. $AD=2s+l=l+l=2l$. Therefore $E:F=l:2l=1:2$.
7. **A.** The ratio of the corresponding sides of similar shapes is equal to the ratio of their perimeters. In this case, the way to find the answer is to ensure that the sides we choose are, in fact, corresponding sides. The only choice of sides that are both across from congruent angles are $\overline{AB}$ and $\overline{BC}$, which are both across from $60°$ angles. Thus, since the ratio between the two is equal to the ratio of the perimeters, the ratio of the perimeters is equal to $AB:BC$.
8. **C.** The triangles are similar, but are mirror images to each other. The ratio of TU to XY is equal to the ratio of UV to ZY. We set this up as $\frac{11}{10}=\frac{8.25}{ZY}$. We cross multiply to get $11ZY=82.5$, and divide by 11 to get $ZY=7.5$ or $7\frac{1}{2}$.
9. **D.** The ratio of $\triangle HKL$ to $\triangle HMN$, 5 to 8, is equal to the ratio of KL to MN. We express this as $\frac{KL}{MN}=\frac{5}{8}$. Since we are given that the ratio of IJ to MN is 3 to 10, that is, $\frac{IJ}{MN}=\frac{3}{10}$, we can combine the fractions to get our desired ratio, $\frac{KL}{IJ}$. We first flip the IJ to MN ratio so that the MN's cancel out: $\frac{MN}{IJ}=\frac{10}{3}$. We then multiply $\frac{KL}{MN}*\frac{MN}{IJ}=\frac{5}{8}*\frac{10}{3}=\frac{50}{24}=\frac{25}{12}$. Since KL and 25 are the numerators, they correspond, and the same goes for IJ and 12. Thus, $KL:IJ=25:12$.
10. **D.** The ratio of DC to AD is equal to the ratio between EB and AE. $\frac{DC}{AD}=\frac{EB}{AE}$. We know that $AD=AE+ED=16+20=36$. With other values given in the diagram, we can express our ratios as $\frac{DC}{36}=\frac{12}{16}$. Cross multiply to get $16DC=432$, and divide by 16 to get $DC=27$.