CHAPTER

4 FRACTIONS

SKILLS TO KNOW

- How to use the "FRAC" function on your calculator
- Comparing fractions, decimals, and percents
- How to multiply, divide, add, and subtract fractions
- Setting up and solving fraction word problems

THE "FRAC" FUNCTION

TI-84 calculators (and many other calculators) have the amazing ability to turn decimal answers into fractions. On the ACT, this is a very valuable tool. To use this function on a TI-84, click the MATH key on the upper left on your calculator, and then hit ENTER to select FRAC on the menu and ENTER again to turn the last number on your calculator into a fraction. This series of steps will turn whatever decimal is in your calculator into a fraction.

We can't cover every calculator in this book, so if you have a different one, get on the internet and look up how to convert decimals to fractions. It's a must know trick for tackling ACT® fraction problems.

Knowing this function allows you to bypass the perils of fraction problems and treat them as you would any problem with integers or decimals, using your calculator for any calculations.

For examples of how to incorporate this idea, see the word problems section at the end of this chapter, and look for the calculator icon.

FRACTIONS VS DECIMALS VS PERCENTS

Remember, a fraction bar means DIVIDE. All you need to do to compare fractions and decimals is simply use your handy calculator and divide the top number by the bottom number. Remember percent values can be converted to decimals by moving the decimal point over two to the left. 25% is $.25$ or $\frac{25}{100}=\frac{1}{4}$.

Which of the following gives the fractions $-\frac{4}{9}$, $-\frac{3}{5}$, and $-\frac{5}{8}$ in order from least to greatest?

A. $-\frac{5}{8}<-\frac{3}{5}<-\frac{4}{9}$ **B.** $-\frac{4}{9}<-\frac{3}{5}<-\frac{5}{8}$ **C.** $-\frac{4}{9}<-\frac{5}{8}<-\frac{3}{5}$

D. $-\frac{3}{5}<-\frac{4}{9}<-\frac{5}{8}$ **E.** $-\frac{3}{5}<-\frac{5}{8}<-\frac{4}{9}$

To solve, we just divide the top number (numerator) by the bottom number (denominator) in our calculator—i.e. for $-\frac{4}{9}$, punch in $-4 \div 9$ into your calculator:

$$-\frac{4}{9} = -0.\overline{4}$$

$$-\frac{3}{5} = -0.6$$

$$-\frac{5}{8} = -0.625$$

Now remember that a negative number which has a BIG absolute value is actually less than a negative number with a smaller absolute value. Don't get confused! Create the proper order based on your decimals, then plug in the fractions:

$$-0.625 < -0.6 < -0.\overline{4}$$

$$-\frac{5}{8} < -\frac{3}{5} < -\frac{4}{9}$$

Answer: **A**.

OPERATIONS WITH FRACTIONS

First, remember that on the ACT® you can use a calculator. Oftentimes that means you can make your calculator do all the work, and won't need to do the work of finding common denominators, etc. However, because some problems have variables or other challenges, it's important to know the basics, too. Below is a quick summary of the basics of fractions.

Multiplication

To multiply fractions, multiply the numerators and denominators separately. It doesn't matter whether the denominators are the same number or not. Here's an example:

$$\frac{3}{5} * \frac{6}{7} = \frac{3*6}{5*7} \rightarrow \frac{18}{35}$$

Division

To divide by a fraction, multiply by the reciprocal.

When you divide by a fraction, simply multiply by the reciprocal (the fraction turned upside-down), i.e.:

$$5 \div \frac{1}{12} = 5 * \frac{12}{1}$$

$$5 * 12 = 60$$

Just flip the fraction AFTER the division sign upside down and multiply—remember the order MUST stay the same:

$$\frac{2}{3} \div -\frac{4}{5} = \frac{2}{3} * -\frac{5}{4}$$

$$\frac{2}{2+\frac{1}{2-\frac{1}{2}}}=?$$

With these types of problems, always start with the most buried fraction and work your way up. Solve out each piece and substitute back in to the expression. Here we'll start with the $2-\frac{1}{2}$ at the bottom:

On the left we have the original expression: On the right we solve out each small piece:

$$\frac{2}{2+\frac{1}{\boxed{2-\frac{1}{2}}}} \longrightarrow 2-\frac{1}{2}=\frac{4}{2}-\frac{1}{2}=\frac{3}{2}$$

$$\frac{2}{2+\boxed{\frac{1}{\left(\frac{3}{2}\right)}}} \longrightarrow \frac{1}{\left(\frac{3}{2}\right)}=1\div\frac{3}{2}=1\times\frac{2}{3}=\frac{2}{3}$$

$$\frac{2}{\boxed{2+\frac{2}{3}}} \longrightarrow 2+\frac{2}{3}=\frac{6}{3}+\frac{2}{3}=\frac{8}{3}$$

$$\frac{2}{\left(\frac{8}{3}\right)} \qquad \frac{2}{\left(\frac{8}{3}\right)}=2\div\frac{8}{3}=2\times\frac{3}{8}=\frac{6}{8}=\frac{3}{4}$$

Addition and Subtraction

For us to add or subtract fractions, they need to have the same denominator. This can be achieved by multiplying both fractions by another fraction equal to one, "de-simplifying" them so as to change the way that they are numerically represented but not changing their values.

$$\frac{4}{7}+\frac{1}{3}=?$$

First, we multiply the fractions by a fraction equal to one which has the opposite denominator so the two fractions have a common denominator. For more information on finding the least common denominators, check out Chapter 3 on GCF/LCM.

For this problem, the common denominator between 7 and 3 is 21.

We multiply by the version of "one" that can convert each denominator to 21:

$$\frac{4}{7}*\frac{3}{3} \text{ and } \frac{1}{3}*\frac{7}{7}$$

This is mathematically valid because both fractions are multiplied by 1, which does not change their numerical values.

This gives us:

$$\frac{12}{21}+\frac{7}{21}=\frac{19}{21}$$

Answer: $\frac{19}{21}$.

$$\frac{\frac{5}{3}-\frac{3}{4}}{\frac{5}{3}+\frac{3}{4}}=?$$

Un-simplify the fractions that we are adding or subtracting so that they have the same denominator. We need to add the two fractions in the denominator and subtract the two in the numerator (remember the top of a fraction or bottom is essentially in "parenthesis" so we add and subtract our elements within the numerator and denominator first before further simplifying).

$$\frac{\frac{5}{3}-\frac{3}{4}}{\frac{5}{3}+\frac{3}{4}}$$

The least common denominator of 3 and 4 is 12, so we multiply to convert all the fractions to equivalent fractions with a denominator of 12:

$$\frac{\frac{5}{3}\left(\frac{4}{4}\right)-\frac{3}{4}\left(\frac{3}{3}\right)}{\frac{5}{3}\left(\frac{4}{4}\right)+\frac{3}{4}\left(\frac{3}{3}\right)}=\frac{\frac{20}{12}-\frac{9}{12}}{\frac{20}{12}+\frac{9}{12}}=\frac{\frac{11}{12}}{\frac{29}{12}}$$

If the fraction format is confusing, we can also write it like this:

$$=\frac{11}{12}\div\frac{29}{12}$$

Now, we are trying to divide this fraction. As mentioned before, we do this by multiplying the top or first fraction by the reciprocal of the bottom or second one.

$$=\frac{11}{12}\times\frac{12}{29}$$

We can cross out the 12's because they cancel out, which leads us to the answer:

$$\frac{11}{29}$$

Answer: $\frac{11}{29}$.

For all real numbers x such that $x \neq 0$, $\frac{3}{4}+\frac{7}{x}=?$

A. $\frac{10}{4+x}$ **B.** $\frac{21}{4x}$ **C.** $\frac{3x+28}{4x}$ **D.** $\frac{10}{4x}$ **E.** $\frac{7x+12}{4+x}$

For this problem we need to simply apply the same rules of fractions we use in fraction problems with numbers only to the variable. We first find our least common denominator:

$$4x$$

We'll add these fractions by converting each fraction into an equivalent fraction that shares this denominator:

$$\left(\frac{3}{4}\right)\left(\frac{x}{x}\right)+\left(\frac{7}{x}\right)\left(\frac{4}{4}\right)$$

$$\left(\frac{3x}{4x}\right)+\left(\frac{28}{4x}\right)$$

Now that the denominators match, we can add the top of the fractions:

$$\frac{3x+28}{4x}$$

Answer: **C**.

WORD PROBLEMS WITH FRACTIONS

For word problems with fractions, sometimes we need to translate the words into an equation and then solve. These are very similar to problems with percents. Just as with percent problems, you'll also sometimes need to distinguish between what you're taking the fraction "of" so you don't calculate the wrong amount. As long as you "translate" the problem correctly, you should be fine.

TIP: Some problems that include fractions are rates problems, so if you struggle with those, check out our chapter on ratios and rates.

What fraction of an empty 750 ml boiling flask can be filled by a 1250 ml volumetric flask filled halfway?

Translate this one step at a time, just as we did in the percent chapter.

"What fraction" → write n

(We simply know that n will be a fraction, but it's just like any other unknown number, don't let the word "fraction" throw you off. Unlike percents, fractions don't require you to put the number over 100 or anything. "What fraction" just means make up a variable).

Of → write the sign for multiply
750 ml flask → write 750
Can be filled by → write =
A 1250 ml Flask → write 1250
Filled half-way → multiply by $\frac{1}{2}$

What fraction of an empty 750 ml flask can be filled by a 1250 ml flask filled halfway?

$$n \times 750 = 1250 \times \frac{1}{2}$$

$$750n = \frac{1}{2}(1250)$$

$$750n = 625$$

$$n = \frac{625}{750}$$

To reduce this fraction, you can use the FRAC function on your calculator: type in $625 \div 750$ then hit ENTER, MATH, be sure FRAC is selected, and hit ENTER again. This process will show you the reduced fraction form:

$$n = \frac{5}{6}$$

Answer: $\frac{5}{6}$.

For English Literature class, Amelia must read Ethan Frome in 7 days. She reads $\frac{1}{10}$ of the book each of the first three days. For the remaining four days, what fraction of the book must Amelia read per day, on average?

This question is both a fraction and a rates problem. Remember we want fraction per day – that's the total fraction of the book left, divided by the number of days left (remember PER means DIVIDE).

First, let's figure out what fraction of the book she must read in the last four days.

She's already read $\frac{1}{10}$ of the book three days in a row, so we can add those three tenths together or multiply that fraction times 3 to get:

$$\frac{1}{10}(3) = \frac{3}{10}$$

If she has read $\frac{3}{10}$ then she has $\frac{7}{10}$ to go. We get $\frac{7}{10}$ by subtracting $\frac{3}{10}$ from 1. Remember in word problems, the whole amount or total is always 1 as the fractions add up to that whole.

Now she must read $\frac{7}{10}$ of the book over the course of 4 days. To figure out the fraction per day we take the fraction she has left to read and divide it by 4. You can do this in your calculator ($0.7 * 0.25$, then ENTER, then hit MATH, select FRAC, and hit ENTER), or multiply by $\frac{1}{4}$ to the same effect:

$$\frac{7}{10} \times \frac{1}{4} = \frac{7}{40}$$

Answer: $\frac{7}{40}$.

$\frac{1}{5}$ of what number is $\frac{1}{7}$ of 28?

This is another translation problem, which is similar to percent problems. Translate it one piece at at time:

$\frac{1}{5}$	of	what number	is	$\frac{1}{7}$	of	28?
$\frac{1}{5}$	$\times$	n	$=$	$\frac{1}{7}$	$\times$	28

$$\frac{1}{5}n = 4$$

$$n = 20$$

Answer: 20.

As you can see, if you simply translate each word and number, the problem is straightforward.

What fraction of a 9-inch-diameter pizza contains the same amount of pizza as a single slice taken from a 16-inch-diameter pizza cut into 12 slices, assuming both pizzas are of uniform, equal thickness?

For this question, we can apply the same logic as the last problem.

Because they are of equal depth or thickness, we can assume this is just an area problem and calculate everything based on surface area of the pizzas.

"What fraction" indicates that we need a variable (x will do!). We can do an initial set up using words to help us know what we need:

What fraction	of	a 9" diameter pizza	is the same	as a single slice	of	a 16" diameter pizza
x	$\times$	*total area of a* 9" *diameter pizza*	$=$	$\frac{1}{12}$	$\times$	*total area of a* 16" *diameter pizza*

Now we can solve for total areas.

First let's find the total area of the 9" diameter pizza:

Area of a circle: $A=\pi r^2$

$r=4.5$ because radius is half of the diameter (9)

Always be careful not to confuse your radius and diameter!

$$A=\pi\left(4.5\right)^2$$

$$A=20.25\pi$$

I'm going to leave the π in because I think it might cancel out later. Now I just need to figure out the area of the other larger pizza:

$$x\times 20.25\pi=\frac{1}{12}\times \textit{total area of a } 16"\textit{diameter pizza}$$

Area of a circle: $A=\pi r^2$

$r=8$ because radius is half of the diameter (16)

Area of larger pizza: $A=\pi\left(8^2\right)=64\pi$

We can now plug this into the above (initial) equation:

$$x\times 20.25\pi=\frac{1}{12}\times\left(64\pi\right)$$

The π cancels on each side and we get:

$$x\times 20.25=\frac{1}{12}\times\left(64\right)$$

Use your calculator to divide 64 by 12 (you can use the FRAC function described earlier in this chapter or know that $0.3333\ldots$ is $\frac{1}{3}$).

$$20.25x=5\frac{1}{3}$$

Now I use my calculator to divide $5\frac{1}{3}$ by 20.25 and get:

$$0.2633744\ldots$$

To turn this into a fraction, I hit MATH, then make sure FRAC is selected and press ENTER to get the answer:

$$\frac{64}{243}$$

Answer: $\frac{64}{243}$.