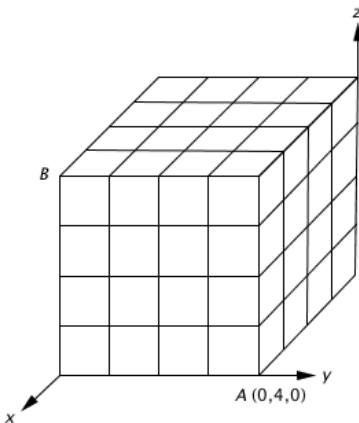


**THE BEST ACT PREP COURSE EVER**

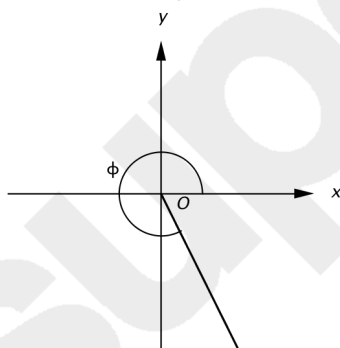
# **COORDINATE GEOMETRY**

*ACT Math: Problem Set*

1. The figure below shows a cube composed of 64 smaller identical cubes in the standard  $(x, y, z)$  coordinate system. The vertex  $A$  has coordinates  $(0, 4, 0)$ . The cube has three vertices at: the origin, on the positive  $x$ -axis, and on the positive  $z$  axis, respectively. What are the coordinates of vertex  $B$ ?

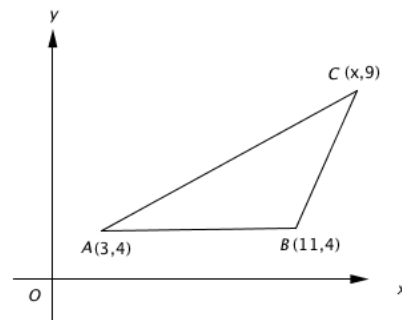


- A.  $(4, 0, 4)$   
 B.  $(4, 0, 0)$   
 C.  $(0, 0, 4)$   
 D.  $(4, 4, 0)$   
 E.  $(4, 4, 4)$
2. The angle formed by the positive  $x$ -axis and the line  $y = -\sqrt{3}x$  is denoted by  $\phi$  in the figure below. What is the value of the angle  $\phi$ ?

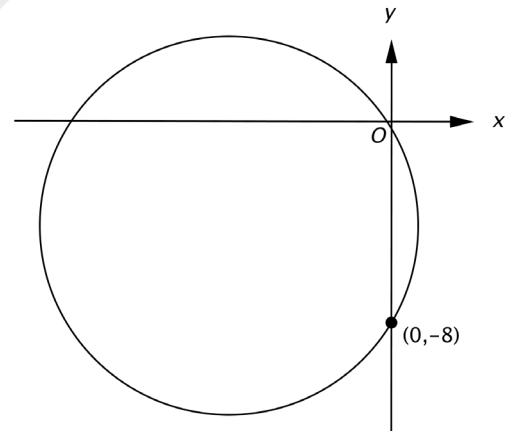


- A.  $-30^\circ$   
 B.  $270^\circ$   
 C.  $300^\circ$   
 D.  $330^\circ$   
 E. Cannot be determined from the information provided

3. The triangle  $\triangle ABC$  is shown below with labeled vertices. What is the area of  $\triangle ABC$ ?

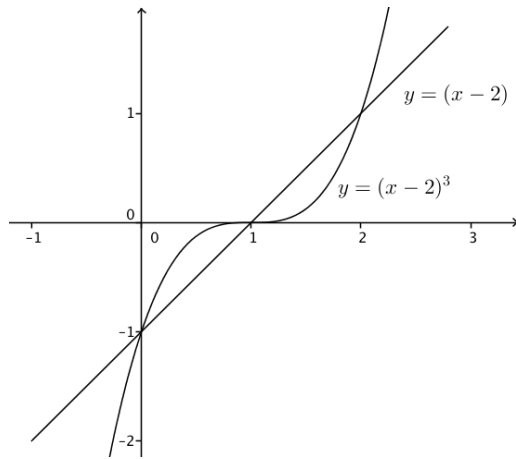


- A. 20  
 B.  $20\sqrt{2}$   
 C. 40  
 D.  $40\sqrt{2}$   
 E. Cannot be determined from the information provided
4. In the figure below, a circle with center  $(-6, -4)$  is shown. What is the equation of a line that is tangent to the circle at point  $(0, -8)$ ?

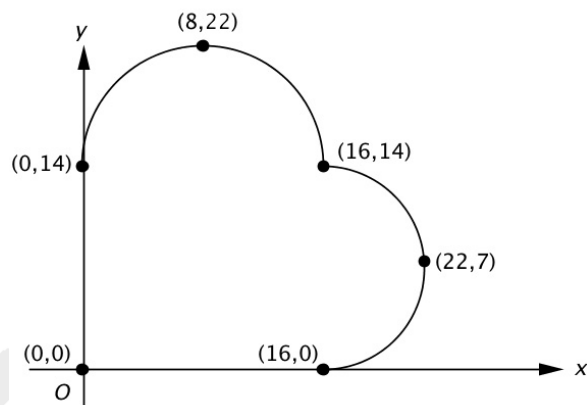


- A.  $y = \frac{3}{2}x - 8$   
 B.  $y = -\frac{3}{2}x - 8$   
 C.  $y = -\frac{3}{2}x + 8$   
 D.  $y = \frac{3}{2}x + 8$   
 E.  $y = \frac{2}{3}x - 8$

5. The graphs of the functions  $y = x - 2$  and  $y = (x - 2)^3$  are shown below. The two graphs intersect at points  $(0, -1)$ ,  $(1, 0)$ , and  $(2, 1)$ . Which values of  $x$  satisfy the inequality  $x - 2 < (x - 2)^3$ ?

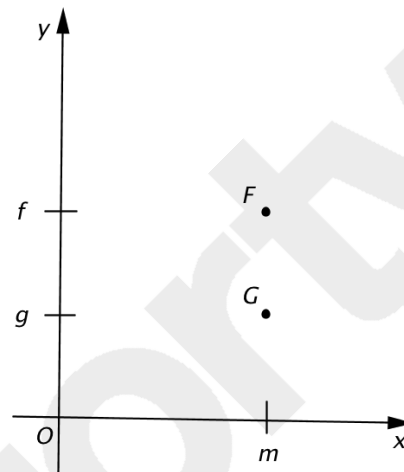


- A. No real values  
 B.  $x < 1$  and  $x > 2$   
 C.  $x < 0$  and  $1 < x < 2$   
 D.  $0 < x < 1$  and  $x > 2$   
 E.  $x > 0$  and  $x > 2$
6. The shape below is formed by combining a rectangle and two semicircles. According to the given vertices, what is the total area of the shape?

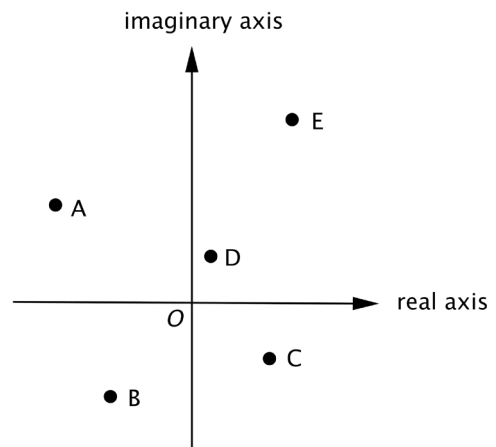


- A.  $224 + 49\pi$   
 B.  $224 + \frac{113}{2}\pi$   
 C.  $224 + 64\pi$   
 D.  $224 + \frac{113}{4}\pi$   
 E.  $224 + 113\pi$

7. In the figure below, the points  $F$  and  $G$  are shown at coordinates  $(m, f)$  and  $(m, g)$  respectively.  $F'$  is the point generated by rotating  $F$   $90^\circ$  counterclockwise about  $G$ . What are the coordinates of  $F'$ ?

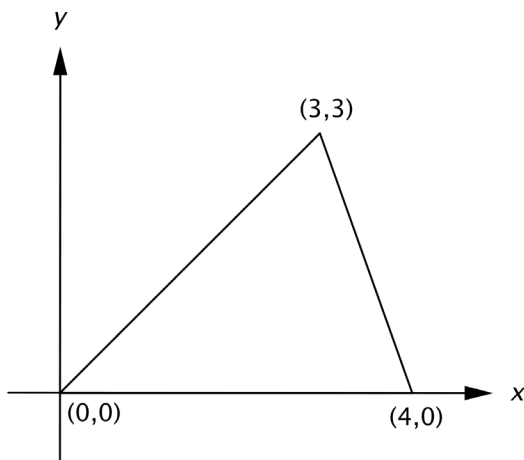


- A.  $(m + f - g, g)$   
 B.  $(m - f + g, g)$   
 C.  $(m + f - g, f - g)$   
 D.  $(m - f + g, f - g)$   
 E.  $(m - g, g)$
8. The figure below depicts a complex plane with the horizontal axis representing real values and the vertical axis representing imaginary values. The modulus of a complex number  $a + bi$  is  $\sqrt{a^2 + b^2}$ . By looking at the points below, which point has the smallest modulus?

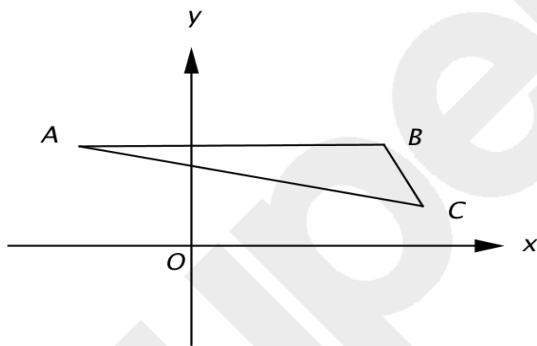


- A. A  
 B. B  
 C. C  
 D. D  
 E. E

9. The figure below depicts a triangle formed by the labeled vertices. If each vertex point of the triangle is multiplied by  $\frac{3}{2}$ , what would be the area of the resulting triangle?



- A. 9  
B. 13.5  
C. 15  
D. 18  
E. 27
10. The triangle below has vertices  $A(-5,5)$ ,  $B(10,5)$ , and  $C(11,2)$ . Point  $C$  can be moved to any point on a certain line, and the area of the triangle will remain the same. What is the slope of this line?



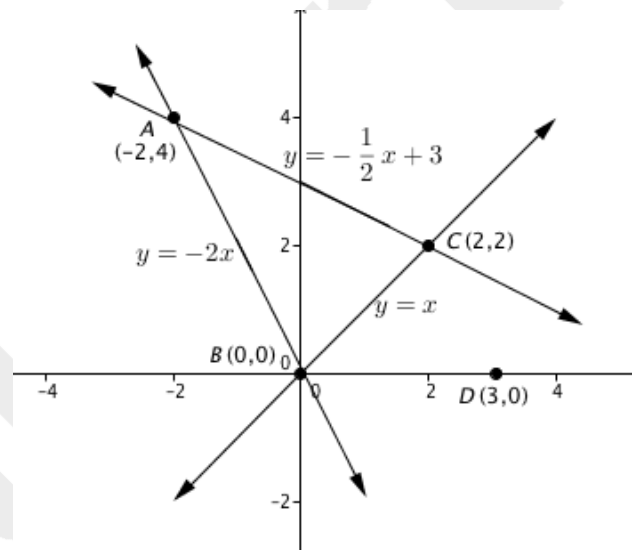
- A. -3  
B.  $-\frac{3}{16}$   
C. 0  
D.  $\frac{3}{16}$   
E. 3

For questions 11-13:

Consider the set of all points  $(x, y)$  that satisfy all 3 of the conditions below:

- I.  $y \geq x$   
II.  $y \geq -2x$   
III.  $y \leq -\frac{1}{2}x + 3$

The graph of this set is  $\triangle ABC$  and its interior, which is shown in the standard  $(x, y)$  coordinate plane below. Let this set be the domain of the function  $S(x, y) = 3x + y$ .



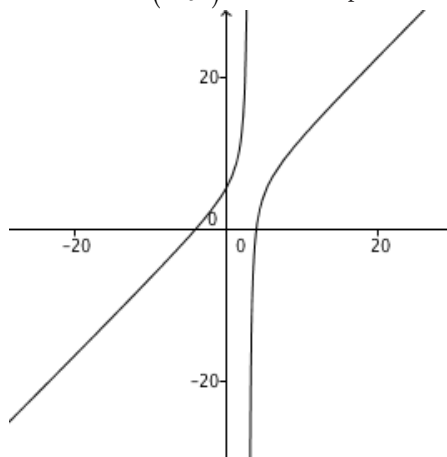
11. What is the minimum value of  $S(x, y)$  when  $x$  and  $y$  satisfy the 3 conditions given?
- A. -5  
B. -3  
C. -2  
D. 0  
E. 8
12. What is the area of  $\triangle ABC$ ?
- A.  $2\sqrt{2}$   
B.  $3\sqrt{2}$   
C. 6  
D. 8  
E. 12

13. What is the  $\cos$  of  $\angle ABD$ ?

- A.  $-\frac{\sqrt{5}}{5}$
- B.  $\frac{\sqrt{5}}{5}$
- C.  $-\frac{2\sqrt{5}}{5}$
- D.  $\sqrt{5}$
- E. Cannot determine from the information given.

For questions 14-16:

Consider the rational function  $f(x) = \frac{x^2 - 16}{x - 3}$ , whose graph is shown in the standard  $(x, y)$  coordinate plane below.



14. What is the value of  $f(x)$  at  $x = 5$ ?

- A. 3
- B. 4.5
- C. 5
- D. 9
- E. 20

15. What is the range of  $f(x)$ ?

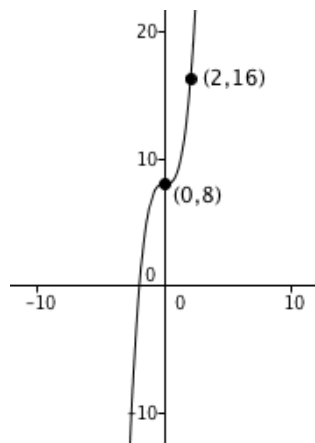
- A. All real values except 3
- B. All real values except  $\pm 4$
- C. All real values except  $\pm 4$  and 3
- D. All real values except  $\frac{4}{3}$
- E. All real values

16. How many horizontal and/or vertical asymptotes does the graph of  $f(x)$  have?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

For questions 17-19:

The graph of  $y = g(x)$  is shown in the standard  $(x, y)$  coordinate plane below with two points labeled.



17. What is the  $x$ -intercept of the graph of  $y = g(x)$ ?

- A. -8
- B. -2
- C. 0
- D. 2
- E. 8

18. The function  $y = g(x)$  can be classified as one of the following types of functions. Which one is it?

- A. Constant
- B. Linear
- C. Quadratic
- D. Cubic
- E. Trigonometric

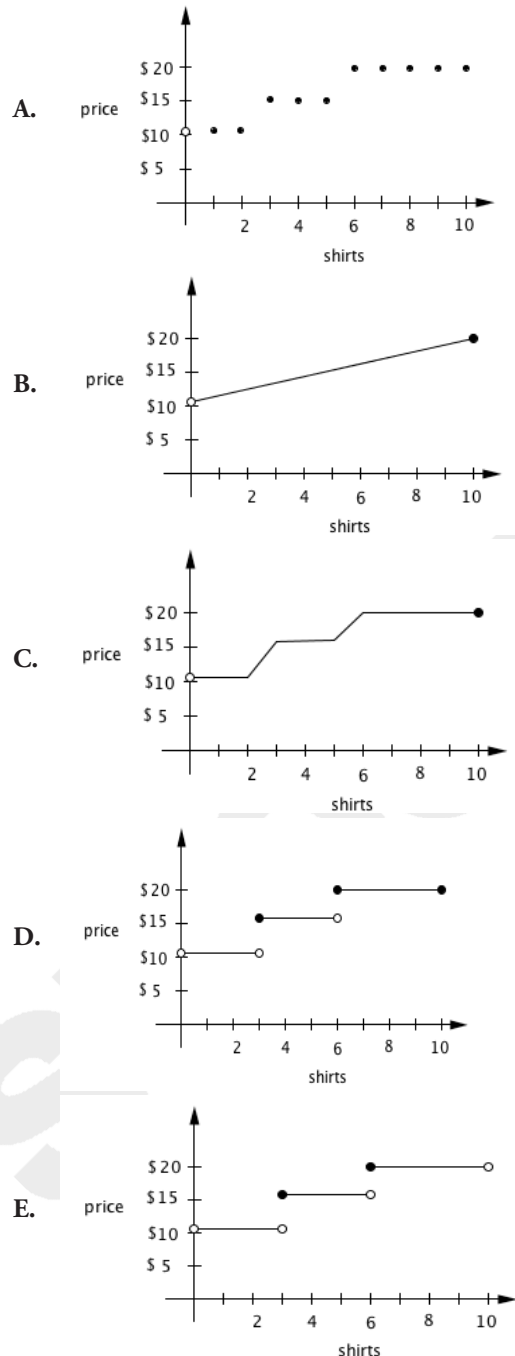
19. Which of the following equations corresponds to the reflection of  $y = g(x)$  across the line  $y = x$ ?

- A.  $y = x^3 - 8$
- B.  $y = (x - 8)^3$
- C.  $y = \sqrt[3]{x + 8}$
- D.  $y = \pm \sqrt[3]{x - 8}$
- E.  $y = \sqrt[3]{x - 8}$

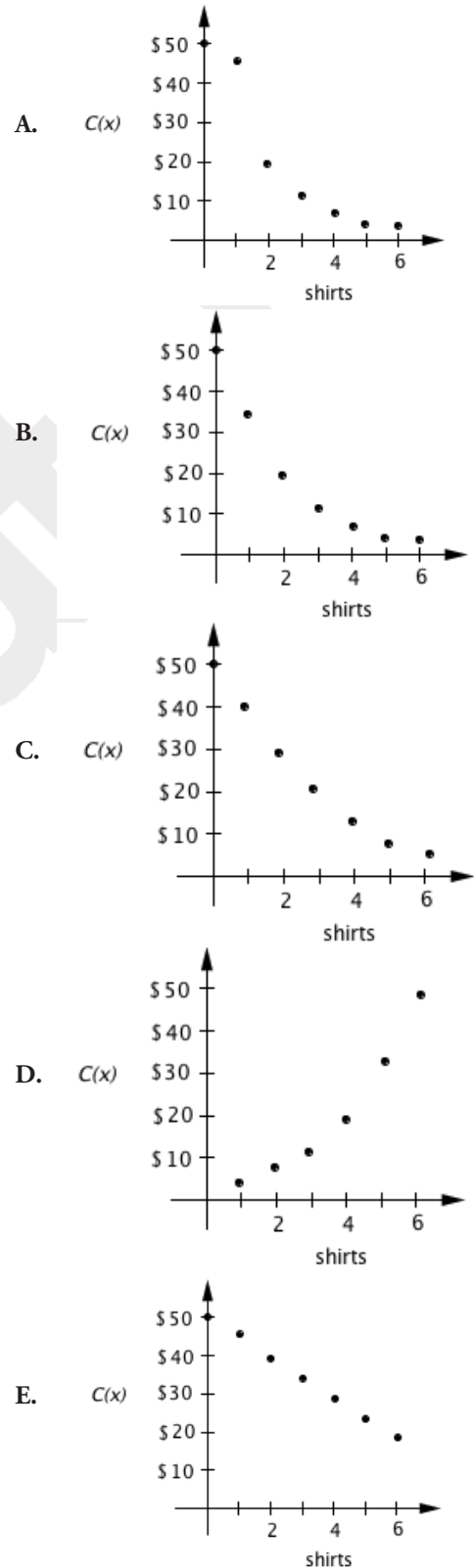
20. The table below gives the price of riding a vehicle through a safari park per passenger. The price of the trip depends on the number of passengers.

# of Passengers	Price
$0 < x < 3$	\$11.60
$3 \leq x < 6$	\$16.30
$6 \leq x \leq 10$	\$20.05

Which of the following graphs best represents this information?



21. Tina is printing t-shirts. Her total setup cost is \$50 and the t-shirts sell for \$5 each. Her profit is found by subtracting her expenses from her income. Which of the following graphs represents how much money she needs to recoup her costs as a function of the number of shirts she sells?



22. An angle in standard position in the standard  $(x,y)$  coordinate plane has its vertex at the origin and its initial side on the positive  $x$  - axis. If the angle measures  $-2493^\circ$  in standard position, it has the same terminal side as all of the following angles except:
- A.  $-2133^\circ$
  - B.  $-1413^\circ$
  - C.  $-1053^\circ$
  - D.  $-513^\circ$
  - E.  $27^\circ$
23. Four points,  $X,Y,Z,W$ , lie on a circle that has a circumference of 23 units.  $X$  is 4 units clockwise from  $W$  and  $Z$  is 15 units counterclockwise from  $W$ .  $Y$  is 9 units counterclockwise from  $X$ . Starting clockwise from  $X$ , what is the correct order of points?
- A.  $X,Z,Y,W$
  - B.  $X,Z,W,Y$
  - C.  $X,Y,Z,W$
  - D.  $X,W,Y,Z$
  - E.  $X,W,Z,Y$
24. A diameter of a circle in the standard  $(x,y)$  coordinate plane has end points at  $(3,8)$  and  $(-3,4)$ . Which of the following points must also be on the circle?
- A.  $(0,6)$
  - B.  $(3,4)$
  - C.  $(-3,8)$
  - D.  $(3,6)$
  - E.  $(0,12)$
25. In the standard  $(x,y)$  coordinate plane, the equation  $x^2 + y^2 = 169$  represents a circle. At what two points does the circle cross the  $x$  - axis?
- A.  $(13,0);(-13,0)$
  - B.  $(0,13);(0,-13)$
  - C.  $(5,12);(-5,-12)$
  - D.  $(12,0);(5,0)$
  - E.  $(16,0);(9,0)$
26. A circle in the standard  $(x,y)$  coordinate plane is centered at  $(5,-2)$  and passes through  $(2,8)$ . What is the area, in square coordinate units, of the circle?
- A.  $109\pi$
  - B.  $\sqrt{109}\pi$
  - C. 109
  - D.  $11881\pi$
  - E. 11881
27. A circle in the standard  $(x,y)$  plane is tangent to the  $y$  -axis and has its center at  $(4,7)$ . Where is the point  $(3,1)$ ?
- A. Inside the circle
  - B. On the circle
  - C. Outside the circle
  - D. On the  $y$ -axis
  - E. Cannot be determined from this information
28. A circle in the standard  $(x,y)$  coordinate plane crosses the points  $(-6,2)$  and  $(5,2)$ . If the radius of the circle is 6 units, which of the following could be the center of the circle?
- I.  $(-1,2)$
  - II.  $(-1,5)$
  - III.  $(-1,-15)$
- A. I only
  - B. I and II only
  - C. I and III only
  - D. II and III only
  - E. I, II, and III
29. There is a right triangle in the standard  $(x,y)$  coordinate plane. If its vertices are at  $(-4,0)$ ,  $(3,0)$ , and  $(-4,10)$ , what is the length of the hypotenuse in coordinate units?
- A.  $-\sqrt{149}$
  - B. 5
  - C. 12
  - D. 17
  - E.  $\sqrt{149}$

30. In the standard  $(x, y)$  coordinate plane,  $\triangle PQR$  is isosceles.  $PQ$  is congruent to  $QR$ . Vertex  $P$  is at  $(-5, 4)$  and vertex  $R$  is at  $(1, 4)$ . What is the  $x$ -coordinate of  $Q$ ?
- A.  $-5$   
B.  $-2$   
C.  $1$   
D.  $3$   
E.  $4$
31. The sides of a square are 5 inches long. One vertex of the square is at  $(3, 4)$ . Which of the following cannot be a vertex of the square?
- A.  $(8, 4)$   
B.  $(3, 9)$   
C.  $(-2, 4)$   
D.  $(3, 1)$   
E.  $(-1, 1)$
32. Points  $X(4, 1)$ ,  $Y(7, 1)$ , and  $Z(4, 6)$  are in the standard  $(x, y)$  coordinate plane. If  $XYZW$  is a rectangle, what is the length, in coordinate units, of  $\overline{XW}$ ?
- A.  $3$   
B.  $4$   
C.  $5$   
D.  $\sqrt{34}$   
E. Impossible to determine from the given information.
33. What is the perimeter of a quadrilateral in the  $(x, y)$  coordinate plane with vertices at  $(5, 5)$ ,  $(6, 8)$ ,  $(9, 9)$ , and  $(8, 6)$ ?
- A.  $4\sqrt{10}$   
B.  $8$   
C.  $2\sqrt{10} + 6$   
D.  $2\sqrt{10} + 8$   
E.  $2\sqrt{10} + 10$
34. A polynomial function  $F(x)$  has degree  $a$ . If the graph of  $F(x)$  touches or crosses the  $x$ -axis exactly 5 times, which of the following cannot be the value of  $a$ ?
- A.  $8$   
B.  $7$   
C.  $6$   
D.  $5$   
E.  $4$
35. In the standard  $(x, y)$  coordinate plane, what is the distance between the points  $(x, x-3)$  and  $(3x, x+5)$ ?
- A.  $2x+8$   
B.  $\sqrt{68}$   
C.  $2\sqrt{x^2+16}$   
D.  $\sqrt{4x^2+4}$   
E.  $2x+2$
36. Let  $6x-8y=11$  be an equation of line  $M$  in the standard  $(x, y)$  coordinate plane. Line  $N$  has a slope that is half of the slope of line  $M$  and a  $y$ -intercept that is 4 more than the  $y$ -intercept of line  $M$ . Which of the following equations is line  $N$ ?
- A.  $y = \frac{3}{4}x - \frac{11}{8}$   
B.  $y = \frac{3}{8}x - \frac{15}{8}$   
C.  $y = \frac{3}{8}x + \frac{43}{8}$   
D.  $y = \frac{3x+21}{8}$   
E.  $y = \frac{3x-15}{8}$
37. A plane contains 14 horizontal and 14 vertical lines that divide the plane into multiple disjoint regions. How many of these regions have a finite, nonzero area?
- A.  $28$   
B.  $169$   
C.  $196$   
D.  $210$   
E.  $225$



38. All of the following create a unique plane in 3-dimensional Euclidean space except:
- A. 2 lines that intersect at only 1 point
  - B. 3 collinear points
  - C. 2 parallel lines that do not touch
  - D. 3 different non-collinear points
  - E. 3 vertices of a scalene triangle
39. Plane  $D$  contains line segment  $K$ , which measures 5 centimeters. How many lines in  $D$  are perpendicular to  $K$ ?
- A. 2
  - B. 3
  - C. 4
  - D. 5
  - E. Infinitely many
40. Every summer Andre drives to his cousin's house to visit. If he maintains a steady speed, he can reach his cousin's house in  $a+3$  hours. If Andre drives for  $b-2$  hours where  $b$  is less than  $a+5$ , what portion of his drive does he have left?
- A.  $\frac{-2}{a+3}$
  - B.  $a-b+5$
  - C.  $b-a+5$
  - D.  $\frac{a-b+5}{a+3}$
  - E.  $\frac{b-a-5}{a+3}$

**ANSWER KEY**

1. C	2. C	3. A	4. A	5. D	6. B	7. B	8. B	9. C	10. C	11. C	12. A	13. B	14. E
15. B	16. B	17. D	18. E	19. A	20. A	21. D	22. A	23. A	24. A	25. A	26. C	27. D	28. E
29. B	30. D	31. D	32. A	33. E	34. C	35. D	36. B	37. B	38. E	39. D	40. D	41. D	42. C

**ANSWER EXPLANATIONS**

- C.** As can be seen in the diagram,  $B$  is directly above the point where the  $x$  and  $y$  axis meet at the origin. Thus,  $B$ 's  $x$  and  $y$  values are 0. Since  $B$  is 4 cubes up along the  $z$  axis its  $z$  coordinate is 4, thus its  $(x, y, z)$  coordinate is  $(0, 0, 4)$ .
- C.** We can construct a right triangle whose hypotenuse is the line shown by drawing an altitude from the line up to the  $x$ -axis. Since the slope of the line is  $-\sqrt{3}$ , it moves down  $\sqrt{3}$  units for every 1 unit it moves to the right. Labeling the corresponding legs of the right triangle we drew, it becomes apparent that this is a  $30^\circ - 60^\circ - 90^\circ$  right triangle since the length of the sides follow the  $1 - \sqrt{3} - 2$  pattern. The angle whose vertex is at the origin is  $60^\circ$ , so the exterior angle  $\Phi$  is  $300^\circ$ .
- A.** The area of a triangle is  $A = \frac{1}{2}bh$ . The base of this triangle, if we define it as the side parallel to the  $x$  - axis, is  $11 - 3 = 8$ , and the height is the distance from the base to the top of the triangle,  $9 - 4 = 5$ . It doesn't matter what the  $x$  - value of the top of the triangle is. Plugging in the base and height into our formula yields  $A = \frac{1}{2} * 8 * 5 = 20$ .
- A.** The line tangent to the edge of a circle has a slope perpendicular to the radius of the circle at that point and passes through the same point on the edge of the circle. We can find the slope of the radius at  $(0, -8)$  by finding the change in  $y$  divided by the change in  $x$  between the desired point and the center of the circle at  $(-6, -4)$ :  $\frac{(-4) - (-8)}{(-6) - (0)} = -\frac{2}{3}$ . The slope tangent to this is the negative reciprocal:  $\frac{3}{2}$ . Since we have the  $y$  - intercept,  $-8$ , the easiest equation for a line to use is point-intercept form:  $y = mx + b$ . Plugging in our slope and  $y$  - intercept yields  $y = \frac{3}{2}x - 8$ .
- D.** The question asks for all the points where  $x - 2 < (x - 2)^3$ . This represents all values of  $x$  for which the corresponding  $y$  - value of the line in the graph is lower than the corresponding  $y$  - value of the cubic function in the graph. Looking at the graph, we can tell that this is between 0 and 1 as well as every value greater than 2. This becomes  $0 < x < 1$  and  $x > 2$ .
- B.** The shape in the graph is formed by a rectangle and two semicircles of different radii. The base and height of the rectangle are 16 and 14 and the radii of the circles are half of their diameters: 7 and 8, respectively. The area of a rectangle is  $A = bh$  and the area of a semicircle is half of a full circle, giving us:  $A = \frac{1}{2}\pi r^2$ . Solve for the respective shapes and simplify to yield  $224 + \frac{113}{2}\pi$ .
- B.** The point generated by rotating  $F$   $90^\circ$  counterclockwise about  $G$  will be at the same height and slightly to the left of  $G$ . The same height means we have the same  $y$  - value,  $g$ . The distance to the left is the difference between the heights of the original  $F$  point and  $G$ :  $f - g$ . Since this is the distance the new point is to the left of  $G$ , the  $x$  - value of the new point will be  $G$ 's  $x$  - value minus this, or,  $m - (f - g)$ . Simplification yields  $m - f + g$ . We put our  $x$  and  $y$  values into a coordinate pair to get  $(m - f + g, g)$ .
- D.** The given formula is equivalent to that of the distance formula, and D is the closest point to the graph.
- B.** The area of a triangle is  $\frac{1}{2}bh$ . Multiplying each vertex point of the triangle by  $\frac{3}{2}$  will yield a base and height  $\frac{3}{2}$  their original measure. Thus, our new area is  $\frac{1}{2}\left(\frac{3}{2}b\right)\left(\frac{3}{2}h\right)$ . Plugging in our values of base and height that can be seen in the diagram, we get  $\frac{1}{2}\left(\frac{3}{2} * 4\right)\left(\frac{3}{2} * 3\right)$ . Simplification yields  $\frac{27}{2} = 13.5$ .

10. **C.** The height of the triangle is not changed as long as the point is moved parallel to the line  $\overline{AB}$ . The slope of  $\overline{AB}$  is  $0$ , and it is the same as the slope of the line along which point  $C$  can be moved.
11. **C.** Since the  $x$  coordinate is multiplied by  $3$ , we can increase the  $y$  - value as long as  $3x$  is decreasing as fast or faster. Thus, we look to the point  $(-2,4)$ , since  $3$ , times the  $x$  - value,  $-6$ , outweighs the added  $4$  to the  $y$  - value. Plugging in this point to the equation for  $S$  yields  $-2$ .
12. **C.** We can tell that  $AB = AC$  because the distances it moves in the  $x$  - axis and the  $y$  -axis are of each correspond to the opposite axis in the other line. The total displacement is the same distance in a different direction. (Imagine taking  $5$  steps forward and then  $1$  step right, then returning where you started and taking  $1$  step forward and  $5$  steps right. You've moved the same distance from where you started each time, even though the direction you moved was different.) Since  $AB = AC$ , the triangle  $\triangle ABC$  is an isosceles triangle. The base is  $BC$ , which we can calculate using the Pythagorean theorem to be  $2\sqrt{2}$  and the height is the distance from  $A$  to the midpoint of  $BC$ , since the altitude of an isosceles triangle bisects the triangle. The midpoint of  $BC$  is  $(1,1)$ , so we can calculate the height of the triangle to be  $\sqrt{18} = 3\sqrt{2}$ . Plugging our base and height into our equation for the area of a triangle:  $A = \frac{1}{2}bh = \frac{1}{2} * 2\sqrt{2} * 3\sqrt{2} = 6$ .
13. **A.**  $\angle ABD$  is coterminal with the angle formed by  $\overline{AB}$  and the  $x$ -axis. We can construct a right-triangle that contains this angle by drawing an altitude from point  $A$  directly down to the  $x$  - axis. The triangle formed by  $\overline{AB}$  the altitude we just drew, and the  $x$  - axis can be used to solve for the cosine of the angle whose vertex is on the origin. Remember SohCahToa. The cosine of an angle equals the adjacent leg divided by the hypotenuse. Finding these values for the triangle we just drew, we get  $\cos(\angle ABD) = \frac{-2}{\sqrt{20}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ . We can be certain that the value should be negative because the cosine of all angles in the  $2^{nd}$  quadrant of the unit circle are negative.
14. **B.** Simply plug in  $x=5$  into the function to get  $f(5) = \frac{5^2 - 16}{5 - 3} = \frac{25 - 16}{5 - 3} = \frac{9}{2} = 4.5$ .
15. **E.** The range of the function contains all the  $y$  - values of the function. From looking at the graph, we can see that the function extends infinitely in the positive and negative directions along the  $y$  - axis, and does not have any horizontal asymptotes. Thus, its range is all real values.
16. **B.** From examining the graph, we can see that there is only one vertical asymptote. The slant asymptote is not included.
17. **B.** The graph shown is a cubic function. We can determine its exact formula by starting with our basic cubic function  $y = x^3$ . We then move it up  $8$  in order to match the graph's  $y$  - intercept. We now have the equation  $y = x^3 + 8$ . This equation matches the graph perfectly, since at  $x=2$ , we get  $y=16$ . The  $x$  - intercept of this graph can be found by finding where the  $y$  - value of the equation is  $0$ :  $0 = x^3 + 8$ . This becomes  $x^3 = -8$ . Simplify this to yield  $x = -2$ . In addition, because  $y = x^3$  has rotational symmetry, the point  $(-2,16)$  can be rotated about  $(0,8)$  by  $180^\circ$  to reveal that the point  $(-2,0)$  is also on the function.
18. **D.** The function looks like  $y = x^3$ , so we can tell it is a cubic function with just a visual inspection.
19. **E.** A reflection across the line  $y = x$  is the inverse function, where the  $x$  and corresponding  $y$  - values of a function are switched. Thus, from our original equation  $y = x^3 + 8$  we get  $x = y^3 + 8$ . Solving this equation for  $y$  gives us  $y = \sqrt[3]{x - 8}$ .
20. **A.** Our graph must be a discrete function ( $x$  values are only allowed at certain values, typically integers or whole numbers) since it would not make sense to have half a passenger unless you have a morbid imagination. The only choice that constrains our domain to whole numbers is A.

21. **E.** The amount of money that Tina makes has a linear relationship with the number of t-shirts she sells, so the graph must be linear. The only linear graph is E.
22. **D.** We can determine which angles have the same terminal side by adding  $360^\circ$  and checking whether it corresponds to the given list of angles and repeating the process until we've gone to or past the furthest angle. In this case, with  $-2493^\circ$  we get  $-2133^\circ, -1733^\circ, -1413^\circ, -1053^\circ, -693^\circ, -333^\circ, 27^\circ$ .
23. **D.** We can sketch a circle and place  $W$  arbitrarily to begin. Place  $X$  slightly clockwise and label the distance between them  $4$ . Then place  $Z$  a good distance counterclockwise from  $W$ . We know that it does not go past  $X$  because the distance clockwise from  $W$  to get  $Z$ ,  $15$ , plus the distance counterclockwise from  $W$  to get  $X$ ,  $4$ , is less than the full circumference of the circle,  $23$ . Thus, they do not overlap. Then, starting from  $X$ , we place  $Y$   $9$  units counterclockwise, which goes past  $W$ , since that is only  $4$  units. Thus, our order starting counterclockwise from  $X$  is  $X, W, Y, Z$ .
24. **C.** We can find the center of the circle by taking the midpoint of  $(3,8)$  and  $(-3,4)$ . We find that the midpoint is  $(0,6)$ . The answer is  $(-3,8)$  because it is the same distance up as  $(3,8)$ , and an equal and opposite distance in the  $x$ -direction. Since the direction of movement doesn't matter to the distance, the resulting point is the same distance from the center as the diameter's endpoint, which means it is on the circle.
25. **A** If you have a graphing calculator, you can solve for  $y$  and graph the result. However, it is faster to recognize that from the general form for circles  $(x - x_1) + (y - y_1) = r^2$  that this equation represents a circle with an origin at  $(0,0)$  and a radius of  $13$ . Thus, it crosses the  $x$ -axis  $13$  units right and left of the origin, at  $(-13,0)$  and  $(13,0)$ .
26. **A.** The area of a circle is  $\pi r^2$ . We can solve for  $r$  by calculating the distance between the center and the point the circle passes through using the Pythagorean theorem. We find that  $r$  equals  $\sqrt{109}$ . Plugging this into our equation yields  $A = \pi(\sqrt{109})^2 = 109\pi$ .
27. **C.** The leftmost point of the circle must be touching the  $y$ -axis (it must be the leftmost point since the  $y$ -axis is tangent to the circle at this point, and so it cannot cross over twice, which it would do if it were any further left). We then measure the distance from the center of the circle at  $(4,7)$ , to the corresponding point on circle tangent to the  $y$ -axis, which happens to be the  $x$ -value of the center,  $4$ . We check to see if  $(3,1)$  is inside, on, or outside the circle by measuring its distance from the center. Since it is greater than the radius of  $4$ , it is outside the circle.
28. **D.** Because the distance between  $(-1,-1)$  and  $(5,5)$  is  $6$  units in the  $x$  direction and the  $y$  direction, the only two points that will be  $6$  units away from both  $(-1,-1)$  and  $(5,5)$  (possibilities for the center of the circle) are  $(5,-1)$  and  $(-1,5)$ .
29. **E.** Sketching the triangle shows that the two legs of the triangle have lengths  $10$  and  $7$ , respectively. Plugging these values into the Pythagorean theorem yields  $\sqrt{149}$ .
30. **B.** The altitude of an isosceles triangle bisects the base. Since the base has a slope of zero, this means that the peak of the triangle,  $Q$ , lies on the line drawn vertically through the midpoint of  $\overline{PR}$ . The  $x$ -value of the point is thus the average of the  $x$ -values of  $P$  and  $R$ :  $\frac{-5+1}{2} = -2$ .
31. **D.** Answer choices A, B, and C can be disproven by moving  $5$  units right, up, or left from  $(3,4)$ . Answer choice E can be disproven since its distance from  $(3,4)$  is  $5$ , as can be shown in the Pythagorean theorem. This leaves answer choice D.
32. **D.** The diagonals of a rectangle are congruent. Thus, we can solve for  $ZY$ , which has the same value as  $XW$ . Plugging in the difference in  $x$ -values and difference in  $y$ -values into the Pythagorean theorem yields  $\sqrt{34}$ .

- 33. A.** This problem is simply the rote work of adding together the distance between each set of adjacent points of the quadrilateral, using the formula  $\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , the Pythagorean theorem. Plugging in each pair of adjacent points, we find that the distance of each side of the quadrilateral is  $\sqrt{10}$ . Adding all of them together, we get  $4\sqrt{10}$ .
- 34. E.** A  $4^{\text{th}}$  degree polynomial function can only cross the  $x$ -axis a maximum of 4 times. This is because the term with the highest degree will be some term containing  $x^4$ . This means that we cannot express the product of five instances of  $(x \pm n)$  where  $n$  is some constant, and so we cannot have five  $x$ -intercepts. Therefore, a  $4^{\text{th}}$  degree polynomial cannot cross or touch the  $x$ -axis five times.
- 35. C.** We find the distance between two points using the Pythagorean theorem, as expressed in the formula  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ . Plugging in the points from the equation, we get  $\sqrt{(3x - x)^2 + ((x + 5) - (x - 3))^2} = \sqrt{(2x)^2 + (8)^2} = \sqrt{4x^2 + 64} = \sqrt{4x^2 + 16} = 2\sqrt{x^2 + 16}$ .
- 36. D.** The easiest way to solve this problem is to first express line  $M$  in point-slope form. We isolate the  $y$  term from the rest of the equation by moving the  $x$  term to the other side and get  $-8y = -6x + 11$ . We then isolate  $y$  completely by dividing by its coefficient and get  $y = \frac{-6}{-8}x + \frac{11}{-8}$ . This simplifies to  $y = \frac{3}{4}x - \frac{11}{8}$ . Now we find the equation for line  $N$  by expressing a line with half the slope and a  $y$ -intercept for higher than in line  $M$ . The slope of line  $M$  is  $\frac{3}{4}$ , so the slope of line  $N$  is half this,  $\frac{3}{8}$ . The  $y$ -intercept of line  $N$  is 4 greater than line  $M$ 's:  $-\frac{11}{8} + 4 = -\frac{11}{8} + \frac{32}{8} = \frac{21}{8}$ . Thus, the equation for line  $N$  is  $y = \frac{3}{8}x + \frac{21}{8}$ , or  $y = \frac{3x + 21}{8}$ .
- 37. B.** The easiest way to solve this problem is to discover the general rule behind it. Imagine a tic-tac-toe board. It is made of 2 horizontal and 2 vertical lines. There is only one box that is completely bounded on all sides, and thus has finite, nonzero area. If you drew a larger tic-tac-toe board by using 3 horizontal and 3 vertical lines, you can see quite easily by sketching it that it contains 4 completely bounded boxes. The general rule we can deduce is that the number of bounded regions is the square of 1 less than the number of lines that are horizontal and vertical. In this case, there are 14 horizontal and vertical lines, so the solution is  $(14 - 1)^2 = 169$ .
- 38. B.** A plane in geometry is constrained by at least three non-collinear points. This is all you need to know to answer this question since all of the correct answers are some extension of this. For example, 2 lines that intersect at only one point have infinitely many non-collinear points, since there are an infinite number of points on one line that are not on the other line. The same is true of 2 parallel lines that do not touch. The 3 vertices of a scalene triangle, of any triangle, are non-collinear points. The only answer choice that does not contain 3 non-collinear points is B. 3 collinear points.
- 39. E.** There are an infinite number of points on a line segment, and by extension, there are an infinite number of points for a perpendicular line to cross through. Crossing through a different point makes these lines unique, so there are an infinite number of perpendicular lines.
- 40. D.** The fraction of the drive time remaining is the drive time left divided by the total drive time. The drive left is how long he needs to drive minus the time he has already driven:  $(a + 3) - (b - 2)$ . The total drive time is  $(a + 3)$ , so our formula can be expressed here as  $\frac{(a + 3) - (b - 2)}{a + 3}$ . Simplifying this yields  $\frac{a - b + 5}{a + 3}$ .