- 1. In a science class, Bob scored 95 on one test, 89 on another test, and had an average test score of 83 in the class before taking those two tests. If Bob's average test score for the entire class, including all tests, is 85, and each test he takes in the course is weighted equally, how many tests has Bob taken in the class?
  - **A.** 3
  - **B.** 9
  - **C.** 7
  - **D.** 14
  - E. Cannot be determined from the given information
- 2. Mary's test average after 8 tests is 80. Her score on the 9th test was 71. If all 9 tests are weighted equally, what is Mary's test average for all 9 tests?
  - **A.** 71
  - **B.** 75.5
  - C. 77
  - **D.** 79
  - E. 80
- 3. Theo has taken 6 of 7 equally weighted tests in his Chemistry class and has an average score of exactly 81 points. What must he score on the 7<sup>th</sup> test to bring his average up to 83 points?
  - **A.** 95
  - **B.** 93
  - C. 85
  - **D.** 83
  - E. 82
- 4. The mean of 6 numbers is 27. The smallest of the 6 numbers is 12. What is the mean of the other 5 numbers?
  - A. 27
  - B. 30
  - C.  $30\frac{3}{4}$
  - **D.**  $41\frac{2}{3}$
  - E. 42

- **5.** The 7 positive integers *a*, *a*, *a*, *a*, *a*, *b*, *c* have an average of *a*. Which of the following equations must be true?
  - A. b = c
  - **B.** b + c = a
  - C. b+c=2a
  - **D.** c = -b
  - **E.** b + c = 0
  - Each element in a data set is increased by 3 then divided by 7. If  $\mu$  is the mean of the final data set, what is the mean of the original set?
    - A.  $\frac{\mu}{7} + 3$
    - **B.**  $\frac{\mu + 3}{7}$
    - C.  $\frac{\mu 3}{7}$
    - **D.**  $7\mu 3$
    - **E.**  $7\mu + 3$
- 7. The average of a set of 8 numbers is 13. When a 9<sup>th</sup> number is added to the set, the average increases to 16. What is the 9<sup>th</sup> number?
  - **A.** 16
  - **B.** 20
  - C. 29
  - **D.** 32
  - E. 40
- 8. A music concert was rated on a 5 point scale by the audience. 10% gave a 1, 18% gave a 2, 33% gave a 3, 20% gave a 4, and 19% gave a 5. To the nearest tenth, what is the average rating given by the audience?
  - **A.** 2.5
  - **B.** 2.8
  - **C.** 3.2
  - **D.** 3.4
  - E. 4.6

- 9. In a town of 600 people, 250 males have an average age of 43, and 350 females have an average age of 38. To the nearest whole number, what is the average age of the entire town?
  - **A.** 38
  - **B.** 39
  - C. 40
  - **D.** 41
  - E. 42
- 10. Each of 16 students took a test and received a whole number of points. The median of the scores was 78, and 25% of the students scored 74 or below. No student received a score of 78. How many students scored 75, 76, or 77?
  - **A.** 2
  - **B.** 3
  - **C.** 4
  - **D.** 5
  - E. 8
- 11. Nick has 30 collectible coins. He paid \$36.50 for each coin 2 years ago. The coins are currently valued at \$38.15. To the nearest cent, how much *more* must the average value per coin rise for the combined value of these 30 coins to be \$350.00 more than Nick paid for them?
  - **A.** \$8
  - **B.** \$8.50
  - C. \$10.02
  - **D.** \$11.67
  - E. \$300.50
- 12. A data set has 20 elements. A second data set of 20 elements is obtained by adding 7 to each element of the first set. A third data set of 20 elements is obtained by multiplying each element of the second data set by 3. The median of the third data set is 63. What is the median of the first data set?
  - A. 7
  - B. 14
  - C. 21
  - **D.** 196
  - E. 210

13. What is the median of the following test scores?

- **A.** 64
- **B.** 73
- **C.** 78
- **D.** 83
- E. 92
- 14. Each number on a list of 8 numbers is multiplied by 13 to produce a 2<sup>nd</sup> list of 8 numbers. Each of the 8 numbers on the second list is increased by 4 to produce a 3<sup>rd</sup> list of 8 numbers. The median of the 3<sup>rd</sup> list is *x*. What is the median of the first list?
  - A.  $\frac{x-4}{13}$
  - **B.**  $\frac{x}{13}$
  - C. x 4
  - D.  $\frac{x}{13} 4$
  - E. Cannot be determined from the information given
- 15. List B contains all the elements of List A including integers x, y, and z, where  $x \ge 43$ , y = z,  $y \le 18$ . What is the median of List B?

- **A.** 27
- **B.** 30
- **C.** 33
- **D.** 36
- E. 39

16. The following table shows the total number of cabinets built in a workshop over 50 consecutive days. What is the average number of cabinets built per day, rounded to the nearest tenth?

Number of cabinets built in a day (Output)	Number of days with this output	
0	5	
1	11	
2	10	
3	18	
4	6	

- **A.** 2
- **B.** 2.1
- C. 2.2
- **D.** 3
- E. 3.3

Use the following information to answer questions 17-19.

Mr. Rivera gives his 20 students a progress report once a month that gives a student their scores on tests, quizzes, and homework, and the average score of all 20 students. The following is a progress report for Isaac Shemtov.

Student: Isaac Shemtov					
Task	Possible Points	Student score	Class average		
Homework #1	100	89	93		
Homework #2	100	94	90		
Quiz #1	100	75	80		
Quiz #2	100	87	85		
Quiz #3	100	100	95		
Quiz #4	100	89	78		
Test #1	100	85	83		
Test #2	100	93	85		
Test #3	100	95	90		

- 17. Isaac is about to take Test #4. What score does he need to get an average test score of 93?
  - **A.** 91
  - **B.** 93
  - C. 95
  - **D.** 97
  - E. 99

- **18.** In Mr. Rivera's class, a homework assignment that has not been turned in receives a score of 0. Of the 20 students in his class, what is the maximum number of students who could have not turned in homework #2?
  - **A.** 0
  - **B.** 1
  - C. 2 D. 3
  - E. Cannot be determined from the given information
- 19. What is Isaac's average quiz score to the nearest point?
  - **A.** 86
  - **B.** 87
  - C. 88
  - **D.** 89
  - E. 90
- **20.** The 6 positive integers a, a, b, b, c, and d have an average of c. What is the value of a + b?
  - A.  $\frac{5c-a}{2}$
  - **B.** 5c + d
  - C.  $\frac{5c+d}{2}$
  - **D.** 3c + d
  - E. Cannot determine from the given information.
- 21. In a biology course, a student scored 94 on one test, 100 on another test, and 90 on each of the other tests. The student's final test average was 91.75. How many tests did the student take?
  - **A.** 6 **B.** 7
  - B. 7
  - **C.** 8
  - D. 9E. 10
- **22.** What is the median of the following set?

$$-5,8,3,17,7,12,100,7,0,20$$

- **A.** 7
- **B.** 7.5
- C. 8
- **D.** 9.5
- E. 16.9

3

- 23. The median of a set of data containing 12 items, all of different values, was found. Three data items, a,b, and c, were added to the set. a was greater than the original median, but less than all of the original data greater than the median. b was greater than all of the original data. c was equal to the original median. What is the new median of the set?
  - **A.** *a*
  - **B.** *b*
  - C. c
  - **D.** The average of a and c
  - E. Cannot be determined from the given information.
- **24.** What is the mode of the following data?

$$-3,-1,0,3,5,5,8,9,10$$

- **A.** 0
- B. 4
- **C.** 5
- **D.** 5.7
- **E.** 6
- **25.** Mr. Harker organized his students' test grades into a stem and leaf plot in order to see the frequency of different letter grades. What is the range of the set of test grades?

Stem	Leaf		
6	7 9		
7	2 3 6		
8	002		
9	1459		

- **A.** 2
- **B.** 9
- **C.** 32
- **D.** 40
- E. 166
- **26.** To decrease the mean of 5 numbers by 2, by how much would the sum of the 5 numbers have to decrease?
  - **A.** 2
  - **B.** 5
  - C. 7
  - **D.** 10
  - E. 17
- **27.** A data set has 4 members. The mode of the data set is both 6 and 8. What are the mean and median of the data set, respectively?
  - **A.** 2,7
  - **B.** 6,8
  - C. 7,7
  - **D.** 24,6
  - E. 24,7

**28.** What is the mode of the data given below?

- **A.** 12.5
- **B.** 13
- C. 16.5
- **D.** 20.5
- E. 31
- **29.** The stem and leaf plot below shows the number of walkins to The Doctor's Office during a 30-day observation period. What is the median number of daily walk-ins?

Stem	Leaf
2	2 3 3 4 7 9
3	2 3 3 4 7 9 1 1 4 4 5 5 9
4	0 0 2 3 7 7 7 8
5	0 3 4 6 6
6	1 3 7 8
	Kev: 2   2 = 22

- A. 41
- **B.** 42
- **C.** 43
- D. 44E. 47
- **30.** Mr. Eames was hired for a new job and was requested to keep a log of all his working hours. He recorded the number of hours he worked each day in a table, as shown below. What was the mean number of hours he spent per day?

ĺ	Day	Monday	Tuesday	Wednesday	Thursday	Friday
	Hours Worked	7	4	6	4	9

- **A.** 4
- **B.** 6
- **C.** 7
- D. 9E. 13
- **31.** Which of the following statements is correct concerning the data set below?

- A. Its mean is 67
- **B.** Its median is approximately 63.8
- C. Its range is 98
- **D.** Its mode is 47
- E. Its median is not a member of the data set

- **32.** The signs of *a* and *d* are positive, but the signs of *b* and *c* are negative. If it can be determined, what are the signs of the mean and median of the four numbers, respectively?
  - A. Both positive
  - **B.** Both negative
  - **C.** Both neither (zero)
  - **D.** The mean is neither, the median is negative
  - E. Cannot be determined from the given information
- **33.** The 8 consecutive integers below add up to 332.

x-3

x-2

x-1

X

x + 1

x + 2x + 3

x + 4

What is the value of x + 2?

**A.** 40

**B.** 41

**C.** 42

**D.** 43

E. 44

- 34. A new show was in town for the past 7 days. The average attendance of the slowest and busiest night was 224 people. The average attendance for the other 5 days was 311. How many people attended the show over the course of 7 days?
  - **A.** 1704
  - **B.** 1405
  - C. 2093
  - **D.** 2003
  - E. 1967

AVERAGES ANSWERS

## **ANSWER KEY**

1. B 2. D 3. A 4. B 5. C 6. D 7. E 8. C 9. C 10. C 11. C 12. B 13. C 14. A 15. C 16. C 17. E 18. C 19. C 20. A 21. C 22. B 23. A 24. C 25. C 26. D 27. C 28. B 29. A 30. B 31. D 32. E 33. D 34. D

## ANSWER EXPLANATIONS

- 1. **B.** If the average of Bob's test scores for the entire class is 85, that means the sum of his test scores can be represented as 85x, where x = the number of tests he took in total. We know two of his test scores are 95 and 89, and the average of the test scores before those two tests was 83. So, the sum of his test scores can also be represented by 83(x-2)+95+89. Setting the two equations expressions to each other, we get 85x = 83(x-2)+95+89. Distributing the 83 on the right hand side, we get 85x = 83x 166 + 95 + 89. Combining like terms, we get 2x = 18. So, x = 9.
- 2. **D.** Mary's average after 8 tests was 80, so the sum of those 8 test scores is 80\*8=640. Now, adding the  $9^{th}$  test score, we have 640+71=711. So, the average of her 9 test scores is the sum of the 9 test scores divided by 9. That is,  $\frac{711}{9}=79$ .
- 3. A. Theo's average after 6 tests was 81, so the sum of those 6 test scores is 81\*6=486. If he wants the average of 7 tests to be 83, then the sum of the 7 tests should be 83\*7=581. Since we already know the sum of her 6 test scores, the  $7^{th}$  test score can be calculated as 581-486=95.
- 4. **B.** If the mean of 6 numbers is 27, then the sum of the 6 numbers is 27\*6=162. We know that the smallest number is 12, so if we take away that number, we know that the sum of the remaining 5 numbers is 162-12=150. This means that the average of those 5 numbers is  $\frac{150}{5}=30$ .
- 5. C. We can use the formula for finding the average in order to solve this problem. First, the average, a, is equal to  $\frac{a+a+a+a+b+c}{7}$ . Through algebraic calculation, we can find the 7a = 5a+b+c. Therefore, 2a = b+c.
- 6. **D.** If every element in a data set is increased by 3 and then divided by 7, then the average would also be increased by 3 and divided by 7. So, if  $\mu$  is the mean of the final data set, then  $\mu = \frac{x+3}{7}$  is true for the original mean = x. Solving for x now, we get  $7\mu = x+3$  so  $x = 7\mu 3$ .
- 7. E. If the average of 8 numbers is 13, then the sum of those 8 numbers is 8\*13=104. When a  $9^{th}$  number is included, the average changes to 16, so the sum of those 9 numbers is 9(16)=144. The  $9^{th}$  number can then be calculated as 144-104=40.
- 8. C. In this case—to make the math easier—we can view each percent of the audience as one count (one person). If 10% of the audience gave the concert a 1, then the concert has 10\*1=10 points from the 10%. Multiplying out the other percentages and points, we have 18\*2=36, 33\*3=99, 20\*4=80, and 19\*5=95. Adding these points together, we get the total number of points accumulated by the ratings. We get 10+36+99+80+95=320. The average can then be calculated by dividing this sum by the total of 100 counts.  $\frac{320}{100}=3.2$ .
- 9. C. Since the average of 250 males is 43, then the sum of their ages is 250\*43=10750. Likewise, the average of 350 females is 38, so the sum of their ages is 350\*38=13300. Adding these two sums together we get 10750+13300=24050. To find the average of the total 600 people, we divide this sum by 600 to get  $\frac{24050}{600}=40.08$ . Rounding this to the nearest whole number, we get the average age = 40.

6 CHAPTER 8

- 10. C. If 25% of the students scored 74 or below, then 4 students scored 74 or below. If the median score was 78, but no one scored a 78, then the 8<sup>th</sup> and 9<sup>th</sup> numbers average out to be 78. This means that the 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, and 8<sup>th</sup> numbers are either 75, 76, or 77. This means 4 students scored 75, 76, or 77.
- 11. C. \$350.00 more than what Nick paid for the coins is 350+36.5(30)=1445. This means that each coin must be worth  $\frac{1445}{30}=48.17$ . This is 48.17-38.15=10.02 more than the coins' current value.
- 12. **B.** As we learned in the lesson, operations on a set of elements don't change the order of the set, so the median for a previous set of numbers can simply be found by applying those operations to the newest median in reverse order. In this case, we can divide 63 by 3 and then subtract by 7.  $63 \div 3 = 21$ . 21 7 = 14. The median of the first set of elements is 14. If we didn't know that, we can also solve this questions algebraically.  $x_1, x_2, x_3 \dots x_{20}$  denotes the 20 elements in the first data set, then the second data set is  $x_1 + 7, x_2 + 7 \dots x_{20} + 7$  and the third data set is  $3(x_1 + 7), 3(x_2 + 7), 3(x_2 + 7), 3(x_{20} + 7)$ . The median of the third data set is 63, which means the average of the two middle terms is 63.  $\frac{3(x_{10} + 7) + 3(x_{11} + 7)}{2} = 63$ . We want to manipulate this equation to find the median of the first data set, which is  $\frac{x_{10} + x_{11}}{2}$ . Distributing the 3's in the numerator and simplifying, we get  $\frac{3x_{10} + 3x_{11} + 42}{2} = 63$ . Multiplying by 2 on both sides, we get  $\frac{x_{10} + x_{11}}{2} = 14$ . Subtracting 42 on both sides, we get  $\frac{x_{10} + x_{11}}{2} = 14$ .
- 13. C. Putting the scores in numerical order, we get  $\{40,56,61,64,78,83,88,92,93\}$ . Now, canceling out the largest and smallest numbers one by one, we arrive at the median.  $\{40,56,61,64,78,83,88,92,93\}$ . So, the median is 78.
- 14. A. This question can be solved the same way question 12 was solved. Because we know that the operations don't change the order of the elements in the set, the old median can be found by applying the operations to the new medians in reverse order. For this problem, we take x and subtract 4 to get the median of the second list: x-4. Then, we divide it all by 13 to get the median of the first list:  $\frac{x-4}{13}$ .
- 15. C. Integer x comes somewhere after 43 in list B, and both y and z come before 18. Because these reference points are at least a few numbers away from the current median, we do not need to know their specific locations to find the median. Let us arbitrarily place the integers into the list as such:  $\{12, y, z, 17, 18, 30, 30, 36, 42, 43, 48, 48, x, 51\}$ . It does not matter where we place x, y, and z as long as they satisfy their inequalities, again, because they are far enough from the center to not impact the median in an unpredictable way. Canceling out the largest and smallest numbers one by one, we are left with  $\{30,36\}$ . Since we have two numbers, we take the average between them and get the median: 33. If this problem used reference points that were not so clearly on the left or right of the median, only then would we be unable to find a solution.
- 16. C. The average number of cabinets built per day is equal to the TOTAL number of cabinets built over the time period divided by the TOTAL number of days. We multiply each of the possible number of cabinets built in a day by the number of days when that output was reached and add together each of our results to get the total number of cabinets built: 5\*0+11\*1+10\*2+18\*3+6\*4=109. We divide by the number of days:  $\frac{109}{50}=2.18$ , which rounds to 2.2.
- 17. E. Isaac's average test score after he takes another test will be  $\frac{85+93+95+n}{4}$  where n is his score on the newest test. We can find out what n must be to get an average of 93 by setting the equation equal to 93:  $\frac{85+93+95+n}{4} = 93$ . This becomes 85+93+95+n=372. Isolating n, n=99.

- 18. C. We can find the maximum number of students who could have not turned in homework #2 by seeing how many students with the maximum score it would take to reach or exceed the class average. The idea is if we want some kids to have zero points, then the rest must make up for those zeroes as much as possible. This is  $\frac{n(0)+x(100)}{20} \ge 90$  where n is the number of students who did not turn in the homework, and x is the number who did and received 100. Together, x+n=20 Solving this gives us  $x\ge 18$ . Since x has a minimum value of 18, x has a maximum value of 2.
- 19. C. Isaac's average quiz score is  $\frac{75+87+100+89}{4} = \frac{351}{4} = 87.75$ , which rounds to 88.
- **20.** A. The average of the 6 integers is  $\frac{a+a+b+b+c+d}{6} = c$ . This becomes a+a+b+b+c+d=6c. We simplify to 2(a+b)+c+d=6c. Isolate (a+b): 2(a+b)=5c-d.  $a+b=\frac{5c-d}{2}$ .
- 21. C. Let be the number of tests the student scored 90 on. The average score of all the tests will be  $\frac{91+100+n(90)}{n+2}$ . Setting this equal to 91.75 and restructuring the equation gives us 94+100+n(90)=91.75(n+2). Simplify: 90n+194=91.75n+183.5. Simplifying further, 1.75n=10.5. Then, n=6. Since the student took 2 tests besides the ones she scored 90 on, the total number of test is n+2=8.
- 22. B. Reorganizing the set in ascending order gives us -5, 0, 3, 7, 7, 8, 12, 17, 20, 100. Crossing off the lowest and highest number one by one leaves us with 7,8. Since we have two numbers left, we average them to get 7.5.
- 24. C. The mode is the data value that appears most often. In this case, 5 occurs twice, and all other values occur once, making 5 the mode.
- 25. C. The stem in a stem and leaf plot is typically the largest place digit in common with the rest of the data set. Thus, if a number which has a 3 in the stem column and a 2 in the leaf column represents a 32. The range of a set of data is the difference between the greatest and smallest number. Therefore, the range for this stem plot would be 99-67=32.
- 26. **D.** The best way to do this problem would be to pick the simplest numbers and plug them into the problem. Let us use 1,2,3,4, and 5. The mean of this set would be  $\frac{15}{3}$ , or 3. If we decrease this mean by 2, we get the mean of 1. This means that the new sum, or x, divided by the number of numbers, 5, is now equal to 1. Through algebraic calculation, we know that the new sum, x, would have to be 5. From the original sum of 15 to the new sum of 5, there would have to be a decrease of 10.
- 27. C. In a data set of four members, 2 of the members must be 6 and the other 2 of the members must be 8 in order to have a mode of both 6 and 8. If there were only 1 member each of 6 and 8, and the 2 remaining members were different numbers, then each member would occur exactly once and there would be no mode. Therefore, the data set is 6,6,8,8. The mean of the set is  $\frac{28}{4}$ , or 7. The median of the set, since there are two remaining numbers, is the average of 6 and 8, or 7.

8 CHAPTER 8

- 28. B. The mode of a set is the number that occurs the most. In this set the number 13 occurs twice, and all other numbers occur only once. The mode is 13.
- 29. A. The median is the number in the middle of an ordered set. Thanks to the stem and leaf plot, the data set is already in order. All that is left to do is to begin crossing out numbers from the beginning, 22,23,23, and then cross out the same amount of numbers from the end, 68,67,63, until we reach 40 and 42, which we average to get the median: 41.
- 30. B. The mean is the statistical term for the average. To find the mean in a data set, we add all the numbers, then divide by the total number of members in the set. In this case,  $\frac{4+4+6+7+9}{5} = 6$ .
- 31. D. One value appears more often than any other values in the data set: 47. This makes it the mode.
- 32. E. A common mistake would be to assume that two negative and two positive numbers always cancel each other out by summing to zero, when that is not always the case. We know that when summing positive and negative numbers, the answer could be positive *or* negative. Diving by the total number of numbers to get the mean will not change the sign of that sum. Thus, the mean's sign cannot be determined. The median, because we are dealing with an even count of numbers (4 numbers), is the mean of the two center numbers in the set. This mean is again the average of a combination of positive and negative numbers, so its sign cannot be determined either. Another approach would be to make up several cases of numbers that have differing means, medians, etc.
- 33. D. An easy way to add the 8 consecutive integers is to add up the x values and constant values separately, and only then add those two sums together. Adding up the x values, we see that there are 8 x's, so their sum is 8x. Then we add the constants: -3-2-1+0+1+2+3+4. -3-2-1 cancels out with 1+2+3, so the sum of the constants is equal to 4. Now, we can add the values and constant values together to get as the total sum. This, we are given, is equal to 332. So solving this equation, we get  $8x+4=332 \rightarrow 8x=328 \rightarrow x=41$ . We are asked to find the value of x+2 which is 41+2=43.
- 34. D. The average of the slowest and busiest nights was 224 which means that the total attendance in those two days was 2\*224=448. The average of the other 5 days was 311 which means that the total attendance for those 5 days was 5\*311=1555. Then, the total attendance for the week is 448+1555=2003. Remember, sum=average(# of items).