CHAPTER

22 GRAPH BEHAVIOR

SKILLS TO KNOW

NOTE: Much of this chapter does not occur on every ACT. Focus on other areas first if you're scoring below a 30 on the math.

- Finding domain and range
- Recognizing when a function is undefined
- Recognizing horizontal and vertical asymptotes
- Recognize basic parent graphs/functions and understand transformations
- How coefficients and constants affect functions
- How to identify and execute compressions
- Less common but still relevant topics:
    - o Trigonometric functions and graphs (these have their own chapter in Book 2 and are not directly covered in this chapter, except for rules that apply to all graphs)
    - o Even, odd, and one-to-one functions
- Understand translations and reflections, along with associated vocabulary

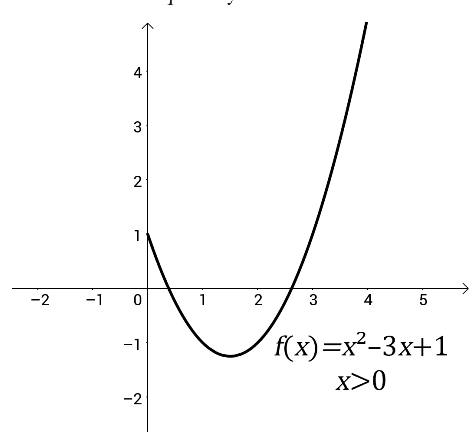
Note: We cover some graph behavior topics in Chapter 16: Quadratics and Polynomials. We recommend doing that chapter first.

DOMAIN AND RANGE

Two terms you'll need to know to interpret graphs on the ACT® are domain and range.

DOMAIN: all the possible values of x in a function or relation. If it's a possible value of x, it is part of the domain. You can find the domain of a function if you have its equation, its full set of values, or its graph.

Domains can be continuous. The graph below represents a function in which all values of x are greater than zero, so the domain of the inequality is $x > 0$.



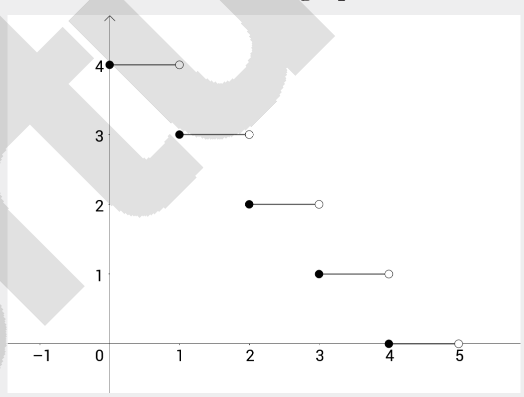
Domains can also be piecewise, meaning made up of a collection of different intervals of values or individual values, or a set of specific numbers. In such cases, the domain is only the specific x -values for which the function is defined, not the values' entire range.

The chart below shows points that represent values of the given function. What is the domain of the function?

x	y
1	3
2	4
3	5
4	8
5	2

The only possible x -values are all real integers between zero and 6, i.e. 1, 2, 3, 4, 5. So the domain of this function is $\{1,2,3,4,5\}$.

One of the following sets is the domain for the function graphed below. Which set is it?



A. $\{0,1,2,3,4\}$

B. $\{1,2,3,4,5\}$

C. $\{x \mid 0 \le x \le 5\}$

D. $\{x \mid 0 \le x < 4\}$

E. $\{x \mid 0 \le x < 5\}$

We can eliminate choices A and B because they indicate that the function is only defined at integer values, which we can clearly see is not true because of the line segments that extend from each dot on the graph.

Although this piecewise function is composed of several separate parts, if you choose any point for $0 \le x < 5$, you will always find a unique y value to match it to, so therefore the function is defined for all $x \mid 0 \le x < 5$. Though you may be tempted by answer choice C, this choice erroneously includes $x = 5$ in the domain. This point should not be included because there is an open circle at $x = 5$, which indicates that 5 is not a possible value of x.

Answer: **E.**

RANGE: the complete set of y –values (or $f(x)$ values) where a function or relation is defined.

TIP: Remember that the word **RANGE** means **three different things** in math:

1. **Range** can simply mean a **span of values**. For example **a word problem** might say "At a drugstore, the cost of a box of tissues **ranges** from \$2.00 to \$5.00, inclusive, that means that boxes of tissues can be as inexpensive as \$2.00, as expensive as \$5.00, or anything in between.
2. When you are dealing with **data sets** and measures of central tendency (i.e. **averages, medians, modes, etc.**) the **range** is the **largest number in a set minus the smallest number** in the set. In these cases "range" will be a single number.
3. When you are dealing with **a function** (or relation, i.e. anything that can be written as an equation, denoted by a graph in the standard coordinate plane, or written as a set of paired values), as we are in this chapter, the **range** is **all the possible values of** y **or** $f(x)$. (Remember $f(x)$ and y are interchangeable ideas).

Because the ACT® is a multiple-choice test, if you don't know which "range" you are being asked for, you can usually figure out which meaning of range is at play by looking at the answer choices.

What is the range of the function $\frac{3x^2-2x+4}{x+2}$ on the interval $(2<x<5)$?

A. $y \neq -2$ **B.** $(3<y<\infty)$ **C.** $\left(3<y<\frac{69}{7}\right)$ **D.** $\left(-\infty<y<\frac{69}{7}\right)$ **E.** $(-\infty<y<\infty)$

This question is simply asking what y-values the function takes on the given interval. First, we plug the function into a graphing calculator to make sure that there isn't a local maximum within the interval. After discovering that the function is roughly linear on that interval, we can plug in the endpoints to get our range. Plugging in $x=2$ gives $f(2)=3$ and $x=5$ gives $f(5)=\frac{69}{7}$. Therefore our range is from 3 to $\frac{69}{7}$, non-inclusive.

Answer: **C**.

GENERAL RULES FOR POLYNOMIALS

Domain & Range

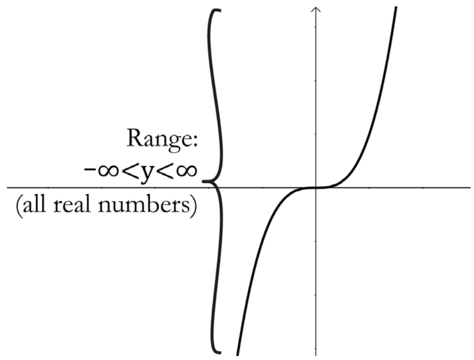
The **domain** of the following types of functions is **all real numbers**:

- Linear Equations, ex: $2x+1$
- Polynomials, ex: x^7+2x
- Power Functions, ex: $y=x^9$

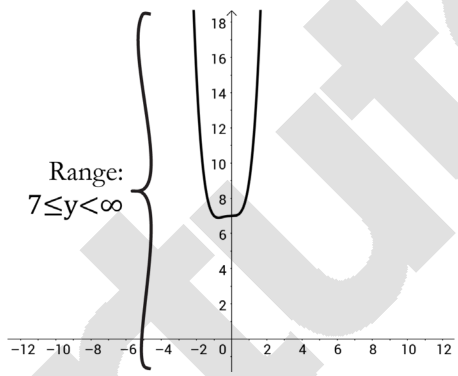
In other words, you can plug ANY VALUE of x into these types of equations and expect to get an answer.

Range of polynomial functions:

- The range of **odd degree polynomial** functions (Example: $y = x^3$) is **all real numbers**.



- The range of **even degree polynomial functions** (Example: $y = x^4 + x^3 + 7$) is **all numbers greater than the minimum value or less than the maximum value**.



In other words, there will be some curve's vertex, such as the vertex of a parabola, that is a minimum or maximum y-value. The range is all the values to one side of that vertex such that the function is defined.

Undefined Values in Domain

For rational functions, the domain excludes points when the denominator equals zero. Whatever values of x make that denominator equal to zero, thus, are excluded from the domain.

There are two cases possible when a denominator equals zero: **holes** and **vertical asymptotes**.

- These points denote a **"hole"** or removable discontinuity in the graph. A hole occurs when the denominator is a factor of the numerator, and when simplified, the factored element in the numerator cancels with the expression in the denominator.
  - For example, $\dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x-2)(x+2)}$.

  Because $(x-2)$ occurs in the numerator and denominator, when this element is equal to zero, the graph will have a "hole" at that point, i.e. when $x = 2$. We can reduce this by canceling these repeating elements and adding in a stipulation, that $x \neq 2$: $\dfrac{1}{x+2}$.

- In all other cases, the graph has a vertical asymptote at any x-values that make the denominator equal to zero.
  - o For example, $\frac{x-2}{x-4}$ cannot be factored or reduced in any way. As a result, $x=4$, which makes the denominator equal to zero, defines a vertical asymptote.

TIP: To find when a function is undefined, simply set the denominator to zero and solve for x!

When is the function $y=\frac{x-4}{x^3+x^2-6x}$ undefined?

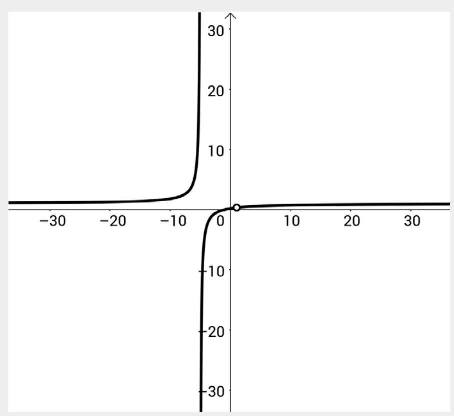
To find where this function is undefined, let's set the denominator equal to zero and solve. Factoring out an x we get that $x\left(x^2+x-6\right)=0$, so the function is undefined at $x=0$. Further simplifying we get that $(x)(x+3)(x-2)=0$, so the function is also undefined at $x=-3$ and 2. Notice that **the numerator has no part in determining where the function is undefined**. The numerator does indicate, however, in conjunction with the denominator, whether this undefined point is a hole or a vertical asymptote. For this problem, we don't need that information, though.

Answer: The function is undefined at $x=-3$, 0, and 2.

The equation $\frac{\left(x^2-1\right)}{x^2+4x-5}$ is graphed on the standard (x,y) coordinate plane below.

At what values of x is the function undefined?

A. –1 and 1
B. 1 and –5
C. –1 and 5
D. 1 only
E. –5 only



This question is tricky because of the graph. Looking at it, it appears that there is only one place where the function is undefined because of the presence of only one vertical asymptote. However, if we take a closer look at the function itself, factoring the numerator and denominator, we get:

$$\frac{(x+1)(x-1)}{(x+5)(x-1)}$$

We can clearly see that the function is undefined at both $x=-5$, which is clear on the graph, and $x=1$, which is much less obvious on the graph.

The reason for this is that the term $(x-1)$ exists both in the numerator and denominator, basically multiplying the expression $\frac{(x+1)}{(x+5)}$ by $\frac{(x-1)}{(x-1)}$ or 1. This adds a **hole**, or a removable discontinuity, to the graph of $\frac{(x+1)}{(x+5)}$, not altering the shape of the graph in any way but making the function undefined when x is 1. Therefore, our function is undefined at $x=-5$ and 1.

Answer: **B.**

Horizontal and Slant Asymptotes

Occasionally, the ACT® may ask you to calculate a **horizontal or slant asymptote** given a rational equation. Please note these questions are rare on the ACT®, and students scoring below a 33 should focus on other skills first. Students also taking the SAT® Math Level II Exam may find these skills helpful, also. In any case, we'll keep this brief.

TIP: remember that you always have your graphing calculator. If memorizing the ideas below is too much for you, you can plug the equation for an asymptote question into your calculator, let it work while you move on, and come back to it to trace and estimate the answer.

If you have a rational expression where $f(x)=\frac{ax^m}{bx^n}$, if:

- $m>n$ and the difference is more than 1, there is no horizontal asymptote. The graph will start at $\pm\infty$ and end up at $\pm\infty$.
- $n>m$ then the horizontal asymptote is $y=0$. As x approaches $\pm\infty$, y approaches 0.
- $m=n$ then the horizontal asymptote is the ratio of the leading coefficients, $\frac{a}{b}$.
- $m>n$ but only by 1, there is potentially a slant (or oblique) asymptote (this one is less important to know than the others).

In the standard (x,y) coordinate plane, when $w\neq 0$ and $z\neq 0$, the graph of $f(x)=\frac{2x-4x^2-4}{x^2+3x+2}$ has a *horizontal* asymptote at:

A. $y=2$
B. $y=-2$
C. $y=4$
D. $y=-4$
E. There is no horizontal asymptote.

In order to determine whether or not there is a horizontal asymptote we need to look at the degrees of the two polynomials. Be careful! The terms are not listed in order from greatest degree to least! The numerator has a highest degree of 2 in the form of $-4x^2$. The denominator also has a highest degree of 2 in the form of x^2. Based on the rules stated earlier, this means that there is a horizontal asymptote at the ratio of the coefficients of the two terms. $-4x^2$ has a coefficient of -4 and x^2 has a coefficient of 1, so our horizontal asymptote is at $y=\frac{-4}{1}=-4$.

Answer: **D**.

BASIC PARENT FUNCTIONS AND GRAPHS

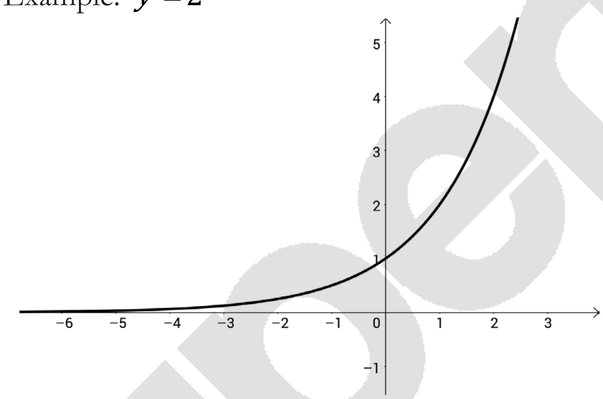
MEMORIZE THESE! It is important and extremely useful to know the basic shapes of linear, parabolic, n - degree polynomial, exponential, logarithmic, and absolute value functions. That being said, you always have your calculator! If you forget the shape of a graph, you can always try graphing the function. Still, memorizing the basic shapes below can help you save precious time and move quickly.

TIP: Also see our chapters on **CONICS (this book)**, **TRIG GRAPHS (Book 2)**, and **COORDINATE GEOMETRY (this book)** for more examples of graphs and their standard forms.

Graph of an exponential function:

Standard Form: $f(x)=a^x$

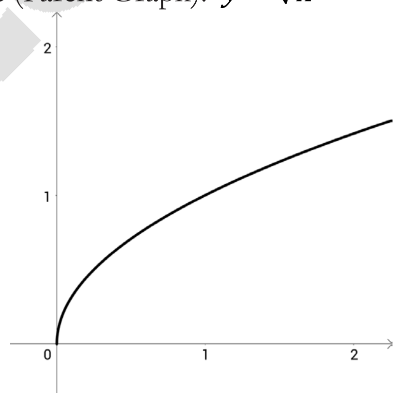
Example: $y=2^x$



Graph of a square root:

Standard Form: $a\sqrt{x}$

Example (Parent Graph): $y=\sqrt{x}$



Graph of a cubic function:

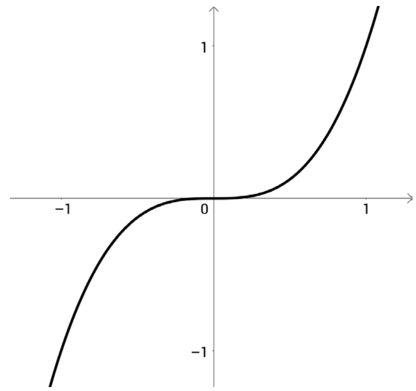
Vertex Form: $y=a(x-h)^3+k$

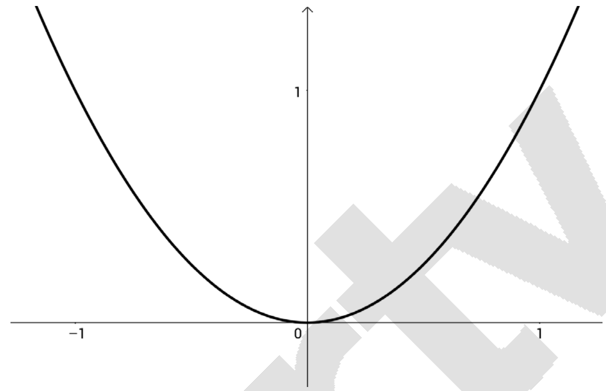
Example (Parent Graph): $y=x^3$

Graph of a parabola (quadratic function):

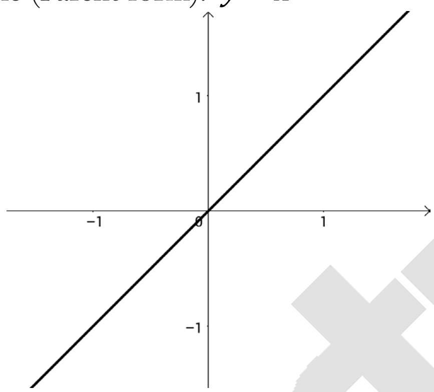
Vertex Form: $f(x)=a(x-h)^2+k$

Example (Parent Graph): $y=x^2$

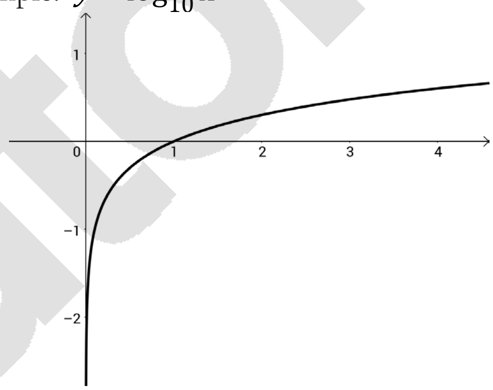




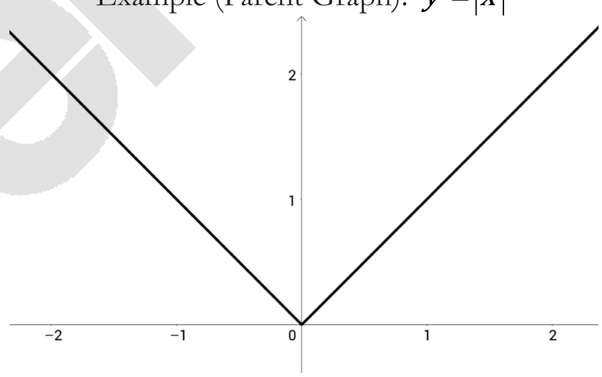
Graph of a linear equation:
Slope-Intercept Form: $y = mx + b$
Example (Parent form): $y = x$



Graph of a logarithmic function:
Standard Form: $y = \log_a x$
Example: $y = \log_{10} x$



Graph of an absolute value function:
Vertex Form: $y = a|x - h| + k$
Example (Parent Graph): $y = |x|$



Horizontal and Vertical Shifts

Additionally, you'll need to know **how to "move"** the graph of a parent function. Though exactly what constants or coefficients "move" these parent functions or adapt them, there is one principal that is universal to graph translations: h **and** k .

Horizontal Shift $(x - h)$:

For ALL functions, if you replace **all instances of** x in a function with "$x - h$," you'll find that the graph moves "h" **units to the right.**

Vertical Shift $(y-k)$:

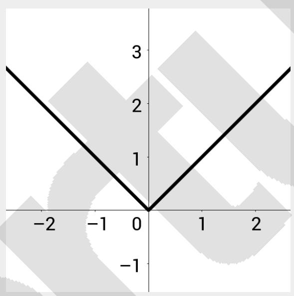
If you replace **all instances of** y in a function with "$y-k$," you'll find that the graph moves "k" **units upward.**

What is great about h and k is that they do the same "shift" no matter what function you apply this rule to. For instance, the parent graph of a parabola is: $y=x^2$. You might know the vertex form of a parabola is $y=a(x-h)^2+k$.

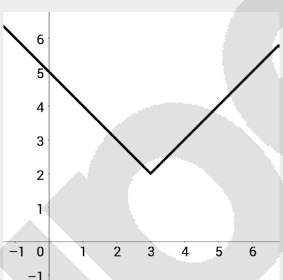
But what you might not realize, is that the x and y shift follow the rule above. Just move the k to the other side of the equation, and you'll see that the original y has been replaced with $y-k$, and the original x has been replaced by $x-h$: $y-k=a(x-h)^2$

Again this trick for vertical and horizontal shift works for EVERY type of function. Even trig functions!
$F(x)=\sin x$, for example, can move h units to the right simply by writing $F(x)=\sin(x-h)$.

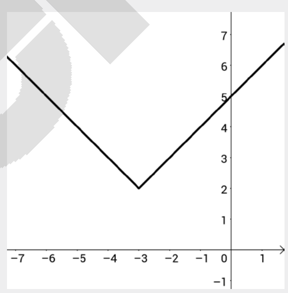
The graph of $f(x)=|x|$ is shown below. One of the following graphs is the graph of $y=f(x-3)+2$. Which one?



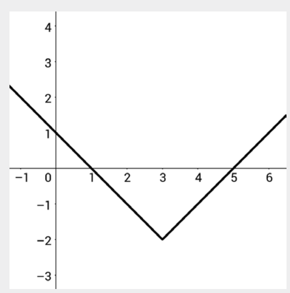
A.



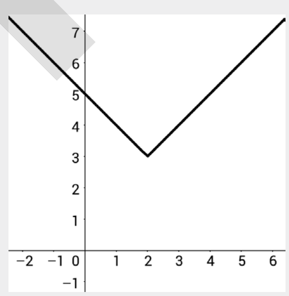
B.



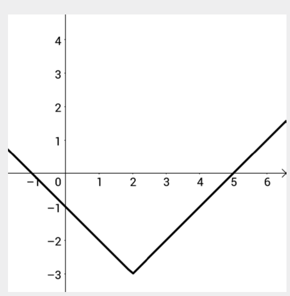
C.



D.



E.



For this problem, we simply apply our h and k rule. When we move to the new equation, we have replaced every instance of x with $x-3$. When we do that, we move the graph 3 units to the right. That means only A and C remain as viable choices.

Now let's think about the y value. If I take the plus two and move it next to y, I get $y-2=f(x-3)$. Let's think about this for a second. $f(x-3)$ was the original equation, but we replaced x with $x-3$. Still, that whole f(something) is the value of the entire equation that equals the y value. Now in this version, we replace $f(x)$ (what y equals) with $y-2$. That means we move 2 "upwards" on the graph. Clearly choice A is correct. It moves the vertex of our "dart" shaped graph up 2 and 3 to the right.

Again the idea is to **replace every x with** $x-h$, and **replace every y with** $y-k$, and you've **moved h to the right and k up,** no matter the graph or equation!

Answer: **A**.

In the standard (x,y) coordinate plane, the graph of $y=x^3$ is moved four units to the right and three units down. Which of the following equations represents this new function?

A. $f(x)=(x-4)^3+3$

B. $f(x)=(x+4)^3-3$

C. $f(x)=(x-4)^3-3$

D. $f(x)=(x-3)^3+4$

E. $f(x)=(x+3)^3-2$

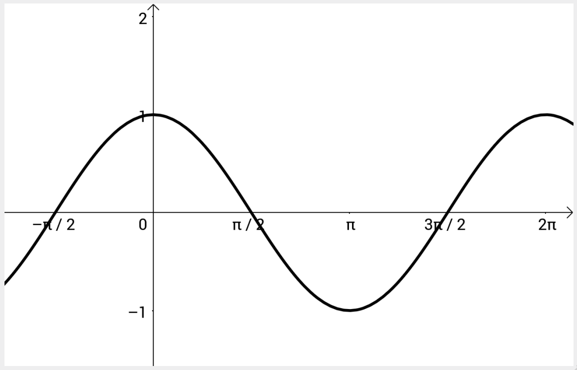
Here we simply apply our h, k rule. H is the units to the right, or 4. K is the units up, or negative 3 (because we move "down" instead of up, we change the sign to negative). We replace the x in $y=x^3$ with $(x-h)$ or $(x-4)$ and we replace the y value with $(y-k)$ or $(y--3)$ or $(y+3)$.

$$y+3=(x-4)^3$$

$$y=(x-4)^3-3$$

Answer: **C**.

The graph of function $y = f(x)$ is graphed in the standard (x, y) coordinate plane below.



The points on the graph of the function $y = 4 + f(x+2)$ can be obtained from the points of $y = f(x)$ by a shift of:

A. 4 units to the right and 2 units up
B. 4 units to the left and 2 units down
C. 2 units to the right and 4 units up
D. 2 units to the left and 4 units up
E. 2 units to the left and 4 units down

To solve, we can simply think about our h and k shifts. The "h" value always "hugs" the x. That means $(x+2)=(x-h)$.

Because $2=-h$, we know that $h=-2$. In other words, we aren't moving 2 to the right, but 2 to the left. When we have x plus a value, we move that distance to the left, not to the right. Now we can eliminate choice (C), (A), and (B), as none of these are 2 units to the left.

For the k movement, we can move the k to the side of the equation where y is, and get: $y-4=f(x+2)$.

Our "$y-k$" is now "$y-4$". Thus we are moving 4 units UPWARD. So only answer choice (D) remains.

Remember, if we replace y with $y-k$ and we replace x with $x-h$, we shift the graph h units to the right and k units up. If the signs in front of k or h are different (i.e. positive), then we are moving left or down respectively.

True, this is a trigonometric graph, but you don't need to know ANYTHING about trig to get this right. If you know the h, k rule, you will be set.

Answer: **D**.

WHAT "a" DOES: DIRECTION, SLOPE, AND STEEPNESS OF GRAPHS

Direction

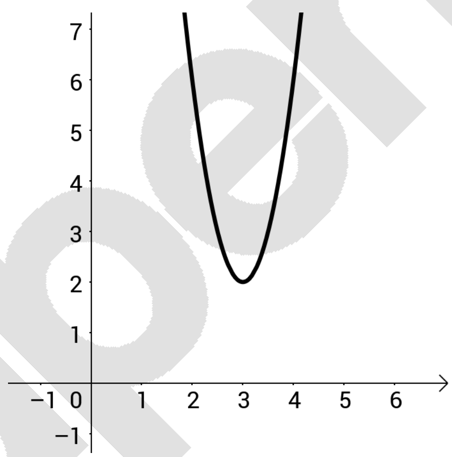
Often, standard or vertex forms of polynomials will include a coefficient or number a, typically the coefficient of the largest power term in a polynomial or linear function. The value by which an entire function is multiplied can also be a version of "a" or create similar effects (flipping a graph, widening it, or stretching it vertically).

For most functions with a vertex form (as noted on parent graphs above) or standard form, the sign (positive or negative) of "a" (typically the coefficient of the largest polynomial term) dictates whether the function is "right side up" or "upside down" as well as the slope or "steepness" of the graph. Exactly what effect a has will vary depending on the specific type of graph, and the exact placement of this variable (or another variable in a similar position) but this general principle is good to know. This letter is something like a "slope" for the graph.

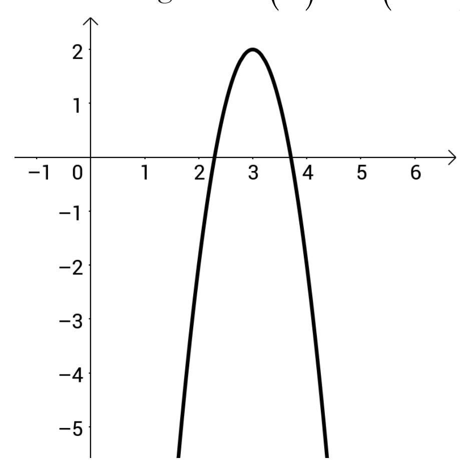
As explained below in "end behavior," paying attention to this coefficient is key to see if you need to do a vertical "flip" of your graph. If "a" is negative, and you're dealing with a polynomial or linear equation, or you can clearly see that an entire function has been multiplied by -1 (for example, the graph of $-f(x)$), flip the shape of the parent graph vertically "upside down" or mirror it across the x-axis before applying other shifts.

For example, a parabola is in the form $f(x)=a(x-h)^2+k$.

Thus the graph of: $f(x)=4(x-3)^2+2$ ...



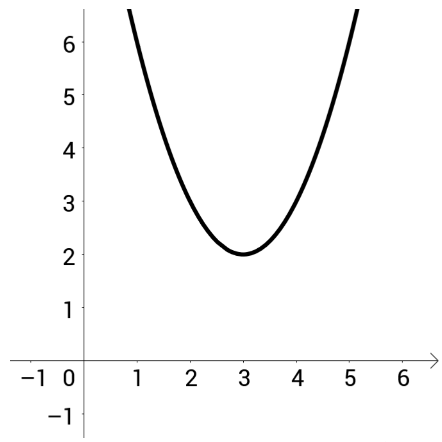
...Would be flipped upside down if the value for a (4) were made negative: $f(x)=-4(x-3)^2+2$



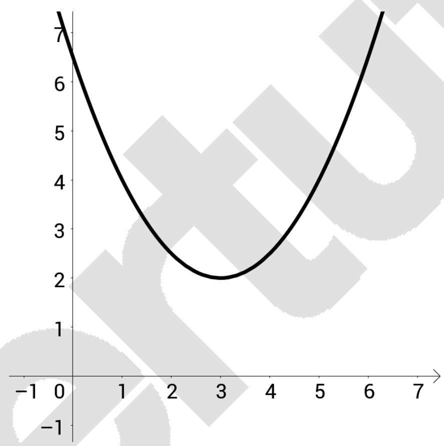
Steepness or Slope

This value also dictates the steepness of the graph, and it functions similarly to slope. The greater the absolute value of a slope, the steeper the line. The same is true for other styles of graphs: if you multiply the largest term or all the terms by a number, if that number's absolute value is greater than one, you'll be making the graph steeper or taller in some way, and if that number's absolute value is fractional, you'll be making that graph flatter or wider in some way.

For example, compare the graph below of $f\left(x\right)=\left(x-3\right)^2+2$ with a value of $a=1$ with the two graphs above. The below graph is less "steep" than the graphs in which $a=4$ or -4 above.



Fractional values of a, on the other hand, would flatten lines and curves, making them less "steep." See $f\left(x\right)=\frac{1}{2}\left(x-3\right)^2+2$ below:

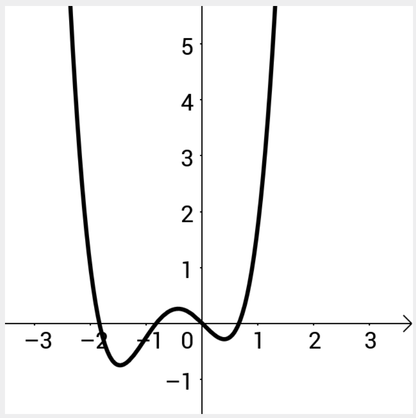


This principle holds for polynomials and absolute value equations, among others.

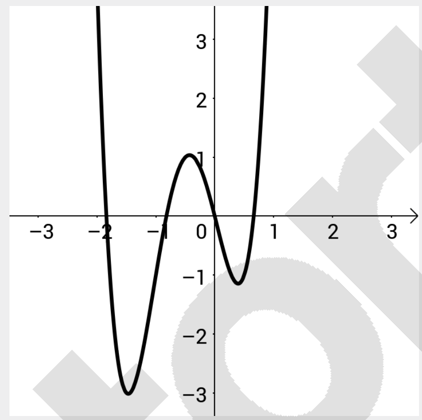
TIP: If you're ever unsure of what a value in an equation is doing on a graph, make up some numbers and test your theory using your graphing calculator. It is a huge resource on this style of question.

The graphs of $f(x)$ and $g(x)$ are shown in the standard (x,y) coordinate planes below. One of the following expresses $g(x)$ in terms of $f(x)$. Which one?

$y=f(x)$



$y=g(x)$



A. $g(x)=\frac{1}{4}f(x)$ **B.** $g(x)=4f(x)$ **C.** $g(x)=f(x)-4$

D. $g(x)=f(x+4)$ **E.** $g(x)=f(x-4)+4$

At first glance, you might think this is a nearly impossible question. We have two polynomials and have literally no idea what polynomial either is! But don't freak out just yet. Look down and analyze what we have to work with. We know these functions are related, and that they have basic shape parameters that are the same. We also know this is NOT a simple h or k shift. If it were, the shape in each function would be EXACTLY THE SAME. The shapes are not. So that eliminates choices C through E, which are all h and k only shifts. We need an "a" of some sort, or essentially different coefficients or different monomial elements within the polynomial to create a different shape. Therefore, we are down to A and B.

Choice A has an "a" value of $\frac{1}{4}$. That means $g(x)$ would be "flatter" and "wider" than $f(x)$. In fact, the opposite is true; $g(x)$ looks taller and skinnier.

Choice B has an "a" value of 4. That means $g(x)$ would be "taller" and "skinner" than $f(x)$, with "steeper" sloping curves.

That aligns with the visual difference in the pictures. Therefore, only choice B makes sense and it is correct.

If in doubt, we could also pluck some points and see if this pattern works. At $(0,0)$, multiplying by 4 shouldn't move that point. Both graphs appear to cross this point at about the same curve/place on the graph, so that works. I then see the graph's minimum point around $(-1.5,-.75)$. These are ballpark estimates, but I can use this point and multiply the y-value by 4 to get -3. Indeed, $g(x)$ appears to be around -3 when $x=-1.5$. By checking individual, plucked points in this manner, I can couple my instincts with mathematical evidence. True, I often don't have time to do all this on the ACT®, but knowing that it's a strategy can help if another method is uncertain.

Answering this question without multiple-choice answers would be nearly impossible, but given the choices and what we know about graph behavior, we can deduce the answer fairly easily if we know our rules.

Answer: **B**.

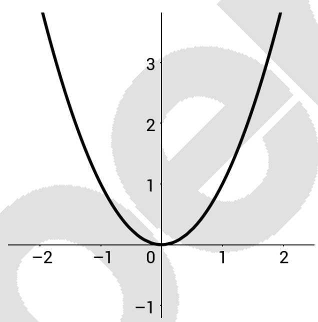
END BEHAVIOR: EVEN VS. ODD DEGREE POLYNOMIAL FUNCTIONS

End Behavior is the behavior of a graph as x approaches the far left or far right of the graph, i.e. or (negative or positive infinity).

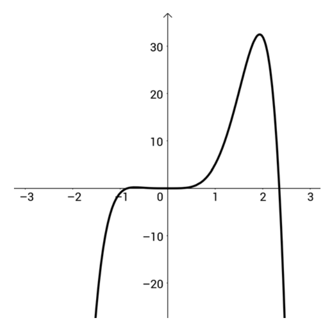
Even Degree Polynomial Functions

Functions whose highest degree is an **even exponent** (i.e. the highest degree term in the polynomial is x^2, x^4, x^6, etc.) start and end with the end points of the function **both pointing upward** or **both pointing downward**. As the x-value gets larger over time or smaller over time, as you move to the right or left on the graph, $f(x)$ will approach one of $\pm\infty$, depending on whether the coefficient of the leading term (i.e. the term with the highest numbered exponent) is positive or negative. See a summary of this concept below.

Degree: even
Leading Coefficient: positive
End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing)
Domain: all reals
Range: all reals $\geq$ maximum
Example: $y = x^2$



Degree: even
Leading Coefficient: negative
End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing)
Domain: all reals
Range: all reals $\leq$ maximum
Example: $y = -2x^6 + 3x^5 + 4x^4$



When the **coefficient of the leading term** (i.e. the highest degree polynomial term in the function) is positive, the ends of an even degree polynomial point **upwards**. I remember this as it looks like a "happy face" and happy faces are **positive**. Because the coefficient of the leading (and only) term of $y = x^2$ is positive one, the parabola above on the left is upward facing.

In contrast, the polynomial graphed above at the right, $y = -2x^6 + 3x^5 + 4x^4$, has ends that both point downward. As a result, it looks like a funky sad face, because the **negative** coefficient (-2) in front of the leading term make the graph look **"sad."**

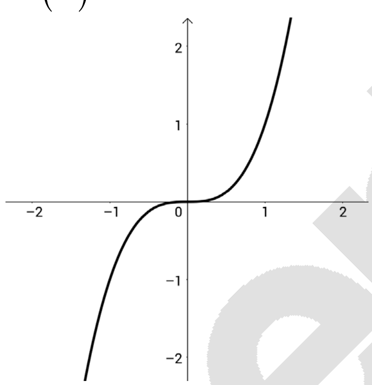
Though **even degree polynomials** of greater degree may not form an exact "u" shape, and may be more complex in the middle, the end behavior will still hold the same pattern, both shooting upward or downward.

Odd Degree Polynomial Functions

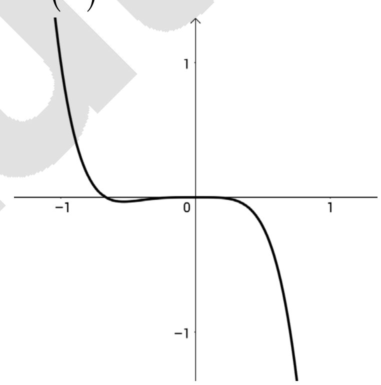
Odd degree polynomial functions (functions whose largest degree term includes an odd exponent, such as x^3, x^5, etc.) with a positive coefficient in front of the greatest polynomial term (leading term) start low and end high; in other words, they start at $f(x) = -\infty$ and end at $f(x) = +\infty$. If the leading term has a negative coefficient, these functions start high and end low; in other words, they start at $f(x) = +\infty$ and end at $f(x) = -\infty$.

TIP: To remember this idea, think about the end behavior of a line: it is essentially an odd degree polynomial. Positive sloped lines have a positive coefficient going "uphill" from left to right. Negative sloped lines have a negative coefficient and go "downhill" from left to right. More complex polynomials are less simple, but their end behavior is the same, sloping uphill or downhill.

Degree: odd
Leading Coefficient: positive
End Behavior: At graph left, $f(x) \to -\infty$. At graph right, $f(x) \to +\infty$ (upward sloping)
Domain: all reals
Range: all reals
Example: $f(x) = x^3$



Degree: odd
Leading Coefficient: negative
End Behavior: At graph left, $f(x) \to +\infty$. At graph right, $f(x) \to -\infty$ (downward sloping)
Domain: all reals
Range: all reals
Example: $f(x) = -3x^5 - 2x^4$



For example, the graph of $f(x) = -3x^5 - 2x^4$ (above right) shows a function that begins at positive infinity and ends at negative infinity.

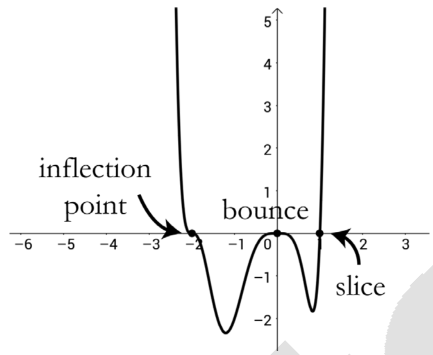
Because the function has a negative coefficient in front of its leading term (i.e. the three in front of x^5 is negative), the overall endpoint behavior will start high and end low. If the coefficient were positive, the graph would start low and end high, as is the case above to the left, $f(x) = x^3$.

"Doing the Disco": How To Remember End Behavior Trends

I remember end behavior by thinking of the iconic dance move in the 1970's, disco dancing. In traditional disco dancing, you might see a dancer strike a pose something like the fellow in this picture. As you can see, one arm is **up** and the other **down**. In the same way that **"7" in 1970's is an odd number**, I remember that **odd exponents** create end points such that one goes **"up"** and the other **"down."**

BOUNCES, INFLECTIONS, AND SLICES

The behavior of a graph along the x-axis is another element that may be tested. When roots occur once in a polynomial, the graph "**slices**" through the x-axis. **Inflection points** occur with multiple roots that occur an odd number of times. "**Bounces**" occur with multiple roots that occur an even number of times.



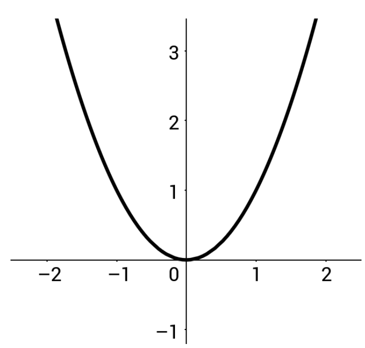
$x^4(x-1)(x+2)^3$		
x^4	$(x-1)$	$(x+2)^3$
4th degree (even)	1st degree (single)	3rd degree (odd)
Bounce ("kisses" the x-axis, reverses direction)	**Slice** (cuts through in a near straight manner)	**Inflection** (curves in different directions—concave vs convex—but does not "change" overall direction)

EVEN, ODD, AND ONE-TO-ONE FUNCTIONS

These are three classifications/properties of functions that are tested infrequently but still useful to know for those overachievers out there (seeking 34+ scores).

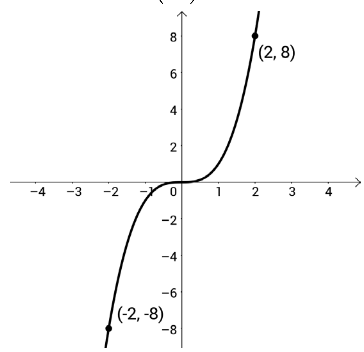
Even Functions

Even functions have graphs that are symmetric about the y-axis, such that for all x, $f(x)=f(-x)$. For example, $f(x)=x^2$ is an even function.



Odd Functions

These are the exact opposite of even function in that for every x, $f(x)=f(-x)$. In other words, the function is symmetrical about the origin. $f(x)=x^3$ is an odd function.

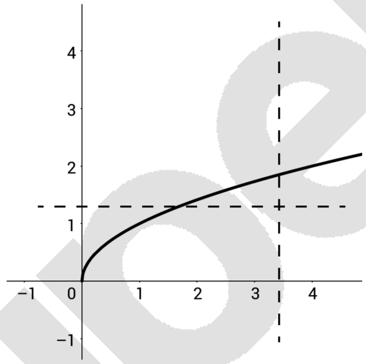


For example, in this graph, when $x=2$, $y=8$ and when $x=-2$, $y=-8$. Every ordered pair that is positive corresponds to an ordered pair with two negative values that are otherwise numerically the same.

One-To-One Functions

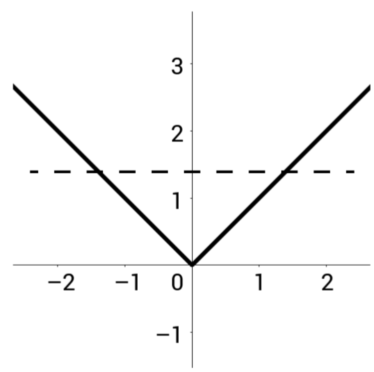
One-to-one functions are functions that have only one x-value for every y-value. In other words, not only must the function pass the required **vertical line test**, but it must also pass the **horizontal line test** to be considered a one-to-one function. Functions that reverse directions, with multiple peaks and valleys, are never one-to-one functions. Relatively linear functions are often one-to-one functions.

Example of a one-to-one function:



passes the vertical and horizontal line tests

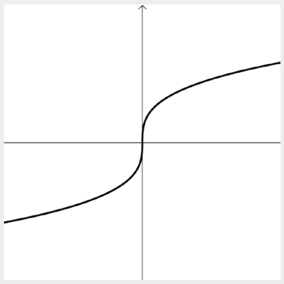
Example of a function that is NOT one-to-one:



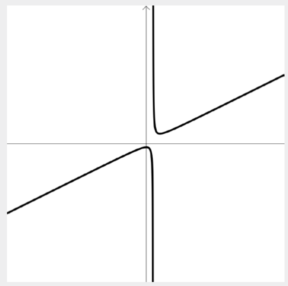
fails the horizontal line test

A function if a *one-to-one* function if and only if each x in the domain of $f(x)$ corresponds to a unique $f(x)$ and each $f(x)$ corresponds to a unique x. Which of the following graphs is a *one-to-one* function?

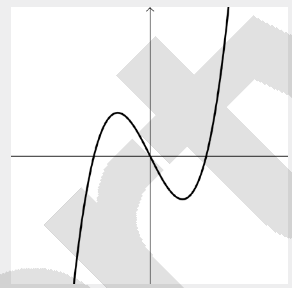
A.



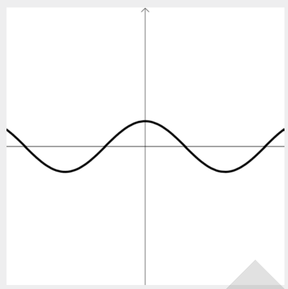
B.



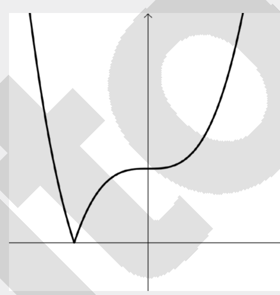
C.



D.



E.



Remember that when it comes to one-to-one functions we have to use the two different line tests. All of these graphs depict functions, and therefore satisfy the vertical line test. However, only one of the functions satisfies the horizontal line test, where every y-value has its own singular x-value, and that is choice A.

Answer: **A**.