CHAPTER

6 DISTANCE AND MIDPOINT

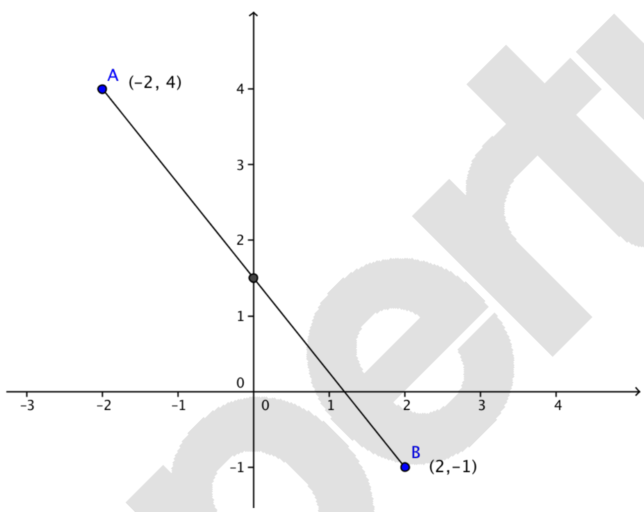
SKILLS TO KNOW

- How to find the midpoint of a line (Midpoint Formula)
- How to find the distance between two coordinate points (Distance Formula)

MIDPOINT

You may not remember midpoint from coordinate geometry, but don't worry, you can do these problems without a complicated formula by using a little common sense to memorize the idea.

The midpoint is essentially the **average** of the x values and the **average** of the y values.



Average of the x-values:

$$\frac{(-2+2)}{2}=x$$

$$\frac{0}{2}=x$$

$$0=x$$

Average of y-values:

$$\frac{(4+-1)}{2}=y$$

$$\frac{3}{2}=y$$

$$1.5=y$$

Answer: $(0,1.5)$

That's right. It's that easy. If you have two points you add the x's together and divide by two, and then add the y's together and divide by two. Above you take the 2 and -2 and find halfway between—that's the average of 2 and -2, which is 0. Same with the y—find the halfway point or average between 4 and -1, which is 1.5.

THE MIDPOINT FORMULA

The midpoint of two coordinate points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$$

TIP: Many questions on the ACT® that deal with midpoint will NOT give you the end points, but rather may give you one end point, the midpoint, and ask for the other endpoint. Always read the questions carefully! Use algebra to solve after setting up an equation.

In the standard (x, y) coordinate plane, the midpoint of a line segment is $(5,7)$ and an endpoint of that segment is located at $(9,-3)$. If (x, y) are the coordinates of B, what is the value of $x + y$?

Here we know our midpoint and endpoint, so we set up our formula to solve for the missing endpoint, (x, y).

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right) = \text{(midpoint } x \text{ value, midpoint } y \text{ value)}$$

$$\left(\frac{(x+9)}{2}, \frac{(y-3)}{2}\right) = (5,7)$$

$$\frac{(x+9)}{2} = 5 \qquad \frac{(y-3)}{2} = 7$$

$$x+9=10 \qquad y-3=14$$

$$x=1 \qquad y=17$$

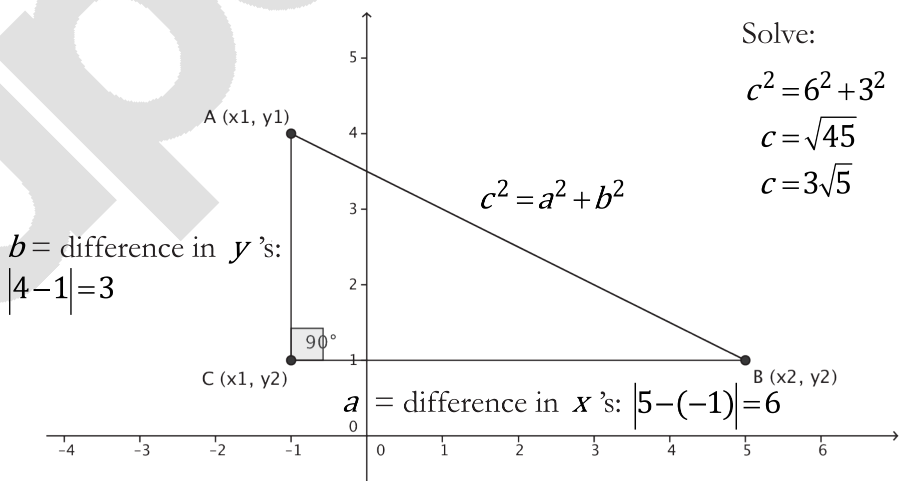
$$x+y=1+17$$

$$x+y=18$$

Answer: 18.

THE DISTANCE FORMULA

To find the distance between two coordinate points, think about the Pythagorean Theorem. Essentially, it's the same thing. If you wanted to find distance between A & B in the picture below, you could draw a right triangle and then use the Pythagorean Theorem:



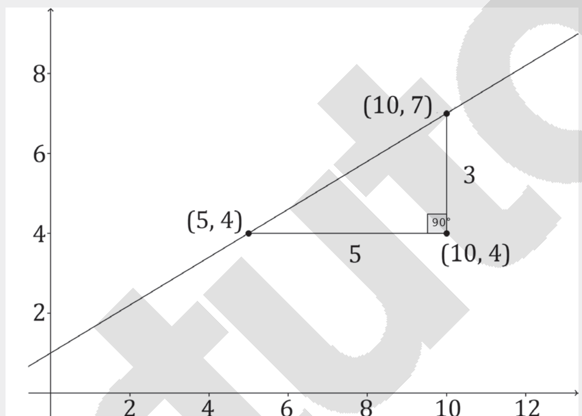
In fact, as long as you remember that the distance formula is essentially the Pythagorean Theorem, you can rely on that idea to solve. If you like formulas, however, there's a formula you can memorize. You can even program that formula into your calculator if you like (see chapter on Calculator Programs).

THE DISTANCE FORMULA

Given two points, (x_1, y_1) and (x_2, y_2), the distance between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

What is the distance in coordinate units between the points $(5,4)$ and $(10,7)$ in the standard (x, y) coordinate plane?



The easiest way to approach this, if you get confused by formulas or tend to make careless errors, is to sketch it out. Then count out the difference from 4 to 7 and from 5 to 10—voila! You get 3 and 5.

Enter the Pythagorean Theorem: $a^2 + b^2 = c^2$

Square each, and you get $9 + 25 = 34 = c^2$ Solve for c and you get $\sqrt{34}$.

Now sometimes on the test things won't be that easy.

(x, y) and $(-4,2)$ are 10 units away from each other in the standard coordinate (x, y) plane. If y is eight less than x, than which of the following could be the value of x?

A. 4 **B.** –10 **C.** –2 **D.** 6 **E.** 8

You could do this problem by sketching it out and using the answers, but the most reliable method is algebraic.

Regardless, first translate the "y is eight less than x" into $y = x - 8$. Then substitute $x - 8$ for y into (x, y). Now you have $(x, x - 8)$ as the point you are looking for.

Algebraic method

Plug in to the ordered pair: $(x, x-8)$ and your original pair, $(-4,2)$ into the distance formula:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$10=\sqrt{(-4-x)^2+(2-(x-8))^2}$$

$100=(-1(4+x))^2+(2-x+8)^2$	Square both sides, factor out -1, distribute negative sign to $(x-8)$
$100=(-1)^2(16+8x+x^2)+(10-x)^2$	Expand: $(a+b)^2=a^2+2ab+b^2$ (special product), distribute exponent, add like terms
$100=x^2+8x+16+100-20x+x^2$	Expand: $(a-b)^2=a^2-2ab+b^2$ (special product); Commutative property
$0=2x^2-12x+16$	Simplify
$0=x^2-6x+8$	Divide both sides by 2
$0=(x-4)(x-2)$	Factor
$0=(x-4)$ *or* $0=(x-2)$	Apply Zero Product Property
$x=4$ *or* 2	Solve for x

Because only 4 is an available answer (A), it is the correct answer.

Answer: **A**.