CHAPTER

5 INTERCEPTS AND SLOPES

SKILLS TO KNOW

- How to find the slope of a line from two points or a graph.
- How to put any equation into slope-intercept form, and from that, derive the slope or y-intercept.
- How to find the equation of a line from two points or a point and a slope.
- How to find the x- or y-intercept(s) of linear (and non-linear) equations.
- How to use information about parallel or perpendicular lines in finding the equation of a line.

When it comes to slopes, intercepts, and linear equations, you are expected to know just about everything you learned in Algebra 1—with less emphasis on actual methods of solving (you won't need point-slope form, for example) and vocabulary, but with full expectation of proficiency in the area of manipulation and problem solving.

SLOPE OF A LINE FROM TWO POINTS

SLOPE FORMULA

For points (x_1, y_1) and (x_2, y_2), $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope Shortcuts

1. If you're given a **GRAPH** instead of points, **pluck points off the graph** and you can still use this formula. OR simply count RISE over RUN but remember downhill lines are negative, and uphill lines are positive.

2. You could also **program this one in your calculator.** More on that in the skills & hacks chapter.

What is the slope of the line containing the points $(10,7)$ and $(14,19)$ in the standard (x,y) coordinate plane?

A. -3 **B.** $-\dfrac{1}{3}$ **C.** $\dfrac{1}{3}$ **D.** 3 **E.** $\dfrac{26}{24}$

To solve this one, simply plug in your numbers into the slope formula.

Let $(10,7) = (x_1, y_1)$ and $(14,19) = (x_2, y_2)$.

Then plug in and solve:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{19-7}{14-10}$$

$$\frac{12}{4} = 3$$

Answer: **D**.

SLOPE-INTERCEPT FORM

You need to know what this means, and how to use it.

SLOPE-INTERCEPT FORM

$$y = mx + b$$

Where m is the **slope** of the line, and b is the **y-intercept** of the line at point $(0,b)$.

What is the slope-intercept form of $8x - 2y + 6 = 0$?

A. $y = -8x - 6$ **B.** $y = -8x + 6$ **C.** $y = 8x + 6$ **D.** $y = 4x + 3$ **E.** $y = 4x - 3$

If you're asked to put an equation in this form, you must isolate the "y" value.

$8x - 2y + 6 = 0$	Add $2y$ to both sides
$8x + 6 = 2y$	"Flip" the equation (put the y on the left)
$2y = 8x + 6$	Divide both sides by 2.
$y = 4x + 3$	

Answer: **D**.

Here's another example:

What is the slope of the line $\frac{y}{3} = \mathbf{x}$?

Here we can put the problem into slope-intercept form again by isolating the y value. If we multiply both sides by 3 we get:

$$\frac{y}{3} = x$$

$$y = 3x$$

$$3 = m$$

The answer is 3.

EQUATION OF A LINE FROM TWO POINTS OR A POINT AND A SLOPE

Here you'll use the slope-intercept form to help again; Yes, you could memorize point-slope form, but ultimately, it's easier to just memorize one equation.

> Which of the following is an equation for the line passing through $(2,2)$ and $(8,4)$ in the standard (x,y) coordinate plane?
>
> **A.** $y=3x$ **B.** $y=3x-20$ **C.** $y=\frac{1}{3}x$ **D.** $y=\frac{1}{3}x+\frac{11}{3}$ **E.** $y=\frac{1}{3}x+\frac{4}{3}$

As you can see, the answers are all in slope-intercept form. Even if they weren't, we could still use this method and then manipulate the equation to the other form (i.e. standard form, etc.) later.

First, we find the slope:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{8-2}=\frac{2}{6}=\frac{1}{3}$$

Now we plug in that slope into $y=mx+b$:

$$y=\frac{1}{3}x+b$$

Finally, we plug in one of the pairs of numbers—it doesn't matter which pair you choose. I'll choose $(8,4)$. That means $x=8$ and $y=4$. I simply substitute these into the equation above since we know that they are a solution, and in the process, I solve for b:

$$4=\frac{1}{3}(8)+b$$

$$b=4-\frac{8}{3}=\frac{12}{3}-\frac{8}{3}=\frac{4}{3}$$

Upon finding $b=\frac{4}{3}$, I plug it back into our earlier equation:

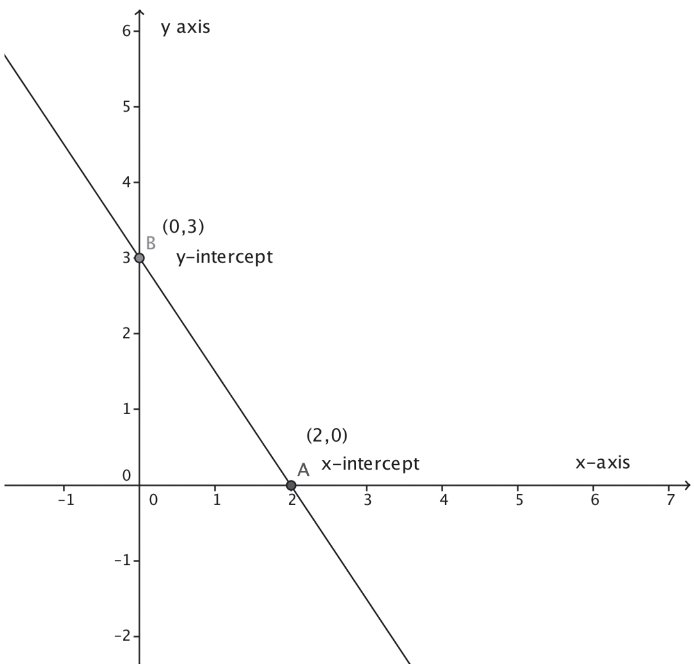
$$y=\frac{1}{3}x+b$$

$$y=\frac{1}{3}x+\frac{4}{3}$$

Answer: **E**. At this point, if the answers are in a different form, you can manipulate the equation to match that form.

You can use simple calculator programs to help with problems such as this one.
See: https://supertutortv.com/blog-resources/act-calculator-programs for more info.

X AND Y INTERCEPTS



Intercepts

Intercepts are the points at which your line, shape, or other element crosses through the y-axis (y-intercept) or x-axis (x-intercept).

The most common error that students make on intercept questions is mixing up the y-axis and x-axis (i.e. when asked for an x-intercept, they put a y-intercept).

The memory trick that I use is to think about "y-i" and "x-i."

I let the "i" stand for IS: it is IS something, not nothing (0).

If you are looking for a y-intercept, the y-is the number, and $x=0$; i.e. $(0,3)$ gives the y-intercept.

If you are looking for an x-intercept, the x-is the number, and $y=0$; i.e. $(2,0)$ gives the x-intercept.

If you tend to confuse these, the other trick you can pull is to quickly sketch a graph—look at the y (rhymes with high—up and down) and x (a-"cross" (get it—x looks like a cross)—left to right) and figure out which you want and which value is 0. In general, if you are getting coordinate geometry questions wrong, you probably need to draw more often. Little sketches are quick and often a great strategy!

To find an intercept, you have a few options:

For the y-intercept: put the equation into slope-intercept form $y = mx + b$ and solve for b.

What is the y-intercept of $y = 2x + 4$?

This equation is already in slope-intercept form, or $y = mx + b$. Thus $b = 4$, and b stands for the y-intercept, so the answer is 4.

Answer: 4.

But what if you need the **x-intercept**? Or if it's a pain to put the equation in that form? Or you have a quadratic or insane-looking polynomial!? In that case, plug in 0 for the appropriate variable (for y-intercept—plug in 0 for x; for x-intercept—plug in 0 for y) and solve for the remaining variable.

Find the y-intercept of $y = x^4 + x^3 + 2x - 7$.

Remember, y-intercept means y-IS the number—so $x = 0$.

Plug in 0 and you get:

$$y = 0^4 + 0^3 + 2(0) - 7$$
$$y = -7$$

You can also solve intercept problems at times using your graphing calculator. If you have an equation you can graph, plug it in, and then hit "trace." Look for the points where the function crosses the axis ($x = 0$ or $y = 0$).

In the (x, y) coordinate plane, the x-intercept of the line $y = -3x + 5$ is represented by:

A. -3 **B.** 3 **C.** 5 **D.** -5 **E.** $\frac{5}{3}$

If x "is" the number than y is zero. Plug in 0 for y and you get:

$$0 = -3x + 5$$
$$-5 = -3x$$
$$\frac{5}{3} = x$$

Answer: **E**.

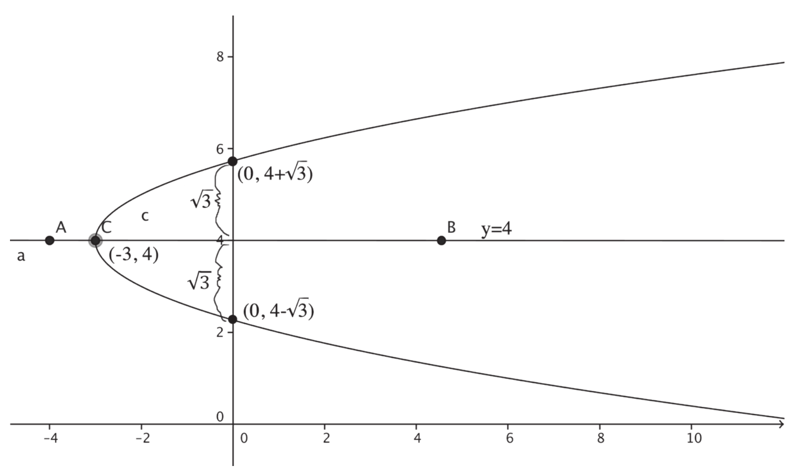
A parabola with vertex $(-3, 4)$ and an axis of symmetry at $y = 4$ crosses the y-axis at $(0, 4 + \sqrt{3})$. At what other point, if any, does the parabola cross the y-axis?

A. $(0, 4 - \sqrt{3})$ **B.** $(0, -4 - \sqrt{3})$ **C.** $(0, -4 + \sqrt{3})$

D. No other point **E.** Cannot be determined from the given information

We could simply sketch a graph knowing the parameters and use the idea of symmetry to find the point on the opposite side of about the line of symmetry, the point $(0, 4 + \sqrt{3})$. Since the line of symmetry is at 4 you're looking above and below that line by $\sqrt{3}$. So $(0, 4 - \sqrt{3})$ is correct.

Answer: **A**.



This above method is probably the fastest—but you may not see it.

Another way to approach this problem is to memorize the standard vertex form of a parabola:

PARABOLA FORMULA

$$y = a(x-h)^2 + k \text{ OR } x = a(y-k)^2 + h$$

You probably deal with parabolas that are vertically oriented (the first equation) far more often, so this problem is a bit tough on that note alone.

We find out from the axis of symmetry at $y=4$ that it's a horizontally facing parabola. "y" is how "high" a line is—so this is a horizontal line across a consistent "height" (high and y rhyme if that helps!).

And thus the parabola is symmetric about that horizontal line. So we need to use the 2nd equation.

We plug in $(-3,4)$ as our vertex at (h,k) and (x,y) our pair at $\left(0,4+\sqrt{3}\right)$ and can solve this algebraically:

$x = a\left(y-k\right)^2 + h$	Plug in: $x=0$, $y=4+\sqrt{3}$, $h=-3$, and $k=4$
$0 = a(4+\sqrt{3}-4)^2 - 3$	Be sure not to confuse equations or variables—h always follows the x, k always corresponds to the y.
$0 = (y-4)^2 - 3$	The 4 and -4 cancel.
$0 = 3a - 3$	Square the $\sqrt{3}$ to get 3.
$3 = 3a$	Add 3 to both sides.
$1 = a$	Divide by 3.

Now that we know $a=1$, and, again $h=-3$ and $k=4$, we know the equation is:

$$x = 1\left(y-4\right)^2 - 3$$

Then we can:

A. Plug in "0" for x and solve for the other point that crosses through the y-axis:

$0=\left(y-4\right)^2-3$	
$3=\left(y-4\right)^2$	Add 3 to both sides.
$\pm\sqrt{3} = y-4$	Take the square root of both sides.
$4+\sqrt{3} = y$ or $4-\sqrt{3} = y$	Add 4 to both sides.

$\left(0,4-\sqrt{3}\right)$ is the point we don't have yet, so that is the answer.

OR

B. Graph on a graphing calculator and trace to the intersection point between the parabola and the y-axis.

Answer: **A**.

PARALLEL AND PERPENDICULAR LINES

Parallel lines share the same slope.
As such, if you know the slope or slope-intercept form of one line, and that another line is parallel to it, you can find its slope as well.

What is the slope of any line parallel to the x-axis in the (x,y) coordinate plane?

A. -1
B. 0
C. 1
D. Undefined
E. Cannot be determined from the given information

So this is a bit of a trick question, but let's think about what happens in a parallel line equation:

The "y" value (or height) of the equation is fixed. For example, $y=6$ is a line parallel to the x-axis. What is its slope? Well it is the same as $y=0x+6$, right? So the slope is 0 and the answer is **B**!

Answer: **B**.

I know it can be confusing to remember that horizontal lines have a slope of zero. Another way to think of this is with the slope equation—there is no rise, only run. That means the slope will be zero over some number.

Perpendicular lines have slopes that are opposite reciprocals of each other.

What is the slope of a certain line perpendicular to the line $4x+8y=20$ in the standard (x,y) coordinate plane?

A. -4 **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** 2 **E.** 4

$4x+8y=20$	First we must put the line into slope-intercept form.
$8y=20-4x$	Move x to the other side to isolate the variable y.
$y=\frac{20}{8}-\frac{4}{8}x$	Divide both sides by 8.

$y = -\dfrac{x}{2} + \dfrac{5}{2}$ Use commutative property to keep the x next to the equal sign.

$y = -\dfrac{1}{2}x + \dfrac{5}{2}$

Now, from slope-intercept form, we know the slope is $-\dfrac{1}{2}$.

Since perpendicular lines have slopes that are opposite reciprocals and the slope of this line is $-\dfrac{1}{2}$, the slope of the perpendicular line is 2.

Answer: **D**.