MATRIX ALGEBRA

ACT Math: Lesson and Problem Set

SKILLS TO KNOW

- Adding and subtracting matrices
- Multiplying a matrix by a scalar
- Solving for variable values in equivalent matrices
- Multiplying matrices
- Finding basic determinants (when given the formula)*

1. MATRIX BASICS

Many students may not have had matrix algebra in school, others forget the rules of matrices, and thus these questions can pose difficulty for many students. These questions do not occur frequently on the test—on any given exam you have perhaps less than a 50% chance of encountering one of them. Nonetheless, you are responsible for knowing the several matrix related tasks above.

First let's familiarize ourselves with the matrix:

m x n Matrix

3 x 3

$$\begin{array}{cc} & \text{Columns} \\ \text{Cells, Rows} & \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \end{array}$$

A matrix is a rectangular array of quantities or values that is usually enclosed by brackets. We usually name matrices with capital letter variables – for example A, B, C.

Matrices are measured RISE by RUN – RISE counts the number of rows, and RUN the number of columns. The dimensions of a few matrices are listed below:

$1\times 3\,Matrix : \begin{bmatrix} 4 & 3 & 5 \end{bmatrix}$ – ONE row by THREE columns

$4\times 2\,Matrix : \begin{bmatrix} 3 & 4 \\ 6 & 5 \\ 8 & 2 \\ 3 & 4 \end{bmatrix}$ – FOUR rows by TWO columns

Sometimes we refer to the individual positions in a matrix based on the row and column position:

See this 2×2 matrix: $\begin{bmatrix} Row1, Column1 & Row1, Column2 \\ Row2, Column1 & Row2, Column2 \end{bmatrix}$

For two matrices to be equal, they must not only have the same dimensions, but also have the same values in every position.

2. ADDING AND SUBTRACTING MATRICES

To be able to ADD or SUBTRACT matrices they must have the same dimensions. Then, you simply add together (or subtract) the values that occupy the corresponding positions.

If $M = \begin{bmatrix} -6 & 3 \\ 5 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 0 & -3 \\ 4 & 1 \end{bmatrix}$ Find $M+N$ and $M-N$

A. Find $M+N$

Simply add the values in the corresponding positions: $\begin{bmatrix} -6 & 3 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6+0 & 3+-3 \\ 5+4 & 2+1 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 9 & 3 \end{bmatrix}$

B. Find $M-N$

(Subtraction is similar. Just make sure you don't reverse the order—in subtraction order matters.)

Simply subtract the values in the corresponding positions: $\begin{bmatrix} -6 & 3 \\ 5 & 2 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6-0 & 3--3 \\ 5-4 & 2-1 \end{bmatrix} = \begin{bmatrix} -6 & 6 \\ 1 & 1 \end{bmatrix}$

3. MULTIPLYING MATRICES BY A SCALAR

A scalar is a single value, number, or expression. When multiplying a matrix by a scalar, the scalar is multiplied by every individual value in a matrix. Unlike matrices, which are generally denoted by capital letters, a scalar is a lower case letter such a k or m. It can also be placed to the left of a matrix (just as a number to the left of parenthesis means multiply by each item in the parenthesis).

Here's what a scalar problem looks like:

If $k = 7$ and $M = \begin{bmatrix} 4 & 5 \\ 3 & 2 \\ 9 & 1 \end{bmatrix}$ find kM.

To solve this problem, we'll multiply each item in M by 7:

$$7\begin{bmatrix} 4 & 5 \\ 3 & 2 \\ 9 & 1 \end{bmatrix} = \begin{bmatrix} 28 & 35 \\ 21 & 14 \\ 63 & 7 \end{bmatrix}$$

4. EQUIVALENT MATRICES

When two matrices are equivalent, match up each corresponding position to solve for unknowns and create separate equations.

Solve for a if: $\begin{bmatrix} 4 & 2n & a+3 \end{bmatrix} = \begin{bmatrix} 4 & a+6 & 3n \end{bmatrix}$

Match up the middle terms and set them equal: $\begin{bmatrix} 4 & 2n & a+3 \end{bmatrix} = \begin{bmatrix} 4 & a+6 & 3n \end{bmatrix}$

$$a+6 = 2n$$

Match up the last terms and set them equal: $\begin{bmatrix} 4 & 2n & a+3 \end{bmatrix} = \begin{bmatrix} 4 & a+6 & 3n \end{bmatrix}$

$$a+3 = 3n$$

Solve by elimination (you can also solve by substitution):

$$\begin{array}{r} a+3=3n \\ -\quad a+6=2n \\ \hline \end{array}$$

Subtract

$$0-3=n$$

$$-3=n$$

Substitute

$$a+3=3(-3)$$

$$a+3=-9$$

$$a=-6$$

5. MULTIPLYING MATRICES

Though multiplying matrices perhaps appears on only $10-20\%$ of ACT exams, you still need to know how to perform this pesky task.

First, understand that matrix multiplication does NOT follow the same, simple rules of matrix addition and subtraction. If you assume it does, you will likely find a nice wrong answer to select.

Instead, it's a bit more complicated. Let's break it into steps:

STEP 1: IDENTIFY YOUR MATRIX SIZES

Remember matrices are measured RISE by RUN (or ROWS by COLUMNS):

Here we have a 1×2 matrix: $\begin{bmatrix}5 & 1\end{bmatrix}$ and here we have a 2×1 matrix: $\begin{bmatrix}4 \\ 3\end{bmatrix}$

STEP 2: TEST TO MAKE SURE YOU CAN MULTIPLY

Before multiplying, you need to make sure you *can* multiply the matrices. Only matrices in which the COLUMN NUMBER of the first matrix matches the ROW NUMBER of the 2^{nd} matrix can be multiplied. That sounds a bit confusing, but in practice, it is useful to think that matrices must "handshake" in order to be multipliable.

We can multiply a 1×2 matrix times a 2×1 matrix because the two "2's" in the center match or "handshake."

On the other hand, we can also reverse the order and multiply the 2×1 matrix with the 1×2 because the 1's are the same, or "handshake" in the middle of the orientation.

> Just because two matrices $A\times B$ can be multiplied does not mean $B\times A$ can be as well.

For example, a 1×4 matrix can be multiplied by a 4×5 matrix (the fours match), but the 4×5 matrix cannot be multiplied by a 1×4 matrix (5 and 1 are not equal).

STEP 3: SET UP YOUR DESTINATION MATRIX

When you set up your multiplication problem, you find the dimensions of the resulting matrix by taking the FIRST and LAST values from the matrix dimensions. So for a 1×2 by 2×1 we get a 1×1 matrix result $\begin{bmatrix}5 & 1\end{bmatrix}\times\begin{bmatrix}4 \\ 3\end{bmatrix}=[\ldots]$

For a 1×4 by $4\times 3,$ we get a 1×3 matrix result:

$$\begin{bmatrix}1 & 3 & 4 & 5\end{bmatrix}\times\begin{bmatrix}1 & 2 & 3 \\ 4 & 5 & 3 \\ 7 & 1 & 2 \\ 4 & 5 & 3\end{bmatrix}=\begin{bmatrix}\ldots & \ldots & \ldots\end{bmatrix}$$

STEP 4: MULTIPLY!

Multiply each row value times a single corresponding column value and then add the products of these products together. Remember you are multiplying ROWS times COLUMNS. You only take TWO elements at a time to multiply together—those in corresponding order—in order horizontally from the first matrix to those in order vertically in the second matrix—i.e. you take the first item horizontally times the first item vertically, then add the second item horizontally times the second item vertically. If that was confusing for you, just pay attention to the matrix multiplication examples below:

Row 1 item 1 times Column 1 item 1, then Row 1 item 2 times Column 1 item 2:

$$\begin{bmatrix}5 & 1\end{bmatrix} \times \begin{bmatrix}4 \\ 3\end{bmatrix} = \begin{bmatrix}5 \times 4 + 1 \times 3\end{bmatrix} = \begin{bmatrix}20 + 3\end{bmatrix} = \begin{bmatrix}23\end{bmatrix}$$

Row 1 item 1 times Column 1 item 1, then Row 1 item 2 times Column 1 item 2, then Row 1 item 3 times Column 1 item 3 …etc. Add together the items for the 1^{st} row and 1^{st} column into a single blank that bears both features (row 1 column 1 position).

$$\begin{bmatrix}a & b & c & d\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12\end{bmatrix} = \begin{bmatrix}1a + 4b + 7c + 10d & 2a + 5b + 8c + 11d & 3a + 6b + 9c + 12d\end{bmatrix}$$

$$\begin{bmatrix}a & b \\ c & d\end{bmatrix} \times \begin{bmatrix}1 & 2 \\ 3 & 4\end{bmatrix} = \begin{bmatrix}1a + 3b & 2a + 4b \\ 1c + 3d & 2c + 4d\end{bmatrix}$$

Matrix multiplication is a bit confusing–once you figure out the pattern it's fine, but that pattern can be tough to depict in a book.

6. FINDING A BASIC DETERMINANT

Finally, you may be asked to find or use the definition of a determinant.

A determinant is usually written as a matrix with "straight" brackets as so: $\begin{vmatrix}a & b \\ c & d\end{vmatrix}$

Or with standard brackets: $\det\begin{bmatrix}a & b \\ c & d\end{bmatrix}$

The great news on the ACT is that you will never (at least not if the test remains consistent with past tests) be required to memorize the formula for a determinant. Rather, if you have a determinant question, the question will include the formula itself. In such a case, all you have to do is use that formula! Plug in numbers that are in the same positions as the variables in the given expression.

These might "look" like matrix problems, and that's why they have been put here.

The determinant of a matrix $\begin{bmatrix}a & b \\ c & d\end{bmatrix}$ is $ad - bc$. If the determinant of $\begin{bmatrix}6 & 4 \\ x & x\end{bmatrix}$ is 24, what is the value of x?

Assume $a = 6$, $b = 4$, $c = x$, $d = x$ (matching items in same position) then plug into the expression $ad - bc$ and set equal to 24:

$$6x - 4x = 24$$

$$2x = 24$$

$$x = 12$$

1. What is the matrix product $\begin{bmatrix}012\end{bmatrix}\begin{bmatrix}x\\3x\\4x\end{bmatrix}$?

A. $\begin{bmatrix}11x\end{bmatrix}$

B. $\begin{bmatrix}12x\end{bmatrix}$

C. $\begin{bmatrix}6x\end{bmatrix}$

D. $\begin{bmatrix}32x\end{bmatrix}$

E. $\begin{bmatrix}8x\end{bmatrix}$

2. For which values of a and v is the following matrix equation true?

$$\begin{bmatrix}1 & 4\\ 2b & \frac{1}{3}b\end{bmatrix}-\begin{bmatrix}3 & -2\\ 6 & -8\end{bmatrix}=\begin{bmatrix}-2 & 6\\ a & a\end{bmatrix}$$

A. $a=\frac{-54}{5}, b=\frac{42}{5}$

B. $a=\frac{54}{5}, b=\frac{-42}{5}$

C. $a=\frac{54}{5}, b=\frac{42}{5}$

D. $a=\frac{-54}{5}, b=\frac{-42}{5}$

E. $a=\frac{54}{4}, b=\frac{42}{5}$

3. Matrix $P=\begin{bmatrix}-3 & 6\\ -2 & 11\end{bmatrix}$ and $Q=\begin{bmatrix}1 & 4\\ 7 & -5\end{bmatrix}$. What is $P-2Q$?

A. $\begin{bmatrix}-4 & 2\\ -9 & 16\end{bmatrix}$

B. $\begin{bmatrix}-2 & 10\\ 5 & 6\end{bmatrix}$

C. $\begin{bmatrix}2 & -10\\ -5 & -6\end{bmatrix}$

D. $\begin{bmatrix}-8 & 4\\ -18 & 16\end{bmatrix}$

E. $\begin{bmatrix}-5 & -2\\ -16 & 21\end{bmatrix}$

4. The determinant of a matrix $\begin{bmatrix}x & y\\ z & w\end{bmatrix}$ is $xw-yz$.

If the determinant of $\begin{bmatrix}2a & 13\\ a & a\end{bmatrix}$ is 24, which of the following is a value of a?

A. 4

B. 24

C. 8

D. -8

E. $\frac{3}{2}$

5. The 3x3 matrix $\begin{bmatrix}-3 & 6 & -5\\ 9 & 2 & -2\\ 4 & 3 & 1\end{bmatrix}$ is multiplied by a scalar n.

The resulting matrix is $\begin{bmatrix}9 & -18 & 15\\ -27 & a & 6\\ -12 & -9 & -3\end{bmatrix}$. What is a?

A. 2

B. -6

C. -3

D. 3

E. 6

6. The determinant of any 2x2 matrix $\begin{bmatrix}x & y\\ z & w\end{bmatrix}$ is $xw-yz$. If the determinant of $\begin{bmatrix}(x+4) & 4\\ 9 & (x-3)\end{bmatrix}$ is 8. What are all the possible values of x?

A. -4 and 3

B. 8 and -7

C. -8 and 7

D. 4 and -3

E. 7 and 8

7. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} b & c \\ d & a \end{bmatrix} =$

A. $\begin{bmatrix} ab & bc \\ cd & da \end{bmatrix}$

B. $\begin{bmatrix} ab+bd & ac+ba \\ cb+d^2 & c^2+da \end{bmatrix}$

C. $\begin{bmatrix} a+b & b+c \\ c+d & d+a \end{bmatrix}$

D. $\begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

E. $\begin{bmatrix} ab+c^2 & b^2+cd \\ ad+ca & bd+da \end{bmatrix}$

8. Which of the following matrices for $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ makes the following expression true?**

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ 23 & 37 \end{bmatrix}$

A. $\begin{bmatrix} 7 & \frac{13}{3} \\ \frac{23}{5} & \frac{37}{7} \end{bmatrix}$

B. $\begin{bmatrix} 6 & 10 \\ 18 & 30 \end{bmatrix}$

C. $\begin{bmatrix} 7 & 0 \\ 1 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

** denotes problem may be more challenging than typical ACT problems

9. If det $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$, then det $\begin{bmatrix} -c & -a \\ d & b \end{bmatrix}$ = ?

A. $-ad-bc$

B. $bc+ad$

C. $ad-cb$

D. $-cb-ad$

E. $-ad+cb$

10. Given the matrix equation shown below, what is $\frac{b}{a-b}$?

(Note: Whenever n is a positive integer, the notation of n! represents the product of the integers from n to 1. For examples, $3! = 3*2*1$)

$\begin{bmatrix} 4! \\ 2! \end{bmatrix} + \begin{bmatrix} 2! \\ 3! \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

A. $\frac{4}{9}$

B. $\frac{13}{9}$

C. $\frac{6}{5}$

D. 5

E. $\frac{1}{7}$

11. $\begin{bmatrix} a & a \\ b & b \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} \frac{1}{a+b} & \frac{1}{a+b} \\ \frac{1}{b+c} & \frac{1}{b+d} \end{bmatrix}$

A. $\begin{bmatrix} \frac{2a}{a+b} & 1 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 2a+\frac{1}{a+b} & a+b+\frac{1}{a+b} \\ b+c+\frac{1}{b+c} & b+d+\frac{1}{b+d} \end{bmatrix}$

C. $\begin{bmatrix} 2a+ab & 2a+2b \\ 2b+2c & 2b+2d \end{bmatrix}$

D. $\begin{bmatrix} \frac{aa}{a+b} & \frac{ab}{a+b} \\ \frac{bb}{b+c} & \frac{cd}{b+d} \end{bmatrix}$

E. $\begin{bmatrix} \frac{1}{3a+b} & \frac{1}{a+2b} \\ \frac{1}{2b+2c} & \frac{1}{2b+2d} \end{bmatrix}$

12. Emmy owns 2 juice bars (X and Y) and stocks 3 flavors of juice (A, B, and C). The matrices below show the numbers of each flavor of juice in each shop, and the cost for each flavor. Using the values given below, what is the difference between the value of juice inventories for the two shops?

$$\begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 25 & 50 & 20 \\ 50 & 100 & 25 \end{bmatrix} \end{array} \qquad \text{Cost} \quad \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} \$5 \\ \$10 \\ \$15 \end{bmatrix}$$

A. \$900
B. \$700
C. \$2850
D. \$2550
E. \$1275

13. The number of students who practice an art at a certain conservatory can be shown by the following matrix

$$\begin{array}{cccc} band & choir & painting & pottery \end{array}$$
$$\begin{bmatrix} 80 & 60 & 60 & 100 \end{bmatrix}$$

The head of the conservatory estimates the ratio of the number of art awards that will be earned to the number of students participating with the following matrix.

$$\begin{array}{r} band \\ choir \\ painting \\ photography \end{array} \begin{bmatrix} .2 \\ .4 \\ .1 \\ .3 \end{bmatrix}$$

Given this, which is the best estimate for the number of art awards that will be earned for the year?

A. 73
B. 74
C. 75
D. 76
E. 77

14. The determinant of any 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad-bc$.

The determinant of $\begin{bmatrix} 3 & (x-1) \\ (x+4) & 2 \end{bmatrix}$ is equal to 0.

What are all possible values of x?

A. $-\frac{\sqrt{17}-3}{3}$ and $\frac{\sqrt{17}+3}{3}$

B. 5 and -2

C. -5 and 2

D. $\frac{\sqrt{17}-3}{3}$ and $-\frac{\sqrt{17}+3}{3}$

E. none of the above

15. What value of x satisfies the matrix equation below? (Assume x is a scalar)

$$x\begin{bmatrix}3 & 2\\4 & 1\end{bmatrix}+\begin{bmatrix}2 & 2\\5 & 1\end{bmatrix}=\begin{bmatrix}11 & 8\\17 & 4\end{bmatrix}$$

A. $\frac{11}{5}$

B. $\frac{20}{9}$

C. 3

D. $\frac{11}{3}$

E. 4

16. $4\begin{bmatrix}0 & 1\\4 & -2\end{bmatrix}-2\begin{bmatrix}3 & -3\\-1 & 2\end{bmatrix}=?$

A. $\begin{bmatrix}0 & 4\\16 & -8\end{bmatrix}$

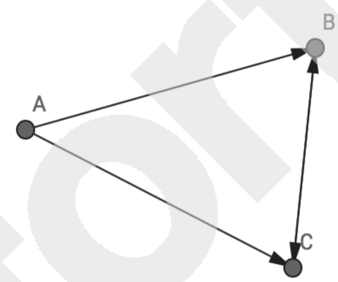
B. $\begin{bmatrix}-3 & 5\\9 & -6\end{bmatrix}$

C. 19

D. -22

E. $\begin{bmatrix}-6 & 10\\18 & -12\end{bmatrix}$

17. Graph theory is often used to represent connections between different points. Three satellites in space communicate with one another, as represented by the drawing below. For example, the arrows indicate that satellite A communicates with satellite B and C, but satellite C only communicates with satellite B. The same relationships are demonstrated in the matrix, where, because satellite A communicates with satellite B, there is a 1 in the A row and B column, but because satellite A does not communicate with itself, there is a 0 in A row and A column,



$$\begin{array}{c c}
 & \begin{matrix}A & B & C\end{matrix}\\
\begin{matrix}A\\B\\C\end{matrix} & \begin{bmatrix}0 & 1 & 1\\0 & 0 & 1\\ & & \end{bmatrix}
\end{array}$$

Which of the following is the third row of the matrix?

A. 1 1 1

B. 1 1 0

C. 1 0 0

D. 0 1 0

E. 0 0 0

18. For what (x, y) pair is the matrix equation below true?

$$\begin{bmatrix} y & 3x \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & 3 \\ 4 & \frac{x}{2} \end{bmatrix} = \begin{bmatrix} 21 & 9 \\ -4 & -1 \end{bmatrix}$$

A. $(4,24)$

B. $(24,4)$

C. $(1,2)$

D. $(2,1)$

E. $\left(-\frac{3}{4},-7\right)$

19. The 2×4 matrix $\begin{bmatrix} 1 & 2 & 8 & 4 \\ 2 & 5 & 6 & 1 \end{bmatrix}$ represents quadrilateral ABCD, with vertices $A(1,2), B(2,5)$, $C(8,6), D(4,1)$ in the standard coordinate plane. After the quadrilateral was reflected over the x – axis,

the matrix representing the translated triangle is $\begin{bmatrix} 1 & 2 & 8 & 4 \\ m & -5 & 3m & -1 \end{bmatrix}$. What is the value of m?

A. -2

B. 2

C. 3

D. -1

E. -3

20. $3\begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 0 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 2 & 1 \\ -3 & 8 \end{bmatrix} = ?$

A. $\begin{bmatrix} -1 & -6 \\ 2 & 7 \end{bmatrix}$

B. $\begin{bmatrix} -3 & -18 \\ 6 & 21 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 6 & -12 \\ 12 & 15 & 24 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 12 \\ 6 & 15 \\ -12 & 24 \end{bmatrix}$

E. $\begin{bmatrix} -12 & 45 \\ -4 & 84 \end{bmatrix}$

ANSWER KEY

1. A **2. C** **3. A** **4. B** **5. C** **6. D** **7. E** **8. C** **9. C** **10. C** **11. C** **12. B** **13. C** **14. A**

15. C **16. C** **17. E** **18. C** **19. C** **20. A**

ANSWER EXPLANATIONS

1. **A.** Here we have a 1×3 matrix and a 3×1 matrix respectively. To multiply, we must ensure the middle two numbers of the matrix dimensions match (they do: 3 and 3). Our solution will be a matrix made up of the first and last digits of the dimensions (1×1). To multiply matrices, multiply the rows times the columns: Row 1 of the first matrix times column 1 of the second. Add the product of the first item in the row times the first item in the column $(0\times x)$, the product of the second item in the row times the second item in the column $(1\times 3x)$ and the product of the third item in the row times the third item in the column $(2\times 4x)$. Simplify this to $0x+3x+8x=11x$. Write your answer in matrix form: $\begin{bmatrix}11x\end{bmatrix}$.

2. **C.** This problem essentially is four different equations written in one matrix based form. When we add matrices, we look only at the positions—same position means we add together those elements. I.e. in the matrix problem here $\begin{bmatrix} a & d \\ g & k \end{bmatrix}-\begin{bmatrix} b & e \\ h & l \end{bmatrix}=\begin{bmatrix} c & f \\ j & m \end{bmatrix}$, a,b, and c are all elements in the same position—so they form the following relationship – $a-b=c$. The same is true for all other "same position" letters in the above matrix. Let's now take the matrix at hand and reorganize it into a series of equations based on elements in the same position: $\begin{bmatrix} 1 & 4 \\ 2b & \frac{1}{3}b \end{bmatrix}-\begin{bmatrix} 3 & -2 \\ 6 & -8 \end{bmatrix}=\begin{bmatrix} -2 & 6 \\ a & a \end{bmatrix}$

   We get:

   $1-3=-2$

   $4--2=6$

   Those first two only state the obvious—not helping us much—but at least you can use these to verify that you're setting the problem up right.

   The last two will help more:

   $2b-6=a$ and $\frac{1}{3}b-(-8)=a$

   Now we have a system of two equations with two unknowns. We can solve by elimination or substitution. Here we will use substitution, substituting for a: First create a positive from the double negative to get $2b-6=\frac{1}{3}b+8$. Solving for b gives us $b=\frac{42}{5}$. Finally, plug in this fraction using the first (or second) equation to find a. $2\left(\frac{42}{5}\right)-6=a$ so $\frac{84}{5}-\frac{30}{5}=\frac{54}{5}=a$.

3. **E.** To understand the basics of how to subtract two matrices, see explanation at question 2. $2Q$ implies that we must first use scalar multiplication before performing the subtraction. To do so, multiply the scalar (2) by each item in the matrix. Let's solve for $2Q$:

   $$2\begin{bmatrix} 1 & 4 \\ 7 & -5 \end{bmatrix}=\begin{bmatrix} 2*1 & 2*4 \\ 2*7 & 2*-5 \end{bmatrix}=\begin{bmatrix} 2 & 8 \\ 14 & -10 \end{bmatrix}.$$

   Now we can set up $P-2Q$, subtracting numbers in the same positions:

   $$\begin{bmatrix} -3 & 6 \\ -2 & 11 \end{bmatrix}-\begin{bmatrix} 2 & 8 \\ 14 & -10 \end{bmatrix}=\begin{bmatrix} -3-2 & 6-8 \\ -2-14 & 11--10 \end{bmatrix}=\begin{bmatrix} -5 & -2 \\ -16 & 21 \end{bmatrix}$$

4. **C.** For this problem just apply the formula given and you get

$2a \times a - 13 \times a = 24$

$2a^2 - 13a = 24$

$2a^2 - 13a - 24 = 0$

To solve, use the quadratic formula or factoring. I'll use the quadratic formula (you should have this memorized). I suggest using your calculator for the multiplication and roots.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+13 \pm \sqrt{13^2 - 4(2)(-24)}}{2(2)} = \frac{13 \pm \sqrt{169 + 192}}{4} = \frac{13 \pm \sqrt{361}}{4} = \frac{13 \pm 19}{4}$$

Now we can split into two solutions. $\frac{13-19}{4} = \frac{-6}{4} \rightarrow -1.5$ OR $\frac{13+19}{4} = \frac{32}{4} \rightarrow 8$

8 is the only answer available, so C is correct. You could also "backsolve" this problem, using the answers, or pro gram your calculator to do the quadratic equation (for limitations on calculator programs see the ACT website (http://www.actstudent.org/faq/cas_functionality.html)).

5. **B.** A scalar is a single number that then is multiplied by each individual value in a matrix. To multiply the given matrix by n:

$$n\begin{bmatrix} -3 & 6 & -5 \\ 9 & 2 & -2 \\ 4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3n & 6n & -5n \\ 9n & 2n & -2n \\ 4n & 3n & 1n \end{bmatrix}$$

Now we can set this equal to the result:

$$\begin{bmatrix} -3n & 6n & -5n \\ 9n & 2n & -2n \\ 4n & 3n & 1n \end{bmatrix} = \begin{bmatrix} 9 & -18 & 15 \\ -27 & a & 6 \\ -12 & -9 & -3 \end{bmatrix} 9$$

Playing "match" we can match up each value in each position and form up to different equations. We actually will only need two, though. <u>We need to find a not n</u> -- but we'll need n to find a. To find n, we have many choices, but let's try for the first row / column value:

$$-3n = 9$$

$$n = -3$$

<u>Now don't go looking for that as the answer choice!</u> Plug in that value into another equation – the one that involves $a: 2n = a$. Again we found this equation by matching up values in the same positions in the equivalent matrices. We then simplify:

$$2(-3) = \boldsymbol{a} = -6$$

6. **E.** Here apply the formula given, set equal to 8, then FOIL and simplify:

$$(x+4)(x-3)-(4)(9)=8$$

$$x^2+4x-3x-12-36=8$$

$$x^2+x-48=8$$

$$x^2+x-56=0$$

Now we can apply the quadratic equation or factor. Remember you can program your calculator to do the quadratic equation as long as it's under 25 logical lines of code.

Here I'll factor to : $(x+8)(x-7)=0$

Which gives the answers $x=7$ or $x=-8$

7. **B.** This problem essentially asks the definition of how to multiply matrices. If you know what you're doing, this should be easy. (A) is wrong—you do not multiply in the same way you add and subtract matrices—you cannot just multiply each item in the same position together. Who knows why this isn't true but it's just the definition of matrix multiplication. (B) is correct—it adds the products of rows times columns. (C) adds the two matrices (D) multiplies the first matrix by a scalar of 2 and (E) mixes up rows and columns—it's rows times columns not columns times rows. If you missed this, go back and review the basics of matrix multiplication.

8. For this problem you must multiply the matrices. Let's first just focus on the setup to the left of the equals sign and simplify that into a single matrix. If you don't know how to do this, review the example problems earlier in the chapter. Take rows times columns and add the products:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} a(1)+b(5) & a(3)+b(7) \\ c(1)+d(5) & c(3)+d(7) \end{bmatrix} = \begin{bmatrix} a+5b & 3a+7b \\ c+5d & 3c+7d \end{bmatrix}$$

Now let's take what we have and put it with what the original problem was equal to, and match each corresponding position to make four equations:

$$\begin{bmatrix} a+5b & 3a+7b \\ c+5d & 3c+7d \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ 23 & 37 \end{bmatrix}$$

$$a+5b=7$$

$$c+5d=23$$

$$3a+7d=13$$

$$3c+7d=37$$

Now we could do the ridiculous task of solving each of these out—but it's a better idea to backsolve a bit—as most of the matrices are VERY different answers. We'll use the first of these equations ONLY – and check each one. Let's try (A): $a=7$ and $b=\frac{13}{3}$ -- no way will that give us the integer value 7. It's out. Let's try (B) $a=6$ and $b=10$. $6+50\neq 7$. Let's try (C) $7+5(0)=7$, but we can't assume this is right yet. (D) yields $0+5\neq 7$. (E) works too though-- $2+5=7$.

Now we've narrowed. Let's try another – this time let's use the last equation $3c+7d=37$: for (E) $c=3$ and d=4, so $3(3)+7(4)=9+28=37$—yes. But for (C), $c=1$ and $d=4$, so $3(1)+7(4)=3+28=31$--not 37.

Note that this problem tests skills for which you are responsible on the ACT but is more difficult than most ACT problems.

9. **C.** This is a very straightforward problem: finding the determinant, made even more simple by giving the definition of a determinant in the question. The trick will be in the format of the answers, as the answers will be rearranged to trick you. If approaching the problem traditionally det $\begin{bmatrix} -c & -a \\ d & b \end{bmatrix} = (-c)(b)-(-a)(d)$ which simplified becomes $-cb+ad$. While that is not an answer, answer (C), $ad-cd$, is the same by simply rearranging the order of the terms.

10. **A.** This requires multiple steps to solve. First apply the factorial (the !), and then sum the matrices properly, summing the corresponding entries until you find the final values for a and b, which are 26 and 8 respectively. Finally, plug those values in correctly to the expression desired. The result is $\frac{8}{18}$, which simplifies into $\frac{4}{9}$, answer (A).

11. **B.** This questions looks more complicated than it really is. You might assume that you will have to simplify the sum, but that's not even necessary. If you take the sum correctly, summing the corresponding entries, B is the obvious answer. If you know how to properly add matrices, then really all this question is testing is your fraction addition skills.

12. **B.** To solve this, you have to find the inventory for each shop, and then subtract to find the difference (it doesn't matter which you subtract from, as the difference is the absolute value).

    To find the total inventory for each shop, you have to multiply the matrices. (just in general, when given word problems with matrices whose inner dimensions match, you usually end up multiplying them for some reason or other). Think of the matrices as tables that organize this information. If you have 25 of Juice A in shop X, and according to the cost matrix juice A costs $\$5$, it makes sense that you have $\$125$ worth of Juice A in shop X $(\$25*\$5)$. Since you have three juices in each shop, you must sum the totals from each juice. Ultimately, it's just as if you were multiplying the matrices, and the resultant matrix tells you the value of the inventory in each shop.

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} X \\ Y \end{array} & \begin{bmatrix} 25 & 50 & 20 \\ 50 & 100 & 25 \end{bmatrix} \end{array} \quad * \quad \begin{array}{cc} & \text{Value} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} \$5 \\ \$10 \\ \$15 \end{bmatrix} \end{array} \quad = \quad \begin{array}{cc} & \text{Value} \\ \begin{array}{c} X \\ Y \end{array} & \begin{bmatrix} \$925 \\ \$1625 \end{bmatrix} \end{array}$$

    (shop x flavor) $*$ (flavor x value) = shop x value

    Then, $\$1625-\$925=\$700$, which is answer (B).

13. **D.** For this question, again you multiply the matrices, necessary in this problem to find out how many students per department receive an award. To find how many total are award across all the departments, sum the elements in that final matrix.

    You are multiplying:

$$\begin{bmatrix} 80 & 60 & 60 & 100 \end{bmatrix} \quad * \quad \begin{array}{cc} \begin{array}{c} band \\ choir \\ painting \\ pottery \end{array} & \begin{bmatrix} .2 \\ .4 \\ .1 \\ .3 \end{bmatrix} \end{array} \quad = \quad \begin{bmatrix} 16 & 24 & 6 & 30 \end{bmatrix}$$

    school $*$ art department $*$ dept. $*$ award = school $*$ students awarded per dept.

    $16+24+6+30=76$, which is answer (D).

14. **C.** With the formula given, all you need to do is substitute the values given with the corresponding variables.

$$\begin{bmatrix} 3 & (x-1) \\ (x+4) & 2 \end{bmatrix} = 0$$

Following the formula we get $6-(x+4)(x-1)=0$ or $6-(x^2+3x-4)=0.$ After distributing the subtraction sign and simplifying we get $6-x^2-3x-(-4)=6-x^2-3x+4=-x^2-3x+10=0.$ After factoring we get $(-x-5)(x-2)=0$ so, $x=-5$ and 2.

15. **C.** Answer (C) correctly solves for x. The trick with problems like this is that you don't have to solve the entire matrix. Selecting the top left element, you multiply the matrix by x and get the following:

$$\begin{bmatrix} 3x & 2x \\ 4x & 1x \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 17 & 4 \end{bmatrix}$$

But since we are only going to focus on one address, the top left, all that matters is: $3x+2=11$

Solve this and $x=3$, which is answer (C).

16. **E.** Answer (E) is correct because per the rules of scalar multiplication, it multiplies all the elements in the matrix by the scalar, and then it subtracts the elements with the corresponding ones on the second matrix.

17. **D.** Satellite C receives messages from both satellite A and B, but it only sends to satellite B, as seen by the double arrow line between C and B, so the correct answer is **0 1 0.**

band choir painting pottery

18. **D.** In order to solve this problem, you have to pick the right place to start. Although convention dictates that you multiply matrices starting with the top left element, and go left to right, top to bottom, the order doesn't really matter. In this case, you must first find x, which is possible by creating the equation for the bottom right element in the final matrix. In that case, you get the equation $0(3)+(-1)\left(\frac{x}{2}\right)=-1,$ and when solved tells you that $x=2.$ From there you already know your answer, (D), since no other answer has 2 for $x.$

19. **A.** In this question the matrices merely serve as a way to organize information, in this case coordinate points. Reflecting a polygon over the x axis means that the coordinates stay the same while the y coordinates become negative. Since the top row of the matrix represents the x coordinates, those stay the same, while the bottom row elements, which represents the y coordinates, all become negative. Thus, $2,$ the left most bottom element, negated becomes $-2.$ Looking at the translated matrix, m takes the place of where -2 should be, so m is clearly $-2.$

20. B. This problem combines scalar and matrix multiplication. Scalar multiplication is commutative, but matrix multiplication is not. The scalar factor, $3,$ can be applied either before or after the two matrices are multiplied correctly (row by column, adding the products), but if the numbers get to big, it's better to wait until after

$$3\begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 0 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 2 & 1 \\ -3 & 8 \end{bmatrix} = 3\begin{bmatrix} (2)(-2)+(0)(2)+(-1)(-3) & (2)(1)+(0)(1)+(-1)(8) \\ (3)(-2)+(4)(2)+(0)(3) & (3)(1)+(4)(1)+(0)(8) \end{bmatrix} = 3\begin{bmatrix} -1 & -6 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -3 & -18 \\ 6 & 21 \end{bmatrix}.$$