SOH CAH TOA

ACT Math: Lesson and Problem Set

DEFINITIONS TO KNOW

Trigonometric function	Is equal to
sin	Opposite leg divided by hypotenuse
cos	Adjacent leg divided by hypotenuse
tan	Opposite leg divided by adjacent leg

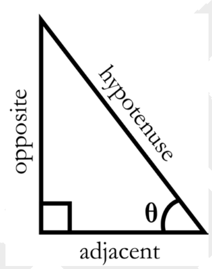
CONCEPTS TO KNOW

- Word problems
- Choosing which trig function to use
- Given angle or right triangle diagram, use SOHCAHTOA to find missing side length or angle
- Given the $\sin/\cos/\tan/etc$ of an unknown angle, find the $\sin/\cos/\tan/etc$ of that same angle

SOHCAHTOA BASICS

Before we dive into other trig topics, it is important that we are first familiar with the basics, which are best explained through using a triangle and "SOHCAHTOA". This acronym defines the ratios that the three most common trigonometric functions describe.

As explained above, SOHCAHTOA stands for:
SOH: **S**ine = **O**pposite over **H**ypotenuse
CAH: **C**osine = **A**djacent over **H**ypotenuse
TOA: **T**angent = **O**pposite over **A**djacent
This can be visualized with this diagram:



So, for example, when a question asks for the $\sin(67^\circ)$, it is asking for the ratio of the length of the opposite to the length of the hypotenuse for any right triangle with an angle of 67°. This value will always be the same, regardless of the scale of the triangle.

In case clarification is needed, just remember:

- The Hypotenuse is always the longest side of the triangle
- The Opposite is the side farthest from the angle of interest
- The Adjacent is the side closest to the angle that isn't the hypotenuse

WORD PROBLEMS

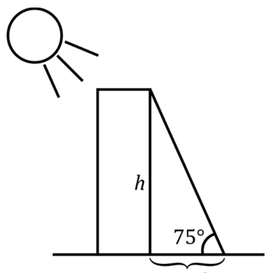
Don't be scared by all the text in word problems; pick out the numbers and pertinent info and you'll be able to solve every problem!

TIP: Trigonometric word problems tend to be much easier to solve if you draw it out so we don't get any numbers mixed up.

Example:

> At a time when the sun's rays are striking the ground at $75°$, a skyscraper casts a shadow that is 300 feet long. To the nearest foot, how tall is the skyscraper?

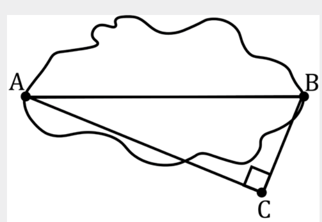
Let's draw this out first:



Now that we can visualize everything, we know we're trying to find height "h" of the building. The angle given, $75°$, is opposite to "h" and adjacent to the given shadow length, 300 ft. Thus, we can solve for "h" by using tangent. We can't use sine or cosine because we don't know the length of the hypotenuse. Remember, tan is opposite over adjacent, so we can set up our equations like so: $\tan(75°) = \frac{h}{300}$. Isolating the h, we get $h = 300\tan(75°)$. Solving for h, we get $h \approx 1112$ ft.

CHOOSING WHICH TRIG FUNCTION TO USE

You can do this by looking at what the question gives you. For example, if only the hypotenuse is given, you know you won't be using the tan function since you won't be able to use any of the given values.

> Scientists attempting to measure the width of a mountain, represented in the figure below by $\overline{AB}$, use the measurements represented in the figure below. The distance from A to C is 5 km. The measure of $\angle CBA$ is $43°$. What is the width of the mountain?
>
> 
>

$$\sin(43°) = \frac{AC}{AB}$$
$$\sin(43°) = \frac{5 \text{ km}}{AB}$$
$$AB = \frac{5 \text{ km}}{\sin(43°)}$$
$$AB = 7.33 \text{ km}$$

FINDING THE MISSING SIDE LENGTH OR ANGLE

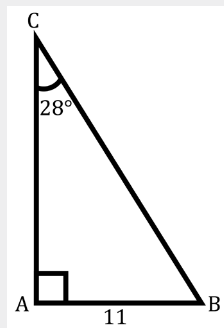
Problems will not always provide all side lengths and angles so you can just plug in numbers and arrive at your answers. The ACT may ask you to use the values of trig functions that you can get from your calculator on demand to solve for a missing side using SOHCAHTOA or find a missing angle using inverse trig functions!

FINDING A MISSING SIDE

One common scenario you may be faced with on the ACT consists of a right triangle with one other angle and at least one side given. What do we do if we are asked to find a missing side?

Luckily, we can make use of our calculators, knowledge of right triangle anatomy, and SOHCAHTOA to find any missing side. Remember, the trig functions give us rations between sides of a triangle, so if we have an angle and any side, we can find any other side by setting up the ratio and solving for our unknown. Let's look at an example:

Find the missing side $\overline{BC}$ in the triangle given below.



Using our knowledge of trig functions, we can set up a ratio between the side we are given and the unknown side and then solve for the unknown. But which trig function can we use? Well, we are given the length of the side opposite the angle, and we are looking for the hypotenuse. What a coincidence! The sine function is a ratio between the opposite and the hypotenuse. Now let's set up our equation.

$$\sin\left(x^\circ\right) = \frac{\text{opposite}}{\text{hypotenuse}} \rightarrow \sin\left(28^\circ\right) = \frac{11}{\overline{BC}}$$

Now we can rearrange the equation to solve for $\overline{BC}$, our unknown side.

$$\overline{BC} = \frac{11}{\sin\left(28^\circ\right)}$$

Plugging this into our calculator, we get that $\overline{BC} = 23.43.$

FINDING A MISSING ANGLE

Another common scenario on the ACT is being given a right triangle with enough information to get all of the side lengths, but no angles (except for the given right angle). This gets a little more complicated because we cannot use the same process that we used to find a missing side length. This is where we introduce you to the inverse trigonometric functions.

INVERSE TRIG FUNCTIONS

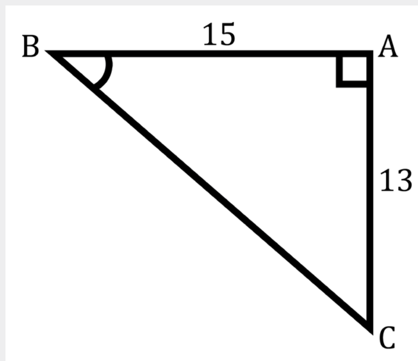
We use inverse trigonometric functions to work backwards, essentially. Instead of inputting an angle and getting a ratio back, it's the other way around!

Inverse functions are usually denoted by a "-1" superscript or an "arc" prefix.

Examples: $\sin^{-1}\left(\frac{12}{13}\right)$, $\arctan\left(\frac{3}{4}\right)$, $\cos^{-1}(0.385)$

Now let's go through an example to see how we apply these functions in the context of a real problem:

Find $\measuredangle B$ in the triangle given below.



First let's see what sides we are given; we are given the side opposite of the unknown angle and the side adjacent to the unknown angle. The tangent function relates the opposite to the adjacent, so we will use that.

$$\tan(B) = \frac{13}{15}$$

This is a situation where the use of the inverse function is required.
However, since we do not have the angle and only the side lengths, we must use the $\arctan$ or $\tan^{-1}$ function. The above expression can be rewritten as:

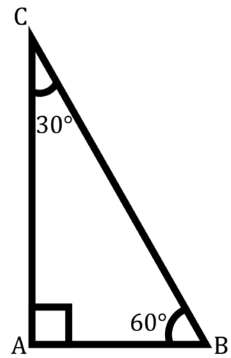
$$\arctan\left(\frac{13}{15}\right) = B$$

or

$$\tan^{-1}\left(\frac{13}{15}\right) = B$$

Plugging this into our calculator, we get that $\measuredangle B = 40.91^\circ$.

1. In the $30°-60°-90°$ right triangle below, $AB=10$ cm. What is the length, in centimeters, of $\overline{BC}$?



A. 20

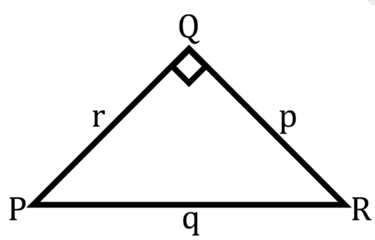
B. $10\sqrt{3}$

C. 10

D. $\frac{10\sqrt{3}}{3}$

E. 5

2. For right triangle $\triangle PQR$ shown below, what is $\cos(P)$?



A. $\frac{p}{r}$

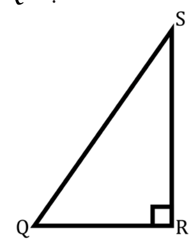
B. $\frac{p}{q}$

C. $\frac{q}{r}$

D. $\frac{r}{p}$

E. $\frac{r}{q}$

3. The hypotenuse of the right triangle $\triangle QRS$ shown below is 18 feet long. The cosine of $\angle Q$ is $\frac{2}{3}$. About how many feet long is $\overline{QR}$?



A. 9

B. 12

C. 13.4

D. 17.3

E. 27

4. A zipline is to be built so that a taut cable is at a $12°$ incline relative to the level ground. If the zipline's starting point is to be 11 meters above the ground at its starting point and end at ground-level, how far horizontally will the endpoint be from the starting point?

A. $11\sin 12°$

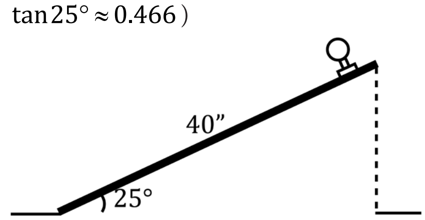
B. $11\cos 12°$

C. $11\tan 12°$

D. $\frac{11}{\tan 12°}$

E. $11\cot 12°$

5. A door is propped open $25°$ with a bar as shown in the diagram below. If the door is 40 inches long, how long does the bar need to be?
(Note: $\sin 25° \approx 0.423$
$\cos 25° \approx 0.906$
$\tan 25° \approx 0.466$)



A. 17

B. 19

C. 36

D. 44

E. 95

6. The sides of a right triangle measure 20 inches, 21 inches, and 29 inches. What is the cosine of the angle opposite the side that measures 20 inches?

A. $\dfrac{20}{29}$

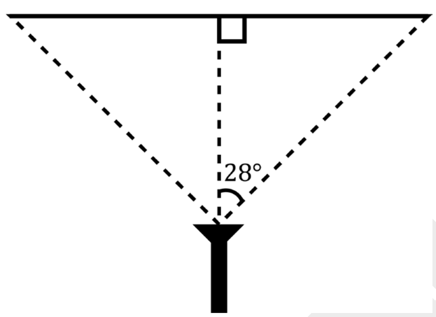
B. $\dfrac{29}{20}$

C. $\dfrac{21}{29}$

D. $\dfrac{29}{21}$

E. $\dfrac{21}{20}$

7. A flashlight emits a cone of light as shown below. If the flashlight is projecting a circle of light on the wall that is 80 cm in diameter, how far is the flashlight from the wall in centimeters?



A. $40\tan 20°$

B. $\dfrac{\tan 20°}{40}$

C. $\dfrac{40}{\tan 20°}$

D. $\dfrac{80}{\tan 20°}$

E. $80\tan 20°$

8. A right triangle has legs of length $49\sin\theta$ units and $49\cos\theta$ units for some angle θ that satisfies $0° \le \theta \le 90°$. What is the length, in units, of the longest side of the triangle?

A. θ

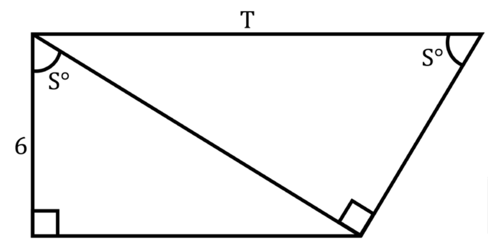
B. 1

C. 7

D. 49θ

E. 49

9. In the figure below, $\cos S = \dfrac{3}{5}$. What is the approximate value of T using the given information and the diagram below?



A. 10

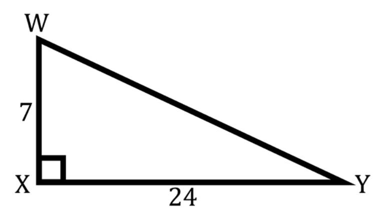
B. 13.5

C. 0.60

D. 12.5

E. 53.1

10. In the right triangle shown below, the length of $\overline{WX}$ is 7 feet and the length of XY is 24 feet. For $\angle Y$, the value of which of the following trignometric expressions is $\dfrac{7}{25}$?



A. $\cos Y$

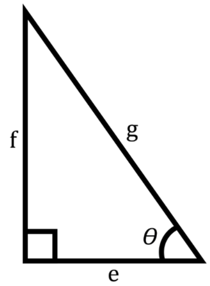
B. $\sin Y$

C. $\tan Y$

D. $\sec Y$

E. $\csc Y$

11. The dimensions of the right triangle shown below are given in meters. What is $\cos\theta$?



A. $\frac{e}{g}$

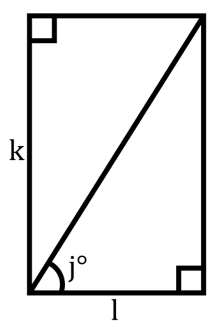
B. $\frac{f}{g}$

C. $\frac{g}{e}$

D. $\frac{g}{f}$

E. $\frac{f}{e}$

12. Which of the following trigonometric equations is valid for the measurements indicated below for the two sides and angle formed by a rectangle and its diagonal?



A. $\tan j° = \frac{l}{k}$

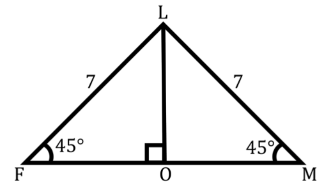
B. $\cos j° = \frac{k}{l}$

C. $\cos j° = \frac{l}{k}$

D. $\cot j° = \frac{k}{l}$

E. $\sec j° = \frac{l}{k}$

13. In right triangle $\triangle LMF$ below, distances are shown in yards. How many yards long is $\overline{LO}$?



A. $\frac{14\sqrt{2}}{2}$

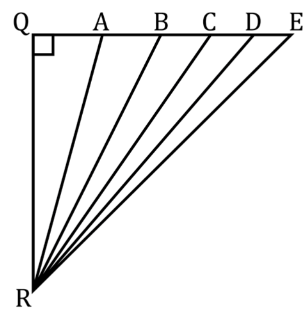
B. $14\sqrt{2}$

C. 7

D. $7\sqrt{2}$

E. $\frac{7\sqrt{2}}{2}$

14. In the figure below, A, B, C, and D are on $\overline{QE}$. Which of the following angles has the smallest cosine?



A. $\angle QRA$

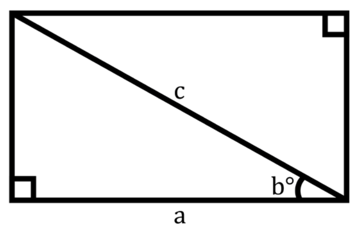
B. $\angle QRB$

C. $\angle QRC$

D. $\angle QRD$

E. $\angle QRE$

15. Which of the following trigonometric equations is valid for the side measurement a nanometers, diagonal measurement c nanometers, and angle measurement $b°$ in the rectangle shown below?



A. $\sin b° = \frac{a}{c}$

B. $\cos b° = \frac{a}{c}$

C. $\cos b° = \frac{c}{a}$

D. $\tan b° = \frac{a}{c}$

E. $\sec b° = \frac{a}{c}$

16. An angle in a right triangle has measure β. If $\cos\beta = \frac{5}{13}$ and $\tan\beta = \frac{12}{5}$, then $\sin\beta = ?$

A. $\frac{12}{5}$

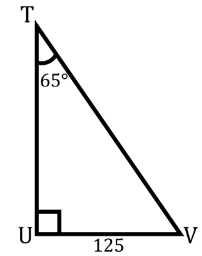
B. $\frac{12}{13}$

C. $\frac{13}{12}$

D. $\frac{12}{\sqrt{194}}$

E. $\frac{12}{\sqrt{313}}$

17. In right triangle $\triangle TUV$ below, $\overline{UV}$ is 125 yards long and $\angle T = 65°$. What is the measure of $\overline{TV}$?



A. $125\sin 65°$

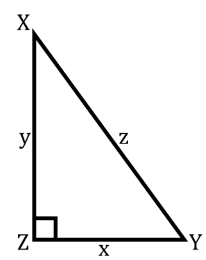
B. $\frac{125}{\sin 65°}$

C. $\frac{\sin 65°}{125}$

D. $125\cos 65°$

E. $\frac{125}{\cos 65°}$

18. For any right triangle as shown, $(\cos X)(\tan Y) = ?$ (All side lengths are given in meters, all angle measurements are given in degrees)



A. $\frac{y^2}{xz}$

B. $\frac{x^2}{yz}$

C. $\frac{x}{z}$

D. $\frac{y}{z}$

E. $\frac{z}{x}$

19. An arbologist (a scientist who studies trees) comes across a tree that juts out of the ground at $12°$. When the sun is directly overhead, so that its rays are perpendicular to the ground, the tree's shadow is 8 meters long. If it can be determined, what is the length of the tree?

A. $\dfrac{8}{\cos 12°}$

B. $8\cos 12°$

C. $\dfrac{8}{\tan 12°}$

D. $8\tan 12°$

E. $\dfrac{8}{\sin 12°}$

20. In $\triangle HIJ$, $\angle J$ is a right angle and the measure of $\angle I$ is $63°$. Another triangle, $\triangle QRS$ is being constructed such that $\angle R$ is a right angle and the measure of $\angle S$ is one third the measure of $\angle I$. What is the measure of $\angle Q$?

A. 21

B. 22.5°

C. 45°

D. 69°

E. 159°

ANSWER KEY

1. A 2. E 3. B 4. D 5. A 6. C 7. C 8. E 9. D 10. B 11. A 12. C 13. E 14. E 15. B 16. B 17. B 18. A 19. A 20. D

ANSWER EXPLANATIONS

1. **A.** Remember that the corresponding ratio of the sides of a $30°-60°-90°$ triangle is $1:\sqrt{3}:2$, respectively. Since 10 is opposite of $30°$, and the side we are looking for is opposite $90°$, the ratio of 10 to the side we're solving for is equal to $1:2$. $\frac{10}{n}=\frac{1}{2}$, where n is the side we are seeking. Simple algebra shows that $n=20$.

2. **E.** Remember that the cosine of an angle in a right triangle is equal to the adjacent side divided by the hypotenuse. The side adjacent to $\angle P$ is r, and the hypotenuse is q. Thus, $\cos(P)=\frac{r}{q}$.

3. **B.** The cosine is equal to the adjacent side divided by the hypotenuse. The cosine of $\angle Q=\frac{\overline{QR}}{18}=\frac{2}{3}$. Algebra shows that $\overline{QR}=12$.

4. **D.** We can sketch the triangle formed by the starting point, the ground below it, and the ending point. The zipline forms the hypotenuse, and the height of the starting point and the horizontal distance to the ending point form the two legs. The distance we are given is the height of the starting point, and we want to find the horizontal distance. The height of the starting point divided by the horizontal distance, d, will be equal to $\tan 12°$. Thus $\frac{11}{d}=\tan 12°$, so $d=\frac{11}{\tan 12°}$.

5. **A.** The relation between the length of the bar, b, and the length of the door is the relation between the side opposite the $25°$ angle and the hypotenuse. $\frac{b}{40}=\sin 25°\approx 0.423$. Thus, $b\approx 40\left(0.423\right)=16.92\approx 17$.

6. **C.** The cosine of the angle opposite the side that measures 20 inches is the measure of the side adjacent the angle divided by the hypotenuse. We know that the hypotenuse must be the 29 because the hypotenuse is always the longest side of a right triangle. That leaves the adjacent side as 21. Thus, the cosine is $\frac{21}{29}$.

7. **C.** Recognize that the side of the right triangle opposite $20°$ is the radius of the circle, not the diameter. Thus, $\sin 20°=\frac{40}{d}$, where d is the distance from the wall, not $\frac{80}{d}$. Once you have this it is easy to rearrange it to $d=\frac{40}{\sin 20°}$.

8. **E.** The longest side of the right triangle is the hypotenuse, which we can find using the Pythagorean theorem. Plug in the the measures we are given: $\sqrt{\left(49\sin\theta\right)^2+\left(49\cos\theta\right)^2}=\sqrt{c^2}$, where c is the hypotenuse. We can simplify the right side and combine the 49's of the right to get $c=\sqrt{49^2\left(\sin\theta^2+\cos\theta^2\right)}$. We know that $\sin\theta^2+\cos\theta^2=1$, so this becomes $c=\sqrt{49^2\left(1\right)}=49$.

9. **D.** Because we know that the ratio for $\cos S=\frac{3}{5}$, and that cosine is adjacent over hypotenuse, we can set up a ratio with our side length 6: $\frac{3}{5}=\frac{6}{h}$, with h being the hypotenuse length of the bottom triangle. We also can figure out the angle of S itself with $\cos S=\frac{3}{5}$. Taking the arccos of 0.6, we solve that $S=53.1°$. Now, we can find out T by setting up this equation: $\sin 53.1=\frac{10}{T}$. Solving this, we get $T\approx 12.5$.

10. **B.** We can recognize that this triangle is a $7-24-25$ right triangle (or solve for the hypotenuse manually). For $\angle Y$, the 7 is opposite, and the 25 foot side is the hypotenuse. This makes $\frac{7}{25}$ the opposite divided by the hypotenuse, which is the sine.

11. **A.** The cosine of an angle is the measure of the adjacent side divided by the measure of the hypotenuse. The side adjacent to $\angle\theta$ is e, and the hypotenuse is g. By plugging in these values we find that $\cos\theta = \frac{e}{g}$.

12. **C.** The side of measure l is adjacent to the angle of measure j, and the side of measure k is a hypotenuse. The trigonometric function for the relationship between an adjacent side and the hypotenuse is the cosine. The cosine is the adjacent side, l, divided by the hypotenuse, k. This is expressed as $\cos j° = \frac{l}{k}$.

13. **E.** One easy way to solve this problem is to recognize that the diagram shown depicts half of a square, cut across the diagonal. The diagonal of a square is equal to the side times $\sqrt{2}$, and since the center line of the diagram is the diagonal cut in half, it is equal to the side of the square times $\frac{\sqrt{2}}{2}$. Plugging in the side, 7, we get $\frac{7\sqrt{2}}{2}$.

14. **E.** The cosine is equal to the adjacent side divided by the hypotenuse. The smallest cosine will be the one with the smallest measure for cosine and largest measure for hypotenuse. All of the angles we can choose as answers have the same adjacent side, $\overline{QR}$, so the only difference will be in the length of the hypotenuse. The largest hypotenuse will have the smallest cosine, and we can tell just by looking at the diagram that $\angle QRE$ has the largest hypotenuse, so it is our answer.

15. **B.** The two sides we are given are the adjacent side and hypotenuse relative to the angle of measure $b°$. The relationship between these two sides is expressed in the cosine. The cosine is the adjacent side divided by the hypotenuse, which in our case is $\frac{a}{c}$. Our answer is $\cos b° = \frac{a}{c}$.

16. **B.** The sine of an angle is the opposite side divided by the hypotenuse. We can find the measure of the opposite side from the tangent we are given, $\frac{12}{5}$, which is the opposite side divided by the adjacent side. If the adjacent side is 5, the opposite side is 12. We can find the hypotenuse from the cosine we are given, $\frac{5}{13}$, which is the adjacent side divided by the hypotenuse. If the adjacent side is 5, the hypotenuse is 13. Let us assume that in our triangle the adjacent side is 5. This won't affect our answer since an angle's sine is the same no matter what the sides are. The opposite side is 12, and the hypotenuse is 13, so the sine (opposite/hypotenuse) is $\frac{12}{13}$.

17. **B.** We are given the measure of an angle, the measure of its opposite side, and we are looking for the measure of the hypotenuse. The relationship between an angle's opposite side and hypotenuse is expressed as the sine of that angle. The sine of $65°$ here is $\frac{125}{n}$ where n is the measure we are looking for. Since, $\sin 65° = \frac{125}{n}$, we can rearrange this into $n = \frac{125}{\sin 65°}$.

18. **A.** The cosine of X is the adjacent side divided by the hypotenuse: $\frac{y}{z}$. The tangent of Y is the opposite side divided by the adjacent side, $\frac{y}{x}$. Plugging these expressions in for the trigonometric functions, we are evaluating $\left(\frac{y}{z}\right)\left(\frac{y}{x}\right) = \frac{y^2}{xz}$.

19. **A.** The important thing to remember in this problem is that the length of the tree is the hypotenuse, not the side. We are given the angle made by the tree and the ground and the length of its shadow, which relative to the angle is the adjacent side. We now have enough information to solve the problem. The cosine is the adjacent side (what we are given) divided by the hypotenuse (what we are looking for). $\cos 12° = \frac{8}{n}$, where n is the length of the tree. Rearranging this, we get $n = \frac{8}{\cos 12°}$.

20. **D.** The measure of $\angle S = \frac{1}{3}\angle I = \frac{1}{3}(63°) = 21°$. We are given that $\angle R = 90°$. We know that $\angle Q + \angle R + \angle S = 180°$, since the interior angles of a triangle on a plane sum to $180°$. Substitute in the values we have: $\angle Q + 90° + 21° = 180° \rightarrow \angle Q + 111° = 180° \rightarrow \angle Q = 69°$.