CONICS: CIRCLES AND ELLIPSES

ACT Math: Problem Set

1. In the standard (x,y) coordinate plane, what is the radius of the circle $(x-7)^2+(y+9)^2=169$?
   A. 169
   B. 3
   C. $\sqrt{7}$
   D. 7
   E. 13

2. A certain circle has equation of $\left(x-\sqrt{5}\right)^2+\left(y+2\sqrt{3}\right)^2=43$ in the standard (x,y) coordinate plane. What are the coordinates of the center of the circle, and the radius of the circle, in coordinate units?

	Center	Radius
A.	$\left(\sqrt{5},-2\sqrt{3}\right)$	$\sqrt{43}$
B.	$\left(-\sqrt{5},2\sqrt{3}\right)$	$\sqrt{43}$
C.	$\left(\sqrt{5},-2\sqrt{3}\right)$	43
D.	$\left(\sqrt{5},-2\sqrt{3}\right)$	21.5
E.	$\left(-\sqrt{5},2\sqrt{3}\right)$	21.5

3. A circle in the standard (x,y) coordinate plane has center $(-4,7)$ and radius 3 units. Which of the following equations represents this circle?
   A. $(x+4)^2+(y-7)^2=9$
   B. $(x-4)^2+(y+7)^2=9$
   C. $(x-4)^2+(y+7)^2=3$
   D. $(x+4)^2+(y-7)^2=3$
   E. $(x+4)^2-(y-7)^2=9$

4. A circle in the standard (x,y) coordinate plane has center $(-12,8)$ and radius 11 units. Which of the following equations represents this circle?
   A. $(x-12)^2-(y+8)^2=121$
   B. $(x-12)^2-(y+8)^2=11$
   C. $(x-12)^2+(y+8)^2=11$
   D. $(x+12)^2+(y-8)^2=121$
   E. $(x+12)^2+(y-8)^2=11$

5. What is the equation of a circle in the standard (x,y) coordinate plane that has a radius of 8 and the same center as the circle described as $x^2-4x+y^2+8y+10=0$?
   A. $x^2+y^2=64$
   B. $(x+4)^2+(y-2)^2=64$
   C. $(x-4)^2+(y+2)^2=64$
   D. $(x-2)^2+(y+4)^2=64$
   E. $(x+2)^2+(y-4)^2=64$

6. In the standard (x,y) coordinate plane, what are the coordinates of the center of the circle whose equation is $x^2+12x+y^2-2y-15=0$?
   A. $(6,-1)$
   B. $(-1,-6)$
   C. $(1,-6)$
   D. $(-6,1)$
   E. $(6,1)$

7. A circle with an area of 9π square in the standard (x,y) coordinate plane has its center at $(8,7)$. Which of the following is an equation for this circle?
   A. $(x-7)^2+(x-8)^2=81$
   B. $(x-7)^2+(y-8)^2=9$
   C. $(x-7)^2-(y-8)^2=81$
   D. $(x-8)^2+(y-7)^2=9$
   E. $(x-7)^2+(y-8)^2=81$

8. In the (x,y) coordinate plane, what is the radius of the circle having the points $(0,-8)$ and $(-24,24)$ as endpoints of a diameter?
   A. 10
   B. 20
   C. 40
   D. 49
   E. 60

9. A circle in the standard (x, y) coordinate plane is tangent to the x-axis at -7 and tangent to the y-axis at -7. Which of the following is an equation of the circle?

A. $x^2 + y^2 = 7$

B. $x^2 + y^2 = 49$

C. $(x+7)^2 + (y+7)^2 = 7$

D. $(x+7)^2 + (y+7)^2 = 49$

E. $(x-7)^2 + (y-7)^2 = 49$

10. What is the center of the circle with the equation $(x+3)^2 + (y-3)^2 = 27$ in the standard (x, y) coordinate plane?

A. $(-\sqrt{3}, \sqrt{3})$

B. $(-3,3)$

C. $(\sqrt{3}, -\sqrt{3})$

D. $(3,-3)$

E. $(3,3)$

11. Which of the following is an equation of the largest circle that can be inscribed in an ellipse with the equation $\frac{(x+5)^2}{36} + \frac{(y-7)^2}{25} = 1$?

A. $(x+5)^2 + (y-7)^2 = 900$

B. $(x+5)^2 + (y-7)^2 = 36$

C. $(x+5)^2 + (y-7)^2 = 25$

D. $x^2 + y^2 = 36$

E. $x^2 + y^2 = 25$

12. Which of the following points is a x-intercept of the graph of $\frac{x^2}{81} + \frac{y^2}{49} = 1$?

A. $(81,0)$

B. $(49,0)$

C. $(15,0)$

D. $(9,0)$

E. $(7,0)$

13. Which of the following points is a y-intercept of the graph of $\frac{x^2}{13} + \frac{y^2}{21} = 1$?

A. $(0,21)$

B. $(0,13)$

C. $(0,-\sqrt{13})$

D. $(0,\sqrt{21})$

E. $(0,\sqrt{13})$

14. Tyler's Tattoo Parlor's logo consists of three concentric circles. The radius of the innermost circle of the logo is 1 foot. The distance between the innermost circle and the outermost circle is 5.5 feet and the distance between the outermost circle and middle circle is 2.5 feet. Which of the following is an expression for the area, in square feet, of the middle circle of the logo of Tyler's Tattoo Parlor?

A. $(4)^2\pi$

B. $(1+4)\pi$

C. $(1^2+4^2)\pi$

D. $(1+5.5)^2\pi$

E. $(1+5.5)\pi$

15. In the standard (x, y) coordinate plane, the graph of the equation $x^2 + 8x - 5y^2 - 10y + 12 = 0$ is a(n):

A. Linear line
B. Parabola
C. Circle
D. Ellipse
E. Hyperbola

16. A circle in the standard (x, y) coordinate plane has an equation of $x^2 + (y+2)^2 = 24$. What are the radius of the circle and the coordinate of the center of the circle?

	radius	center
A.	24	$(0,2)$
B.	24	$(0,-2)$
C.	$\sqrt{24}$	$(0,-2)$
D.	$\sqrt{24}$	$(0,2)$
E.	12	$(0,2)$

17. A circle in the standard (x, y) coordinate plane has center $(-4,3)$ and is tangent to the y–axis. The point (x, y) is on the circle if and only if x and y satisfy which of the following equations?

A. $(x+4)^2+(y+3)^2=16$
B. $(x-4)^2+(y+3)^2=16$
C. $(x+4)^2+(y-3)^2=16$
D. $(x-4)^2+(y+3)^2=9$
E. $(x+4)^2+(y-3)^2=9$

18. In the standard (x, y) coordinate plane there is only one circle centered at the point $(5,2)$ that also passes through the point $(9,-1)$. Which of the following is the equation for that circle?

A. $(x+5)^2+(y+2)^2=25$
B. $(x-5)^2+(y+2)^2=5$
C. $(x-5)^2+(y+2)^2=25$
D. $(x-5)^2+(y-2)^2=25$
E. $(x-5)^2+(y-2)^2=5$

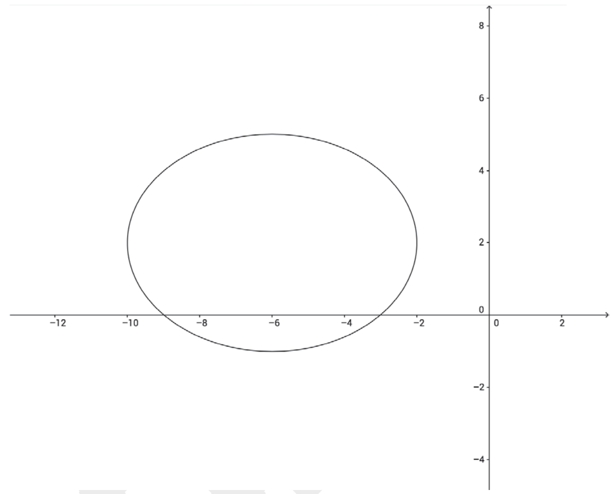
19. In the standard (x, y) coordinate plane, what is another way of writing the equation for a circle whose equation is $x^2+y^2+4x-2y=4$?

A. $(x-1)^2+(y+2)^2=9$
B. $(x+2)^2+(y+1)^2=3$
C. $(x-1)^2+(y-2)^2=3$
D. $(x+2)^2-(y-1)^2=9$
E. $(x+2)^2+(y-1)^2=9$

20. What is the largest value of x for which there exists a real value of y such that $x^2+y^2=100$?

A. 6
B. 8
C. 10
D. 99
E. 100

21. Which of the following is the equation of the ellipse that is graphed in the standard (x, y) coordinate plane below?



A. $\dfrac{(x-6)^2}{16}+\dfrac{(y-2)^2}{9}=1$
B. $\dfrac{(x-6)^2}{4}+\dfrac{(y+2)^2}{3}=1$
C. $\dfrac{(x+6)^2}{16}+\dfrac{(y+2)^2}{9}=1$
D. $\dfrac{(x+6)^2}{16}+\dfrac{(y-2)^2}{9}=1$
E. $\dfrac{(x+6)^2}{4}+\dfrac{(y-2)^2}{3}=1$

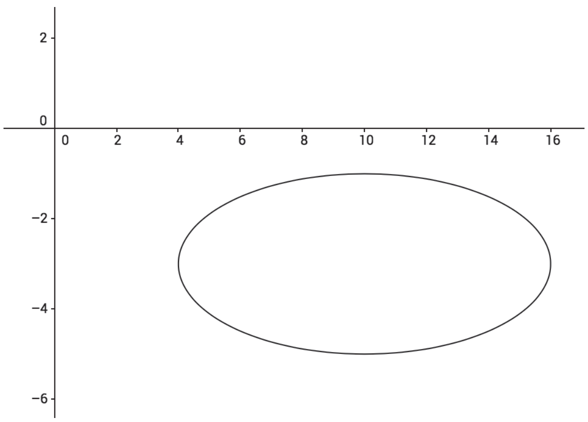
22. Jenny decides to frame the edge of her elliptical mirror. The perimeter of an ellipse is given by the formula $p=\frac{\pi}{2}\sqrt{2\left(h^2+w^2\right)}$, where h is the height and w is the width, as shown in the diagram below. If the mirror has an outside height equal to 5 feet and an outside width equal to 2 feet, what is the outside perimeter, in feet?

A. $\frac{\pi}{2}\sqrt{29}$
B. $\pi\sqrt{29}$
C. $\pi\sqrt{58}$
D. $\frac{\pi}{2}\sqrt{58}$
E. $\frac{\pi}{2}\sqrt{98}$

23. How many points lie on an ellipse?

A. 4
B. 2
C. 1
D. 0
E. Infinitely many

24. One of the following equations determines the graph in the standard (x, y) coordinate plane below. Which one?



A. $4(x-10)^2+36(y+3)^2=144$

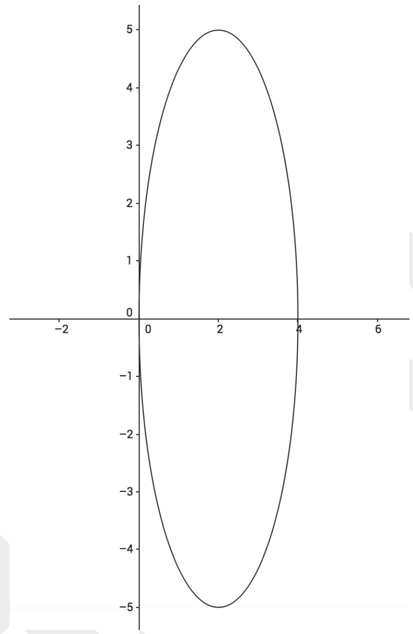
B. $4(x+10)^2+36(y-3)^2=144$

C. $\frac{(x+10)^2}{4}+\frac{(y-3)^2}{36}=1$

D. $\frac{(x-10)^2}{4}+\frac{(y+3)^2}{36}=1$

E. $\frac{(x+10)^2}{36}+\frac{(y-3)^2}{4}=1$

25. Which of the following is the equation of the ellipse that is graphed below?



A. $25x^2-100x+y^2=50$

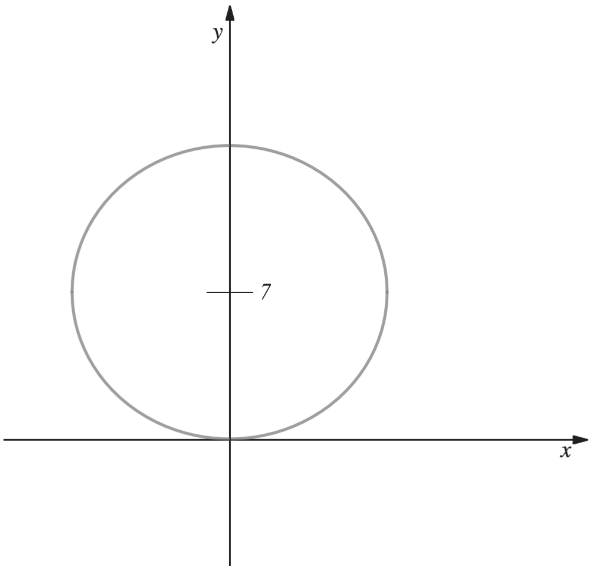
B. $4(x-2)^2+25y^2=100$

C. $25(x-2)^2+4y^2=100$

D. $25x^2-100x+4y^2=100$

E. $\frac{(x+2)^2}{4}+\frac{y^2}{25}=1$

26. In the standard (x, y) coordinate plane, the center of the circle shown below lies on the y-axis at $y = 7$. If the circle is tangent to the x-axis, which of the following is an equation of that circle?



A. $x^2 + (y-7)^2 = 49$

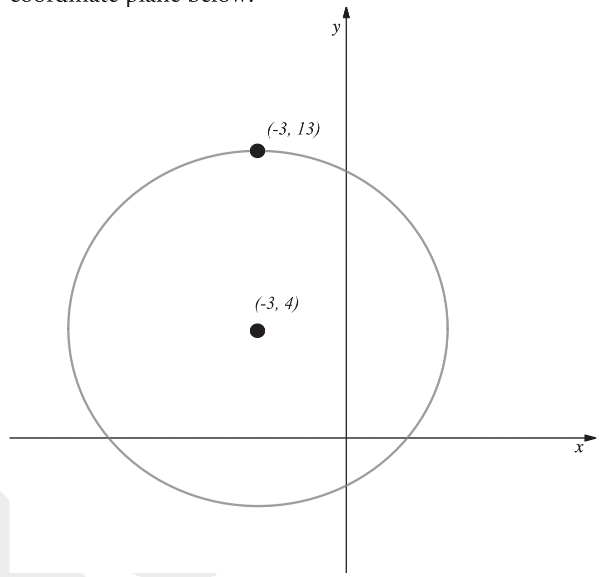
B. $x^2 + (y+7)^2 = 49$

C. $x^2 + (y-7)^2 = 7$

D. $x^2 + (y+7)^2 = 7$

E. $(x-7)^2 + y^2 = 49$

27. Which of the following equations represents the circle with center $(-3, 4)$ shown in the standard (x, y) coordinate plane below?



A. $(x-3)^2 + (y+4)^2 = 81$

B. $(x+3)^2 + (y-4)^2 = 81$

C. $(x-3)^2 + (y+4)^2 = 9$

D. $(x+3)^2 + (y-4)^2 = 9$

E. $(x+3)^2 - (y-4)^2 = 81$

ANSWER KEY

1. B	2. A	3. A	4. D	5. D	6. D	7. D	8. B	9. D	10. B	11. C	12. D	13. D	14. A
15. E	16. C	17. C	18. D	19. E	20. C	21. D	22. D	23. E	24. A	25. C	26. A	27. B	

ANSWER EXPLANATIONS

1. **E.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. From the equation they give us, we see that $r^2=169$ so $r=13$.

2. **A.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. From the equation they give us, we see that the center is at $(\sqrt{5},-2\sqrt{3})$, and $r^2=43$ so $r=\sqrt{43}$.

3. **A.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, the equation of a circle with center at $(-4,7)$ and radius 3 is $(x-(-4))^2+(y-7)^2=3^2$. This can be simplified to be $(x+4)^2+(y-7)^2=9$.

4. **D.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, the equation of a circle with center at $(-12,8)$ and radius 11 is $(x-(-12))^2+(y-8)^2=11^2$. This can be simplified to be $(x+12)^2+(y-8)^2=121$.

5. **D.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, we wish to write the circle described as $x^2-4x+y^2+8y+10=0$ in that form. First, we must complete the square in both the x and y terms. We do this by adding constant terms to make the x and y polynomials perfect squares. These constant terms can be calculated by taking the coefficient of the x term and y term, dividing it by 2, and then squaring it. So, in the case of the x term we have $\left(-\frac{4}{2}\right)^2=4$ and in the case of the y term, we have $\left(\frac{8}{2}\right)^2=16$. $(x^2-4x+4)+(y^2+8y+16)+10-4-16=0$. This is factored into $(x-2)^2+(y+4)^2=10$. So, the center of this circle is at $(2,-4)$. We now want to construct the equation for a circle with the same center but with radius 8. This turns out to be $(x-2)^2+(x+4)^2=8^2$ or $(x-2)^2+(x+4)^2=64$.

6. **D.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, we wish to write the circle described as $x^2+12x+y^2-2y-15=0$ in that form. First, we must complete the square in both the x and y terms. We do this by adding constant terms to make the x and y polynomials perfect squares. These constant terms can be calculated by taking the coefficient of the x term and y term, dividing it by 2, and then squaring it. So, in the case of the x term we have $\left(-\frac{12}{2}\right)^2=36$ and in the case of the y term, we have $\left(-\frac{2}{2}\right)^2=1$. $(x^2+12x+36)+(y^2-2y+1)-15-36-1=0$. This is factored into $(x+6)^2+(y-1)^2=52$. So, the center of this circle is at $(-6,1)$.

7. **D.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. We find the radius of the circle first using the formula $A=\pi r^2$. Plugging in $A=9\pi$, we get $9\pi=\pi r^2 \rightarrow 9=r^2 \rightarrow r=3$. So, the circle with radius 3 and center at $(8,7)$ has the equation $(x-8)^2+(y-7)^2=3^2 \rightarrow (x-8)^2+(y-7)^2=9$.

8. **B.** To find the radius of the circle, we first find the diameter of the circle and divide it by 2 to get the radius. The end points of the diameter are given, so we can find the length of the diameter by finding the distance between the two points $(0,-8)$ and $(-24,24)$. The distance formula is $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. Following this formula, we get $\sqrt{(0-(-24))^2+(-8-24)^2} \rightarrow \sqrt{24^2+32^2} \rightarrow \sqrt{576+1024} \rightarrow \sqrt{1600}=40$. So, $r=\frac{d}{2}=\frac{40}{2}=20$.

9. **D.** If the circle is tangent to the x-axis at -7 and tangent to the y-axis at -7, then the circle is in the third quadrant with a center at $(-7,-7)$ and radius 7. The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, the equation of this circle is $(x-(-7))^2+(y-(-7))^2=7^2 \rightarrow (x+7)^2+(x+7)^2=49$.

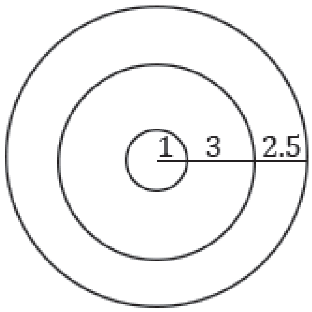
10. **B.** The equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$ where a and b represent the displacement of the center of the circle from the origin and r is the circle's radius. So, the center of the circle represented by $(x+3)^2+(y-3)^2=27$ is at $(-3,3)$.

11. **C.** The equation of an ellipse is in the form $\frac{(x-a)^2}{c^2}+\frac{(y-b)^2}{d^2}=1$ where a and b represent the displacement of the center of the circle from the origin and c and d represent the horizontal and vertical radii of the ellipse respectively. So, the ellipse represented by $\frac{(x+5)^2}{36}+\frac{(y-7)^2}{25}=1$ has a horizontal radius of 6, a vertical radius of 5, and a center at $(-5,7)$. Since the ellipse has a shorter vertical radius, the biggest circle that can be inscribed in the ellipse is limited to the vertical radius. So, this circle will have radius 5 and center at $(-5,7)$. The equation for this circle is $(x+5)^2+(y-7)^2=25$.

12. **D.** The equation of an ellipse is in the form $\frac{(x-a)^2}{c^2}+\frac{(y-b)^2}{d^2}=1$ where a and b represent the displacement of the center of the circle from the origin and c and d represent the horizontal and vertical radii of the ellipse respectively. So, the circle represented by $\frac{x^2}{81}+\frac{y^2}{49}=1$ has center at $(0,0)$ with a horizontal radius of $\sqrt{81}=9$. So, the ellipse intersects the x-axis at $x=9$. This is at the point $(9,0)$.

13. **D.** The equation of an ellipse is in the form $\frac{(x-a)^2}{c^2}+\frac{(y-b)^2}{d^2}=1$ where a and b represent the displacement of the center of the circle from the origin and c and d represent the horizontal and vertical radii of the ellipse respectively. So, the circle represented by $\frac{x^2}{13}+\frac{y^2}{21}=1$ has center at $(0,0)$ with a vertical radius of $\sqrt{21}$. So, the ellipse intersects the x-axis at $y=\sqrt{21}$. This is at the point $(0,\sqrt{21})$.

14. A. The radius of the innermost circle is 1 and the distance between the innermost circle and the outermost circle is 5.5, and the distance between the middle circle and the outermost circle is 2.5. So the distance between the middle circle and the innermost circle is $5.5-2.5=3.$ The radius of the middle circle is then $3+1=4.$ The area of the middle circle can then be calculated as $A=\pi r^2=(4)^2\pi.$ The diagram below illustrates the logo and its dimensions.



15. E. Completing the square for the equation, we get $x^2+8x+16-5\left(y^2+2y+1\right)+12-16+5=0.$ This is simplified to be $(x+4)^2-5(y+1)^2=1.$ The equation of a hyperbola is in the form $\frac{(x-a)^2}{c^2}-\frac{(y-b)^2}{d^2}=1$, so our equation matches this form.

16. C. The equation provided follows the format of the basic circle formula, where $(x-a)^2+(y-b)^2=r^2$, and the circle's center is at (a,b). Thus, in our equation, $r^2=24$, so $r=\sqrt{24}$. $a=0$ and $b=-2$, since the addition implies double negation (subtracting a negative number), placing the circle's center at $(0,-2)$.

17. C. When given the description of a conic without a picture, it's usually to your benefit to draw a little diagram for yourself. This circle is tangent to the y–axis, which means it must have a radius of 4 (because the center is 4 units away from the y axis). Looking at the same equation from the question above and plugging in -4 for a, 3 for b, and 4 for r yields answer (C).

18. D. Finding the answer requires first finding the distance between the two points given, the center and the second point, which will be our radius. Using the distance formula (which is basically the conic equation for a circle) $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$, we see that $d=5$, which means the radius is five. Going back to the basic equation of a circle, after plugging in 5 for a, 2 for b, and 5 for r, D is the obvious answer. The remaining answers try to confuse us by changing the signs and forgetting the square the radius, but that only works if one doesn't know the basic equation for a circle well enough.

19. E. To find the conventional form of the formula, complete the square. Start by rearranging the equation given into two distinct parts, putting the terms with x next to each other and the terms with y next to each other. $x^2+4x+y^2-2y=4$. To complete the square with the x terms, add a 4 to either side (because $\left(\frac{4}{2}\right)^2=1$), and to complete the square with the y terms, add a 1 to either side (because $\left(\frac{2}{2}\right)^2=1$). $(x^2+4x+4)+(y^2-2y+1)=4+4+1.$

Now that we have perfect squares, we can factor those 2 separate parts. $(x+2)^2+(y-1)^2=9.$

This equation looks exactly like the formula, so this is our answer. The rest of the answers rely on a careless student who doesn't notice a sign change or a swapping of the numbers, or perhaps make a silly mistake when factoring.

20. C. If we plug in $y=0$, we maximize the value of x. This value is found to be $x^2+0^2=100 \rightarrow x^2=100 \rightarrow x=10$.

21. D. From the graph, ellipse is shown to be centered $(-6,2)$, with vertices at $(-10,2)$ and $(-2,2)$ and covertices at $(-6,-1)$ and $(-6,5)$. The standard equation for an ellipse where $a>b$ is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, with a center at (h,k), vertices at $(h \pm a,\ k)$, and co-vertices at $(h,\ k \pm b)$. In the graph, our $h=-6$, $k=2$, $a=4$, and $b=3$, which plugged in yields answer (D).

22. D. We are given a formula $\left(p=\frac{\pi}{2}\sqrt{2\left(h^2+w^2\right)}\right)$ and some values $(h=5, w=2)$. Plug in and solve to get answer (D), $\frac{\pi}{2}\sqrt{58}$.

23. E. A central concept to ellipse is that they contain an infinite number of points, just like circles. Although they have a defined center and vertices and co-vertices, there are infinite values in between on the ellipse, so the answer is (E).

24. A. The graph shows an ellipse centered at $(10,-3)$, with vertices at $(4,-3)$ and $(16,-3)$ and co-vertices at $(10,-1)$ and $(10,-5)$. The standard equation for an ellipse where $a>b$ is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, with a center at (h,k), vertices at $(h \pm a,\ k)$, and co-vertices at $(h,\ k \pm b)$, so the equation for the ellipse shown must be $\frac{(x-10)^2}{6^2}+\frac{(y-(-3))^2}{2^2}=1$, which simplified becomes $\frac{(x-10)^2}{36}+\frac{(y+3)^2}{4}=1$. However, that version is not a possible answer, so try rewriting the formula. Multiply the entire expression by both denominators to eliminate the fractions. $36*4*\left(\frac{(x-10)^2}{36}+\frac{(y+3)^2}{4}\right)=1*36*4 \rightarrow$ $4(x-10)^2+36(y+3)^2=144$, answer (A).

25. C. The ellipse is centered at $(2,0)$, extends 2 units in either x-direction, and extends 5 units in either y-direction, implying that $h=2, k=0, a=2$ and $b=5$. Plugging those values into the formula gives us $\frac{(x-2)^2}{4}+\frac{y^2}{25}=1$. Simplify that by multiplying both side by the least common denominator to get $25(x-2)^2+4y^2=100$, answer (C).

26. A. The center of the circle is at $(0,7)$ and the radius is 7 because the circle is tangent to the x-axis, meaning that one point on the circle is exactly 7 units away from the center, which is essentially the definition of a radius. Consequently, the circle equation with these values inputted is $x^2+(y-7)^2=49$.

27. B. The center is at $(-3,4)$ and the distance between the center and the given point on the circle (found visually or with the distance equation if you feel you need to) is 9, which means the radius is 9. Thus, the circle equation filled in becomes $(x-(-3))^2+(y-4)^2=81 \rightarrow (x+3)^2+(y-4)^2=81$.