

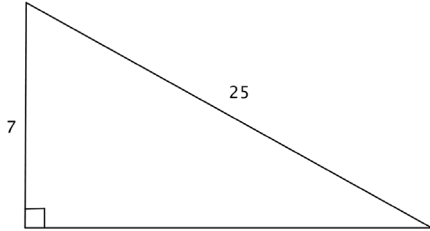
THE BEST ACT PREP COURSE EVER

SOLIDS

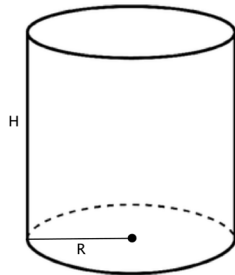
ACT Math: Problem Set

1. The right triangle shown below with hypotenuse 25 inches long and vertical leg 7 inches long is rotated 360° around the vertical leg to form a right circular cone. What is the volume of this cone, to the nearest

cubic inch? (Note: $V = \frac{1}{3}\pi r^2 h$)

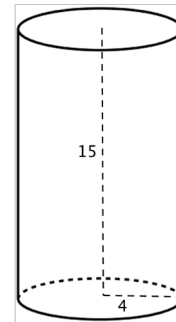


- A. 1322
B. 1344
C. 3878
D. 4222
E. 15080
2. The volume, V , of a right circular cylinder with radius R and height H is given by the formula $V = \pi R^2 H$. The right circular cylinder below has radius R , height H , and volume V . A second right circular cylinder has radius $3R$ and height $6H$. What is the volume of the second cylinder in terms of V ?

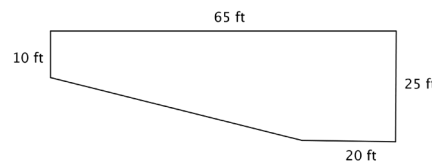


- A. $15V$
B. $18V$
C. $27V$
D. $54V$
E. $108V$

3. The radius and height of a right circular cylinder are given below in feet. What is the approximate volume of the cylinder?

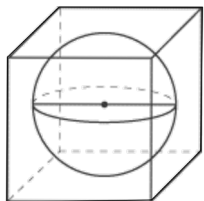


- A. 188
B. 240
C. 754
D. 815
E. 900
4. The figure below is a cross section of a pool that is 20 ft wide and 65 ft long. The sides are vertical, and the bottom begins at an incline, and then becomes level. What is the volume of water, in cubic feet, it would take to fill the pool so that it is 10 ft deep at one end and 25 ft deep at the other end, as shown?

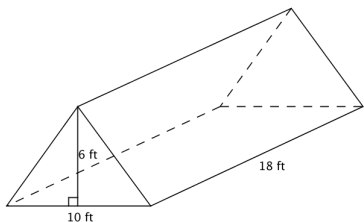


- A. 1137.5
B. 1287.5
C. 21250
D. 22750
E. 25750

5. A circle with radius 3 inches long is inscribed in a cube, as shown below. What is the length of a diagonal of the cube (i.e. the front top right corner to the back bottom left corner)?

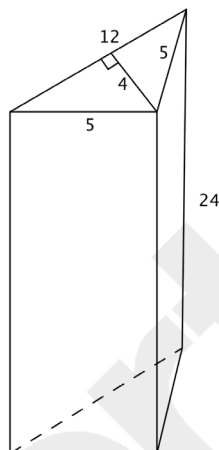


- A. $3\sqrt{2}$
B. $3\sqrt{3}$
C. 6
D. $6\sqrt{2}$
E. $6\sqrt{3}$
6. A right triangular prism is 18 inches deep, 10 inches wide, and 6 inches tall. What is the volume of the prism in cubic inches?

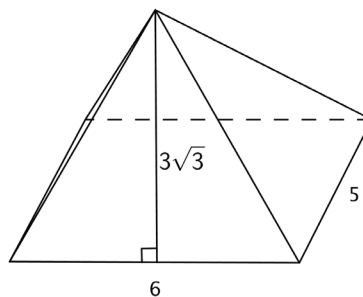


- A. 300
B. 360
C. 540
D. 720
E. 1080

7. What is the total surface area, in inches, of the right triangular prism shown below?

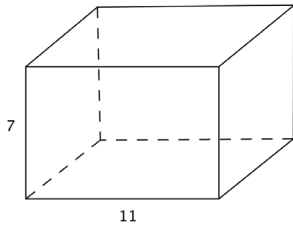


- A. 500
B. 528
C. 552
D. 576
E. 624
8. A right rectangular pyramid is shown in the figure below. The slant height is $3\sqrt{3}$ meters and the length of the bases sides are 6 meters and 5 meters, respectively. What is the total length, in meters, of all 8 edges of the pyramid?

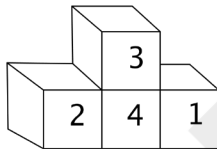
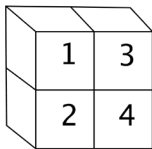


- A. 22
B. 43
C. 46
D. 54
E. 64

9. The right rectangular prism shown below has a face that is 7 meters by 11 meters and a volume of 616 cubic meters. What is the total surface area of the prism?

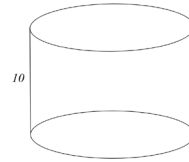


- A. 221
B. 330
C. 365
D. 407
E. 442
10. 4 cubes of equal volume can be rearranged in the 2 orientations shown below. To the nearest percent, the total surface area of the orientation on the left is what percent less than the total surface area of the orientation on the right?



- A. 11%
B. 20%
C. 27%
D. 89%
E. 94%
11. Which of the following is closest to the diameter, in me-

ters, of a cylindrical container, shown below, with height 10 m and volume 1540 cubic meters?



(Note: The volume of a cylinder with radius r and height h is $\pi r^2 h$.)

- A. 4
B. 7
C. 10
D. 13
E. 14
12. A right circular cylinder has diameter 4 cm and height 8 cm. What is the total surface area of this cylinder, in square centimeters?
(Note: The total surface area of a cylinder is given by $2\pi r^2 h + 2\pi rh$ where r is the radius and h is the height.)
- A. 10π
B. 20π
C. 30π
D. 40π
E. 96π
13. Steve decides to put away a standard gym mat (15 feet wide and 20 feet long) after practice. The mat is rolled up so that the two shorter edges just meet, forming a circular tube (cylinder) 15 feet tall. Given that there is no top or bottom to the cylinder, what is the cylinder's surface area?
- A. $\frac{20}{2\pi}$
B. 40π
C. 225π
D. 300
E. 300π
14. Cubes each having a side length of 1.5 in are put together to form a rectangular solid with 4 layers. Each layer has 12 cubes. What is the volume, in cubic centimeters, of the

rectangular solid?

- A. 48
- B. 72
- C. 108
- D. 162
- E. 170

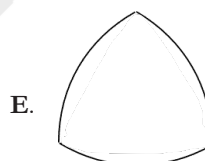
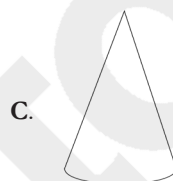
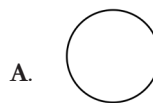
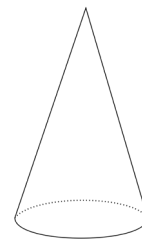
15. The volume of a cube is 216 cubic feet. What is the total surface area, in square feet, of the cube?

- A. 24
- B. 36
- C. 72
- D. 216
- E. 1296

16. In order to build a new parking lot next to the beach, the city removes 2,000 cubic meters of sand from a lot. They decide to use the sand removed to create an indoor volleyball court of sand. If this sand was spread in an even layer over a volleyball court (which measures 18 meters by 9 meters), how many yards deep would the sand layer be?

- A. Less than 9
- B. Between 12 and 14
- C. Between 16 and 18
- D. Between 18 and 20
- E. More than 20

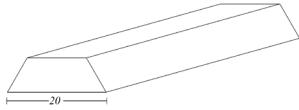
plane section?



17. A solid right circular cone, picture below, is sliced horizontally, parallel to its base and perpendicular to its central line of height, somewhere between the base and the top of the cone. Which of the following best represents the possible

18. A chocolate company develops a chocolate bar that is in the shape of a trapezoidal prism. The length of the bottom side of the trapezoid base is 20 millimeters, and the length of the top side is 14 millimeters. What is the millimeters, in

inches, of the median of the trapezoid?



- A. 14
- B. 16
- C. 17
- D. 18
- E. 20

19. A solid beam of wood has 6 rectangular faces. The length, width, and height of the beam are h , j , and k feet respectively. Which of the following represents the beam's surface area, in square feet?

- A. $2h + 2j + 2k$
- B. $2(hj + jk + hk)$
- C. $2hj + 4jk$
- D. $(h + j + k)^2$
- E. $6hjk$

ANSWERS

1. D 2. D 3. C 4. E 5. E 6. C 7. D 8. C 9. E 10. A 11. E 12. E 12. D 14. D
 15. D 16. B 17. A 18. C 19. B

ANSWER EXPLANATIONS

- D.** The radius of the cone is the bottom leg of the right triangle, which we can either recognize from the 7-24-25 Pythagorean triple, or as $\sqrt{25^2 - 7^2} = \sqrt{576} = 24$. The height of the cone will be the vertical leg of the triangle, 7. Plugging our values into the equation for volume of a cone gives us $\frac{1}{3}\pi 24^2(7) = 1344\pi \approx 4222$.
- D.** The volume of the second cylinder will be $\pi(3R)^2(6H) = 54\pi R^2 H$. Since $\pi R^2 H = V$, the volume of the second cylinder is $54V$.
- C.** The volume of a cylinder is $\pi r^2 h$. Plugging in our radius of 4 and height of 15 gives us $\pi 4^2 * 15 = 240\pi \approx 754$.
- E.** We will first find the area of the polygon, and then multiply by the width of the pool to get the volume. Draw a line towards the right of the figure that divides it into a trapezoid and a rectangle. The bases of the trapezoid are 10 and 25, and its height is $65 - 20 = 45$. The area of the trapezoid is $\frac{10+25}{2} * 45 = 787.5$. The area of the rectangle is $20 * 25 = 500$. The total area of the figure is $787.5 + 500 = 1287.5$. We multiply by 20 to get the total volume: $1287.5 * 20 = 25750$.
- E.** The diagonal of the cube is going to be the hypotenuse of a right triangle with one leg, an edge of the cube, and the other, a diagonal across the face of the cube. The length of an edge is 6 inches since the diameter of the cube is 6 inches, and that spans the entire cube. Drawing a diagonal across a face of the cube forms a 45-45-90 triangle, so the diagonal equals $6\sqrt{2}$, according to the ratio of sides of a 45-45-90 triangle. Since the diagonal of the cube (not the face) is the hypotenuse of a right triangle, and we now have the lengths of the legs, we can solve for it: $\sqrt{(6)^2 + (6\sqrt{2})^2} = \sqrt{36 + 72} = \sqrt{108} = \sqrt{36} \sqrt{3} = 6\sqrt{3}$.
1.
- C.** The volume of a prism equals the area of the face that corresponds to the type of prism times the depth. In this case, the area of the triangle is $\frac{1}{2} 10 * 6 = 30$. So the volume is $30 * 18 = 540$.
2.
- D.** There are 5 faces of the prism: 2 triangles and 3 rectangles. The 2 triangles are congruent, and 2 of the rectangles are congruent, since they have the same side lengths. The area of each triangle is $\frac{1}{2} 12 * 4 = 24$. The area of one of the rectangles is $12 * 24 = 288$ and the area of both of the other rectangles is $5 * 24 = 120$. Adding together all of the faces' areas gives us $24 + 24 + 120 + 120 + 288 = 576$.
- C.** The perimeter of the base of the pyramid is $6 + 6 + 5 + 5 = 22$. The only remaining edges we need to find the total length of all 8 edges are the 4 edges from the vertices of the square to the apex of the pyramid. In a right triangle, these are all equal. We can solve for one of them as the hypotenuse with legs the length of the slant of height $3\sqrt{3}$ and of half the 6 unit base, which is 3. These form a 30-60-90 triangle with a hypotenuse of 6. Thus, we add $4 * 6 = 24$ to the perimeter of the base to get the total length of all 8 edges, which is $24 + 22 = 46$.
- E.** Let l be the height, 7. Let w be the width, 11. Let d be the depth. lwd equals the volume, 616. By substitution, $d * 7 * 11 = 616$. Using division, we find that $d = 8$. The surface area of a rectangular prism equals $2lw + 2ld + 2dw$. Plugging in our numbers, we get $2(7)(11) + 2(7)(8) + 2(8)(11) = 154 + 112 + 176 = 442$.
- A.** Let f be the area of one face of a cube. The cubes are all congruent, since they have the same volume. The number of exposed faces in the left figure is 16, so the surface area is $16f$. The number of exposed faces in the right figure is 18, so the surface area is $18f$. Thus, the percent of surface area shown on the left out of the surface area on the right is $\frac{16f}{18f} = \frac{16}{18} = 88.9\%$. Thus, the percent surface area not shown, the percent less, is $100\% - 88.9\% \approx 11\%$.
- (41) E.** Using the given information and the formula for the volume of a cylinder, we can arrive at the equation $1540 = \pi r^2 10$. If we divide both sides by 10π , we can see that the square of the radius equals about 49 cubic meters. If we square root both sides of the new equation, we find that the radius is 7 meters. However, we must carefully note that the problem asks for the diameter—the answer is 14 meters.

12. **E.** The question tells us that the formula for surface area is $SA = 2\pi r^2 h + 2\pi rh$. Although we are given that the diameter is 4, we must solve for the radius of 2 in order to plug into the problem. Then, we can use the equation $SA = 2\pi(2)^2 8 + 2\pi(2)(8)$ to find that the SA is 96π .
13. **D.** The surface area is equal to the circumference of the cylinder times the height plus the top and bottom circles. However, since there is no top or bottom the surface is only the circumference times the height. In this case, the gym mat is said to be 20 feet long. Once wrapped around in a cylinder, this would become the circumference of the circle. Therefore, we can find the surface area by multiplying 20 times the length of the mat, 15, to get 300.
14. **D.** The volume of each individual cube is $(1.5)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$. The volume of the solid is the sum of the volume of all the cubes that are in the solid. Because there are 4 layers of 12 cubes each, there is a total of $4(12) = 48$ cubes. Thus, the total volume is $\frac{27}{8}(48) = 162$.
15. **D.** If the volume is 216, then the length of one side s is equal to the cube root of 216: $s = \sqrt[3]{216} \rightarrow s = 6$. The Surface area is the sum of the surface area of each side. The surface area of one side of this cube is $6(6) = 36$. Since a cube has 6 equal sides, the surface area is $36(6) = 216$.
16. **B.** You are essentially given a volume, a length, and a width, and are told to find the height, as height and depth fulfill the same function when finding volume of a rectangular solid. Thus we have: $2000 = (18)(9)h$. Solve for h to get $h = 12.346$, which is between 12 and 14.
17. **A.** The cone is sliced horizontally. Because it is a circular cone, meaning it's base is a circle, any cross sections that are parallel with the base must be circles as well.
18. **C.** The median is the average of the two bases, so the median of this trapezoid is $\frac{14+20}{2} = 17$ millimetres.
19. **B.** The surfaces area is the sum of each individual side's area. This beam has 3 pairs of identical sides, one pair whose sides are h and j , giving it an area of hj , another whose sides are j and k , giving it an area of jk , and the final pair whose sides are h and k , giving it an area of hk . Thus the total surface area is $2hj + 2jk + 2hk = 2(hj + jk + hk)$.