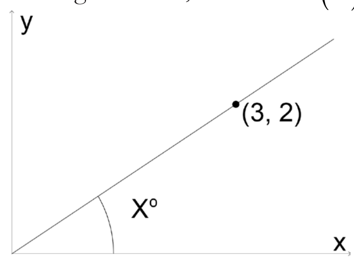
THE BEST ACT PREP COURSE EVER

SINE COSINE TANGENT

ACT Math: Problem Set

1. In the figure below, what is $\cos(x)$?



A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{3}{\sqrt{13}}$

D. $\frac{2}{\sqrt{13}}$

E. $\frac{4}{9}$

2. Roman is standing 55 feet from a cell-phone tower, on level ground. He can see the tip of the tower at an angle of inclination of $38°$. How many feet is the tip of the cell-phone tower from his eyes?

A. $\frac{55}{\sin 38°}$

B. $\frac{55}{\sin 142°}$

C. $\frac{55}{\sin 52°}$

D. $\frac{\sin 52°}{55}$

E. $\frac{\sin 38°}{55}$

3. If $\sin(A) = \frac{10}{26}$, which of the following values could $\cos A$ equal?

A. $\frac{24}{26}$

B. $\frac{16}{26}$

C. $\frac{26}{24}$

D. $\frac{10}{24}$

E. 24

4. If $\tan(x) = \frac{8}{15}$ and $\cos(x) = -\frac{15}{17}$, then $\sin(x) = ?$

A. $\frac{8}{17}$

B. $-\frac{7}{17}$

C. $\frac{8}{15}$

D. $-\frac{8}{17}$

E. $-\frac{7}{15}$

5. If $90° < \theta < 180°$ and $\sin\theta = \frac{12}{20}$ then $\cos\theta = ?$

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{4}{5}$

D. $-\frac{5}{4}$

E. $-\frac{5}{3}$

6. For right triangle $\triangle XYZ$, $\sin\angle X = \frac{5}{6}$. If $\angle Z$ is a right angle, what is $\tan\angle Y$?

A. $\frac{\sqrt{11}}{5}$

B. $\frac{5}{\sqrt{11}}$

C. $\frac{6}{5}$

D. $\frac{11}{5}$

E. $\frac{\sqrt{61}}{5}$

7. If $\sin\theta = 0.6$, what is $\cot\theta$?

A. $\frac{6}{\sqrt{164}}$

B. $\frac{3}{5}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

E. $\frac{4}{5}$

8. If $\sin\beta = \frac{4}{7}$, what is $\cos\beta$?

A. $\frac{\sqrt{33}}{4}$

B. $\frac{\sqrt{33}}{7}$

C. $\frac{4}{\sqrt{33}}$

D. $\frac{7}{\sqrt{33}}$

E. $\frac{\sqrt{23}}{7}$

9. If $\frac{\pi}{2} < \alpha < \pi$ and $\csc\theta = \frac{29}{21}$, what is $\cos\theta$?

A. $\frac{-20}{29}$

B. $\frac{20}{29}$

C. $\frac{29}{20}$

D. $\frac{21}{\sqrt{1282}}$

E. $\frac{-21}{29}$

10. Given that $\tan\theta = \frac{9}{40}$, what are all possible values of $\sec\theta$?

A. $\frac{41}{9}$

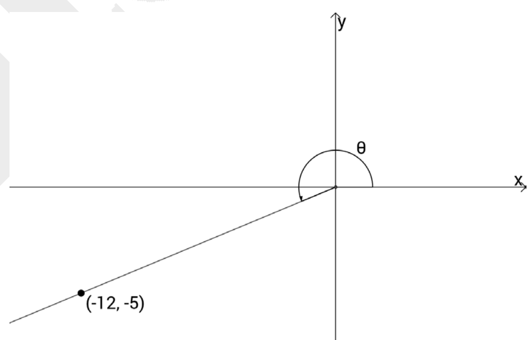
B. $\frac{41}{40}$

C. $\frac{41}{40}$ & $\frac{-41}{40}$

D. $\frac{41}{9}$ & $\frac{-41}{9}$

E. $\frac{39}{40}$

11. In the standard (x, y) coordinate place below, an angle is shown whose vertex is the origin. One side of this angle with measure θ passes through $(-12, -5)$ and the other side includes the positive x-axis. What is the sine of θ?



A. $-\frac{5}{13}$

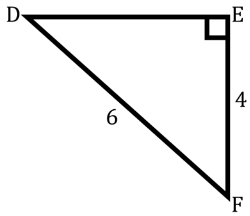
B. $\frac{5}{13}$

C. $\frac{5}{12}$

D. $\frac{12}{13}$

E. $-\frac{12}{13}$

12. What is the sine of angle F in right triangle ΔDEF below?



A. $\frac{2}{3}$

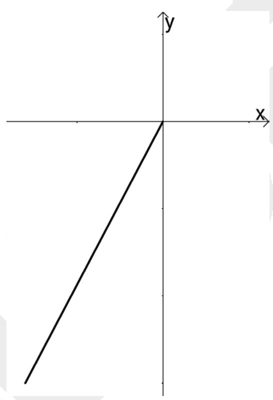
B. $\frac{3}{2}$

C. $\frac{3}{\sqrt{5}}$

D. $\frac{\sqrt{5}}{2}$

E. $\frac{\sqrt{5}}{3}$

13. An angle with measure γ such that $\cos\gamma = \frac{-15}{17}$ is in standard position with its terminal side extending into Quadrant III as shown in the standard (x, y) coordinate plane below. What is the value of $\sin\gamma$?



A. $-\frac{17}{8}$

B. $\frac{15}{17}$

C. $-\frac{15}{17}$

D. $\frac{8}{17}$

E. $-\frac{8}{17}$

14. What are the values of θ between 0 and 2π such that $\cot\theta = -1$?

A. $\frac{\pi}{4}$ and $\frac{7\pi}{4}$

B. $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

C. $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

D. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

E. $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

15. For an angle with measure ϕ in a right triangle such that $\sin\phi = \frac{13}{85}$ and $\sec\phi = \frac{85}{84}$ $\tan\phi$?

A. $\frac{13}{84}$

B. $\frac{84}{13}$

C. $\frac{13}{\sqrt{6887}}$

D. $\frac{\sqrt{6887}}{13}$

E. $\frac{13}{\sqrt{7225}}$

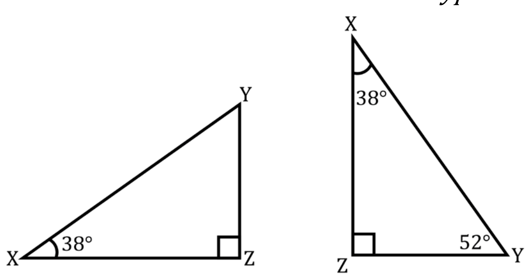
ANSWER KEY

1. C 2. C 3. A 4. D 5. C 6. A 7. D 8. B 9. A 10. C 11. A 12. E 13. E 14. C 15. A

ANSWER EXPLANATIONS

1. **C.** If we draw a triangle with a side on the x-axis, a side on the line $x=3$ and the hypotenuse going from the origin to point $(3,2)$, we get a right triangle with $2=$ the side opposite x and $3=$ the side adjacent angle x. Since $\cos(x)=\frac{adjacent}{hypotenuse}$, we need to find the length of the hypotenuse first by using the Pythagorean theorem $a^2+b^2=c^2$. Plugging in $a=3$ and $b=2$ and solving for c, we get $3^2+2^2=c^2 \to 9+4=c^2 \to 13=c^2 \to c=\sqrt{13}.$ Now, we can plug in $adjacent=3$ and $hypotenuse=\sqrt{13}$ to get $\cos(x)=\frac{3}{\sqrt{13}}$.

2. **C.** We can draw a triangle $\triangle XYZ$ using the points from his eyes (X) to the tip of the tower (Y) to the point on the tower that matches his eye-level (Z). The angle at Z is equal to 90 because the tower is perpendicular to the ground and $\overline{XZ}$ is parallel to the ground, so angle $\angle Y=52°$. It would be easier to use cosine, but because none of the answer choices include a cosine, we must use sine. In order to use sine, we must look at the triangle in another way. If we rotate the triangle by $90°$ and look at $\angle Y$, by SOCAHTOA, $\sin\angle X=\sin 52°=\frac{opposite}{hypotenuse}=\frac{55}{\overline{XY}}$, so $\overline{XY}$, the distance we want to find, is equal to $\frac{55}{\sin 52}$.



3. **A.** Since we know that $\sin(A)=\frac{10}{26}$, we know by SOHCAHTOA that the opposite and hypotenuse sides are 10 and 26, respectively. We can solve for the adjacent side, a, by using the Pythagorean Theorem $a^2+b^2=c^2$:
$a^2+10^2=26^2 \to a^2+100=676 \to a^2=576 \to a=24$. Then, $\cos(A)=\frac{adjacent}{hypotenuse}=\frac{24}{26}$. Notice it can save time to know the 5-12-13 right triangle.

4. **D.** By SOHCAHTOA, we know that $\tan(x)=\frac{opposite}{adjacent}$ and $\cos(x)=\frac{adjacent}{hypotenuse}$ so the opposite, adjacent, and hypotenuse sides are $-8, -15,$ and 17 respectively. We know that the opposite and adjacent sides are negative because $\cos(x)$, which involve the adjacent side and the hypotenuse, is negative, which means that either but not both the adjacent side and the hypotenuse must be negative, but $\tan(x)$, which involve the adjacent and hypotenuse side, is positive, which means that both the adjacent and opposite side must be negative and cancel each other out). So, $\sin(x)=\frac{opposite}{hypotenuse}=\frac{-8}{17}$.

5. **C.** By SOHCAHTOA, we know that $\sin(x)=\frac{opposite}{hypotenuse}$. So, the opposite and hypotenuse sides are 12 and 20, respectively. We can solve for the adjacent side, a, by using the Pythagorean Theorem ($a^2+b^2=c^2$)
$a^2+12^2=20^2 \to a^2+144=400 \to a^2=256 \to a=16$. Then $\cos(x)=\frac{adjacent}{hypotenuse}=\frac{16}{20}$, which simplifies to $\frac{4}{5}$.
However, since $90°<\theta<180°$, it is in the 2nd quadrant, which means the adjacent side is negative. So, $\cos(x)=-\frac{4}{5}$.

6. **A.** If $\angle Z$ is the right angle, then the hypotenuse is opposite of $\angle Z$. Since we know $\sin\angle X = \frac{5}{6}$, according to SOHCAHTOA, the hypotenuse is equal to 6 and the side opposite of $\angle X$ is equal to 5. We now use the Pythagorean Theorem $a^2+b^2=c^2$ to find the adjacent side. We plug in $a=5$ and $c=6$ to get $5^2+b^2=6^2 \rightarrow 25+b^2=36 \rightarrow b^2=11 \rightarrow b=\sqrt{11}$. So, the adjacent side of $\angle X$ is $\sqrt{11}$. Now, if we look at the graph from a different perspective, taking $\angle Y$ as our featured angle but keeping the same lengths on either side of it, the adjacent side becomes 5 and the opposite side becomes $\sqrt{11}$. So, $\tan\angle Y = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{11}}{5}$.

7. **D.** If $\sin\theta = 0.6 = \frac{6}{10}$ and by SOHCAHTOA we know that $\sin\theta = \frac{opposite}{hypotenuse}$, so we can assign the opposite side to be equal to 6 and the hypotenuse to be equal to 10. Then, using the Pythagorean Theorem $a^2+b^2=c^2$, we plug in $a=6$ and $c=10$ to solve for the adjacent side b. We have $6^2+b^2=10^2 \rightarrow 36+b^2=100 \rightarrow b^2=64 \rightarrow b=8$. So, the adjacent side is equal to 8. By SOHCAHTOA, we know that $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{opposite}{adjacent}} = \frac{adjacent}{opposite} = \frac{8}{6} = \frac{4}{3}$. It is helpful to know the $3-4-5$ right triangle in this problem because $6-8-10$ is a just multiple.

8. **B.** If $\sin\beta = \frac{4}{7}$, by SOHCAHTOA, we know that the opposite side is equal to 4 and the hypotenuse is equal to 7. Then, using the Pythagorean Theorem $a^2+b^2=c^2$, we plug in $a=4$ and $c=7$ to solve for the adjacent side b. We have $4^2+b^2=7^2 \rightarrow 16+b^2=49 \rightarrow b^2=33 \rightarrow b=\sqrt{33}$. So, the adjacent side is equal to $\sqrt{33}$. By SOHCAHTOA, we know that $\cos\theta = \frac{adjacent}{hypotenuse} = \frac{\sqrt{33}}{7}$.

9. **A.** Since $\frac{\pi}{2} < \alpha < \pi$, the triangle is in the second quadrant, which means it is on the negative side of the x-axis. This means the adjacent side is negative. Since $\csc\theta = \frac{29}{21}$, we know by SOHCAHTOA that $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{opposite}{hypotenuse}} = \frac{hypotenuse}{opposite}$. So, the hypotenuse is 29 and the opposite side is 21. We use the Pythagorean Theorem $a^2+b^2=c^2$ to solve for the adjacent side b, which will be negative. Plugging in $a=21$ and $c=29$, we get $21^2+b^2=29^2 \rightarrow b^2=400 \rightarrow b=20$. So, the adjacent side is -20. We can now solve $\cos\theta = \frac{adjacent}{hypotenuse} = -\frac{20}{29}$.

10. **C.** Given $\tan\theta = \frac{9}{40}$, by SOHCAHTOA, we know that $\tan\theta = \frac{opposite}{adjacent}$, so the opposite side is 9 and the adjacent side is 40 in the first quadrant, or -9 and -40 if θ is in the third quadrant, since tangent is only positive for an angle in the first and third quadrant. Using the Pythagorean Theorem and plugging in $a=\pm 9$ and $b=\pm 40$ (because we are squaring the side lengths, it doesn't matter if they are positive or negative) then solving for the hypotenuse c, we get $(\pm 9)^2+(\pm 40)^2=c^2 \rightarrow 1681=c^2 \rightarrow c=41$.(Although the sides might be negative, the hypotenuse is always positive, both mathematically and on graphs). Then, the possible values of $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{adjacent}{hypotenuse}} = \frac{hypotenuse}{adjacent}$ are $\frac{41}{40}$ or $\frac{41}{-40}$.

11. **A.** The sine of this angle is the same as the sine of its reference angle (the angle the segment makes with the closest side of the x-axis, which in this case is the negative x-axis). It we draw a triangle with vertices at $(0,0)$, $(0,-12)$, and $(-12,-5)$, we can use the Pythagorean theorem to find the length of the hypotenuse, which is 13. Using the triangle, we see that the sine of the reference angle is $\frac{5}{13}$. However, because the angle is, in reality, not the same as the reference angle because lies in the third quadrant, we know it is negative (because sine is negative in the third quadrant). Thus, our answer is $-\frac{5}{13}$.

12. **E.** First we need to find $\overline{DE}$ by using the Pythagorean theorem: $\overline{DE}^2+4^2=6^2 \rightarrow \overline{DE}=\sqrt{20}=2\sqrt{5}$. By SOCAHTOA, $\sin\angle F=\frac{opposite}{adjacent}=\frac{\overline{DE}}{6}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$.

13. **E.** In Quadrant III, sine is negative. We can find the length of the opposite side using the Pythagorean theorem and the lengths of the adjacent side and hypotenuse we are given from the cosine. $a^2+b^2=c^2 \rightarrow a^2+(-15)^2=17^2 \rightarrow$ $a^2+225=289 \rightarrow a^2=64 \rightarrow a=8$. However, since we are in Quadrant III, sine must be negative, so $a=-8$. Now we plug that value in with the value of the hypotenuse to find the sine: $\sin\gamma=\frac{-8}{17}$.

14. C. $\cot\theta=\frac{1}{\tan\theta}$. If we want the cotangent to be equal to -1, then the tangent must be equal to -1. $\tan\theta=\frac{sin\theta}{\cos\theta}$, so it will only equal -1 when sine and cosine have the same magnitude but opposite signs, which we can express as $\sin\theta=-\cos\theta$. The Quadrants where sine and cosine have opposite signs are Quadrants II and IV. Sine and cosine have equal magnitude when both sides of a right triangle are equal (this is because $\sin\theta=\cos\theta \rightarrow$ $\frac{opposite}{hypotenuse}=\frac{adjacent}{hypotenuse} \rightarrow opposite=adjacent$). This makes it an isosceles triangle, and a right-isosceles triangle has angles of $45-45-90$. In terms of radians, the $45°$ angle is $\frac{\pi}{4}$. However, this is only the reference angle. We are looking for the angles that fall within Quadrants II and IV. The angle in Quadrant II will be $\pi-\frac{\pi}{4}=\frac{3\pi}{4}$. The angle in Quadrant IV will be $2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$. Thus, our answer is C.

15. A. If sine is $\frac{13}{85}$, we can define the measure of the opposite side as 13 and the measure of the hypotenuse as 85. Since the secant is $\frac{85}{84}$, we can define the adjacent side as 84, since $\sec\theta=\frac{hypotenuse}{adjacent}$. We have everything we need to find the tangent now. $\tan\phi=\frac{opposite}{adjacent}=\frac{13}{84}$.