

# THE BEST ACT PREP COURSE EVER

## PROBABILITY

*ACT Math: Problem Set*

1. A box contains 10 blue marbles, 30 green marbles, and 24 orange marbles. How many orange marbles must be added so that there is a 60% chance of picking an orange marble at random?  
A. 36  
B. 40  
C. 44  
D. 48  
E. 52
2. A number is chosen from the set  $\{1, 2, 3, 4, 5, \dots, 24\}$ . What is the probability that the number is a factor of 18?  
A.  $\frac{1}{2}$   
B.  $\frac{1}{3}$   
C.  $\frac{1}{4}$   
D.  $\frac{1}{5}$   
E.  $\frac{1}{6}$
3. There are 8 children, and you are assigned to line them up from youngest to oldest. You know which child is the oldest, which child is the second youngest, and which child is the youngest, but the other 5 children's ages are unknown. If you randomly sort the middle 5 children, what is the probability that all the children are ordered correctly?  
A.  $\frac{1}{20}$   
B.  $\frac{1}{120}$   
C.  $\frac{1}{760}$   
D.  $\frac{1}{5}$   
E.  $\frac{1}{25}$
4. In a set of integers from 1 to 50, inclusive, what is the probability of randomly selecting a prime number?  
A.  $\frac{1}{10}$   
B.  $\frac{1}{5}$   
C.  $\frac{2}{5}$   
D.  $\frac{3}{10}$   
E.  $\frac{1}{4}$
5. An integer from 10 through 999, inclusive, is to be chosen at random. What is the probability that the integer has 3 as 2 (not more) of its digits?  
A.  $\frac{1}{3}$   
B.  $\frac{1}{33}$   
C.  $\frac{11}{330}$   
D.  $\frac{2}{55}$   
E.  $\frac{3}{110}$
6. There are 200 paper slips in a hat, each numbered from  $\sqrt{1}, \sqrt{2}, \dots, \sqrt{200}$  with no repeats. What is the probability that the number on a slip drawn at random is irrational?  
A.  $\frac{7}{100}$   
B.  $\frac{6}{100}$   
C.  $\frac{5}{100}$   
D.  $\frac{93}{100}$   
E.  $\frac{94}{100}$

7. O'Shea puts 7 blue marbles in a box. He now wants to add enough yellow marbles so that the probability of drawing a blue marble  $\frac{1}{11}$  is . How many yellow marbles does he need to add?
- A. 50  
B. 60  
C. 70  
D. 80  
E. 90
8. At a party with 100 guests, there is a raffle. Each guest is given a ticket with a number from 00 to 99. There are no repeated numbers. Each guest signs his/her name on the ticket and drops it in a basket. There is a second basket of tickets numbered identically. A guest wins the raffle if his/her ticket is picked from the first basket and a ticket with the same ones digit is picked from the second basket. For example, if a guest has a ticket number 14 and the ticket picked from the first basket is 14 and the ticket from the second basket is 94, the guest wins. If Bernie's ticket number 42 is drawn from the first basket, what is the probability that Bernie will *not* win the raffle?
- A.  $\frac{1}{10}$   
B.  $\frac{9}{10}$   
C.  $\frac{1}{2}$   
D.  $\frac{3}{4}$   
E.  $\frac{2}{3}$
9. Robin has a fake coin that has a 40% chance of landing on tails and a 60% chance of landing on heads. In 4 coin tosses, what is the probability of getting exactly 3 tails?
- A. .064  
B. .936  
C. .216  
D. .784  
E. .153
10. At a start-up company with a staff of 15 people, 6 people are male and 9 people are female. Two people are randomly chosen to be campus representatives. What is the probability that both representatives are male?
- A.  $\frac{6}{15} + \frac{6}{15}$   
B.  $\frac{6}{15} \cdot \frac{6}{15}$   
C.  $\frac{6}{15} \cdot \frac{5}{14}$   
D.  $\frac{9}{15} \cdot \frac{8}{14}$   
E.  $\frac{6}{15} \cdot \frac{5}{15}$
11. In a survey conducted at a university, students were asked to write down the number of campus organizations they are involved in. The results are shown below. What are the odds of a student at the university being involved in at least 3 organizations?
- Distribution of Student Involvement in Campus Organizations.
- |                    |    |    |    |    |    |
|--------------------|----|----|----|----|----|
| # of organizations | 0  | 1  | 2  | 3  | >3 |
| % of students      | 14 | 27 | 39 | 16 | 4  |
- A. 1:25  
B. 1:5  
C. 1:4  
D. 1:20  
E. 4:25
12. 25% of the dogs at a park are Corgis. There are 28 dogs at the park. How many dogs at the park are not Corgis?
- A. 7  
B. 14  
C. 20  
D. 21  
E. 24

13. In a Secret Santa gift exchange, there are 3 gift cards, 5 stuffed animals, and 2 articles of clothing. 5 more people want to join the exchange. How many of the 5 people should bring stuffed animals so that the overall probability of getting a stuffed animal gift is 40%?
- A. 1  
B. 2  
C. 3  
D. 4  
E. 5
14. Sam rolls a 6 sided die painted with 3 sides yellow, 2 sides red, and 1 side white. If Sam rolls the die and records the color of the side facing up repeatedly, how many times should Sam expect to record the color red after 180 rolls?
- A. 30  
B. 60  
C. 90  
D. 120  
E. 150
15. A teacher lines 30 students in a single file line and starts passing out candy at the front of the line. The teacher has 15 lollipops, 10 candy canes, and 5 gumdrops. Lisa is 6<sup>th</sup> in line to get the candy and the students in front of her have received 3 lollipops and 2 candy canes. What is the probability that Lisa will get a lollipop or a gumdrop?
- A.  $\frac{12}{25}$   
B.  $\frac{17}{30}$   
C.  $\frac{2}{3}$   
D.  $\frac{17}{25}$   
E.  $\frac{4}{5}$
16. In a list of 60 songs, there are 13 songs by artist A, 24 songs by artist B, 13 songs by artist C, and 10 songs by artist D. The first song on the playlist is set to play on random. What is the probability that the first song played is by artist B?
- A.  $\frac{1}{5}$   
B.  $\frac{13}{60}$   
C.  $\frac{2}{5}$   
D.  $\frac{3}{5}$   
E.  $\frac{4}{5}$
17. In box of 15 pebbles, 4 are white, 6 are black, and 5 are gray. If a blindfolded person is asked to pick one pebble out by random, what is the probability of the person picking a pebble that is not white?
- A.  $\frac{11}{15}$   
B.  $\frac{9}{15}$   
C.  $\frac{6}{15}$   
D.  $\frac{5}{15}$   
E.  $\frac{4}{15}$
18. Two events are independent if the outcome of one event does not affect the outcome of the other event. One of the following statements does NOT describe independent events. Which one?
- A. An 8 is drawn from a deck of cards, then after replacing the card, an 8 is drawn.  
B. An ace card is pulled from a deck of cards, then, without replacing the card a coin lands tails up.  
C. A coin is flipped and lands heads up, then the same coin is flipped again and lands heads up.  
D. A 4 is drawn from a deck of cards, then after replacing the card, a 3 is drawn.  
E. A 4 is drawn from a deck of cards, then, without replacing the card, a king is drawn.

19. A new version of roulette is played where 2 pockets are green, 9 are red, 9 are black, 9 are blue, and 9 are yellow. If a ball is rolled into one of the pockets at random, what is the probability that it does NOT land in a blue pocket?

A.  $\frac{29}{38}$   
 B.  $\frac{27}{38}$   
 C.  $\frac{9}{28}$   
 D.  $\frac{3}{4}$   
 E.  $\frac{1}{4}$

20. A taxi service has 240 taxis in its service. Based on previous data, the company constructed the table below showing the percent of taxis in use and the probabilities of occurring. Based on the probability distribution in the table, to the nearest whole number, what is the expected number of taxis that will be in use any given day?

Tax Rate Usage	Probability
0.4	0.3
0.6	0.4
0.7	0.2
0.9	0.1

A. 59  
 B. 60  
 C. 142  
 D. 144  
 E. 156

21. For the first 7 possible values of  $x$ , the table below gives the probability,  $P(x)$ , that  $x$  inches of rain will fall in any given month.

$x$ inches of rain	$P(x)$
0	0.3102
1	0.1020
2	0.1567
3	0.2021
4	0.1166
5	0.0621
6	0.0503

Which of the following values is closest to the probability that at least 3 inches of rain will fall in any given month?

A. 0.11  
 B. 0.20  
 C. 0.23  
 D. 0.3  
 E. 0.43

22. To gather information about changing public opinions on foreign policy, a research company called 500 households randomly selected from the census and gathered answers to a pre-prepared survey. 6 months later, another 500 households randomly selected from the census were surveyed and the answers compared to the previous answers. Which of the following phrases best describes the company's methodology?

A. Randomized census  
 B. Randomized experiment  
 C. Nonrandomized experiment  
 D. Randomized sample survey  
 E. Nonrandomized sample survey

23. Let  $X$  and  $Y$  be independent events.  $P(X)$  represents the probability that event  $X$  will occur,  $P(\sim X)$  represents the probability that event  $X$  will not occur, and  $P(X \cap Y)$  represents the probability that both events  $X$  and  $Y$  will occur. Which of the following equations *must* be true?

A.  $P(X) = P(Y)$   
 B.  $P(X \cap \sim Y) = P(\sim X \cap Y)$   
 C.  $P(X) - P(\sim X) = P(Y) - P(\sim Y)$   
 D.  $P(X \cap Y) = P(\sim X \cap \sim Y)$   
 E.  $P(X) \geq P(X \cap Y)$

24. A lab is testing a new machine to diagnose breast cancer. In 50 trials of 800 individuals, the number of false positives (instances when the machine diagnoses a woman with breast cancer who does not actually have it) were recorded. Based on the distribution below, what is the expected number of false positives that will occur among 50,000 tests?

Number, $n$ , of false positives	Probability that $n$ false positives are produced in a trial of 800 people
0	0.2
1	0.4
2	0.15
3	0.15
4	0.1

- A. 1.55  
 B. 63  
 C. 97  
 D. 124  
 E. 500
25. The probability distribution of the discrete random variable  $Y$  is shown in the table below. What is closest to the expected value of  $Y$ ?

$y$	$P(Y = y)$
0	$\frac{2}{9}$
1	$\frac{1}{18}$
2	$\frac{5}{18}$
3	$\frac{1}{6}$
4	$\frac{2}{9}$
5	$\frac{1}{18}$

- A. 1  
 B. 2  
 C. 2.28  
 D. 3  
 E.  $\frac{2}{9}$

26. The table below shows the results of a survey of 300 people who were asked whether they liked spicy food and whether they liked hiking.

	Like spicy food	Do not like spicy food	Total
Like to hike	70	115	185
Do not like to hike	40	75	115
Total	110	190	300

According to the results, which is closest to the probability that a randomly selected person who was surveyed doesn't like spicy food given that they don't like to hike?

- A. 165%  
 B. 65%  
 C. 63%  
 D. 39%  
 E. 38%
27. The probability that a specific event,  $E$ , happens is denoted  $P(E)$ . The probability that this event does not happen is denoted  $P(\text{not } E)$ . Which of the following statements is *always* true?
- A.  $P(\text{not } E) > P(E)$   
 B.  $P(\text{not } E) < P(E)$   
 C.  $P(\text{not } E) = P(E) + 1$   
 D.  $1 - P(E) = P(\text{not } E)$   
 E.  $0 < P(\text{not } E) < P(E)$
28. Suppose that  $a$  will be randomly selected from the set  $\{-3, -1, 0, 1, 2\}$  and that  $b$  will be randomly selected from the set  $\{-3, -2, 0, 1, 2, 3\}$ . What is the probability that  $ab < 0$ ?

- A.  $\frac{1}{3}$   
 B.  $\frac{3}{20}$   
 C.  $\frac{13}{30}$   
 D.  $\frac{4}{15}$   
 E.  $\frac{1}{15}$

**ANSWERS**

1. A    2. C    3. B    4. D    5. E    6. D    7. C    8. B    9. E    10. C    11. C    12. D    13. A    14. B  
 15. D    16. C    17. A    18. E    19. A    20. C    21. E    22. D    23. E    24. C    25. C    26. B    27. C    28. A

**ANSWER EXPLANATIONS**

- A. We have a total of  $10 + 30 + 24 = 64$  marbles, 24 of them being orange marbles, and we are looking to find the number of additional orange marbles to add in order to have a 60% probability of picking an orange marble. Let  $x$  be the number of additional orange marbles needed. Then, we can say that the probability of picking an orange marble after the addition of the  $x$  marbles is  $\frac{24+x}{64+x}$ . We want this to be equal 60%, so we set up the equation  $\frac{24+x}{64+x} = \frac{6}{10}$ . Cross-multiplying opposite sides of the equation gives us  $240 + 10x = 384 + 6x$ . Then, subtracting  $6x$  from both sides, we get  $240 + 4x = 384$ . Subtracting 240 from both sides gives us  $4x = 144$ . Finally, dividing each side by 4 gives us  $x = 36$ . So, we need to add 36 additional orange marbles for there to be a 60% chance of picking an orange marble.
- C. First, we must find all factors of 18 (integers that 18 can be divided by). They are 1, 2, 3, 6, 9, and 18. We see that 18 has 6 factors and all of these numbers are included in the set  $\{1, 2, 3, 4, \dots, 24\}$ . The set  $\{1, 2, 3, 4, \dots, 24\}$  has 24 numbers, so the probability of choosing a factor of 18 from these 24 numbers is  $\frac{6}{24} = \frac{1}{4}$ .
- B. In order to line the children up from youngest to oldest, we start with the youngest. Since we already know which children are the two youngest, we know which one child to place in the first spot, and which one child to place in the second spot. The same goes for the last spot since we know which child is the oldest. For the remaining spots 3-7, we have 5 children left (original 7 - 1 oldest - 2 youngest) who need to be placed. For spot three there are 5 children who could randomly be placed there. Once one child is randomly placed in the third spot, there are 4 children left who could be placed in the fourth spot, and then 3 children for the fifth spot, 2 choices for the sixth spot, and one remaining child at the end who will take the 7th spot. So, the total possible ways of ordering the children is calculated by  $1 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 120$ . Since only one of these line-ups is the correct order, the probability that the children are ordered correctly is  $\frac{1}{120}$ .
- D. We must first list all the prime numbers between 1 and 50. We find these to be 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47. That means 15 out of the 50 integers from 1 to 50 are primes. So, the probability of selecting a prime number is  $\frac{15}{50} = \frac{3}{10}$ .  
 A good way to determine if a number is prime is to divide that number by all integers less than or equal to its square root, and if the number cannot be divided by any of these integers other than 1, it is prime. For example, 13 is prime because the square root of 13 is approximately 3.60555, and 13 is not divisible by 3 or 2.
- E. First, we determine that there are  $999 - 10 + 1 = 990$  integers from 10 through 999 (we add 1 because the set is inclusive). Then we count the number of integers that have 3 as exactly two of its digits. If we let  $x$  represent any digit from 0-9 excluding 3, then the numbers we want to count can be represented in the following forms:  $x33$ ,  $3x3$ , and  $33x$ . Since there are 9 possible choices for  $x$  (0, 1, 2, 4, 5, 6, 7, 8, 9) while the other two digits in our numbers only have one possible choice respectively (3), the number of possible permutations could be calculated for each of the forms.  $x33$  has  $9 \cdot 1 \cdot 1 = 9$  possible outcomes,  $3x3$  has  $1 \cdot 9 \cdot 1 = 9$  possible outcomes, and  $33x$  has  $1 \cdot 1 \cdot 9 = 9$  possible outcomes. This gives us a total of  $9 + 9 + 9 = 27$  numbers that satisfy our condition of having 3 as 2 of its digits. So, the probability of choosing such an integer out of a total of 990 numbers (calculated earlier) is  $\frac{27}{990} = \frac{3}{110}$ .
- D. Since there are 200 integers from 1 to 200, there are also 200 integers from  $\sqrt{1}, \sqrt{2}, \dots, \sqrt{200}$ . We must now find the number of values from  $\sqrt{1}, \sqrt{2}, \dots, \sqrt{200}$  that are irrational. An irrational number is a number that cannot be expressed as a fraction or in the form  $\frac{n}{d}$  where  $n$  and  $d$  are integers. Numbers of the form  $\sqrt{x}$  are rational only if  $x$  is a perfect square. So, since there are 14 numbers from 1 to 200 that are perfect squares (1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, and 196), the probability of drawing an irrational number is  $1 -$  (the probability of drawing a rational number).  $1 - \left(\frac{14}{200}\right) = \frac{186}{200} = \frac{93}{100}$ .

7. C. The box initially only has 7 blue marbles. Let  $x$  be the number of yellow marbles we want to add in order for the probability of drawing a blue marble to be  $\frac{1}{11}$ . This means  $\frac{7}{7+x} = \frac{1}{11}$ . Cross-multiplying this equation, we get  $77 = 7 + x$ . Subtracting 7 from both sides, we get  $70 = x$ . Accordingly, we need to add 70 yellow marbles in order for the probability of drawing a blue marble to be equal to  $\frac{1}{11}$ .
8. B. Bernie will not win the raffle if the ticket drawn from the second basket does not end in the same ones digit as 42. So, if the second ticket does not end with a 2, Bernie will not win. The possible ticket numbers that will not allow Bernie to win have 10 possible numbers (0-9) for the first digit and 9 possible numbers (0-9 excluding 2) for the second digit (ones place). This gives  $10 \cdot 9 = 90$  possible outcomes out of a total of  $99 - 0 + 1 = 100$  tickets to choose from for Bernie to lose. The probability of choosing such a ticket is  $\frac{90}{100} = \frac{9}{10}$ .
9. E. There are 4 orders for which Robin can get 3 tails out of 4 tosses: TTTH, TTHT, THTT, or HTTT. First we find the probability of each possible outcome. Each coin toss is an independent "AND" event, so the probability that she will get, for example, tails in the first toss AND tails in the second toss AND heads in the third toss AND tails in the fourth toss, is found by multiplying the probability of each independent coin toss. The probability of each of these outcomes is, respectively,  $(0.4)(0.4)(0.4)(0.6)$ ,  $(0.4)(0.4)(0.6)(0.4)$ ,  $(0.4)(0.6)(0.4)(0.4)$ , and  $(0.6)(0.4)(0.4)(0.4)$ . Now, the probability that she will get one of the four desired outcomes is an independent "OR" event, the probability that she will get TTTH OR TTHT OR THTT OR HTTT, so we must sum the individual probabilities we found before. Thus, the total probability of getting 3 tails out of 4 is  $(0.4)(0.4)(0.4)(0.6) + (0.4)(0.4)(0.6)(0.4) + (0.4)(0.6)(0.4)(0.4) + (0.6)(0.4)(0.4)(0.4)$ , or  $4 \cdot (0.4)^3(0.6)$ . This simplifies to  $4 \cdot 0.0384 = 0.1536$ .
10. C. When calculating the probability of choosing two males, we are looking for the probability of selecting one male and selecting another male from the remaining staff. The probability of selecting the first male is  $\frac{6}{15}$ . Taking out the first selected male, the pool of candidates now consists of 5 males and 9 females, so the probability of selecting a second male from the staff not including the first male is  $\frac{5}{14}$ . We multiply the two probabilities to find the probability of both occurrences happening, so the probability that both representatives are male is  $\frac{6}{15} \cdot \frac{5}{14} = \frac{6}{15} \cdot \frac{6}{15}$ . A common mistake made is multiplying  $\frac{6}{15} \cdot \frac{6}{15}$ . This is the probability of selecting two males with replacement, which means that it is possible to select the same person twice. Since the staff is selecting two different people, they are selecting without replacement.
11. C. The odds of a student being involved in at least 3 organizations is the percentage of the student being in 3 or  $>3$  against the percentage of the student being in 0, 1, or 2. The percentage of a student being in 3 or  $>3$  is  $16\% + 4\% = 20\%$  and the percentage of students in the remaining tallies are  $14\% + 27\% + 39\% = 80\%$ . So, the odds are  $20:80$ , which reduces to  $1:4$ .
12. D. Out of the 28 dogs,  $25\%$  or  $\frac{28}{4} = 7$  are Corgis. The number of dogs at the park that are NOT Corgis is  $28 - 7 = 21$ .
13. A. Initially, there are a total of  $3 + 5 + 2 = 10$  gifts, and 5 of these gifts are stuffed animals. With the addition of 5 more gifts, the total number of gifts in the exchange is now  $10 + 5 = 15$ . If we want  $40\%$  of those 15 gifts to be stuffed animals, then  $0.4 \cdot 15 = 6$  of the gifts must be stuffed animals. We already know that there are 5 stuffed animal gifts from the original 10 people, so we only need  $6 - 5 = 1$  out of the 5 people joining to bring a stuffed animal gift.
14. B. To find the expected number of rolls that are red side up, we must first determine the probability of rolling a red. Since 2 out of the 6 sides are red, the probability of rolling a red is  $\frac{2}{6} = \frac{1}{3}$ . The expected number of rolls that are red side up is then the probability of rolling a red multiplied by the total number of rolls made. Out of the total 180 rolls,  $\frac{1}{3}$  of them are expected to be red:  $180 \cdot \frac{1}{3} = 60$  rolls.



15. D. The teacher started out with 15 lollipops, 10 candy canes, and 5 gumdrops. At the time the teacher reaches Lisa, there are  $15 - 3 = 12$  lollipops left,  $10 - 2 = 8$  candy canes left, and  $5 - 0 = 5$  gumdrops still remaining. The total number of candies by the time the teacher reaches Lisa is now  $30 - 5 = 25$ . So the probabilities of getting a lollipop, candy cane, and gumdrop are  $\frac{12}{25}$ ,  $\frac{8}{25}$ , and  $\frac{5}{25}$  respectively. The probability of getting a lollipop or a gumdrop is  $\frac{12}{25} + \frac{5}{25} = \frac{17}{25}$ .
16. C. There are 24 songs out of 60 that are by artist B, so the probability of the first song to be by artist B is  $\frac{24}{60} = \frac{12}{30} = \frac{6}{15} = \frac{2}{5}$ .
17. A. We wish to find the probability of the pebble being NOT white, which is  $1 - (\text{probability of picking a white pebble})$ . The probability of picking a white pebble is  $\frac{4}{15}$ , so the probability of picking a pebble that is *not* white is  $1 - \frac{4}{15} = \frac{15}{15} - \frac{4}{15} = \frac{11}{15}$ .
18. E. For each answer choice, the events described are independent except for answer choice E because if a 4 is drawn from a deck, and without replacement, a king is drawn, then the probability of drawing the king depends on whether or not the first card drawn from the deck was also a king. All other answer choices describe events that are independent because each draw made from a deck is replaced, making the next draw not dependent on the previous draw. Each coin toss also does not depend on the previous toss.
19. A. The probability that an event does not occur is equal to the probability that any mutually exclusive events will occur added together. However, since we may come across a problem where there are too many alternate events to calculate in the time we have, it's better to know how to calculate the probability that an event will not happen as 100%, or 1, minus the probability that the event will happen. In this case, that is  $1 - \frac{9}{38}$ , since there are 9 chances for the ball to land in blue out of 38 possibilities. This gives us  $\frac{38}{38} - \frac{9}{38} = \frac{29}{38}$ .
20. C. The expected value is equal to the sum of all possible values, each multiplied by its probability. Our expected taxi usage rate is  $0.4(0.3) + 0.6(0.4) + 0.7(0.2) + 0.9(0.1) = 0.59$ . We expect the taxi service to be using 59% of its taxis at any given time. To find the expected number of taxis, not the rate, we multiply the number of taxis, 240, by the rate:  $240(59\%) = 240(0.59) = 141.6 \approx 142$ .
21. E. We are looking for the probability that *at least* 3 inches of rain will fall. The solution is the sum of all probabilities of  $X \geq 3$ . From the chart, these probabilities are 0.2021, 0.1166, 0.0621, and 0.0503. Added together:  $0.2021 + 0.1166 + 0.0621 + 0.0503 = 0.4311$ . The closest value is answer choice (E).
22. D. Answer choice (A) is wrong because a census targets the entire population. The company only calls a sample of the population. Answer choices (B) and (C) are wrong because in an experiment, the tester changes variables between two groups and compares the differences. The research company does not change any of the variables between the two groups. This leaves us with answer choices (D) and (E). We know that the answer is (D) because the houses are selected randomly, making this a randomized sample survey.
23. E. It is a fact of statistics that the likelihood of a given event happening is always greater than or equal to the probability of that event happening alongside a second event (and only equal when the probability of the second event is 100%!) This is a useful fact to memorize as there may be ACT questions that ask about this exact relationship.
24. C. The expected number of false positives in a group of 800 people will be equal to the sum of each number  $n$  of false positives times its probability. The expected number is  $0(0.2) + 1(0.4) + 2(0.15) + 3(0.15) + 4(0.1) = 0 + 0.4 + 0.3 + 0.45 + 0.4 = 1.55$ . However, we are looking for the number of false positives in a group of 50,000 people. There are  $\frac{50,000}{800} = 62.5$  groups of 800 people in the total 50,000 people. The number of false positives is equal to  $1.55(62.5) = 96.875 \approx 97$  false positives.

25. C. The expected value of a variable is the sum of all of its possible values multiplied by their respective probabilities. The expected value of  $Y$  equals  $0\left(\frac{2}{9}\right) + 1\left(\frac{1}{18}\right) + 2\left(\frac{5}{18}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{2}{9}\right) + 5\left(\frac{1}{18}\right) = \frac{0}{9} + \frac{1}{18} + \frac{10}{18} + \frac{3}{6} + \frac{8}{9} + \frac{5}{18} \approx 2.28$ .
26. B. Since we are given that the person doesn't like to hike, we can get rid of all of the individuals who do like to hike, leaving us with a population of 115. In this group, there are 75 people who don't like spicy food, so our probability is  $\frac{75}{115} \approx 0.65 = 65\%$ .
27. C. The probability of an event occurring and the probability of it not occurring must always equal 1, since those are the only two possible outcomes ever. Given this, we know that  $1 = P(E) + P(\text{not } E)$ , which makes  $1 - P(E) = P(\text{not } E)$  also true.
28. A. The probability that  $ab < 0$  is equal to the probability that  $a < 0$  and  $b > 0$  plus the probability that  $a > 0$  and  $b < 0$ . There are 5 possibilities for  $a$ , of which 2 are negative and 2 are positive. There are 6 possibilities for  $b$ , of which 2 are negative and 3 are positive. The probability that  $a < 0$  and  $b > 0 = \frac{2}{5} \cdot \frac{3}{6} = \frac{6}{30} = \frac{1}{5}$ . The probability that  $a > 0$  and  $b < 0 = \frac{2}{5} \cdot \frac{2}{6} = \frac{4}{30} = \frac{2}{15}$ .  $\frac{1}{5} + \frac{2}{15} = \frac{3}{15} + \frac{2}{15} = \frac{5}{15} = \frac{1}{3}$ .