CHAPTER

8 INEQUALITIES: ADVANCED

SKILLS TO KNOW

- How to graph/shade inequalities in a coordinate plane
- How to think abstractly about math involving inequalities
- How to solve "must be" and "could be" true problems with inequalities
- How to solve absolute value inequalities (see: Absolute Value chapter in Part Two: Advanced Algebra)

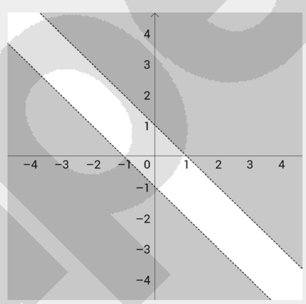
INEQUALITIES ON A COORDINATE PLANE

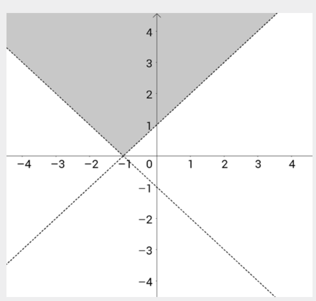
These can seem complicated, but they don't have to be. A few tips to remember:

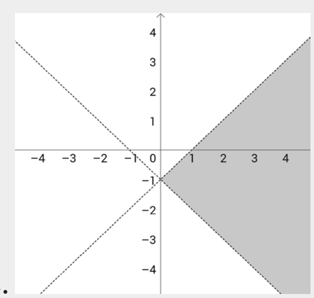
1. You can always "pluck points."
Most of the time on a multiple choice question you'll be given a system of inequalities and every picture will have the same lines—only the shading will be different. As such, you can always choose test points, plug them in, and use the process of elimination to then eliminate choices. $(0,0)$ is almost always the easiest point to test.

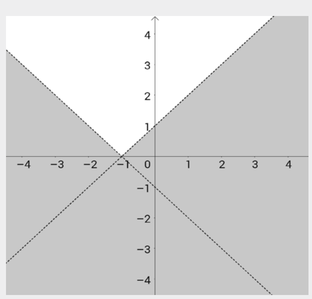
2. Remember—get y isolated. If the y is **greater** than the line, **shade above the line.** If y is **less than** the line, **shade below.** Even if you're in function notation, $f(x)$, the same rule applies.

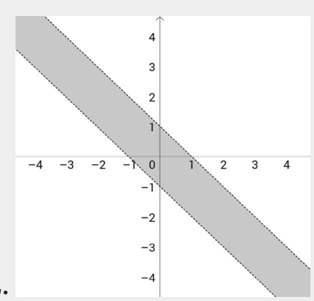
The shaded portion of one of the following graphs is the set of all x and y such that $-1 < x + y < 1$. Which one?

A.


B.


C.


D.


E.


First we need to split this into two inequalities and solve for each:

$-1 < x + y < 1$ can become two:

$-1 < x + y$ and $x + y < 1$.

Isolate the y value for each:

$$-1 < x + y \qquad x + y < 1$$
$$-x - 1 < y \qquad y < -x + 1$$
$$y > -x - 1$$

Now let's think about what these mean. The first of these ($y > -x - 1$) says to shade "up" (above the y) from the line $-x - 1$: that line is going to have a negative slope and a y-intercept of -1. Choices (A) and (E) above seem to have that line.

The second of these, $y < -x + 1$, indicates that we should shade "under" the line $y = -x + 1$. That line has a y-intercept of 1 and a slope of -1.

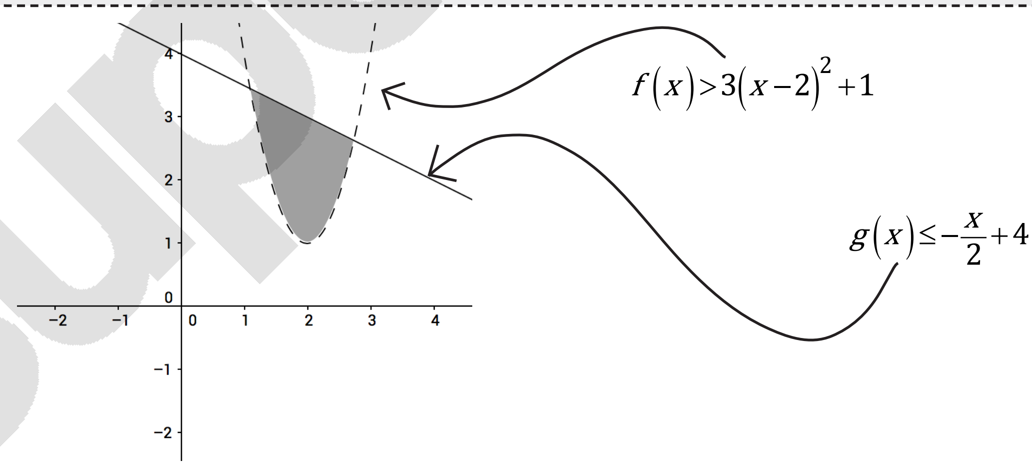
Since both lines we have show the same slope, we're looking for a set of parallel lines—clearly (A) & (E). But only choice (E) has us shading above the line that has a y-intercept of -1 and below the one with a y-intercept of 1. As such (E) is correct.

Answer: **E.**

This idea can also be applied to parabolas. Just use a vertical line and draw it through any x value—you're going to be shading the portion in the proper direction from that point of intersection from the vertical line.

What is the graph of the system:

$$f(x) > 3(x-2)^2 + 1$$
$$g(x) \le -\frac{x}{2} + 4$$



3. You always have your calculator.
The TI-84 series and TI-Nspire (note: only TI-Nspire non-CAS is permitted for the ACT®) can graph equations or inequalities to help you.

INEQUALITIES ON A COORDINATE PLANE

Oftentimes, questions that have inequalities in them are testing your understanding of basic math concepts or number be**havior, such as how positive or negative numbers react in different situations. When you have these problems:**

1. Try to simplify algebraically first
2. Plug in numbers if you get stuck

Let p and q be numbers such that $|p| \leq 0 < q$. Which of the following expressions *must be* true for all such p and q?

A. $q \div p < 0$ **B.** $p + q < 0$ **C.** $pq = 0$ **D.** $p - q > 0$ **E.** $p + q < q$

Let's start with the first part of this inequality:

$$|p| \leq 0$$

Now let's think about what this means: the distance between p and 0 is at most 0.

If I drew this on a number line, the absolute value of p would be either 0 or a negative number right? But absolute values can only be positive or 0. As such, p will have to be 0.

$$p = 0$$

From that conclusion, we can look at the answer choices and see which requires p to equal 0, which are answers A, B, and C.

(A) Mathematically impossible—you can't divide anything by 0.

(B) Doesn't work—q must be positive and p is 0—that makes a positive number.

(C) From the zero product property: $pq = 0$ implies that either p or q must equal 0. If p equals 0, then this expression must be true. P is 0, so this expression is true.

Answer: **C.**

Now the 2nd way we can attempt that problem, if we can't see the logical, algebraic relationship, is to make up some numbers.

Let's try $p = 5$:

$$|5| \leq 0 < q$$

That doesn't work—5 isn't less than or equal to zero! What about -5?

$$|-5| = 5$$

That's still not less than or equal to zero!

You can keep trying numbers—but a good rule of thumb is to always try from the following list:

LIST OF NUMBERS TO TRY

(for must be true/could be true problems)

1) A negative number (-5)
2) A positive number (4)
3) Zero
4) One
5) Negative one
6) A positive fraction $(1/4)$
7) A negative fraction $(-1/9)$
8) A huge number $(1{,}000{,}000{,}000\ etc.)$
9) A very small number $(-1{,}000{,}000\ etc.)$

Here, by trying #3, we'll get to zero, see that works and would continue on as we did above.