



An intensify Harris Hawks optimizer for numerical and engineering optimization problems



Vikram Kumar Kamboj ^{a,*}, Ayani Nandi ^b, Ashutosh Bhaduria ^c, Shivani Sehgal ^b

^a Domain of Power Systems, School of Electronics and Electrical Engineering, Lovely Professional University, Phagwara, Punjab, India

^b Ph.D Research Scholar, School of Electronics and Electrical Engineering, Lovely Professional University, Phagwara, Punjab, India

^c Department of Electrical & Instrumentation Engineering, Sant Longowal Institute of Engineering & Technology, Longowal, Punjab, India

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ABSTRACT

Recently developed Harris Hawks Optimization has virtuous behavior for finding optimum solution in search space. However, it easily get trapped into local search space for constrained engineering optimization problems. In order to accelerate the global search phase of existing Harris Hawks optimizer and to stuck it out of local search space, the proposed research aims to explore the exploration phase of the existing optimizer, the hybrid variant of Harris Hawks optimizer has been developed using sine-cosine algorithm and named as *Hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA)*. The effectiveness of the proposed optimizer has been tested for various nonlinear, non-convex and highly constrained engineering design problem. In order to validate the results of the proposed algorithm, 65 standard benchmark problems including CEC2017, CEC2018 and eleven multidisciplinary engineering design optimization problems has been taken into consideration. After verification it has been observed that the outcomes of the proposed hHHO-SCA optimization algorithm is much better than standard sine-cosine optimization algorithm, Harris Hawks Optimizer, Ant Lion Optimizer algorithm, Moth Flame Optimization algorithm, grey wolf optimizer algorithm, and others recently described metaheuristics, heuristics and hybrid type optimization search algorithm and proposed algorithm endorses its effectiveness in multi-disciplinary design and engineering optimization problems.

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1. Introduction

Artificial intelligence and Machine learning are fast growing era now a days, as most of the real world problems, which are either continuous or discrete, constrained or unconstrained in nature can be easily tackled with the help of artificial intelligence and machine learning techniques [1,2]. For these varieties of characteristics, there arose some difficulties to tackle these types of problems using conventional approaches with mathematical or numerical programming including sequential quadratic programming, quasi-Newton method, fast steepest and conjugate gradient [3,4]. There are several existing research, where it has been experimentally tested that all of these methods are not enough effective or efficient to deal with numerous types of non-differentiable, non-continuous problem and also in so many large scale real world multimodal problem [5]. Thus, meta-heuristics algorithm has been taken into consideration and has been used to tackling so many problems, which is mostly simple in nature and can be easily implemented. In optimization, inhabitants based

techniques are basically used to find some solution based on sub-optimal and optimal that can be same with the exact optimal value, which situated in its nearby point or neighborhood. By generating population set of the individuals, the optimization process of the algorithm start and in the population each individuals represent candidate solution of the problem based on optimization technique. By replacing the population of the best position of the current location, the population will be changed iteratively and generate a new population using some operators, which are stochastic in nature [6,7]. This process of optimization is continued until it can satisfy the maximum iteration. Now a days, there is fast growing awareness and interest in efficient, inexpensive and successful application of such types of metaheuristics algorithm in the recent modern research.

In the proposed research, a new hybrid metaheuristics optimization technique, hHHO-SCA algorithm, which is based on nature-inspired and mathematical functions sine and cosine, has been developed to compete with the other recent types of metaheuristics optimizer. The main impression behind this kind of proposed optimization technique is encouraged from the cooperative natural behavior of the most intelligent bird, named Harris Hawks for its natural hunting behavior and the escaping or avoiding nature of the prey (rabbit) [8]. Thus, a novel mathematical

* Corresponding author.

E-mail address: vikram.23687@lpu.co.in (V.K. Kamboj).

model, which is stochastic and metaheuristics in nature has been developed to tackle different types of optimization problem and experimentally tested in the proposed research.

2. Literature review

Optimization technique is enormous area of research and research is going on very fast. The researchers are doing continues research work on different problems in order to implement various types of new techniques on different problems and are capable to find the results successfully. The work is profitable on to find the new algorithms and also the algorithms with its hybrid form to mitigate any types of drawback in the present in existing methods. In the proposed research, contemporary research papers has been selected to investigate the shortfalls of existing algorithms.

Some of these research paper includes Ant Colony Optimization (ACO) algorithm [9] was one of the nature inspired meta-heuristics optimization method mimics the foraging behavior of ant species. The ants deposit pheromone on the ground to mark some favorable pathway which should be followed by other ants of the colony. This similar mechanism are used to solve optimization problems. Adaptive gbest-Guided Search Algorithm (AGG) [10] is a heuristic evolutionary search algorithm which inspired by gravitational forces between masses in nature. The Ant Lion Optimizer (ALO) [11] mimics the hunting mechanism of ant lions in nature. The main steps are such as hunting of prey like random walk of ants, building traps, entrapment of ants in traps, catching preys, and re-building traps were implemented. Artificial Neural Network [12] and an adaptive collocation strategy was implemented to solve the partial differential equation. This method raised the strength of the network estimate and the result were important for computational savings, particularly when the solution was non-smooth. Branch and Bound (BB) [13] is a new method based on branch-and-bound techniques and has been used to solve unit commitment problem. Bat Algorithm (BA) [14] is a novel optimization technique is based on Levy flights path and differential operator which is presented to increase speed of convergence. Binary Bat Algorithm (BBA) [15] is a nature-inspired feature selection technique which based on the bats behavior used to combine the exploration power of the bats considering the speed of the Optimum-Path Forest classifier to find the set of features which increased accuracy in the validating set. Biogeography Based Optimization (BBO) [16] technique motivates the application of biogeography to solve optimization problems. This unique optimization technique was used to solve the problem based on a real-world sensor selection for aircraft engine health estimation. Binary Gravitational Search Algorithm (BGSA) [17] is one of the optimization technique which based on the mass interactions and law of gravity. Bacterial Foraging Optimization Algorithm (BFOA) [18] is a global optimization algorithm inspired by the social foraging behavior of Escherichia coli which is used to solve the real-world optimization problems rising in various application domains. Bird Swarm Algorithm (BSA) [19] is a bio-inspired algorithm which was based on the swarm intelligence extracted from the social behaviors and social interactions in bird swarms. Backtracking Search Optimization (BSO) [20] is one of the stochastic new evolutionary algorithm based on the strategy to generate a trial solution with two new operators including mutation and crossover which are used to maintain the amplitude of the search-space boundaries and search-direction matrix, that improved the capabilities of the powerful exploration and exploitation. Colliding Bodies Optimization (CBO) [21] algorithm is a multi-agent metaheuristic algorithm, which was conceptualized using the one-dimensional collisions between bodies including each agent solution considered as body or object with mass. Cultural Evolution Algorithm (CEA) [22] was the population-based

algorithm based on cultural evolution goal. Chaotic Krill Herd Algorithm (CKHA) [23] is a metaheuristic optimization algorithm which introduced the chaos theory into Krill Herd Optimization process with the acceleration of its global convergence speed. Cuckoo Search Algorithm (CS) [24] is one of the metaheuristic nature inspired algorithm based on the obligate brood parasitic behavior of some cuckoo species in combination with the Levy flight behavior of some birds. Dragonfly Algorithm (DA) [25] is inspired from the static and dynamic swarming behaviors of dragonflies in nature. Elephant Herding Optimization (EHO) [26] is a swarm-based metaheuristic optimization method inspired by the herding behavior of the group of elephant. Exchange Market Algorithm (EMA) [27] is a new approach of evolutionary algorithm for continuous non-linear optimization problems that inspired by the procedure of trading the shares on stock market. Electromagnetic Field Optimization (EFO) [28] is a physics inspired metaheuristic optimization method, which is based on the behavior of electromagnets with different polarities. Earthworm Optimization Algorithm (EOA) [29] is a bio-inspired metaheuristic algorithm to solve the global optimization problems. Fireworks Algorithm (FA) [30] is a swarm intelligence optimization technique based on two types of processes and the mechanisms in explosion for keeping diversity of sparks were also well designed. Firefly Algorithm (FFA) [31] is a nature inspired optimization algorithm which had been significant developed and its applications based on real-world problem. Forest Optimization Algorithm (FOA) [32] is inspired by few trees in forest which can survive for several decades, while other trees could live for a limited period. Flower Pollination Algorithm (FPA) [33] is inspired by the pollination process of flowers. Grasshopper Optimization Algorithm (GOA) [34] mimics the behavior of grasshopper swarms in nature for solving real problems. Gravitational Search Algorithm (GSA) [35] is one of the heuristic search algorithms inspired by swarm behaviors in nature. This algorithm was based on the law of gravity and mass interactions. grey Wolf Optimizer (GWO) [36] is a meta-heuristic optimization technique inspired by grey wolves which mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. Genetic Algorithm (GA) [37] motivated from the biological evolution using comparisons of based on principles mechanisms such as natural selection, genetic recombination and survival of the fittest. Lightning Search Algorithm (LSA) [38] is a novel metaheuristic optimization method which used to solve constraint optimization problems, which is based on the natural phenomenon of lightning and the mechanism of step leader proliferation using the idea of fast particles known as projectiles. Hopfield method [39] technique is used to solve ramp rate constrained for unit commitment problem. Human Group Optimizer (HGO) [40] swarm intelligence optimization method is inspired by pretending human behaviors and especially human searching/foraging behaviors with local search technique. Imperialist Competitive Algorithm (ICA) [41] is one of the evolutionary search algorithm is based on imperialist competition. Isogeometric analysis (IGA) [42] and density mapping methods for topology optimization of flexoelectric or piezoelectric materials. League Championship Algorithm (LCA) [43] was a stochastic population based algorithm which was used to solve continuous global optimization problem. Interior search algorithm (ISA) [44] is a metaheuristic algorithm inspired by interior design and decoration. Invasive Weed Optimization (IWO) [45] a numerical stochastic optimization algorithm which was inspired from colonizing weeds and this was proposed for Electromagnetic applications. Krill Herd Algorithm (KHA) [46] is based on the simulation of the herding behavior of krill individuals. Mine Blast Algorithm (MBA) [47] is a population-based optimization algorithm followed by the mine bomb explosion concept. Monarch Butterfly Optimization

(MBO) [48] is a nature-inspired metaheuristic algorithm based on the updated positions of the monarch butterflies. Moth-Flame Optimization (MFO) [49] strategy depends on moths which fly in night by keeping fixed angle with respect to moon and it was a very effective mechanism for traveling in a straight line for long distances. Multi-Verse Optimizer (MVO) [50] is a nature-inspired heuristic search algorithm which was based on three concepts in Cosmology: wormhole, black hole and white hole. A multi-material level set (LS) [51] based topology optimization of flexoelectric amalgams were designed to extend the point wise density mapping technique. Optics Inspired Optimization (OIO) [52] optimization technique is based on the law of reflection. Particle Swarm Optimization (PSO) [53] is a stochastic algorithm used for the optimization problem on the global and local best particles by introducing the mutation operators and improved its performance. Runner-Root Algorithm (RRA) [54] was a metaheuristic optimization method which inspired by runners and roots of plants in nature. Simulated Annealing (SA) [55] is a deep and useful connection between statistical mechanics and multivariate or combinatorial optimization. Sine-Cosine Algorithm (SCA) [56] is a population based optimization technique which created multiple initial random solutions. Stochastic Fractal Search (SFS) [57] is a metaheuristic algorithm used a mathematic concept known as fractal which explore the search space more efficiently. Shuffled Frog-Leaping Algorithm (SFLA) [58] is a population-based cooperative search metaphor inspired by natural memetics. Search Group Algorithm (SGA) [59] is a metaheuristic optimization technique to deal with the optimization of truss structures. Seeker Optimization Algorithm (SOA) [60] is based on the perception of simulating the act of humans' intelligent search with their experience, memory and uncertainty reasoning. Random Walk grey Wolf Optimizer (RW-GWO) [61] is a new algorithm in the arena of swarm intelligence to solve continuous optimization problems and real world optimization problems. The grey Wolf Optimizer was the only algorithm in the category of swam intelligence, which is based on leadership hierarchy. To improve the search capability by grey wolf a modified algorithm RW-GWO based on random walk. Symbiotic Organisms Search (SOS) [62] is a robust and powerful metaheuristic algorithm depend upon the strategies adopted by organisms to survive and propagate in the ecosystem. Salp Swarm Algorithm (SSA) [63] is a bio inspired heuristic search algorithm for solving optimization problems with single and multiple objectives. Tabu Search (TS) [64] is the metaheuristic optimization technique which gives good results on combinatorial optimization problems such as quadratic assignment. Teaching-Learning-Based Optimization (TLBO) [65] is based on the teaching-learning phenomenon of a classroom to solve multi-dimensional, linear and nonlinear problems with appreciable efficiency. Virus Colony Search (VCS) [66] is a nature-inspired technique which simulates diffusion and infection strategies for the host cells adopted by virus to survive and propagate in the cell environment. Water Cycle Algorithm (WCA) [67] is inspired from nature and based on the observation of water cycle process and how rivers and streams flow to the sea in the real world. Wind Driven Optimization (WDO) [68] is nature inspired optimization algorithm used to solve global optimization problems and inspired by the motion of wind in the Earth's atmosphere. Whale Optimization Algorithm (WOA) [69] is a nature-inspired meta-heuristic optimization algorithm which mimics the social behavior of humpback whales. Weighted Superposition Attraction (WSA) [70] is based on two basic mechanisms, "superposition" and "attracted movement of agents". Water Wave Optimization (WWO) [71] is inspired by the beautiful phenomena of water waves, such as propagation, refraction, and breaking. Modified Dragonfly Optimization Algorithm [72] is a nature inspired algorithm based on random flying

behavior of dragonflies. Artificial Flora (AF) Optimization Algorithm [73] is a stochastic optimization method Inspired by the procedure of reproduction and migration of flora. Chicken Swarm Optimization algorithm [74] is a metaheuristic bio-inspired optimization technique, which mimics the nature of the chicken swarm. Crow Particle Optimization Algorithm [75] is the hybrid form of Particle swarm optimization and crow search algorithm. Improved Electromagnetic Field Optimization algorithm [76] was a metaheuristic physics inspired method, which mimics the performance of electromagnets considering different polarities. A significant research on multi-material level set-based topology optimization of flexoelectric composites [51], Computational Machine Learning Representation for the Flexoelectricity Effect in Truncated Pyramid Structures [77], design methodology based on a combination of isogeometric analysis (IGA), level set and point wise density mapping techniques for topology optimization of Piezoelectric/flexoelectric materials [42,78] and Sensitivity and uncertainty analysis for flexoelectric nanostructures has been reported in recent research articles [79]. Quasi-Opposition-Based Learning and Dimensional Search algorithm [80] is inspired by the attachment process of lightning in environment. Supernova Optimizer [81] is inspired by the supernova phenomena in nature. Manta ray foraging optimization [82] is a recently developed bio-inspired optimization technique, which is based on intelligent performances of manta rays. Hybrid Artificial Grasshopper Optimization (HAGO) [83] is a metaheuristics optimization algorithm inspired by grasshopper to recover the exploration and exploitation in given search space. Harris Hawks optimization (HHO) [8] is recently proposed nature-inspired search algorithm, which is encouraged from the cooperative natural behavior of the most intelligent bird, named as Harris Hawks of its natural hunting behavior and the escaping or avoiding nature of the prey (rabbit). The brief reviews of the various meta-heuristics stochastic and heuristic search algorithms has been depicted in Table 1.

All the recently developed heuristics, metaheuristics, evolutionary and nature inspired algorithms has its own pros and cons and most of these search algorithms are not applicable to every kind of optimization problems and hence universally cannot be accepted. Further, from No Free Lunch Theorem (NFL) [84], all types of algorithms based on optimization techniques recommended and show average equivalent performance, if it is applied to all probable types of tasks based on optimization technique. According to NFL theorem, it cannot consider theoretically an algorithm as universally best type of optimizer in general purpose. Hence, NFL theorem motivates for penetrating and rising more effective algorithm based on optimization technique. Motivated from these, in the proposed research, the initiative has been taken to provide another powerful optimizer, which is based on natural hunting behavior of Harris Hawks and trigonometric functions sine and cosine and names as hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA).

3. Mathematical approach for hybrid Harris Hawks optimizer

In this paper, HHO algorithm includes exploratory and exploitative phases, which is encouraged by surprise attack, the nature of exploration of a prey and different strategies based on attacking phenomenon of Harris hawks. This is one of the gradient-free and inhabitants based algorithms for optimization technique, which will be useful to formulate on any types of optimization problem. HHO have some major techniques in the phase of exploration. Not only this, but also HHO have a major strategy in conversion from phase of the exploration to the phase of the exploitation.

Table 1

Review of various meta-heuristics and heuristics type search algorithms.

Algorithm name	No. of benchmark problems	Year	Reference. no.	Author's name	Problem type
Animal Migration Optimization	23	2013	[85]	Li X.	NA
Artificial Bee Colony Algorithm	NA	2019	[86]	Xiao-long Chen	Economic Load Dispatch
Artificial Flora (AF) Optimization Algorithm	6	2019	[73]	Long Cheng, Xue-han Wu, Yan Wang	Standard benchmark
An upgraded Artificial Bee Colony Algorithm	5	2012	[87]	Ivona Brajevic	Constrained Optimization
Ameliorated grey Wolf Optimization	15	2018	[88]	Diljinder Singh, J.S. Dhillon	Economic load dispatch
Barnacles Mating Optimizer	23	2018	[89]	Mohd Herwan Sulaiman	Engineering Design Optimization
Binary grey Wolf Optimization	18	2015	[90]	E. Emary, Hossam	Feature Selection
Biogeography-Based Optimization	14	2008	[16]	Simon D.	Real World
Binary PSO-GSA	22	2014	[91]	Mirjalili S.	NA
Bird Swarm Algorithm	18	2015	[19]	Meng Bing Z.	NA
Binary whale optimization algorithm	NA	2018	[92]	Srikanth Reddy K.	Profit-Based Unit Commitment
BBO Train Multilayer perceptron	6	2014	[93]	Mirjalili S.	Bio-medical Optimization
Coyote Optimization Algorithm	40	2018	[94]	Juliano Pierezan	Engineering Design Optimization
Crow Particle Optimization Algorithm	6	2019	[75]	Ko-Wei Huang, Ze-Xue Wu	Six standard benchmark
Cultural Evolution Algorithm	7	2013	[22]	Kuo H.C.	Reliability Engineering
Chaotic Krill Herd Algorithm	14	2014	[23]	Wang G.	NA
Chicken Swarm Optimization algorithm	20	2019	[74]	Sanchari Deb, Xiao-Zhi Gao, Kari Tammi, Karuna Kalita	18 standard benchmark problems and 2 engineering problems
Competition Over Resources	8	2014	[95]	Mohseni S.	NA
Exchange Market Algorithm	12	2014	[27]	Ghorbani N.	NA
Electromagnetic Field Optimization	30	2015	[28]	Beheshti Z.	NA
Electro-Search algorithm	10	2017	[96]	Amir Tabari Arshad Ahmad	Engineering Design Optimization
Emperor Penguin Optimizer	44	2018	[97]	Gaurav Dhiman	Engineering Design
Elephant Herding optimization	15	2015	[26]	Wang G.	NA
Firework Algorithm	9	2010	[30]	Tan Ying	NA
Fuzzy Optimization Technique	NA	2015	[98]	Mohammad Shoaib Shahriar	Optimization of Unit Commitment
Forest Optimization Algorithm	4	2014	[32]	Ghaemi M.	Feature weighting
Gravitational Search Algorithm	23	2009	[35]	Rashedi E.	NA
Grid Classification Based Algorithm	NA	2016	[99]	Tripatjot Singh Panag	Wireless Sensor in Network Monitored Area
Grasshopper Optimization Algorithm	19	2017	[34]	Saremi S.	Engineering Design Optimization
grey Wolf Optimizer	29	2014	[36]	Mirjalili S.	Engineering Design Optimization
GWO-SCA	22	2017	[100]	Singh N.	Bio-medical Optimization
Harris Hawks Algorithm	29	2019	[8]	Ali Asghar Heidari	Standard Benchmark

(continued on next page)

Table 1 (continued).

Algorithm name	No. of benchmark problems	Year	Reference. no.	Author's name	Problem type
Hybrid Artificial Grasshopper Optimization (HAGOA)	19	2018	[83]	Brahm Prakash Dahiya, Shaveta Rania and Paramjeet Singhb	Standard benchmarks
Interior Search Algorithm	14	2014	[44]	Gandomi A.	Engineering Design Optimization
Improved Electromagnetic Field Optimization	13	2019	[76]	Alkin Yurtkuran	Standard Benchmark
Krill Herd	20	2012	[46]	Gandomi A.	NA
Lightning Search Algorithm	24	2015	[38]	Shareef H.	NA
Lion Optimization Algorithm	NA	2017	[101]	Narendrasinh B Gohil	Engineering Design Optimization
Manta ray foraging optimization	39	2019	[82]	Weiguo Zhao, Zhenxing Zhang and Liying Wang	31 Benchmark functions and 8 constrained engineering design problems
Moth-flame Optimization Algorithm	29	2015	[49]	Mirjalili S.	Engineering Design Optimization
Modified Dragonfly Optimization Algorithm	21	2019	[72]	Çiğdem İnan Açı Hakan Gülcen	15 single objective and 6 multi objective problems
Multi-start methods	NA	2012	[102]	Rafael Martí	Combinatorial Optimization
Multi-verse Optimizer	19	2015	[50]	Mirjalili S.	Engineering Optimization
Oppositional Artificial Bee Colony Algorithm	NA	2016	[103]	Kamalpreet Kaur Dhaliwal	Optimization of Digital IIR Filter
Quasi-Opposition-Based Learning and Dimensional Search Algorithm	32	2019	[80]	Tongyi Zheng and Weili Luo	unimodal, multimodal, and CEC 2014 functions
Stochastic Fractal Search	23	2014	[57]	Salimi H.	Engineering Design Optimization
Supernova Optimizer	20	2019	[81]	Amjad A. Hudaib & Hussam N. Fakhouri	unimodal, multimodal, and CEC 2005 functions
Symbiotic Organism Search	26	2014	[62]	Cheng M.	Engineering Design Optimization
Sine–Cosine Algorithm	19	2016	[56]	Mirjalili S.	Aircraft Wing Design
Salp Swarm Algorithm	19	2017	[63]	Mirjalili S.	Engineering Design Optimization
Self-adaptive differential artificial bee colony algorithm	28	2019	[104]	Xu Chen	global Optimization
The Sailfish Optimizer	20	2019	[105]	S. Shadravan	Engineering Design Optimization
Virus Colony Search	30	2016	[66]	Li Dond M.	Engineering Design Optimization
Volleyball Premier League Algorithm	23	2017	[106]	Reza Moghdani	Global Optimization
Variable neighborhood search	16	2007	[107]	Krzysztof Fleszar	Open Vehicle Routing
Water Cycle Algorithm	19	2012	[67]	Eskandar H.	Engineering Design Optimization
Whale Optimization Algorithm	29	2016	[69]	Mirjalili S.	Engineering Design Optimization

3.1. Exploration and its conversion

In this section the mechanism of HHO is discussed. Considering the normal hunting strategy of Harris hawks, they detect the prey and track it by using their dominant eyes through which the victim cannot be realize it easily. Thus, after several hours the Harris hawks delay some time by observing and monitoring the site to detect the prey. Now for HHO, the Harris hawks

are taken as best solution in each step which is taken as the projected prey and which is also in nearly optimum region. In this optimization technique Harris hawks settle randomly on some positions and wait to detect and locate the prey based on two types of strategies. Now considering the equal chance w for each balancing strategy which is based on the locations of the another members of the family to near enough to them when attacking and the rabbit as a prey, shown in Eq. (1) for such condition where

$w < 0.5$ or balance on random positions which is shown in Eq. (2) where $w \geq 0.5$ and Eq. (3) is determine the average location of Harris hawks.

$$H(\text{iter} + 1) = H_{\text{rand}}(\text{iter}) - e_{s1} |H_{\text{rand}}(\text{iter}) - 2e_{s2}H(\text{iter})|, w \geq 0.5 \quad (1)$$

$$H(\text{iter} + 1) = H_{\text{rabbit}}(\text{iter}) - H_m(\text{iter}) \\ - e_{s3}(L_{\text{Bound}} + e_{s4}(U_{\text{Bound}} - L_{\text{Bound}})), w < 0.5 \quad (2)$$

$$H_m(\text{iter}) = \frac{1}{N} \sum_{i=1}^N H_i(\text{iter}) \quad (3)$$

where, e_{s1} and w are arbitrary numbers in between (0, 1) and these are improved in each iteration, $H_{\text{rabbit}}(\text{iter})$ = Rabbit's position, $H_{\text{rand}}(\text{iter})$ = Number of Harris hawks are selected from recent population, N = Total number of Harris hawks

This model is basically used to generate location randomly in between the range of upper and lower boundary. This optimization technique consist of some rules, firstly, the solution which is generated basically based on other hawks including its random location, secondly form Eqs. (1) and (2) usually to find out the best value of different location and the component of random scale which is based on the range of the upper and the lower bound of the variables where e_{s3} is represented as coefficient scale which used to advance increase the rule of nature randomly, while the value of e_{s4} is mostly near about 1 and this pattern is distributed in similar manner including average position. According to this rule, algorithm of HHO add the movement of scale length up to lower bound L_{Bound} . After that to provide more exploration in different section of space of feature which is considered as randomization of coefficient of scaling. There is also possible to create different types of updated rules, but here we developed simple rule which has the capability to copycat the nature of the hawks. The ordinary location of the Harris Hawks is achieved by using the Eq. (3).

Conversion from the phase of exploration to the phase of exploitation is given below:

In this algorithm, based on HHO optimization technique can transference from exploration condition to exploitation condition and after that alteration between various types of nature base on exploitative behavior which is based on the avoidance energy of the prey. Due to this avoidance behavior, it decreases the energy of the prey. The equation based on the behavior of the energy of the prey is given below:

$$EG = 2EG_0(1 - \frac{\text{iter}}{\text{iter}_{\text{max}}}) \quad (4)$$

where, EG=Avoidance energy of the prey, EG_0 = Initial condition of the energy, iter_{max} = Maximum iteration

Here for this proposed algorithm EG_0 change randomly in between the interval $(-1, 1)$ for each number of iteration. If the number of iteration reduces from 0 to -1 , that means the nature of the physically flagging behavior of rabbit is represented and if the number of iteration rises from 0 to 1, that means the establishment nature of the rabbit is represented. The avoidance energy EG is dynamic in nature and it also has a tendency to go down in decreasing manner during the running condition of the iterations. If the avoidance energy $|EG| \geq 1$, that means to explore the location of the rabbit, the Harris hawks search various different kinds of regions, hence this algorithm can perform the phase of exploration and if, $|EG| < 1$, that means during the stage of exploitation, the nature of the algorithm is to exploit the nearby solutions. That means, we can say exploration occurs when $|EG| \geq 1$ while the exploitation occurs when $|EG| < 1$.

3.2. Exploitation strategy with Soft encircle and Hard encircle

In this section, the performance of Harris hawks is behave like the surprise attack by attacking nature upon projected prey identified in the previous phase. Though, preys often try to escape or avoid from dangerous conditions. Hence, various types of hunting styles happen in real circumstances. According to avoiding and escaping nature of the victim and the policies of hunting of Harris hawks, there are four types of probable strategies which are presented in the optimization technique based on HHO to implement the stage of attacking strategy. The main behavior of such kind of preys are try to avoid or escape from this kind of situation which is basically based on threatening conditions. Let e_s is the probability of the prey can escape successfully when $e_s < 0.5$ or cannot escape successfully when $e_s \geq 0.5$ before surprise the pounce. The Harris hawks will give their performance soft or hard encircle to catch the prey at any conditions whatever the prey does. That means, hawks will surround the prey from the different types of various directions softly or hardly which is depending upon prey's energy itself. In real time operation or for real situation by performance of the surprise attack, the Harris hawks get nearer to the projected prey to rise their probabilities in attacking and after that killing the rabbit. After some time, the avoiding or escaping prey must lose their energy, then the Harris hawks increase encircle process to smoothly catch the shattered prey. To implement this tactic and allow HHO to change over between hard and soft encircle process, the constraint EG is applied for that purpose. The soft encircle occurs for $|EG| \geq 0.5$ and hard encircle occurs for $|EG| < 0.5$. In case of Soft encircle, if $e_s \geq 0.5$ and $|EG| \geq 0.5$, that means the rabbit still has its enough energy through which it can try to avoid or escape by jumps randomly but it unable to go forward into its safe condition. Due to this softly encircle is occurred by Harris hawks, for which the victim rabbit tends to more and more exhausted and after that execute the surprise attack. This kind of natural behavior is demonstrated by some rules which is given below:

$$H(\text{iter} + 1) = \Delta H(\text{iter}) - EG |K H_{\text{rabbit}}(\text{iter}) - H(\text{iter})| \quad (5)$$

$$\Delta H(\text{iter}) = H_{\text{rabbit}}(\text{iter}) - H(\text{iter}) \quad (6)$$

where, $\Delta H(\text{iter})$ = Difference between the iteration based on the present location and the vector based on the position of that victim rabbit

e_s = Random number inside (0, 1), $K = 2(1 - e_s)$, this signify the strength of jump randomly throughout the procedure of avoiding and escaping. The value of K randomly changes for each iteration to simulate the behavior of the motion of rabbit. In case of Hard encircle, if $e_s \geq 0.5$ and $|EG| < 0.5$, that means the rabbit is so shattered and at this condition it has small avoidance or avoidance energy. In this situation the Harris Hawks performs scarcely enclose the projected victim to do finally shock attack. At this condition, the current locations are changed and updated by using the equation no. (7)

$$H(\text{iter} + 1) = H_{\text{rabbit}}(\text{iter}) - EG |\Delta H(\text{iter})| \quad (7)$$

3.2.1. Soft encircle with advanced fast dives

If still $|EG| \geq 0.5$, but at this time $e_s < 0.5$, that means the rabbit has sufficient energy to avoid or escape successfully and before the surprise attack a soft encircle is created. This process is much more intellectual than the previous process.

The levy flight (LFT) conception is applied in HHO algorithm through which we can understand the mathematical model of the leapfrog movements and the patterns of avoiding or escaping of prey. It helps to detect the LFT based on such patterns which can

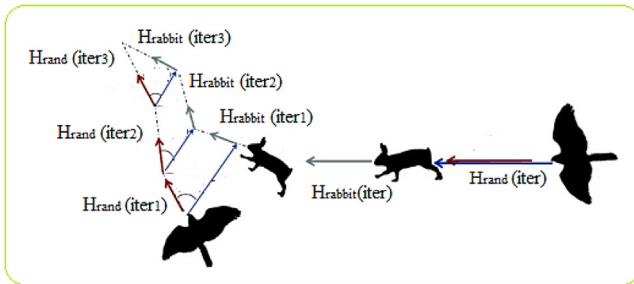


Fig. 1(a). Surprise Attack.

be identified the activity of chasing of such kind of animals like sharks and monkeys [44–47]. Hence the LFT based patterns can utilized in this technique of the phase of HHO algorithm. It is also developed to mimic the actual zigzag movements of victims (rabbits) during in the phase of escaping and asymmetrical abrupt and advanced fast dives of Harris hawks around the avoiding the prey. Basically Harris hawks performs numerous team rapidly dives round the prey (rabbit) and then try to increasingly correct their position and location as well as the directions with respect to the pretended motion of the rabbit. This type of mechanism is also maintained by observations in real world in other reasonable situations in the behavior of nature. With the help of this nature the hawks can increasingly select the best probable dive towards the rabbit at that time when they desire to catch the rabbit in the economic situations. So, for better performance of a soft encircle, the Harris hawks can decide their next movement (M_N), which is based on a rule which is given in the equation number (8):

$$M_N = H_{\text{rabbit}}(\text{iter}) - EG |KH_{\text{rabbit}}(\text{iter}) - H(\text{iter})| \quad (8)$$

After that they can relate the probable result of such kind of movement to the earlier dives which detect that it will be better dive or not. If they saw that the performance (motion) of the prey (rabbit) was more deceptive in nature, which means they also start to accomplish abrupt, rapid and irregular dives when oncoming the rabbit. The dive based upon the LFT patterns based which follow the given rule in equation number (9) [108].

$$P = M_N + R_S \times LF_T(D_{IM}) \quad (9)$$

where, D_{IM} = Problem's dimension, R_S = Size of random vector by size $1 \times D_{IM}$

$$LF_T(x) = 0.01 \times \frac{\alpha \times \delta}{|\mu|^{\frac{1}{\gamma}}} \quad (10)$$

$$\delta = \left(\frac{\Gamma(1 + \gamma) \times \sin(\frac{\pi\gamma}{2})}{\Gamma(\frac{1+\gamma}{2}) \times \gamma \times 2(\frac{\gamma-1}{2})} \right)^{\frac{1}{\gamma}} \quad (11)$$

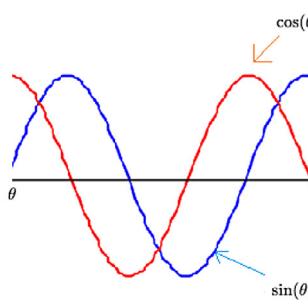


Fig. 1(b). Basic principle of SCA.

In the proposed research, the parameters α and μ are decided heuristically between the random search space of (0, 1) and the parameter γ has been set to 1.5 as a default constant.

So, including all of these the actual and final strategy for updating the actual location of Harris hawks in the phase of soft encircle can be achieved by Eqs. (12) and (13) which is given below.

$$H(\text{iter} + 1) = \{M_N \text{ if } F(M_N) < F(H(\text{iter})) \quad (12)$$

$$H(\text{iter} + 1) = \{P \text{ if } F(P) < F(H(\text{iter})) \quad (13)$$

3.2.2. Hard encircle with advanced fast dives

If $|EG| < 0.5$ and $e_s < 0.5$, at this moment the prey has not sufficient energy to escape or avoid and a hard encircle is created before the surprise attack (Fig. 1) to catch and kill the rabbit. At this situation the Harris hawks try to reduce the distance with the escaping rabbit. So this rule can be performed in basis of hard encircle condition and followed by Eqs. (12) and (13), where the value of M_N and P can obtain using new rule in Eqs. (14) and (15) which contain next position for next iteration.

$$M_N = H_{\text{rabbit}}(\text{iter}) - EG |KH_{\text{rabbit}}(\text{iter}) - H_m(\text{iter})| \quad (14)$$

$$P = M_N + R_S \times LF_T(D_{IM}) \quad (15)$$

where, $H_m(\text{iter})$ can obtain from equation number (3).

4. Sine–Cosine Algorithm (SCA)

The proposed method Sine–Cosine Algorithm is one of the population based optimizer which is recognized established on the mathematical functions of sine as well as cosine shown in Fig. 1(b). Similarly to the other types searching process by randomly constructing the solutions set. After that, all these types of solution attained randomly so extreme which signifies it as the point of destination and the resolutions or solutions. With the help of this optimization technique, this algorithm stores that solution which is more better and signifies it as the point of destination and the point of the solutions, which update to build a new resolution according to the function depend on sine and cosine, shown in the equation number (16) through equation number (17)

$$H_{i,\text{iter}+1} = H_{i,\text{iter}} + re_1 \times \sin(re_2) \times |re_3 p_{i,\text{iter}} - H_{i,\text{iter}}| \quad (16)$$

$$H_{i,\text{iter}+1} = H_{i,\text{iter}} + re_1 \times \cos(re_2) \times |re_3 p_{i,\text{iter}} - H_{i,\text{iter}}| \quad (17)$$

where $H_{i,\text{iter}}$ is used to identify the position of the present solution in i th dimension at the iter iteration and the random numbers

INPUTS: The population size is taken as N and maximum iteration number is taken as $iter_{max}$

OUTPUTS: The position of prey (rabbit) and its value of fitness

Initialization of random population $H_i (i = 1, 2, 3, \dots, N)$

While ($iter < iter_{max}$)

- Calculation of the fitness value of Harris hawks
- Set the parameter H_{rabbit} as the best position of the prey (rabbit)
- for** (each Harris hawks (H_i))

 - Do** $EG_0 = 2rand() - 1, K = 2(1 - rand())$ → Update energy at initial condition EG_0
 - Update EG using equation number (4)
 - if** $|EG| \geq 1$ then → **Phase of Exploration**

 - Update the position vector using equation (1) and (2)

 - if** $|EG| < 1$ then → **Phase of Exploitation**

 - if** $(e_s \geq 0.5) \text{ and } |EG| \geq 0.5$ then → **Soft encircle**

 - Location vector updated using equation number (5)

 - else if** $(e_s \geq 0.5) \text{ and } |EG| < 0.5$ then → **Hard encircle**

 - Location vector updated using equation number (7)

 - else if** $(e_s < 0.5) \text{ and } |EG| \geq 0.5$ then → **Soft encircle with advanced fast dives**

 - Location vector updated using equation number (12) and (13)

 - else if** $(e_s < 0.5) \text{ and } |EG| < 0.5$ then → **Hard encircle with advanced fast dives**

 - Location vector updated using equation number (14) and (15)

 - end**

- end**

- end**

Initialize the starting position of the search agents using final position obtained through Harris Hawks optimizer

Do

Evaluate each of the search agents using objective functions

Update the best fitness obtained so far ($H \in X$)

Update the random numbers re_1, re_2, re_3 and re_4

if $re_4 < 0.5$

Update the position of search agents using $H_{i,iter+1} = \left\{ H_{i,iter} + re_1 \times \sin(re_2) \times |re_3 p_{i,iter} - H_{i,iter}| \right\}$

else

Update the position of search agents using $H_{i,iter+1} = \left\{ H_{i,iter} + re_1 \times \cos(re_2) \times |re_3 p_{i,iter} - H_{i,iter}| \right\}$

2nd stage of improvement of Exploration and Exploitation

end

While ($iter < iter_{max}$)

Return the best optimal solution

Record the mean, best, worst fitness and standard deviation.

Save the global optimum value obtained through successive trial runs

Fig. 1(c). PSEUDO code of proposed hybrid HHO-SCA algorithm.

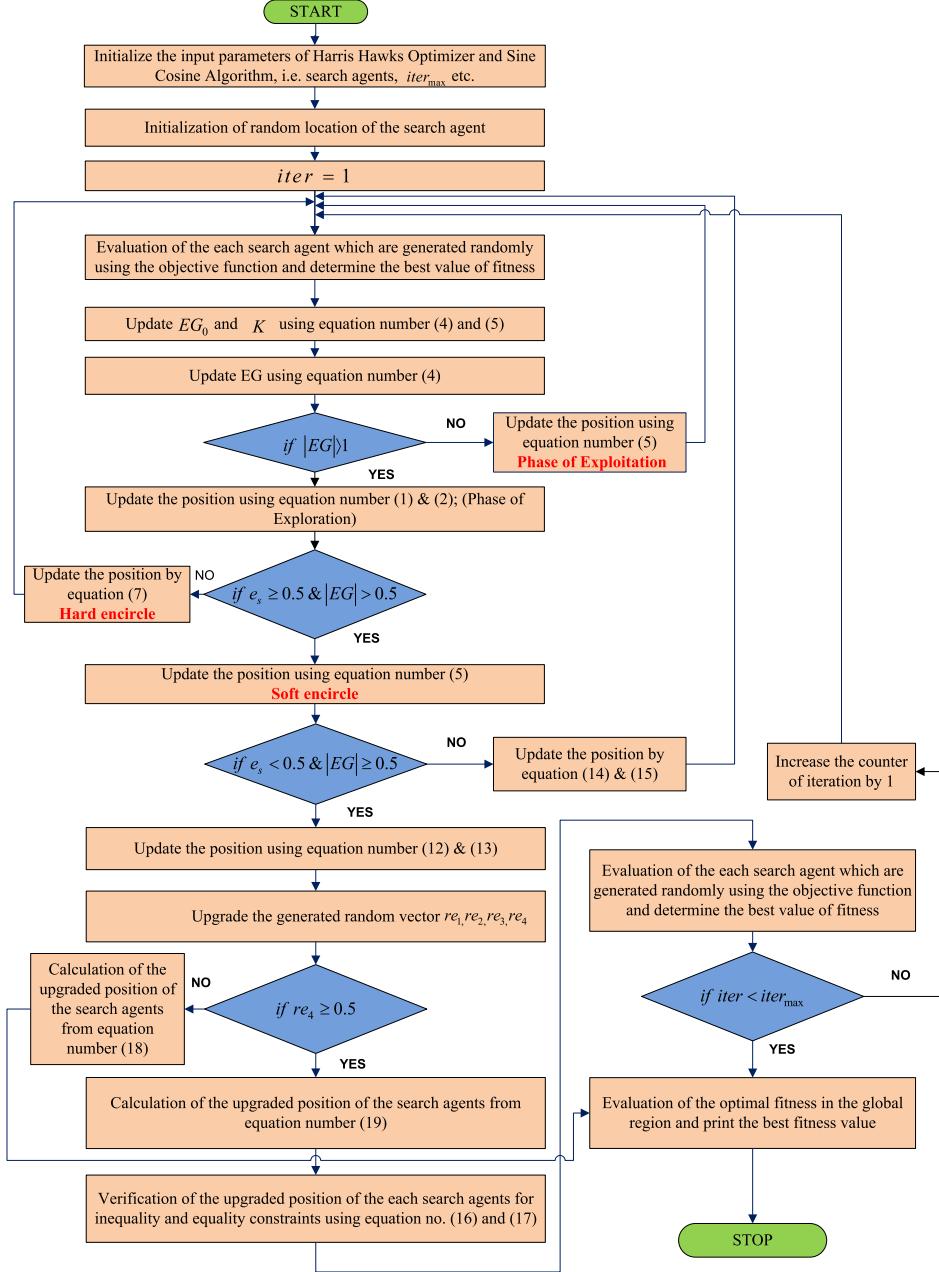


Fig. 1(d). Flow chart of proposed hHHO-SCA algorithm.

are denoted as re_1 , re_2 and re_3 respectively. $p_{i,iter}$ is the position of the point of the destination with i th number of dimension and absolute vale is indicated using the symbol of \parallel . After combining those two equations (16) and (17) are shown in Eq. (18).

$$H_{i,iter+1} = \{H_{i,iter} + re_1 \times \sin(re_2) \times |re_3 p_{i,iter} - H_{i,iter}|, re_4 < 0.5\} \quad (18)$$

$$H_{i,iter+1} = \{H_{i,iter} + re_1 \times \cos(re_2) \times |re_3 p_{i,iter} - H_{i,iter}|, re_4 \geq 0.5\} \quad (19)$$

where re_4 is the random number [0,1].

The optimization technique based on this algorithm can balance the phase of exploration as well as exploitation to solve the favorable regions in the search space and finally converge to the solution of global optimum. To balance the phase of exploitation

and exploration, the range of cosine and sine from the Eqs. (18) and (19) are improved adaptively using the Eq. (20).

$$re_1 = U - \frac{U \times iter}{iter_{max}} \quad (20)$$

where U is a constant and $iter_{max}$ is maximum number of iteration. After attained the last condition, the optimization procedure of this projected algorithm stops after sustaining the maximum number of iterations. The complexity of computation of this proposed algorithm mainly be subject to on three major process, such as, at first initialization, after that evaluation of fitness and at last updating of Harris hawks. It must be noted that I number of Harris hawks, the complexity of computation of the process of initialization is $U(I)$ For the updating mechanism the complexity of computation is $U(iter_{max} \times I) + U(iter_{max} \times I \times D_{IM})$, which is collected for penetrating for the best position and again update the position vector of all the Harris hawks, where $iter_{max}$ is representing the maximum iteration and the dimension of the problem

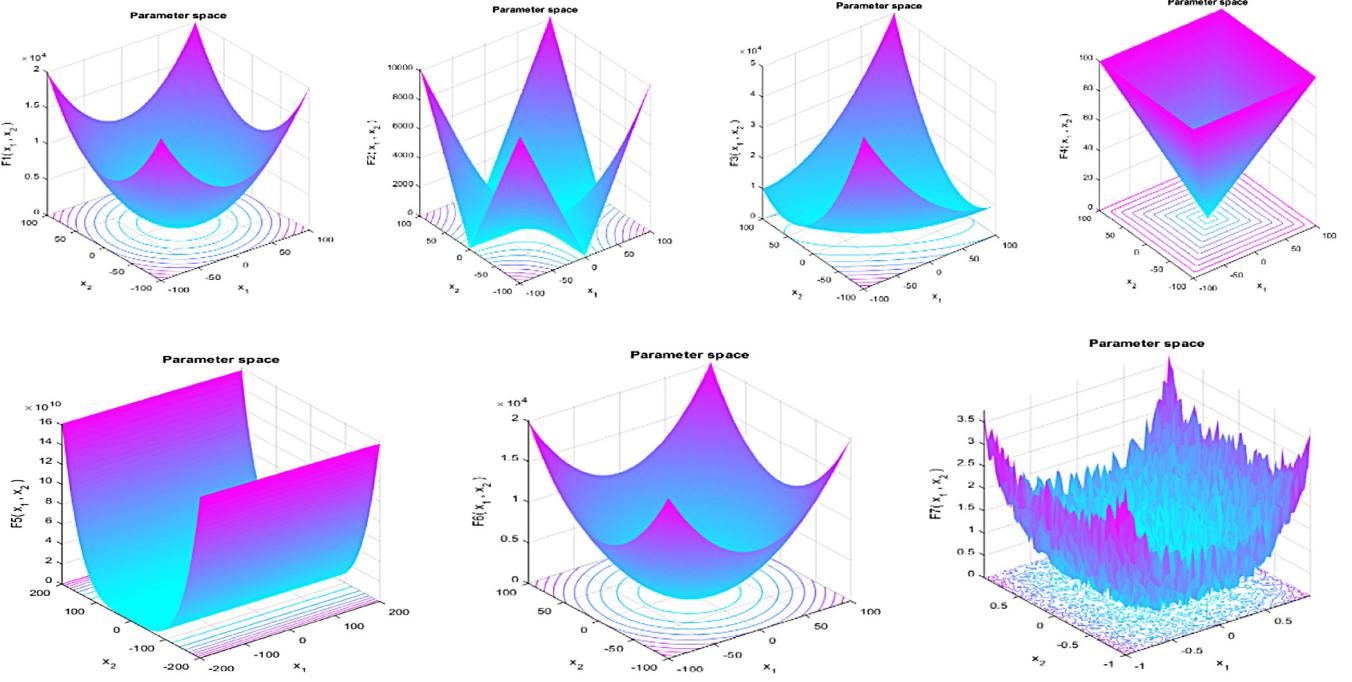


Fig. 1(e). 3D view of unimodal benchmark problem.

Table 2
Unimodal benchmark function.

Function	Dim	Range	f_{\min}
$f_1(x) = \sum_{i=1}^n x_i^2$	30	[-100, 100]	0
$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	30	[-10, 10]	0
$f_3(x) = \sum_{i=1}^n (\sum_{j=1}^i x_j)^2$	30	[-100, 100]	0
$f_4(x) = \max_i\{ x_i , 1 \leq i \leq n\}$	30	[-100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	[-30, 30]	0
$f_6(x) = \sum_{i=1}^n (x_i + 0.5)^2$	30	[-100, 100]	0
$f_7(x) = \sum_{i=1}^n ix_i^4 + \text{random}[0, 1]$	30	[-1.28, 1.28]	0

is representing as D_{IM} . Thus the complexity of computation of this proposed algorithm is $U(I(\text{iter}_{\max} + \text{iter}_{\max}D_{IM} + 1))$.

In proposed hybrid HHO-SCA algorithm, the steps of SCA are applied sequentially after the HHO algorithm to improve the exploitation and exploration to further extent. The Pseudo code and the flow chart of the proposed hybrid optimizer has been shown in Figs. 1(c) and 1(d).

5. Benchmark functions

To examine the effectiveness of the proposed hHHO-SCA optimization technique, a well-studied set of various benchmark functions are taken [109,110]. These set of benchmark are consist of three main group of benchmark function, such as, unimodal, multimodal and fixed dimension. In proposed research, the performance of the proposed hybrid optimizer has been tested for unimodal, multi-modal, fixed dimension, CEC2017 and CEC2018 benchmark functions. The mathematical formulation of unimodal,

multi-modal and fixed dimension benchmarks are shown in Table 2, Table 3 and Table 4 respectively. The 3D view of unimodal and multi-modal benchmark functions is shown in Fig. 1(e), Fig. 1(f) and Fig. 1(g) respectively.

6. Results and discussion

To validate the results, 30 trial runs are taken into consideration to overcome the stochastic nature of projected hHHO-SCA algorithm and each objective function has been estimated for average value, best values, standard deviation and worst value. In order to approve the phase of exploitation recommended algorithm, unimodal benchmark functions F1, F2, F3, F4, F5, F6 and F7 are taken into consideration. Table 5 shows the solution of unimodal benchmark function using hHHO-SCA algorithm. The estimate results for unimodal benchmark functions has been shown in Table 6, which are compared with others recently established metaheuristics algorithms GWO [36], GSA [35], FEP [109],

Table 3
Multi-modal benchmark functions.

Function	Dim	Range	f_{\min}
$F_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	30	[-500, 500]	-418.98295
$F_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	30	[-5.12, 5.12]	0
$F_{10}(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) + 20 + c)$	30	[-32, 32]	0
$f_{11}(x) = 1 + \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos \frac{(x_i)}{\sqrt{i}}$	30	[-600, 600]	0
$F_{12}(x) = \frac{\pi}{n} \left\{ 10 \sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right\} + \sum_{i=1}^n u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i+1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	30	[-50, 50]	0

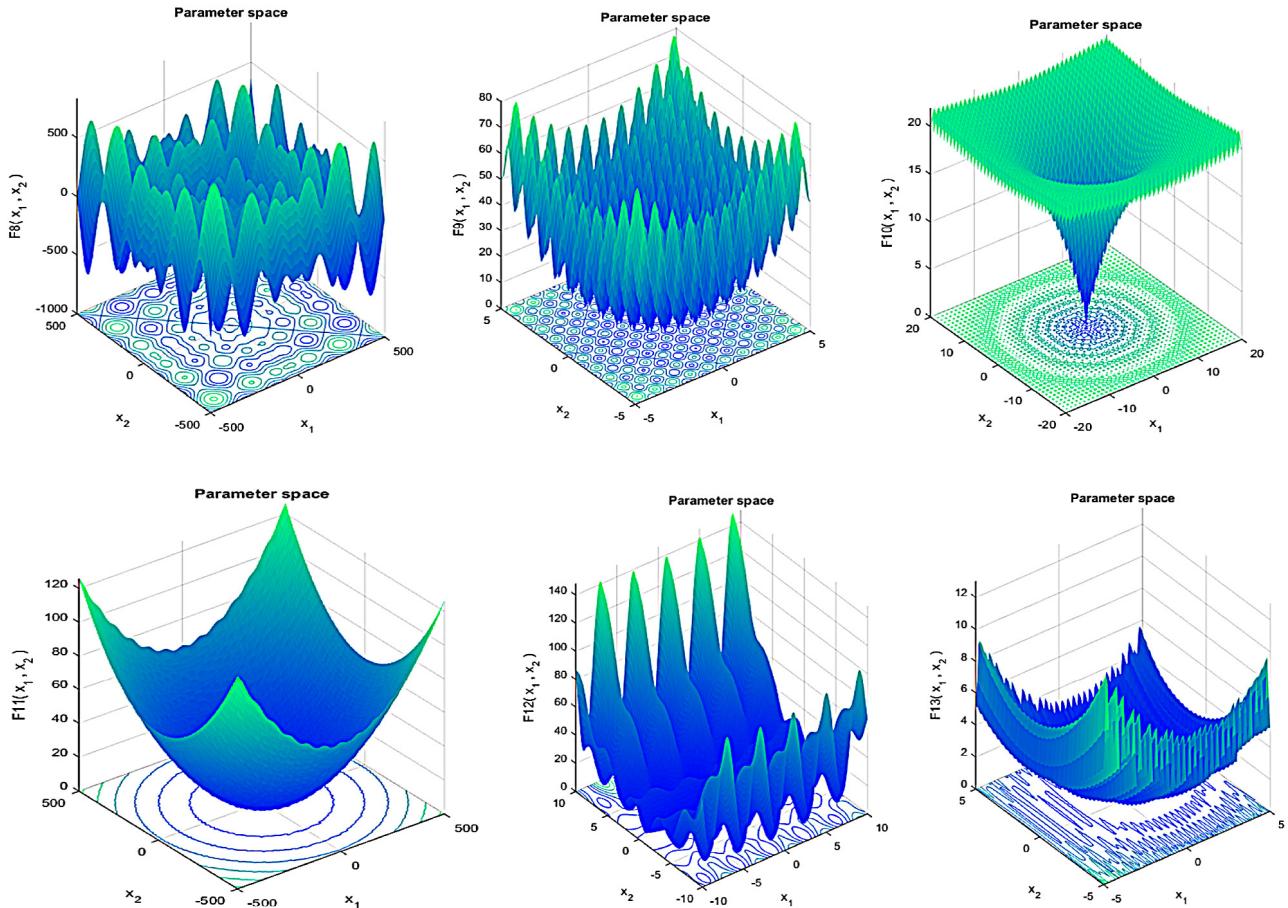


Fig. 1(f). Dimension of multimodal benchmark problem.

ALO [11], SMS [111] [112], BA [14], FPA [33], CS [113] [31], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], BDA [25], BPSO [118], BGSA [17], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of average and standard deviation. The convergence curve of hHHO-SCA and its comparative analysis with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO and HHO is shown in Fig. 1(h) and the trial solutions for unimodal benchmark functions are shown in Fig. 2. In order to validate the phase of exploration of submitted

algorithm, the multi-modal benchmark functions F8, F9, F10, F11, F12 and F13 are taken into concern, as these functions have many local optimal search with the number increasing exponentially with dimension. For effective analysis of results, similar number of iterations and runs has been taken into consideration, while comparing the results with others developed methodologies.

Table 7 shows the solution of multi-modal benchmark function using hHHO-SCA algorithm. The comparison results for multi-modal benchmark functions has been shown in Table 8, which

Table 4
Fixed dimension benchmark functions.

Function	Dim	Range	f_{\min}
$F_{13}(x) = 0.1 \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \right\} + \sum_{i=1}^n u(x_i, 5, 100, 4)$	30	[-50,50]	0
$f_{14}(x) = \left[\frac{1}{500} + \sum_{j=1}^2 5 \frac{1}{j + \sum_{i=1}^n (x_i - a_{ij})^6} \right]^{-1}$	2	[-65.536, 65.536]	1
$F_{15}(x) = \sum_{i=11}^{11} \left[a_i - \frac{x_1 (b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$	4	[-5, 5]	0.00030
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	[-5, 5]	-1.0316
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos x_1 + 10$	2	[-5, 5]	0.398
$F_{18}(x) = [1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	[-2,2]	3
$f_{19}(x) = - \sum_{i=1}^4 c_i \exp \left(- \sum_{j=1}^3 a_{ij} (x_j - p_{ij})^2 \right)$	3	[1, 3]	-3.32
$f_{20}(x) = - \sum_{i=1}^4 c_i \exp \left(- \sum_{j=1}^6 a_{ij} (x_j - p_{ij})^2 \right)$	6	[0, 1]	-3.32
$f_{21}(x) = - \sum_{i=1}^5 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0,10]	-10.1532
$f_{22}(x) = - \sum_{i=1}^7 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0, 10]	-10.4028
$f_{23}(x) = - \sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0, 10]	-10.5363

Table 5
Test results for unimodal benchmark functions using hHHO-SCA algorithm.

Functions	Mean	SD	Best	Worst	Median	p-Value
F1	1.8568E-91	9.48882E-91	9.8005E-117	5.20089E-90	1.0278E-102	1.7344E-06
F2	2.46034E-51	1.11204E-50	1.1796E-58	6.03651E-50	2.01442E-54	1.7344E-06
F3	8.88008E-72	4.86382E-71	1.7853E-105	2.66402E-70	7.22098E-86	1.7344E-06
F4	8.01525E-49	2.82586E-48	1.38633E-56	1.17982E-47	8.00088E-53	1.7344E-06
F5	0.014313995	0.020188417	0.000237175	0.104630448	0.006851675	1.7344E-06
F6	0.000223702	0.000338117	3.18673E-09	0.001181842	4.05202E-05	1.7344E-06
F7	0.00012246	0.000110326	3.25707E-06	0.000469272	8.27434E-05	1.7344E-06

are compared with others recently developed metaheuristics search algorithms GWO [36], GSA [35], FEP [109], ALO [11], SMS [111], BA [14], FPA [33], CS [113], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], BDA [25], BPSO [118], BGSA [17], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of average and standard deviation. The convergence curve of hHHO-SCA for multi-modal benchmark functions are shown in Fig. 3 and corresponding their trial solutions are shown in Fig. 4.

For fixed dimension benchmark problems the test results are shown in Table 9 through Table 10. The comparison results for fixed dimension benchmark functions has been shown in Table 10, which are compared with others recently developed metaheuristics search algorithms GWO [36], GSA [35], DE [120], FEP [109], ALO [11], SMS [111], BA [14], FPA [33], CS [113], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of standard deviation and average. The trial solutions for fixed dimension benchmark functions and their convergence curve are shown in Figs. 5 and 6 respectively. For statistical analysis of the proposed algorithm, the

non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level.

For statistical analysis of the proposed algorithm, the non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level. The p -value from Wilcoxon rank-sum has been recorded for CEC2017 multimodal benchmark functions and shown in Table 11 and it has been experimentally found that the low p -value of Wilcoxon rank-sum validate the research study at 0.05 significant level.

For statistical analysis of the proposed algorithm, the non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level. The p -value from Wilcoxon rank-sum has been recorded for CEC2017 Hybrid Benchmark functions and shown in Table 12 and it has been experimentally found that the low p -value of Wilcoxon rank-sum validate the research study at 0.05 significant level.

In order to verify the parametric variations of hHHO-SCA algorithm, the parameters α and μ are selected heuristically between the search space rand(0,1) and two different values of α has been taken into consideration for comparative analysis for CEC2017 benchmark functions and comparative results are recorded for

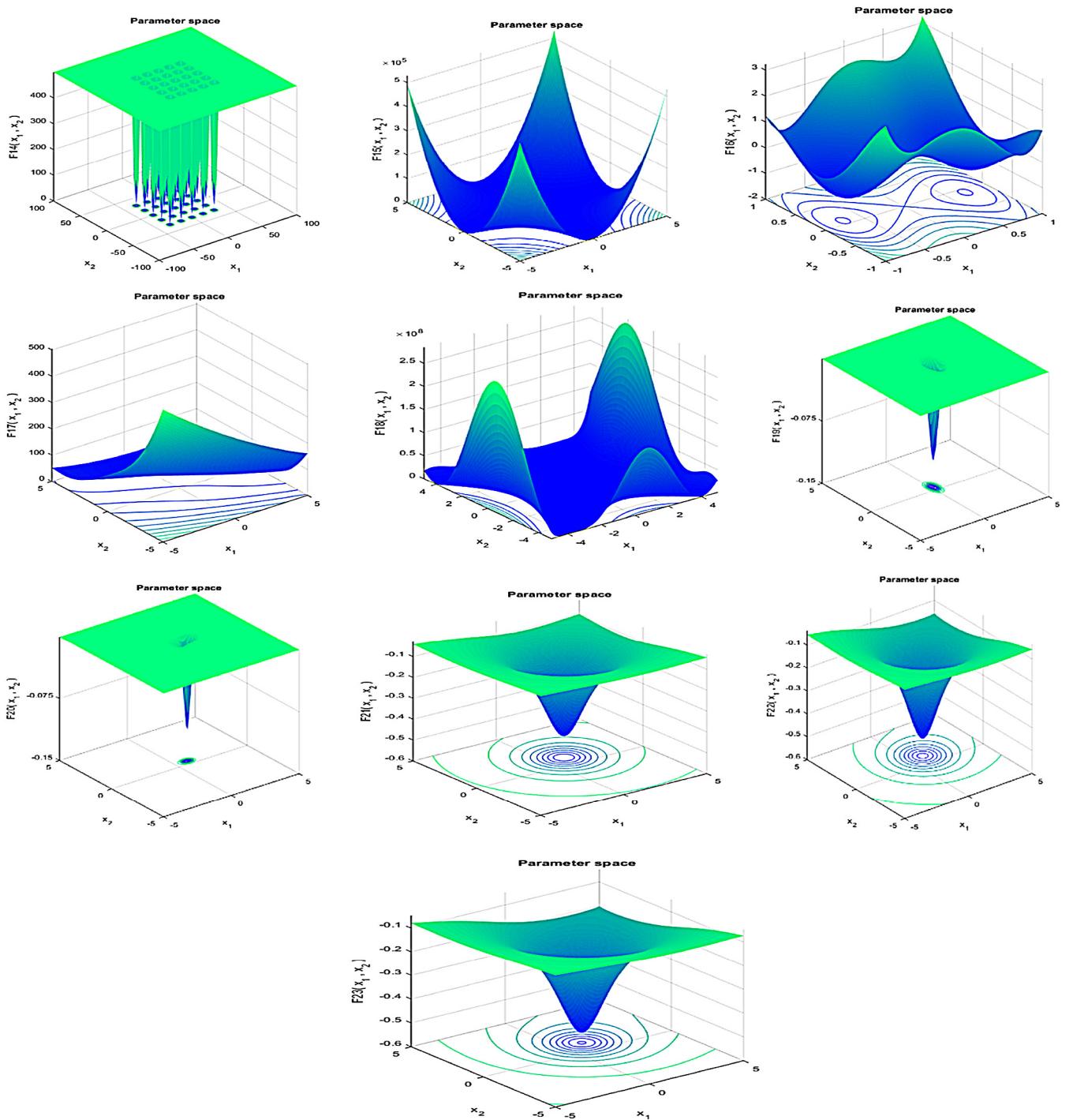


Fig. 1(g). Dimension of Fixed Dimension modal.

$\alpha = 0.93$ and $\alpha = 0.88$ in Table 14. It has been experimentally observed that the best value of α is 0.93 for proper tuning of hHHO-SCA algorithm. Also, the computational time recorded for CEC2017 benchmark functions and compared with TLBO-FL and PSO-GWO algorithm as shown in Tables 13 and 14 (see Table 15).

7. Multidisciplinary engineering design optimization problems

To validate the performance of suggested hHHO-SCA algorithm in the field of multidisciplinary engineering design optimization problems, eleven types of problems of engineering

design are taken into consideration in which Pressure vessel problem, Three-bar truss problem, welded beam problem, Cantilever Beam Design problem, Tension/compression spring design problem, Gear Train Design problem, Speed reducer, Belleville spring, coil compression and multidisc clutch are included shown in Tables 16–18. The complete description of those types of engineering design problems has been mentioned in the following sections.

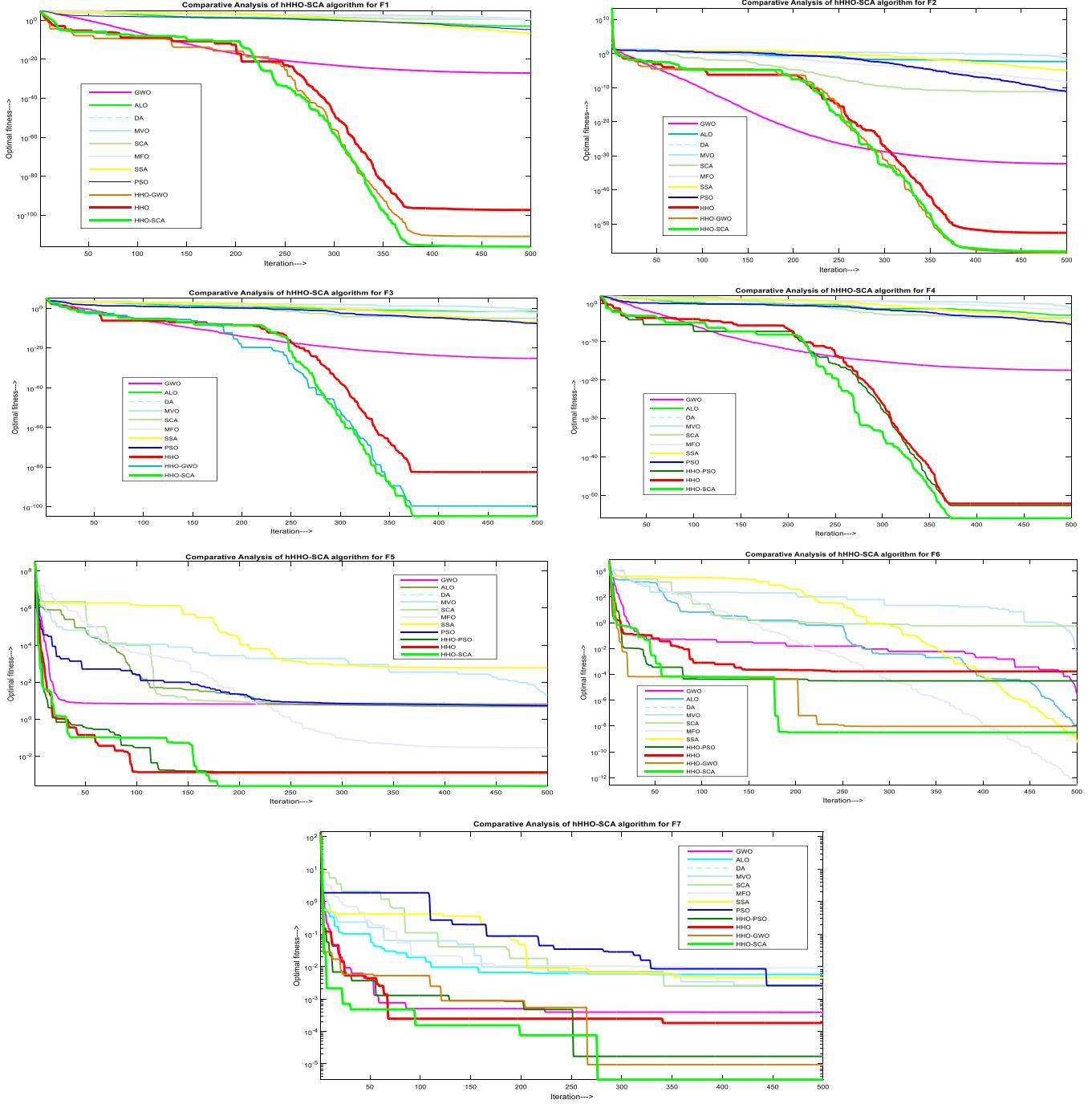


Fig. 1(h). Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for unimodal benchmark functions.

7.1. Three-bar truss engineering design problem

To verify the results of the suggested hHHO-SCA algorithm for the optimization problem based on engineering design shown in Fig. 7, the problem of Three-Bar Truss engineering design has taken into consideration [47,127]. The objective of fitness value is to decrease or minimize the weight. It consists of three types of constraints, like buckling constraints, deflection constraints and stress constraints. The numerical model for the Three-Bar Truss problem are given below in Eqs. (21) through Eq. (22). The comparison results are shown in Table 19.

$$\text{Consider } \vec{x} = [x_1, x_2] = [A_1, A_2] \quad (21)$$

$$\text{Minimize } f(\vec{x}) = (2\sqrt{2}x_1 + x_2) * l, \quad (22)$$

$$\text{Subject to } g_1(\vec{x}) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2x_1^2 + 2x_1x_2}}P - \sigma \leq 0 \quad (22a)$$

$$g_2(\vec{x}) = \frac{x_2}{\sqrt{2x_1^2 + 2x_1x_2}}P - \sigma \leq 0 \quad (22b)$$

$$g_3(\vec{x}) = \frac{1}{\sqrt{2x_2 + x_1}}P - \sigma \leq 0 \quad (22c)$$

Variable range $0 \leq x_1, x_2 \leq 1$.

where $l = 100 \text{ cm}$, $P = 2 \text{ KN/cm}^2$, $\sigma = 2 \text{ KN/cm}^2$.

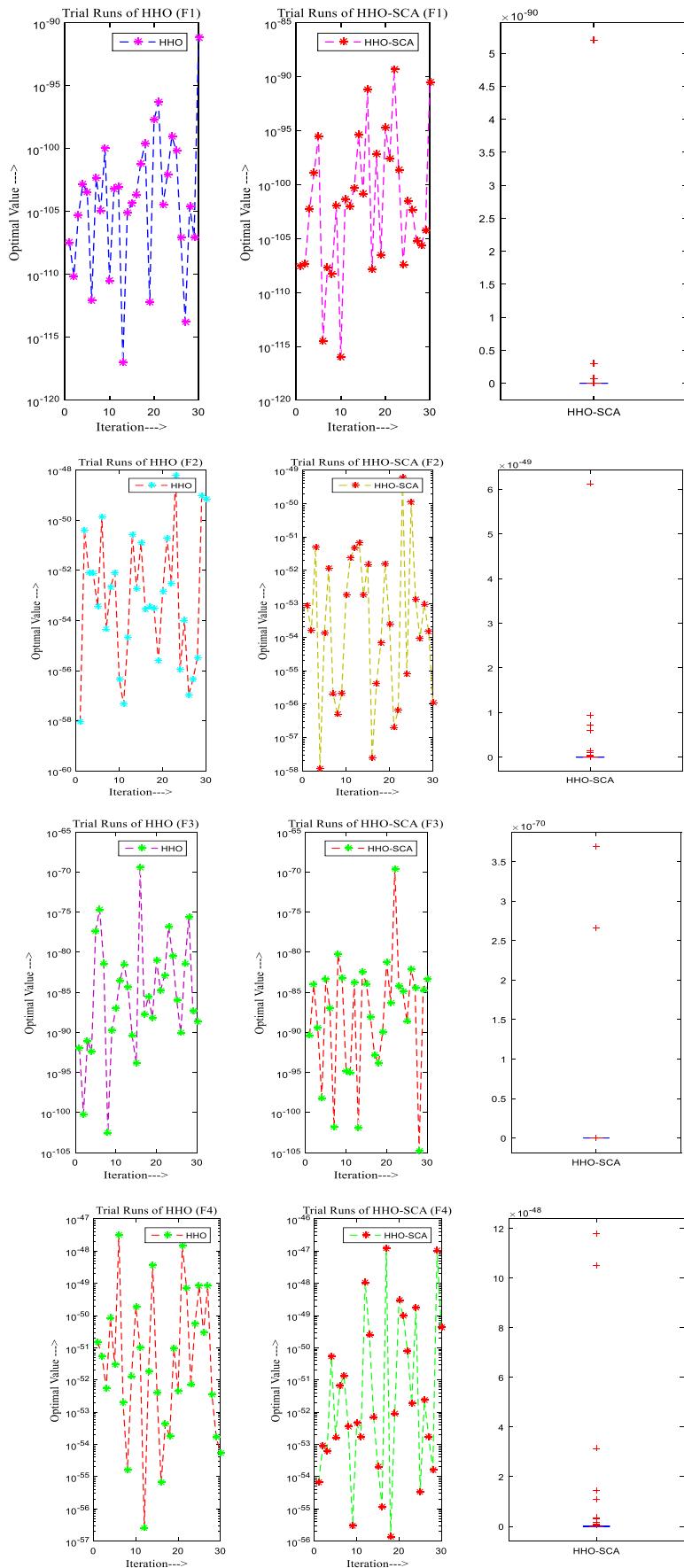
**Fig. 2.** Trial solutions for unimodal benchmark functions.

Table 6
Comparison of unimodal benchmark functions.

Algorithms	Parameters	Unimodal benchmark functions						
		F1	F2	F3	F4	F5	F6	F7
GWO [36]	Mean	0.00	0.00	0.00	0.00	26.81	0.82	-6120.00
	SD	0.00	0.03	79.15	1.32	69.91	0.00	-4090.00
GSA [38]	Mean	0.00	0.06	896.54	7.36	67.54	0.00	-2820.00
	SD	0.00	0.19	318.96	1.74	62.23	0.00	493.00
FEP [97]	Mean	0.00	0.01	0.02	0.30	5.06	0.00	-12600.00
	SD	0.00	0.00	0.01	0.50	5.87	0.00	52.60
ALO [12]	Mean	0.00	0.00	0.00	0.00	0.35	0.00	-1610.00
	SD	0.00	0.00	0.00	0.00	0.11	0.00	314.00
SMS [98]	Mean	0.06	0.01	0.96	0.28	0.09	0.13	-4.21
	SD	0.02	0.00	0.82	0.01	0.14	0.09	0.00
BA [14]	Mean	0.77	0.34	0.12	0.19	0.33	0.78	-1070.00
	SD	0.53	3.82	0.77	0.89	0.30	0.67	858.00
FPA [34]	Mean	0.00	0.00	0.00	0.00	0.78	0.00	-1840.00
	SD	0.00	0.00	0.00	0.00	0.37	0.00	50.40
CS [100]	Mean	0.01	0.21	0.25	0.00	0.01	0.00	-2090.00
	SD	0.00	0.04	0.02	0.00	0.01	0.00	0.01
FA [101]	Mean	0.04	0.05	0.05	0.15	2.18	0.06	-1250.00
	SD	0.01	0.01	0.02	0.03	1.45	0.01	353.00
GA [102]	Mean	0.12	0.15	0.14	0.16	0.71	0.17	-2090.00
	SD	0.13	0.05	0.12	0.86	0.97	0.87	2.47
GOA [35]	Mean	0.00	0.00	0.00	0.00	0.00	0.00	1.00
	SD	0.00	0.00	0.02	0.00	0.00	0.00	0.00
MFO [50]	Mean	0.00	0.00	696.73	70.69	139.15	0.00	-8500.00
	SD	0.00	0.00	188.53	5.28	120.26	0.00	726.00
MVO [51]	Mean	2.09	15.93	453.20	3.12	1272.13	2.30	-11700.00
	SD	0.65	44.75	177.10	1.58	1479.48	0.63	937.00
DA [26]	Mean	0.00	0.00	0.00	0.00	7.60	0.00	-2860.00
	SD	0.00	0.00	0.00	0.00	6.79	0.00	384.00
BDA [26]	Mean	0.28	0.06	14.20	0.25	23.60	0.10	-924.00
	SD	0.42	0.07	22.70	0.33	34.70	0.13	65.70
BPSO [105]	Mean	5.59	0.20	15.50	1.90	86.40	6.98	-989.00
	SD	1.98	0.05	13.70	0.48	65.80	3.85	16.70
BGSA [18]	Mean	83.00	1.19	456.00	7.37	3100.00	107.00	-861.00
	SD	49.80	0.23	272.00	2.21	2930.00	77.50	80.60
SCA [56]	Mean	0.00	0.00	0.04	0.10	0.00	0.00	1.00
	SD	0.00	0.00	0.14	0.58	0.00	0.00	0.00
SSA [63]	Mean	0.00	0.23	0.00	0.00	0.00	0.00	0.06
	SD	0.00	1.00	0.00	0.66	0.00	0.00	0.81
WOA [69]	Mean	0.00	0.00	0.00	0.07	27.87	3.12	-5080.00
	SD	0.00	0.00	0.00	0.40	0.76	0.53	696.00
hHHO-PSO	Mean	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	SD	0.00	0.00	0.00	0.00	0.01	0.00	0.00
hHHO-GWO	Mean	0.00	0.00	0.00	0.00	0.02	0.00	0.00
	SD	0.00	0.00	0.00	0.00	0.02	0.00	0.00
hHHO-SCA	Mean	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	SD	0.00	0.00	0.00	0.00	0.02	0.00	0.00

Table 7
Test results for multi-modal benchmark functions using hHHO-SCA algorithm.

Functions	Mean	SD	Best	Worst	Median	p-Value
F8	-12569.08113	0.76690716	-12569.48662	-12566.13644	-12569.41349	1.7344E-06
F9	0	0	0	0	0	1
F10	8.88178E-16	0	8.88178E-16	8.88178E-16	8.88178E-16	4.32046E-08
F11	0	0	0	0	0	1
F12	1.13E-05	1.5E-05	9.73E-10	6.46E-05	4.86E-06	1.734E-06
F13	0.000113	0.000166	1.49E-06	0.000674	5.53E-05	1.734E-06

7.2. Pressure vessel engineering design problem

Another types of multidisciplinary engineering design problem is taken as Pressure Vessel Design problem [47,127] shown in Fig. 8. The main objective function of this engineering design problem is to decrease or minimize the total cost including the

material cost, welding and forming of the vessel which is in cylindrical form, shown in Fig. 11. There are four different types of variables used to design the pressure vessel problem, such as, head thickness (T_h), shell thickness (T_s), without considering head, the length of cylindrical unit (L). The both ends of the pressure vessel has capped and the head of this pressure vessel has taken

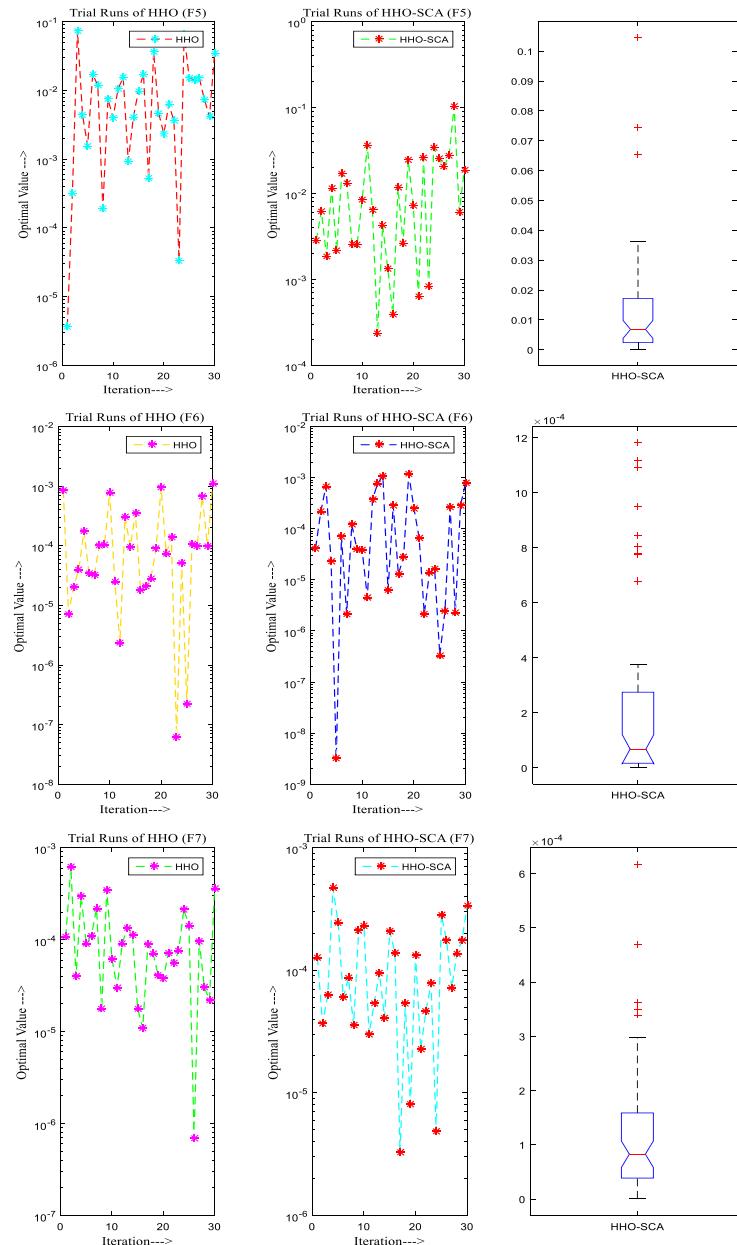


Fig. 2. (continued).

as hemi spherical shape. The above mentioned engineering design problem is subjected to four types of constraints and the mathematical construction of the pressure vessel design problem are shown in Eqs. (23) through (24d). The comparison results are shown in Table 20.

Consider,

$$\vec{x} = [x_1 x_2 x_3 x_4] = [T_s T_h RL] \quad (23)$$

$$\begin{aligned} \text{Minimize } f(\vec{x}) &= 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 \\ &+ 3.1661x_1^2x_4 + 19.84x_1^2x_3 \end{aligned} \quad (24)$$

$$\text{Subjected to } g_1(\vec{x}) = -x_1 + 0.0193x_3 \leq 0 \quad (24a)$$

$$g_2(\vec{x}) = x_3 + 0.00954x_3 \leq 0 \quad (24b)$$

$$g_3(\vec{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0 \quad (24c)$$

$$g_4(\vec{x}) = x_4 - 240 \leq 0 \quad (24d)$$

Variable ranges are: $0 \leq x_1 \leq 99$; $0 \leq x_2 \leq 99$; $10 \leq x_3 \leq 200$; $10 \leq x_4 \leq 200$.

7.3. The tension/compression spring engineering design problem

The Tension/Compression Spring Design optimization Problem is the part of multidisciplinary engineering optimization problem shown in Fig. 9, which is a portion of mechanical engineering problem [47,127]. The main objective of this type of problem is to decrease or minimize the spring weight. To get solution of this kind of problem the Tension/Compression Spring Design problem needs three types of design variables, such as, the diameter of the wire (d), the diameter of the mean coil (D) and the active coils number (N). The design problem is subjected to constraints depends on surge frequency, minimum deflection and constraints based on shear stress. The mathematical model of this type of engineering design problem is shown in the Eqs. (25) through (26e). The comparison results are shown in Table 21.

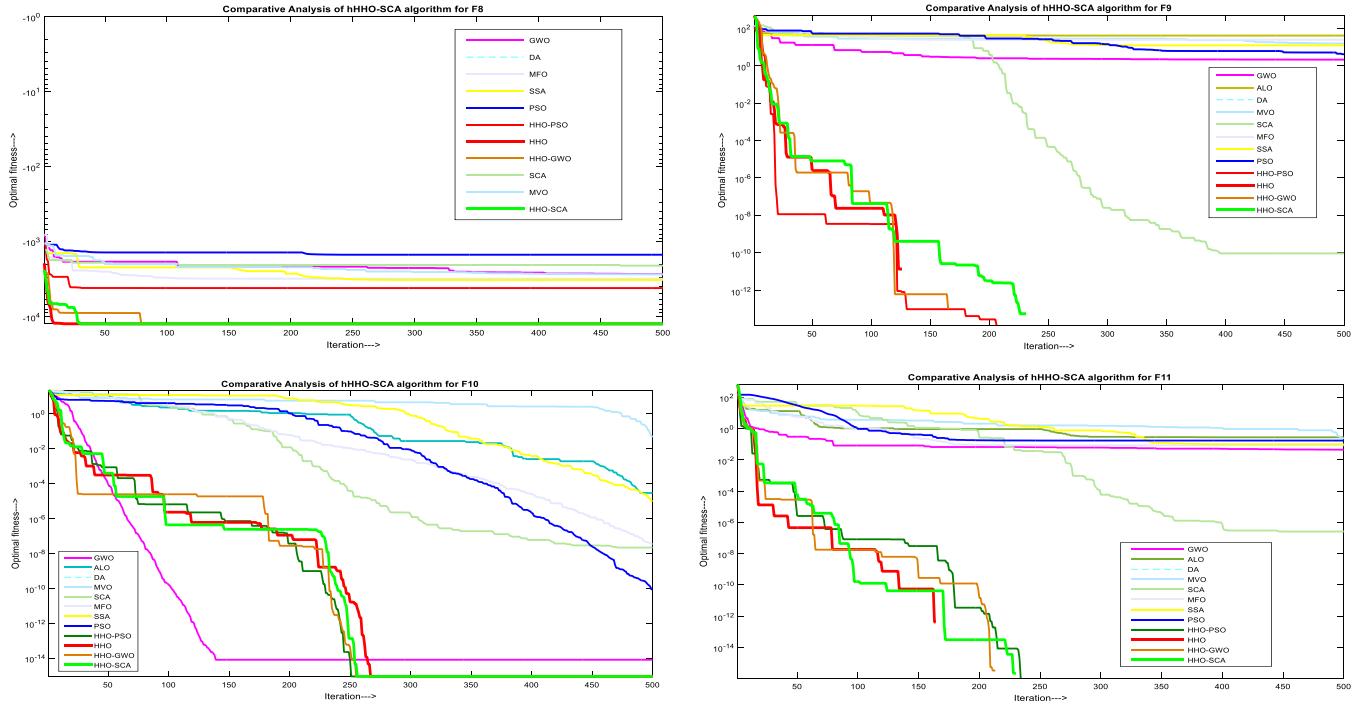


Fig. 3. Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for multimodal benchmark functions.

Table 8
Comparison of multi modal benchmark functions.

Algorithms	Parameters	Multi modal benchmark functions					
		F8	F9	F10	F11	F12	F13
GWO [36]	Mean	-6.123E+224	0.310521	1.06E-13	0.004485	0.053438	0.654464
	SD	-4087.44	47.35612	0.077835	0.006659	0.654464	0.004474
PSO	Mean	-4841.29	46.70423	0.276015	0.009215	0.006917	0.006675
	SD	1152.814	11.62938	0.50901	0.007724	0.026301	0.008907
GSA [35]	Mean	-2821.07	25.97	0.06	27.70	1.80	8.90
	SD	493.04	7.47	0.24	5.04	0.95	7.13
DE [120]	Mean	-11080.10	69.20	0.00	0.00	0.00	0.00
	SD	574.70	38.80	0.000	0.000	0.00	0.00
FEP [109]	Mean	-12554.50	0.05	0.02	0.02	0.00	0.00
	SD	52.60	0.01	0.00	0.02	0.00	0.00
hHHO-PSO	Mean	-12568.81154	0.00	8.88178E-16	0.00	1.12558E-05	0.000113306
	SD	0.946744548	0.00	0.00	0.00	1.49725E-05	0.000166039
hHHO-GWO	Mean	-1.26E+04	0.00E+00	8.88E-16	0.00E+00	NA	NA
	SD	1.04E+00	0.00E+00	0.00E+00	0.00E+00	NA	NA
hHHO-SCA	Mean	-12569.08113	0	8.88178E-16	0	1.13E-05	0.000113
	SD	0.76690716	0	0	0	1.5E-05	0.000166

Table 9
Test results for fixed dimension multi-modal benchmark functions using hHHO-SCA algorithm.

Functions	Mean	SD	Best	Worst	Median	p-Value
F14	1.26E+00	4.47E-01	9.98E-01	1.99E+00	9.98E-01	1.73E-06
F15	3.45E-04	4.03E-05	3.07E-04	4.68E-04	3.26E-04	1.73E-06
F16	-1.03E+00	1.80E-09	-1.03E+00	-1.03E+00	-1.03E+00	1.73E-06
F17	3.98E-01	2.15E-05	3.98E-01	3.98E-01	3.98E-01	1.73E-06
F18	3.00E+00	9.98E-07	3.00E+00	3.00E+00	3.00E+00	1.73E-06
F19	-3.86E+00	3.00E-03	-3.86E+00	-3.86E+00	-3.86E+00	1.73E-06
F20	-3.09E+00	1.09E-01	-3.30E+00	-2.79E+00	-3.12E+00	1.73E-06
F21	-5.21E+00	8.95E-01	-9.95E+00	-5.03E+00	-5.05E+00	1.73E-06
F22	-5.25E+00	9.26E-01	-1.02E+01	-5.07E+00	-5.09E+00	1.73E-06
F23	-5.28E+00	8.65E-01	-9.86E+00	-5.11E+00	-5.13E+00	1.73E-06

Consider

Minimize

$$\vec{x} = [x_1 x_2 x_3] = [dN]$$

$$(25) \quad f(\vec{x}) = (x_3 + 2)x_2 x_1^2$$

$$(26)$$

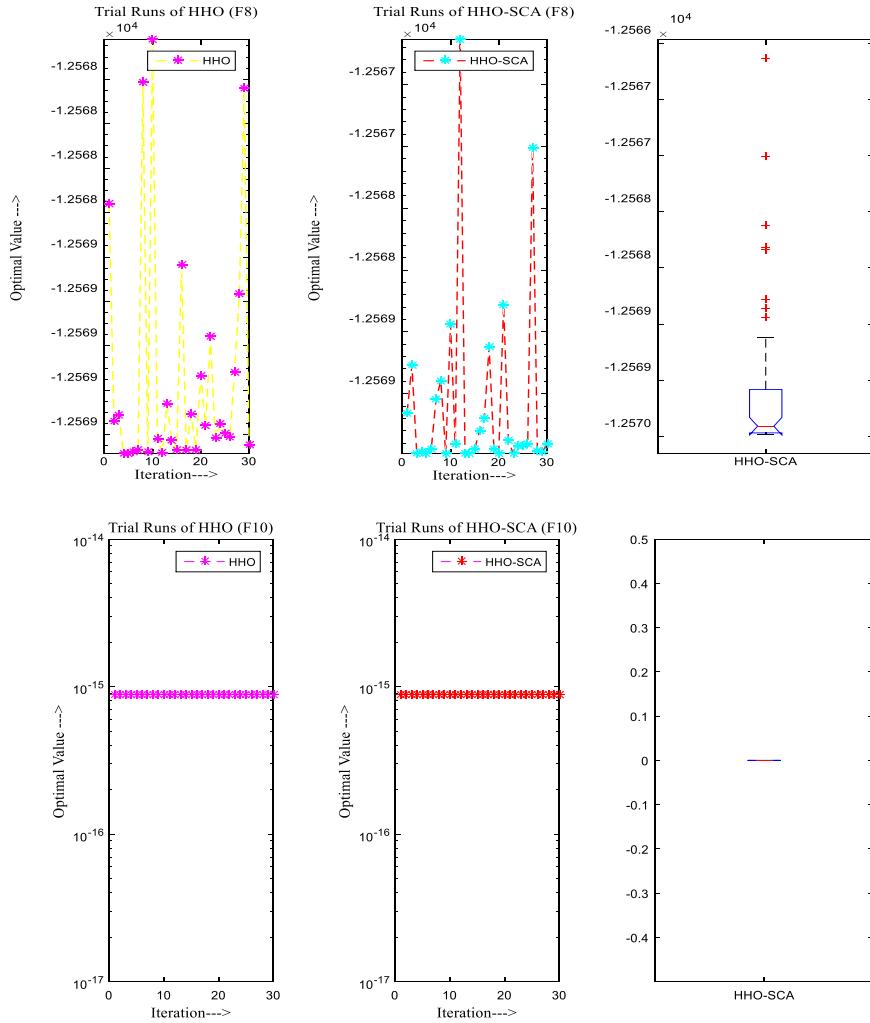


Fig. 4. Trial solutions for multimodal benchmark functions.

Table 10
Comparison of fixed dimension benchmark function.

Algorithms	Parameters	Multi modal benchmark functions									
		F14	F15	F16	F17	F18	F19	F20	F21	F22	F23
GWO [36]	Mean SD	4.04 4.25	0.00 0.00	-1.03 -1.03	0.40 0.40	3.00 3.00	-3.86 -3.86	-3.29 -3.25	-10.15 -9.14	-10.40 -8.58	-10.53 -8.56
PSO	Mean SD	3.63 2.56	0.00 0.00	-1.03 0.00	0.40 0.00	3.00 0.00	-3.86 0.00	-3.27 0.06	-6.87 3.02	-8.46 3.09	-9.95 1.78
GSA [35]	Mean SD	5.86 3.83	0.00 0.00	-1.03 0.00	0.40 0.00	3.00 0.00	-3.86 0.00	-3.32 0.02	-5.96 3.74	-9.68 2.01	-10.54 0.00
DE [120]	Mean SD	1.00 0.00	0.00 0.00	-1.03 0.00	0.40 0.00	3.00 0.00	N/A N/A	N/A N/A	-10.15 0.00	-10.40 0.00	-10.54 0.00
FEP [109]	Mean SD	1.22 0.56	0.00 0.00	-1.03 0.00	0.40 0.00	3.02 0.11	-3.86 0.00	-3.27 0.06	-5.52 1.59	-5.53 2.12	-6.57 3.14
hHHO-PSO	Mean SD	1.359860826 1.255144819	0.000394039 0.000217743	-1.031628452 4.80343E-09	0.397904414 3.49021E-05	3.000000204 5.27912E-07	-3.861232979 0.003410542	-3.095084545 0.103220758	-5.049396926 0.006604414	-5.083324889 0.004116707	-5.122724703 0.005765656
hHHO-GWO	Mean SD	1.26E+00 9.32E-01	3.44E-04 3.14E-05	-1.03E+00 3.97E-10	3.98E-01 1.86E-05	3.00E+00 2.65E-07	-3.86E+00 4.30E-03	-3.11E+00 1.21E-01	-5.22E+00 8.97E-01	-5.14E+00 1.10E+00	-5.30E+00 9.55E-01
hHHO-SCA	Mean SD	1.26E+00 4.47E-01	3.45E-04 4.03E-05	-1.03E+00 1.80E-09	3.98E-01 2.15E-05	3.00E+00 9.98E-07	-3.86E+00 3.00E-03	-3.09E+00 1.09E-01	-5.21E+00 8.95E-01	-5.25E+00 9.26E-01	-5.28E+00 8.65E-01

Subjected to

$$g_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2 x_3} \leq 0 \quad (26d)$$

$$g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0 \quad (26a) \quad g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \leq 0 \quad (26e)$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} \leq 0 \quad (26b)$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} \leq 0 \quad (26c)$$

Variable range are: $0.005 \leq x_1 \leq 2.00$; $0.25 \leq x_2 \leq 1.30$; $2.00 \leq x_3 \leq 15.0$.

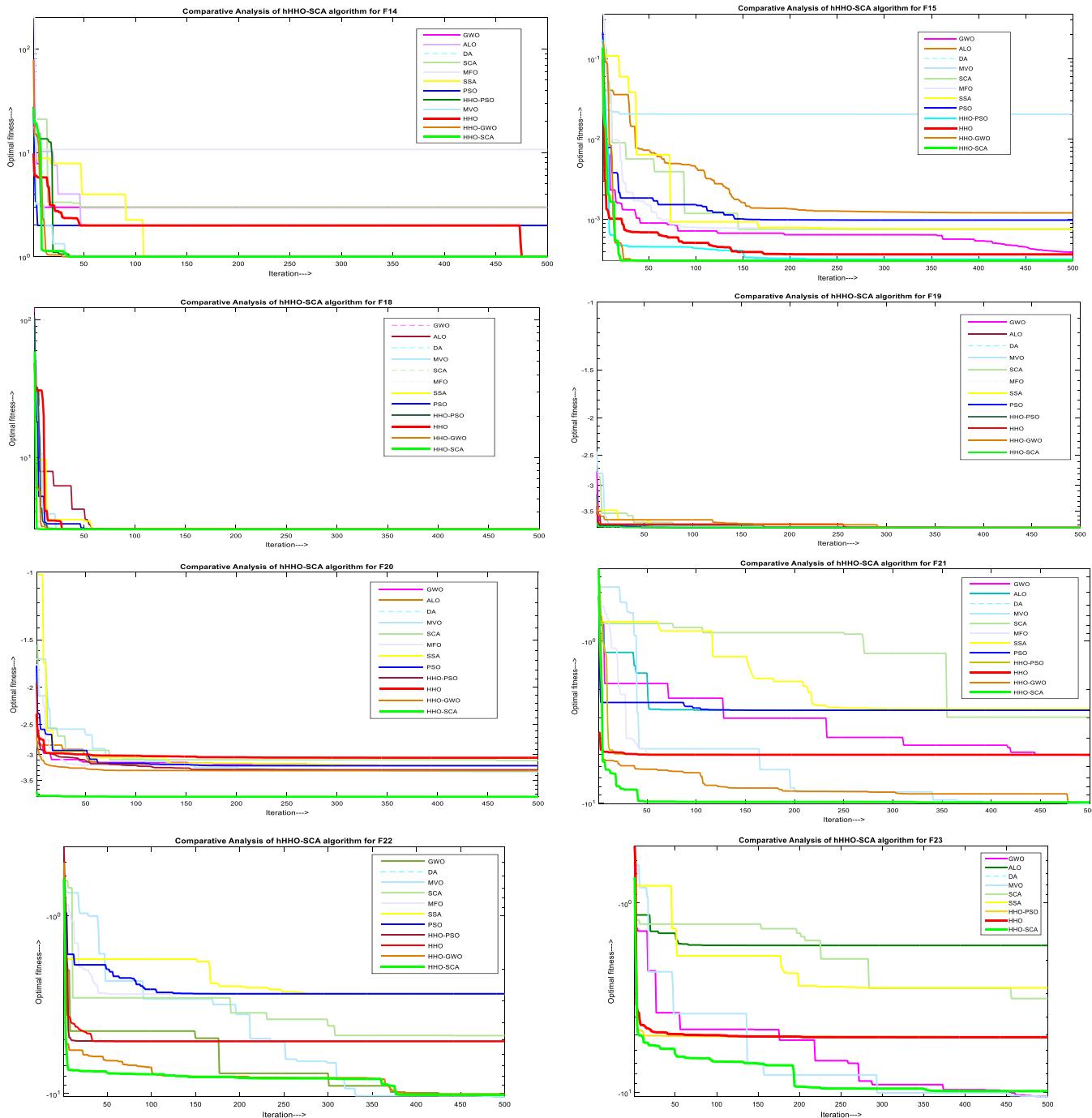
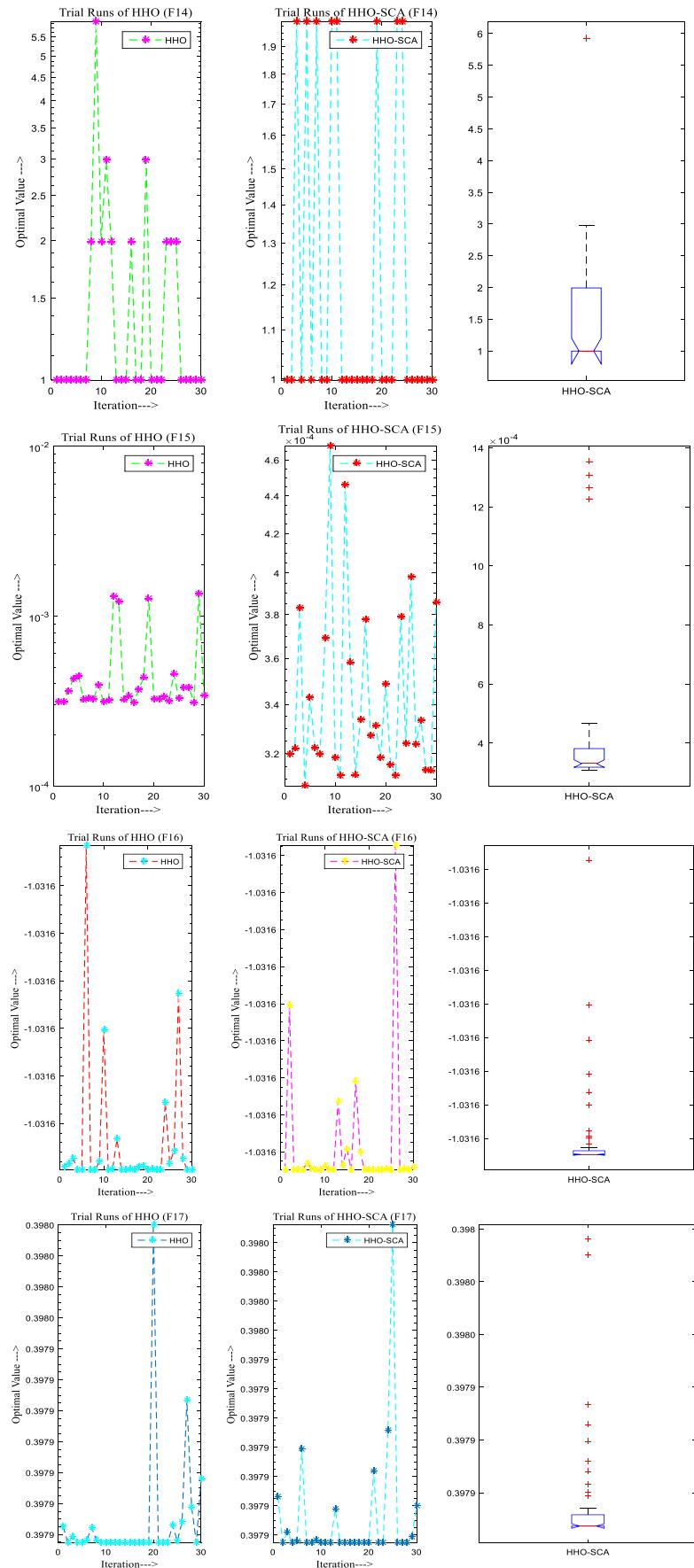
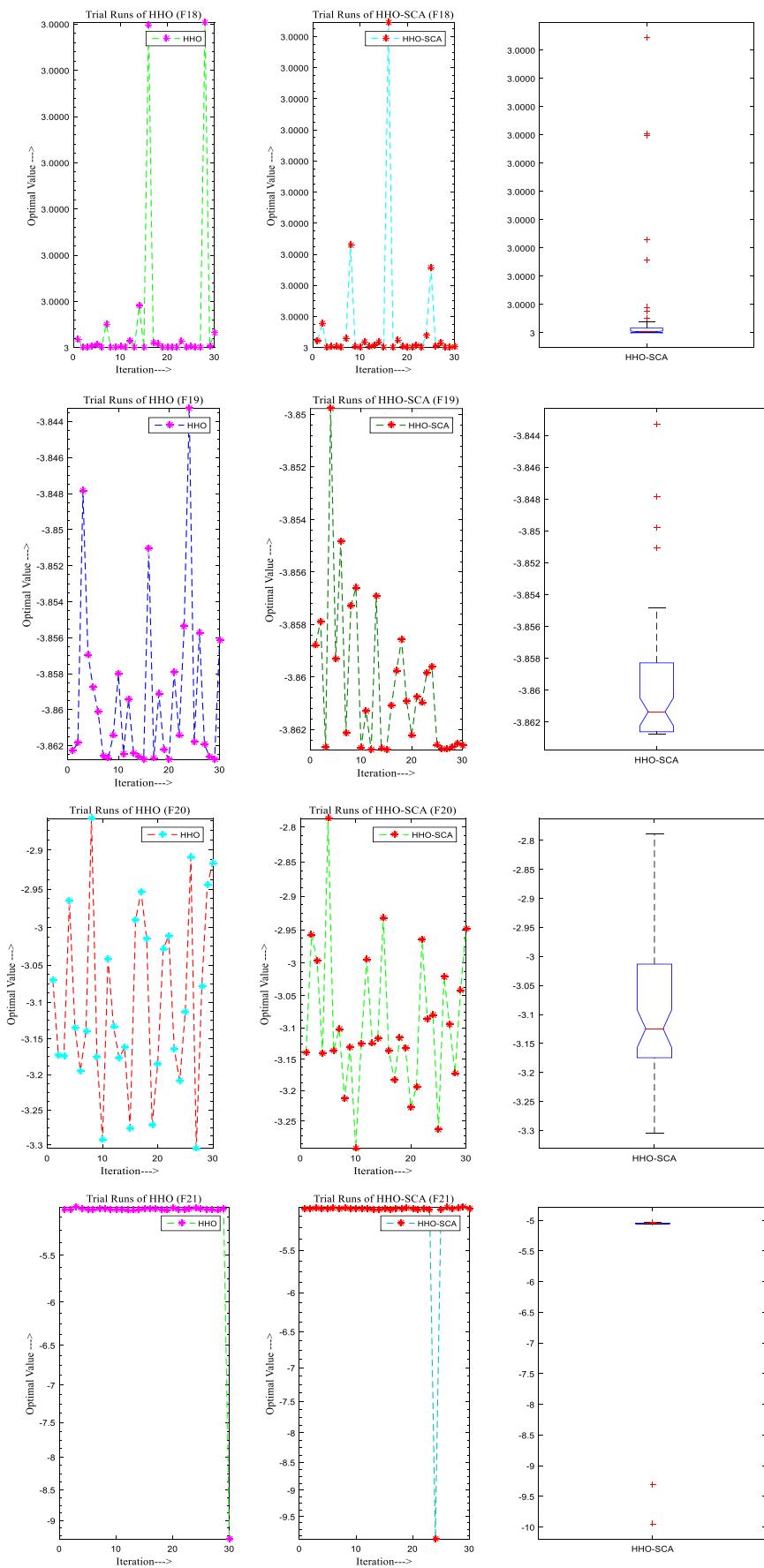


Fig. 5. Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for Fixed Dimension Benchmark functions.

Table 11
Comparison of CEC 2017 multimodal benchmark functions.

CEC2017 benchmark functions	hHHO-SCA	jSO [121]	DES [122]	TLBO-FL [123]	PPSO [124]
CEC2017(F1)	0	0.00E+00	0	3.50E+03	7.48E+02
CEC2017(F2)	8.9401	8.94E+00	5.88E-02	8.50E+16	5.32E+01
CEC2017(F3)	0.000002389	2.3912E-06	0	3.00E+03	1.13E+00
CEC2017(F4)	189.6287	1.90E+02	5.69E+01	9.00E+01	4.39E+01
CEC2017(F5)	43.9065	4.39E+01	4.64E+00	4.00E+01	1.12E+02
CEC2017(F6)	0.000202421	2.0244E-04	3.34E-07	4.90E-01	2.03E+01
CEC2017(F7)	144.897	1.45E+02	3.57E+01	1.40E+02	1.35E+02
CEC2017(F8)	42.151653	4.22E+01	4.55E+00	3.70E+01	8.10E+01
CEC2017(F9)	0.045903546	4.5904E-02	0	3.40E+01	1.36E+03
CEC2017(F10)	9704.36758	9.70E+03	1.39E+02	6.70E+03	3.13E+03

**Fig. 6.** Trial solutions for Fixed dimension benchmark functions.

**Fig. 6. (continued).**

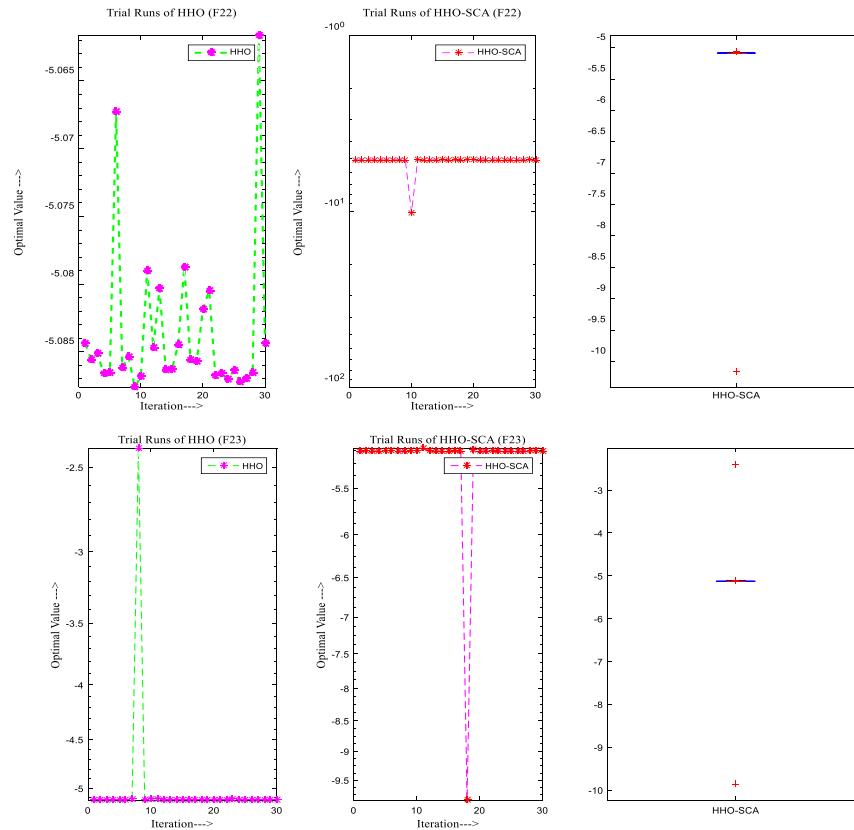


Fig. 6. (continued).

Table 12
Comparison of CEC 2017 hybrid benchmark functions.

CEC2017 hybrid benchmark functions	hHHO-SCA		jSO [121]	DES [122]	TLBO-FL [123]	PPSO [124]
	Mean value	p-Value				
CEC2017(F11)	104.1954	0.0234	1.04E+02	2.73E+01	8.20E+01	8.43E+01
CEC2017(F12)	17033	0.1746	1.70E+04	1.21E+03	5.70E+04	2.77E+04
CEC2017(F13)	139.57953	0.3258	1.40E+02	4.87E+01	2.00E+04	3.21E+03
CEC2017(F14)	63.84568	0.477	6.39E+01	2.66E+01	7.10E+03	2.32E+03
CEC2017(F15)	164.67964	0.4282	1.65E+02	3.24E+01	2.20E+04	2.13E+03
CEC2017(F16)	1881.6784	0.3794	1.88E+03	7.64E+01	4.90E+02	8.46E+02
CEC2017(F17)	1273.1457	0.4306	1.27E+03	5.54E+01	1.40E+02	3.31E+02
CEC2017(F18)	156.92753	0.0818	1.57E+02	3.51E+01	3.70E+05	6.99E+04
CEC2017(F19)	106.34753	0.233	1.06E+02	1.64E+01	1.10E+04	1.71E+03
CEC2017(F20)	1380.9854	0.3842	1.38E+03	7.06E+01	2.20E+02	3.48E+02

Table 13
Comparison of CEC 2017 composite benchmark functions.

CEC2017 composite benchmark functions	hHHO-SCA	jSO [121]	DES [122]	TLBO-FL [123]	PPSO [124]
CEC2017(F21)	263.5574	2.64E+02	2.07E+02	2.30E+02	3.05E+02
CEC2017(F22)	10661.8734	1.07E+04	1.00E+02	1.00E+02	1.00E+02
CEC2017(F23)	571.18453	5.71E+02	3.50E+02	4.00E+02	6.81E+02
CEC2017(F24)	902.60964	9.03E+02	4.18E+02	4.70E+02	7.39E+02
CEC2017(F25)	761.62547	7.62E+02	3.87E+02	4.00E+02	3.85E+02
CEC2017(F26)	3281.0964	3.28E+03	5.74E+02	1.40E+03	2.04E+03
CEC2017(F27)	586.458356	5.86E+02	5.10E+02	5.30E+02	7.08E+02
CEC2017(F28)	523.664348	5.24E+02	3.18E+02	4.30E+02	3.27E+02
CEC2017(F29)	1237.89454	1.24E+03	4.44E+02	6.20E+02	7.80E+02
CEC2017(F30)	2317.0746	2.32E+03	2.16E+03	2.60E+04	3.32E+03

7.4. Welded beam engineering design problem

The Welded Beam Design Problem is another most important engineering design problem shown in Fig. 10, which is taken into consideration [47,127]. The main objective of this types of design problem is to decrease or minimize the cost of the fabrication of

the welded beam which consists of four types of variables, such as, bar thickness (b), bar length including attached part (l), weld thickness (h) and the bar height (h). This problem is subjected to four types of constraints including Buckling constraints of bar (Pc), Side constraints, End deflection of beam (d), Bending stress of the beam (h) and stress of shear (s). The mathematical

Table 14

Comparison of CEC 2017 composite benchmark functions for parameters variations and computational time.

CEC2017 composite benchmark functions	Effect of variations of parameters of hHHO-SCA		Computational time (in seconds)		
	hHHO-SCA ($\alpha = 0.93$)	hHHO-SCA ($\alpha = 0.88$)	hHHO-SCA	TLBO-FL [123]	PSO-GWO [125]
CEC2017(F11)	263.5574	263.560	1.23E+02	1.42E+02	1.57E+02
CEC2017(F12)	10661.8734	10662.000	1.12E+02	1.31E+02	1.45E+02
CEC2017(F13)	571.1845	571.190	1.24E+01	3.10E+01	4.54E+01
CEC2017(F14)	902.6096	902.610	1.36E+02	1.55E+02	1.69E+02
CEC2017(F15)	761.6255	761.630	1.69E+02	1.87E+02	2.02E+02
CEC2017(F16)	3281.0964	3281.100	1.88E+02	2.07E+02	2.21E+02
CEC2017(F17)	586.4584	586.460	1.95E+02	2.14E+02	2.28E+02
CEC2017(F18)	523.6643	523.670	1.85E+02	2.04E+02	2.18E+02
CEC2017(F19)	1237.8945	1237.900	1.75E+03	1.77E+03	1.78E+03
CEC2017(F20)	2317.0746	2317.100	1.74E+02	1.93E+02	2.07E+02

Table 15

Comparison of CEC 2018 benchmark function for 05, 10 and 15 objectives problems.

CEC 2018 benchmark functions	No. of objectives = 05		No. of objectives = 10		No. of objectives = 15	
	hHHO-SCA	NSGA-III _{TS} -NL [126]	hHHO-SCA	NSGA-III _{TS} -NL [126]	hHHO-SCA	NSGA-III _{TS} -NL [126]
CEC2018 (F01)	9.73E-04	1.00E-03	0.0011078	1.11E-03	0.0014780	1.50E-03
CEC2018 (F02)	8.72E-04	8.72E-04	0.0018785	1.88E-03	0.0027376	2.74E-03
CEC2018 (F03)	1.63E-04	1.63E-04	0.0017698	1.77E-03	0.0020386	2.04E-03
CEC2018 (F04)	2.14E-04	2.14E-04	0.0001768	1.77E-04	0.0004609	4.61E-04
CEC2018 (F05)	1.56E-04	1.56E-04	0.0003370	3.37E-04	0.0006180	6.18E-04
CEC2018 (F06)	1.30E-04	1.21E-04	0.0026696	2.67E-03	0.0037696	3.77E-03
CEC2018 (F07)	1.43E-03	1.43E-03	0.0023589	2.36E-03	0.0023780	2.38E-03
CEC2018 (F08)	4.87E-04	4.86E-04	0.0006467	6.50E-04	0.0015287	1.53E-03
CEC2018 (F09)	2.20E-04	2.11E-04	0.0014178	1.42E-03	0.0013185	1.32E-03
CEC2018 (F10)	2.24E-03	2.24E-03	0.0020657	2.07E-03	0.0027754	2.78E-03
CEC2018 (F11)	3.62E-04	3.62E-04	0.0007388	7.40E-04	0.0013075	1.31E-03
CEC2018 (F12)	4.47E-04	4.47E-04	0.0008578	8.58E-04	0.0020796	2.08E-03
CEC2018 (F13)	4.19E-04	4.19E-04	0.0005488	5.49E-04	0.0012388	1.24E-03
CEC2018 (F14)	3.49E-02	3.49E-02	0.0980867	9.81E-02	0.0680864	6.81E-02
CEC2018 (F15)	1.67E-03	1.67E-03	0.0024560	2.50E-03	0.0080765	8.10E-03

Table 16

Abbreviation and name of engineering problems.

Engineering problem abbreviation	Problem name	Objective function
ENGG 1	Three-bar truss problem	@SPECIAL1
ENGG 2	Pressure Vessel	@SPECIAL2
ENGG 3	Spring Design	@SPECIAL3
ENGG 4	Welded Beam	@SPECIAL4
ENGG 5	Cantilever Beam Design	@SPECIAL5
ENGG 6	Gear Train	@SPECIAL6
ENGG 7	Speed Reducer problem	@SPECIAL7
ENGG 8	Belleville Spring	@SPECIAL8
ENGG 9	Rolling Element Bearing	@SPECIAL9
ENGG 10	Multiple Disk Clutch Brake (Discrete variables)	@SPECIAL10
ENGG 11	I beam design	@SPECIAL11

Table 17Results of engineering design problems using **hHHO-SCA** algorithm.

ENGG PROBLEM	Mean fitness	Std	Best fitness	Worst fitness	Median	p-Value
ENGG 1	264.1683669	0.32426697	263.8958665	264.954366	264.035028	1.7344E-06
ENGG 2	3696.691485	659.2181603	3029.873076	5053.181732	3340.706919	1.7344E-06
ENGG 3	6960.69725	381.1146136	6393.092794	7589.688312	6846.846771	1.7344E-06
ENGG 4	0.014325517	0.001551857	0.012822904	0.017773563	0.013568707	1.7344E-06
ENGG 5	2.093154146	0.332409903	1.779032249	3.146845758	1.983147741	1.7344E-06
ENGG 6	-71372.54227	15842.67551	-84491.62424	-42057.01431	-80828.62664	1.7344E-06
ENGG 7	0.449799283	0.055402317	0.389653842	0.571013688	0.43931432	1.7344E-06
ENGG 8	0	0	0	0	0	1
ENGG 9	2.29698E+22	2.6716E+22	1.98170396	5.30072E+22	2.090597201	1.56097E-06
ENGG 10	1.307608751	0.003105711	1.30412236	1.316613773	1.307056295	1.7344E-06
ENGG 11	0.006625958	1.31572E-16	0.006625958	0.006625958	0.006625958	1.57099E-06

description of the above mentioned engineering design problem is discussed in the following equations (27) through (29f). The comparison results are shown in Table 22.

Consider

Minimize

$$f(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4 (14.0 + x_2) \quad (28)$$

Subject to

$$g_1(\vec{x}) = \tau(\vec{x}) - \tau_{\max} \leq 0, \quad (28a)$$

$$\vec{x} = [x_1 x_2 x_3 x_4] = [hlbt] \quad (27)$$

Table 18

Simulation time for engineering design problems.

Engineering problem	Best time	Mean time	Worst time
ENGG 1	2.390625	2.452604167	2.5625
ENGG 2	3.34375	3.4625	3.53125
ENGG 3	2.390625	2.469791667	2.625
ENGG 4	2.546875	2.618229167	2.6875
ENGG 5	2.90625	3.106770833	3.5
ENGG 6	4.265625	4.4375	5.328125
ENGG 7	3.40625	3.477604167	3.71875
ENGG 8	1.796875	1.9578125	2.1875
ENGG 9	3.53125	3.886979167	4.21875
ENGG 10	2.125	2.402604167	2.640625
ENGG 11	2.5625	2.748958333	3.03125

Table 19

Comparison of three-bar truss problem.

Algorithm	hHHO-SCA	CS [128]	Ray and Sain [129]	Tsa [129]
Optimal values for variables	$x_1 = \mathbf{0.788498}$ $x_2 = \mathbf{0.40875}$	0.789 0.409	0.795 0.395	0.788 0.408
Optimal weight	263.8958665	263.972	264.3	263.68

Table 20

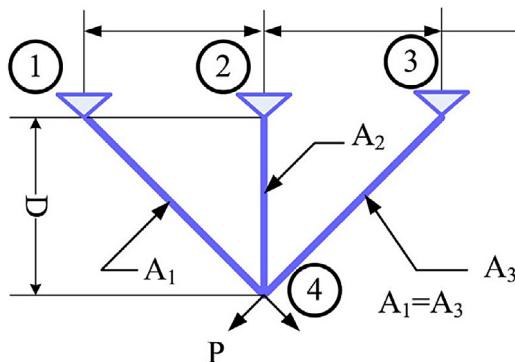
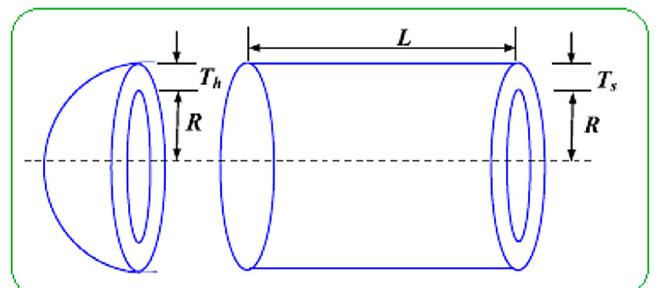
Comparison of pressure vessel problem.

Algorithm	hHHO-SCA	GWO [130]	GSA [35]	PSO [131]	GA [132]	GA (Coello and Montes) [132]	GA (Deb and Gene) [133]	ES (Montes and Coello)	DE [134]	ACO [9]	Lagrangian Multiplier [135]	Branch-bound [13]
Optimum Value	$T_s = \mathbf{0.945909}$ $T_h = \mathbf{0.447138}$ $R = \mathbf{46.8513}$ $L = \mathbf{125.4684}$	0.8125 0.4345 42.0892 176.7587	1.125 0.625 55.9887 84.4542	0.8125 0.4375 42.0913 176.7465	0.8125 0.4345 40.3239 200	0.9375 0.5 42.0974 176.6541	0.8125 0.4375 48.329 112.679	0.8125 0.4375 42.0981 176.641	0.8125 0.4375 42.0984 176.6377	0.8125 0.4375 42.1036 176.5727	1.125 0.625 58.291 43.69	1.125 0.625 47.7 117.701
Optimum cost	6393.092794	6051.564	8538.84	6061.078	6288.745	6059.946	6410.381	6059.75	6059.734	6059.089	7198.043	8129.1

Table 21

Comparison of Tension/compression spring engineering design problem.

Algorithm	hHHO-SCA	GWO	GSA	PSO (Ha and Wang)	ES (Coello and Montes)	GA (Coello)	HS (Mahdavi et al.)	DE (Huang et al.)	Mathematical optimization	Constraint correction
AGASDM RW	0.054693	0.0516	0.0503	0.0517	0.052	0.0515	0.0512	0.0516	0.0534	0.05
Optimum D Variables	0.433378	0.3567	0.3237	0.3576	0.364	0.3517	0.3499	0.3547	0.3992	0.3159
N	7.891402	11.2889	13.5254	11.2445	10.8905	11.6322	12.0764	11.4108	9.1854	14.25
Optimum weight	0.012822904	0.01267	0.0127	0.01267	0.01268	0.0127	0.01267	0.01267	0.01273	0.01283

**Fig. 7.** Three-Bar Truss engineering design problem.**Fig. 8.** Pressure Vessel engineering design problem.

$$g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{\max} \leq 0 \quad (28b)$$

$$g_3(\vec{x}) = \delta(\vec{x}) - \delta_{\max} \leq 0 \quad (28c)$$

$$g_4(\vec{x}) = x_1 - x_4 \leq 0 \quad (28d)$$

$$g_5(\vec{x}) = P - P_c(\vec{x}) \leq 0 \quad (28e)$$

$$g_6(\vec{x}) = 0.125 - x_1 \leq 0 \quad (28f)$$

$$g_7(\vec{x}) = 1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0 \quad (28g)$$

Variable range $0.1 \leq x_1 \leq 2$; $0.1 \leq x_2 \leq 10$; $0.1 \leq x_3 \leq 10$; $0.1 \leq x_4 \leq 2$
where

$$\tau(\vec{x}) = \sqrt{(\tau')^2 + 2\tau'\tau'' \frac{x_2}{2R} + (\tau'')^2}, \quad (29a)$$

$$\tau' = \frac{P}{\sqrt{2x_1x_2}}, \quad \tau'' = \frac{MR}{J}, \quad M = P \left(L + \frac{x_2}{2} \right), \quad (29b)$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2} \right)^2}, \quad (29c)$$

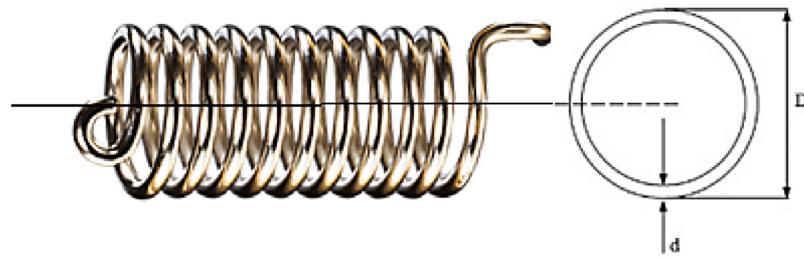


Fig. 9. The Tension/Compression Spring engineering design problem.

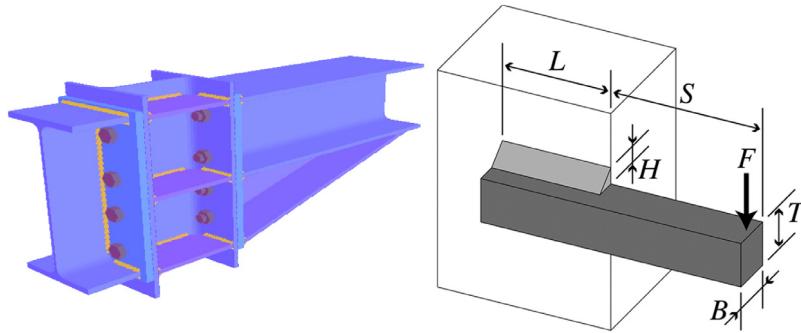


Fig. 10. Welded Beam engineering design problem.

$$J = 2 \left\{ \sqrt{2}x_1x_2 \left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2} \right)^2 \right] \right\}, \quad (29d)$$

$$\sigma(\vec{x}) = \frac{6PL}{x_4x_3^2}, \delta(\vec{x}) = \frac{6PL^3}{Ex_2^2x_4}, \quad (29e)$$

$$P_c(\vec{x}) = \frac{4.013E \frac{\sqrt{x_3^2x_4^6}}{36}}{L^2} \left(1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}} \right), \quad (29f)$$

$$P = 6000lb, L = 14 \text{ in}, \delta_{\max} = 0.25 \text{ in}, E = 30 \times 10^6 \text{ psi},$$

$$G = 12 \times 10^6 \text{ psi}, \tau_{\max} = 13600 \text{ psi}, \sigma_{\max} = 3000 \text{ psi}$$

7.5. Cantilever beam engineering design problem

Another engineering optimization problem is a part of civil engineering which is named as Cantilever Beam Design problem shown in Fig. 11. This problem consists of five types of hollow element including its square-shaped cross section [127]. The main objective of this type of optimization is to decrease or reduce the weight of the beam shown in equation number (29). This kind of engineering design problem each elements can defined by one variable and the structure of the overall design consists of five types of structural parameters and where the beam thickness is taken constant. In this suggested design, the load due to vertical end is applied to the free end position of the beam (6th node) and the side of the right portion of the beam (1st node) which is supported rigidly. The comparison results are shown in Table 23. In the final design for optimal solution this types of cantilever beam design problem the displacement of vertical constraint must take into consideration which should not violate for design of final optimal solution shown on equation no (30) to Eq. (31). The mathematical formula is given below:

The problem formulation is as follows:

Consider $\vec{x} = [x_1 x_2 x_3 x_4 x_5]$,

$$\text{Minimize } f(\vec{x}) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5), \quad (30)$$

$$\text{Subject to } g(\vec{x}) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \leq 1 \quad (31)$$

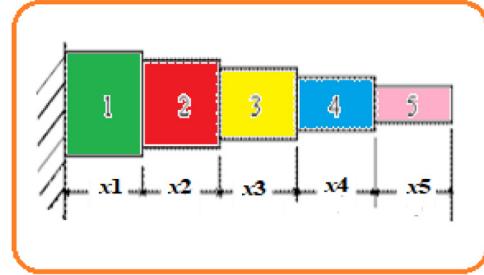


Fig. 11. Cantilever Beam engineering design problem.

$$\text{Variable range } 0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100$$

7.6. Gear train engineering design problem

Another optimization types of engineering design optimization problem is named as Gear Train Design problem, which have four types of parameters shown in Fig. 12 [47]. The actual objective of this type of design problem is that to minimize the scalar value and the teeth ratio of the gear. Thus the decision variable is included by the number of teeth on each of the gear. The comparison results are shown in Table 24. The mathematical model is given below:

Let, considering:

$$\vec{g} = [g_1 g_2 g_3 g_4] = [M_A M_B M_C M_D] \quad (32)$$

$$\text{Minimizing; } f(\vec{g}) = \left(\frac{1}{6.931} - \frac{g_3 g_4}{g_1 g_4} \right)^2 \quad (33)$$

$$\text{Subjected to; } 12 \leq g_1, g_2, g_3, g_4 \leq 60$$

Table 22

Comparison of welded beam problem.

Algorithm	hHHO-SCA	GWO	GSA	GA (Coello)	GA (Deb)	GA (Deb)	HS (Lee and Geem)	Random	Simplex	David	APPROX
Optimum Variables	h 0.190086	0.2057	0.1821	N/A	N/A	0.2489	0.2442	0.4575	0.2792	0.2434	0.2444
	l 3.696496	3.4784	3.857	N/A	N/A	6.173	6.2231	4.7313	5.6256	6.2552	6.2189
	t 9.386343	9.0368	10	N/A	N/A	8.1789	8.2915	5.0853	7.7512	8.2915	8.2915
	b 0.204157	0.2058	0.2024	N/A	N/A	0.2533	0.2443	0.66	0.2796	0.2444	0.2444
Optimal cost	1.779032249	1.7262	1.88	1.8245	2.38	2.4331	2.3807	4.1185	2.5307	2.3841	2.3815

Table 23

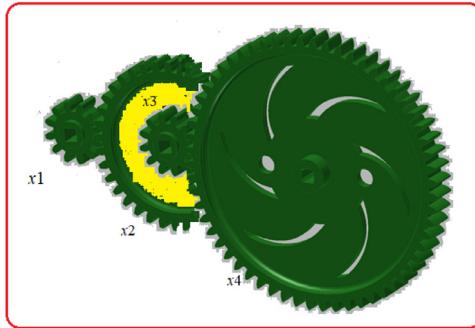
Comparison of cantilever beam engineering design problem.

Algorithm	hHHO-SCA	ALO [136]	SOS [62]	CS [128]	MMA [137]	GCA_I [137]	GCA_II [137]
Optimal values for variables	x1 5.937725	6.0181	6.0188	6.0089	6.01	6.01	6.01
	x2 4.85041	5.3114	5.3034	5.3049	5.3	5.304	5.3
	x3 4.622404	4.4884	4.4959	4.5023	4.49	4.49	4.49
	x4 3.45347	3.4975	3.499	3.5077	3.49	3.498	3.49
	x5 2.089114	2.1583	2.1556	2.1504	2.15	2.15	2.15
Optimum weight	1.30412236	1.33995	1.33996	1.33999	1.34	1.34	1.34

Table 24

Comparison of gear train engineering design problem.

Algorithm	hHHO-SCA	GeneAS [133]	Kannan and Kramer [133]	Sandgren [133]
Optimal values for variables	x1 58.22844	50	41	60
	x2 57.73199	33	33	45
	x3 40.41797	14	15	22
	x4 12	17	13	18
Optimum fitness	0	0.144242	0.144124	0.146667

**Fig. 12.** Gear Train engineering design problem.

7.7. Speed reducer engineering design problem

Another most important engineering design problem is speed reducer, shown in Fig. 13 [47]. Speed reducer consists of the width of face s_1 , teeth module s_2 , number of the pinion teeth s_3 , in between the bearings of the first shaft length s_4 , in between the bearings of the second shaft length s_5 , the first shaft diameter s_6 and second shaft diameter s_7 ; all types of variables are continuous in nature except one variable s_3 , which is an integer. The main objective is to decrease or reduce the weight of the speed reducer which is subjected to some constraints which is depended on surface stress and bending stress of the teeth of the gear, shaft's transverse movement or deflection and stress of it on the shaft. The comparison results are shown in Table 25. The mathematical model is given below:

Minimizing;

$$\begin{aligned} f(\vec{s}) = & 0.7854s_1s_2(3.3333s_3^2 + 14.9334s_3 - 43.0934) \\ & - 1.508s_1(s_6^2 + s_7^2) + 7.4777(s_6^3 + s_7^3) \\ & + 0.7854(s_4s_6^2 + s_5s_7^2) \end{aligned}$$

$$\text{Subjected to; } g_1(\vec{s}) = \frac{27}{s_1s_2^2s_3} - 1 \leq 0 \quad (34)$$

$$g_2(\vec{s}) = \frac{397.5}{s_1s_2^2s_3^2} - 1 \leq 0 \quad (34a)$$

$$g_3(\vec{s}) = \frac{1.93s_4^3}{s_2s_3s_6^4} - 1 \leq 0 \quad (34b)$$

$$g_4(\vec{s}) = \frac{1.93s_5^3}{s_2s_3s_7^4} - 1 \leq 0 \quad (34c)$$

$$g_5(\vec{s}) = \frac{1}{110s_6^3}\sqrt{\left(\frac{745.0s_4}{s_2s_3}\right)^2 + 16.9 \times 10^6} - 1 \leq 0 \quad (34d)$$

$$g_6(\vec{s}) = \frac{1}{85s_7^3}\sqrt{\left(\frac{745.0s_5}{s_2s_3}\right)^2 + 157.5 \times 10^6} - 1 \leq 0 \quad (34e)$$

$$g_7(\vec{s}) = \frac{s_2s_3}{40} - 1 \leq 0 \quad (34f)$$

$$g_8(\vec{s}) = \frac{5s_2}{s_1} - 1 \leq 0 \quad (34g)$$

$$g_9(\vec{s}) = \frac{s_1}{12s_2} - 1 \leq 0 \quad (34h)$$

$$g_{10}(\vec{s}) = \frac{1.5s_6 + 1.9}{12s_2} - 1 \leq 0 \quad (34i)$$

$$g_{11}(\vec{s}) = \frac{1.1s_7 + 1.9}{s_5} - 1 \leq 0 \quad (34j)$$

where $2.6 \leq s_1 \leq 3.6$, $0.7 \leq s_2 \leq 0.8$, $17 \leq s_3 \leq 28$, $7.3 \leq s_4 \leq 8.3$, $7.8 \leq s_5 \leq 8.3$, $2.9 \leq s_6 \leq 3.9$ and $5 \leq s_7 \leq 5.5$.

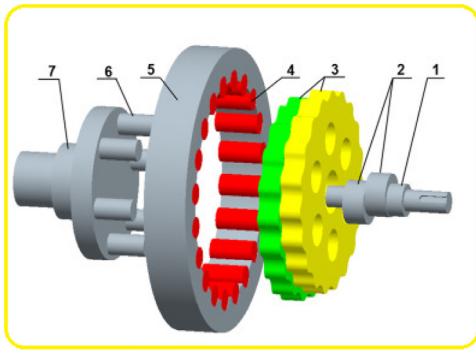
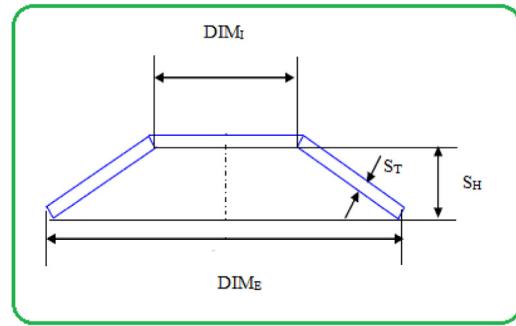
7.8. Belleville spring engineering design problem

Another types of engineering problem is Belleville spring design problem shown in Fig. 14 [47]. It is a minimization of the problem through which one existing parameter in the constraints is accordingly to be selected to the designed variable ratios.

Table 25

Comparison of speed reducer engineering design problem.

Algorithm	hHO-SCA	HEAA	MDE	PSO-DE	MBA
Optimal values for variables	x1 3.506119	3.500022	3.50001	3.50	3.5
	x2 0.7	0.70000039	0.7	0.7	0.7
	x3 17	17.000012	17	17	17
	x4 7.3	7.300427	7.300156	7.3	7.300033
	x5 7.99141	7.715377	7.800027	7.8	7.715772
	x6 3.452569	3.35023	3.350221	3.350214	3.350218
	x7 5.286749	5.286663	5.286685	5.2866832	5.286654
Optimum fitness	3029.873076	2994.49911	2996.35669	2996.34817	2994.48245

**Fig. 13.** Speed reducer engineering design problem.**Fig. 14.** Belleville Spring Engineering Design.

The main objective function is to design Belleville spring which having the minimum weight and it should satisfy number of constraints. This type of designed problem has four types of designed variables, such as, diameter of the internal part of the Belleville spring (DIM_I), diameter of the external part of the Belleville spring (DIM_E), Spring height (S_H) and The thickness of the Belleville spring (S_T). The comparison results are shown in **Table 26**. The subjected constraints are concerned the deflection, compressive type of stresses, deflection height, diameter of outer and inner portion and the slope. The mathematical expressions are given below:

$$\text{Minimizing: } f(x) = 0.07075\pi(DIM_E^2 - DIM_I^2)t \quad (35)$$

Subjected to;

$$b_1(x) = G - \frac{4P\lambda_{\max}}{(1 - \delta^2)\alpha DIM_E} \left[\delta(S_H - \frac{\lambda_{\max}}{2}) + \mu t \right] \geq 0 \quad (36)$$

$$b_2(x) = \left(\frac{4P\lambda_{\max}}{(1 - \delta^2)\alpha DIM_E} \left[(S_H - \frac{\lambda}{2})(S_H - \lambda)t + t^3 \right] \right)_{\lambda_{\max}} - P_{\max} \geq 0 \quad (36a)$$

$$b_3(x) = \lambda_1 - \lambda_{\max} \geq 0 \quad (36b)$$

$$b_4(x) = H - S_H - t \geq 0 \quad (36c)$$

$$b_5(x) = DIM_{MAX} - DIM_E \geq 0 \quad (36d)$$

$$b_6(x) = DIM_E - DIM_I \geq 0 \quad (36e)$$

$$b_7(x) = 0.3 - \frac{S_H}{DIM_E - DIM_I} \geq 0 \quad (36f)$$

where,

$$\alpha = \frac{6}{\pi \ln J} \left(\frac{J-1}{J} \right)^2$$

$$\delta = \frac{6}{\pi \ln J} \left(\frac{J-1}{\ln J} - 1 \right)$$

$$\mu = \frac{6}{\pi \ln J} \left(\frac{J-1}{2} \right)$$

$$P_{MAX} = 5400lb$$

$$P = 30e6 \text{ psi}, \lambda_{\max} = 0.2 \text{ in}, \delta = 0.3, G = 200 \text{ Kpsi},$$

$$H = 2 \text{ in}, DIM_{MAX} = 12.01 \text{ in}, J = \frac{DIM_E}{DIM_I}, \lambda_1 = f(a)a, a = \frac{S_H}{t}$$

7.9. Rolling element bearing engineering design problem

Rolling element bearing design problem is another most important part of multidisciplinary engineering design problem shown in **Fig. 15** [47]. The main objective of this type of engineering design problem is to increase the dynamic nature of load carrying capacity of the bearing of the rolling element as shown in the **Fig. 15**. This engineering design problem includes ten numbers of decision variable such as, diameter of the pitch (DIM_P), diameter of the ball (DIM_B), ball numbers (N), coefficient of raceway curvature for outer and inner part (R_I and R_O) etc. The five latter variables are only appeared in constraints and affect indirectly to the internal portion of the geometry. The design variable of number of the rolling balls (N) is discrete in nature and the remainder design variables are continuous in nature. In this types of design problem constraints are executed based on manufacturing and kinematic conditions.

The mathematical model is given below:

Maximizing;

$$C_D = f_C N^{2/3} DIM_B^{1.8} \text{ if } DIM \leq 25.4 \text{ mm} \quad (37a)$$

$$C_D = 3.647 f_C N^{2/3} DIM_B^{1.4} \text{ if } DIM \geq 25.4 \text{ mm} \quad (37b)$$

$$\text{Subjected to: } r_1(x) = \frac{\theta_0}{2 \sin^{-1} \left(\frac{DIM_B}{DIM_{MAX}} \right)} - N + 1 \geq 0 \quad (38)$$

$$r_2(x) = 2DIM_B - K_{DIM_{MIN}}(DIM - dim) \geq 0 \quad (38a)$$

$$r_3(x) = K_{DIM_{MAX}}(DIM - dim) \geq 0 \quad (38b)$$

Table 26

Comparison of Belleville spring engineering design problem.

Algorithm	hHHO-SCA	TLBO	MBA
Optimal values for variables	x1 11.98603 x2 10.0002 x3 0.204206 x4 0.2	12.01 10.03047 0.204143 0.2	12.01 10.030473 0.204143 0.2
Optimum fitness	1.98170396	0.198966	0.198965

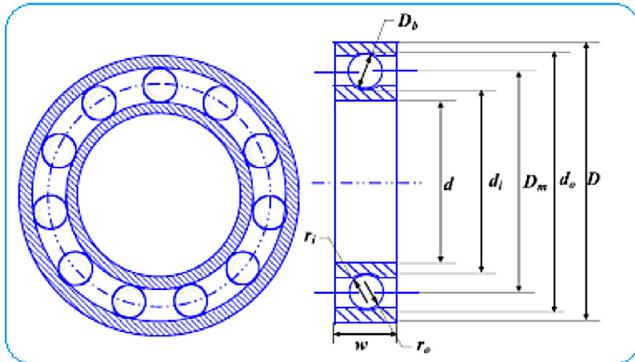


Fig. 15. Rolling element bearing engineering design problem.

$$r_4(x) = \beta B_W - DIM_B \leq 0 \quad (38c)$$

$$r_4(x) = DIM_{MAX} - 0.5(DIM + dim) \geq 0 \quad (38d)$$

$$r_5(x) = DIM_{MAX} - 0.5(DIM + dim) \geq 0 \quad (38e)$$

$$r_6(x) = (0.5 + re)(DIM + dim) \geq 0 \quad (38f)$$

$$r_7(x) = 0.5(DIM - DIM_{MAX} - DIM_B) - \alpha DIM_B \geq 0 \quad (38g)$$

$$r_8(x) = f_l \geq 0.515 \quad (38h)$$

$$r_9(x) = f_0 \geq 0.515 \quad (38i)$$

where,

$$f_c = 37.91 \left[1 + \left\{ 1.04 \left(\frac{1-\varepsilon}{1+\varepsilon} \right)^{1.72} \left(\frac{f_l(2f_0-1)}{f_0(2f_l-1)} \right)^{0.41} \right\}^{10/3} \right]^{-0.3}$$

$$\times \left[\frac{\varepsilon^{0.3}(1-\varepsilon)^{1.39}}{(1+\varepsilon)^{1/3}} \right] \left[\frac{2f_l}{2f_l-1} \right]^{0.41}$$

$$\theta_0 = 2\pi - 2\cos^{-1}$$

$$\times \left(\frac{[(DIM-dim)/2-3(t/4)]^2 + (DIM/2-t/4-DIM_B)^2 - \{dim/2+t/4\}^2}{2 \{ (DIM-dim)/2 - 3(t/4) \} \{ D/2 - t/4 - DIM_B \}} \right)$$

$$\varepsilon = \frac{DIM_B}{DIM_{MAX}}, f_l = \frac{R_l}{DIM_B}, f_0 = \frac{R_0}{DIM_B}, t = DIM - dim - 2DIM_B$$

$$DIM = 160, dim = 90, B_W = 30, R_l = R_0 = 11.033$$

$$0.5(DIM + dim) \leq DIM_{MAX} \leq 0.6(DIM + dim),$$

$$0.15(DIM - dim) \leq DIM_B \leq 0.45(DIM - dim), 4 \leq N \leq 50$$

$$0.515 \leq f_l \text{ and } f_0 \leq 0.6$$

$$0.4 \leq K_{DIM_{MIN}} \leq 0.5, 0.6 \leq K_{DIM_{MAX}} \leq 0.7, 0.3 \leq re \leq 0.1, 0.02 \leq re \leq 0.1, 0.6 \leq \beta \leq 0.85$$

7.10. Multidisc clutch brake engineering design problem

One of the most important engineering problem is Multidisc clutch brake design problem shown in Fig. 16 [138]. The main object of this type of optimization problem is to minimize or

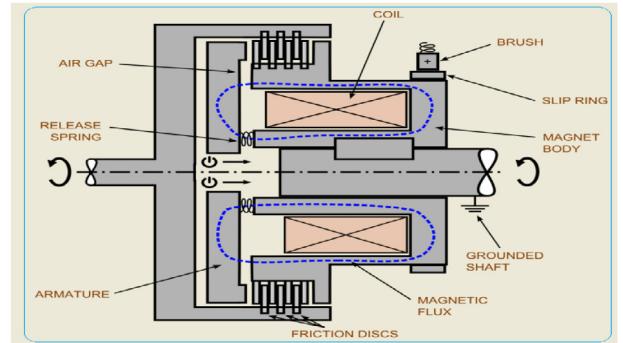


Fig. 16. Multidisc clutch break design.

decrease the weight and it consists of five numbers of discrete variables, such as, radius of the inner surface (R_{in}), radius of the outer surface (R_o), disc's thickness (Th), actuating type of force (F_{ac}) and number of the surface of friction (S_f).

The mathematical formulation for this engineering optimization problem is given below;

$$\text{Minimizing; } f(R_{in}, R_o, S_f, Th) = \pi Th \gamma (R_o^2 - R_{in}^2) (S_f + 1) \quad (39)$$

where,

$$R_{in} \in 60, 61, 62, \dots, 80; \quad R_o \in 90, 91, \dots, 110;$$

$$Th \in 1, 1.5, 2, 2.5, 3; \quad F_{ac} \in 600, 610, 620, 1000;$$

$$S_f \in 2, 3, 4, 5, 6, 7, 8, 9$$

$$\text{Subjected to; } m_1 = R_o - R_{in} - \Delta R \geq 0 \quad (40)$$

$$m_2 = L_{MAX} - (S_f + 1)(Th + \alpha) \geq 0 \quad (40a)$$

$$m_3 = PM_{MAX} - PM_\pi \geq 0 \quad (40b)$$

$$m_4 = PM_{MAX} Y_{MAX} + PM_\pi Y_{SR} \geq 0 \quad (40c)$$

$$m_5 = Y_{SR_{MAX}} - Y_{SR} \geq 0 \quad (40d)$$

$$m_6 = t_{MAX} - t \geq 0 \quad (40e)$$

$$m_7 = DC_h - DC_f \geq 0 \quad (40f)$$

$$m_8 = t \geq 0 \quad (40g)$$

where,

$$PM_\pi = \frac{F_{ac}}{\pi (R_o^2 - R_{in}^2)}$$

$$Y_{SR} = \frac{2\pi n (R_o^3 - R_{in}^3)}{90 (R_o^2 - R_{in}^2)}$$

$$t = \frac{i_x \pi n}{30 (DC_h + DC_f)}$$

For the active analysis of the effect of results of hHHO-SCA are estimated for trial runs 30 and statistical hypothesis test which is non-parametric in nature i.e. Wilcoxon rank sum test which

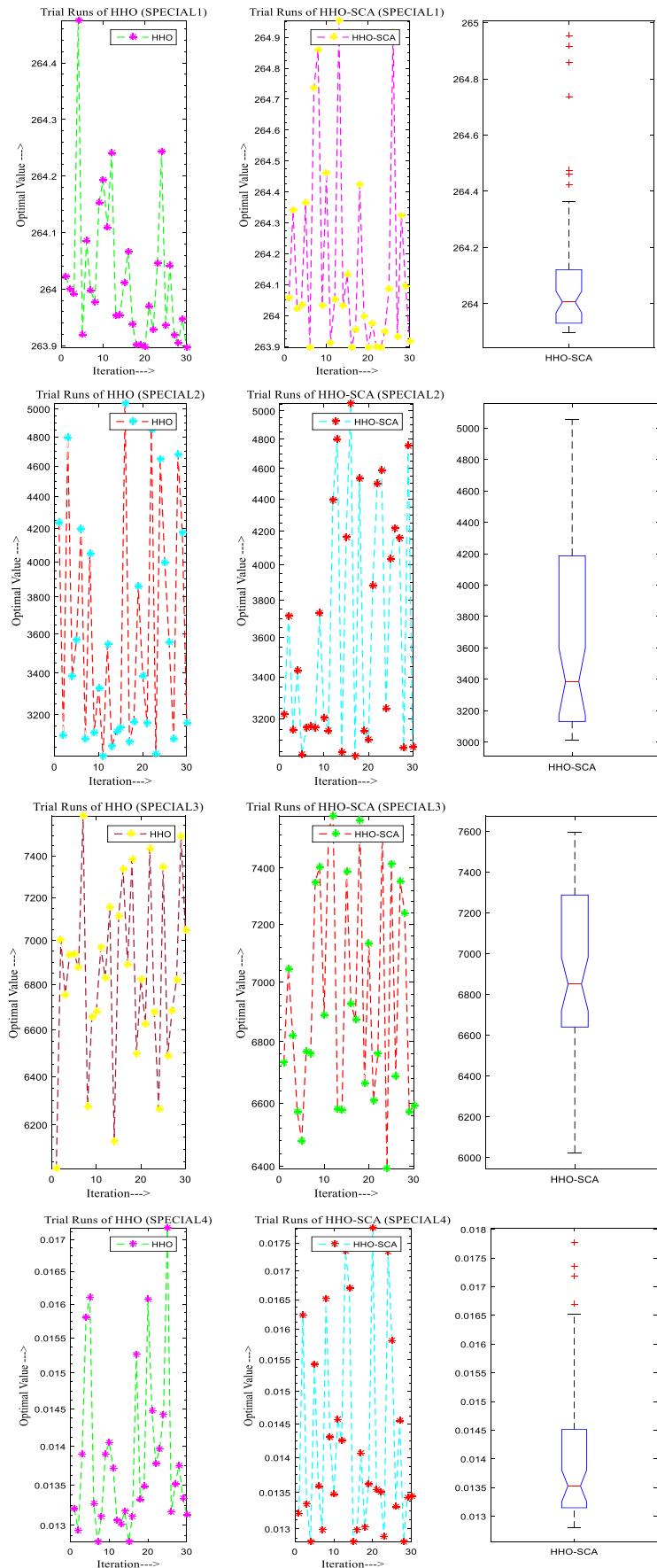
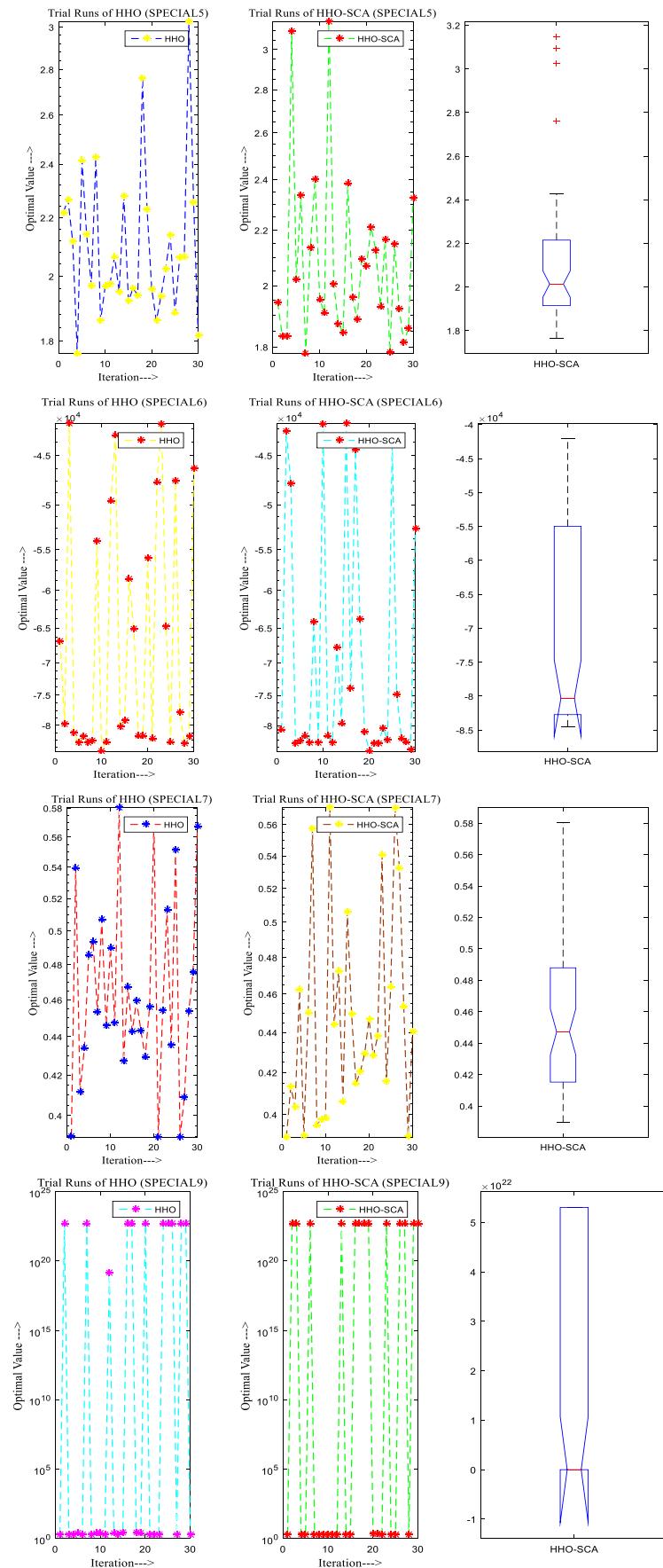


Fig. 17. Trial runs with box plot for multidisciplinary engineering design problems.

**Fig. 17. (continued).**

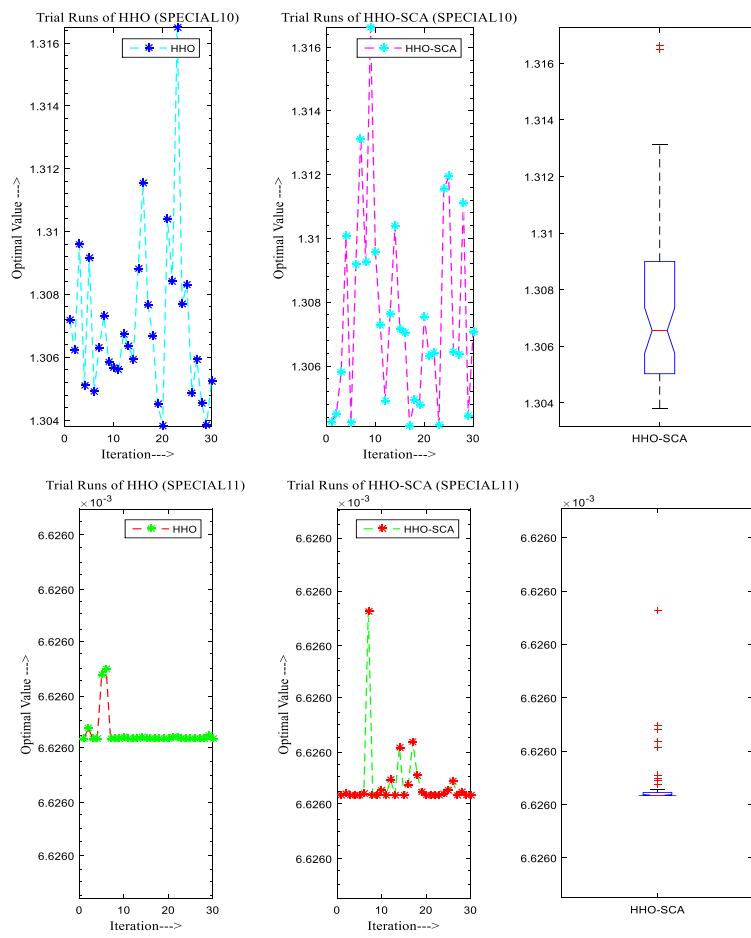


Fig. 17. (continued).

Table 27
Comparison of multiple disc clutch brake problem.

Algorithm	hHHO-SCA	NSGA-II	TLBO	AMDE
Optimal values for variables	x1 70	70	70	70.00
	x2 90	90	90	90
	x3 2.312785	3	3	3
	x4 1.5	1.5	1	1
	x5 1000	1000	810	810
Optimum fitness	0.389653842	0.4704	0.3136566	0.3136566

has been applied. The main consequence of that test is to observe the samples' distribution, that means weather the two types of dependent samples are selected from the number of populations which having the same number of distribution or not. The outputs of the multi-disciplinary engineering design problems are verified with the respect of the best value, worst value, *p*-value and standard deviation, also shown the variations of best time and worst time. The comparison of the results are also shown in the above Table 27. The results of the multi-disciplinary engineering design problems are the strongly indication of the qualities of the hHHO-SCA algorithm to solve the problems with the unknown types of search space. The output of the proposed optimization algorithm also shows the effectiveness of the problems including discrete and continuous types. The trial runs with box plot of the engineering optimization problems are shown in Fig. 17, which shows the lead of hHHO-SCA over HHO.

8. Conclusion

In the proposed research, the phase of exploitation of existing Harris Hawks Optimization has been upgraded successfully using Sine–Cosine algorithm and the developed hybrid hHHO-SCA has been successfully tested for continuous, discrete, highly constrained, nonlinear and non-convex engineering design and optimization problems include several benchmark function and engineering design problems. Experimentally, it has been found that the minimum weight for three-bar truss engineering design problem is 263.8958665, optimal cost for Pressure Vessel Design problem is 6393.092794, optimal weight for Tension/Compression Spring Design optimization Problem is 0.012822904, fabrication cost of welded beam of welded beam design problem is 1.779032249. Also, the experimental results shows that the reduced weight of Cantilever Bream Design problem and multidisc clutch brake design problem is 1.30412236 and 0.389653842

respectively. From the comparative analysis, it has been experimentally observed that the outcome of the suggested hybrid hHHO-SCA algorithm are superior than the existing Harris Hawks Optimization algorithm, sine-cosine optimizer algorithm, Ant Lion Optimization algorithm, Moth Flame Optimizer algorithm and other recently proposed meta heuristic, evolutionary, nature inspired and heuristic search algorithms and thus the proposed algorithm approves its efficiency in the research field of the meta heuristic algorithm inspired by nature for various types of optimization problems including multi-disciplinary engineering design optimization problems.

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to <https://doi.org/10.1016/j.asoc.2019.106018>.

CRediT authorship contribution statement

Vikram Kumar Kamboj: Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation. **Ayani Nandi:** Visualization, Writing - original draft. **Ashutosh Bhaduria:** Conceptualization, Data curation, Formal analysis, Funding acquisition. **Shivani Sehgal:** Writing - review & editing.

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