Applied Soft Computing Journal 89 (2020) 106134

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

Failure mode and effects analysis (FMEA) for risk assessment based on interval type-2 fuzzy evidential reasoning method

Jindong Qin [a,*], Yan Xi [a], Witold Pedrycz [b]

[a] School of Management, Wuhan University of Technology, 430070, Hubei, China
[b] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6R 2G7, Canada

ARTICLE INFO

Article history:
Received 9 April 2019
Received in revised form 14 January 2020
Accepted 23 January 2020
Available online 30 January 2020

Keywords:
Failure mode and effects analysis (FMEA)
Interval type-2 fuzzy sets (IT2FSs)
Evidential reasoning (ER)
Risk assessment

ABSTRACT

Failure mode and effect analysis (FMEA) has been widely adopted to define, identity, and remove potential and recognized hazards. As an indicator in traditional FMEA, the risk priority number (RPN) is an effective tool for measuring risk and the calculation of RPN is also very simple. Nevertheless, there are many drawbacks in the conventional FMEA method. It is necessary to seek approaches that can make up for the deficiency of traditional FMEA method and strengthen assessment capability of ranking failure modes according to three relevant risk factors. This paper presents a way to combine interval type-2 fuzzy sets (IT2FSs) with evidential reasoning (ER) method, which is able to overcome some disadvantages of the conventional FMEA approach and deal with uncertainties more efficiently. First, we give a more precise expression of the risk factors in the form of IT2FSs and gain the relative weight of three risk factors. Second, one can judge the failure modes in relation to each risk factors with belief structures. Finally, the ER method is used to combine the belief structures under the weight of the three risk factors. To verify the feasibility of the method, an application for steam valve system is performed and the obtained results show the effectiveness of the method.

1. Introduction

Failure mode and effects analysis (FMEA) used in systems, designs, and product has drawn much attention [1]. Unlike other risk assessment tools that look for solutions after failure occurred, the main functions of FMEA include identifying various potential failures and assessing their risk. Then precautions may be taken to decrease the likelihood and severity of failure or avoid dangerous accidents. FMEA was proposed by aeronautical engineering in the 1960s to inform risk management decisions [2]. When used in critical analysis, it is also known as the failure mode, effects and criticality analysis (FMECA) [3,4]. In general, a team of experts who have a good command of expertise in some certain fields are required to examine and quantify failure modes, impacts, reasons and come up with present countermeasures comprehensively in FMEA.

A classic FMEA is made up of five implementing procedures: ***preparation***, ***identification***, ***prioritization***, ***risk reduction***, and ***re-evaluation*** [5]. Among them, prioritization between each failure mode by suitable way is the main task of this paper. The RPN is a useful tool for measure risk, which takes the occurrence of failure modes (O), the severity of failures effect (S) and the probability of not detecting the failure (D) into account [6]. Normally, analysts or experts rate the three risk factors from 1 to 10, then RPN is obtained as the product of three risk factors. A failure mode with a higher RPN is regarded more dangerous and worth giving greater attention [7].

The FMEA method has been widely applied to automotive [8], medical and health [9–11], electronics [1], aerospace [12] and other fields. However, various deficiencies of the original RPN formula have also attracted attention, for instance, there is the difficulty in dealing with the complex uncertainty in risk assessment [4]. The uncertainties may present in many ways, for instance, limited professional knowledge and background leading to inaccurate and incomplete evaluation of FMEA team members [13]. Up to now, uncertainty has been studied widely with the use of various uncertainty theories, such as fuzzy set theory [14,15], evidence reasoning theory [13,16], grey theory [17], D-numbers approach [18] and others.

Fuzzy sets theory provides a means to express the uncertainty, which can well deal with fuzzy concepts. Considerable research efforts have been devoted to fuzzy sets theory in FMEA approach [5,19–21]. Meanwhile, various methods under fuzzy environment applied to handle FMEA problem have widely received attention. For example, Liu et al. [20] developed a fuzzy VIKOR approach for prioritizing failure modes. Comparing with type-1 fuzzy sets (T1FSs), that IT2FSs are capable of coping with the intra-uncertainty and inter-uncertainty in risk assessment

* Corresponding author.
E-mail address: qinjindongseu@126.com (J. Qin).

https://doi.org/10.1016/j.asoc.2020.106134
1568-4946/

problems [5] also makes it widely used in numerous models, while few studies have concentrated on FMEA problem.

In other way, evidential reasoning (ER) method is a kind of decision-making method firstly proposed on the basis of Dempster–Shafer (D–S) evidence theory and decision theory. The ER method makes up for the traditional MCDM method by establishing a unified confidence framework to describe all kinds of uncertainties in the problem. In general, the D–S theory can well handle many synthesis problems with fuzzy and uncertain information, and has certain advantages when aggregating different experts' opinions. But it is often necessary to consider the impact of multiple risk factors on the failure mode. Hence, it is reasonable to apply ER method which uses weights to correct the evidence and improves D–S evidence theory in FMEA. The ER method integrated with other theories have also had intensive applications in FMEA. For instance, Liu et al. [22] used fuzzy evidential reasoning (FER) method and grey theory to modify the traditional risk assessment model. Du et al. [23] developed a fuzzy FMEA method adopting ER and TOPSIS, in which ER is to present the evaluation information of FMEA team members and TOPSIS is taken to rank the risk priority of failure mode. Li and Chen [24] came up with an evidential method combining fuzzy BPAs and grey relational projection method

The main motivation of this study is as follows: For one thing, there are a large quantity of uncertainties in FMEA. ER in dealing with uncertain or incomplete decision information has a unique advantage. For another, combination rule of ER can effectively integrate the consistent part of evidence and strengthen the result of the consistent part. Using such formula one can fuse the individual risk information of O, S and D and have a comprehensive understanding of risk level. This paper attempts to come up with a FMEA approach on the basis of interval type-2 fuzzy evidential reasoning method, which combines the IT2FSs and evidence theory and measures risk more precisely.

Aiming at the uncertainty and fuzziness in the course of risk assessment, this paper uses the linguistic variable that being expressed as IT2FSs to appraise the fuzzy related importance about risk factors and the fuzzy rating of failure mode. According to the risk factors in failure mode analysis, the ER method is used to synthesize interval risk evidence and a new failure mode priority ranking method is found. The major contributions of this paper are summarized up as follows: (1) Risk factors reflect the source or influence of risk from three different perspectives. The relative importance given by experts as the weight of each attributes is effective to deal with extremely conflicting evidence [25] because D–S evidence theory may draw a counter-intuitive conclusion. (2) As an extension of the T1FSs, the extra one-dimensional membership of IT2FSs makes it more flexible than the T1FSs in expressing the fuzziness. So IT2FSs can express the fuzzy language evaluation information more efficiently. By synthesizing each expert's opinion in the form of IT2FSs and assigning the collective IT2FSs to its two adjacent levels, the belief structure becomes an interval that includes all possible belief degree value, which makes it more flexible in expressing the fuzziness than a certain belief degree value. (3) By integrating the risk information of O, S and D, the value of final RPN in the shape of interval could be gained. The interval RPN scales are continuous and most of them are unique, which can improve the weakness of traditional methods and correctly reflect the subtle differences in evaluation.

The paper is organized as follows. In the following section, the previous studies concerned with FMEA, ER method and IT2FSs are reviewed. In Section 3, A brief description of some basic concepts related to IT2FSs and ER method are introduced. Section 4 presents the proposed FMEA method in detail, which combines IT2FSs and ER for prioritization. A specific example with the application of presented FMEA method is put into use in Section 5 to demonstrate the feasibility of the method. Finally, Section 6 covers conclusions of this paper and points out future research directions.

2. Literature review

This section reviews various approaches applied to FMEA in the first place, and ER method and IT2FSs for FMEA are reviewed as well.

2.1. FMEA

RPN is critical for risk prioritization in conventional FMEA method, the formulation of which can be denoted as [26]:

$$RPN = O \times S \times D \tag{1}$$

The formula of RPN allows not only to represent the impact of the risk information on the failure mode, but also to help rank all the failure modes. Notably, it remains several drawbacks in the traditional FMEA, which are described below [27]:

(1) The numerical range of RPN is discontinuous, which signifies many of the points ranging from 1 to 1000 cannot be formed.
(2) There is likely to be identical RPN for various combinations of the risk factors (e.g., 12 may be made up of the sets of 1, 2, 6 or 2, 2, 3 of O, S and D).
(3) The weights of three risk indicators are ignored, while the value of weights will influence the final priorities.
(4) The accurate and direct numeric judgments for risk factors are hard to obtain.
(5) It is hard to solve the problem of uncertainty and subjectivity inherent in assessments.

In view of these shortcomings of RPN, many scholars have proposed approach to enhance FMEA assessment capabilities [28–30]. Beside, The methods for FMEA under uncertainty have been intensively investigated, especially some multiple criteria decision-making (MCDM) methods [20,31–34]. On the basis of the review conducted by Liu et al. [27], fuzzy rule-based approach is the most frequently applied owing to its several advantages. But it remains doubtful in setting up a number of necessary rules and membership functions when using this approach, because it highly relies on expert knowledge. There are a lot of excellent reviews in the literature putting forward FMEA method, which are summarized in Table 1.

As shown in Table 1, one can find that the fuzzy set theory has numerous applications to FMEA problems. Yet type-2 fuzzy environment for decision making problem in FMEA has received little attention. So, it continues to be a great impetus to research FMEA with ER method processing inaccurate and uncertain information under type-2 fuzzy environment.

2.2. ER in FMEA

Traditional D–S evidence theory, put forward by Dempser [42] and generalized by Shafer [43], failed to solve the issue of conflicting evidence effectively. In addition, it requires complete independence of evidence. Yang and Singh [44] came up with ER method, which had been widely used into decision assessment case [4,45]. Wang et al. [45] further generalized evidential reasoning operators and applied range, and constructed interval evidential reasoning method. Denoeux and Yaghlane [46] had extended the evidence theory to fuzzy sets. Jiang et al. [1] applied a fuzzy evidential method and handled conflicting evidence in FMEA. The ER method applied to FMEA has been demonstrated in a few studies so far, and there is still room for improvement. For instance, Chin et al. [13] developed ER method for multiple attribute decision in FMEA. Liu et al. [38] presented a model with FER and belief rule-based (BRB), in which FER was better utilized to evaluate and aggregate uncertain information. Yang and Wang [47] introduced ER approach into a maritime system to get the safety assessment of all failure modes.

Table 1
Summary of various methods related to FMEA.

Category	Method	Literature
Traditional method	RPN methodology	McDermott et al. [6]
Fuzzy MCDM model	AHP ANP VIKOR DEMATEL TOPSIS	Hu et al. [31] Chemweno et al. [32] Liu et al. [20] Zhou et al. [18] Braglia et al. [33]
Statistical decision making	Evidential reasoning Bayesian decision theory	Chin et al. [13]; Du et al. [23]; Chemweno et al. [35]
Uncertain theory	Grey theory Rough set theory	Zhou and Thai [17] Song et al. [12]
Mathematical optimization	Linear programming DEA	Wang et al. [7]; Liu et al. [21] Yousefi et al. [36]
Fuzzy logic methods	Rule-base system	Kumar et al. [37]; Liu et al. [38]
Integrated methods	AHP and TOPSIS cloud model and TOPSIS VIKOR and AHP FER and Grey theory TOPSIS and DEMATEL	Kutlu and Ekmekcioglu [10] Liu et al. [30] Liu et al. [3] Liu et al. [22] Chang et al. [39]
Other methods	Digraph and matrix method Choquet integral	Liu et al. [40] Wang et al. [41]

2.3. Type-2 fuzzy in FMEA

Little work so far has focused on FMEA when it comes to IT2FSs. Bozdag et al. [5] proposed a fuzzy FMEA method in view of IT2FSs, which was able to capture uncertainty within and between individuals. Chai et al. [48] proposed a new model making use of IT2FSs to express linguistic uncertainties and vagueness of words. But neither takes the relative weights of risk factors into consideration. Wang et al. [49] proposed a new way to prioritize risk for FMEA based on the extended MULTIMOORA under interval type-2 fuzzy environment.

From the literature review, one can conclude that a number of improved approaches developed can cope with the defects of the conventional FMEA. The method used most often is proved difficult in setting up plenty of necessary rules and membership functions. And there is a tendency to adopt more than one MCDM method to deal with uncertainty. At present, there are a few studies that apply IT2FSs in FMEA. ER is capable of making use of risk information and discovering relationship between data, while IT2FSs can be used to solve complex and uncertain problems, so integrating the two strengthens the effectiveness and empirical validity of risk assessment results.

3. Preliminaries

In this section, we introduce some fundamental concepts concerned with IT2FSs and ER, and also refer to some of their arithmetic operational laws, which will be used throughout the paper.

3.1. General type-2 fuzzy sets (GT2FSs)

GT2FSs is the extension of T1FSs, which means its membership itself is a T1FSs. It is a quantitative expression of higher-order uncertainty and can greatly facilitate the representation of uncertain information. However, due to the computational complexity of T2FSs, there are few studies in terms of GT2FSs in MCDM problem at present.

Definition 1. GT2FSs A in the universe of discourse X is defined as [50]:

$$A=\left\{((x,u),\mu_A(x,u))\,\middle|\,\forall x\in X,\forall u\in J_x\subseteq[0,1]\right\} \tag{2}$$

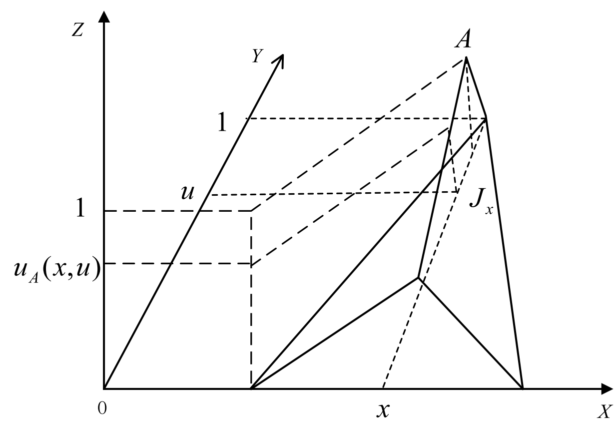

Fig. 1. GT2FSs A.

A can also be expressed as follows:

$$A=\int_{x\in X}\int_{u\in J_x}\mu_A(x,u)/(x,u)=\int_{x\in X}\frac{\int_{u\in J_x}\mu_A(x,u)/u}{x} \tag{3}$$

Where x denotes the primary variable, $J_x\in[0,1]$ denotes the primary membership function at x, u is the secondary variable, and $\int_{u\in J_x}\mu_A(x,u)/u$ is the secondary membership function at x. That integral sign $\int\int$ means the traversal for all available x and u. The geometric representation of GT2FSs A is shown in Fig. 1.

3.2. Interval type-2 fuzzy sets

Definition 2. If $\mu_{\widetilde{A}}(x,u)=1$ for all u, then $\widetilde{A}$ is called IT2FSs. It is described as follows [50]:

$$\widetilde{A}=\int_{x\in X}\int_{u\in J_x}1/(x,u)=\int_{x\in X}\left[\int_{u\in J_x}1/u\right]/x \tag{4}$$

Where $\int_{u\in J_x}1/u$ is the secondary membership function at x. It is obvious that the GT2FSs is a general form of IT2FSs.

Definition 3. A bounded region, made up of the primary membership function of $\widetilde{A}$, is named as the footprint of uncertainty (FOU)

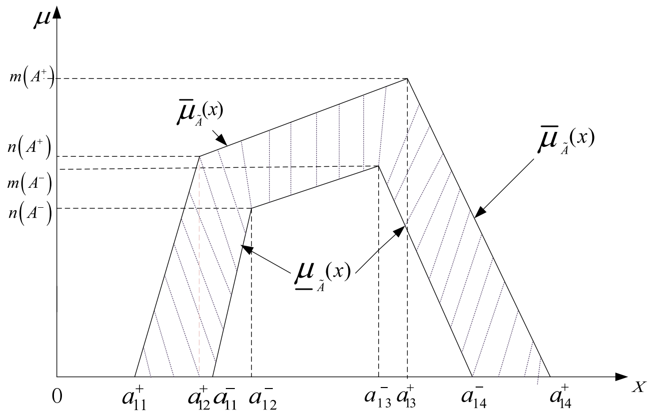

Fig. 2. IT2FSs $\tilde{A}$.

of $\widetilde{A}$. It is expressed as follows [51,52]:

$$FOU\left(\widetilde{A}\right)=\bigcup_{x\in X}J_x \tag{5}$$

IT2FSs $\widetilde{A}$ can be expressed as: $\widetilde{A}=\left(\left(a_{11}^+,a_{12}^+,a_{13}^+,a_{14}^+;m(A^+),n(A^+)\right),\left(a_{11}^-,a_{12}^-,a_{13}^-,a_{14}^-;m(A^-),n(A^-)\right)\right)$. The $FOU\left(\widetilde{A}\right)$ described as the shadow region in Fig. 2 is enclosed by an upper membership function (UMF) $\overline{\mu}_{\widetilde{A}}(x)$ and a lower membership function (LMF) $\underline{\mu}_{\widetilde{A}}(x)$. The membership of each element of an IT2FSs is an interval $\left[\underline{\mu}_{\widetilde{A}}(x),\overline{\mu}_{\widetilde{A}}(x)\right]$.

Definition 4. Giving two IT2FSs $\widetilde{A}_1=\left(\widetilde{A}_1^+,\widetilde{A}_1^-\right),\widetilde{A}_2=\left(\widetilde{A}_2^+,\widetilde{A}_2^-\right)$, the arithmetic operations between them are displayed as follows [53,54]:

$$\begin{aligned}\widetilde{A}_1\oplus\widetilde{A}_2=&\left(\left(a_{11}^++a_{21}^+,a_{12}^++a_{22}^+,a_{13}^++a_{23}^+,a_{14}^++a_{24}^+;\right.\right.\\&\min\left(m\left(A_1^+\right),m\left(A_2^+\right)\right),\\&\left.\min\left(n\left(A_1^+\right),n\left(A_2^+\right)\right)\right),\\&\left(a_{11}^-+a_{21}^-,a_{12}^-+a_{22}^-,a_{13}^-+a_{23}^-,a_{14}^-+a_{24}^-;\right.\\&\min\left(m\left(A_1^-\right),m\left(A_2^-\right)\right),\\&\left.\left.\min\left(n\left(A_1^-\right),n\left(A_2^-\right)\right)\right)\right)\end{aligned}\tag{6}$$

$$\begin{aligned}\widetilde{A}_1\otimes\widetilde{A}_2=&\left(\left(a_{11}^+\times a_{21}^+,a_{12}^+\times a_{22}^+,a_{13}^+\times a_{23}^+,a_{14}^+\times a_{24}^+;\right.\right.\\&\left.\min\left(m\left(A_1^+\right),m\left(A_2^+\right)\right),\min\left(n\left(A_1^+\right),n\left(A_2^+\right)\right)\right),\\&\left(a_{11}^-\times a_{21}^-,a_{12}^-\times a_{22}^-,a_{13}^-\times a_{23}^-,a_{14}^-\times a_{24}^-;\right.\\&\left.\left.\min\left(m\left(A_1^-\right),m\left(A_2^-\right)\right),\ \min\left(n\left(A_1^-\right),n\left(A_2^-\right)\right)\right)\right)\end{aligned}\tag{7}$$

$$\begin{aligned}\lambda\widetilde{A}_1=&\left(\left(\lambda a_{11}^+,\lambda a_{12}^+,\lambda a_{13}^+,\lambda a_{14}^+;m\left(A_1^+\right),n\left(A_1^+\right)\right),\right.\\&\left.\left(\lambda a_{11}^-,\lambda a_{12}^-,\lambda a_{13}^-,\lambda a_{14}^-;m\left(A_1^+\right),n\left(A_1^+\right)\right)\right)\quad(\lambda>0)\end{aligned}\tag{8}$$

Definition 5. The centroid $C_{\widetilde{A}}$ of IT2FSs $\widetilde{A}$ is a set of the centroids of all admissible embedded T1FSs A_e. $C_{\widetilde{A}}$ is an interval, the length of which indicates the level of uncertainty. i.e., [55]

$$C_{\widetilde{A}}\equiv\bigcup_{\forall A_e}c\left(A_e\right)=\left[c_l\left(\widetilde{A}\right),c_r\left(\widetilde{A}\right)\right]\tag{9}$$

Where $c_l\left(\widetilde{A}\right)$ and $c_r\left(\widetilde{A}\right)$ are the left and right endpoints of the interval $C_{\widetilde{A}}$ respectively and mathematically represented as:

$$c_l\left(\widetilde{A}\right)=\min_{\forall A_e}c\left(A_e\right)=\frac{\sum_{i=1}^{L}x_i\overline{\mu}_{\widetilde{A}}\left(x_i\right)+\sum_{i=L+1}^{N}x_i\underline{\mu}_{\widetilde{A}}\left(x_i\right)}{\sum_{i=1}^{L}\overline{\mu}_{\widetilde{A}}\left(x_i\right)+\sum_{i=L+1}^{N}\underline{\mu}_{\widetilde{A}}\left(x_i\right)}\tag{10}$$

$$c_r\left(\widetilde{A}\right)=\max_{\forall A_e}c\left(A_e\right)=\frac{\sum_{i=1}^{R}x_i\underline{\mu}_{\widetilde{A}}\left(x_i\right)+\sum_{i=R+1}^{N}x_i\overline{\mu}_{\widetilde{A}}\left(x_i\right)}{\sum_{i=1}^{R}\underline{\mu}_{\widetilde{A}}\left(x_i\right)+\sum_{i=R+1}^{N}\overline{\mu}_{\widetilde{A}}\left(x_i\right)}\tag{11}$$

Where x_L and x_R are the switch points which can be figured out by Karnik and Mendel (KM) Algorithms shown in [56], satisfying $x_L\le c_l\left(\widetilde{A}\right)\le x_R$ and $x_L\le c_r\left(\widetilde{A}\right)\le x_R$.

The ranking of $\widetilde{A}$ can be determined in the form:

$$c\left(\widetilde{A}\right)=\frac{c_l\left(\widetilde{A}\right)+c_r\left(\widetilde{A}\right)}{2}\tag{12}$$

(1) If $c(\widetilde{A}_1)>c(\widetilde{A}_2)$, then $\widetilde{A}_1\succ\widetilde{A}_2$;
(2) If $c(\widetilde{A}_1)=c(\widetilde{A}_2)$, then $\widetilde{A}_1=\widetilde{A}_2$;
(3) If $c(\widetilde{A}_1)<c(\widetilde{A}_2)$, then $\widetilde{A}_1\prec\widetilde{A}_2$.

3.3. Evidential reasoning algorithm

As an uncertain reasoning method, ER mainly follows the thought of D–S combination rules but it clarifies the weight of evidence. It has been extended to interval evidence fusion to deal with uncertain evidence [45,57]. To facilitate understanding the ideas, some primary concepts about traditional D–S theory and evidential reasoning are presented here.

Definition 6. Sample space is defined as a frame of discernment Θ, which is a finite nonempty set of mutually exclusive hypothesizes. Usually the frame of discernment is described as follows [42]:

$$\Theta=\left\{\theta_1,\theta_2,\ldots,\theta_N\right\}\tag{13}$$

The power set of Θ is represented by 2^Θ or $P(\Theta)$, and it contains the following 2^N subsets, e.t.,

$$P\left(\Theta\right)=2^\Theta=\left\{\varnothing,\theta_1,\ldots\theta_N,\left\{\theta_1,\theta_2\right\},\ldots,\left\{\theta_1,\theta_2,\theta_3\right\},\ldots,\Theta\right\}\tag{14}$$

Definition 7. Basic probability assignment (BPA), referred to as belief structure as well, can be defined as the following mapping function [42]:

$$m:2^\Theta\to[0,1]\tag{15}$$

Which meets the following conditions:

$$m\left(\varnothing\right)=0\tag{16}$$

$$\sum_{\theta\subseteq\Theta}m\left(\theta\right)=1\tag{17}$$

If $m(\theta)>0$, θ is named as the focal element of the BPA.

Definition 8. The key of D–S evidence theory is the Dempster's combination rule. Assume that there are two BPAs of two independent and completely reliable evidences indicated by m_1 and m_2, the result of combination is shown as follows [42]:

$$m\left(\theta\right)=\left[m_1\oplus m_2\right]\left(\theta\right)=\begin{cases}0 & \theta=\varnothing\\ \dfrac{1}{1-K}\displaystyle\sum_{B\cap C=\theta}m_1\left(B\right)m_2\left(C\right) & \theta\ne\varnothing\end{cases}\tag{18}$$

Where $K=\sum_{B\cap C=\varnothing}m_1\left(B\right)m_2\left(C\right)$. K represents the conflict between the two BPAs m_1 and m_2.

Definition 9. Interval evidence is denoted in the shape of interval probability masses so it corresponds to interval belief structure. Let m_1 and m_2 be two interval belief structure, the result will be also an interval belief structure after combining, which is described as follows [45]:

$$[m_1 \oplus m_2](\theta) = \begin{cases} 0 & \theta = \varnothing \\ \left[(m_1 \oplus m_2)^-(\theta), (m_1 \oplus m_2)^+(\theta)\right] & \theta \neq \varnothing \end{cases} \tag{19}$$

Where $(m_1 \oplus m_2)^+(\theta)$ and $(m_1 \oplus m_2)^-(\theta)$ are the maximum and the minimum, respectively of the following optimization problem:

$$Max/Min\ [m_1 \oplus m_2](\theta) = \frac{\sum_{B_i \cap C_j = \theta} m_1(B_i)\, m_2(C_j)}{1 - \sum_{B_i \cap C_j = \varnothing} m_1(B_i)\, m_2(C_j)}$$

$$s.t. \begin{cases} \sum_{i=1}^{n_1} m_1(B_i) = 1 \\ \sum_{j=1}^{n_2} m_2(C_j) = 1 \\ m_1^-(B_i) \le m_1(B_i) \le m_1^+(B_i) & i = 1, 2, \ldots, n_1 \\ m_2^-(C_j) \le m_2(C_j) \le m_2^+(C_j) & j = 1, 2, \ldots, n_2 \end{cases} \tag{20}$$

The above model extends D–S evidence theory to the combination of interval evidence, but it still ignores the influence of evidence weight. It is important to highlight that the associative law does not apply to combinations of interval evidence, which means $(m_1 \oplus m_2) \oplus (m_3) \neq m_1 \oplus (m_2 \oplus m_3)$. Accordingly, to correctly combine multiple interval belief structures, the three risk factors in this study must be combined simultaneously.

Definition 10. Assume that the weight of two pieces of independent evidence e_1 and e_2 are ω_1 and ω_2 respectively, the following synthesis results are obtained by applying the evidential reasoning algorithm [16].

$$m_i(\theta) = \begin{cases} 0 & \theta = \varnothing \\ \dfrac{\hat{m}_{\theta,e(2)}}{\sum_{D \subseteq \Theta} \hat{m}_{D,e(2)}} & \theta \neq \varnothing \end{cases} \tag{21}$$

$$\hat{m}_{\theta,e(2)} = \left[(1-\omega_2)\, m_{\theta,1} + (1-\omega_1)\, m_{\theta,2}\right] + \sum_{B \cap C = \theta} m_{B,1} m_{C,2}, \quad \theta \subseteq \Theta \tag{22}$$

Where $\hat{m}_{\theta,e(2)}$ represents the degree to which e_1 and e_2 both support the hypothesis θ. It consists of two independent part: the finite weighted sum supported by a single piece of evidence $(1-\omega_2)\, m_{\theta,1} + (1-\omega_1)\, m_{\theta,2}$ and the orthogonal sum $\sum_{B \cap C = \theta} m_{B,1} m_{C,2}$.

4. The proposed IT2FSs evidential reasoning method for FMEA

According to the above brief introduction of the related concepts, this section propounds a new approach to the problem of risk priority, in which the evaluation opinions on failure modes in terms of each risk factor offered by FMEA team members are represented in the form of interval type-2 fuzzy numbers. The point of our approach is to produce a new belief structure by using an optimization model and then integrate all risk information. The process is shown in Fig. 3 and the approach is described in detail below.

4.1. Risk assessment

Sometimes it is impossible to describe the exact risk level with specific numbers owing to the intricacy of the system evaluated

Table 2
Linguistic terms and IT2FSs for rating risk factor weights.

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	[(0,0,0.05,0.25;1), (0,0,0,0.15;0.75)]
Low(L)	[(0,0.2,0.3,0.5;1), (0.1,0.25,0.25,0.4;0.75)]
Medium(M)	[(0.25,0.45,0.55,0.75;1), (0.35,0.5,0.5,0.65;0.75)]
High(H)	[(0.5,0.7,0.8,1;1), (0.6,0.75,0.75,0.9;0.75)]
Very High (VH)	[(0.75,0.95,1,1;1), (0.85,1,1,1;0.75)]

Table 3
Linguistic terms and IT2FSs for rating failure modes.

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	[(0,0,0.5,2.5;1), (0,0,0,1.5;0.75)]
Low(L)	[(0,2,3,5;1), (1,2.5,2.5,4;0.75)]
Medium(M)	[(2.5,4.5,5.5,7.5;1), (3.5,5,5,6.5;0.75)]
High(H)	[(5,7,8,10;1) ;(6,7.5,7.5,9;0.75)]
Very High (VH)	[(7.5,9.5,10,10;1), (8.5,10,10,10;0.75)]

or the shortage of relevant historical information about the risks of the system. In this case, introducing linguistic terms for assessment as an alternative is appropriate. So, in our proposed method, we define all kinds of grades with five linguistic variables to make fuzzy judgments.

Step 1. Find out potential failure modes.

Find out and list possible risks or failure mode in a system based on previous literature or practical experiences. In this process, experts point out possible failure in a certain system and what the reasons and consequences of each failure are.

Step 2. Define failure modes and risk factors with linguistic terms and IT2FSs.

In the real world, risk factors are not easy to assess accurately due to the complexity of evaluation systems and a lack of knowledge or data about the problem areas. Therefore, in this study, we define that five grades of risk assessment information about risk factors and failure modes are expressed as linguistic terms. The corresponding IT2FSs are as well as provided respectively (see Tables 2 and 3). The membership functions of the five grades should satisfy several conditions: **For risk factor weights**, the membership functions of five assessment grades are defined on 0∼1. **For failure modes**, the membership function of VL, L, M, H, VH is defined on the universal discourse, namely 0∼10. Each level is divided evenly, and the membership function of VL and VH, L and H are symmetric.

To gain a better understanding of membership functions for the defined linguistic terms, Figs. 4 and 5 are provided.

4.2. Belief structure formation

Belief degrees (or possibility measures), which is expressed as a value or an interval, is used to denote the reliability assigned to assessment grades [22]. Take a linguistic term such as High for example, it signifies that the level of a failure mode in terms of the risk factor is High, which could be represented as $\{(H_4, 1.0)\}$. Similarly, if a FMEA team member assesses a failure mode at a level between High and Very High, where the belief degree assigned to High is 0.8 and that assigned to Very High is 0.2, then his or her judgment can be equivalently expressed as $\{(H_4, 0.8), (H_5, 0.2)\}$. While belief degree is an interval value such as $\{(H_2, [0.3, 0.6]), (H_3, [0.4, 0.7])\}$, which means that the risk grade is Low with the degree of 0.3–0.6 and Medium with the degree of 0.4–0.7.

Since each expert's view vary greatly, it makes sense to assume that the belief degree distributed to a grade is actually a range rather than an exact number in FMEA. Therefore, all possible belief structures can be considered synthetically in the

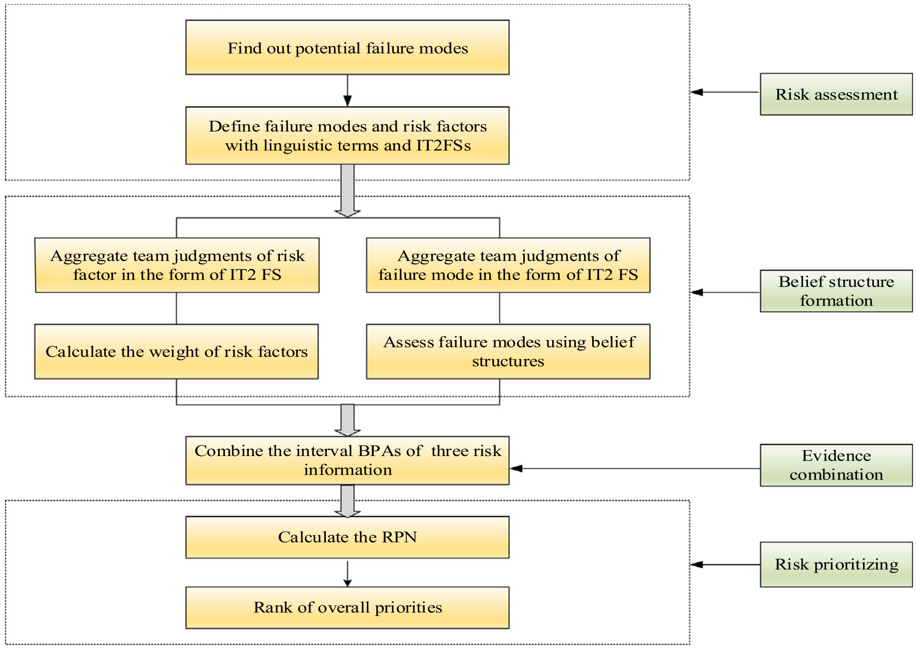

Fig. 3. The process of the proposed FMEA model.

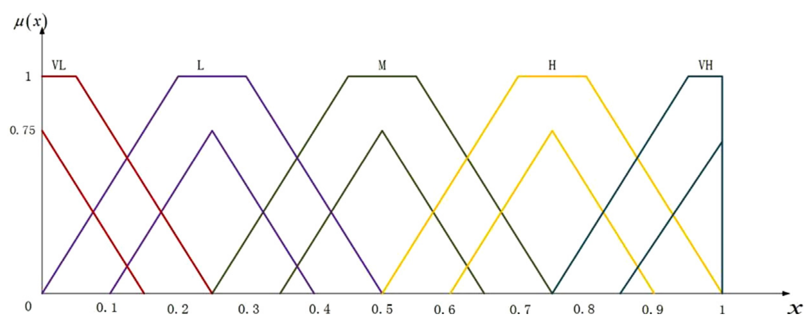

Fig. 4. Membership functions for rating risk factor weights.

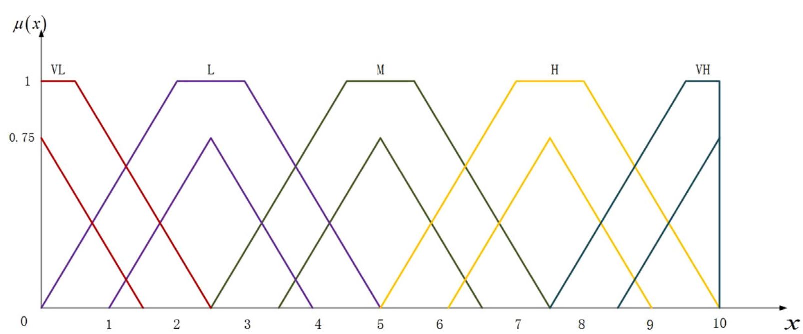

Fig. 5. Membership functions for rating failure modes.

form of interval. Section 4.2 gives an algorithm to acquire its interval belief structure.

Step 3. Calculate the weight of risk factors.

Assume that there are l members $(TM_1, TM_2, \ldots, TM_l)$ in a FMEA team in charge of the evaluation of m failure modes $(FM_1, FM_2, \ldots, FM_m)$ regarding three risk factors (O, S, D). Considering that team members are from different departments and are proficient in kinds of fields, their relative importance in the team is denoted as λ_k ($\lambda_k > 0$ and $\sum_1^l \lambda_k = 1$). Let $\widetilde{A}_R(R = O, S, D)$ be the aggregating team judgment of risk factor in the form of IT2FSs and its centroid is $\left[c_l\left(\widetilde{A}_R\right), c_r\left(\widetilde{A}_R\right)\right]$. Then ω_R reflects the three risk factors' relative importance in risk assessment prioritization.

$$\widetilde{A}_R = \lambda_1\widetilde{A}_{R1} \oplus \lambda_2\widetilde{A}_{R2} \oplus \cdots \oplus \lambda_l\widetilde{A}_{Rl} \tag{23}$$

$$\omega_R = \frac{c_l\left(\widetilde{A}_R\right) + c_r\left(\widetilde{A}_R\right)}{\sum\left(c_l\left(\widetilde{A}_R\right) + c_r\left(\widetilde{A}_R\right)\right)} \quad (R = O, S, D) \tag{24}$$

Step 4. Assess failure modes using belief structures.

In what follows, linguistic terms are used to evaluate failure modes and every assessment grade set is considered as a fuzzy

Table 4
The rules of generating belief structure.

Case	Conditions	Belief Structure
1	$\left[c_l\left(\widetilde{A}_F\right), c_r\left(\widetilde{A}_F\right)\right] = \left[c_l\left(\widetilde{H}_i\right), c_r\left(\widetilde{H}_i\right)\right]$	$\{(H_i, 1.0)\}$
2	$c_l\left(\widetilde{H}_i\right) \le c_l\left(\widetilde{A}_F\right) \le c_r\left(\widetilde{H}_i\right) \le c_r\left(\widetilde{A}_F\right)$	$\left\{\left(H_i, \beta_{i,R}\right), \left(H_{i+1}, \beta_{i+1,R}\right)\right\}$
3	$c_l\left(\widetilde{A}_F\right) \le c_l\left(\widetilde{H}_i\right) \le c_r\left(\widetilde{A}_F\right) \le c_r\left(\widetilde{H}_i\right)$	$\left\{\left(H_{i-1}, \beta_{i-1,R}\right), \left(H_i, \beta_{i,R}\right)\right\}$
4	$c_r\left(\widetilde{H}_i\right) \le c_l\left(\widetilde{A}_F\right) \le c_r\left(\widetilde{A}_F\right) \le c_l\left(\widetilde{H}_{i+1}\right)$	$\left\{\left(H_i, \beta_{i,R}\right), \left(H_{i+1}, \beta_{i+1,R}\right)\right\}$

set H_i, that is:

$$H_i = \{H_1, H_2, H_3, H_4, H_5\} = \{Very\ Low,\ Low,\ Medium,\ High,\ Very\ High\}$$

Remark 1. The judgments of FMEA team members may vary widely. To synthesize the assessments of all the members, we will combine individual assessment into group assessments by Eq. (25). Since the group assessment may be not just the appointed five grades in Section 4.1, we assign its belief degree to its two adjacent grades, which can not only make the evaluation result more accurate, but also simplify the subsequent calculation.

$$\widetilde{A}_F = \lambda_1\widetilde{A}_{F1} \oplus \lambda_2\widetilde{A}_{F2} \oplus \cdots \oplus \lambda_l\widetilde{A}_{Fl} \tag{25}$$

Where $\widetilde{A}_F$ is the aggregating team judgment of failure mode regarding risk factor in the form of IT2FSs. And its centroid is expressed as $\left[c_l\left(\widetilde{A}_F\right), c_r\left(\widetilde{A}_F\right)\right]$.

In this paper, the BPA of three risk factors are fused by ER. But how to define BPA remains an open question. Considering that only one or two assessment grades being assigned to belief degree when fusing three factors can greatly reduce the amount of calculation, we give the rules of generating belief structure that are shown in Table 4. $\left[c_l\left(\widetilde{H}_i\right), c_r\left(\widetilde{H}_i\right)\right]$ is the centroid of $\widetilde{H}_i\,(i = 1, 2, \ldots, 5)$, one of the assessment grade of failure mode in the form of IT2FSs. According to Eqs. (10) and (11), we can obtain the centroids of them, which are [0.5, 0.9], [2, 3], [4.5, 5.5], [7, 8], [9.1, 9.5] respectively.

Where the belief degree can be calculated by the following optimization model (26). $\beta_{i,R}$ and $\beta_{i+1,R}$ are belief degree which are used to assign to two assessment grades adjacent to $\widetilde{A}_F$.

$$\begin{aligned}
&Max/Min\beta_{i,R}\\
&s.t.\begin{cases}\beta_{i,R} + \beta_{i+1,R} = 1\\ c_l\left(\widetilde{A}_F\right) \le C\left(\widetilde{H}_i\right)\cdot\beta_{i,R} + C\left(\widetilde{H}_{i+1}\right)\cdot\beta_{i+1,R} \le c_r\left(\widetilde{A}_F\right)\\ c_l\left(\widetilde{H}_i\right) \le C\left(\widetilde{H}_i\right) \le c_r\left(\widetilde{H}_i\right)\\ c_l\left(\widetilde{H}_{i+1}\right) \le C\left(\widetilde{H}_{i+1}\right) \le c_r\left(\widetilde{H}_{i+1}\right)\end{cases}\\
&Max/Min\beta_{i+1,R}\\
&s.t.\begin{cases}\beta_{i,R} + \beta_{i+1,R} = 1\\ c_l\left(\widetilde{A}_F\right) \le C\left(\widetilde{H}_i\right)\cdot\beta_{i,R} + C\left(\widetilde{H}_{i+1}\right)\cdot\beta_{i+1,R} \le c_r\left(\widetilde{A}_F\right)\\ c_l\left(\widetilde{H}_i\right) \le C\left(\widetilde{H}_i\right) \le c_r\left(\widetilde{H}_i\right)\\ c_l\left(\widetilde{H}_{i+1}\right) \le C\left(\widetilde{H}_{i+1}\right) \le c_r\left(\widetilde{H}_{i+1}\right)\end{cases}
\end{aligned} \tag{26}$$

Therefore, the belief structures provided by TM_i in terms of risk factor is transformed as $\{(H_i, 1.0)\}\,(i = 1, 2, \ldots, 5)$ or $\left\{\left(H_i, \left[\beta^-_{i,R}, \beta^+_{i,R}\right]\right), \left(H_{i+1}, \left[\beta^-_{i+1,R}, \beta^+_{i+1,R}\right]\right), i = 1, 2, 3, 4\right\}$. Then l members' collective evaluation on each failure mode with regard to each risk factor can also be expressed by belief structure, which are referred to as group or collective belief structure. The specific expression is as follows:

$$\begin{aligned}S_R\left(FM_m\right) = \Big\{&\left(H_i, \left[\beta^-_{i,R}\left(FM_m\right), \beta^+_{i,R}\left(FM_m\right)\right]\right),\\ &\left(H_{i+1}, \left[\beta^-_{i+1,R}\left(FM_m\right), \beta^+_{i+1,R}\left(FM_m\right)\right]\right);\\ &R = O, S, D;\ i = 1, 2, 3, 4\Big\}\end{aligned}$$

Table 5
The rule of combination under the weight of risk factors.

	H_1	H_2	H_3	H_4	H_5	H
m_O	$m_O(H_1)$	$m_O(H_2)$	$m_O(H_3)$	$m_O(H_4)$	$m_O(H_5)$	$m_O(H)$
m_S	$m_S(H_1)$	$m_S(H_2)$	$m_S(H_3)$	$m_S(H_4)$	$m_S(H_5)$	$m_S(H)$
m_D	$m_D(H_1)$	$m_D(H_2)$	$m_D(H_3)$	$m_D(H_4)$	$m_D(H_5)$	$m_D(H)$
$m_{O\oplus S\oplus D}$	$m_{O\oplus S\oplus D}(H_i)$					$m_{O\oplus S\oplus D}(H)$

Example 1. Let $c\left(\widetilde{A}\right) = [5.25, 6.25]$ be the centroid of IT2FSs $\widetilde{A}$, then its belief degree could be assigned to the assessment grade $\widetilde{H}_3$ and $\widetilde{H}_4$. The belief degree of $\widetilde{H}_3$ and $\widetilde{H}_4$ can be determined with model (26) and the process is as follow:

$$\begin{aligned}
&Max/Min\beta_3\\
&s.t.\begin{cases}\beta_3 + \beta_4 = 1\\ 5.25 \le C\left(\widetilde{H}_3\right)\cdot\beta_3 + C\left(\widetilde{H}_4\right)\cdot\beta_4 \le 6.25\\ 4.5 \le C\left(\widetilde{H}_3\right) \le 5.5\\ 7 \le C\left(\widetilde{H}_4\right) \le 8\end{cases}\\
&Max/Min\beta_4\\
&s.t.\begin{cases}\beta_3 + \beta_4 = 1\\ 5.25 \le C\left(\widetilde{H}_3\right)\cdot\beta_3 + C\left(\widetilde{H}_4\right)\cdot\beta_4 \le 6.25\\ 4.5 \le C\left(\widetilde{H}_3\right) \le 5.5\\ 7 \le C\left(\widetilde{H}_4\right) \le 8\end{cases}
\end{aligned}$$

From the calculation above, the ranges of β_3 and β_4 are obtained as $\beta_3 \in [0.3, 1]$, $\beta_4 \in [0, 0.7]$. Then the belief structure is denoted as $\left\{\left(H_3, [0.3, 1]\right), \left(H_4, [0, 0.7]\right)\right\}$.

4.3. Evidence combination

One of the primary causes for D–S evidence theory's generating results that reaches counter-intuitive is that the weight of evidence is not determined. So, the relative weight of three risk factors being used to handle basic probability assignment can make results more accurate.

Step 5. Combine the interval BPAs of three risk information.

Under the weight of every risk factor ω_R, the BPA of a failure mode FM_m can be determined as follows, where $m_R(H_i)$ represses BPA on each grade, $m_R(H)$ denotes unallocated BPA that is difficult to assign to each grade.

$$m_R\left(H_i\right) = \omega_R\beta_{i,R}, \left(\beta^-_{i,R} \le \beta_{i,R} \le \beta^+_{i,R}\right) \tag{27}$$

$$m_R(H) = 1 - \sum_{i=1}^{5}\omega_R\beta_{i,R} \in \left[1 - \sum_{i=1}^{5}\omega_R\beta^+_{i,R}, 1 - \sum_{i=1}^{5}\omega_R\beta^-_{i,R}\right] \tag{28}$$

The rules of combination under the weight of risk factors are exhibited in Table 5. The unknown quantity in Table 5 can be calculated by Eqs. (29)–(31), $m_{O\oplus S\oplus D}(H_i)$ represents an interval probability assigned to each grade and $m_{O\oplus S\oplus D}(H)$ is an interval probability assigned to the universal set after fusion (see Box I).

The minimum and the maximum of probability assigned to each grade can be determined by the following model (32). Then

$$m_{O\oplus S\oplus D}\left(H_i\right)=\frac{\prod_R m_R\left(H_i\right)+m_O\left(H_i\right)m_D\left(H\right)\left[m_S\left(H_i\right)+m_S\left(H\right)\right]+m_D\left(H_i\right)m_S\left(H\right)\left[m_O\left(H_i\right)+m_O\left(H\right)\right]+m_S\left(H_i\right)m_O\left(H\right)\left[m_D\left(H_i\right)+m_D\left(H\right)\right]}{1-K} \tag{29}$$

$$m_{O\oplus S\oplus D}\left(H\right)=\frac{\prod_R m_R\left(H\right)}{1-K} \tag{30}$$

$$1-K=\sum_{i=1}^{5}\left\{\prod_R m_R\left(H_i\right)+m_O\left(H_i\right)m_D\left(H\right)\left[m_S\left(H_i\right)+m_S\left(H\right)\right]+m_D\left(H_i\right)m_S\left(H\right)\left[m_O\left(H_i\right)+m_O\left(H\right)\right]+m_S\left(H_i\right)m_O\left(H\right)\left[m_D\left(H_i\right)+m_D\left(H\right)\right]\right\}+\prod_R m_R\left(H\right) \tag{31}$$

Where K represses the conflict between the three BPAs m_O, m_S and m_D.

Box I.

we can obtain the final interval BPAs.

$$\begin{aligned}
&Max/Min\ m_{O\oplus S\oplus D}\left(H_i\right)\\
&s.t.\begin{cases}\sum_{i=1}^{5}\omega_R\beta_{i,R}+m_R\left(H\right)=1\\ \omega_R\beta_{i,R}^{-}\le\omega_R\beta_{i,R}\le\omega_R\beta_{i,R}^{+} & (R=O,S,D)\\ 1-\sum_{i=1}^{5}\omega_R\beta_{i,R}^{+}\le m_R\left(H\right)\le 1-\sum_{i=1}^{5}\omega_R\beta_{i,R}^{-}\end{cases}\\
&Max/Min\ m_{O\oplus S\oplus D}\left(H\right)\\
&s.t.\begin{cases}\sum_{i=1}^{5}\omega_R\beta_{i,R}+m_R\left(H\right)=1\\ \omega_R\beta_{i,R}^{-}\le\omega_R\beta_{i,R}\le\omega_R\beta_{i,R}^{+} & (R=O,S,D)\\ 1-\sum_{i=1}^{5}\omega_R\beta_{i,R}^{+}\le m_R\left(H\right)\le 1-\sum_{i=1}^{5}\omega_R\beta_{i,R}^{-}\end{cases}
\end{aligned} \tag{32}$$

This nonlinear programming model is actually similar to model (20), except that it combines three pieces of evidence simultaneously, could be easily solved by LINGO 11 software package.

4.4. Risk prioritizing

In order to fully prioritize all failure modes, it is often necessary to calculate RPN. Based on the following steps, the final RPN will also be an interval. The midpoint value of the acquired RPN being used for ranking, one can systematically researches the risk concerned with each identified or possible failure mode.

Step 6. Calculate the RPN.

After combining the three pieces of evidence, the BPA on H_i of every failure mode is $m\left(H_i\right)$ and that on H is $m\left(H\right)$. Since BPA on H is difficult to be assigned to any grade, $m\left(H\right)$ is not taken into account in the calculation of RPN. We hence obtain interval RPN through the following model:

$$\begin{aligned}
&Max/Min\ \sum_{i=1}^{5}\frac{c_l\left(\widetilde{H}_i\right)+c_r\left(\widetilde{H}_i\right)}{2}m\left(H_i\right)\\
&s.t.\begin{cases}\sum_{i=1}^{5}m\left(H_i\right)+m\left(H\right)=1\\ m^{-}\left(H_i\right)\le m\left(H_i\right)\le m^{+}\left(H_i\right)\end{cases}
\end{aligned} \tag{33}$$

Step 7. Rank of overall priorities.

After calculating all RPN value of failure modes, rank the overall failure modes and determine which should be taken into consideration. The higher the midpoint value of RPN is, the more concern is supposed to be given to its corresponding failure mode.

Step 8. End.

5. An application of IT2FSs evidential reasoning method to FMEA

In this section, we give an example of steam valve system [12,40] to illustrate the proposed method. As has been discussed in Tables 1 and 2, TMs' judgments with regard to failure modes and risk factors such as "Very Low" are expressed as linguistic terms and their corresponding IT2FSs are also given.

5.1. Problem description of steam valve system

Steam valve system plays a significant role in steam turbine operation, where fault will have a huge effect on stability of the entire power plant. It is stipulated that steam valve switch needs to be turned on and off in time and operated steadily during the whole process. In consideration of the complexity of steam valve system, it is hard to quantitate numerous elements. Therefore, it is feasible for the proposed method to identify failure modes and evaluate risk of the steam valve system.

Suppose that eight failure modes have been discerned in steam valve system. As presented in Table 6, the failure modes in regard to each risk factor are evaluated by four team members (TM_l, $l=1,2,3,4$). Due to the difference of domain knowledge and expertise, the four members are distributed relative weights in FMEA as 0.15, 0.30, 0.35 and 0.20. The importance weight of three risk factors assessed by four members is exhibited in Table 7.

5.2. Implementation

As shown in Table 8, it gives the centroids of group assessments of all failure mode concerning each type of risk and the important weight of risk factors. According to the rule which are defined in Table 4, the belief degrees assigned to one or two assessment grades of three risk factors can be computed and are exhibited in Table 9. Taking the weight of each risk factor into account, the weighted BPAs of O, S and D allocated to one or two assessment grades are computed and shown in Table 10. Next, evidential reasoning rule are adopted to combine the BPAs of three risk information and the ultimate interval belief structures of

Table 6
Assessment of failure modes of team members.

Risk factors		O				S				D			
Team members		TM_1	TM_2	TM_3	TM_4	TM_1	TM_2	TM_3	TM_4	TM_1	TM_2	TM_3	TM_4
Failure modes	1	L	L	M	VL	VH	VH	VH	VH	VH	VH	H	M
	2	L	L	M	L	VL	VL	VL	L	L	M	M	L
	3	H	H	M	M	L	L	M	L	L	L	VL	M
	4	L	VL	L	L	H	H	H	M	M	VL	L	L
	5	L	L	M	M	VH	VH	H	VH	L	L	VL	L
	6	VL	L	VL	L	VH	VH	VH	VH	VL	L	L	M
	7	M	M	H	M	H	H	VH	VH	L	VL	L	L
	8	M	H	M	H	M	M	H	H	L	M	L	L

Table 7
Assessment of risk factors of team members.

Risk factors	Team members			
	TM_1	TM_2	TM_3	TM_4
O	L	M	M	H
S	M	H	H	VH
D	M	H	M	H

Table 8
The centroids of aggregating IT2FSs.

Failure modes	O	S	D
1	[2.575,3.455]	[9.100,9.500]	[7.445,8.175]
2	[2.875,3.875]	[0.800,1.320]	[3.625,4.625]
3	[5.625,6.625]	[2.875,3.875]	[1.975,2.765]
4	[1.460,2.370]	[6.500,7.500]	[1.925,2.745]
5	[3.375,4.375]	[8.365,8.975]	[1.475,2.265]
6	[1.250,1.950]	[9.100,9.500]	[2.275,3.185]
7	[5.375,6.375]	[8.155,8.825]	[1.550,2.370]
8	[5.750,6,750]	[5.875,6.875]	[2.750,3.750]
RF	[0.463,0.563]	[0.705,0.793]	[0.575,0.675]

each assessment grade for each mode are presented in Table 11. Finally, the eight maximal, minimal and mean RPNs which are determined by the Weighted Mean of Maximum (WMM) method [2] are shown in Table 12. Besides, it provides the ranking based on the mean RPNs.

According to Eq. (24), we can work out the relative weight of three risk factors, which are of importance in subsequent steps. The results are shown as follows

$$\omega_O = 0.27 \ \omega_S = 0.40 \ \omega_D = 0.33.$$

Our observation in Table 12 leads us to conclude that the risk prioritization about eight failure modes are $FM_1 > FM_7 > FM_6 > FM_5 > FM_8 > FM_4 > FM_3 > FM_2$. Therefore, failure mode 1 with high risk evaluations should be given the greatest attention. On the contrary, the lowest attention could be paid to failure mode 2 as has the least overall risk.

5.3. Comparisons with other methods

To evaluate the feasibility of proposed method, fuzzy VIKOR method proposed by Liu et al. [3] and fuzzy TOPSIS method proposed by Braglia et al. [33] are adopted to rank the priority and make comparisons with the above result. With these methods, the fuzzy assessment of failure modes and important weights in terms of each risk factors are described by linguistic variables with T1FSs. Take IT2FSs' mid-point, namely $A = \frac{A^L + A^R}{2}$, then IT2FSs degenerate to T1FSs. The linguistic variables and their corresponding T1FSs are shown in Tables 13 and 14.

5.3.1. Fuzzy VIKOR Method

In this method, the values of S, R and Q are calculated and the eight failure modes' risk prioritizing based on the value of Q could be obtained (see Table 15). The formula for calculating Q is as:

$$Q_i = \upsilon \frac{S_i - S^*}{S^- - S^*} + (1 - \upsilon) \frac{R_i - R^*}{R^- - R^*} \tag{34}$$

Where $S^* = \min S_i$, $S^- = \max S_i$, $R^* = \min R_i$, $R^- = \max R_i$. In this paper, the value of υ is set to 0.5 for simplifying the calculation.

Based on Table 15, the fact that the ranking orders for failure modes 1 and 2 are exactly the same as those conducted by the proposed method makes us conclude that the method we proposed in this paper is reasonable. Because they are the failure modes with the highest and lowest overall risk.

5.3.2. Fuzzy TOPSIS method

With the fuzzy TOPSIS method [33], we can get the value of d_i^+ and d_i^- for all failure modes. d_i^+ is the positive Euclidean distance from the failure mode FM_i to the positive ideal solution; d_i^- is the negative Euclidean distance from the failure mode to the negative ideal solution. Then the final ranking could be obtained on the basis of the relative closeness c_i to ideal solution. The values of these three indicators are expressed in the form:

$$d_i^+ = \sqrt{\sum_{j=1}^{3} \left(v_{ij} - 10\right)^2} \tag{35}$$

$$d_i^- = \sqrt{\sum_{j=1}^{3} \left(v_{ij} - 0\right)^2} \tag{36}$$

$$c_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{37}$$

Apparently, the ranking in Table 16 is consistent with the ranking conducted by our proposed method. That also demonstrates the effectiveness of proposed model.

In accordance with the above analysis, the ranking comparison of the eight failure modes calculated by the three listed method is described in Fig. 6. From that comparison diagram we know FM_6 has a lower risk priority than FM_7, FM_5 has a high-risk evaluation compared to FM_8 and FM_3 is with the lower risk than FM_4, which are contrary to the result gained from the fuzzy VIKOR method. That a majority of failure modes have disparate prioritization between the proposed method and fuzzy TOPSIS method indicates that the two FMEA methods differ in the ability to deal with uncertainties. The primary cause for that ranking differences is that ER and VIKOR aggregate different types of risk in different ways. The ER is based on the syncretic formula that aggregates two or more mass functions to one. In this way, we can fully consider the impact of all types of risk on risk priority. While in fuzzy VIKOR method, an aggregate function on behalf of the distance to the ideal solution is introduced, where both group utility and individual regret values are taken into account. As for TOPSIS, it is based on the fact that the distance between the optimal point and the positive ideal solution should be the

Table 9
Belief structures of risk factors on each assessment grade.

Failure modes	Risk factors	Assessment grades				
		H_1	H_2	H_3	H_4	H_5
1	O		[0.418,1]	[0,0.582]		
	S					1
	D				[0.440,1]	[0,0.560]
2	O		[0.250,1]	[0,0.750]		
	S	[0.450,1]	[0,0.550]			
	D		[0,0.750]	[0.250,1]		
3	O			[0.150,0.950]	[0,0.850]	
	S		[0.250,1]	[0,0.750]		
	D	[0,0.490]	[0.510,1]			
4	O	[0,0.730]	[0.270,1]			
	S			[0,0.600]	[0.400,1]	
	D	[0,0.512]	[0.488,1]			
5	O		[0,0.850]	[0.15,0.950]		
	S				[0,0.757]	[0.243,0.940]
	D	[0,0.726]	[0.274,1]			
6	O	[0,0.833]	[0.167,0.967]			
	S					1
	D		[0.526,1]	[0,0.474]		
7	O			[0.250,1]	[0,0.750]	
	S				[0.131,0.896]	[0.103,0.869]
	D	[0,0.690]	[0.310,1]			
8	O			[0.100,0.900]	[0.100,0.900]	
	S			[0,0.850]	[0.150,0.950]	
	D		[0.300,1]	[0,0.700]		

Table 10
Belief structures of risk factors on each assessment grade under the weight of O, S and D.

Failure modes	Risk factors	Assessment grades					
		H_1	H_2	H_3	H_4	H_5	H
1	O		[0.110,0.270]	[0,0.160]			[0.570,0.890]
	S					0.4	0.6
	D				[0.150,0.33]	[0,0.180]	[0.490,0.850]
2	O		[0.062,0.270]	[0,0.202]			[0.528,0.933]
	S	[0.180,0.400]	[0,0.220]				[0.380,0.820]
	D		[0,0.248]	[0.083,0.33]			[0.422,0.917]
3	O			[0.040,0.257]	[0,0.230]		[0.513,0.960]
	S		[0.100,0.400]	[0,0.300]			[0.300,0.900]
	D	[0,0.162]	[0.168,0.330]				[0.508,0.832]
4	O	[0,0.197]	[0.073,0.270]				[0.473,0.927]
	S			[0,0.240]	[0.160,0.400]		[0.360,0.840]
	D	[0,0.170]	[0.160,0.330]				[0.500,0.840]
5	O		[0,0.230]	[0.041,0.257]			[0.513,0.959]
	S				[0,0.303]	[0.097,0.376]	[0.321,0.903]
	D	[0,0.240]	[0.090,0.330]				[0.430,0.910]
6	O	[0,0.225]	[0.045,0.261]				[0.514,0.955]
	S					0.4	0.6
	D		[0.174,0.330]	[0,0.156]			[0.514,0.826]
7	O			[0.068,0.270]	[0,0.202]		[0.528,0.932]
	S				[0.052,0.358]	[0.041,0.348]	[0.294,0.897]
	D	[0,0.228]	[0.102,0.330]				[0.442,0.898]
8	O			[0.027,0.243]	[0.027,0.243]		[0.514,0.946]
	S			[0,0.340]	[0.060,0.380]		[0.280,0.940]
	D		[0.100,0.330]	[0,0.231]			[0.439,0.900]

shortest, and the distance from the negative ideal solution should be the longest.

5.4. Sensitivity analysis

According to the original weights of the three types of risk and variation of ±10%, ±20% and ±30% based on the original weights given in Table 17, the sensitivity analysis is carried out. The main purpose of sensitivity analysis is to investigate whether the weight changes of the three types of risk will make a difference to final ranking of failure modes.

We use the Monte Carlo simulation to generate 100 sets of random weights, and obtain the ranking results when the weights are changed. The results of such study are described in Table 18 and Fig. 7, which plots that most of failure modes are not sensitive to weight changes. FM_5, FM_6 and FM_8 are a little sensitive. Sensitivity analysis shows that the weight of risk factors will influence risk assessment of failure modes. Therefore, it is meaningful and beneficial for risk priority of failure mode and subsequent corrective measures to select the appropriate weight of risk factors in accordance with the actual situation and experts' opinions.

Table 11
Interval BPAs on each assessment grade for each mode.

Failure modes	Risk level					
	H_1	H_2	H_3	H_4	H_5	H
1		[0,0.182]	[0,0.108]	[0,0.219]	[0.229,0.453]	[0.270,0.508]
2	[0,0.372]	[0,0.524]	[0,0.409]			[0.159,0.724]
3	[0,0.145]	[0.130,0.588]	[0,0.420]	[0,0.196]		[0.151,0.726]
4	[0,0.276]	[0,0.467]	[0,0.218]	[0,0.363]		[0.196,0.678]
5	[0,0.225]	[0,0.453]	[0,0.228]	[0,0.290]	[0,0.360]	[0.174,0.801]
6	[0,0.149]	[0.126,0.383]	[0,0.113]		[0.172,0.345]	[0.257,0.517]
7	[0,0.211]	[0,0.305]	[0,0.238]	[0,0.449]	[0,0.330]	[0.166,0.769]
8		[0,0.310]	[0,0.584]	[0,0.500]		[0.122,0.810]

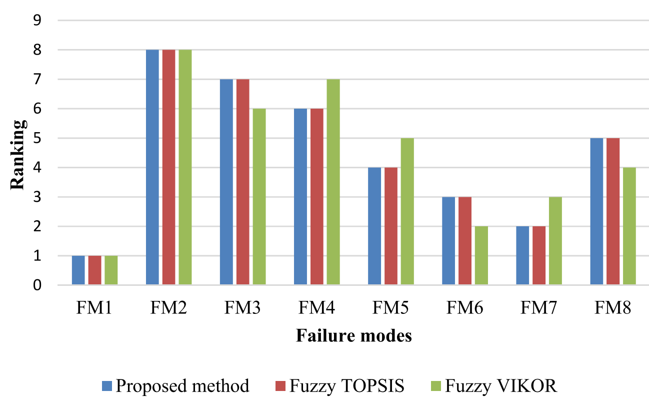

Fig. 6. Ranking comparison for three methods.

Table 12
Risk estimation and risk prioritization of the eight failure modes.

Failure modes	RPN			Ranking
	max	min	mean	
1	6.145	2.990	4.568	1
2	3.125	0.193	1.659	8
3	4.153	0.426	2.290	7
4	4.370	0.308	2.339	6
5	6.403	0.139	3.271	4
6	4.486	2.109	3.298	3
7	6.712	0.198	3.455	2
8	5.640	0.475	3.058	5

Table 13
Linguistic variables and T1FSs for rating failure modes.

Linguistic variables	T1FSs
VL	(0,0,0.25,2;0.875)
L	(0.5,2.25,2.75,4.5;0.875)
M	(3,4.75,5.25,7;0.875)
H	(5.5,7.25,7.75,9.5;0.875)
VH	(8,9.75,10,10;0.875)

Table 14
Linguistic variables and T1FSs for rating risk factor weights.

Linguistic variables	T1FSs
VL	(0,0,0.025,0.2;0.875)
L	(0.05,0.225,0.275,0.45;0.875)
M	(0.3,0.475,0.525,0.7;0.875)
H	(0.55,0.725,0.775,0.95;0.875)
VH	(0.8,0.975,1,1;0.875)

Table 15
The values of S, R and Q and risk ranking of eight failure modes.

Failure modes	S	R	Q	Ranking
1	1.474	0.689	1.000	1
2	0.428	0.230	0.000	8
3	0.798	0.499	0.470	6
4	0.579	0.436	0.297	7
5	0.808	0.589	0.572	5
6	0.833	0.689	0.694	2
7	1.075	0.581	0.692	3
8	1.112	0.504	0.626	4

Table 16
The values of c_i and risk ranking of eight failure modes.

Failure modes	O	S	D	d_i^+	d_i^-	c_i	Ranking
1	0.813	3.730	2.581	13.370	4.608	0.256	1
2	0.911	0.416	1.361	15.782	1.690	0.097	8
3	1.654	1.350	0.779	15.149	2.273	0.130	7
4	0.527	2.800	0.768	15.060	2.951	0.164	6
5	1.046	3.474	0.614	14.521	3.680	0.202	4
6	0.429	3.730	0.900	14.620	3.860	0.209	3
7	1.586	3.401	0.644	14.208	3.808	0.211	2
8	1.688	2.550	1.073	14.293	3.240	0.185	5

Table 17
Variation of weights in the sensitivity analysis.

Weight	ω_O	ω_S	ω_D
−30% (Lower bound)	0.189	0.28	0.231
−20% (Lower bound)	0.216	0.32	0.264
−10% (Lower bound)	0.243	0.36	0.297
0	0.27	0.4	0.33
+10% (Upper bound)	0.297	0.44	0.363
+20% (Upper bound)	0.324	0.48	0.396
+30% (Upper bound)	0.351	0.52	0.429

Table 18
Ranking acceptability indices (%) with interval weights.

Alternatives	1	2	3	4	5	6	7	8
FM_1	100							
FM_2							5	95
FM_3						22	75	3
FM_4						78	20	2
FM_5			28	39	33			
FM_6		15	32	30	23			
FM_7		85	10	5				
FM_8			30	26	44			

5.5. Further analysis

Compared with the previous method, the method presented in this paper also has some advantages as follows.

(1) By comparing H in Tables 10 and 11, we find that the uncertainty is decreased. Take FM_1 for example, the uncertainty values of the three risk factors are respectively [0.570,0.890], 0.6 and [0.490,0.850]. After fusion, the uncertainty value is within [0.270,0.508]. The left and right endpoints of the uncertainty interval are smaller than before, which proves that ER can reduce the uncertainty of evidence and make the fusion result more objective.

(2) Since the RPN value calculated by the proposed method is an interval, there is a very low probability that the same RPN

Table 19
Assessed information of failure modes.

Risk factors		O			S			D		
Team members		TM_1	TM_2	TM_3	TM_1	TM_2	TM_3	TM_1	TM_2	TM_3
Failure modes	1	L	H	M	M	H	VH	H	L	L
	2	M	M	L	H	H	VH	VL	L	L
	3	H	VH	VH	M	H	L	VH	H	H
The weight of risk factors		M	M	L	H	M	VH	L	M	H

Table 20
The ranking result based on the TOPSIS method.

Failure modes	O	S	D	d_i^+	d_i^-	c_i	Ranking
1	1.045	3.525	1.280	14.077	3.893	0.217	3
2	0.825	3.870	0.625	14.479	4.006	0.217	3
3	1.931	1.955	2.575	13.600	3.766	0.217	3

value will be produced, so it has a strong advantage over previous methods. The following Example 2 will illustrate this feature.

Example 2. Suppose that three failure modes have been discerned in a system. Three team members have assessed the information of failure modes on three risk factors and their weight, which are presented in Table 19. And three team members are given relative importance as 0.3, 0.2, 0.5 respectively.

Then the TOPSIS method and the proposed method are used to rank the three failure modes, the results are shown in Tables 20 and 21. From Table 20 we find the relative closeness c_i of three failure modes are very close and are difficult to rank them. While the RPNs figured out by the proposed method are interval and it is not easy to get exactly the same. According to the slight differences between RPNs, we get the ranking shown in Table 21. So the problem that there may be the same RPN for various combinations of risk factors can be well solved by the method proposed in this paper.

(3) Besides, it should be pointed out that taking IT2FSs' midpoint data as T1FSs represses a compromised opinion. If the evaluators are aggressive and consider risks comprehensively, they could let $A = A^R$. In contrast, if the evaluators are conservative and consider less risk, they could let $A = A^L$. In three different cases, the result conducted by fuzzy VIKOR method may be different. In consequence, we consider IT2FSs as an extension of T1FSs. In IT2FSs, we can take all the different T1FSs. Comparing with the original type-1 fuzzy method, type-2 fuzzy environment provides FMEA team members with a greater flexibility to express their judgments concerning uncertainty. So, in comparison with Liu's method [3], the flexibility of our method is much higher. Thus, the proposed method in this study is preferable to figure out risk prioritizing problems in FMEA.

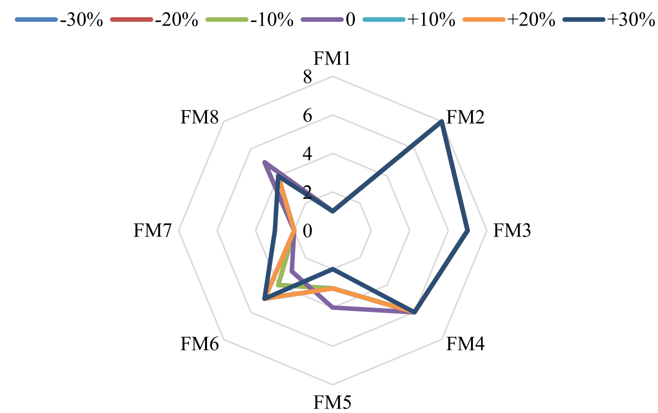

Fig. 7. Ranking orders with weights changing.

6. Conclusions

FMEA is an effective tool for identifying and eliminating risk in many fields, so it is necessary to rank each failure mode accurately. In this paper, we put forward an evidential reasoning method under interval type-2 fuzzy environment that could well represent complicated uncertain information. The method is applied to a steam valve system for risk analysis to certify its validity in the end. The result indicate that the method is effective.

The proposed method in this study has some contributions. Theoretically, it might provide a viable source for generating new interval belief structures with the application of IT2FSs. In management practice, it is able to rank multiple decisions and help decision makers choose the best decisions. For instance, the decision makers are capable of finding out a failure mode with highest risk and give more concern by ranking the RPN value of each failure mode. And there are several advantages of our proposed method. Firstly, it is more precise than conventional method and reduces the probability of producing same RPN value. Secondly, the weight of three risk factors being taken into consideration makes the result more persuasive and comprehensive. Thirdly, IT2FSs as an extension of T1FSs can better express uncertainty. It is worth noting the limitations of this paper that the linguistic terms need to be pre-defined and defining linguistic terms with multiscale, for example, a group of seven linguistic variables (VL, L, ML, M, MH, H, VH) would make the results more accurate.

To future research we will focus more on GT2FSs in FMEA. As a result of its computing complexity, there are relatively few studies devoted to the decision-making approach in view of GT2FSs at present. Next, considering that the interaction of three risk indicators may result in occurrence of failure, we demand to construct a model that is capable of expressing the relevance of the risk factors. Then, to some extent pre-defining limited linguistic terms to experts for judgment will limit the experts to express their opinion. Future research will identify risk factors that can be accurately quantified by experts, and assess risk factors that cannot be quantified by linguistic terms. Finally, experts' judgment is not always rational. We will pay more attention to regret theory, prospect theory and TODIM method to reflect the bounded rationality of decision makers. Further studies and development of the mentioned outlook will be ongoing.

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to https://doi.org/10.1016/j.asoc.2020.106134.

CRediT authorship contribution statement

Jindong Qin: Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **Yan Xi:** Conceptualization, Methodology, Data Curation, Writing - review & editing. **Witold Pedrycz:** Writing - review & editing, Supervision.

Table 21
The ranking result based on the proposed method.

Failure modes	H_1	H_2	H_3	H_4	H_5	H	RPN			Ranking
							max	min	mean	
1		[0,0.344]	[0,0.503]	[0,0.545]	[0,0.261]	[0.188,0.660]	6.545	0.850	3.698	3
2	[0,0.263]	[0,0.528]	[0,0.210]	[0,0.691]	[0,0.579]	[0.127,0.804]	7.590	0.137	3.864	2
3		[0,0.449]	[0,0.641]	[0,0.466]	[0,0.418]	[0.150,0.745]	7.128	0.638	3.883	1

Acknowledgments

The work is supported by the National Natural Science Foundation of China (NSFC) under Project 71701158, MOE (Ministry of Education in China) Project of Humanities and Social Sciences (Project No. 17YJC630114) and the Fundamental Research Funds for the Central Universities 2018IVB036 and 2019VI030, China.

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