Inhaltsverzeichnis

Here everything can be written down that helps with understanding the Non-Linear Wave Equations Lecture and solving the exercises.

1 Norms Used

2 Functional Analysis Theorems

Theorem 1 (Arzelà-Ascoli theorem, source: Wiki)**.** *Consider a sequence of functions (f_n) defined on a compact interval. If this sequence is uniformly bounded and uniformly equicontinuous, then there exists a subsequence (f_{n_k}) that converges uniformly.*

Definition 2 (uniformly equicontinuous)**.** *A sequence of functions (f_n) is said to be uniformly equicontinuous if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that*

$$|f_n(x) - f_n(y)| < \varepsilon \tag{1}$$

whenever $|x - y| < \delta$ for all functions f_n in the sequence.

This is the version of Arzelà-Ascoli used on example sheet 6, problem*, in a specific norm:

Theorem 3 (Arzelà-Ascoli theorem, source: solution to sheet 5)**.** *Suppose there is a sequence (of solutions) $(\phi^{(i)})$ in $C^{k-1}([0,T]\times\mathbb{R}^n)$ that converges to ϕ in a $C^{k-1}([0,T]\times\mathbb{R}^n)$-sense and is uniformly bounded, meaning $\|\partial^k\phi\|_{L^\infty} \leq A$ (A not dependent on i).*
Then, there exists a subsequence $(\phi^{(i_\lambda)})$ that converges in $C^k([0,T]\times\mathbb{R}^n)$.

Theorem 4 (Banach-Alaoglu)**.**

Theorem 5 (Riesz Representation thm)**.**

3 Fourier Transformation

4 Important Inequalities

4.1 Sobolev Embedding

Theorem 6 (Sobolev embedding thm; source: sheet 1)**.** *There exists a constant $C = C(n,s) > 0$ such that for every $f \in H^s(\mathbb{R}^n)$ with $s > \frac{n}{2}$ we have*

$$\|f\|_{L^\infty(\mathbb{R}^n)} \leq C\|f\|_{H^s(\mathbb{R}^n)}. \tag{2}$$

Theorem 7 (second Sobolev embedding thm; source: Wiki)**.** *If $n < pk$ and $r + \alpha = k - \frac{n}{p}$ with $\alpha \in (0,1)$, then one has the embedding*

$$W^{k,p}(\mathbb{R}^n) \subset C^{r,\alpha}(\mathbb{R}^n). \tag{3}$$

5 Basic Analysis

Theorem 8 (Young's Inequality)**.** *Let p and q be positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$. Then, for any non-negative a and b it holds*

$$ab = \frac{a^p}{p} + \frac{b^q}{q}. \tag{4}$$