Option Pricing: Using Bayes Implied Volatility

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1 Introduction

This paper is meant to be a gentle introduction to option prices and their relation to volatility. This is an important topic because, with the proper understanding of the relationship between prices and volatility, the market participant can better navigate his investment strategy for a favorable position. The majority of analysis between the two metrics will be through building mathematical equations and computation using Python. For this project, I wanted to personally be involved in the options market in order to experience first hand the movement of option prices and the changing dynamics of volatility so a small contract position was taken. The paper contains an introduction, a literature review where some basic concepts about options and volatility are introduced. A methodology section that introduces the mathematical and statistical tools employed for finding volatility metrics and other relating metrics, the main findings section where most of the applications and explanations are presented, and ending with concluding remarks about the project.

1.1 Data

For this project, I used daily stock price data for Google, Amazon, Apple and the SPY index downloaded from Yahoo Finance to calculate the historical volatility and periodic daily returns. The reason behind choosing these assets are due to their stability and more or less the predictability of their movements. They may better fit the assumptions held by the BSM model. Also, a year's worth of implied volatility data of calls and puts were taken from MarketChameleon for exploring potential market indicators as there are not as many indicators publicly available when it comes to options; most sites that offered abundant option information were subscription based and quite expensive.

2 Understanding Options

With the introduction of the Black-Scholes-Merton model ("Black & Scholes",1973) calculating implied volatility has become an easy task, and there probably is no model used more frequently for its calculation. Many scholars have since expounded on the model, some have hailed its simplicity and convenience while others not so much ("Haug & Taleb",2010). Nevertheless, it is a great place for any novice to begin as the model introduces the metrics that comprise the price of an option which many are from its underlying asset.

2.1 Options

Options are similar to futures in that an agreement is formed between two parties, one can exercise to buy and the other to sell at predetermined prices. However, with options, the buyer has the right to exercise his position but not the obligation, hence the name options. Although the value of derivatives mainly fluctuates in tandem with the value of the underlying asset, option prices do not always move in this sort of manner; therefore it is important to understand many other factors contributing to the value of the option's price and their effects. As understanding, some of the concepts in this section will be more for practical purposes.

2.2 Option Premium

When options are traded there are those who sell "write" an option contract and those willing to buy them. The option premium is the money charged to the option buyer that is payable to the seller at expiration of the contract; for stock options, premiums is listed as amount of dollars per share and majority of contracts are a ratio of one option contract to 100 shares of stock. The option premium consists of three components, the intrinsic value, time value and the implied volatility (IV) of the underlying asset where intrinsic value is influenced by the IV of the underlying asset, thus we can write an equation for the option premium as:

$$OptionPremium = IntrinsicValue - TimeValue \tag{1}$$

2.3 Intrinsic Value

The intrinsic value is made up of the spot price "current stock price" and the strike price. The strike price is when the buyer and seller of an option make an agreement on the future value of the price of the stock. To calculate the intrinsic value one only needs to subtract the current price of the asset from the strike price:

$$IntrinsicValue = CurrentStockPrice - StrikePrice \tag{2}$$

So the intrinsic value would be what the buyer would receive if the option was exercised today. A \$20 call option would be \$10 in-the-money if the stock was currently at \$30. It is important to note that one option contract is 100 shares of stock so the payout would be \$10 per contract, equalling the amount of profit to \$1000. If the intrinsic value were negative the option buyer would simply forego exercising his option; therefore, the intrinsic value would be zero instead of negative. Below is a table illustrating the intrinsic value of a call and put.

Table 1: Call and put intrinsic value

Strategy	current stock price	strike price	intrinsic value
call	\$100	\$85	\$15
put	\$100	\$120	\$20

Intrinsic value can be a subjective value as investors may speculate on the worth of a company and decide what its fair market value should be. Investors, whether trading options or other assets are always looking to find good deals, which is reflected in the price of an option but in a more complicated way than assets such as stocks and houses.

2.4 Time Value

The more complicated portion of the option is the time value. This is the additional amount an option buyer is willing to pay to buy the right to exercise an option. If the buyer wishes to hold the contract longer the time value will be higher, this is because as time passes there is a greater chance of the contract becoming profitable for the investor due to a favorable move in the underlying asset. Below is a table to show how the time value can be calculated:

Table 2: Call and put time value

Strategy	stock price	$85 strike price trading at	intrinsic value	time value
call	$100	$17	$15	$2
put	$65	$17	$20	$3

In the above table 2 we have simply utilized equation (3) to calculate the time value.

$$TimeValue = OptionPremium - IntrinsicValue \tag{3}$$

2.5 Historical Volatility

When one is looking for options to either buy or sell it is prudent practice to examine the past volatility of the asset. This would help the investor understand the magnitude of risk associated with the given asset, and in turn with the option contract.

Historical volatility is a statistical measure of how volatile the underlying asset has been in the past, there are several ways to calculate it but one common method we introduce here is the standard deviation from the mean. This means historical volatility can tell us how far the price of the asset can move from the average price in a given period. To calculate the historical volatility we first find the periodic daily returns.

$$PeriodicDailyReturns = \ln\left(\frac{s_n}{s_{n-1}}\right) \quad \forall\{s_1, ..., s_n, ..., s_N\} \tag{4}$$

Where s_i represents the stock price at a given period. Next, we calculate the standard deviation of the periodic daily returns.

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} = HistoricalVolatility \tag{5}$$

Here we are mainly interested in the annual historical volatility because it allows us to calculate the IV later which is a component in trading options.

2.6 Implied Volatility

Implied volatility (IV) is the current market's prognosis of the underlying asset's movement and shows correlation to the market which in turn effects option prices ("Ganti",2019). Although in general volatility seems to increase more in bearish markets due to phenomena, a high IV is not an indication of a bear market, nor does it indicate the direction of the movement of the asset. The prognosis is more indicative of the magnitude of the movement, but because IV is a probability it tends to over estimate the magnitude of movement. Nevertheless, IV is used by investors to find favorable trades; such as when volatility is high investors may find a different sector or hedge their current positions. The method by which IV is calculated will be introduced later as it is a useful indicator for investors.

2.7 The "Greeks"

Such as derivatives represent the sensitivity of assets, Greeks represent the sensitivity of options to assets. Things can get confusing pretty fast, and we will not touch too much on this subject in this paper but it is important to introduce them as they are powerful measurements that can be used for risk management strategies when balancing portfolios. The main two Greeks that will be touched on are delta, δ and vega, ν as these play crucial parts when working with the Black-Scholes-Merton model.

Both delta and vega are first-order partial derivative equations of an option pricing model. Delta is used to measure the sensitivity between the option's price and a $1 change in the asset. The scale is measured between 0 and 1 for calls and 0 and -1 for puts, so if a delta of 0.9 is calculated, a $1 increase in the asset would result in a $0.9 increase in the option. Moreover, there are further sophisticated delta position strategies that can be taken; some informative reading on the subject can be found on Charles Schwab's website ("Chiappetta",n.d.).

Vega measures the sensitivity between the asset's IV and its option price. As with vega the ratio is defined by a 1% increase in the asset's volatility to the option price. Since volatility is an influential metric in the pricing of options, a ν close to one would indicate a higher option price which entails more volatility and possibly higher risk. According to Investopedia "Vega is at its maximum for at-the-money options that have longer times until expiration." ("Suma", 2019).

Although there have been only two Greeks briefly discussed in this paper there are many more, some are even second and third order derivatives where they are implemented more into software as the computations can become quite burdensome.

3 Methodology

This chapter briefly explores some of the mathematical and statistical methods used for calculating things such as implied volatility, option prices, and probabilistic outcomes of market speculation which often times gives investors an edge in the market. Those learning about option pricing or derivative pricing, in general will encounter these methodologies or if not, more. The methodologies below were implemented using Python.

For further understanding, there have been a myriad of approaches taken by scholars regarding option pricing; some of the more well known literature such as the article from Black, F., and M. Scholes (1973) where I believe the Black-Scholes equation was introduced, the widely used Markov-Switching model by Hamilton (1989) using Hidden Markov models, and the paper by Kim and Nelson (1999) incorporating a unique Bayesian approach, provide informative insight into the methodology of option pricing.

3.1 Black-Scholes-Merton Model

The Black-Scholes-Merton (BSM) option pricing model is implied from a partial differential equation-PDE called the Black-Scholes equation. This equation was constructed to assess the price movement of European call options and European put options and that one could eliminate risk by hedging the option position through the delta-hedge portfolio strategy. Incorporated in the equation are many fundamental assumptions about asset prices and its movements, this is because option pricing is determined strongly by these factors.

3.2 Asset Price and Risk

Since the price of the asset S_t changes with time t its current value can be estimated by taking yesterdays stock price S_{t-1} and multiplying it by the exponential of periodic daily returns, $\exp(pdr)$:

$$S_t = S_{t-1} exp(pdr) \tag{6}$$

The pdr is the rate the asset changes on a daily bases and is continuously compounded. If there was no risk involved theory is that all markets would increase at the same rate because any discrepancy in market pricing would be removed by risk-less arbitrage. This means with risk removed stock prices would increase at a constant rate that is equal to the risk-free rate such as a government bill or bond. Because risk coexists with the market there is uncertainty that is introduced into the pdr of tomorrows price making it a stochastic component. Under the Black-Scholes equation, this stochastic component of the price movement is assumed to follow a geometric Brownian motion ("Bachelier",1870-) hence, the asset price change can be defined as a stochastic process:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{7}$$

Here the change in our asset price,S_t, is determined by two components a constant drift $\mu S_t dt$ and the stochastic component $\sigma S_t dW_t$ where dW_t is a Brownian motion. Equation (7) is indicating that there is not only the stochastic component $\sigma S_t dw_t$ that is affecting our asset price, but it also is capturing a drift from the mean which would indicate that the mean over time is not constant as well.

3.3 Black-Scholes Equation

In order to obtain the famous BSM model, it is important to note that the value of S_t in equation (7) follows an Ito's process where dW_t is a Brownian motion and μ and σ are parameters that are functions of the asset price S_t. Also, Ito's Lemma shows that we can model any other function, say V, that is of the asset price S_t and derive another Ito's process as follows:

$$dV = \left(\frac{dV}{dS}\mu S + \frac{dV}{dt} + \frac{1}{2}\frac{d^2V}{dS^2}\sigma^2 S^2\right)dt + \frac{dV}{dS}\sigma S dW_t \tag{8}$$

Where the drift rate is in parentheses and the stochastic component is all that is right of the addition sign.

Now we incorporate the delta-hedge portfolio where we are short on our option position and long with $\frac{dV}{dS}$ number of shares ("Chan",n.d.) over a time period and call it H.

$$H = -V + \frac{dV}{dS}S \tag{9}$$

If we substitute (7) and (8) into (9) the stochastic terms will vanish this means that we have created a risk neutral portfolio in theory. Therefore, the rate of return on this portfolio should be the same as a risk-free investment r. This, in turn, would let us set H equal to rH ("Wikapedia Contributors", 2019) and ultimately give us the partial differential equation that Fisher Black and Myron Scholes derived:

$$\frac{dV}{dt} + \frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + rS\frac{dV}{dS} - rV = 0 \tag{10}$$

3.4 The Model

The model being used in this paper for option pricing, the Black-Scholes-Merton model, was not published until later when Robert Myron used stochastic calculus to mathematically interpret the Black-Scholes partial differential equation. The famous model for both a call option and put option are as below:

$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{11}$$

$$P(S,t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \tag{12}$$

Where

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{13}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{14}$$

S_0 is the stock price, t is time to maturity in years, K is the strike price, and σ is the volatility with $N(.)$ indicating the CDF of a normal distribution function. Also It is imortant to note here that $\delta = d_1$.

Today this model is still widely used for calculating IV. In order to do this one would enter the adjusted closing price S, the strike price of the option K, the risk-free rate such as the 10 year treasury rate r, the time to maturity in years t, and the annual historical volatility σ into the model and use Newton-Raphson method to calculate the IV to match the desired target option price. This is where ν is used as it is the sensitivity of options price to the IV.

Although it is a very convenient method, due to the strong assumptions regarding the model it has raised some concerns when pricing options. If one were to plot the derived IV's against their respective strikes a convex curve or "volatility skew" instead of a straight line would show such as in the paper by Rexhepi (2008); this curve is used as an explanation to why the assumption of the BSM does not hold. The curvature would nullify the constant mean and variance assumption which is part of the Brownian motion theory.

3.5 Newton-Raphson Method

The Newton-Raphson is a powerful numerical method for solving equations and is often used to calculate IV of an option price. The formula is a Taylor Series of a function, and the equation is presented below without proof:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{15}$$

The formula expresses an iteration method where we use the previous value to find a better approximation and repeating it until we find a convergence value. In our case we will use this method to isolate the volatility value in our BSM model, making the formula look as follows:

$$\sigma_{n+1} = \sigma_n - \frac{diff}{Vega} \tag{16}$$

where $diff$ would be the difference between the current calculated stock price from the BSM model and the market option price value. $Vega$ is the first partial differential equation to the BSM with respect to the volatility, σ.

$$\begin{aligned} Vega &= \frac{\delta}{\delta\sigma} SN(d_1) - Ke^{-r(T-t)}N(d_2) \\ &= S\sqrt{t}N(d_1) \end{aligned} \tag{17}$$

Finally, a limit of the error term is needed to be defined for the convergence criteria. In this case the error ϵ was set to within 1×10^{-6} accuracy. The final volatility that is used to calculate the option price within the specified accuracy is returned as the IV of the option.

3.6 Bayesian Statistics

Bayesian statistics has been categorized as not just a different method of statistical inference but as an entire category of its own. The differences of approach in analysis are thought to be so drastic that the statistical community has often been divided between Frequentism and Bayesianism. Both are useful and very important to know for statisticians today.

3.7 Bayes Formula

A fundamental theorem to grasp in the field of Bayesian statistics is Bayes Theorem. Named after Thomas Bayes who formulated a specific case of the theorem in his famous 1763 paper "An Essay towards solving a Problem in the Doctrine of Chance".

The derivation of the equation is pretty straight forward. Given two events A and B we can derive an expression of the conditional probability that event A will happen given B. In mathematical terms, this is described as $P(A|B)$, and the equation is given below:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{18}$$

where $P(B \cap A)$ is the joint probability of A and B happening, and that is divided by the conditioning event $P(B)$. It is also true that the opposite can be said about the two events where B is conditional on A.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \tag{19}$$

If we were to define equations (18) and (19) in terms of joint probability, set them equal to each other, and solve for either $P(A|B)$ or $P(B|A)$ we would come up with Bayes Theorem. The steps are shown below.

First, we define our joint probabilities:

$$P(A \cap B) = P(A|B)P(B) \tag{20}$$

$$P(B \cap A) = P(B|A)P(A) \tag{21}$$

Then we set them equal and solve for $P(A|B)$:

$$P(A|B)P(B) = P(B|A)P(A) \tag{22}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{23}$$

Thus equation (23) is Bayes formula. Here A is the unknown and B is our observation; we simply want to know A given our observations B. This implies that observations are the fixed values and not our unknown parameter A, and because A is unknown we can use a distribution to describe how much we know about this unknown parameter. This implies that we are creating a probability model where we assign probability distributions to our data B as well as our unknown parameter A. As you can see from equation (23) this has been done in Bayes formula. $P(B|A)$ is called the likelihood function which expresses the probabilistic likelihood of our evidence given the data. $P(A)$ is our prior, this is the probabilistic belief we currently have toward our unknown parameter A prior to our data. $P(B)$ is the total or sum of all possible outcomes, this is used to keep the outcome as a value between zero and one. Most often $P(B)$ will be calculated using integrals and approximation methods making it computationally burdensome without computers. Lastly, it is important to note that our outcome $P(A|B)$ is called the posterior distribution.

3.8 Bayesian Inference

In Bayesian statistics, there is one estimator equation and we have defined that as Bayes formula, equation (23). The formula is essential to Bayesian inference because this enables us to utilize Bayes Theorem where we constantly update our knowledge pertaining to the subject given more evidence is shown. We simply do this by substituting our posterior as our prior and running it through Bayes formula until a posterior distribution over possible points for the parameter of interest is created. It is this probabilistic interpretation of the parameter that is perhaps the striking difference between a Bayesian approach and Frequentist approach. A great study for one to better understand the power of Bayesian inference is the study done by Richard Fowles on motorcycle helmet laws ("Fowles", 2017).

3.9 Hamiltonian with NO-U-Turn Sampler

Many today are now familiar with powerful statistical analysis methods such as Bayesian using Gibbs sampling (BUGS) and Just another Gibbs sampling (JAGS) which implement Markov chains with a Monte Carlo simulation restricted by a sampling methodology such as Gibbs sampling or Metropolis Hastings. The popularity of these methods has exploded with the computational power of the computer but they have not been without their flaws as well. One very efficient sampling method to improve on some of the flaws was published in an article by Hoffman and Gelman (2014) called the No-U-Turn sampler, where it propounds on the improvement of Hamilton Monte Carlo (HMC). In a simple explanation, HMC restricts the pure randomness and averts correlation of parameters that can at times pose problems in Markov chain Monte Carlo (MCMC) sampling. Although improvements have been made the HMC's performance is largely dependent on the user's specification of two parameters ϵ (step size) and L (number of steps) when setting the acceptance criteria. If ϵ is too small the algorithm will take steps that are too small; if ϵ is too large there will be a low acceptance rate. If L is specified too small the samples will be too similar to one another, and if L is too large the trajectory will loop back (U-turn) and retrace itself. The randomly initiated trajectory for HMC can be expressed as a joint distribution of θ and r below ("Neal", 2011; "Duane et al.", 1987).

$$p(\theta, r) \propto exp\{\mathscr{L}(\theta) - \frac{1}{2} r \cdot r\} \tag{24}$$

With "leapfrog" integrator updates of variables as,

$$\begin{aligned} r^{t+\epsilon/2} &= r^t + (\epsilon/2)\Delta_\theta \mathscr{L}(\theta^t) \\ \theta^{t+\epsilon} &= \theta^t + \epsilon r^{t+\epsilon/2} \\ r^{t+\epsilon} &= r^{t+\epsilon/2} + (\epsilon/2)\Delta_\theta \mathscr{L}(\theta^{t+\epsilon}) \end{aligned} \tag{25}$$

Here $\mathscr{L}$ is the logarithm of joint density of the θ's and $r \cdot r$ is an inner product of vectors. r^t is the momentum at time t, θ^t is the position at time t, and Δ_θ is the gradient with respect to θ. L is specified when α, the acceptance probability, is used to accept or reject the samples from time t to ϵL. The algorithm is very sophisticated so further explanation can be found in the aforementioned article.

Hopefully, by now it is somewhat transparent where the NUTS algorithm gets its name. It is still an MCMC algorithm which has the resemblance of HMC but eliminates the need for specification of the parameters ϵ and L; in other words, it is an expansion of HMC and uses the joint distribution (25). First, in order for L to be a desirable number of steps without specification, a criterion was set as to stop the simulation when the distance between the initial θ and the new $\hat{\theta}$ no longer increased. This distance would be right before the simulation started to make a U-turn. However, the paper ("Hoffman & Gelman", 2014) explains that this would not take into account the time reversibility and therefore, the Hamilton simulation is run both forward and backward in time with the leapfrog integrator to trace out a path. The algorithm introduces a slice variable u to the joint probability from HMC,

$$p(\theta, r, u) \propto \Pi[u \in [0, \exp\{\mathscr{L}(\theta) - \frac{1}{2} r \cdot r\}]] \tag{26}$$

When the expression is true $\Pi[.]$ is 1 and when it is not, 0 u acts as a slicer to stop the trajectory then the algorithm carefully selects a state that is defined within the trajectory, and lastly choosing the next position and momentum uniformly at random.

Now to optimize ϵ, an adaptation of the primal-dual algorithm Nesterov (2009) was recommended by the authors. More can be read by following the reference.

3.10 Hidden Markov Models

Hidden Markov models (HMM) also known as Markov-Switching models is a Bayesian Network model that further augments the Markov Chain. Although the Markov Chain is very useful when computing the probability of sequences for observable events, one can run into difficulty when trying to find probabilities of unobservable or hidden events. This is where HMM comes to application. HMM is applied to find the probability of hidden events given we can observe something. A simple example to illustrate an observable event and a hidden event can be found in the financial market. Say that we wanted to find whether a certain asset's beta value is rising or declining by the observance of the number of good news in its sector; our observations would be the number of good news and the hidden states would be the value of beta. A more general definition can be applied to define the metrics needed for an HMM.

$$S = S_1, ..., S_n \quad \text{with n hidden states}$$

$$O = O_1, ..., O_m \quad \text{with m observables}$$

$$T = a_{11}, ..., a_{ij}, ..., a_{nn} \quad \text{transition matrix}$$

$$E = P(O_i|S_i) \quad \text{emission matrix}$$

$$\pi = \pi_1, ..., \pi_n \quad \text{initial probabilities over hidden states}$$

$$\lambda = \pi TE \quad \text{probability of the state at } t = 1$$

There can be as many as n hidden states if one chooses, T is a matrix that shows the probability of transitioning from one hidden state to another, and E is the likelihood of seeing an observation within a given state. We could then use the transition probabilities and the emission probabilities to construct a Bayesian Network of probabilities. Now, given a number of sequences of observations, we could find the hidden state sequence that caused the observations, this is known as decoding. There are two more problems that characterize the HMM ("Jurafsky & Martin", 2018). The first is the likelihood problem which is also known as the forward algorithm. This algorithm is used to find $P(O|\lambda)$ or in other, words the probability of observing our event given our state probability at $t = 1$. The third fundamental problem is learning/training, also known as the forward-backward algorithm. Instead of inquiring about the sequences, the training algorithm will try to learn the probability matrix T and emission matrix E by using the sequences of observations and the set of states. With these algorithms together HMM becomes a powerful tool for statisticians and computer scientists by helping them to identify relations between observed states and hidden states.

4 Main Findings

4.1 Application

To gain a better understanding of options I started by contacting my brokerage and getting approval for options trading. As I learned, to get approval for options trading level 1 one must answer a series of questions and show that they are competent enough to understand the risks associated with investing in options. Unfortunately, I was not approved so they gave me a level 0 clearance. There are a total of 3 levels and each gives the account holder more sophisticated methods to trade options. With a level 0 clearance, I am not able to buy options but am allowed to write cash covered orders. Some strategies that are associated with a level 0 account are covered calls and puts, protective puts and calls, long and short collars, and cash security equity puts. The strategies presented are mainly for risk-management and can offset losses when investing in assets such as stocks and ETFs.

4.2 Long Collar Strategy

Now that I had opened up an account I decided to put theory to the test. I looked for possible "deals" in the market, some main criteria for selecting an asset were the companies current debt and the potential of an increase in money flow to the sector in the future. Prudent advice when investing can be found from a book by Benjamin Graham ("Graham, 1949"). In order for an investor to buy stock or ETF options the underlying asset must be listed under a respected stock exchange such as the NASDAQ or NYSE, and with my level 0 options account I must have the necessary cash to cover my orders. This means if I wanted to write a single put order, I would need at least enough cash to buy 100 shares of that stock in my brokerage account. As you can tell this can get quite expensive for a student as a single share of some of the companies with strong brands such as Apple, Amazon, and Google can range from $100 to even $1000. I found a stock ranging in price from $2.50 to $3.00. After buying shares of the company I decided to execute a simple long collar strategy which is writing an out of the money (OTM) put and call order.

Table 3: Long Collar Strategy

Strategy	# of Contracts	Sell Price per Share	Premium
write call	1	$0.075	$7.50
write put	1	$0.225	$22.50

The premium that one receives from writing a contract can be calculated as,

$$\text{Premium} = \#\text{ of Contracts} \times 100 \times \text{Sell Price per Share} \tag{27}$$

In the above example with the Long collar strategy, we can hedge up to a 5% move in our underlying asset. Considering most well-established assets move daily on an average of 1% to 2% this is quite effective, and it is not taking into consideration the possible upward asset price movement. Although it is important to note, caution must be used because the stock price does have a possibility of losing all its value, in which case we would lose all of our investment. Before taking larger positions it would be recommended that the investor have more knowledge in the financial market with an arsenal of tools at his disposal.

4.3 Calculating Implied Volatility

At times it can be quite daunting entering into a contract without any idea of the direction of the market. For this, investors familiarize themselves with the IV metric. The IV is a metric that can inform us about the future volatility of the underlying asset. It is derived from calculating the option price; therefore, it is important to keep in mind that the IV gives us information about the underlying asset based on the price changes of the option and not the movement of the underlying asset. Just as upcoming earnings and news affect asset prices it can change option prices independent of asset value. This is because investors will take new positions or alter trading patterns to put them in a favorable position. This sort of movement in the market will only affect the time value of the option price and not the intrinsic value. Therefore, if option prices rise due to more activity IV will also be high. This makes IV more interesting for investors than historical volatility because it is a forward indicator.

One way to calculate the IV of an asset is the well known Black-Scholes-Merton (BSM) model. To do this one must input the current stock price, strike price, the risk-free interest rate, which I used the 10-year treasure value at the given day, the target option price value, and the annual historical volatility. These parameters were then put into a Python BSM function to calculate the option price. This price was then compared with the market value of the option price if the calculated price value was not within a specified accuracy range the calculation would be done again with a newly updated volatility using Newton-Raphson's method. After convergence was met, the volatility to obtain the market option price was returned, this would be the IV.

Three companies, Google, Amazon, and Apple annual price data were used. Later, the obtained IVs were compared with subscription-based websites MarketChameleon's

(MC) and AlphaQuery's (AQ) IV data, below is their comparison.

Table 4: IV comparison in percentage

Stock	BSM IV	MC IV	AQ IV	HV	IV%
GOOG	29.2	28.5	27.7	27.3	85.7
AMZN	30.1	29.8	29.4	35.5	57.5
AAPL	24.3	25.2	25.9	31.1	59.9

As can be seen from table 4 there are some slight differences in IV out comes, this could be due to the subtle difference in calculation methods. MarketChameleon may have calculated the 25 Delta IV for a call and put and used the average of the two. I used the average price of the ask and bid for a call order and put order, then used that as my market price to derive the IV for both a call order and put order. Finally, I took the average of the two for my final IV. Just by observing the percentages alone it would be hard to deduce any meaning because each stock's IV is independent of the other; therefore, IV percent was calculated. This was done by separately taking each stock's annual days where IV was less than the given percentage and divided by the total number of trading days. If the percentage returned back is above 50% the underlying asset would have higher than average volatility compared to its previous annual trading days. This would normalize the IVs and make them more comparable.

The IV can also be used for risk management. Some investors will calculate the 25 Delta IV for a call and put as it is used in the delta hedge strategy to neutralize risk. A simple example would be if an investor had 100 shares of a stock he/she could buy two -50 delta puts to eliminate risk; of course this is theoretical where the market is assumed to be predictable with no volatility skew. I have also taken sixteen put and call IV's of SPY and plotted them with their respective strike price to illustrate a skew that is present in the volatility of options. The volatility skew is due to the difference in IV of the opposite equidistant option. In a world where people place the same value on a call and put the skew would be non-existent. There is a famous saying among investors "Bulls walk up the stairs, bears jump out the window", this is indicative of a humorous metaphor of how the market operates.

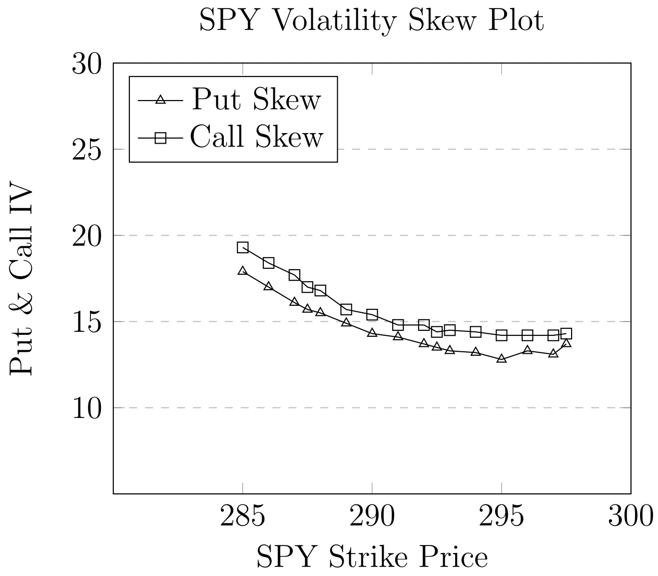

Figure 1: Volatility Skew

When markets move up they tend to do so slowly and steadily but when they fall, they fall hard and fast. The 1987 market crash was a hard lesson for many investors, including large hedge funds where the market made a 20 standard deviation move downward. The probability of such a deviation is close to none, yet it happened. The crash was so significant that since then large investment groups and retail investors alike have put emphasis on owning long, inexpensive put contracts which drives up the price for puts and creates a skew.

4.4 Bayesian Approach

If you look at an option price, the main determining factor of its price is the volatility, if utilized correctly the options trader could benefit from the asset price movement as well as the option price movement. All other things held constant when IV increases the option of the price will increase as well. If a trade was placed and volatility were to increase it would put the buyer in a good position by giving the buyer a higher chance for the contract to be in the money (BTM). Contrarily, if the IV were to decrease after the purchase of a position this would be good for the writer as he could take home the premium in a sideways market. Regardless of what method is used, understanding volatility can minimize exposure to risk and be part of prudent investing.

In this section, I used a Bayesian approach in finding the implied volatility of three companies, Google, Amazon, and Apple, and compared the values with the previously calculated IVs. Since IV is an "implied" metric it captures human behavior, and if traders and investors are using indicators to move in and out of positions there may be recognizable patterns within the market that prove beneficial in finding proper entry and exit points. But in this paper, I am interested in the posterior distribution of the IV's, which is the probability that the IV is in fact what the market is implying given the previous IVs. This would require to find a prior distribution and for this purpose, the least unpredictable companies that could be found were used. They hold a strong brand recognition with a robust market share in their sector, and the companies have already matured to a point where growth is steady.

The simple implementation will consist of taking annual implied volatility data and assume a constant mean and variance over the time period and define the data using a distribution. This could be done because there seemed to be no significant structural breaks, and the company is stable enough that the asset volatility would stay relevantly consistent. This is important as option prices are influenced by historical volatility which is derived from periodic daily returns, and IV is calculated from option prices. The graphs of periodic daily returns for Google, Amazon, and Apple are shown in fig 2, fig 3, and fig 4 (see Appendix A).

Next, the mean and variance were computed and the annual IV data of each company was taken and plotted with three distributions as a probability plot. The distributions were the normal, gamma, and inverse gamma distributions. All three distributions seemed to fit reasonably well, so it was a perfectly good reason to first set the normal distribution as the prior (see Appendix B). Next, the Hamiltonian Monte Carlo and NUTS were implemented to generate a posterior distribution of IV and calculate its mean and variance. There are three parts associated with this technique:

1. Hamiltonian Monte Carlo

2. Markov Chain

3. NUTS Algorithm

The Hamiltonian Monte Carlo is a random number generating technique with step size trajectories to sample the distribution space. The Markov Chain is a chain of numbers where the present number is dependent only on the previous, and the NUTS algorithm is used to choose which proposed number of IV it is to rejected or accepted from the simulation. Also, an additional update is performed by setting the posterior and its parameters as a prior distribution and adding in new IV data to use its distribution as the likelihood. All three companies were tested with a normal prior and an inverse gamma prior.

Table 5: Normal Bayesian IV

Normal Prior				
Company	**Mean**	**STD**	**MAP**	**68%**
Google call	23.3	5.24	$\approx$ 22.5	(18.1,28.5)
Google put	26.1	5.97	$\approx$ 26.8	(20.1,32.1)
Amazon call	29.5	7.8	$\approx$ 33.0	(21.7,37.3)
Amazon put	32.7	8.82	$\approx$ 35.5	(23.9,41.5)
Apple call	23.3	5.23	$\approx$ 24.3	(18.07,28.5)
Apple put	26.1	5.92	$\approx$ 25.3	(20.2,32.0)

Table 6: Inv.Gamma Bayesian IV

Inverse Gamma Prior				
Company	**Mean**	**STD**	**MAP**	**68%**
Google call	23.8	5.80	$\approx$ 23.0	(18.0,29.6)
Google put	26.4	6.76	$\approx$ 24.5	(19.64,33.16)
Amazon call	30.1	8.5	$\approx$ 27.2	(21.6,38.6)
Amazon put	33.2	9.93	$\approx$ 28.5	(23.3,43.1)
Apple call	23.5	5.96	$\approx$ 22.2	(17.54,29.5)
Apple put	26.5	7.56	$\approx$ 23.5	(18.94,34.1)

If we take the averages of the Bayesian call and put IV we would obtain values similar to those derived from the BSM for the exception of Google. Google's BSM IV is more than one standard deviation away from the Bayes call IV derived from a normal prior. This would tell us that there is less than a 68% chance that the underlying asset would move to an asset price indicated by the BSM IV. This is pretty interesting because the IV% on Google is at a high of 85% but the Bayes IV is telling us that the value should be lower. With this indication, the chances of the asset actually moving as implied by the BSM IV may be lower than what the market is implying. From this, we may be able to hypothesize that the current volatility potential with Google is less likely to be as drastic and the option prices may be overpriced and it is not a good buying opportunity.

Table 7: Comparing Bayes IV

Company	Bayes		BSM	MC	AQ
	N	Inv.G			
Google	24.7	25.1	29.2	28.5	27.7
Amazon	31.1	31.7	30.1	29.8	29.4
Apple	24.7	25	24.3	25.2	25.9

Although this is a simple market indicator that needs a lot of tweaks and may not be very applicable to other assets, further experimentation and elaboration for improvement would be interesting. The next security to implement this method would be with the SPY index.

4.5 Going Further...

Going forward from this point would entail incorporating a method that can be used without specifying constant parameters over the entire time series data. One such model taken into consideration is the Hidden Markov model (HMM) where observable events could be used to find distribution parameters. There hasn't been a lot of literature published using HMM without specifying a time series process, in my search I have run into one by Ishijima and Takahira (2005).

5 Conclusion

Within this paper, I have only focused on one small aspect, volatility and its relation to assets and option prices, but the market is comprised of many other parameters that may be of good use for investors. However, volatility is still a very important part of options and implied volatility is especially favored by option contract buyers and sellers. Also, the Bayesian IV may be of interest to contrast with the IV obtained from the BSM as one is forward-looking and the other is derived from past market probability.

Although the Bayesian approach in estimating the IV of an underlying asset showed to be promising, further work is in need for a more robust market indicator because so far we must assume a constant mean and variance over the entire time series. Markov-Switching models are a potential option as well as non-parametric approaches or assigning distributions to daily candlesticks may also be of interest. Also, it can be stated from experience that by understanding covered calls, covered puts and possibly other hedging strategies will reduce the amount of risk the investor is exposed to. Interestingly, it has made me realize that risk management employed by options strategies will help avoid risk not by less market exposure but by market diversity. In fact, one would become more immersed in the market. Although hedging strategies may be complicated at times the overall outcome has made my life simpler in the investing world.

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A APPENDIX

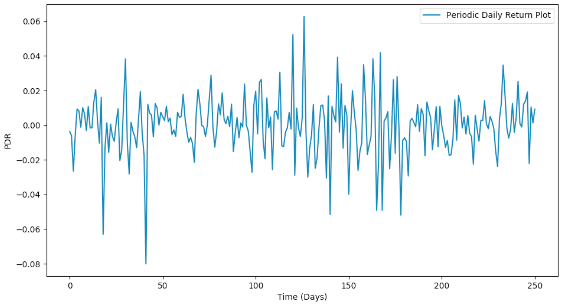

Figure 2: Google PDR

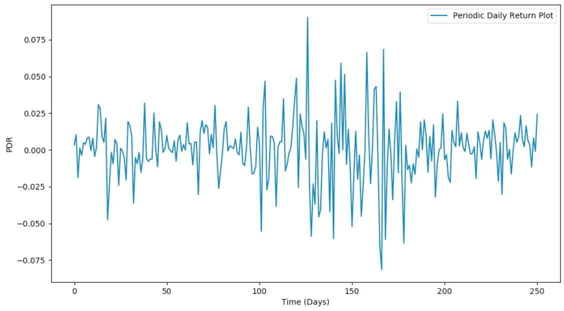

Figure 3: Amazon PDR

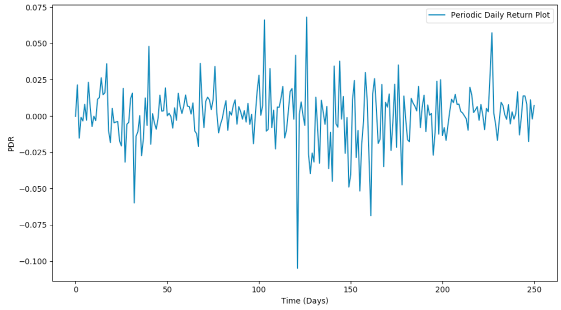

Figure 4: Apple PDR

B APPENDIX

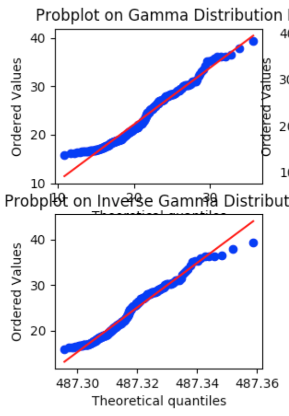

Figure 5: Google Call IV QQplot

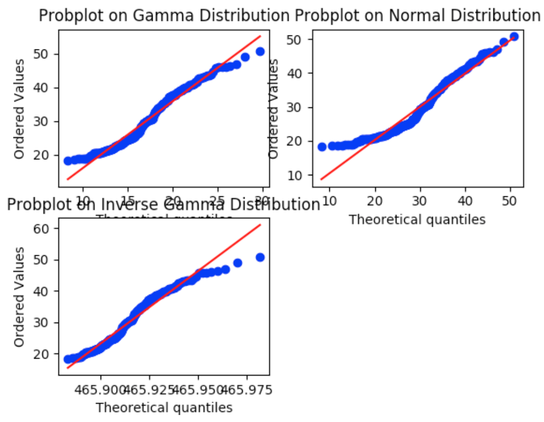

Figure 6: Amazon Call IV QQplot

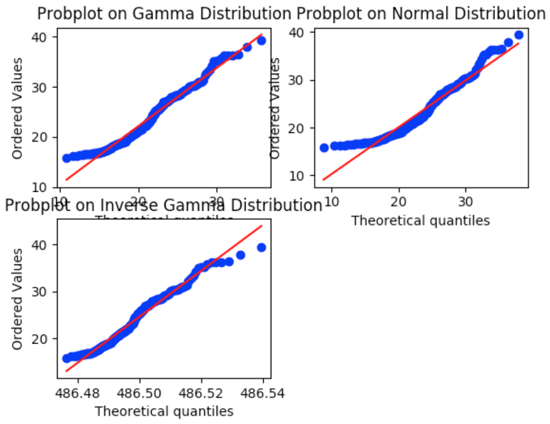

Figure 7: Apple Call IV QQplot

C APPENDIX

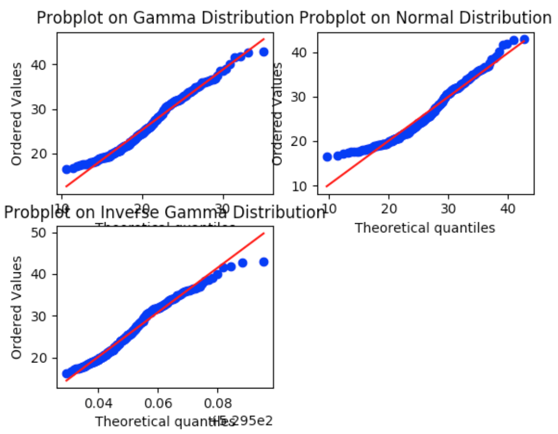

Figure 8: Google Put IV QQplot

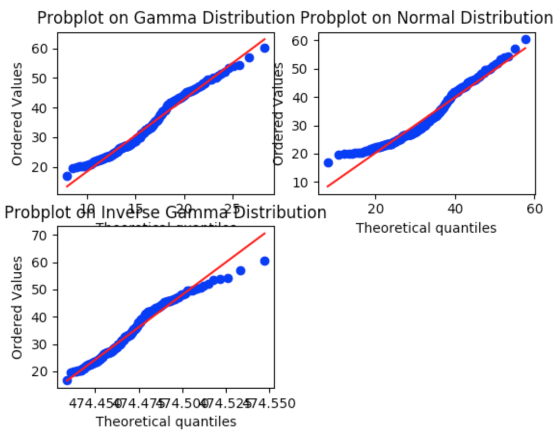

Figure 9: Amazon Put IV QQplot

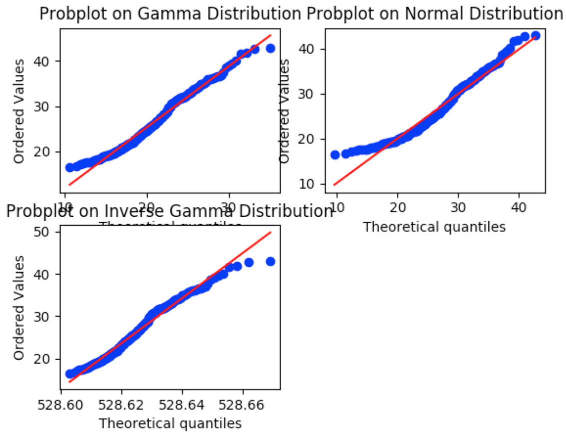

Figure 10: Apple Put IV QQplot