IMPERIAL

On the subject of lattices

and why cryptographers love them

Joshua Limbrey
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Agenda

What schemes are currently used for public key cryptography (signatures, key exchanges, etc.) today?

What is currently used for cryptography?

- Diffie-Hellman
- El Gamal
- RSA
- DSA
- ECDSA
- ECDH
- Ed25519

All public key cryptography relies on what is called a **trap-door function**.

Easy to go one way *(encrypt)*, difficult to go the other *(decrypt)* without knowledge of a secret *(private key)*.

All of the schemes listed to the left are all dependant on the hardness of **prime factorisation** or the **discrete logarithm problem**.

Why are we bored of these?

These schemes have been around for a while (some nearly 50 years). The security against a standard adversary has been extensively studied, and the hardness of the problems fairly well understood.

Challenging to construct new and interesting forms of encryption (such as *homomorphic encryption*) due to the properties of the underlying problems.

Shor's algorithm[a] means that given an adversary with a sufficiently strong quantum computer, these schemes are no longer secure.

[a] P.W. Shor. "Algorithms for quantum computation: discrete logarithms and factoring". In: *Proceedings 35th Annual Symposium on Foundations of Computer Science*. 1994, pp. 124–134. DOI: 10.1109/SFCS.1994.365700.

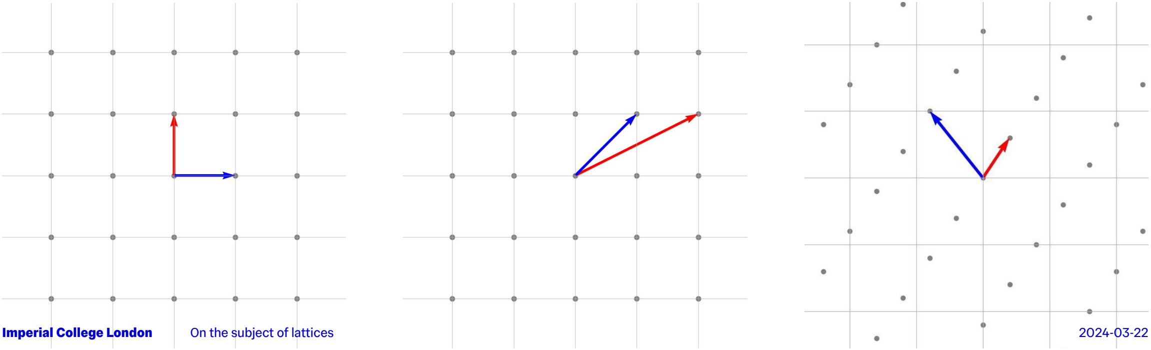
(Above) Cryptographers wanting new toys, circa 2000 (colourised)

What is a lattice?

Definition: Lattice

The set of all linear integer combinations of basis vectors,

$$\mathcal{L} = \{\sum_{i=1}^{d} \mathbf{b}_i \mathbf{x} \mid \mathbf{x} \in \mathbb{Z}^d\}$$



What are hard lattice problems?

Definition: Shortest Vector Problem (SVP)

Given a lattice, find the shortest *non-zero* lattice point.

Definition: Learning With Errors Problem (LWE)

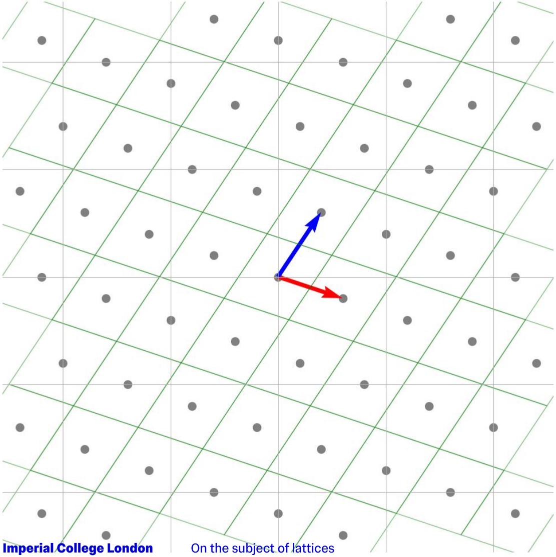
Let, $\begin{bmatrix} \mathbf{b} \end{bmatrix} = \begin{bmatrix} \quad \mathbf{A} \quad \end{bmatrix} \begin{bmatrix} \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{e} \end{bmatrix}$. Knowing *only* the values in green, find $\begin{bmatrix} \mathbf{s} \end{bmatrix}$.

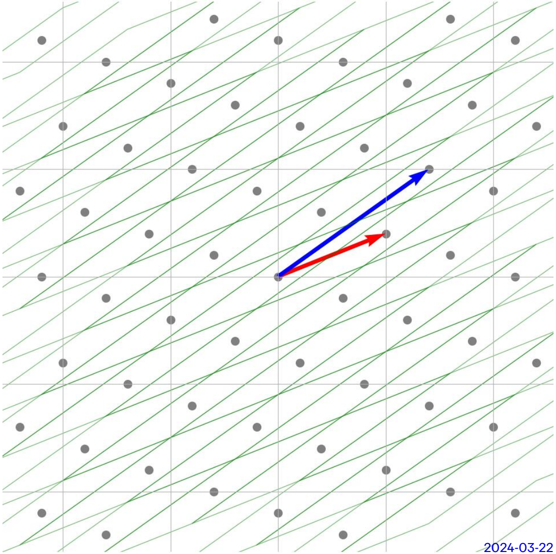
Definition: Lattice Isomorphism Problem (LIP)

Given two lattices, $\mathcal{L}_1$ and $\mathcal{L}_2$, find the scaling factor and rotation to send one to the other (if it exists).

But we also have the Short Integer Solutions problem (SIS), the NTRU problem, the Closest Vector Problem (CVP), and all the many variants of anything mentioned here!

Shortest/Closest Vector Problem



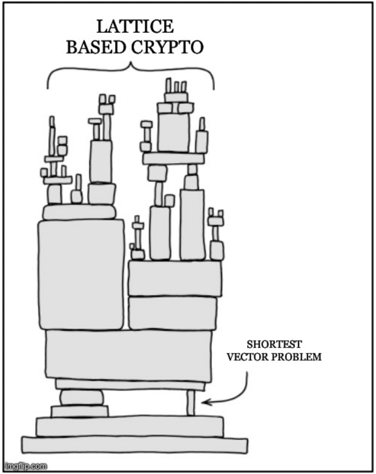


(Above) Cryptographers wanting new toys, circa 2000 (colourised)

It's all SVP?

(Above) Regev discovering the the LWE to γ-SVP reduction, 2005

But don't worry, even though all of these problems are reducible to SVP, they all work in slightly different ways, giving the schemes we build from them different properties (speed, key size, etc.) We also believe that SVP is a *very* hard problem, even for quantum computers.



(Above) xkcd Dependency, lattice based cryptography special edition

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Section Title Examples

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List Examples

Bullet List

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Three-Column Example

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Three-Column With Images Example

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Large Right Image Example

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Tiled Images Example

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Thank you

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Questions?