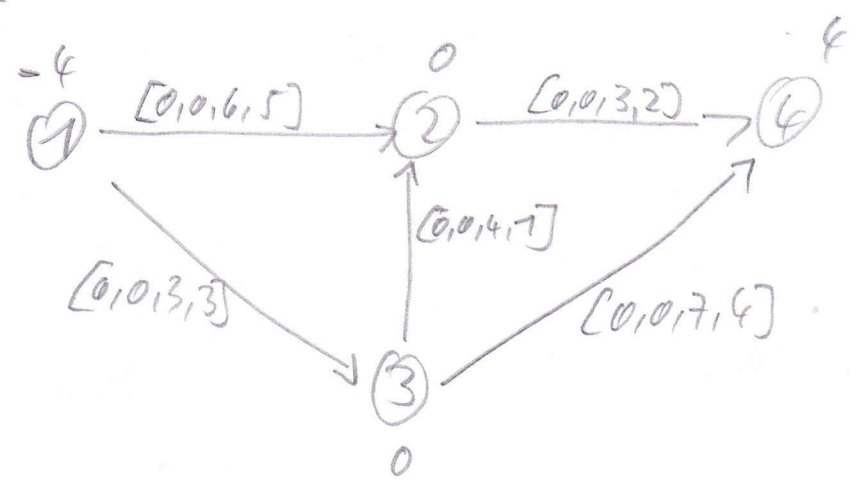
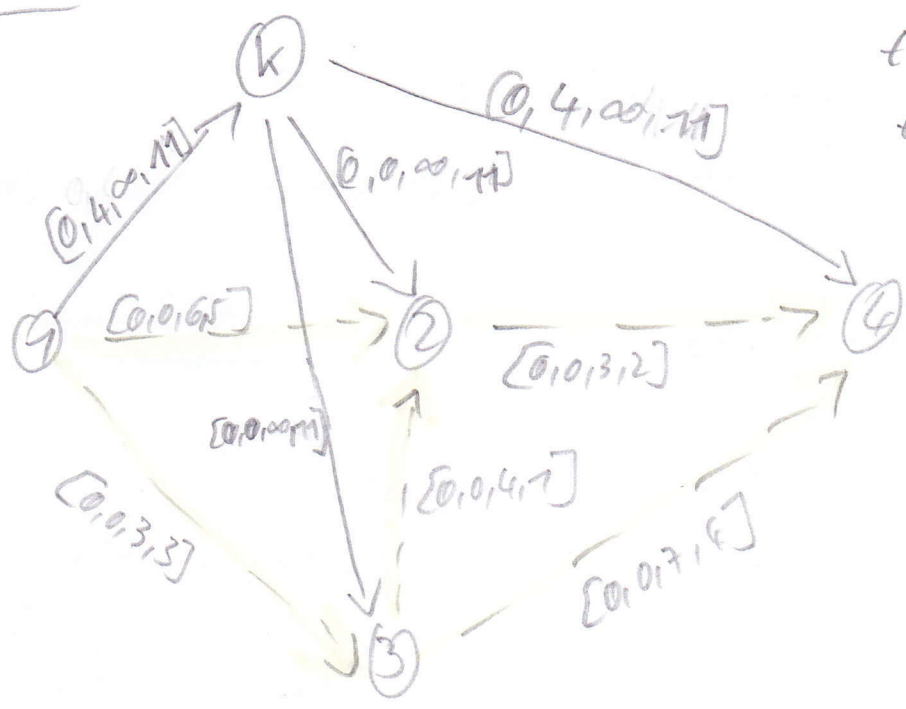


Beispiel test graph 2.txt

Netzwerk:  $[c_{ij}, x_i, u_{ij}, c_{ij}]$



Inhaltsbaum:



$t[k] = -1$        $d(u) = 0$   
 $t[1] = 2$        $d(i) = 1$   
 $t[2] = 3$        $i \in \{1, \dots, 4\}$   
 $t[3] = 4$   
 $t[4] = k$

$p(k) = -1$   
 $p(1) = k$   
 $p(2) = k$   
 $p(3) = k$   
 $p(4) = k$

Funktion:  $y(k) = 0$

$y(1) = y(k) + c_{1k} = 0 + 11 = 11$ , da  $(1, k) \in T$

$y(2) = y(k) - c_{k2} = 0 - 11 = -11$ , da  $(k, 2) \in T$

$y(3) = y(4) = -11$

$\boxed{T:} \bar{c}_{1k} = c_{1k} - y(1) + y(k)$   
 $= 11 - 11 + 0$   
 $= 0$

reduced Costs:  $\bar{c}_{ij} = c_{ij} - y(i) + y(j)$ ,  $\forall (i, j) \in T$   
 $\bar{c}_{k2} = 11 - 0 + (-11) = 0$

$\bar{c}_{k3} = 11 - 0 + (-11) = 0$        $\bar{c}_{k4} = 0$

A

$$\bar{c}_{13} = 3 - 11 + (-11) = -19, \quad \bar{c}_{12} = 5 - 11 + (-11) = -17$$

$$\bar{c}_{32} = 1 - (-11) + (-11) = 1, \quad \bar{c}_{24} = 2 - (-11) + (-11) = 2$$

$$\bar{c}_{34} = 4 - (-11) + (-11) = 4$$

$$\Rightarrow v_{\max}^{LH} = \bar{c}_{13} = -19 \Rightarrow \text{entering arc} = (1, 3)$$

$$\Rightarrow \text{Cycle: } (k, 1, 3, 2, k) \quad \text{Schritt} = k$$

$$\varepsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, x_{k3} - l_{k3})$$

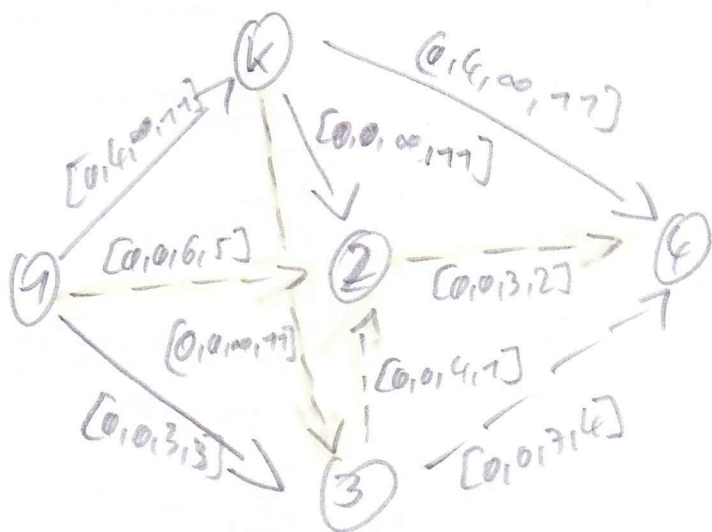
$$= \min (4 - 0 = 4, 3 - 0 = 3, 0 - 0 = 0)$$

$$= 0 \Rightarrow \text{letzter blockierender Bogen: } (k, 3)$$

$\Rightarrow$  keine Flussveränderung

leaving arc: (k, 3)

Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k \quad p(2) = k \quad p(4) = k$$

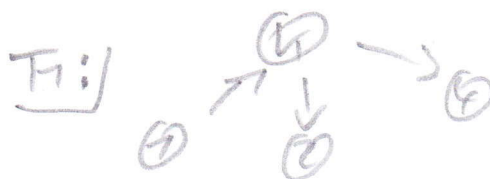
$$p(3) = 1$$

$$\ell(1) = 1 \quad \ell(2) = 3 \quad \ell(4) = 4$$

$$\ell(3) = 2 \quad \ell(5) = k$$

$$d(1) = 0 \quad d(2) = 1$$

$$d(3) = 1 \quad d(4) = 2 \quad d(5) = 1$$



T1:

T2:

(3)

Flow Prices Update:  $\text{change} = -\bar{c}_{13} = +19$   
 $\rightarrow v(3) := v(3) + \text{change} = v(3) + 19 = 8$

fertig, da  $d(\ell(3)) \leq d(3)$

reduced Costs Update:

$$T: \quad \bar{c}_{1k} = 0 \quad , \quad \bar{c}_{13} = 3 \quad -11 + 8 = 0 \\ \bar{c}_{k2} = 0 \quad , \quad \bar{c}_{k4} = 0$$

$$L: \quad \bar{c}_{k3} = 11 - 0 + 8 = 19$$

$$\bar{c}_{12} = -17 \quad , \quad \bar{c}_{32} = 1 - 8 + (-11) = -18$$

$$\bar{c}_{24} = 2 \quad , \quad \bar{c}_{34} = 4 - 8 + (-11) = -15$$

$$\Rightarrow \text{"max"}_{L, u} = \bar{c}_{32} = -18 \Rightarrow \text{entering arc} = (3, 2)$$

$$\Rightarrow \text{Circle} = (k, 1, 3, 2, k) \quad \text{Schritt} = k$$

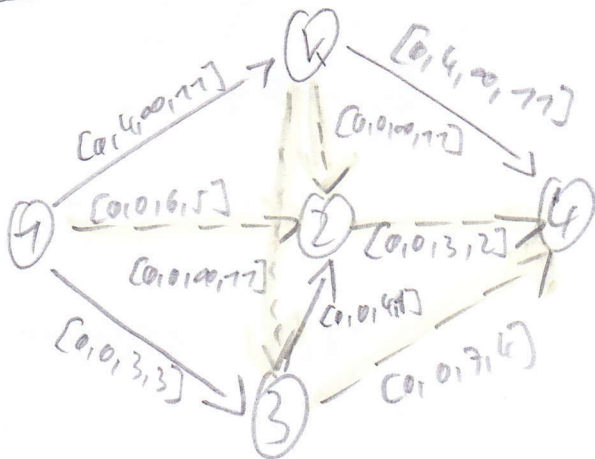
$$\epsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{32} - x_{32}, x_{k2} - l_{k2})$$

$$= \min (4 - 0 = 4; 3 - 0 = 3; 4 - 0 = 4; 0 - 0 = 0)$$

$$= 0 \Rightarrow \text{leaving arc} = (k, 2) \quad (\text{leider blockierender Bogen})$$

$\Rightarrow$  keine Flussänderung!

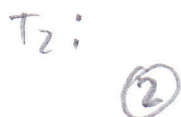
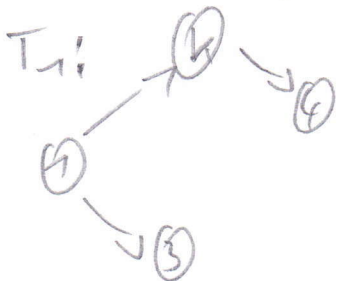
keine Baumstrukturen:



$$p(k) = -1, p(1) = k, p(2) = 3 \\ p(3) = 1, p(k) = k$$

$$t(k) = 1, t(1) = 3, t(2) = 4, t(3) = 2 \\ e(k) = k$$

$$d(k) = 0, d(1) = d(4) = 1 \\ d(2) = 3, d(3) = 2$$



### Fair Price Update:

$$\text{change} = -\bar{c}_{32} = -18 \rightarrow v(2) = v(2) + \text{change} \\ = -11 + 18 = 7$$

fertig, da  $d(f(2)) \leq d(2)$

### Reduced Costs Update:

$$\text{+!} \quad \bar{c}_{1k} = 0, \quad \bar{c}_{k4} = 0, \quad \bar{c}_{13} = 0$$

$$\bar{c}_{32} = -1 - 8 + 7 = 0$$

$$\text{L!} \quad \bar{c}_{k3} = -9, \quad \bar{c}_{34} = -15$$

$$\bar{c}_{12} = 5 - 11 + 7 = 1$$

$$\bar{c}_{k2} = 11 - 0 + 7 = 18$$

$$\bar{c}_{24} = 2 - 7 + (-11) = -16$$

$$\Rightarrow \underset{L, U}{\text{"max"}} = \bar{c}_{24} = -16 \Rightarrow \underline{\text{entering arc}} = (2, 4)$$

$$\Rightarrow \text{cycle} = (k, 1, 3, 2, 4, k) \quad \text{Schritt} = k$$

$$\varepsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{32} - x_{32}, u_{24} - x_{24}, x_{k4} - l_{k4}) \\ = \min (4 - 0 = 4, 3 - 0 = 3, 4 - 0 = 4, 3 - 0 = 3, 4 - 0 = 4)$$

$$\Rightarrow \varepsilon = 3 \Rightarrow \text{lehter blockierender Bogen} = (2, 4) \\ = \underline{\text{leaving arc}}$$

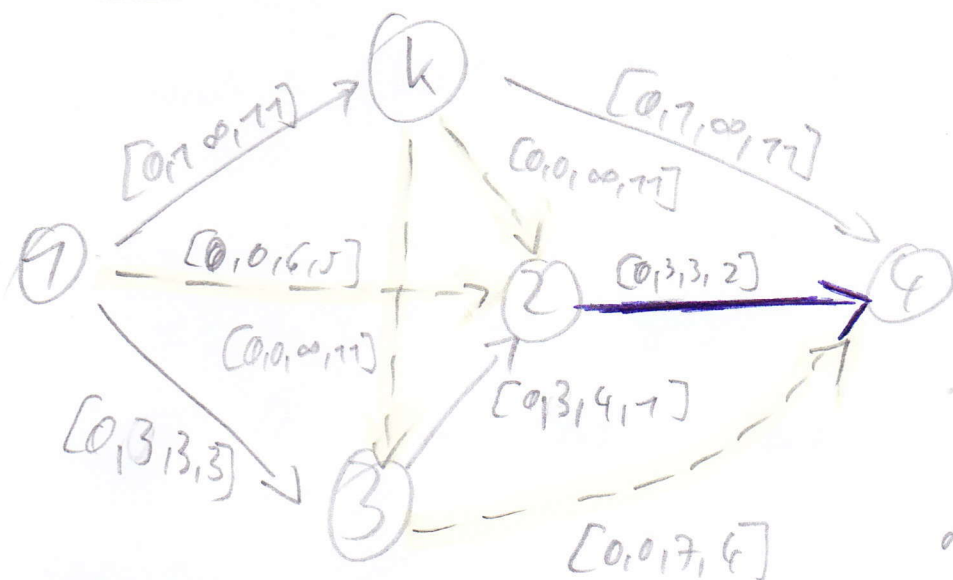
$$\underline{\text{Flussänderung}}: x_{24} := x_{24} + \varepsilon = 3, \quad x_{k4} := x_{k4} - \varepsilon = 1$$

$$x_{1k} := x_{1k} - \varepsilon = 1, \quad x_{13} := x_{13} + \varepsilon = 3$$

$$x_{32} := x_{32} + \varepsilon = 3$$

$$\Rightarrow (2, 4) \text{ und } (2, 4) \text{ in } U$$

# Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k = p(4)$$

$$p(2) = 3, p(3) = 1$$

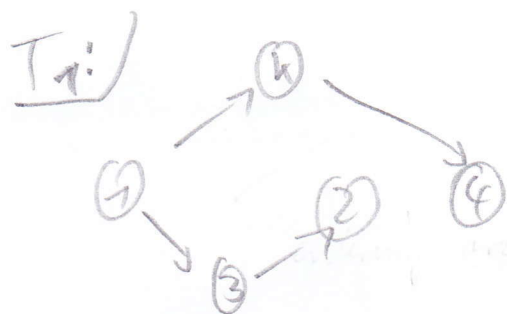
$$t(k) = 1, t(1) = 3$$

$$t(3) = 2, t(2) = 4$$

$$t(4) = k$$

$$d(k) = 0, d(1) = d(4) = 1$$

$$d(2) = 3, d(3) = 2$$



T2:

## Fair Price Update:

reduced Costs: (no update needed)

$$T: \bar{c}_{1k} = 0, \bar{c}_{k4} = 0, \bar{c}_{13} = 0$$

$$\bar{c}_{32} = 0$$

$$L: \bar{c}_{k3} = -19, \bar{c}_{34} = -15, \bar{c}_{12} = 1$$

$$\bar{c}_{k2} = -18$$

$$u: \bar{c}_{24} = -16$$



$$\Rightarrow \text{"max"}_{2,u} = \bar{c}_{34} = -15 \Rightarrow \underline{\text{entering arc} = (3,4)}$$

$$\Rightarrow \text{arc} = (k, 1, 3, 4, k) \text{ Scheitel} = k$$

$$\varepsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{34} - x_{34}, x_{k4} - l_{k4})$$

$$= \min (1 - 0 = 1, 3 - 3 = 0, 7 - 0 = 7, 1 - 0 = 1)$$

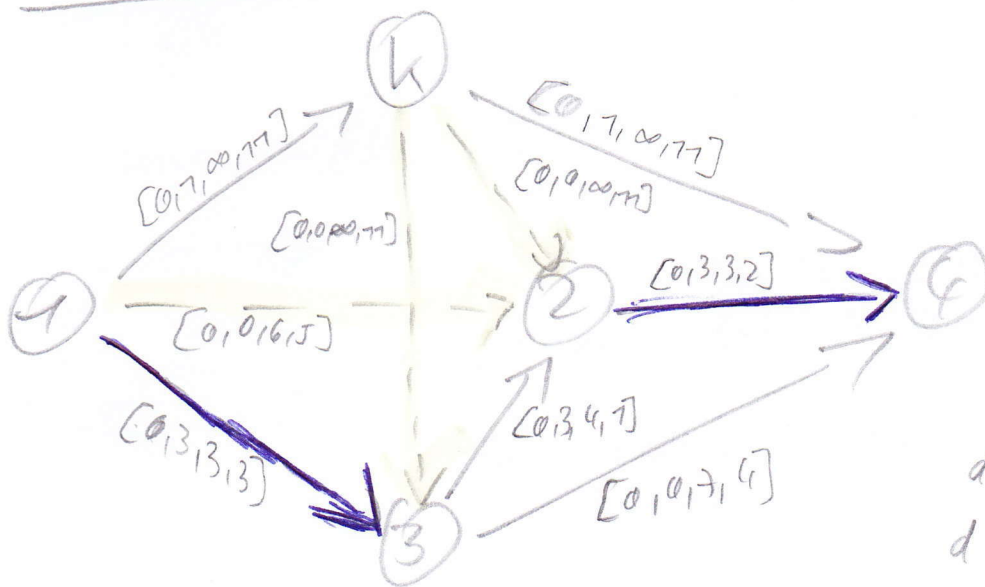
$$= 0 \Rightarrow \text{letzter blockierender Bogen} = (1,3)$$

$$= \underline{\text{leaving arc}}$$

Flussänderung: /

$$\Rightarrow (1,3) \text{ in } U$$

Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k = p(4)$$

$$p(2) = 3, p(3) = 4$$

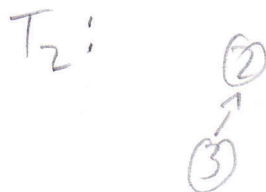
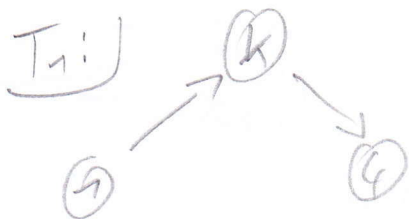
$$t(k) = 1, t(1) = 4$$

$$t(2) = k, t(3) = 2$$

$$t(4) = 3$$

$$d(k) = 0, d(1) = d(4) = 1$$

$$d(2) = 3, d(3) = 2$$



Fair Price Update:  $\text{change} = -\bar{c}_{34} = -15 \quad (3 \in T_2)$

$$y(3) := y(3) + \text{change} = 8 - 15 = -7$$

$$y(2) := y(2) + \text{change} = 7 - 15 = -8$$

### Reduced Costs Update:

$$\begin{array}{l} T: \quad \bar{c}_{1k} = 0, \quad \bar{c}_{k4} = 0, \quad \bar{c}_{32} = 1 - (-7) + (-8) = -02 \\ \bar{c}_{34} = 4 - (-7) + (-11) = 0 \end{array}$$

$$\begin{array}{l} L: \quad \bar{c}_{k2} = 11 - 0 + (-8) = 3, \quad \bar{c}_{12} = 5 - 11 + (-8) = -14 \\ \bar{c}_{k3} = 11 - 0 + (-7) = 4 \end{array}$$

$$\begin{array}{l} U: \quad \bar{c}_{13} = 3 - 11 + (-7) = -15 \\ \bar{c}_{24} = 2 - (-8) + (-11) = -1 \end{array}$$

$$\Rightarrow \underset{L, U}{\text{"max"}} = \bar{c}_{12} = -14 \quad \Rightarrow \text{entering arc} = (1, 2)$$

$$\Rightarrow \text{cycle} = (k, 1, 2, 3, 4, k) \quad \text{Schubst} = k$$

$$\begin{aligned} \varepsilon &= \min (x_{1k} - l_{1k}, u_{12} - x_{12}, x_{32} - l_{32}, u_{34} - x_{34}, x_{k4} - l_{k4}) \\ &= \min (1 - 0 = 1, 6 - 0 = 6, 3 - 0 = 3, 7 - 0 = 7, 1 - 0 = 1) \\ &= 1 \end{aligned}$$

$$\Rightarrow \text{leider blockierender Bogen} = (k, 4)$$

$$= \text{leaving arc}$$

### Flussänderung:

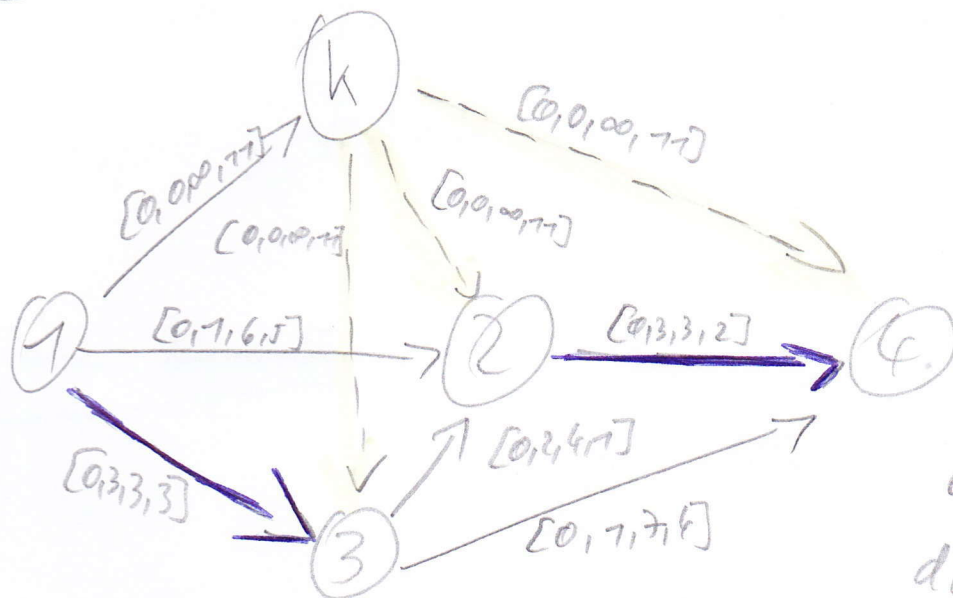
$$x_{1k} := x_{1k} - \varepsilon = 0, \quad x_{k4} := x_{k4} - \varepsilon = 0$$

$$x_{12} := x_{12} + \varepsilon = 1, \quad x_{32} := x_{32} - 1 = 2$$

$$x_{34} := x_{34} + \varepsilon = 1$$

$$\Rightarrow (k, 4) \in L$$

Neue Baumstruktur:



$$p(k) = -1, p(1) = k$$

$$p(2) = 1, p(3) = 2$$

$$p(4) = 3$$

$$t(1) = 1, t(1) = 2$$

$$t(2) = 3, t(3) = 4$$

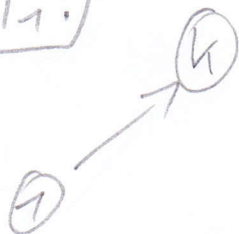
$$t(4) = k$$

$$d(k) = 0$$

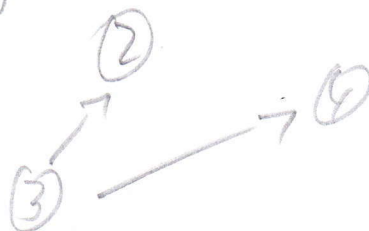
$$d(1) = 1, d(2) = 2, d(3) = 3$$

$$d(4) = 4$$

T<sub>1</sub>:



T<sub>2</sub>:



Fair Price Update:

$$\text{change} = -\bar{c}_{12} = 14, \text{ da } (1) \in T_1$$

$$y(1) := y(1) + \text{change} = -11 + 14 = 3$$

$$y(2) := y(2) + \text{change} = -8 + 14 = 6$$

$$y(3) := y(3) + \text{change} = -7 + 14 = 7$$

Reduced Costs Update:

$$\bar{c}_{1k} = 0, \bar{c}_{12} = 5 - 11 + 6 = 0$$

$$\bar{c}_{32} = 1 - 7 + 6 = 0$$

$$\bar{c}_{34} = 4 - 7 + 3 = 0$$

$$\bar{c}_{13} = 3 - 11 + 7 = -1$$

$$\bar{c}_{24} = 2 - 6 + 3 = -1$$

$$\bar{c}_{k4} = -11 - 0 + 3 = -14, \bar{c}_{k3} = -11 - 0 + 7 = -14$$

$$\bar{c}_{k2} = -11 - 0 + 6 = -15$$

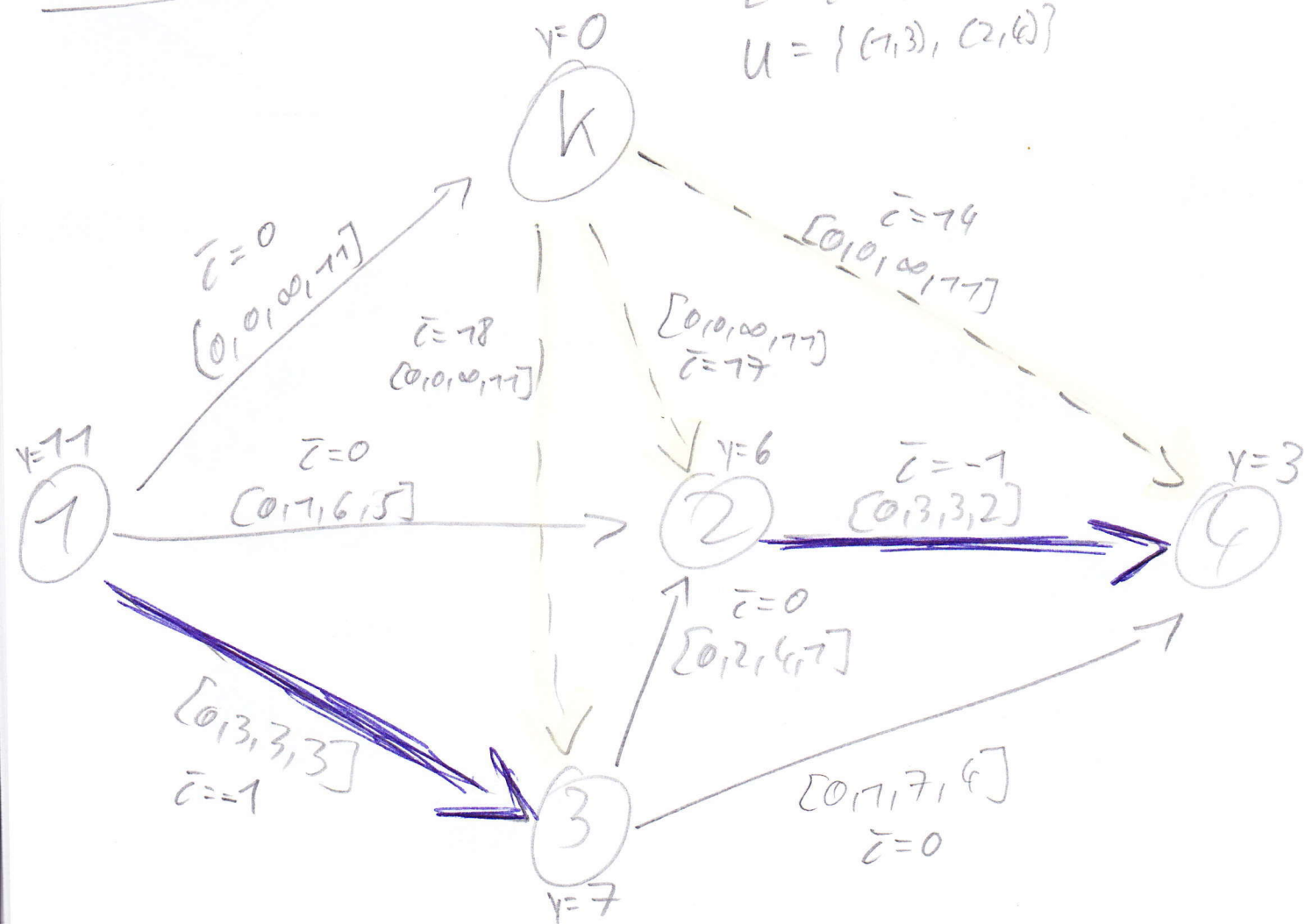
8



$$\left. \begin{array}{l} \bar{c}_{ij} = 0 \quad \forall (i,j) \in T \\ \bar{c}_{ij} \geq 0 \quad \forall (i,j) \in L \\ \bar{c}_{ij} \leq 0 \quad \forall (i,j) \in U \end{array} \right\} \Rightarrow \text{optimale Lösung erreicht, da} \\ x_{ij} = 0 \quad \forall (i,j) \text{ mit } i \neq k \vee j \neq k$$

Ende Baumstruktur:

$$\begin{aligned} T &= \{(1,k), (1,2), (3,2), (3,4)\} \\ L &= \{(k,3), (k,2), (k,4)\} \\ U &= \{(1,3), (2,4)\} \end{aligned}$$



Fluss:  $[c_1, x_1, c_2, c_3]$

