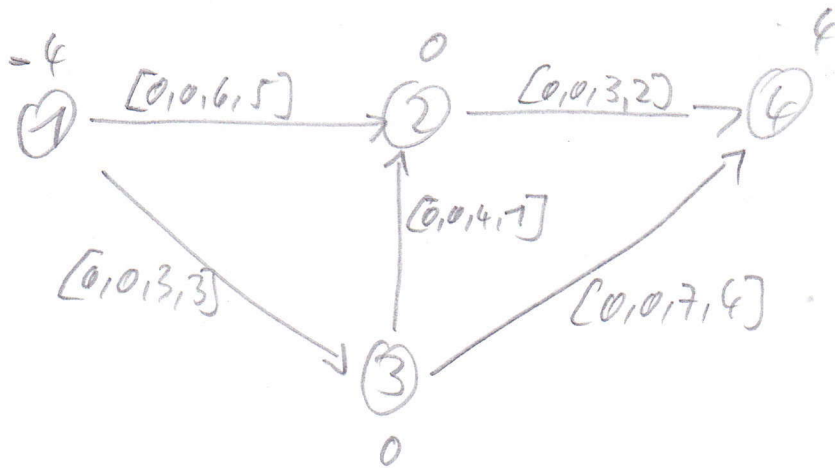


Beispiel test graph 2.txt

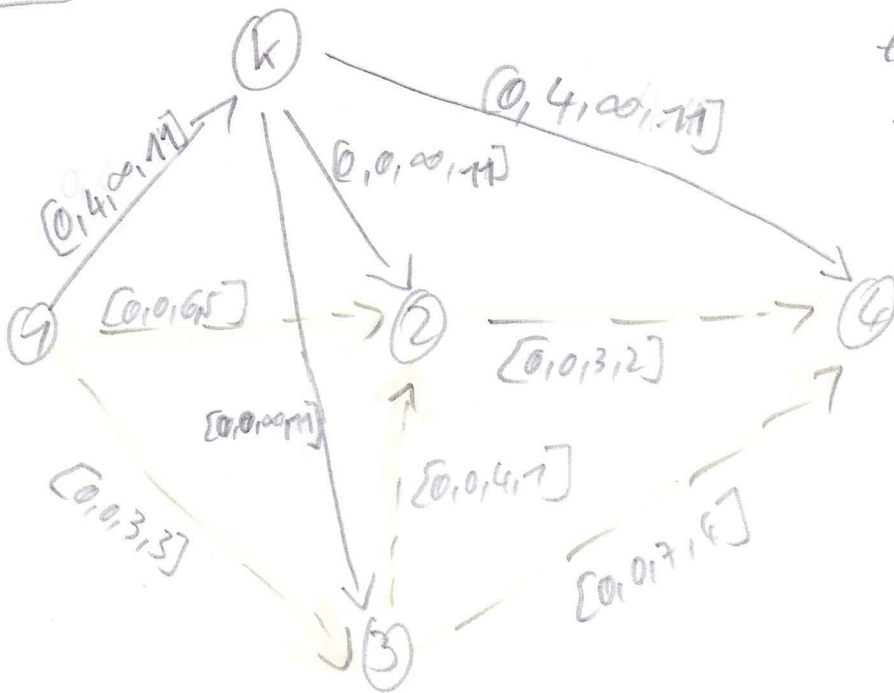
Netzwerk:

$[c_{ij}, u_i, u_j, c_{ij}]$



$b(v)$

Inhaltsbaum:



$t[k] = -1$ $d(u) = 0$
 $t[1] = 2$ $d(i) = 1$
 $t[2] = 3$ $i \in \{1, \dots, 4\}$
 $t[3] = 4$
 $t[4] = k$

$p(k) = -1$
 $p(1) = k$
 $p(2) = k$
 $p(3) = k$
 $p(4) = k$

Funktion:

$$y(k) = 0$$

$$y(1) = y(k) + c_{1k} = 0 + 11 = 11, \text{ da } (1, k) \in T$$

$$y(2) = y(k) - c_{k2} = 0 - 11 = -11, \text{ da } (k, 2) \in T$$

$$y(3) = y(4) = -11$$

$$T: \bar{c}_{1k} = c_{1k} - y(1) + y(k) = 11 - 11 + 0 = 0$$

$$\bar{c}_{k2} = 11 - 0 + (-11) = 0$$

$$\bar{c}_{k3} = 11 - 0 + (-11) = 0 \quad \bar{c}_{k4} = 0$$

reduced Costs:

$$\bar{c}_{ij} = c_{ij} - y(i) + y(j), \forall (i, j) \in T$$

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$$\underline{L:} \quad \bar{c}_{13} = 3 - 11 + (-11) = -19, \quad \bar{c}_{12} = 5 - 11 + (-11) = -17$$

$$\bar{c}_{32} = 1 - (-11) + (-11) = 1, \quad \bar{c}_{24} = 2 - (-11) + (-11) = 2$$

$$\bar{c}_{34} = 4 - (-11) + (-11) = 4$$

$$\Rightarrow \underset{L, u}{v}^{\max} = \bar{c}_{13} = -19 \Rightarrow \underline{\text{entering arc} = (1, 3)}$$

$$\Rightarrow \text{Cycle: } (k, 1, 3, 2, k) \quad \text{Schinkel} = k$$

$$\epsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, x_{k3} - l_{k3})$$

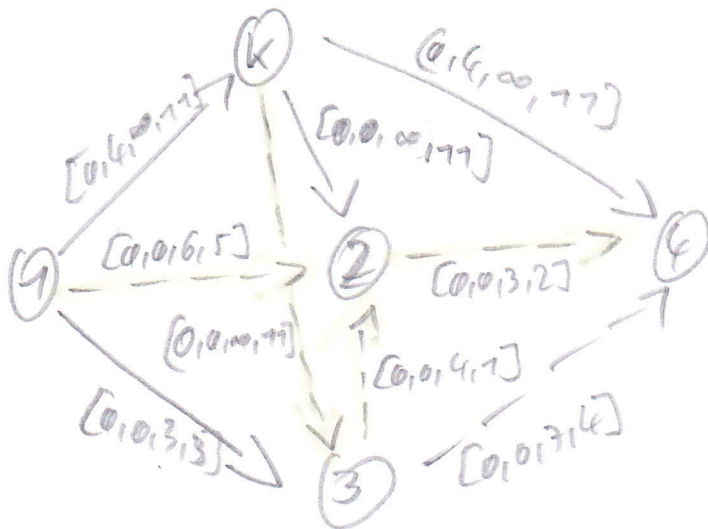
$$= \min (4 - 0 = 4, 3 - 0 = 3, 0 - 0 = 0)$$

$$= 0 \Rightarrow \text{letzter blockierender Bogen: } (k, 3)$$

\Rightarrow keine Flussveränderung

leaving arc: (k, 3)

Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k \quad p(2) = k \quad p(4) = k$$

$$p(3) = 1$$

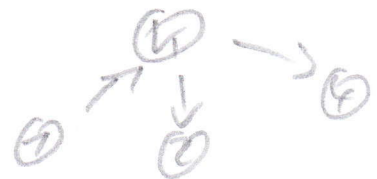
$$\epsilon(k) = 1 \quad \epsilon(1) = 2 \quad \epsilon(2) = 3$$

$$\epsilon(3) = 4 \quad \epsilon(4) = k$$

$$d(k) = 0 \quad d(1) = 1$$

$$d(2) = 1 \quad d(3) = 2 \quad d(4) = 1$$

T1:



T2:



Fair Prices Update: $\text{change} = -\bar{c}_{13} = +19$
 $\rightarrow y(3) := y(3) + \text{change} = y(3) + 19 = 8$

fertig! da $d(\epsilon(3)) \leq d(3)$

reduced Costs Update:

T: $\bar{c}_{1k} = 0$, $\bar{c}_{13} = 3$, $-11 + 8 = 0$
 $\bar{c}_{k2} = 0$, $\bar{c}_{k4} = 0$

L: $\bar{c}_{k3} = 11 - 0 + 8 = 19$
 $\bar{c}_{12} = -17$, $\bar{c}_{32} = 1 - 8 + (-11) = -18$
 $\bar{c}_{24} = 2$, $\bar{c}_{34} = 4 - 8 + (-11) = -15$

\Rightarrow "max" $\bar{c}_{32} = -18 \Rightarrow$ entering arc = (3,2)

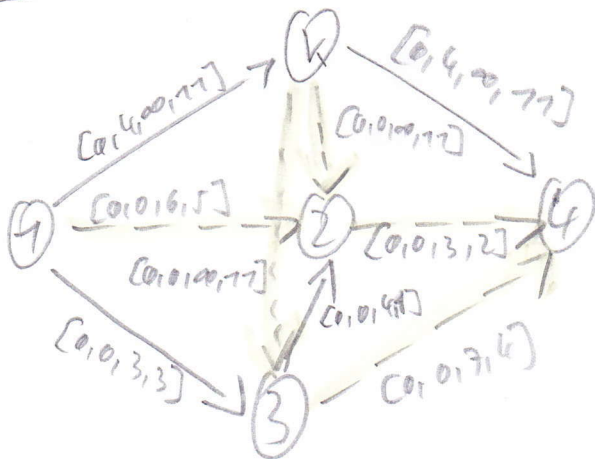
\Rightarrow Circle = (k, 1, 3, 2, k) Scheitel = k

$\epsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{32} - x_{32}, x_{k2} - l_{k2})$
 $= \min (4 - 0 = 4; 3 - 0 = 3; 4 - 0 = 4; 0 - 0 = 0)$

$= 0 \Rightarrow$ leaving arc = (k,2) (letzter blockierender Bogen)

\Rightarrow keine Flussänderung!

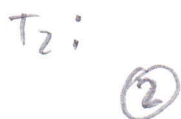
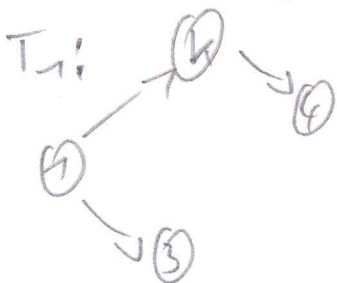
Keine Baumstruktur:



$p(k) = -1, p(1) = k, p(2) = 3$
 $p(3) = 1, p(k) = k$

$t(k) = 1, t(1) = 2, t(2) = 3, t(3) = 4$
 $e(k) = k$

$d(k) = 0, d(1) = d(4) = 1$
 $d(2) = 3, d(3) = 2$



Fair Price Update:

$$\text{change} = -\bar{c}_{32} = 18 \rightarrow y(2) = y(2) + \text{change} \\ = -11 + 18 = 7$$

$$\text{fertig, da } d(f(2)) \leq d(2)$$

Reduced Costs Update:

$$\text{+!} \quad \bar{c}_{1k} = 0, \quad \bar{c}_{k4} = 0, \quad \bar{c}_{13} = 0$$

$$\bar{c}_{32} = 1 - 8 + 7 = 0$$

$$\text{L!} \quad \bar{c}_{k3} = 19, \quad \bar{c}_{34} = -15$$

$$\bar{c}_{12} = 5 - 11 + 7 = 1$$

$$\bar{c}_{k2} = 11 - 0 + 7 = 18$$

$$\bar{c}_{24} = 2 - 7 + (-11) = -16$$

$$\Rightarrow \underset{L, U}{\text{max}} \bar{c}_{24} = -16 \Rightarrow \text{entering arc} = (2, 4)$$

$$\Rightarrow \text{cycle} = (k, 1, 3, 2, 4, k) \quad \text{Schritt} = k$$

$$\varepsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{32} - x_{32}, u_{24} - x_{24}, x_{k4} - l_{k4}) \\ = \min (4 - 0 = 4, 3 - 0 = 3, 4 - 0 = 4, 3 - 0 = 3, 4 - 0 = 4)$$

$$\Rightarrow \varepsilon = 3 \Rightarrow \text{leider blockierender Zyklus} = (2, 4) \\ = \text{leaving arc}$$

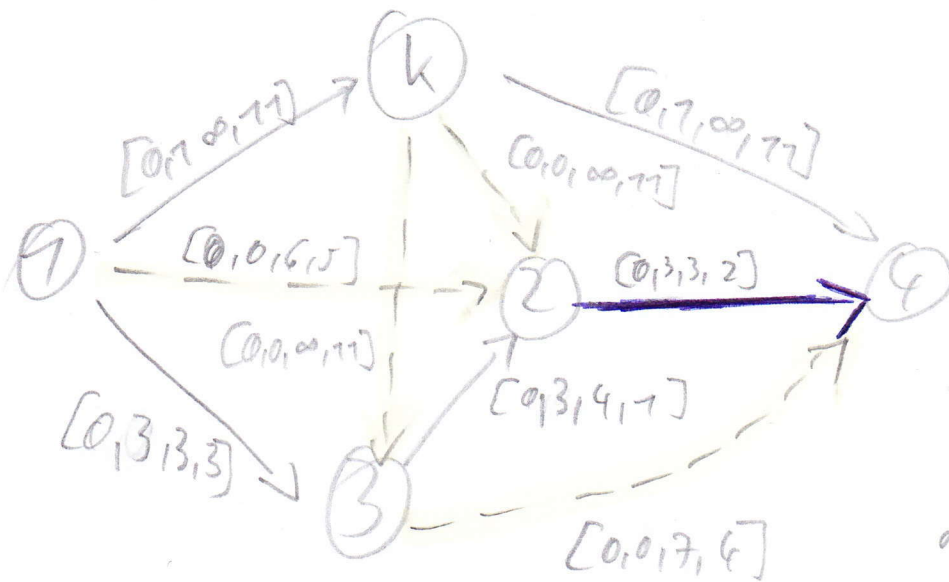
$$\text{Umwandlung: } x_{24} := x_{24} + \varepsilon = 3, \quad x_{k4} := x_{k4} - \varepsilon = 1$$

$$x_{1k} := x_{1k} - \varepsilon = 1, \quad x_{13} := x_{13} + \varepsilon = 3$$

$$x_{32} := x_{32} + \varepsilon = 3$$

$$\Rightarrow (2, 4) \text{ und } (2, 4) \text{ in } U$$

Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k = p(4)$$

$$p(2) = 3, p(3) = 1$$

$$t(k) = 7, t(1) = 2$$

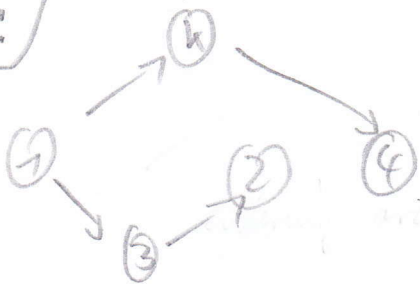
$$t(3) = 4, t(2) = 3$$

$$t(4) = k$$

$$d(k) = 0, d(1) = d(4) = 1$$

$$d(2) = 3, d(3) = 2$$

T1:



T2:

Fair Price Update:

reduced Costs: (no update needed)

$$T: \bar{c}_{1k} = 0, \bar{c}_{k4} = 0, \bar{c}_{13} = 0$$

$$\bar{c}_{32} = 0$$

$$L: \bar{c}_{k3} = -19, \bar{c}_{34} = -15, \bar{c}_{12} = 1$$

$$\bar{c}_{k2} = -18$$

$$U: \bar{c}_{24} = -16$$

$$\Rightarrow \text{"max"}_{2,4} = \bar{c}_{34} = -15 \Rightarrow \underline{\text{entering arc} = (3,4)}$$

$$\Rightarrow \text{cycle} = (k, 1, 3, 4, k) \text{ Schenkel} = k$$

$$\varepsilon = \min (x_{1k} - l_{1k}, u_{13} - x_{13}, u_{34} - x_{34}, x_{k4} - l_{k4})$$

$$= \min (1 - 0 = 1, 3 - 3 = 0, 7 - 0 = 7, 1 - 0 = 1)$$

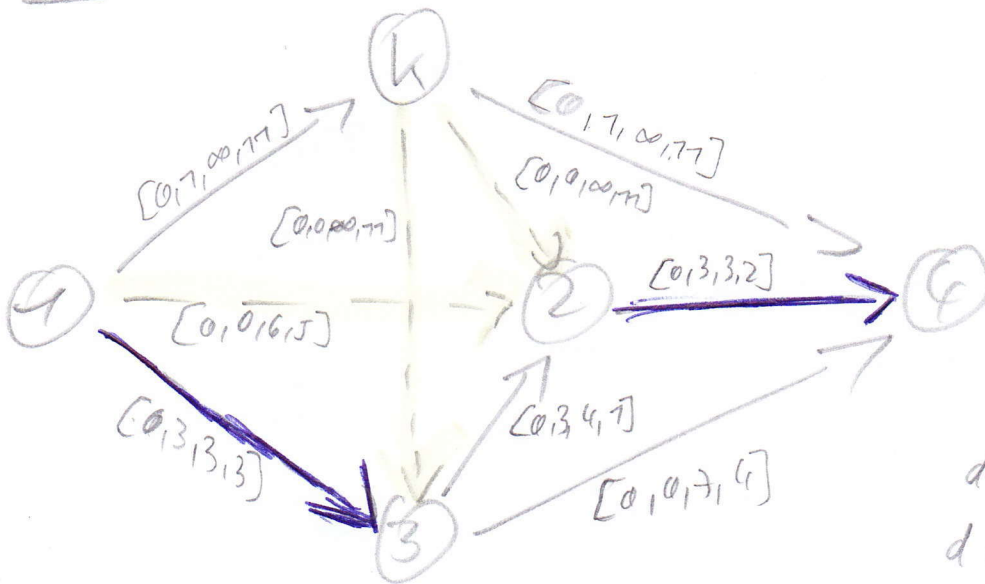
$$= 0 \Rightarrow \text{letzter blockierender Bogen} = (1,3)$$

$$= \underline{\text{leaving arc}}$$

Flussänderung: ✓

$$\Rightarrow (1,3) \text{ in } U$$

Neue Baumstruktur:



$$p(k) = -1$$

$$p(1) = k = p(4)$$

$$p(2) = 3, p(3) = 4$$

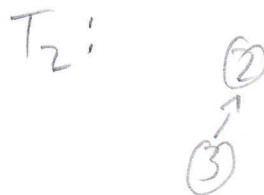
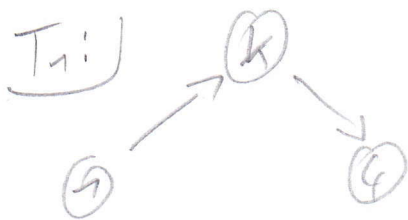
$$t(k) = 1, t(1) = 2$$

$$t(2) = 3, t(3) = 4$$

$$t(4) = k$$

$$d(k) = 0, d(1) = d(4) = 1$$

$$d(2) = 3, d(3) = 2$$



Fair Price Update: $\text{change} = -\bar{c}_{34} = -15 \quad (3 \in T_2)$

$$y(3) := y(3) + \text{change} = 8 - 15 = -7$$

$$y(2) := y(2) + \text{change} = 7 - 15 = -8$$

Reduced Costs Update:

$$\underline{T:} \quad \bar{c}_{1k} = 0, \quad \bar{c}_{k4} = 0, \quad \bar{c}_{32} = 1 - (-7) + (-8) = -02$$

$$\bar{c}_{34} = 4 - (-7) + (-11) = 050$$

$$\underline{L:} \quad \bar{c}_{k2} = 11 - 0 + (-8) = 3, \quad \bar{c}_{12} = 5 - 11 + (-8) = -14$$

$$\bar{c}_{k3} = 11 - 0 + (-7) = 44$$

$$\underline{U:} \quad \bar{c}_{13} = 3 - 11 + (-7) = -15$$

$$\bar{c}_{24} = 2 - (-8) + (-11) = -1$$

$$\Rightarrow \underset{L, U}{\text{"max"}} = \bar{c}_{12} = -14 \quad \Rightarrow \text{entering arc} = (1, 2)$$

$$\Rightarrow \text{cycle} = (k, 1, 2, 3, 4, k) \quad \text{Schubel} = k$$

$$\begin{aligned} \varepsilon &= \min (x_{1k} - l_{1k}, u_{12} - x_{12}, x_{32} - l_{32}, u_{34} - x_{34}, x_{k4} - l_{k4}) \\ &= \min (1 - 0 = 1, 6 - 0 = 6, 3 - 0 = 3, 7 - 0 = 7, 1 - 0 = 1) \\ &= 1 \quad \Rightarrow \text{leider blockierender Bogen} = (k, 4) \\ & \quad \quad \quad = \text{leaving arc} \end{aligned}$$

Flussänderung:

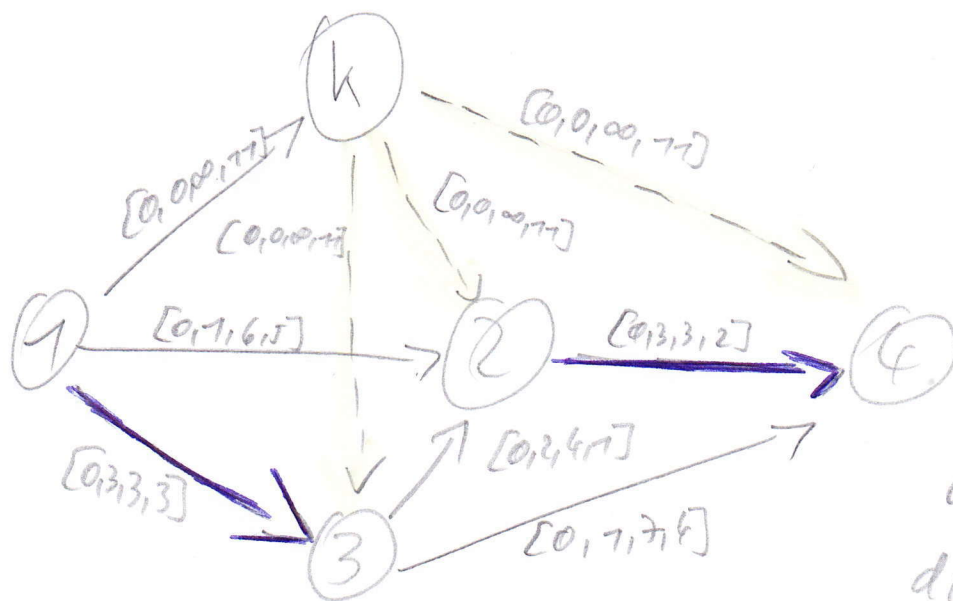
$$x_{1k} := x_{1k} - \varepsilon = 0, \quad x_{k4} := x_{k4} - \varepsilon = 0$$

$$x_{12} := x_{12} + \varepsilon = 1, \quad x_{32} := x_{32} - 1 = 2$$

$$x_{34} := x_{34} + \varepsilon = 1$$

$$\Rightarrow (k, 4) \in L$$

Neue Baumstruktur:



$$p(k) = -1, p(1) = k$$

$$p(2) = 1, p(3) = 2$$

$$p(4) = 3$$

$$t(1) = 1, t(1) = 2$$

$$t(2) = 3, t(3) = 4$$

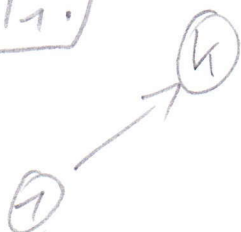
$$t(4) = k$$

$$d(k) = 0$$

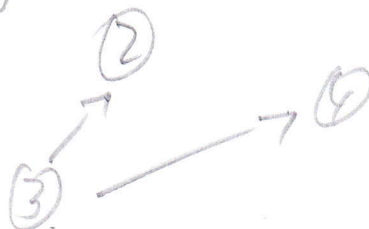
$$d(1) = 1, d(2) = 2, d(3) = 3$$

$$d(4) = 4$$

T₁:



T₂:



Far-Price Update:

$$\text{change} = -\bar{c}_{12} = 14, \text{ da } (4) \in T_1$$

$$y(4) := y(4) + \text{change} = -11 + 14 = 3$$

$$y(2) := y(2) + \text{change} = -8 + 14 = 6$$

$$y(3) := y(3) + \text{change} = -7 + 14 = 7$$

Reduced Costs Update:

$$\underline{T:} \quad \bar{c}_{1k} = 0, \bar{c}_{12} = 5 - 11 + 6 = 0$$

$$\bar{c}_{32} = 1 - 7 + 6 = 0$$

$$\underline{U:} \quad \bar{c}_{13} = 3 - 11 + 7 = -1$$

$$\bar{c}_{24} = 2 - 6 + 3 = -1$$

$$\bar{c}_{34} = 4 - 7 + 3 = 0$$

$$\underline{L:} \quad \bar{c}_{k4} = 11 - 0 + 3 = 14, \bar{c}_{k3} = 11 - 0 + 7 = 18$$

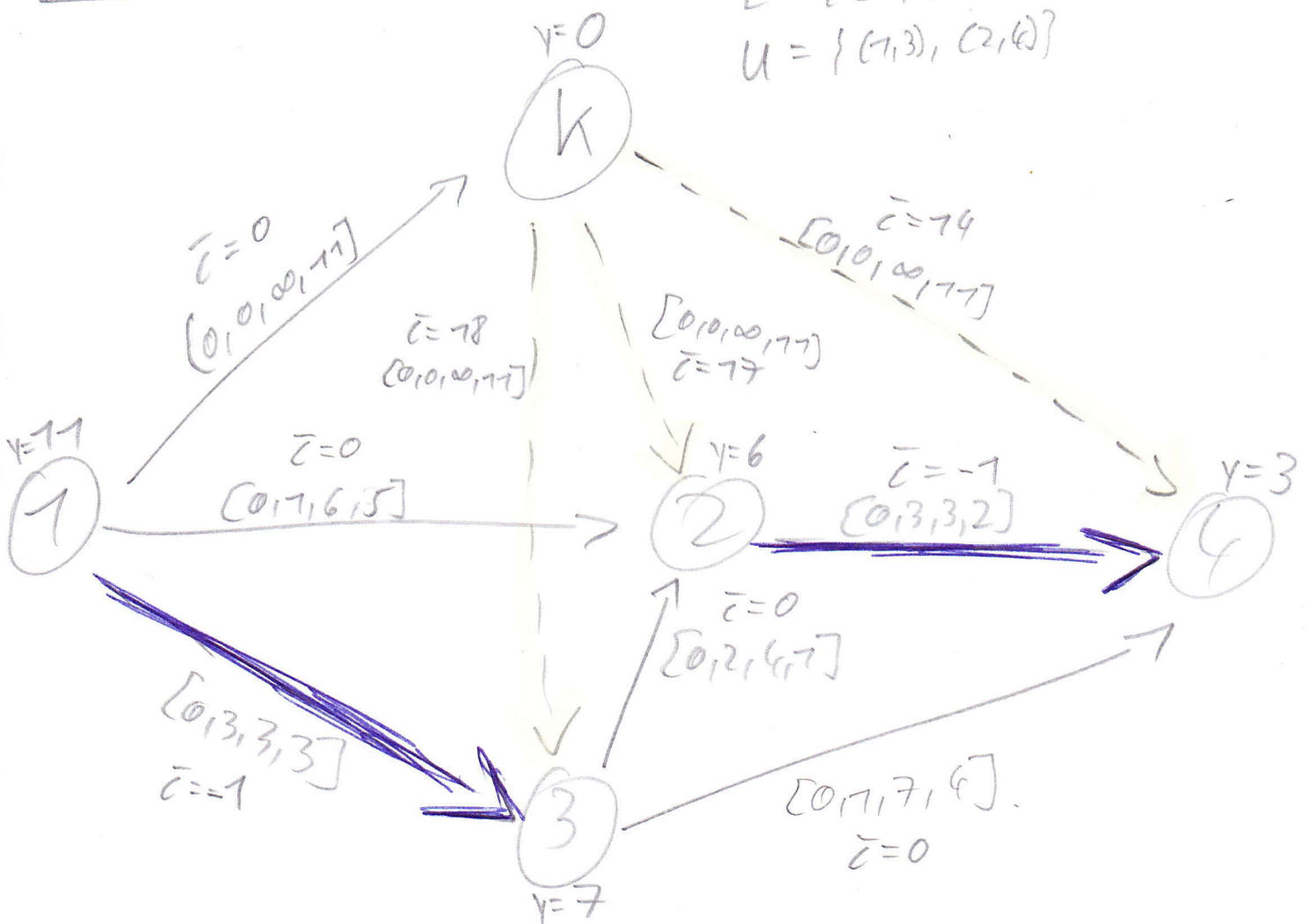
$$\bar{c}_{k2} = 11 - 0 + 6 = 17$$

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$$\Rightarrow \left. \begin{array}{l} \bar{c}_{ij} = 0 \quad \forall (i,j) \in T \\ \bar{c}_{ij} \geq 0 \quad \forall (i,j) \in L \\ \bar{c}_{ij} \leq 0 \quad \forall (i,j) \in U \end{array} \right\} \Rightarrow \text{optimale Lösung erreicht, da} \\ x_{ij} = 0 \quad \forall (i,j) \text{ mit } i=k \vee j=k$$

Finale Baumstruktur:

$$\begin{aligned} T &= \{(1,k), (1,2), (3,2), (3,4)\} \\ L &= \{(k,3), (k,2), (k,4)\} \\ U &= \{(1,3), (2,4)\} \end{aligned}$$



Fluss: $[c_1, x_1, u_1, c_4]$

