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摘　要

本文完全基于 [30] 进行 Navier-Stokes 公式的推导。

关键词： Navier-Stokes Equations, Continuum Mechanics

1 Balance Laws

如果只考虑等温流体，则可以忽略热力学相关的温度 (temperature)、热 (heat)，则一共有 $3+1+9=13$ 个未知数需要求解：

$$\begin{array}{ll} v_i & \text{3 unknowns} \\ \rho & \text{1 unknown} \\ S_{ij} & \text{9 unknowns} \end{array}$$

而为了求解这 13 个未知数，我们手头有的公式目前只有 balance law。又因为忽略了热力学相关因素，所以 Balance Laws 里的热力学第一定律被丢掉了。于是 Balance Laws 中剩下的公理都是和运动学相关的了 (See Section 5.3):

$$\begin{cases} \dot{\rho} + \rho \nabla^x \cdot \boldsymbol{v} = 0 & [\text{质量守恒}] \\ \rho \dot{\boldsymbol{v}} = \nabla^x \cdot \boldsymbol{S} + \rho \boldsymbol{b} & [\text{线动量守恒}] \\ S^T = S & [\text{角动量守恒}] \end{cases}$$

这里我们选取的是 Eulerian Form of Balance Laws，似乎在流体仿真的时候比较流行用 Eulerian Form。

上述三个公式提供了 $1+3+3=7$ 个等式，所以为了求解 13 个未知数，还需要 $13-7=6$ 个等式。这 6 个等式将由本构方程 (constitutive equation) 提供。

2 Constitutive Model

称一个连续体为不可压缩的牛顿流体，若：

1. 密度均匀：$\rho_0(X,t) = \rho_0 > 0(\text{constant})$
2. 不可压缩：$\nabla^x \cdot \boldsymbol{v} = 0$
3. 应力满足：$\boldsymbol{S} = -p\boldsymbol{I} + 2\mu \operatorname{sym}(\nabla^x \boldsymbol{v})$

上式三个条件即为不可压缩牛顿流体的本构方程。

3 Constraints for Constitutive Model

- Result 6.8 中验证了在该本构方程下连续体的 Frame-Indifference 性质依然被满足。
- Section 6.3.4 中验证了在该本构方程下热力学第二定律依然被满足。

4 简化公式

联立 Balance Laws 和 Constitutive Model：

$$
\begin{cases}
\dot{\rho} + \rho \nabla^x \cdot \boldsymbol{v} = 0 & \text{(质量守恒)} \\
\rho \dot{\boldsymbol{v}} = \nabla^x \cdot \boldsymbol{S} + \rho \boldsymbol{b} & \text{(线动量守恒)} \\
S^T = S & \text{(角动量守恒)} \\
\rho_0(X,t) = \rho_0 > 0(\text{constant}) & \text{(密度均匀)} \\
\nabla^x \cdot \boldsymbol{v} = 0 & \text{(不可压缩)} \\
\boldsymbol{S} = -p\boldsymbol{I} + 2\mu \mathrm{sym}(\nabla^x \boldsymbol{v}) & \text{(应力条件)}
\end{cases}
$$

其中 $\boldsymbol{b}(x,t), \rho_0, \mu$ 为给定的 body force field per unit mass, density, absolute viscosity。注意 ρ_0, μ 都为与 x, t 无关的常数。

4.1 角动量守恒

注意到对式 (应力条件) 两边取转置会得到：

$$\boldsymbol{S}^T = -p\boldsymbol{I} + 2\mu \mathrm{sym}(\nabla^x v)$$

所以式 (应力条件) 隐含了 $\boldsymbol{S} = \boldsymbol{S}^T$，已经隐含了式 (角动量守恒)。

4.2 质量守恒

将式（不可压缩）$\nabla^x \cdot \boldsymbol{v}$ 代入式（质量守恒）中会得到：

$$\dot{\rho} = \frac{\mathrm{d}\rho}{\mathrm{d}t} = 0$$

即 ρ 不随 t 发生变化，所以 $\rho(x,t) = \rho_m(X,0)|_{X=\psi(x,t)}$，代入式（密度均匀），即有：

$$\rho(x,t) = \rho_0 > 0(\text{constant})(\text{常数密度})$$

上式隐含了式（质量守恒）和式（密度均匀）。

4.3 线动量守恒

将式（应力条件）和式（常数密度）代入式（线动量守恒）中，消掉 $\boldsymbol{S}, \rho$，得到：

$$
\begin{aligned}
\rho_0 \dot{\boldsymbol{v}} &= \nabla^x \cdot (-p\boldsymbol{I} + 2\mu \mathrm{sym}(\nabla^x \boldsymbol{v})) + \rho_0 \boldsymbol{b} \\
&= -\nabla^x \cdot (p\boldsymbol{I}) + 2\mu \nabla^x \cdot \mathrm{sym}(\nabla^x \boldsymbol{v}) + \rho_0 \boldsymbol{b}
\end{aligned}
\tag{1}
$$

其中各项有：

$$
\begin{cases}
\dot{\boldsymbol{v}} = \dfrac{\partial}{\partial t}\boldsymbol{v} + (\nabla^x \boldsymbol{v})\boldsymbol{v} & \text{[Result 4.7]} \\
\nabla^x \cdot (p\boldsymbol{I}) = p\nabla^x \cdot \boldsymbol{I} + \boldsymbol{I}\nabla^x p = \nabla^x p \\
\nabla^x \cdot \mathrm{sym}(\nabla^x \boldsymbol{v}) = \dfrac{1}{2}(\nabla^x \cdot \nabla^x \boldsymbol{v} + \nabla^x \cdot (\nabla^x \boldsymbol{v})^T) = \dfrac{1}{2}\Delta^x \boldsymbol{v} & \text{[See Below]}
\end{cases}
$$

展开其中对 $\nabla^x \cdot \mathrm{sym}(\nabla^x \boldsymbol{v})$ 的推导：

$$
\nabla^x \cdot \nabla^x \boldsymbol{v} = \Delta^x \boldsymbol{v} \quad \text{[拉普拉斯算子的定义]}
$$

$$
\begin{aligned}
\nabla^x \cdot (\nabla^x \boldsymbol{v})^T &= \frac{\partial(\nabla^x \boldsymbol{v})^T_{ij}}{\partial x_j}\boldsymbol{e}_i && \text{[散度定义]} \\
&= \frac{\partial(\frac{\partial v_j}{\partial x_i})}{\partial x_j}\boldsymbol{e}_i && \text{[梯度定义]} \\
&= \frac{\partial^2 v_j}{\partial x_i \partial x_j}\boldsymbol{e}_i \\
&= \frac{\partial(\frac{\partial v_j}{\partial x_j})}{\partial x_i}\boldsymbol{e}_i && \text{[任意阶导数连续]} \\
&= \nabla^x(\frac{\partial v_j}{\partial x_j}) && \text{[梯度定义]} \\
&= \nabla^x(\nabla^x \cdot \boldsymbol{v}) && \text{[散度定义]} \\
&= \nabla^x \boldsymbol{0} && \text{[不可压缩]} \\
&= 0
\end{aligned}
$$

将这三项带回式 1 中，得到式（线动量守恒）化简后的结果：

$$
\rho_0 \left[\frac{\partial}{\partial t}\boldsymbol{v} + (\nabla^x \boldsymbol{v})\boldsymbol{v}\right] = -\nabla^x p + \mu\Delta^x \boldsymbol{v} + \rho_0 \boldsymbol{b} \tag{2}
$$

上式隐含了式（应力条件）、式（常数密度）、式（线动量守恒）。

4.4 整理公式

最开始通过联立 Balance Laws 与 Constitutive Model 得到的方程组经过前文的化简后变为：（联立式（不可压缩）和式 2）

$$
\begin{cases}
\rho_0 \left[\dfrac{\partial}{\partial t}\boldsymbol{v} + (\nabla^x \boldsymbol{v})\boldsymbol{v}\right] = -\nabla^x p + \mu\Delta^x \boldsymbol{v} + \rho_0 \boldsymbol{b} \\
\nabla^x \cdot \boldsymbol{v} = 0
\end{cases}
$$

上式即为 **Navier-Stokes Equations**。

其中：

- $\boldsymbol{b}, \rho_0, \mu$ 为给定的 body force field per unit mass, density, absolute viscosity。
- 一共有 4 个等式和 4 个未知数：$p, \boldsymbol{v}$，因此可以解出来（笑）。

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