Coordinating Time Series

Methods 3

The plan

- Time series
- Physiological signals
- Interpersonal coordination

- Sales over time
- Densely sampled behaviors
- Opinion polls

- Proxies for
  - Cognitive effort
  - Emotional arousal
  - Brain activity

- Any time you have two temporal phenomena in interaction

The goal

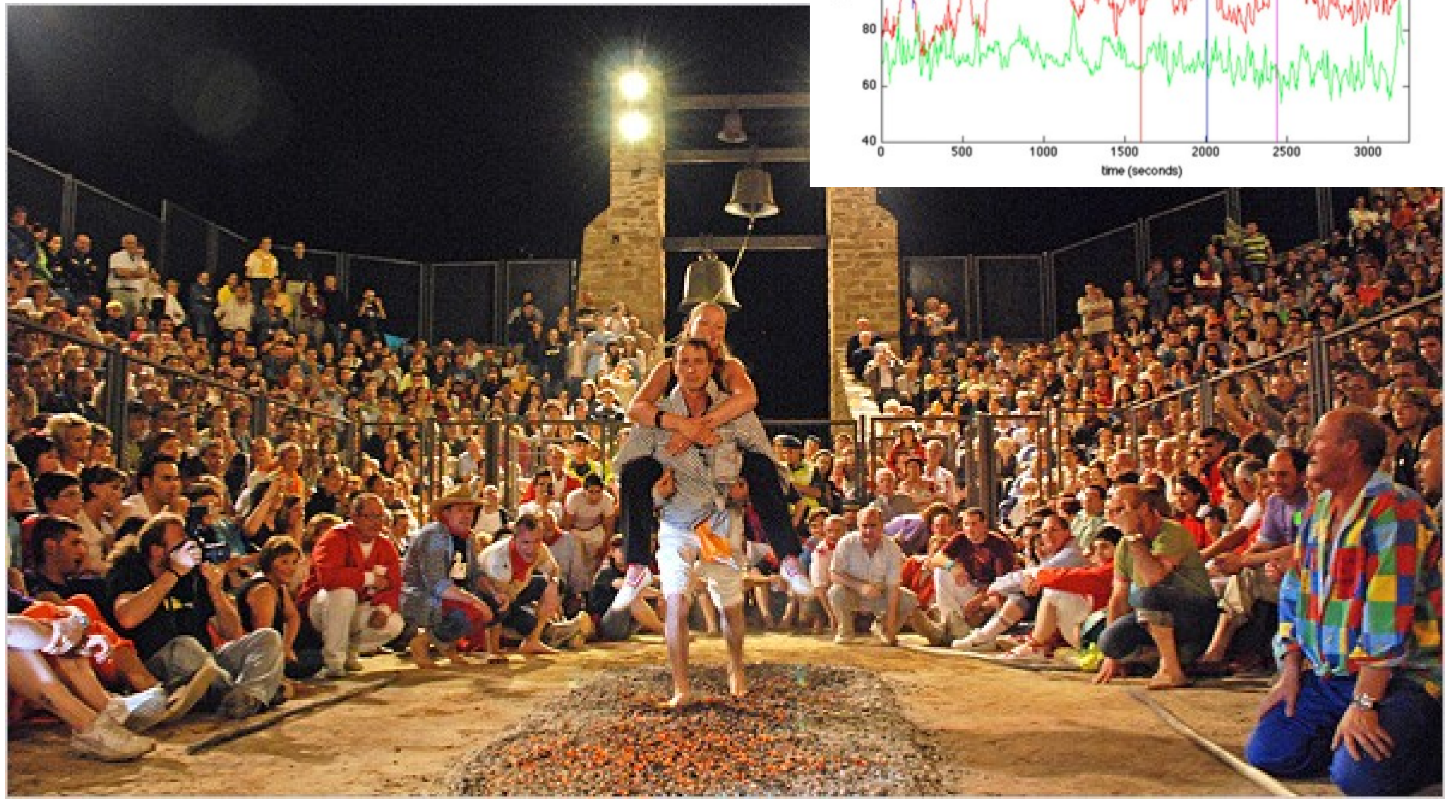
- Able to collect physiological signals (it's not rocket science!)

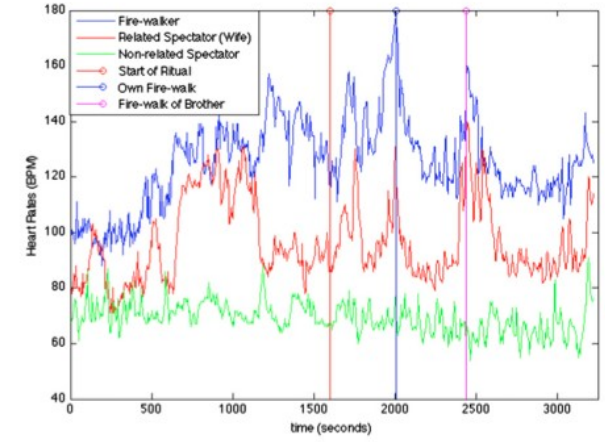
- Able to preprocess the signals (it's not rocket science!)

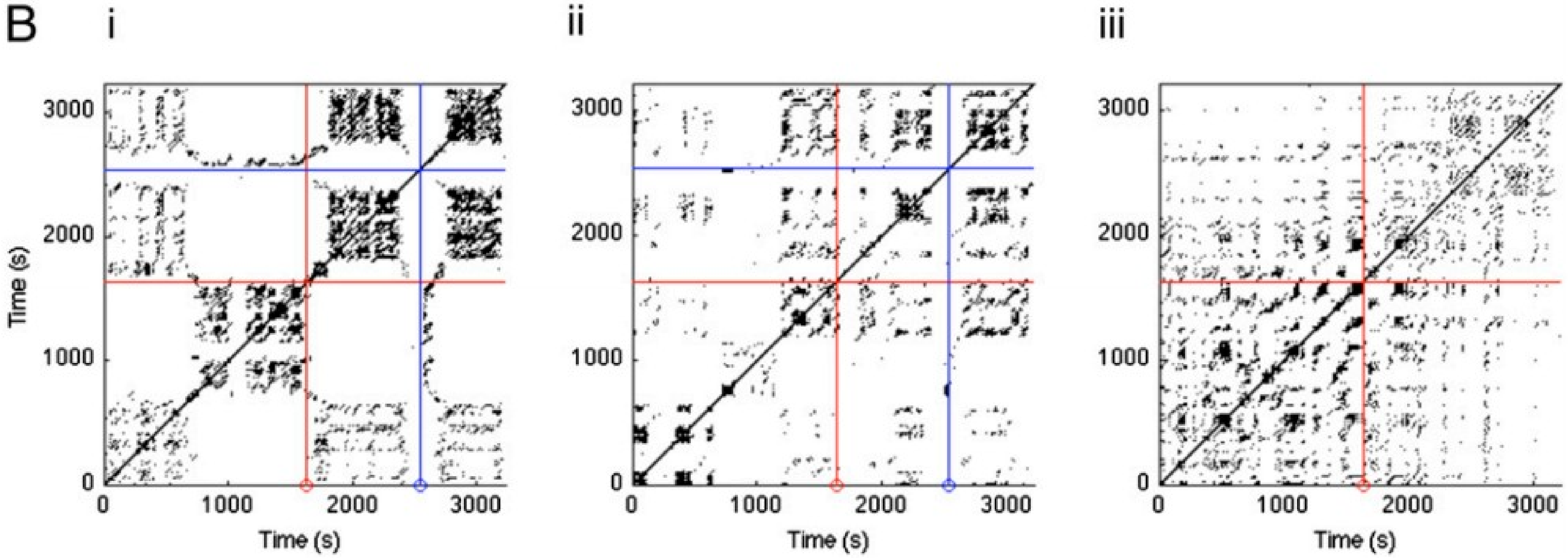
- Able to assess the degree of influence between two temporal phenomena
  - With appropriate controls

- Initial idea of the kind of questions this enables you to ask
  - How do we build a study? How do we run control experiments?

Investigating HR coordination




Fire-walker
Related Spectator (Wife)
Non-related Spectator
Start of Ritual
Own Fire-walk
Fire-walk of Brother
Heart Rates (BPM)
180
160
140
120
100
80
60
40
0
500
1000
1500
2000
2500
3000
time (seconds)



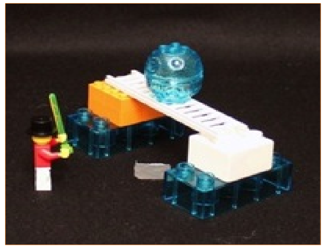
Searching for optimal parameters and 2% RR
Delay: 3 to 21 (optimized by time-series)
Embedding dimensions: 3 to 4 (optimized by time)
Radius: optimized to always get 2%

General phenomenon?
Conditions?
Effects?

Three open questions

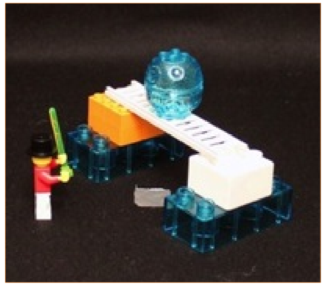
- Does this happen in less extreme contexts?
- What could be driving this?
- Which relation does it have with actual experience of the participants?

The Lego Study

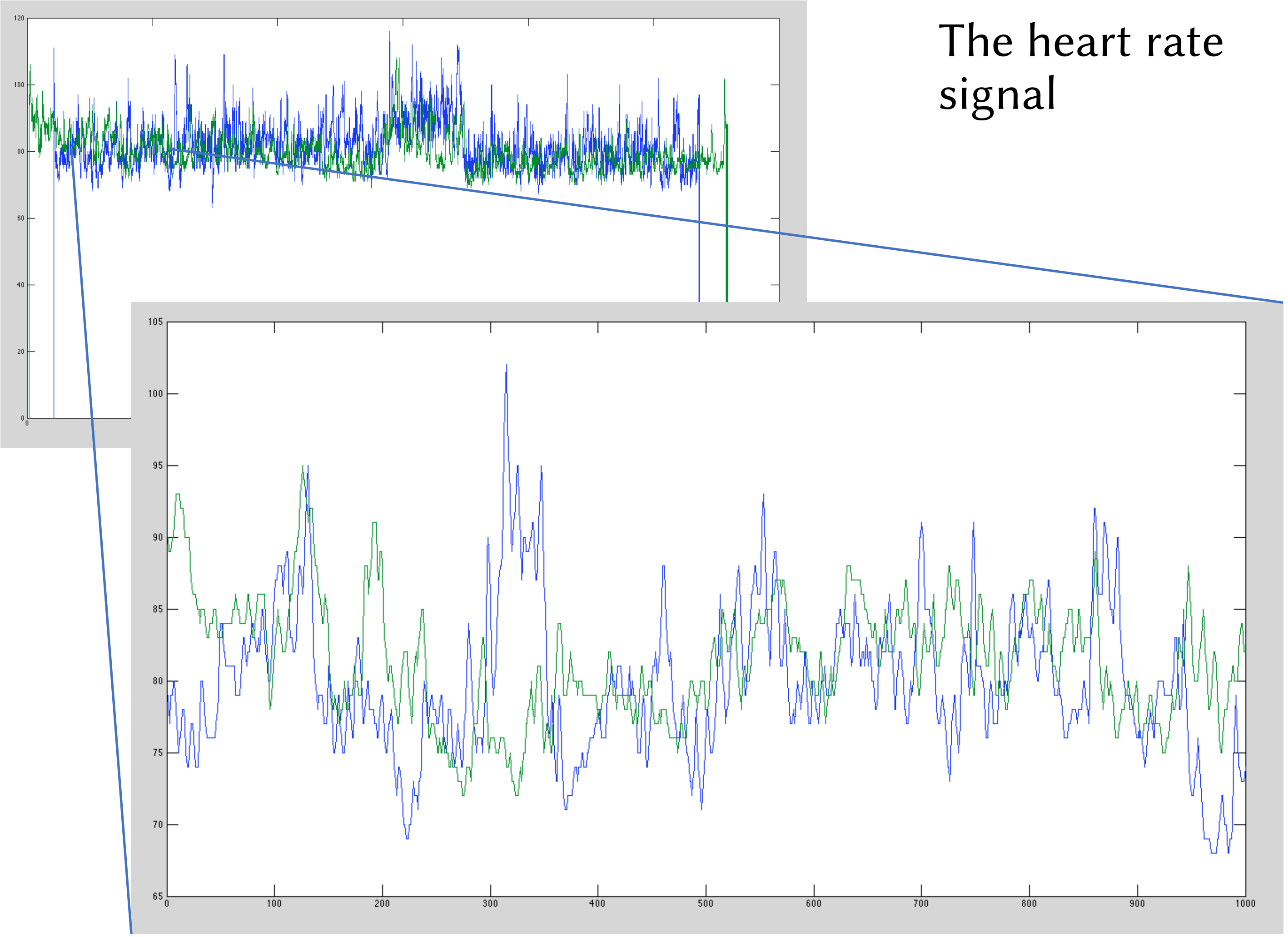


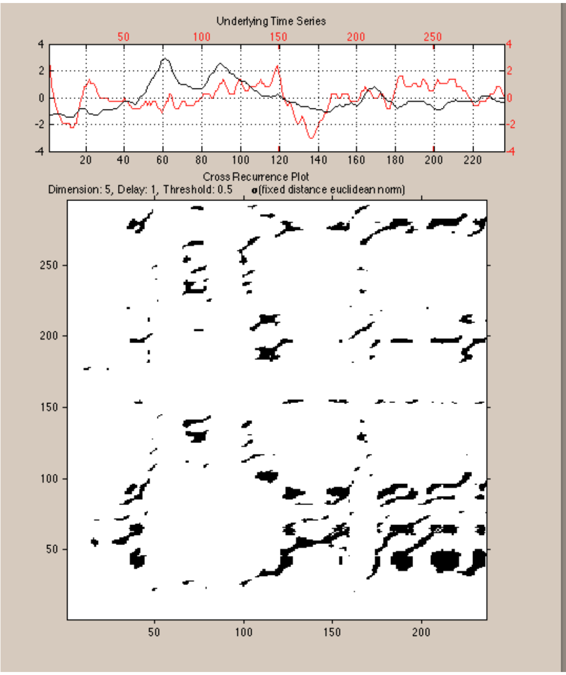
The experimental setting

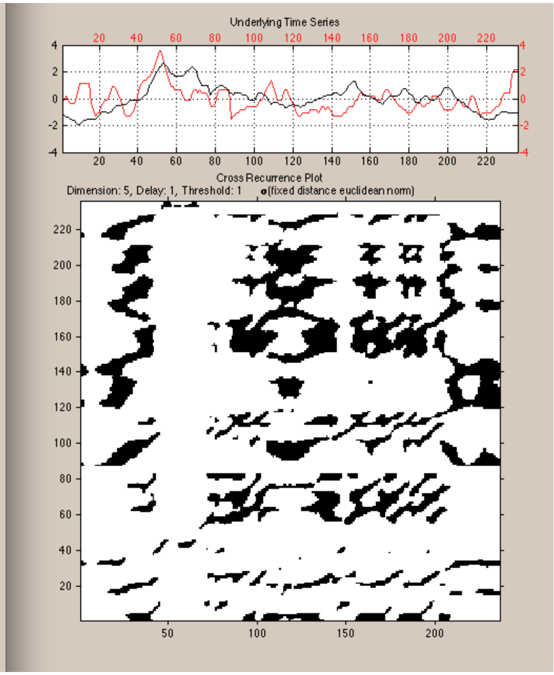
- 6 groups of 5 participants
  - A sequence of 6 trials (5*2 minutes)
    - Negotiating and building their understanding of abstract concepts like “responsibility”, “tolerance”, “safety”, “justice”, etc.
      - Some individual and then collective
      - Some collective and then individual
- We collected:
  - Behavioral Data: Videos of construction sessions
  - Physiological Data: Heart rates with Polar Team transmitter belts
  - Experience Data: The Intrinsic Motivation Inventory (IMI)



The heart rate signal




Underlying Time Series
Cross Recurrence Plot
Dimension: 5, Delay: 1, Threshold: 0.5 (fixed distance euclidean norm)


Underlying Time Series
Cross Recurrence Plot
Dimension: 5, Delay: 1, Threshold: 1 (fixed distance euclidean norm)

Q1: task vs. actual interaction

- Creating surrogate pairs
  - They share the task but not the interaction

	Individual	Collective
Real		✖
Surrogate		

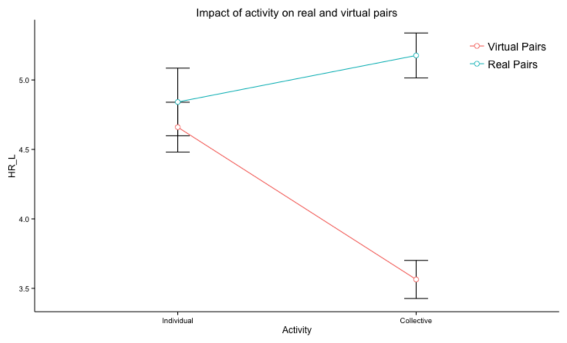
- HR entrainment grows over time

N.B. Repeated measures design with complex interdependencies.
Mixed model analysis:

- With pair and group as random effects
- Session and order of conditions as control fixed effects

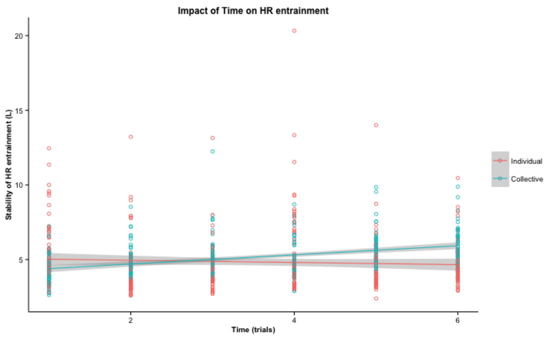
H1

Virtual vs. Real Pairs



$R^2m=0.14$
$p<0.0001$

Development over time



$R^2m=0.04$
$p<0.0001$

Q2: the role of behavior

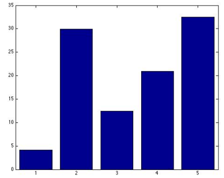
- H2: HR entrainment is behaviorally driven
  - Behavioral coordination predicts HR entrainment

Speech and Action Coding

Videos were coded for onset/duration of Speech events (vocal action) and Construction events (touching LEGO blocks) for each participant/each task with a 1Hz resolution.

Speech Acts

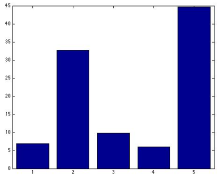
S1:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	… etc
S2:	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	… etc
S3:	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	… etc
S4:	0	0	1	1	0	0	0	0	0	0	0	1	1	1	1	… etc
S5:	1	1	1	1	1	1	0	1	0	0	1	0	0	1	1	… etc
Time(secs)			1	2	3	4	5	6	7	8	9	10	11	12	13	14 15 … etc



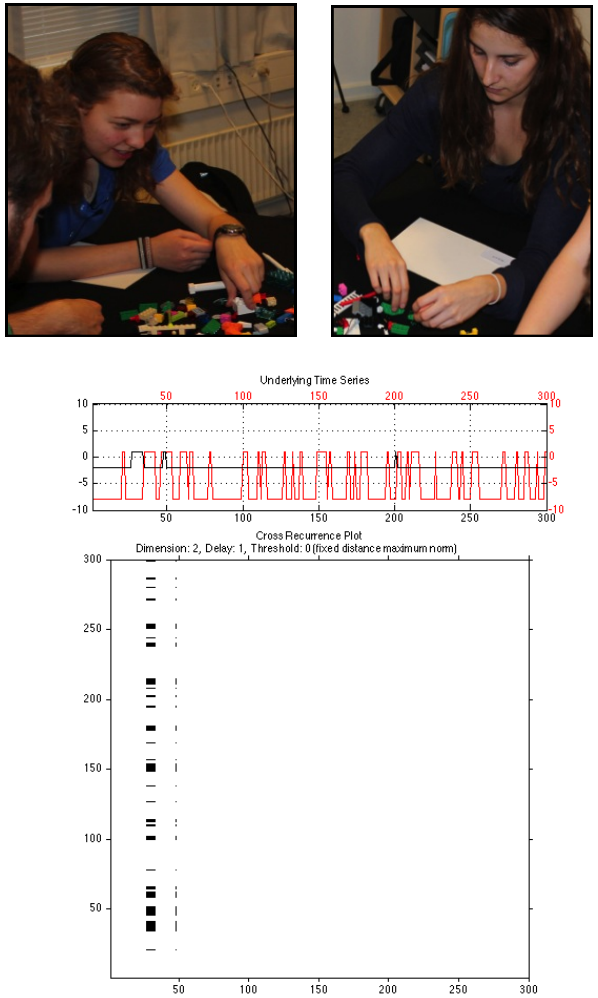
S1 S2 S3 S4 S5

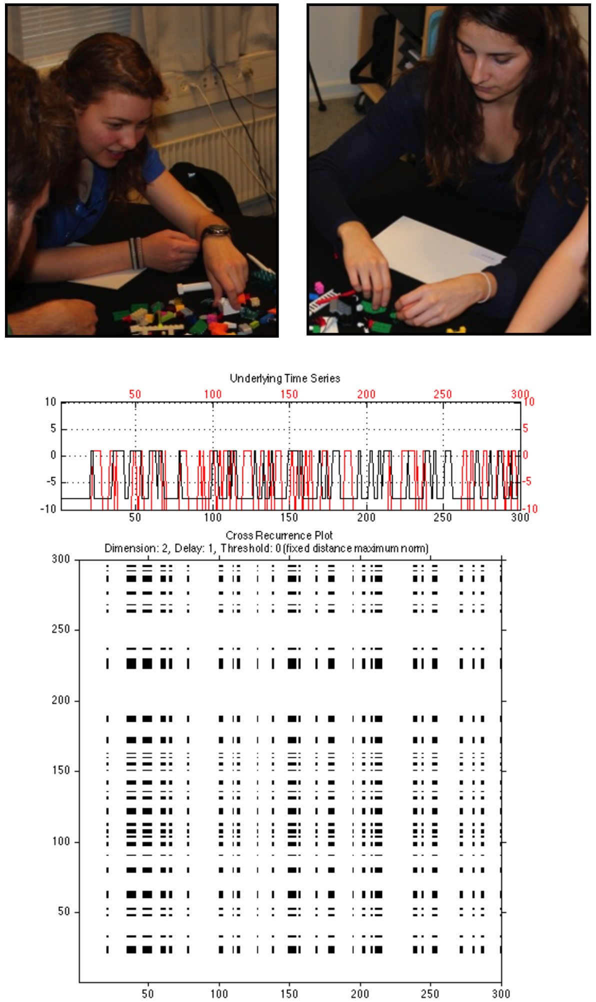
LEGO construction acts

S1:	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	… etc
S2:	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	… etc
S3:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	… etc
S4:	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	… etc
S5:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	… etc
Time(secs)			1	2	3	4	5	6	7	8	9	10	11	12	13	14 15 … etc

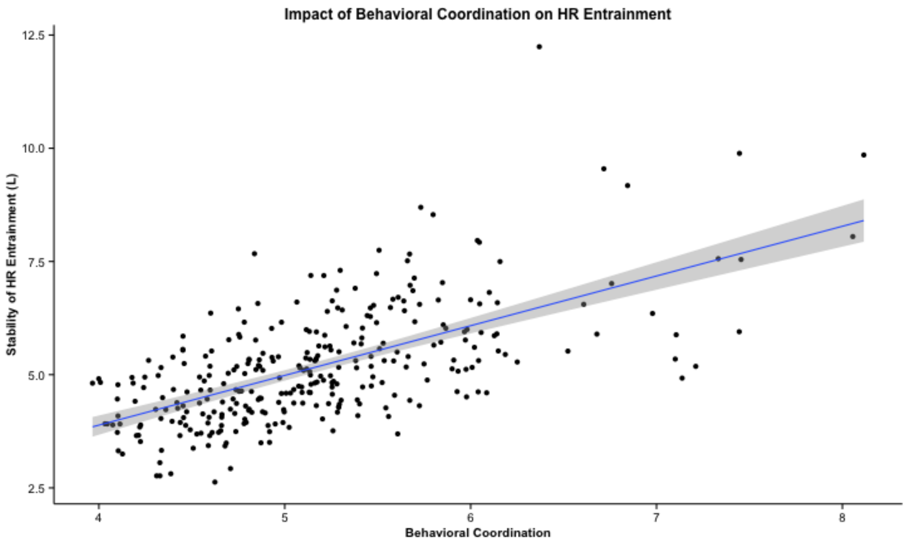


S1 S2 S3 S4 S5


Underlying Time Series
Cross Recurrence Plot
Dimension: 2, Delay: 1, Threshold: 0 (fixed distance maximum norm)


Underlying Time Series
Cross Recurrence Plot
Dimension: 2, Delay: 1, Threshold: 0 (fixed distance maximum norm)

H2:
From behavior to physiology



HR L	Build RR	2.04	4.19	0.63
$R^2m=0.37$	Build L	-0.16	0.09	0.077
$R^2=0.47$	Speech RR	1.77	0. 45	<0.0001*
	Speech L	0.02	≈0	<0.0001*

Q3: Impact on Experience

- H3:
  - If H2 is true, then behavioral coordination is a better predictor of experience than HR
    - IMI Relatedness
    - IMI Group Competence

H3:
What predicts experience?

Relatedness

- HR?
  - Nope
- Behavior?
  - Yups!
  - BA L and SA L
    Adj R2 = 0.45

Group Competence

- HR?
  - Nope
- Behavior?
  - Yups!
  - BA RR
    Adj R2 0.27

How would you make a better analysis?

Putting the heart back in its place?

- HR entrainment takes place even in everyday contexts
- But it seems more related to behavioral coordination…
  - Activity
  - Speech (respiration)
- Than it is to experience
  - Indeed behavior might be what is related to experience

However…

Military research

Business research

Sports research

Back to the board

Understanding the signals:
The heart, ECG and HR

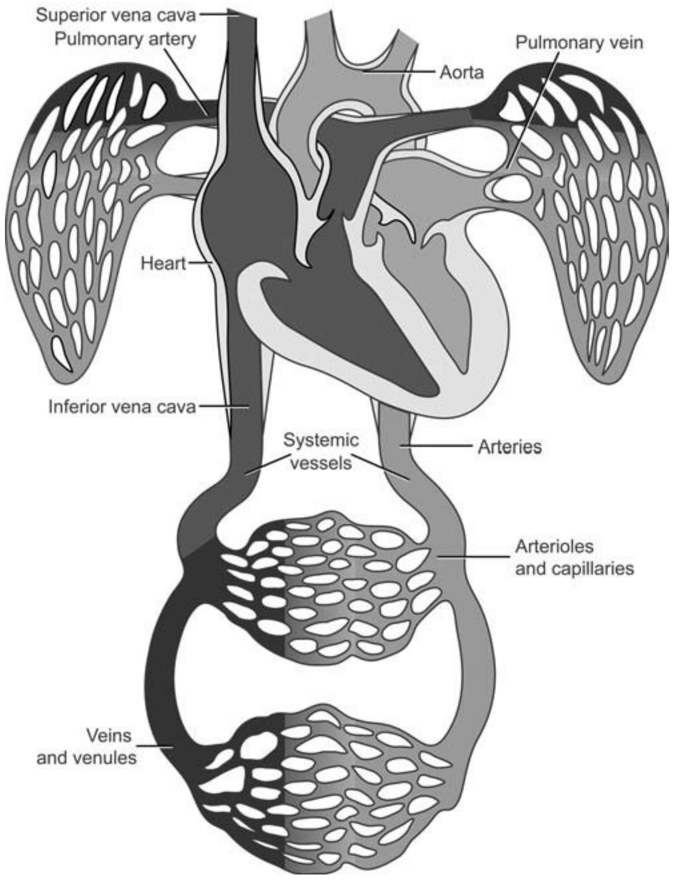
The Cardiovascular System

- Components: Heart (pump) and Vasculature (distribution system)

- Full Circle (Comprising pulmonary circulation and systemic circulation):

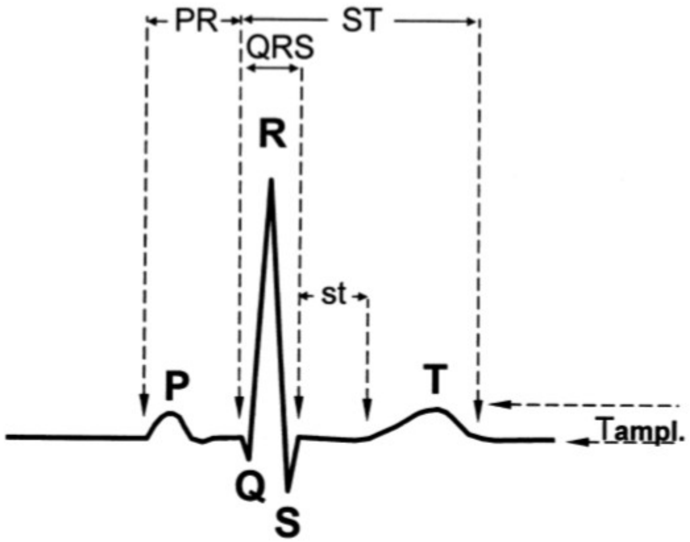
- Deoxygenated blood enters the right ventricle via the right atrium from which it is pumped to the lungs for re-oxygenation. Oxygenated blood then returns via left atrium to left ventricle from which it is pumped into the aorta. The capillary system transports it into the whole body and then back via veins to the heart.

- Start over

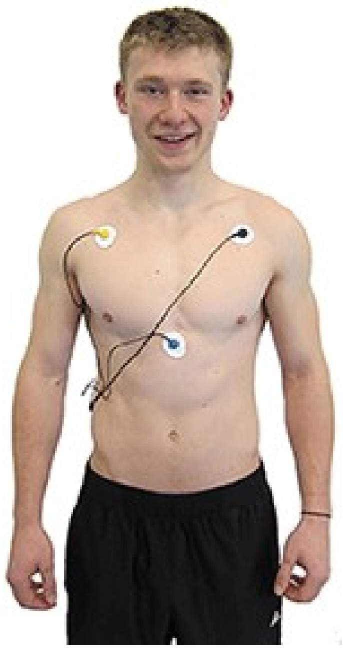


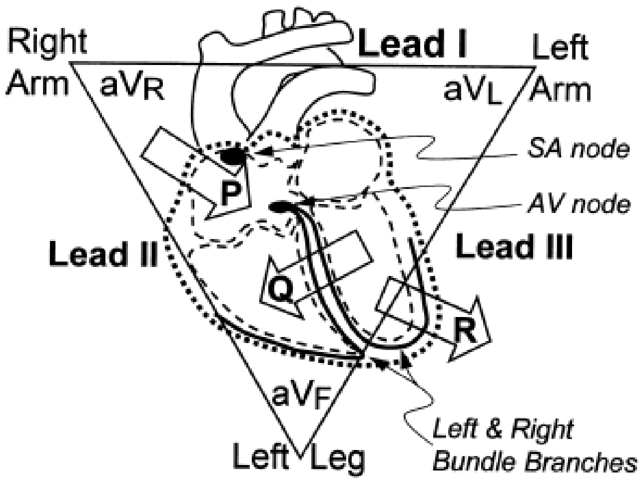
The Electrocardiogramm (ECG)

- Record of electrical activity of the heart by means of electrodes on skin detecting potential changes due to de/re-polarization of cardiac muscles:
  - Depolarization of sinoatrial node in right atrium (in latter part of diastole: blood refill. Circa 300 ml of blood at each time).
  - Wave passing through the atrial muscle. (P wave)
  - Atrial contraction
  - Ventricular contraction (QRS complex marking the latter two=
  - Valves open and close due to pressure conditions that also lead to refill of heart with blood after each systole
  - The initiation of the diastole is marked by the repolarization of the ventricles, which is represented by the T wave in the ECG.



Diastole Systole





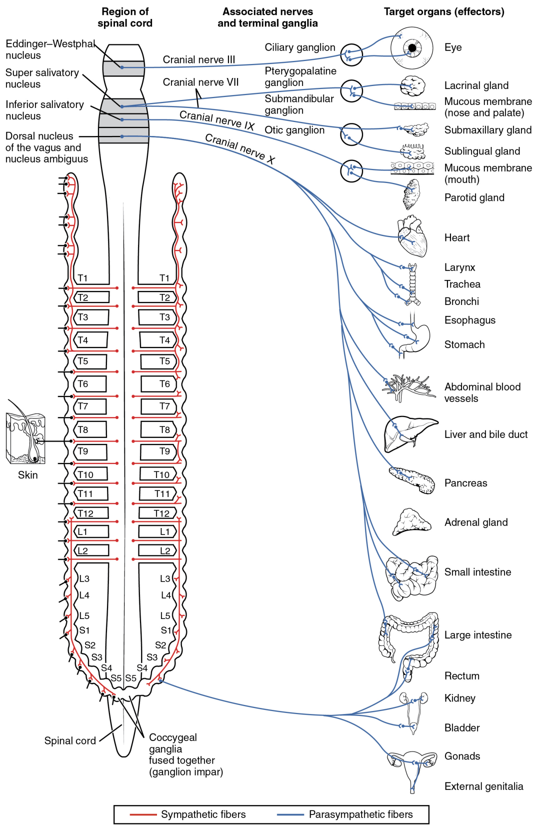
Heart activity regulation

- Intrinsic mechanisms:
  - **Local regulation**: Vasodilation/Vasoconstriction modulated by oxygenation of tissues
  - **Global homeostasis**: Frank Starling mechanism: greater venous return elicits greater contractility (stronger beat)

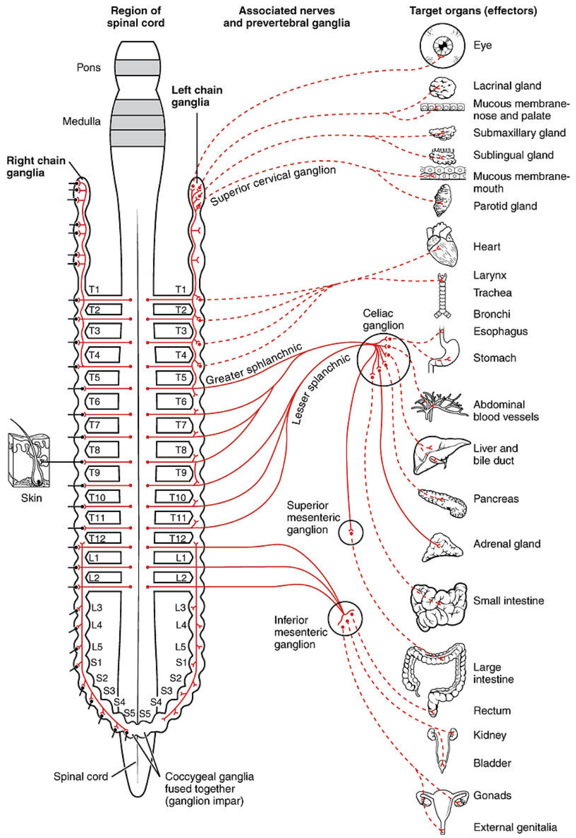
- Extrinsic mechanisms:
  - The **autonomous nervous system** (regulation of unconscious actions). Major cardiovascular center is located in medulla oblongata (low brainstem) and integrates information from proprioceptors, chemoreceptors and mechanoreceptor from the heart as well as from cerebral cortex and limbic system. Heart activity is adjusted via shifts in relative balance between **sympathetic and parasympathetic outflows**.
  - **Hormonal control**: a true mess in the literature

The autonomous system

Parasympathetic



Sympathetic



Promote “Fight and flight”

 slower, diffused but longer lasting effect

Autonomous hearts

Parasympathetic

- It slows heartrate
- Short response time (short postganglionic fibers; fast dissipating acetylcholine as neurotransmitter)
- Short term effects
- Immediate response within cardiac cycle in which it occurs and affects only one or two heartbeats after onset.
- Dominant during resting state (constantly activated).

- It can slow the heart down to 20 or 30 bpm or even briefly stop it in the extreme case. Dynamic range of up to 125 bpm

Sympathetic

- It speeds heart rate
- Slow response time (Long postganglionic fibers; Slowly dissipating noradrenaline as neurotransmitter): delay of up to 5 seconds before HR increase.
- Long term effects: even a short stimulus can affect HR for 5-10 seconds.

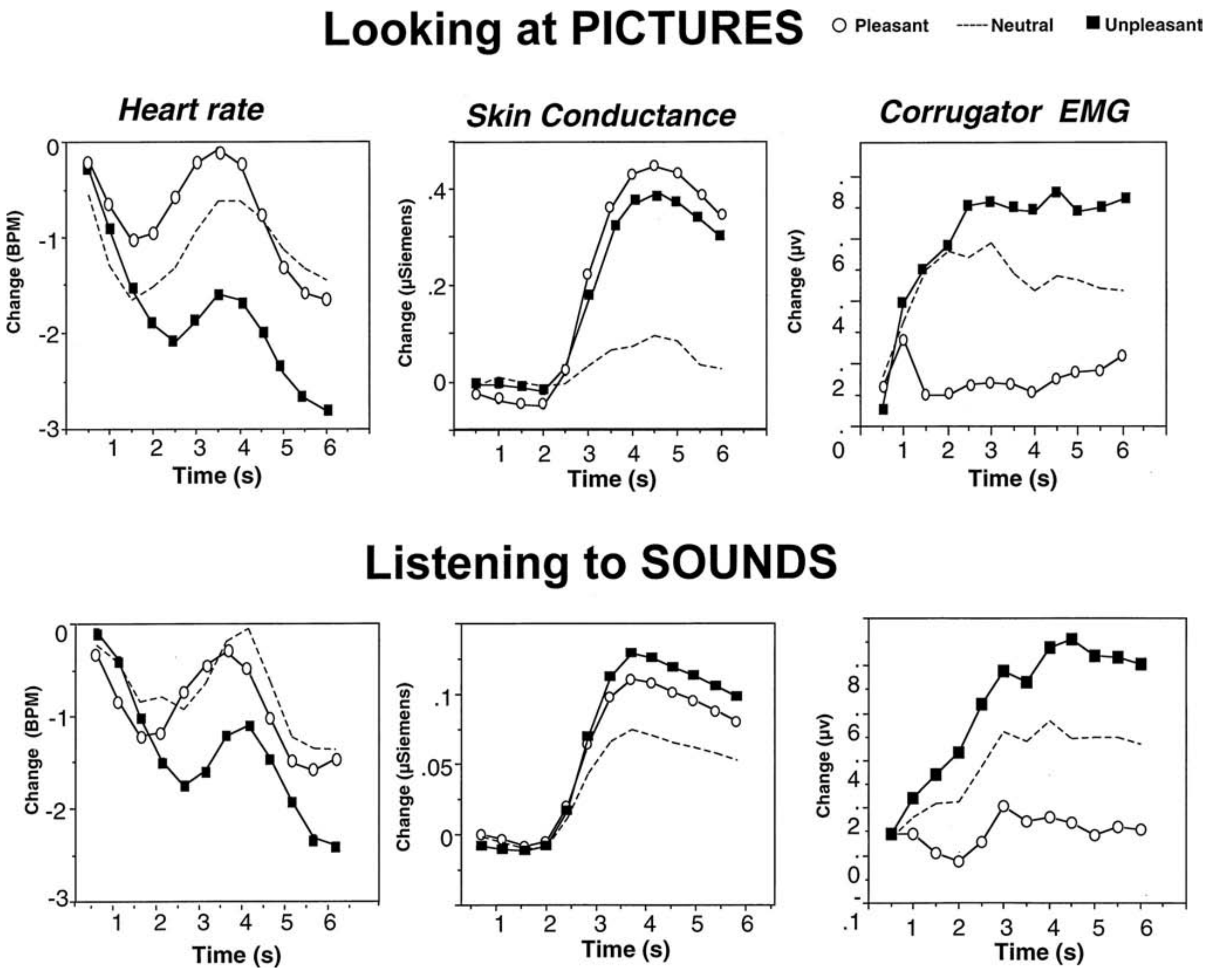
- It can speed the heart up by 5-15 beats per minute only, with a steady peak after 20-30 s when stimulus stays continuously there. Dynamic range of up to 55bpm only

What has been done with that? I

- Study of emotions
  - “Such measures as skin resistance, heart rate [..]have been shown to be remarkably sensitive and responsive measures in a variety of “emotional” states. Conflict, threat and frustration; anxiety, anger and fear; startle and pain; embarrassment, pleasant and unpleasant stimuli; all these produce autonomic changes” (Lacey 1958)

  - Against an idea of discrete emotional states (à la Eckman – Lie to me), towards actions as “predisposition to actions” along continuous dimensions. E.g.:
    - valence (positive vs. negative)
    - intensity (level of arousal) (Schneirla, 1959; Hebb, 1949)

  - Triangulating with direct report and behavioral observation


Looking at PICTURES
Pleasant
Neutral
Unpleasant
Heart rate
Skin Conductance
Corrugator EMG
Change (BPM)
Change (µSiemens)
Change (µv)
Time (s)
Listening to SOUNDS
Change (BPM)
Change (µSiemens)
Change (µv)
Time (s)

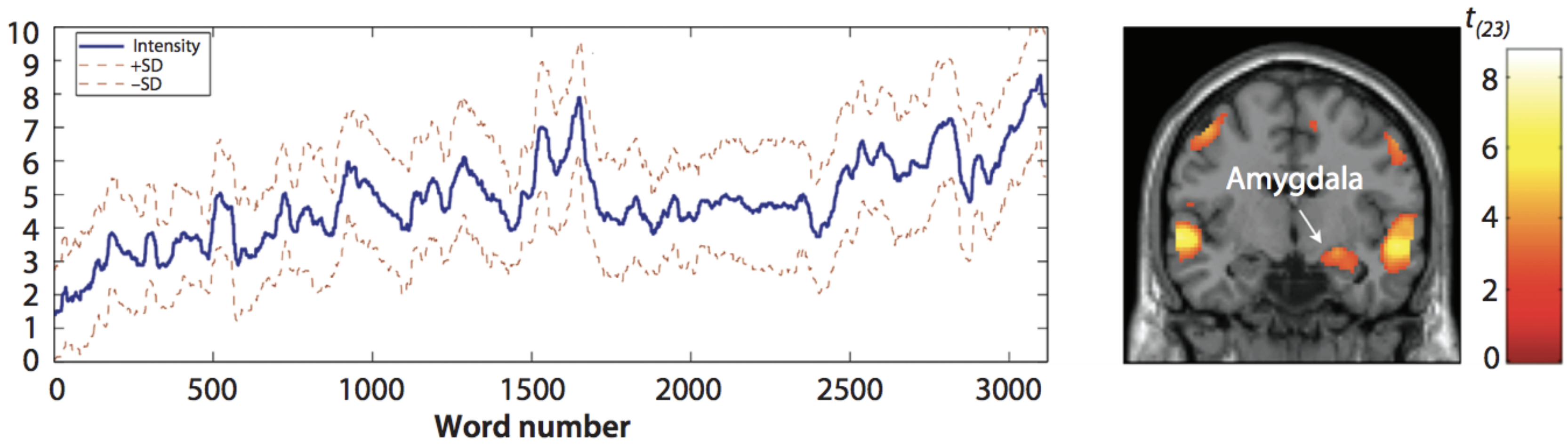
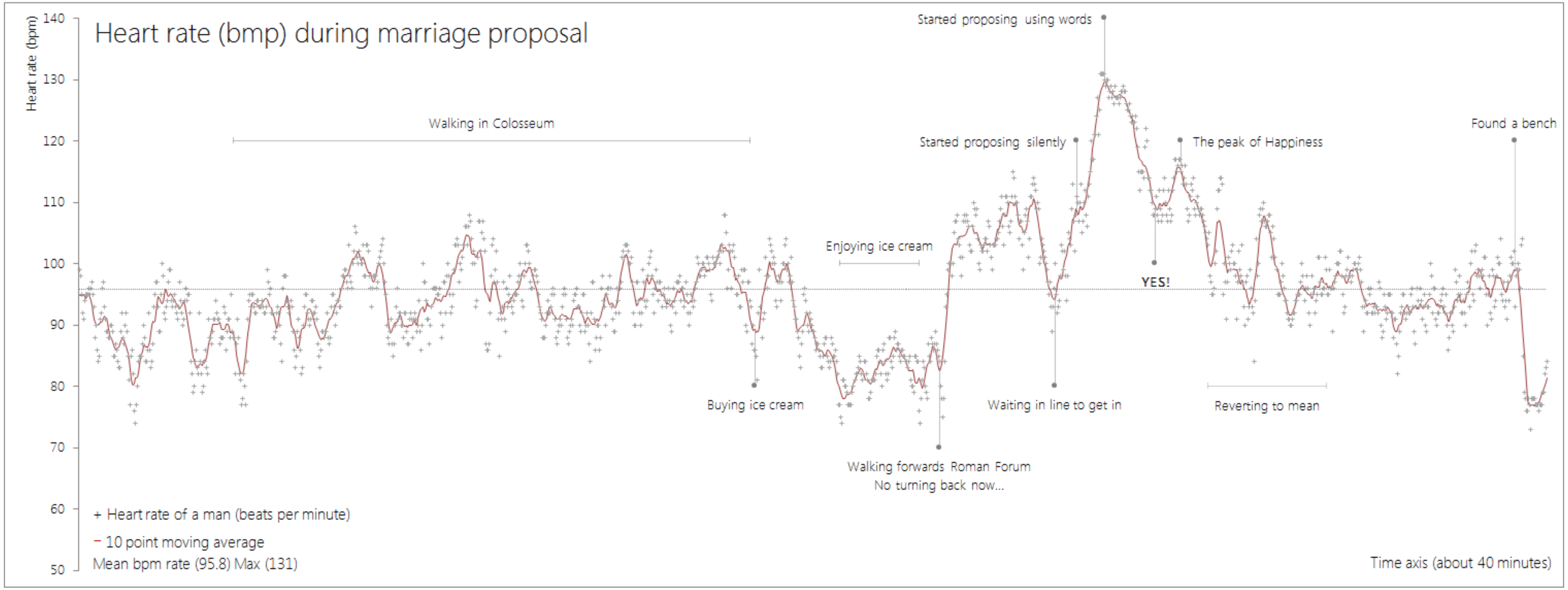
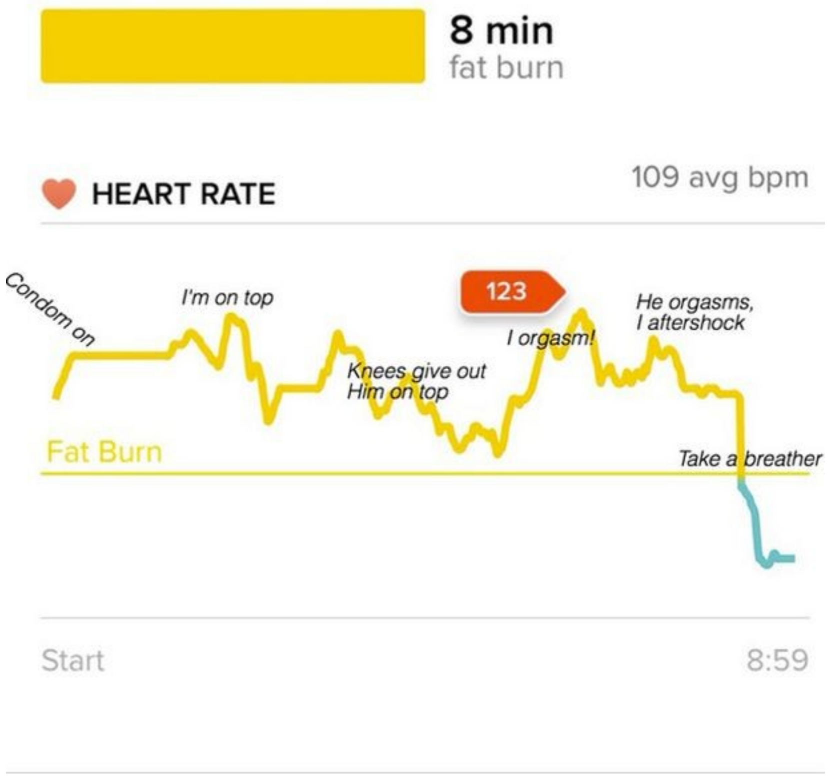


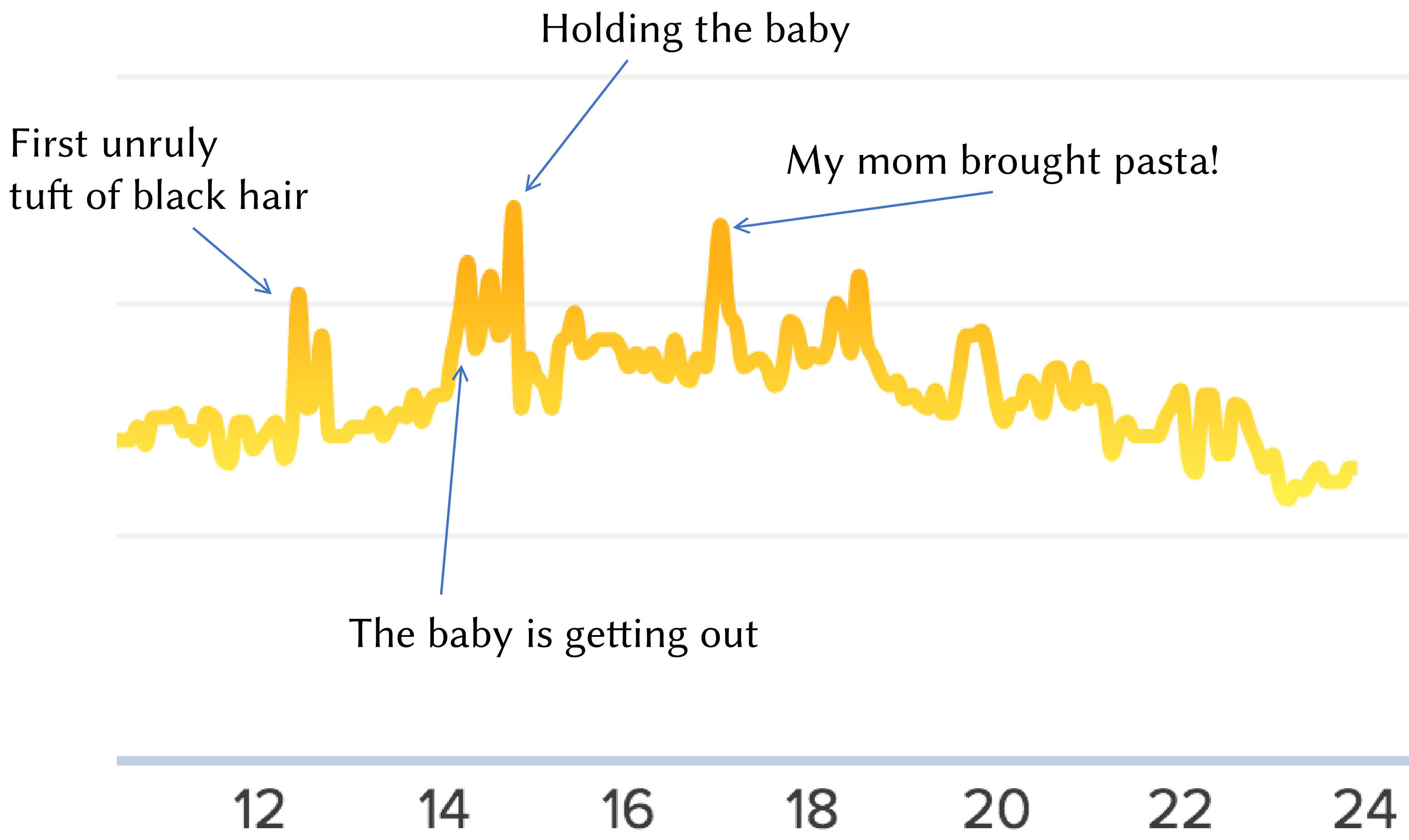
Figure 1. Results from Wallentin et al. *NeuroImage*, 2011a. *Left*: Intensity profile from the Ugly Duckling, as measured by an average experience of 27 raters on a scale from 0 to 10. *Right*: Brain activation in a basic emotion processing network (amygdala, thalamus and primary auditory cortices) correlated with intensity profile in a group of participants listening to the story.

In the wild



Heart rate (bmp) during marriage proposal
Heart rate (bpm)
140
130
120
110
100
90
80
70
60
50
Walking in Colosseum
Buying ice cream
Enjoying ice cream
Walking forwards Roman Forum
No turning back now...
Started proposing silently
Waiting in line to get in
Started proposing using words
YES!
The peak of Happiness
Reverting to mean
Found a bench
+ Heart rate of a man (beats per minute)
− 10 point moving average
Mean bpm rate (95.8) Max (131)
Time axis (about 40 minutes)


8 min
fat burn
HEART RATE
109 avg bpm
Condom on
I'm on top
123
I orgasm!
He orgasms,
I aftershock
Knees give out
Him on top
Fat Burn
Take a breather
Start
8:59

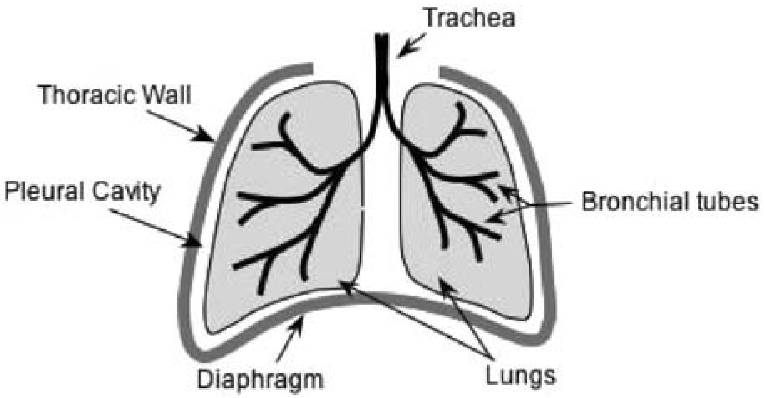


Holding the baby
First unruly
tuft of black hair
My mom brought pasta!
The baby is getting out
12
14
16
18
20
22
24

What has been done with that? II

- Study of cognitive processing
  - Heart rate variability (e.g. SD of R-R) has been shown to be lowered by sustained attention, e.g. cognitive processing during executive tasks (action planning, error detection, automatic response inhibition etc.).
    - Tho' much smaller in physically fit individuals

Thesis idea: a proper review / meta-analysis of relations between HR and cognition (or emotions)



Respiratory System

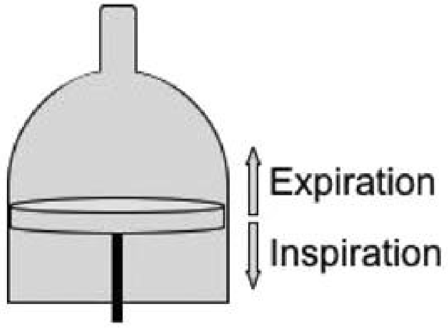
The respiratory system

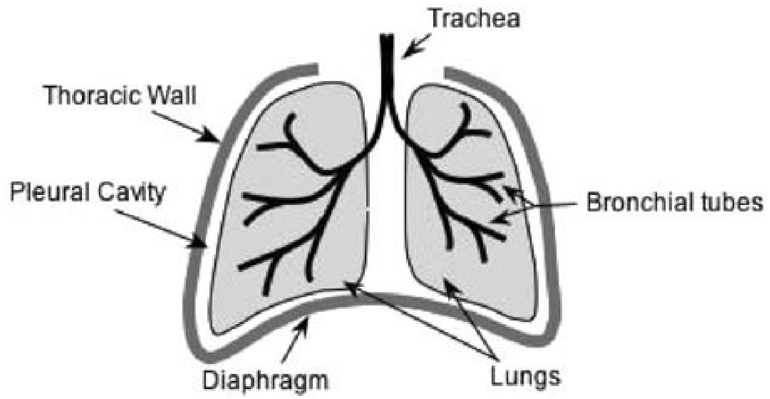
- Phenomenon “caught midway between being conscious and unconscious” (Richards, 1953)

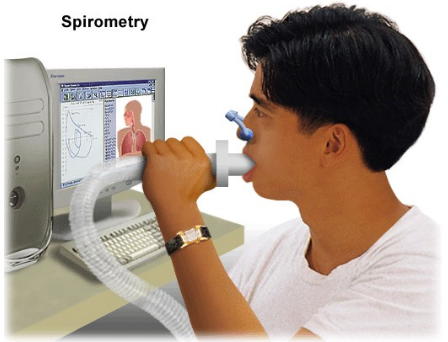
- Functions of respiration:
  - gas exchange of blood,
  - Airflow regulation for speech,
  - pressure necessary to bring odors to olfactory mucosa,
  - anticipating metabolic demands of cognitive and muscular activity etc.

Facts and figures:

- Organ with massive surface area (140m2) due to ca. 30 million alveoli sacks in 5 lung lobes (3 right, 2 left), volume of normal adult lung ca. 5l.
- Respiration enabled by contraction of diaphragm muscle which increases volume of thoracic cavity leading to low pressure and air flowing in. Further help by intercostal muscles.
- Exhalation largely passive process (result of gravity and elasticity of rib fascia and thoracic cavity). Forced Exhalation by rapid relaxation of external intercostal and diaphragm and contraction of inner intercostal and abdominal muscles

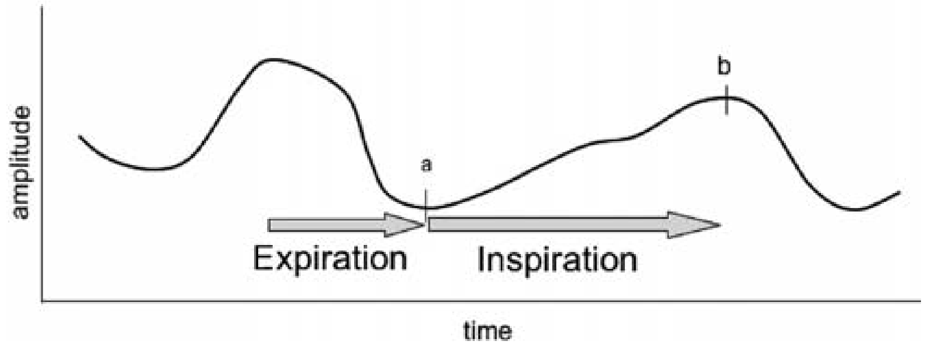






Spirometry = psychologically invasive (centering patients focus on breathing),

Pressure belt = non intrusive, but less informative (no info about blood gasses etc.) AND sensitive to fit & muscle movements)



Typical output respiratory belt

What cognitive stuff can we learn from respiration?

- Historical hype:
  - Feleky (1916): Ratio between inspiration and expiration is intimately connected to psychological state and personality
  - Nielsen & Roth (1929): There are 10 breathing patterns associated to personality traits
  - Sutherland, Wolf, & Kennedy (1938): Respiratory curve as constant and characteristic of an individual as its handwriting.

- Modern downplay:
  - Recording just to account for “artifacts”/baselines, such as coupling to heart rate (respiratory sinus arrhythmia), electrodermal activity, BOLD signal, etc.
    - But is it really an artifact?

What cognitive stuff can we learn from respiration?

- Emotion
  - Boiten (1998) found that positive affect is associated with shorter inspiratory cycles, disgust leads to respiratory pauses.
  - Gomez, Shafy and Danuser (2008) found associations between respiration cycles and both valence and arousal
- Cognitive activity
  - Cognitive load
  - occlusion of the left or right nostril alters cognitive function (I shit you not!)
    - Super p-hacked
  - Executive control: super-controversial
- Speech
  - Infer stuff about the syntax
  - Infer stuff about the reading / cognitive complexity of the speech task
  - Infer stuff about turn-taking

Thesis idea ;-)

Learning more

- Skeptically read:
  - Cacioppo, Tassinary, & Berntson (2007). Handbook of Psychophysiology. Cambridge University Press.

Understanding the signal

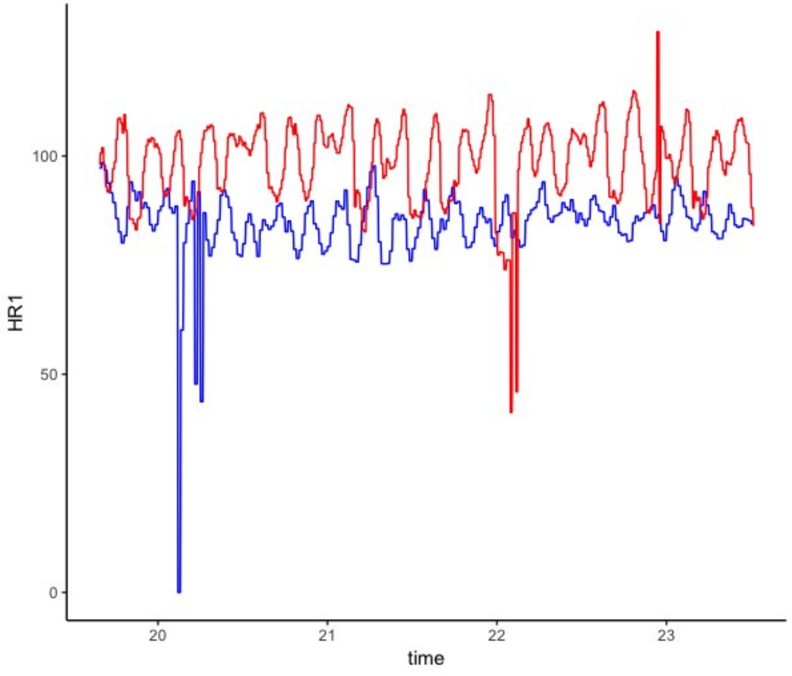
Step 0: Downsampling

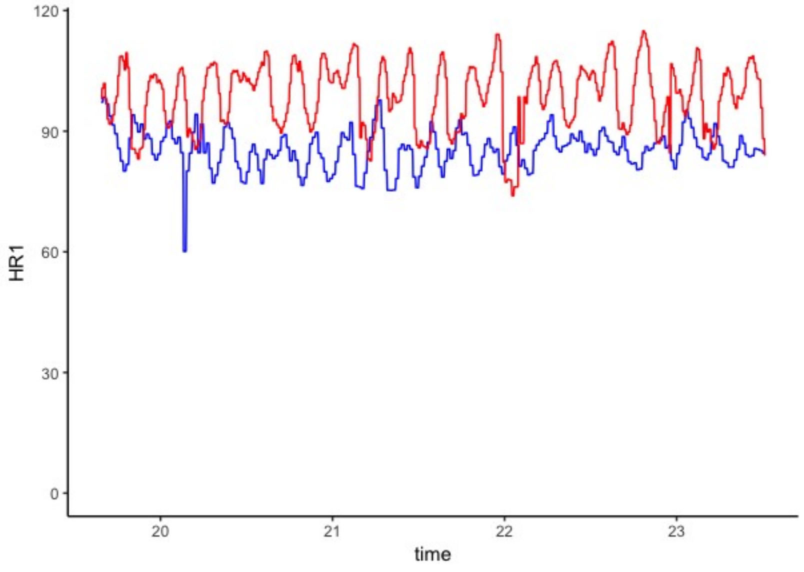
- Simply too much darned data

- p_load(groupdata2) # Thanks to Ludvig/Benjamin

- d1 = d %>%
```
  group(n = 100, method = 'greedy') %>%
  dplyr::summarise(
    time = mean(time,na.rm=T),
    HR1 = mean(HR1,na.rm=T),
    HR2 = mean(HR2,na.rm=T),
    Resp1 = mean(Resp1,na.rm=T),
    Resp2 = mean(Resp2,na.rm=T))
```

Example 1





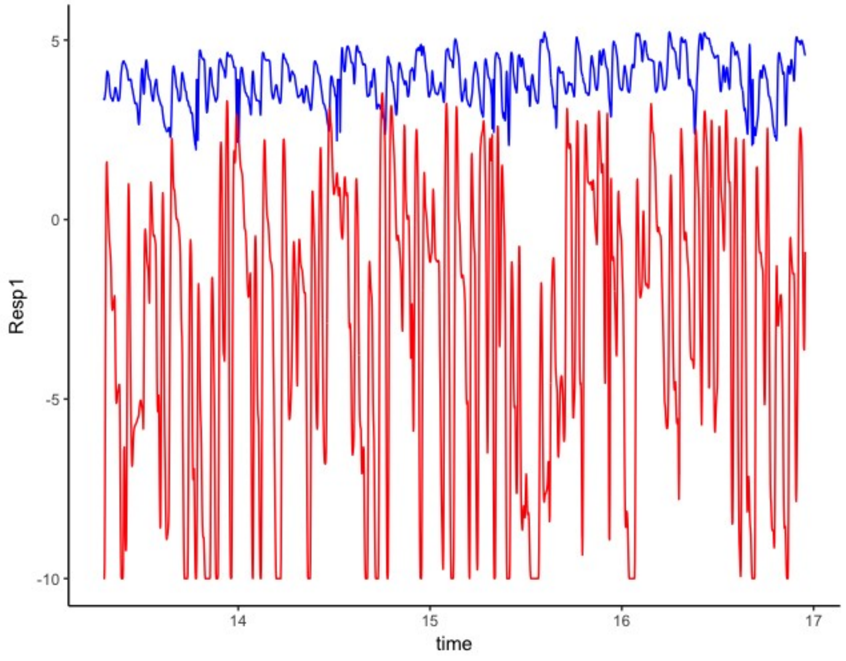
Thoughts?

Example 2 – 1 step at a time

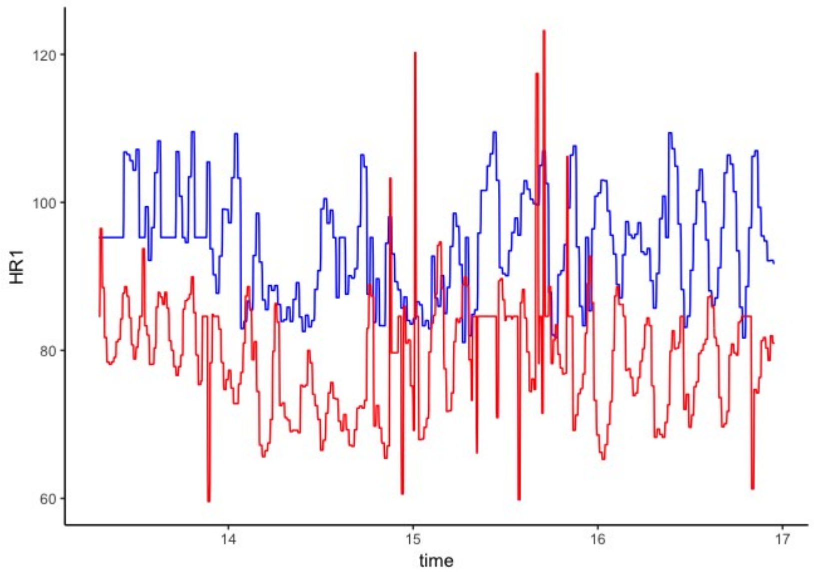
- Outlier removal:
  - ```
    removeOuts <- function(ts,threshold){
       ts[ts > (mean(ts,na.rm=T) +
               (threshold*sd(ts,na.rm=T))) |
            ts < (mean(ts,na.rm=T) –
               (threshold*sd(ts,na.rm=T)))] = mean(ts,na.rm=T)
    return(ts)}
    threshold=2.5
    d$HR1=removeOuts(d$HR1,threshold)
    ```
    ```

Example 2

Respiration



Heart rate



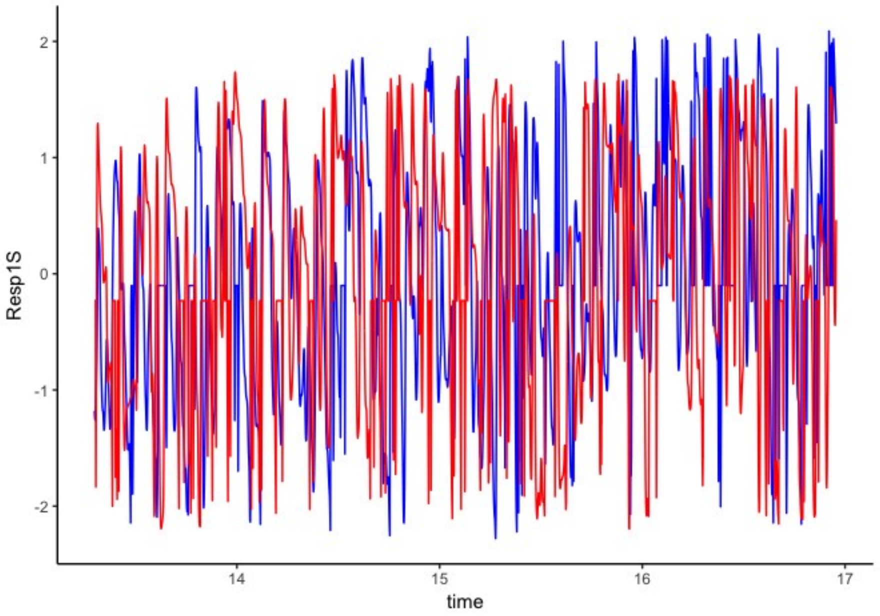
What to you think?

Example 2

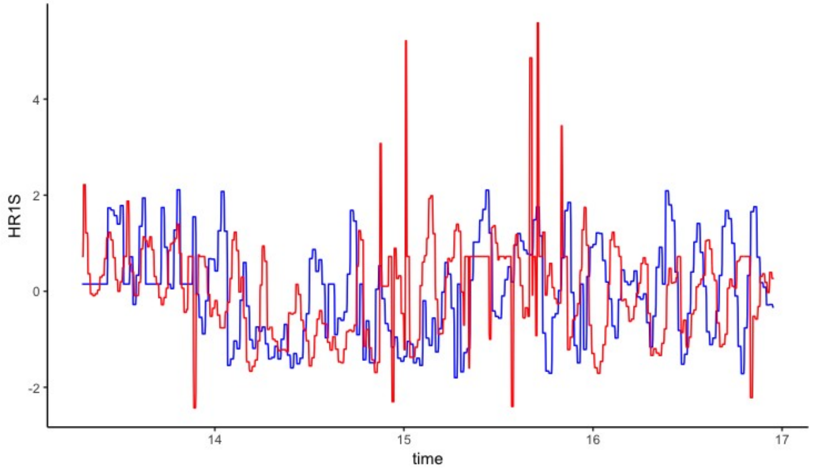
- Scaling the signals:
  - d1$Resp1S=scale(d1$Resp1) d1$Resp2S=scale(d1$Resp2)
    d1$HR1S=scale(d1$HR1)
    d1$HR2S=scale(d1$HR2)

Example 2 – 1st step

Respiration scaled



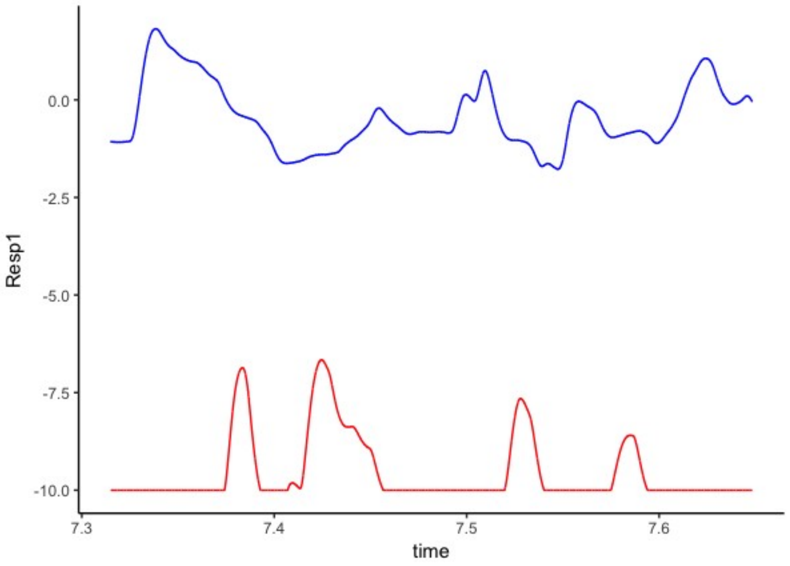
Heart rate scaled



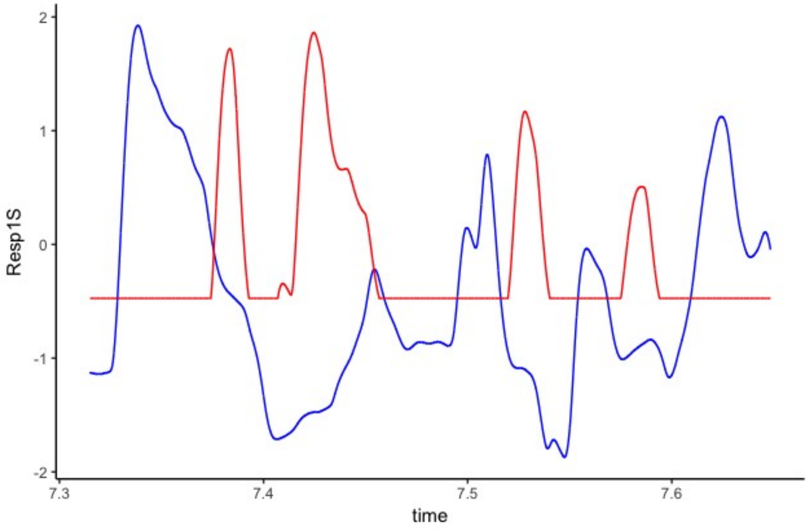
What to you think?

Example3

Respiration



Respiration scaled



What to you think?

Define interpersonal coordination

2 minutes

Some examples

- Having a chat at the bus stop
- Fighting with the guy who just cut the line
- Jointly deciding how to organize a Friday bar
- Team sport matches
- Collectively piloting a battleship
- Stock-option market
- Science

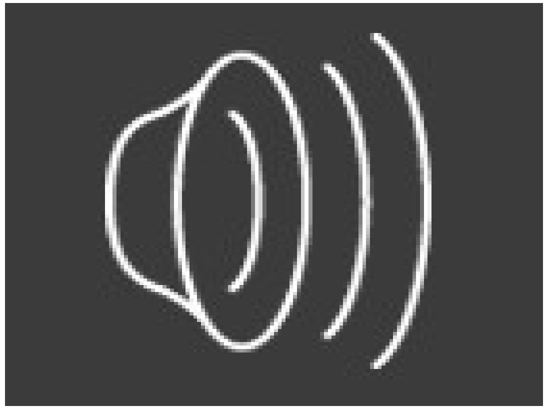
A minimal definition

- at least two agents,
- engaging in a social activity (with goals and affordances)
- most often employing several expressive modalities (e.g. words, prosody, gestures, posture, etc.)
- continually influencing each other, in ways that are difficult to capture when the individual behaviors are analyzed separately

The centipede dilemma

The issue of social coordination: A simple mechanism

- Synchronization-inspired framework:
  - Interacting agents tend to adapt to each other in physiology and behavior (via priming)
  - The more they get similar in behavior and physiology, the better the reciprocal understanding and therefore the coordination.



The chameleon effect

- The chameleon effect (Chartrand & Bargh 1999)
- The devil's advocate effect

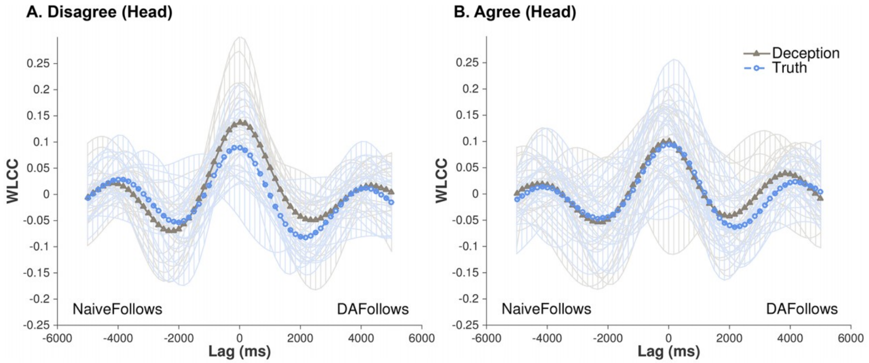
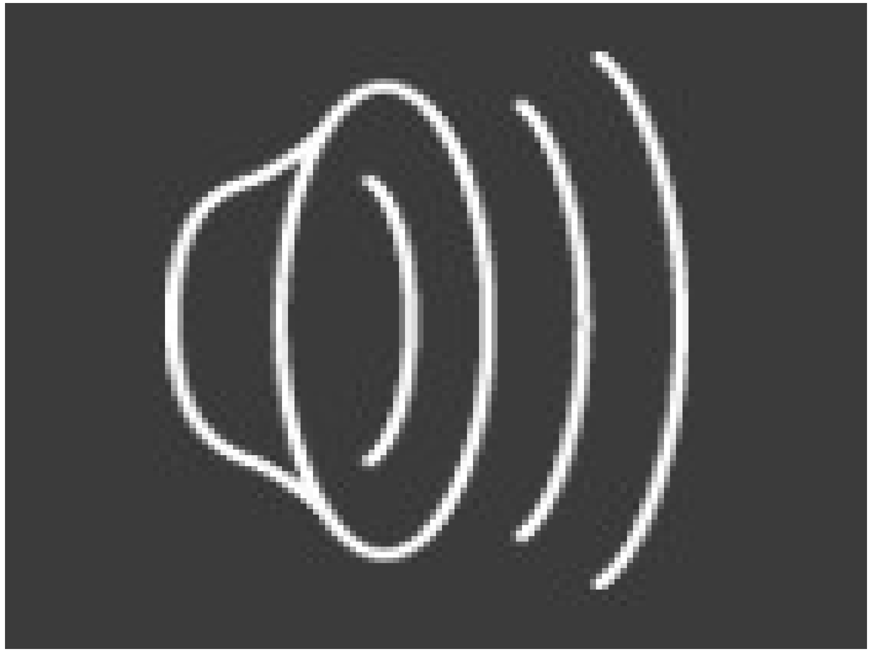


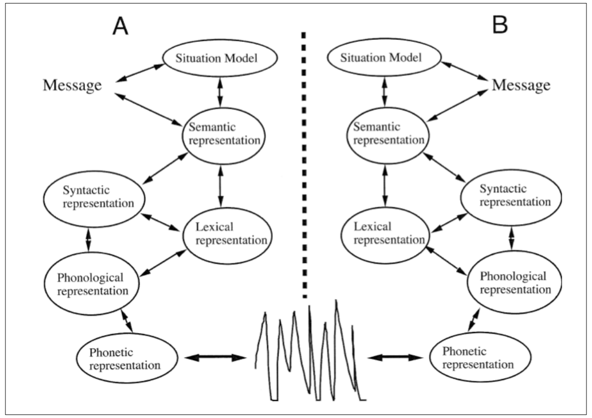
Fig 3. WLCC profiles for actual DAs and naives' head movements. (A) Disagree and (B) Agree conversations involving deception (triangle-solid line) or truth (circle-dashed line). For (A), systematic patterns of synchronization were found, peaked at near-simultaneous shared activity (Lag 0) and decreases as movements were lagged. Figures also show each dyad's contributions (opaque lines) to the average pattern across all conditions.

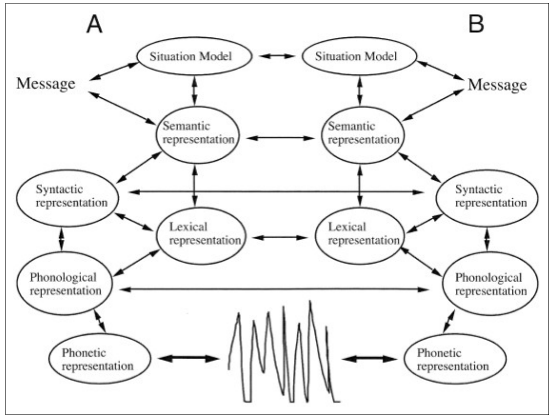
Do people really work this way?



Pickering and Garrod: the interactive alignment model of conversation*

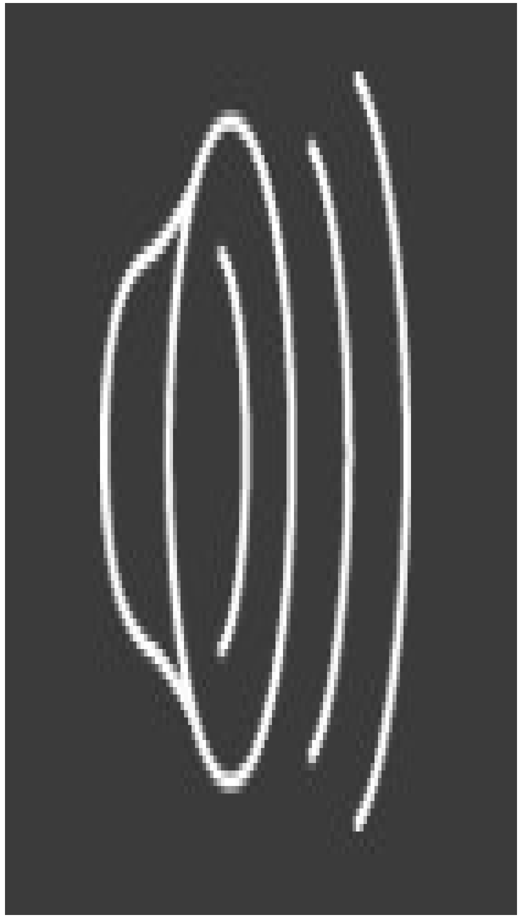
- "… in dialogue, the linguistic representations employed by the interlocutors become aligned at many levels, as a result of a largely automatic process." (2004:169). By hearing a linguistic expression I am primed to re-use it.





*Pickering and Garrod (2004). *Brain and Behavioral Sciences*

Basics of linguistic alignment



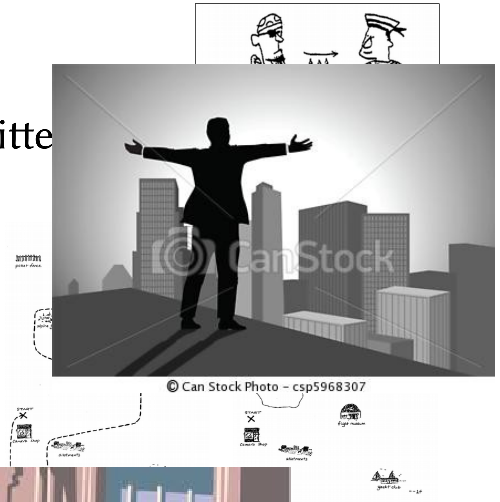
Experimenter confounder?

- https://www.youtube.com/watch?v=_JmA2ClUvUY

Evidence of Linguistic Alignment

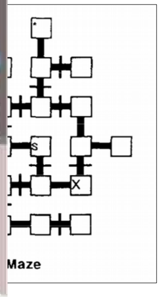
- Syntactic priming (Branigan et al 2007, Reitte

- 6)



g (Garrod and Do

e Matching (Irela



Physiological coordination

- Respiration
- Heart rate
- Galvanic skin response
- fMRI, EEG, MEG signals

Physiological coordination

- construction of a sense of community (Konvalinka et al., 2011),
- empathy and mindreading (Levenson & Gottman, 1983; Levenson & Ruef, 1992),
- group social structure (Cleveland, Finez, Blascovich, & Ginther, 2012),
- team performance (Elkins et al., 2009; Henning, Boucsein, & Gil, 2001; Henning & Korbelak, 2005; Strang, Funke, Russell, Dukes, & Middendorf, 2014; Wallot, Mitkidis, McGraw, & Roepstorff, submitted)

Quantifying coordination

- Tools to quantify coordination
- Appropriate contrasts

Challenges in analyzing interpersonal coordination

- Imagine you have two timeseries, e.g. heart rate, or accelerometer data

- How do you analyze whether they are coordinated?
  - What are the challenges?
  - 5 min

Challenges in analyzing interpersonal coordination

- Interdependent components
- Non-stationary values
- Varying lags of coordination (e.g. due to turn taking)
- Different sources of "coordination"

Cross Recurrence Quantification to the rescue

The basic idea

Cross-recurrence plot

Cross-recurrence quantification analysis:

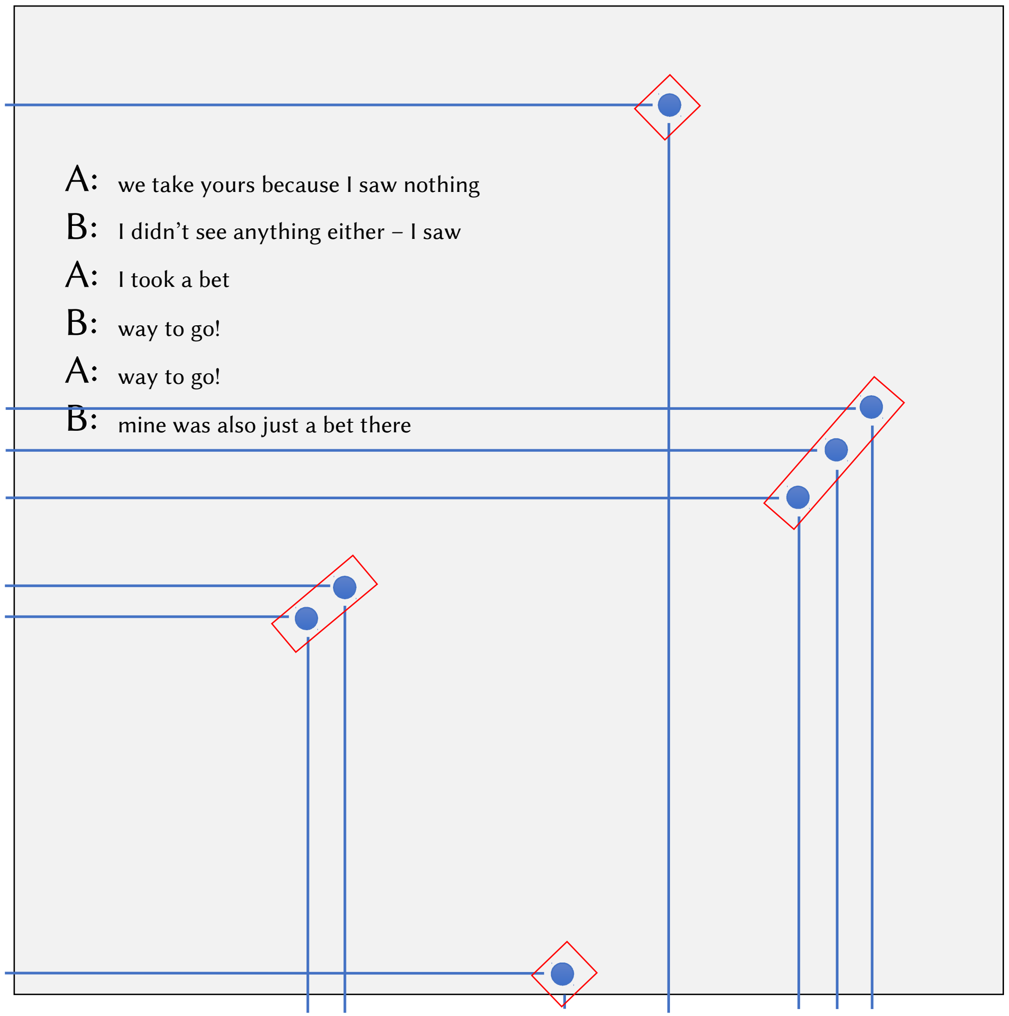
- *Averaged diagonal line length*

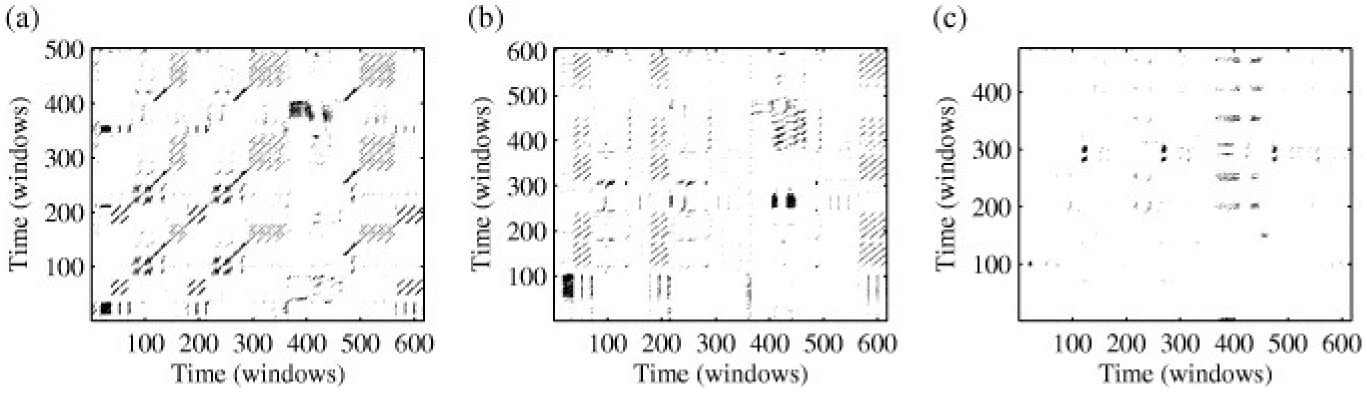
$$L = \frac{\sum_{\ell=\ell_{\min}}^{N} \ell\, P(\ell)}{\sum_{\ell=\ell_{\min}}^{N} P(\ell)}$$

- *The Shannon entropy of line lengths*

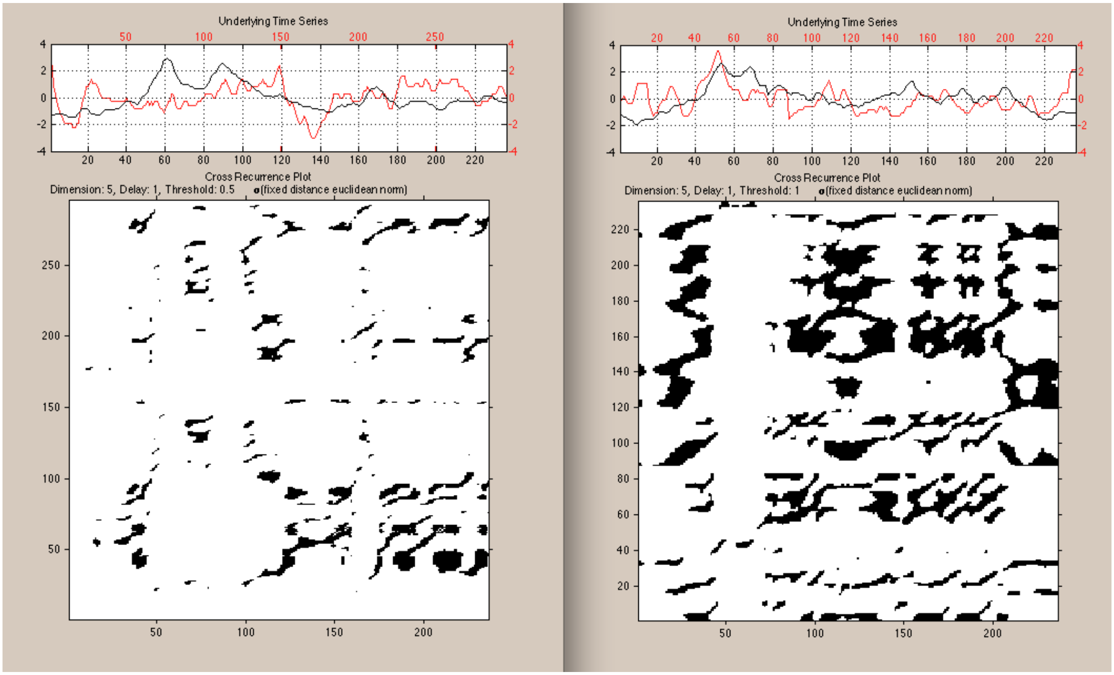
$$\mathrm{ENTR} = -\sum_{\ell=\ell_{\min}}^{N} p(\ell) \ln p(\ell),$$

Same method can be used to assess individual behaviors and synergy (auto recurrence)





CRPs for the song *Gimme, Gimme, Gimme* as performed by the group ABBA, taken as song X, versus three different songs, taken as song Y. These are a cover made by the group A-Teens (a), a techno performance of the song *Hung up* by Madonna (b), and the song *The Robots* by Kraftwerk (c).


Underlying Time Series
Cross Recurrence Plot
Dimension: 5, Delay: 1, Threshold: 0.5
(fixed distance euclidean norm)
Underlying Time Series
Cross Recurrence Plot
Dimension: 5, Delay: 1, Threshold: 1
(fixed distance euclidean norm)

Sign me up!

How do we do this?

Steps

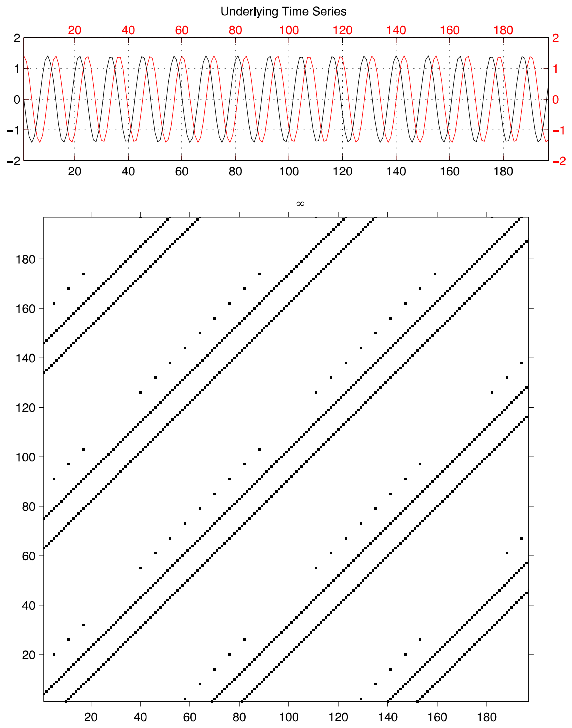
- Delay: autocorrelation
- Embedding dimensions: how long a sequence?
- Radius: how similar should it be to be counted as recurrence?
- CRQA Measures

Cross Recurrence Quantification Analysis

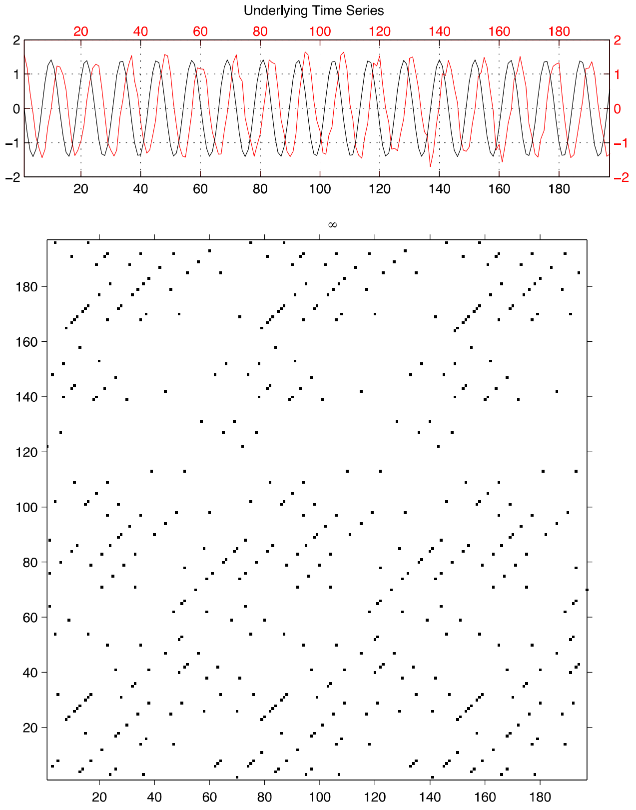
- Reconstructing the phase space of the 2 systems
  - Where does mutual info *between* the two timeseries minimize?
  - Where does the number of false neighbors *btw* the 2 timeseries minimize?

- Assessing whether and how they display similar trajectories
  - Put one timeseries on the x axis, one on the y axis

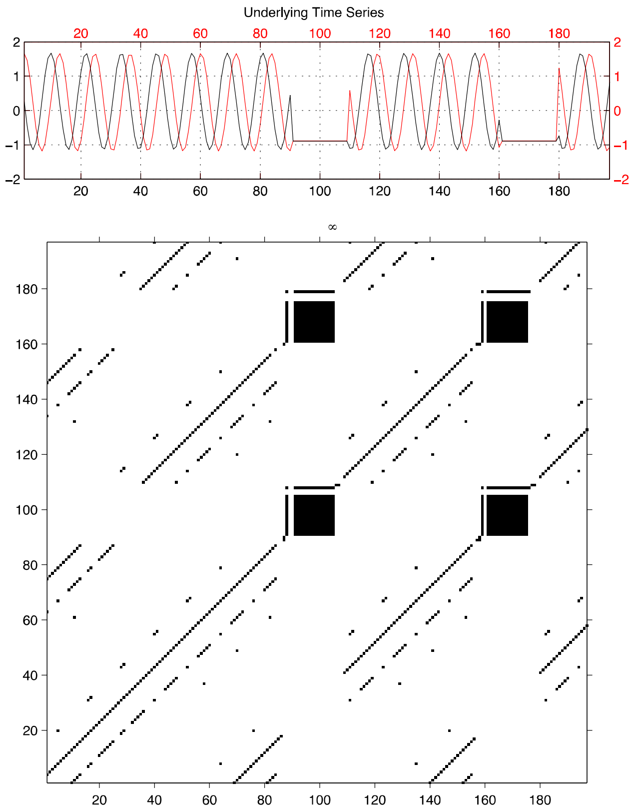
Perfectly lag-coordinated pendula



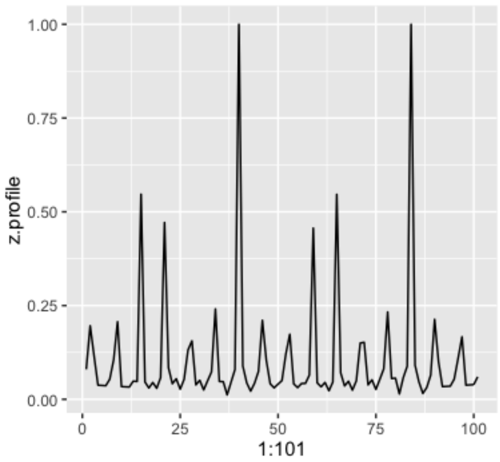
Noisy pendula



Stopped pendula



Diagonal recurrence profile



-50 -25 0 +25 +50

Code?

- Finding the optimal parameters: run on all pairs:
  - par = list(lgM = 50, steps = seq(1, 6, 1), radiusspan = 100, radiussample = 40, normalize = 0, rescale = 0, mindiagline = 2, minvertline = 2, tw = 0, whiteline = FALSE, recpt = FALSE, fnnpercent = 10, typeami = "mindip”)
  - ans = optimizeParam(x1, x2, par, min.rec = 3.5, max.rec = 4.5)

- Choose a common value for delay, emb.dim and radius

- Results=crqa (x1, x2, delay=delay, embed=emddim, radius=radius,normalize=0,rescale=0,mindiagline = 2,minvertline = 2)

- Represent the plot:
  - RP=Results$RP
  - RP = matrix(as.numeric(RP), nrow = ncol(RP))
  - cols = c("white","blue4”)
  - image(RP, xlab = "", ylab = "", col = cols)

- Explore the lags of coordination:
  - Profile=drpdfromts(x1, x2,datatype = 'continuous',ws=50,radius=radius)
  - timecourse = round(seq(-5000,5000,100)/1000, digit = 1)
  - maxlag = Profile$maxlag/1000
  - profile = Profile$profile*100
  - Prof=data.frame(profile)
  - ggplot(Prof, aes(timecourse,profile))+geom_line()+ geom_vline(xintercept = timecourse[maxlag], colour='red')

Different sources of coordination

- Assume you've established that the two signals seem coordinated. Where does coordination come from? Aka, what could cause two signals being similar?

- Components of "coordination":
  - The signals involve similar values (e.g. speaking very fast minimizes variability in the signal)
  - Task related coordination (music example)
  - Actual coordination (music example)
  - Long term routines coordination

Challenges in analyzing interpersonal coordination

- How do we establish whether there is “significant” coordination?
  - Signal similarity:
    - Chance level: real vs. shuffled time-series
  - Task-related coordination:
    - Real vs. surrogate pairs
  - Within pair manipulation

Cracks in the wall

Is alignment always good?

- Escalating a fight
- Optimally aligned interaction
- Some contrary results:
  - More lexical alignment, less collaborative benefit (Fusaroli et al 2012)
  - More speech rate and pitch alignment, more awkwardness (Michael et al 2015)
  - More speech rate and head movement alignment, deception (Duran et al 2017)
  - More physiological alignment, less effective team coordination (Strang et al; Wllot et al; Mønster et al)

Is coordination just alignment?

- Turn-taking
- Complementarity
- Routines

Alternative approaches?

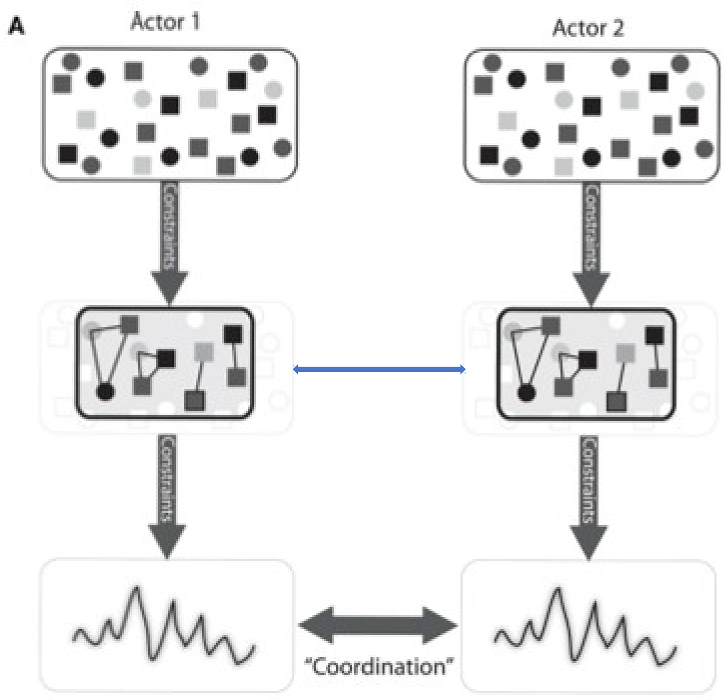
Interpersonal synergies

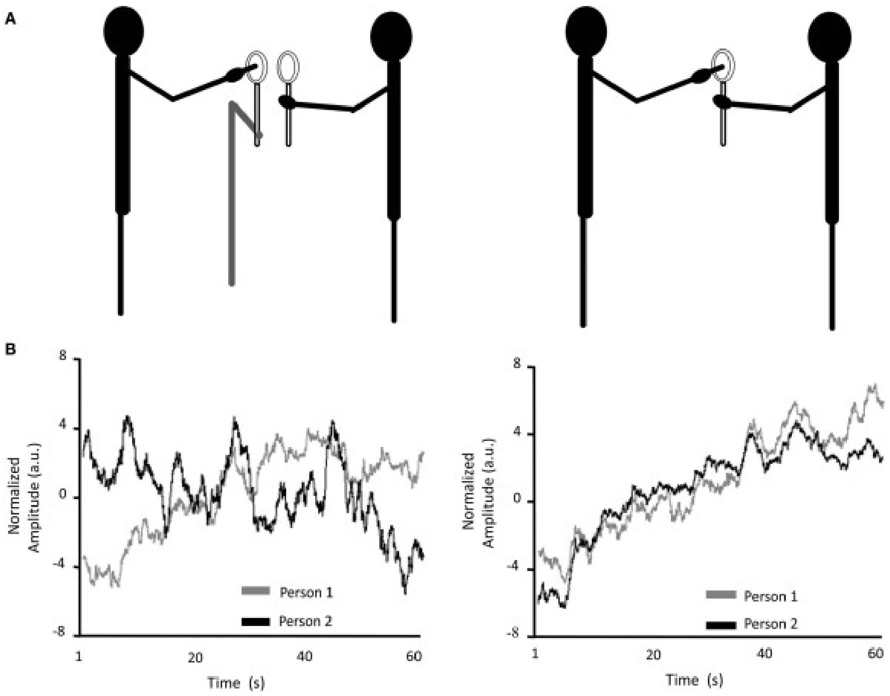
Bernstein 1956
Latash 2008

Interpersonal Synergies

- Functionally driven (Depending on the task-constraints!)
- Coordination as Interpersonal Reduction of Degrees of Freedom
  - **Task-oriented** Alignment
  - Complementary dynamics (e.g. taking different roles)
- The appropriate unit of analysis for linguistic behavior in conversations is the complementary structures making up the conversation as whole.

Riley et. al. 2011
Kelso 2010
Fusaroli et al. (2014) / Dale et al (2014)

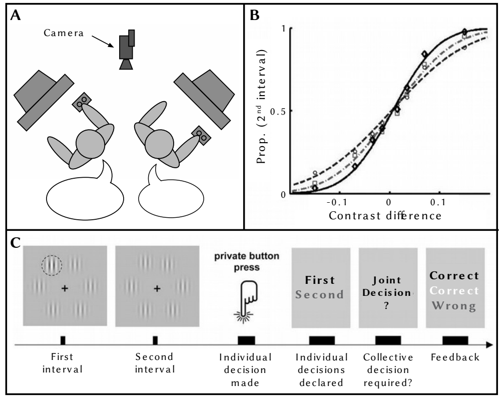




And in language?

The corpus of task-oriented conversations

- Two interlocutors repeatedly discussing and solving a problem together
- Objective index of collective performance (fostered by linguistic coordination): joint performance/best individual performance
- Approximately 20 hours of video of 16 pairs engaged in 1470 short interactions (joint decision trials)



Interaction 43(S)

A	(0:02:42.1)	we take yours because I saw nothing
B	(0:02:43.7)	I didn’t see anything either – I saw ...
A	(0:02:46.2)	I saw I took a bet
B	(0:02:47.2)	[way to go!
A	(0:02:47.3)	[way to go!
B	(0:02:48.5)	mine was also just a bet there

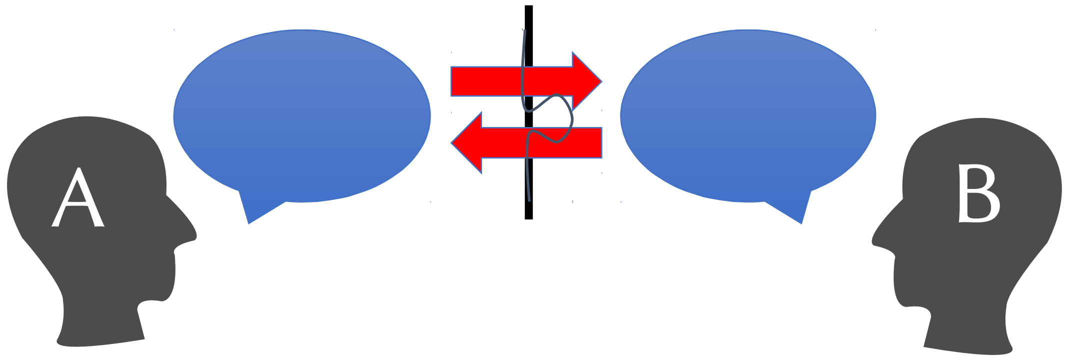
Interaction 44(F)

B	(0:02:58.3)	((laughs)) I don’t know
A	(0:02:59.4)	I don’t know either
A	(0:03:00.3)	I saw something both in the left corner and in the center on the right in both of them
B	(0:03:04.6)	okay, I think it was over in the left side, but oehm I’ll pass
A	(0:03:13.6)	no!
B	(0:03:16.3)	we ruin the scores – now we must...
A	(0:03:18.0)	yeah, now we must pull ourselves together

We want to quantify
- linguistic coordination
- its effects on collective benefit

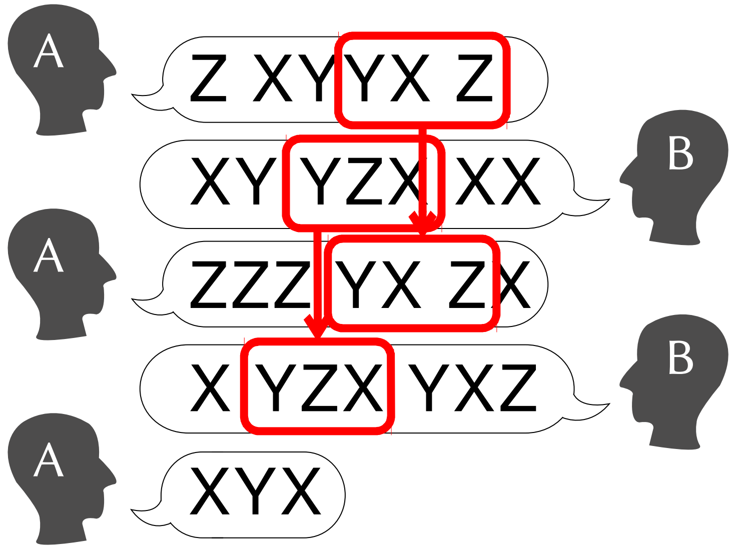
Levels of linguistic coordination

- Individual behaviors
- Interactive alignment
- Interpersonal synergies



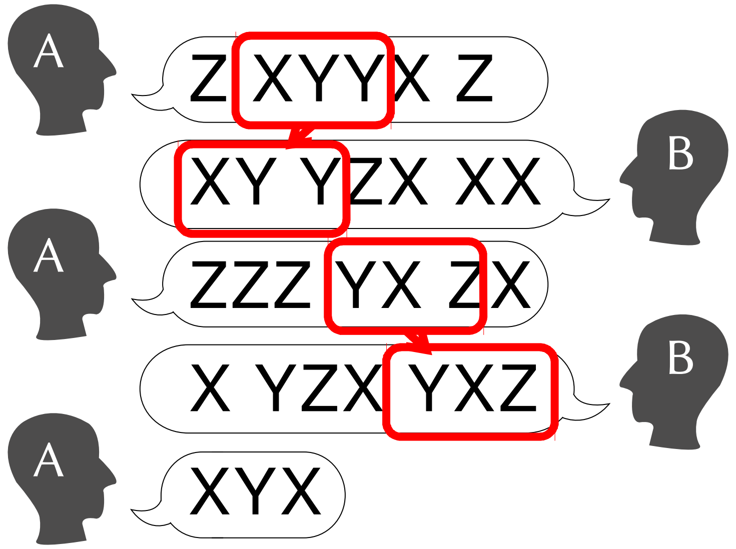
Individual linguistic behavior

"*A powerful coordination strategy is to be as predictable as possible because it allows individuals to rely on and build up common ground.*" Vesper, van der Wel, Knoblich and Sebanz (2011)



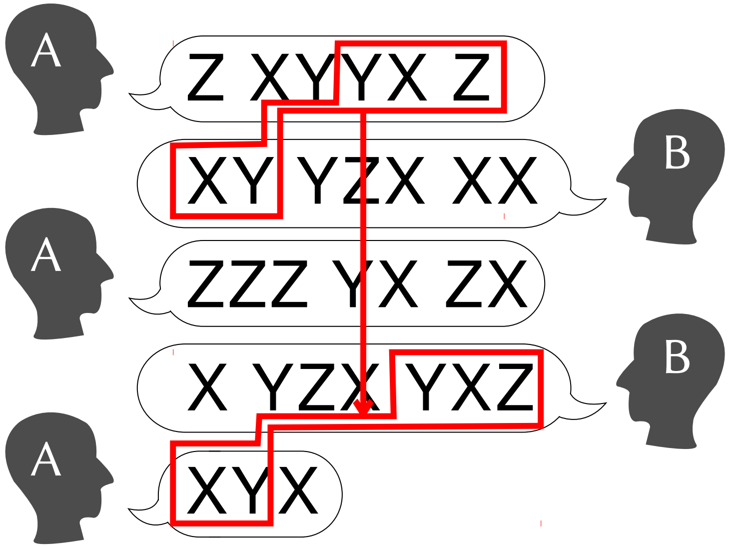
Interactive alignment in conversation

"*... in dialogue, the linguistic representations employed by the interlocutors become aligned at many levels, as a result of a largely automatic process*". (Pickering and Garrod (2004)

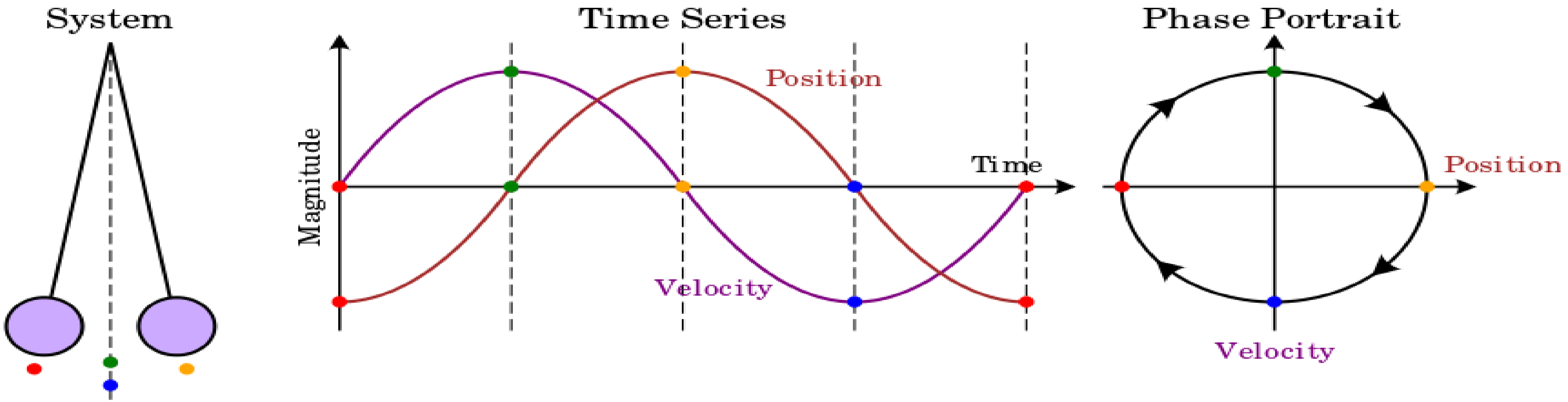


Interpersonal synergy

"... *linguistic processes cannot be reduced to the workings of individual cognitive systems but must be approached also at the interpersonal level*". (Fusaroli, Raczaszek-Leonardi & Tylén 2013)



Phase space

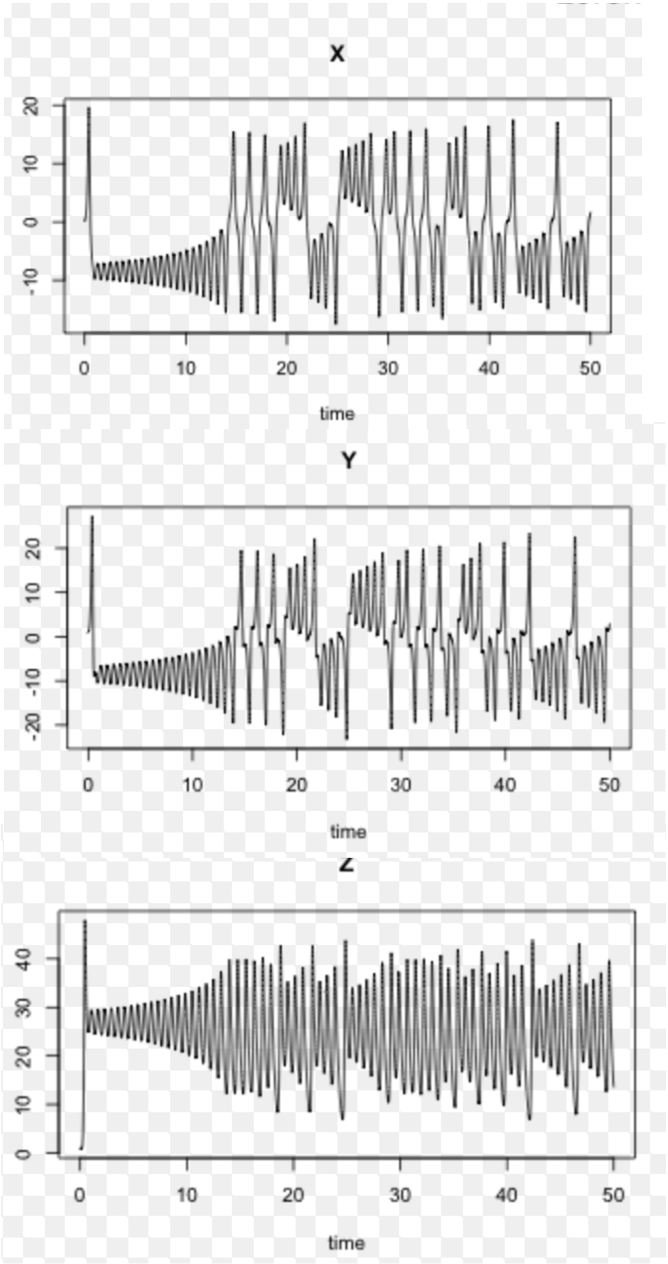


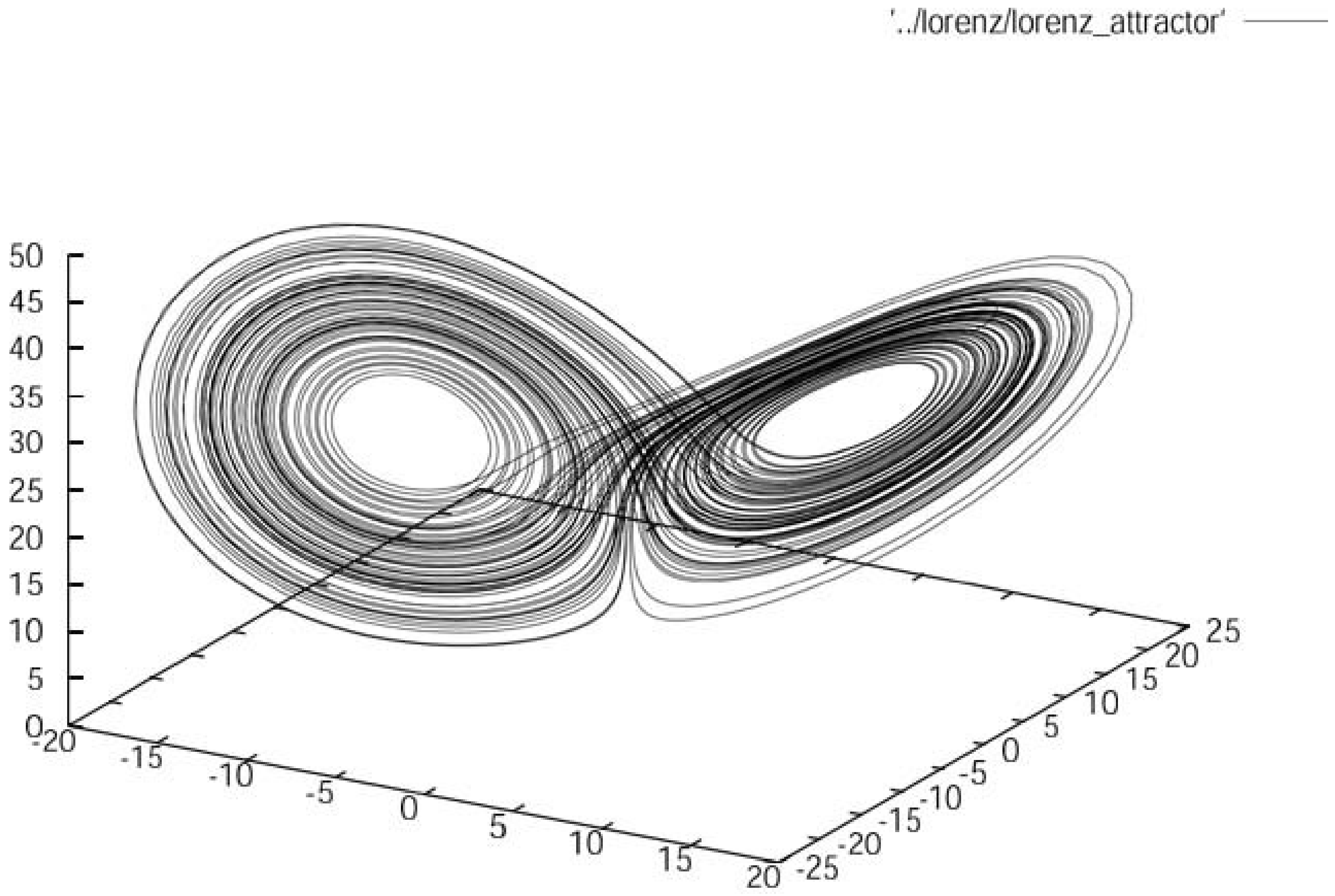
System
Time Series
Phase Portrait
Position
Magnitude
Time
Velocity
Position
Velocity

A more complex phase space

- Lorenz attractor

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$





'../lorenz/lorenz_attractor'
50
45
40
35
30
25
20
15
10
5
0
-20
-15
-10
-5
0
5
10
15
20
-25
-20
-15
-10
-5
0
5
10
15
20
25

Let’s go back to our example, pitch

- But the system has many components and we only have one variable! What should we do?

- Can we use the fact that the components are interdependent?

Takens’ theorem

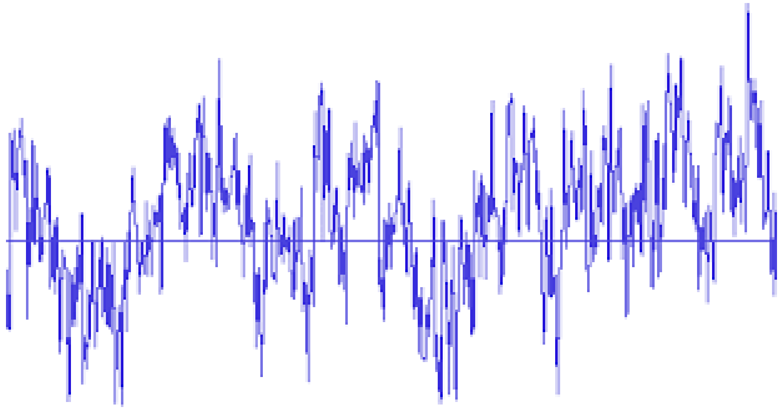
Takens' theorem

- Topological features of any higher-dimensional system consisting of multiple coupled variables can be reconstructed from but a single measured variable of that system.

-

- The reconstruction is performed by defining time-delayed vectors (Vi) of M points (Pi) that are delayed or offset in time (τ).
  - Find the delay
  - Find how many variables should be reconstructed

How do we calculate the delay?



- Ideally we would like the dimensions of the phase-space to be orthogonal, providing new, unconfounded information about the dynamics
- To that end, we search for a delay at which the time-series is uncorrelated – or minimally correlated – with itself

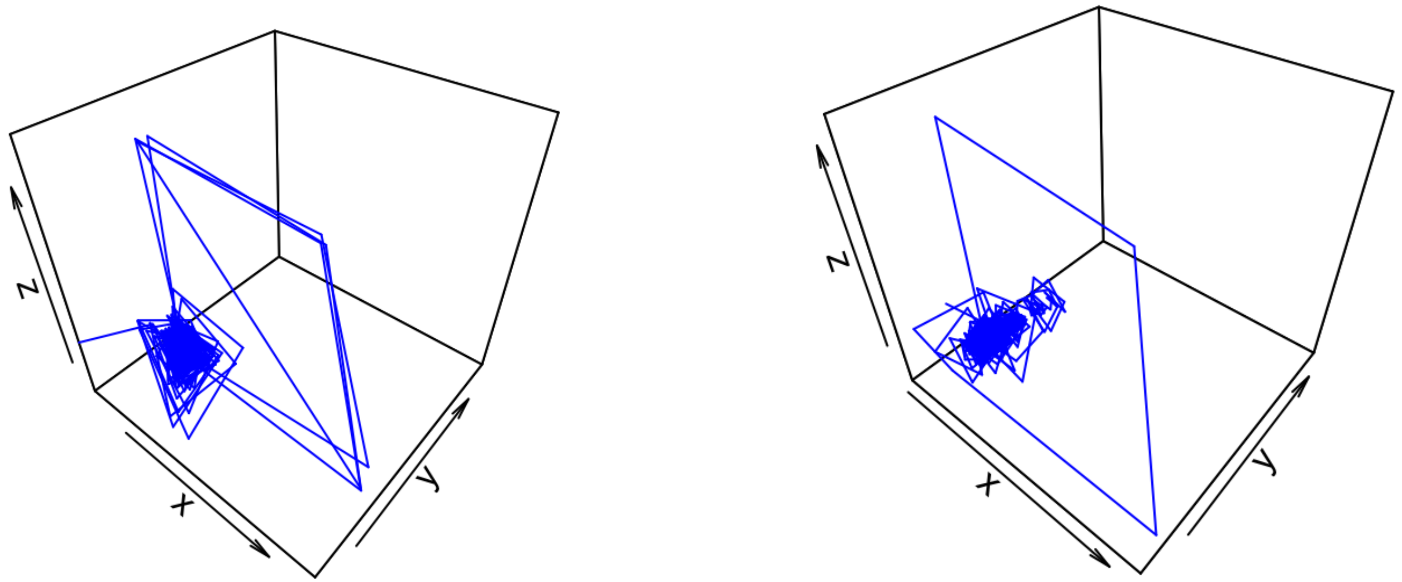
Calculating the number of variables to reconstruct (embedding dimensions)

- how many times the time-series has to be plotted against itself at a delay *d* to obtain a proper phase-space portrait
  - A phase space that reflects the characteristics of the underlying system
- As many dimensions that we exhaust all information about higher-dimensional dynamics contained in the time-series, but not more

False nearest neighbors method

- Look at neighbors with 2d, with 3d, etc. and see how fast they drop with sparsity of dimensions

Now we have the phase space



How do we get a recurrence plot?

- http://www.recurrence-plot.tk/glance.php?show_intro=1

Radius / Threshold

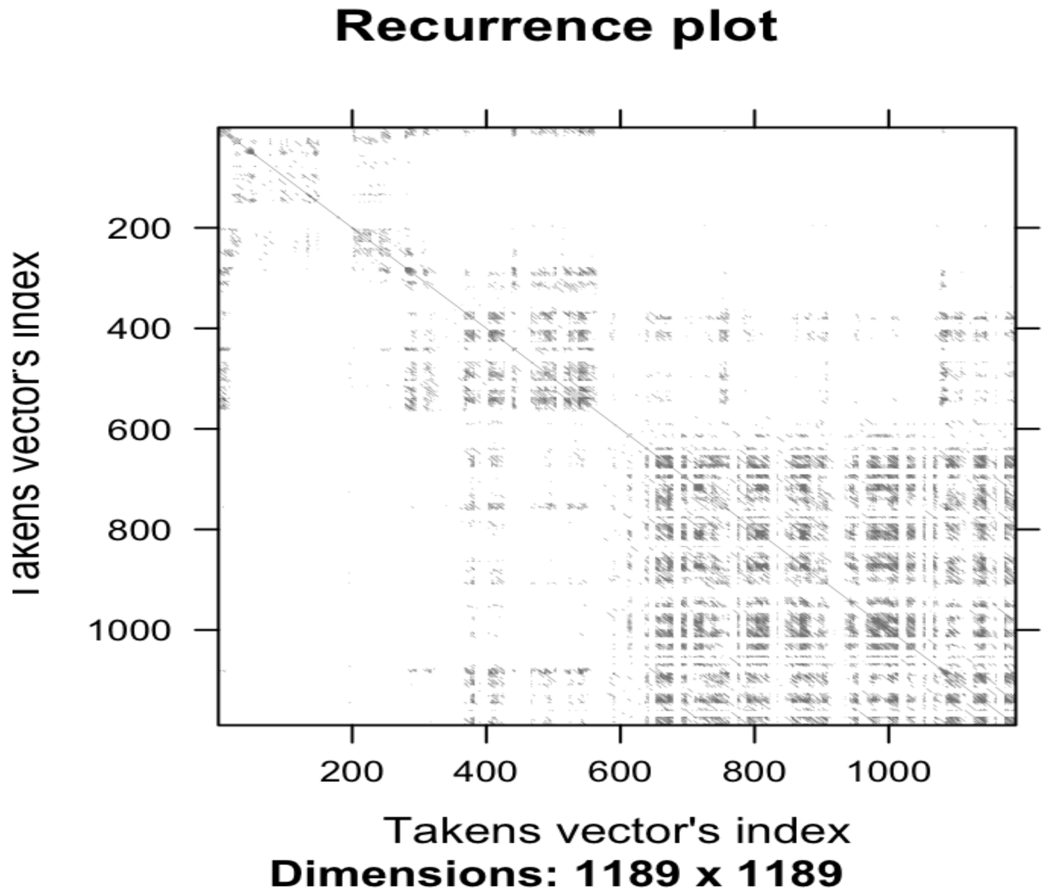
- The size of the neighborhood in phase-space, within which we count two points as either being recurrent (when they lie within the same radius) or not (when they lie farther apart than the size of the radius).
- The higher the radius, the more points will be counted as neighbors. The aim is to find a radius that does not yield too many, nor to few recurrences, as we will lose information about the dynamics of the time-series in both cases.

- Rule of thumb: 4% or 0.04 ca.

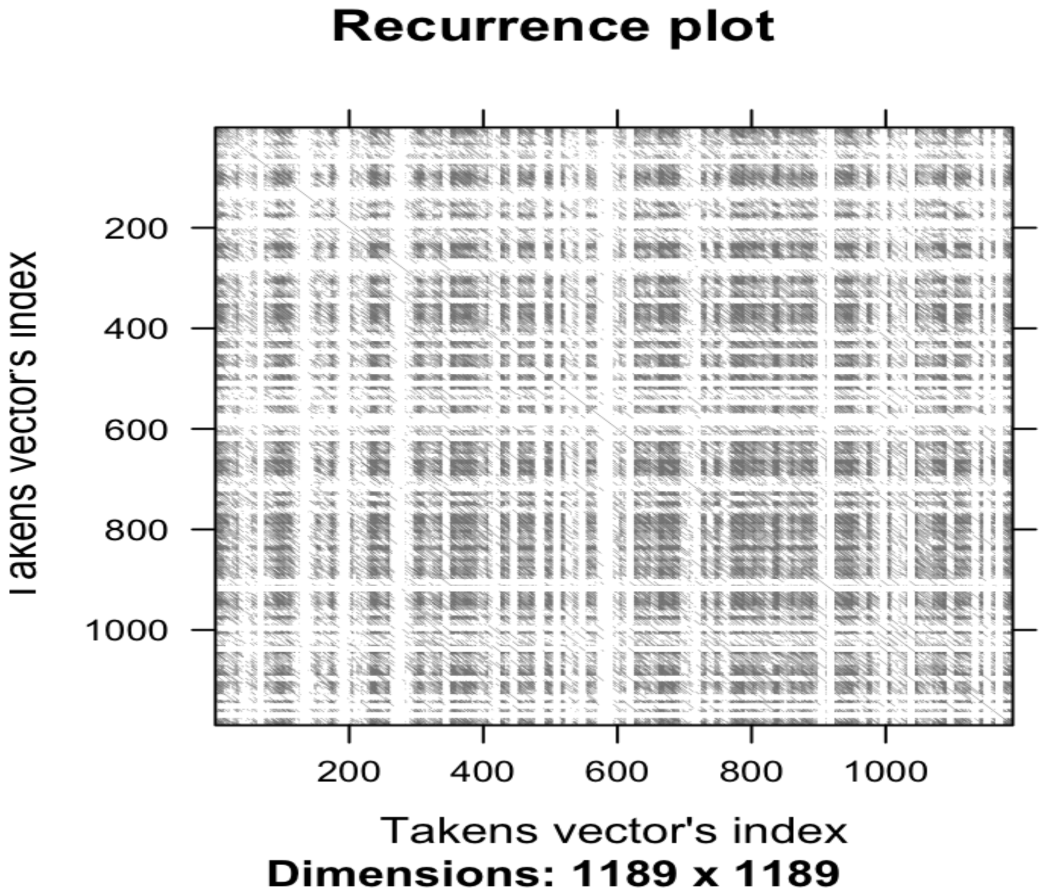
A code guideline

- Finding the optimal parameters: run on all pairs:
  - par = list(lgM = 50, steps = seq(1, 6, 1), radiusspan = 100, radiussample = 40, normalize = 0, rescale = 0, mindiagline = 2, minvertline = 2, tw = 0, whiteline = FALSE, recpt = FALSE, fnnpercent = 10, typeami = "mindip")
  - ans = optimizeParam(x, x, par, min.rec = 3.5, max.rec = 4.5)
- Choose a common value for delay, emb.dim and radius
- Results=crqa (x, x, delay=delay, embed=emddim, radius=radius,normalize=0,rescale=0,mindiagline = 2,minvertline = 2)
- Represent the plot:
  - RP=Results$RP
  - RP = matrix(as.numeric(RP), nrow = ncol(RP))
  - cols = c("white","blue4")
  - image(RP, xlab = "", ylab = "", col = cols)
- Explore the lags of coordination:
  - Profile=drpdfromts(x, x,datatype = 'continuous',ws=50,radius=radius)
  - timecourse = round(seq(-5000,5000,100)/1000, digit = 1)
  - maxlag = Profile$maxlag/1000
  - profile = Profile$profile*100
  - Prof=data.frame(profile)
  - ggplot(Prof, aes(timecourse,profile))+geom_line()+ geom_vline(xintercept = timecourse[maxlag], colour='red')

Let's plot!



Recurrence plot
200
400
600
800
1000
Takens vector's index
200
400
600
800
1000
Takens vector's index
Dimensions: 1189 x 1189



Recurrence plot
200
400
600
800
1000
Takens vector's index
200
400
600
800
1000
Takens vector's index
Dimensions: 1189 x 1189

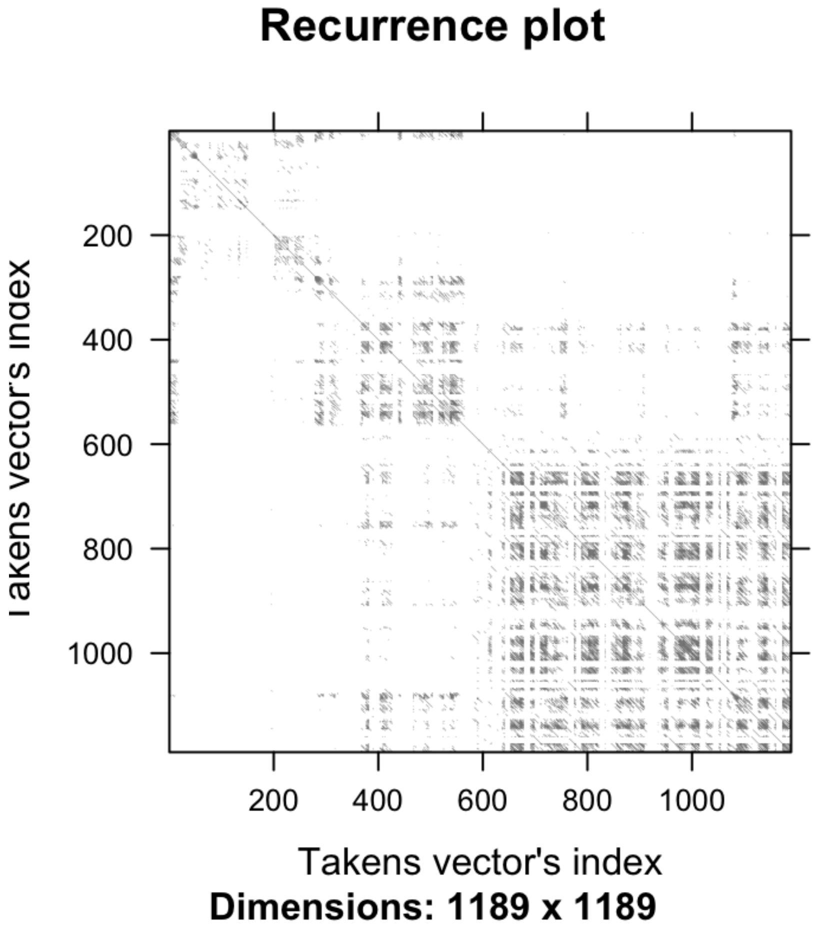
Is this enough?

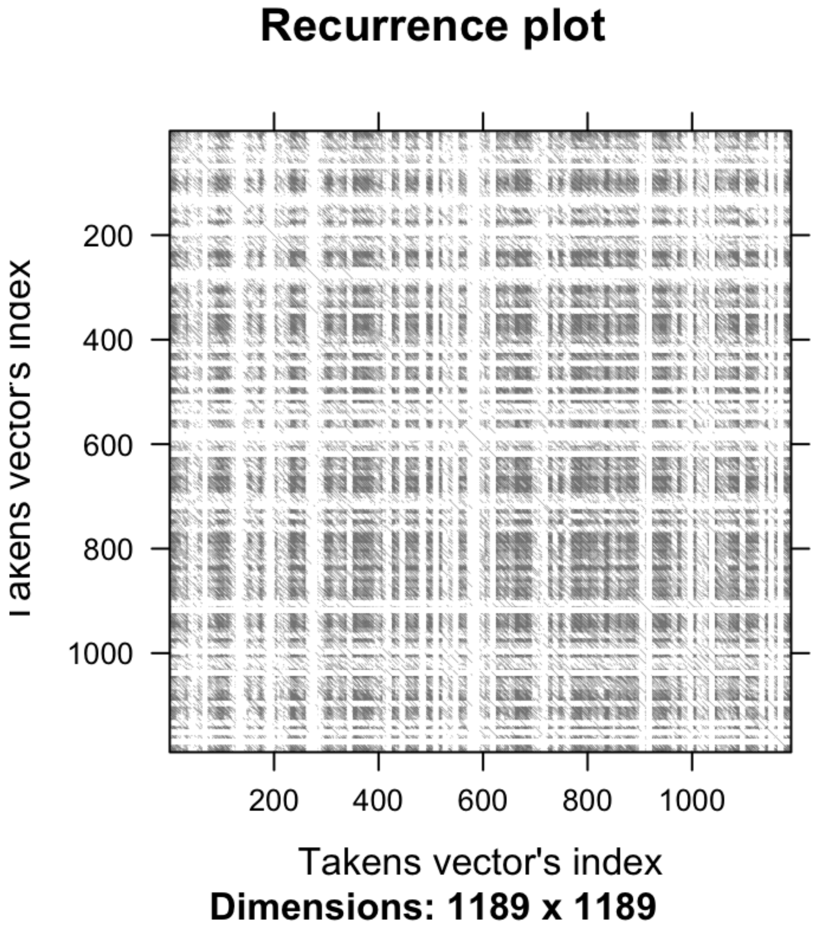
Quantifying the plots

RR

$$\mathrm{RR} = \frac{1}{N^2} \sum_{i,j=1}^{N} \mathbf{R}(i,j).$$

- s

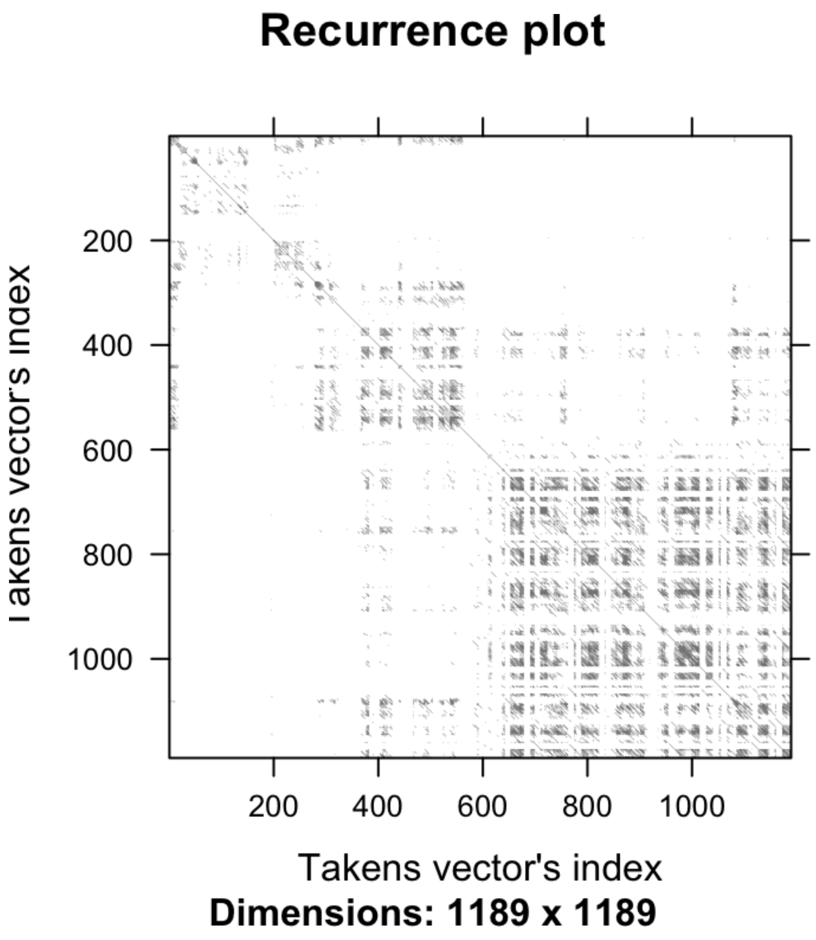


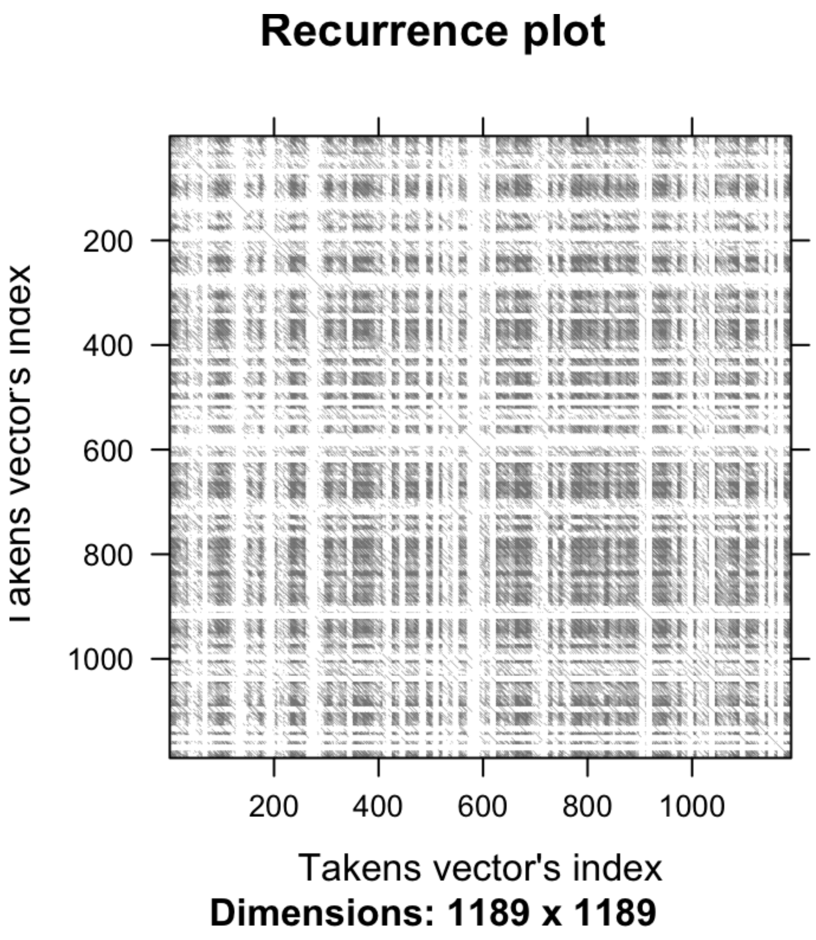


- High RR: lots of similar values in the timeseries
- Low RR: the timeseries contains many different values

$$\mathrm{DET} = \frac{\sum_{\ell=\ell_{\min}}^{N} \ell\, P(\ell)}{\sum_{i,j=1}^{N} R(i,j)},$$

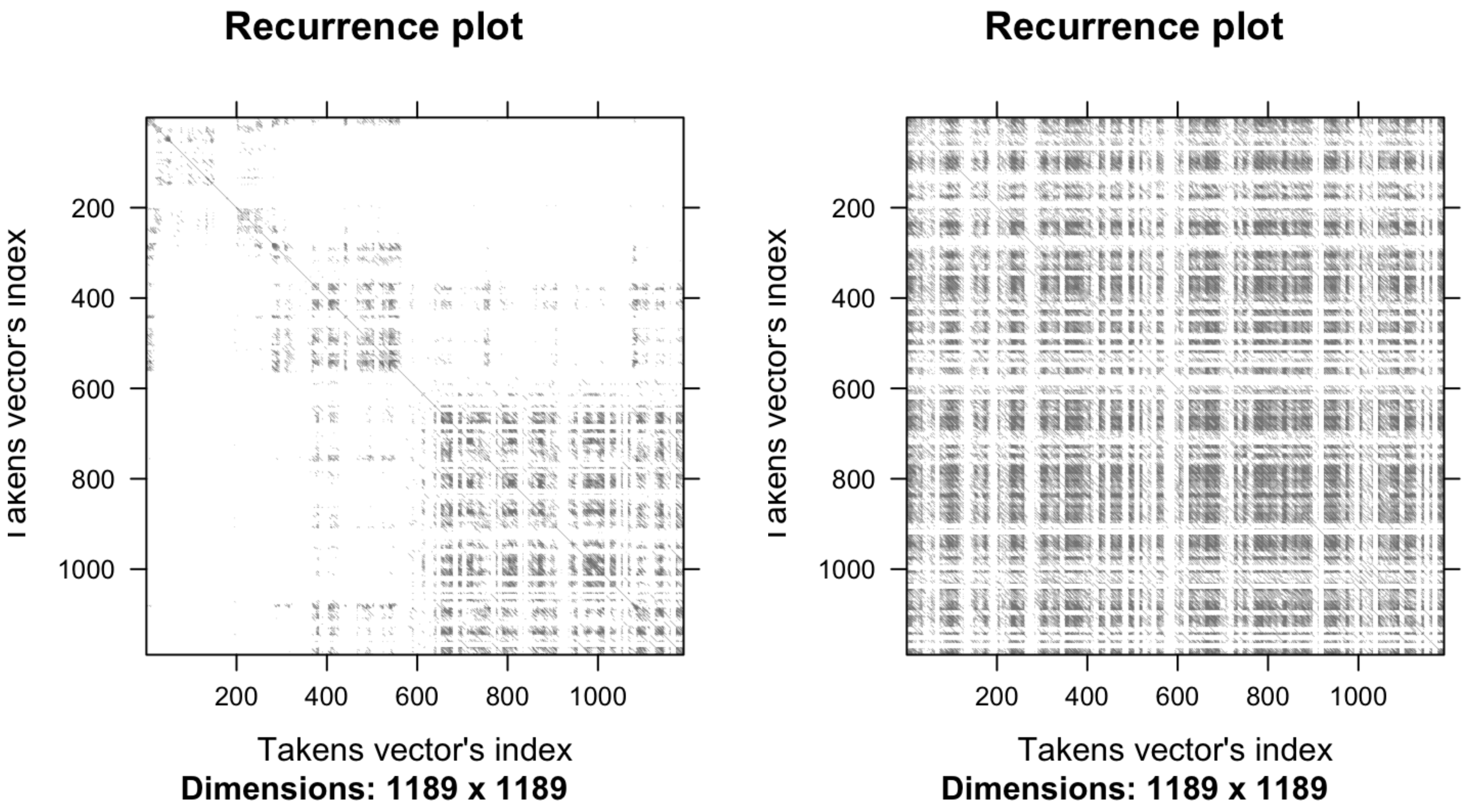
- ss





- High DET: when the system recurs, it tends to do that for at least two states in phase space
- Low DET: when the system recurs, it tends to do that for only 1 state

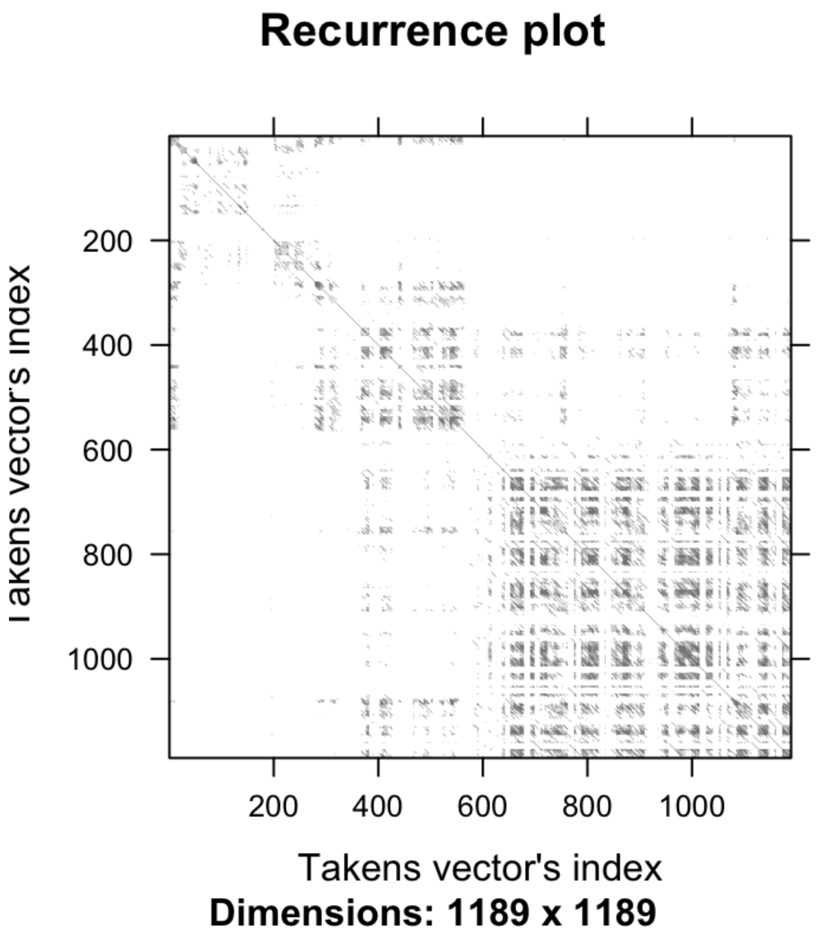
$$\mathrm{L} = \frac{\sum_{\ell=\ell_{\min}}^{N} \ell\, P(\ell)}{\sum_{\ell=\ell_{\min}}^{N} P(\ell)}$$

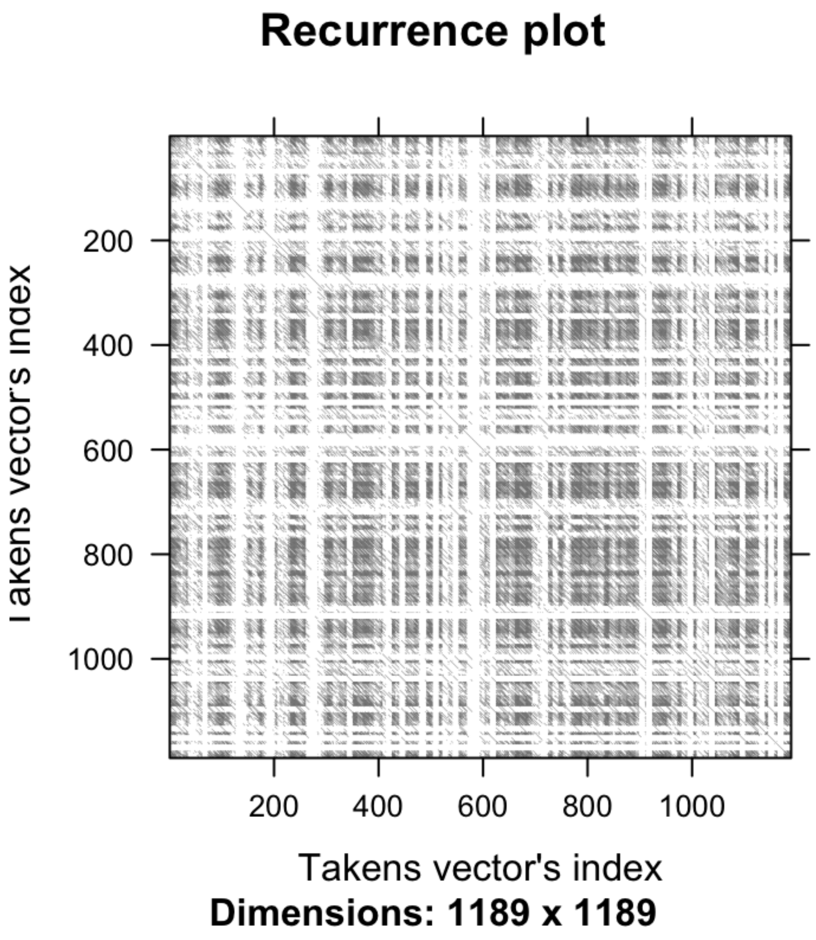


- High L: When the system recurs, it tends to do so in “long” structured sequences of states

- Low L: short structured sequences

Max (L)


Recurrence plot
200
400
600
800
1000
Takens vector's index
Dimensions: 1189 x 1189

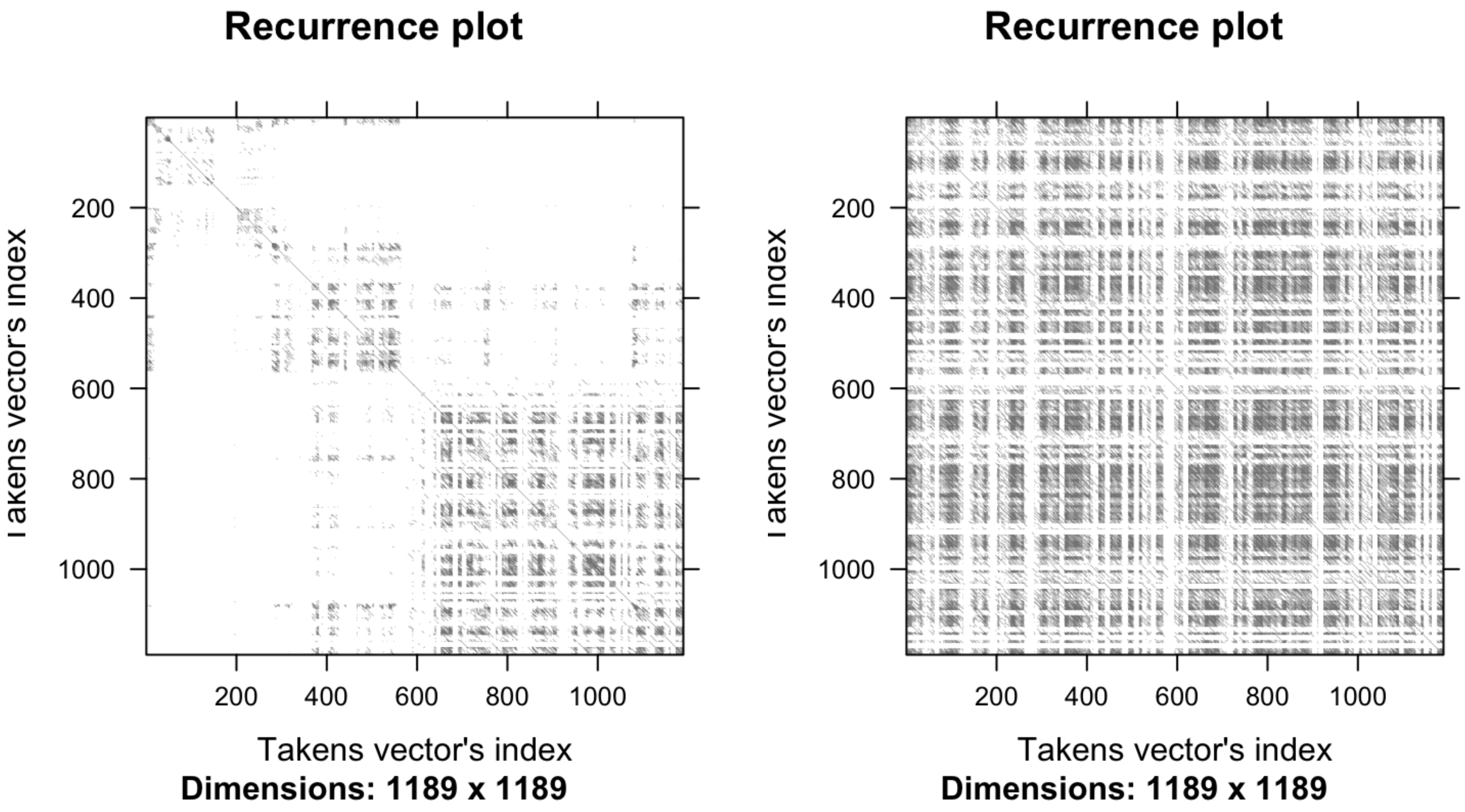

Recurrence plot
200
400
600
800
1000
Takens vector's index
Dimensions: 1189 x 1189

- High MaxL: highly stable system: the system can repeat a very long sequence without being perturbed

- Low MaxL: highly unstable system: the system tends to be perturbed

$$\mathrm{ENTR} = -\sum_{\ell=\ell_{\min}}^{N} p(\ell) \ln p(\ell),$$

- a

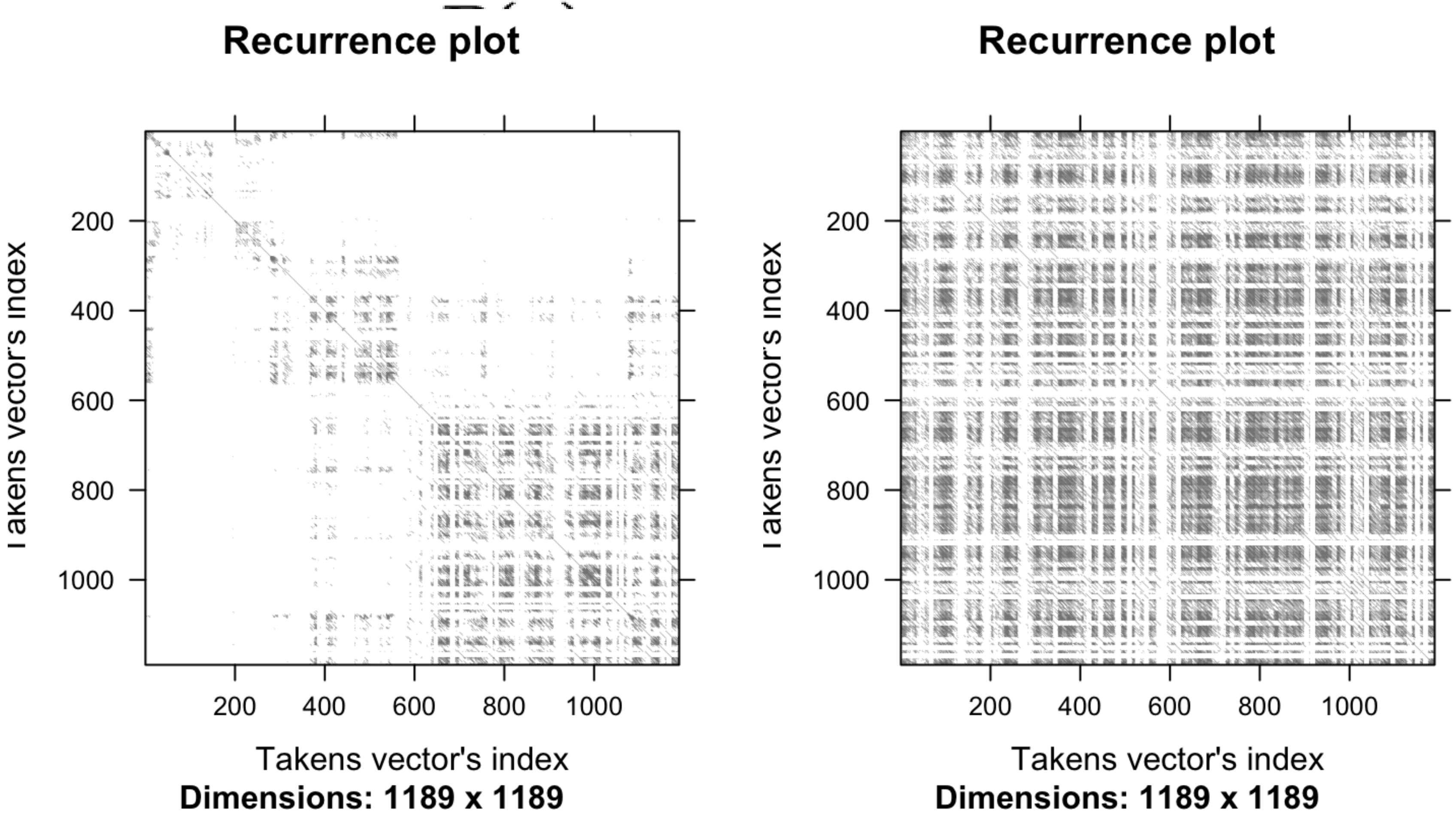


- High Entropy: the system recurs in many different ways: sometimes short sequences, sometimes long sequences.

- Low Entropy: the system recurs in stereotyped ways: always the same length

$$\mathbf{LAM} = \frac{\sum_{v=v_{\min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)},$$

- w

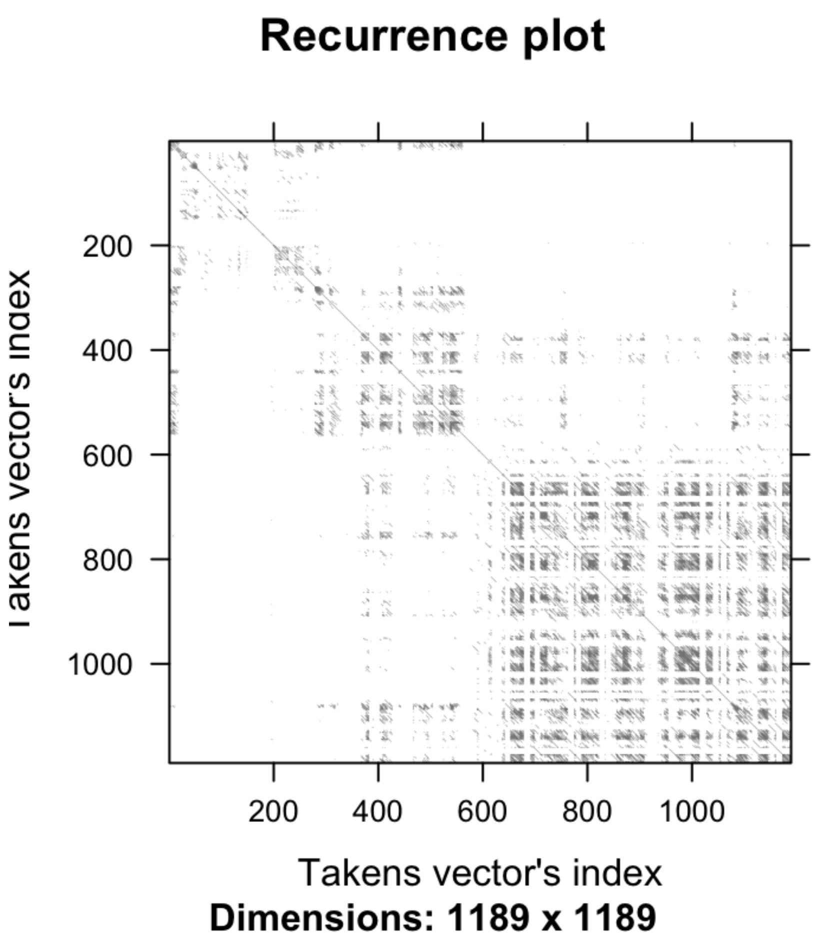


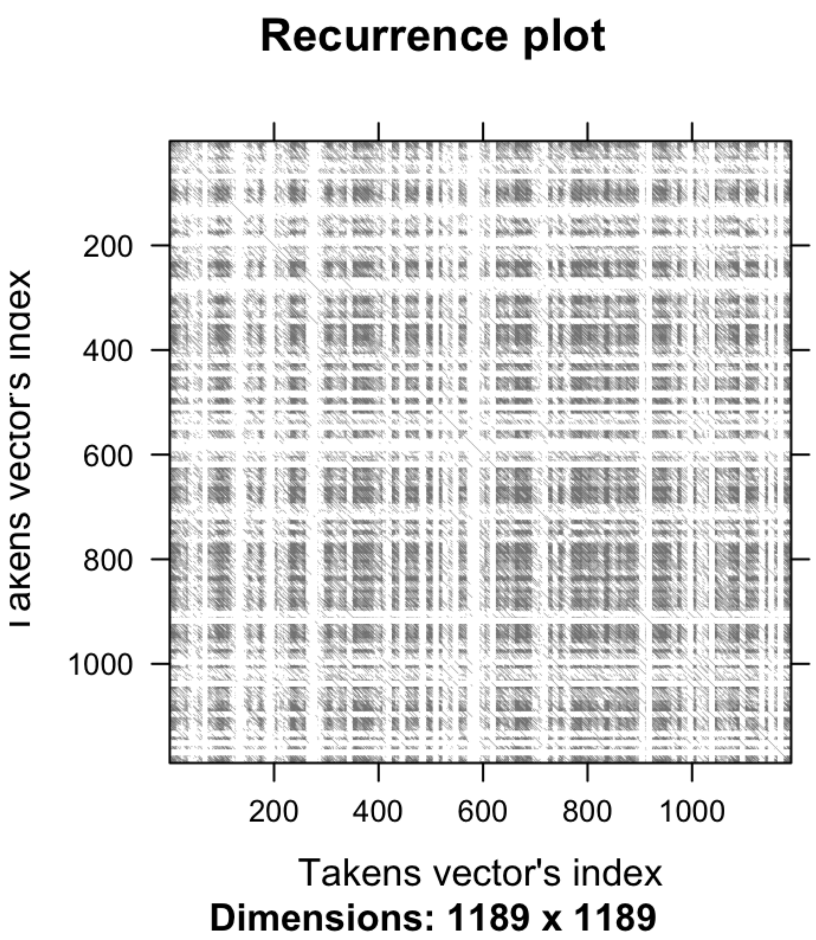
- High LAM: sequences tend to be composed of the same state repeated over and over

- Low LAM: sequences if present (cf. DET) tend to be composed of different states

$$TT = \frac{\sum_{v=v_{\min}}^{N} vP(v)}{\sum_{v=v_{\min}}^{N} P(v)}$$

- s

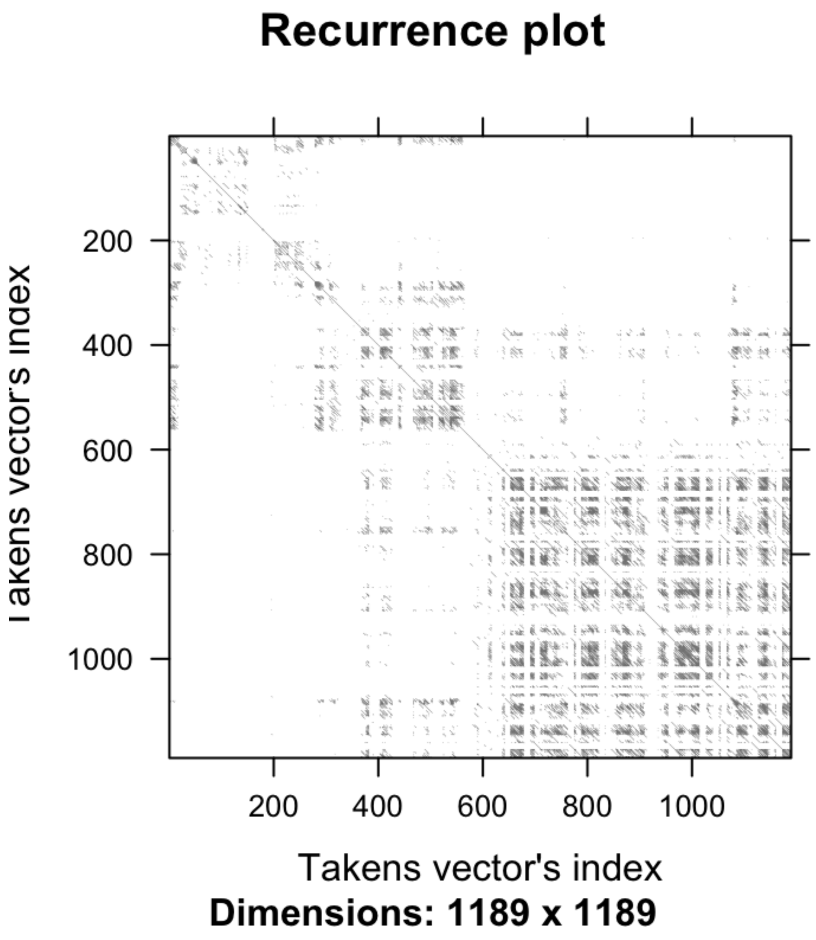




- High TT: when recurring a state the system tends to be trapped into it, repeating it again and again for long periods

- Low TT: the system tends not to repeat the same state

Max V


Recurrence plot
Takens vector's index
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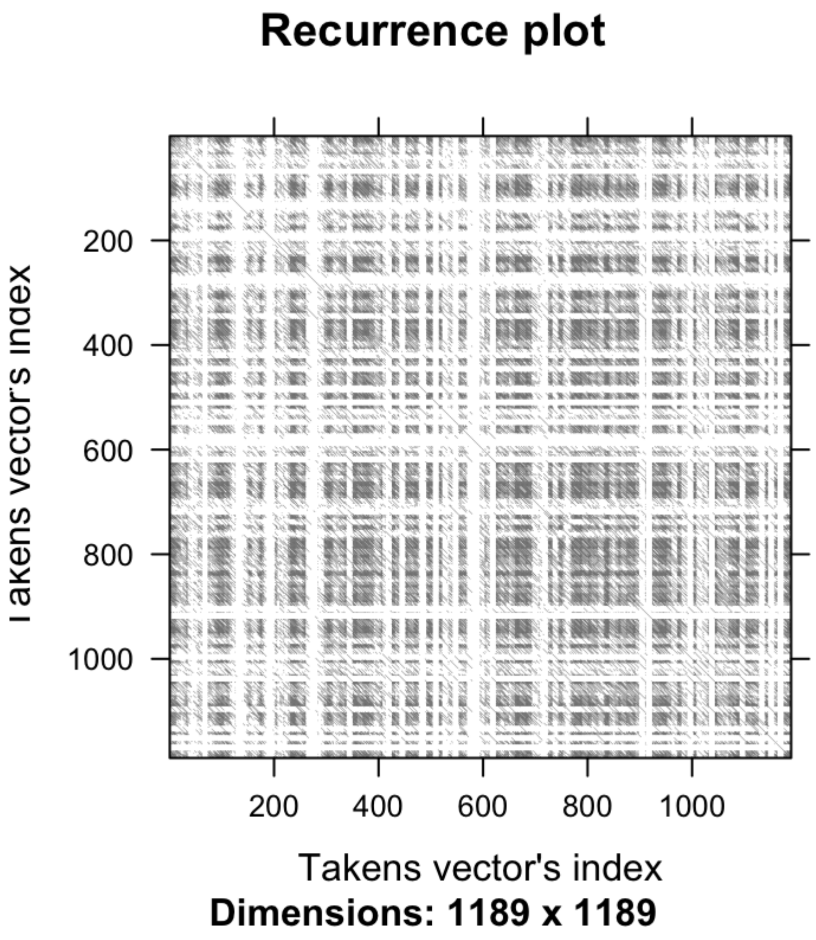
Takens vector's index
Dimensions: 1189 x 1189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