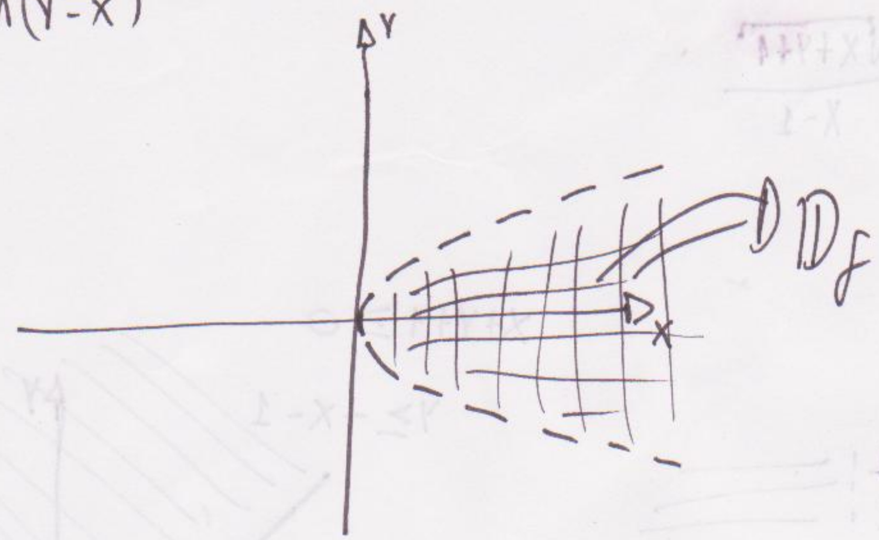
② $f(x,y) = x\ln(y^2 - x)$

Solución:

$y^2 - x > 0$

$y^2 > x$

$x < y^2$



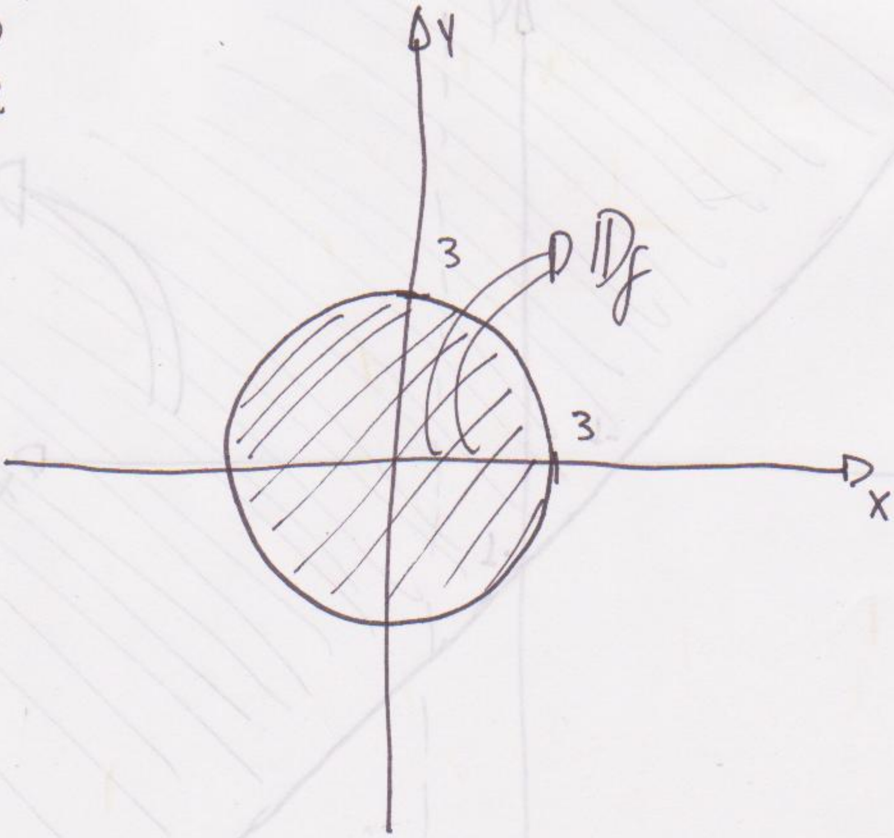
③ $g(x,y) = \sqrt{9 - x^2 - y^2}$

Solución

$9 - x^2 - y^2 \geq 0 \quad (-1)$

$x^2 + y^2 - 9 \leq 0$

$x^2 + y^2 \leq 3^2$

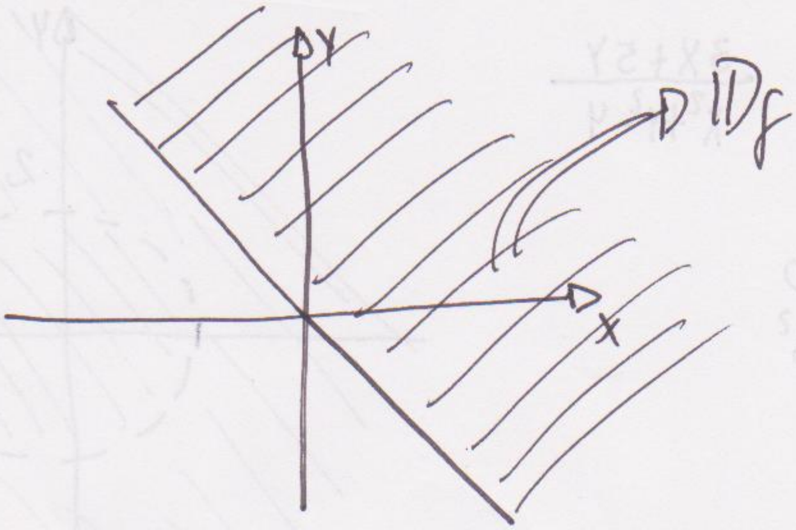


④ $f(x,y)=\sqrt{x+y}$

Solución:

$x+y \geq 0$

$y \geq -x$



⑤ $f(x,y)=\ln(9-x^2-9y^2)$

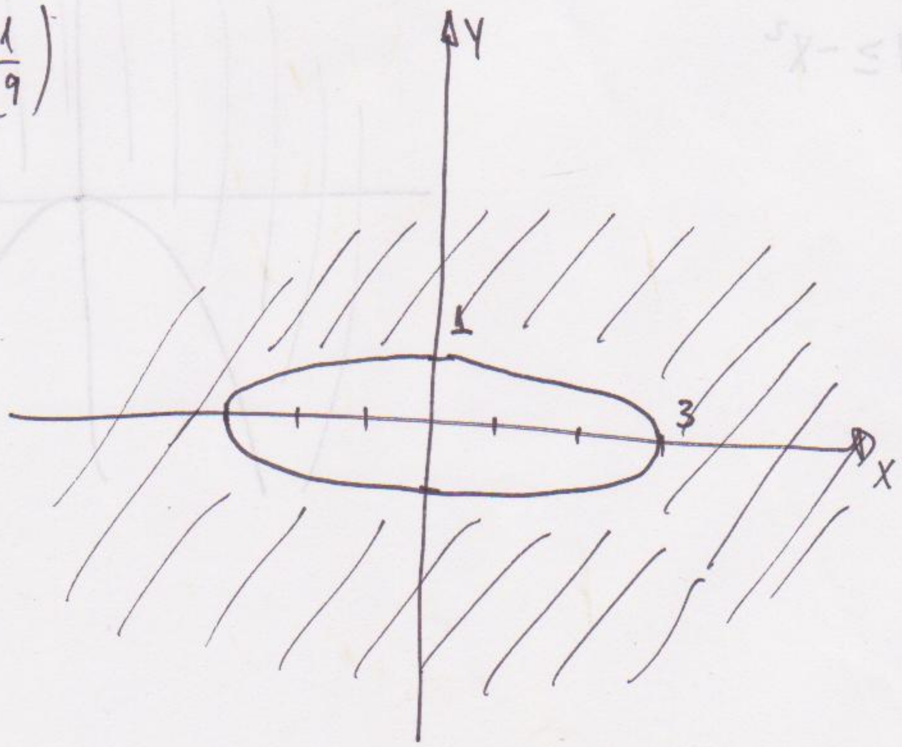
Solución:

$9-x^2-9y^2 > 0 \quad (-1)$

$x^2+9y^2-9>0$

$x^2+9y^2>9 \quad \left(\frac{1}{9}\right)$

$\frac{x^2}{3^2}+\frac{y^2}{1^2}>1$

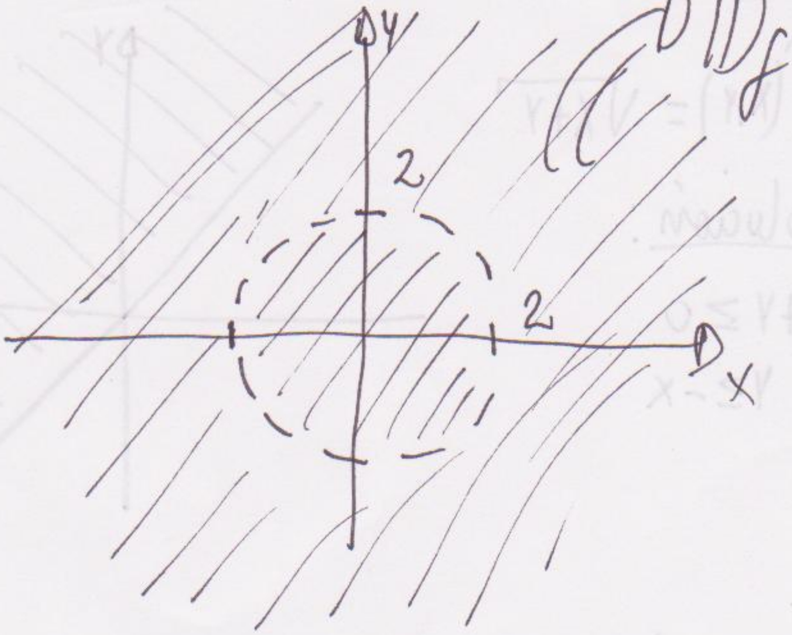


(6) $f(x,y) = \dfrac{3x+5y}{x^2+y^2-4}$

<u>Solución</u>:

$x^2+y^2-4 \neq 0$

$x^2+y^2 \neq 2^2$

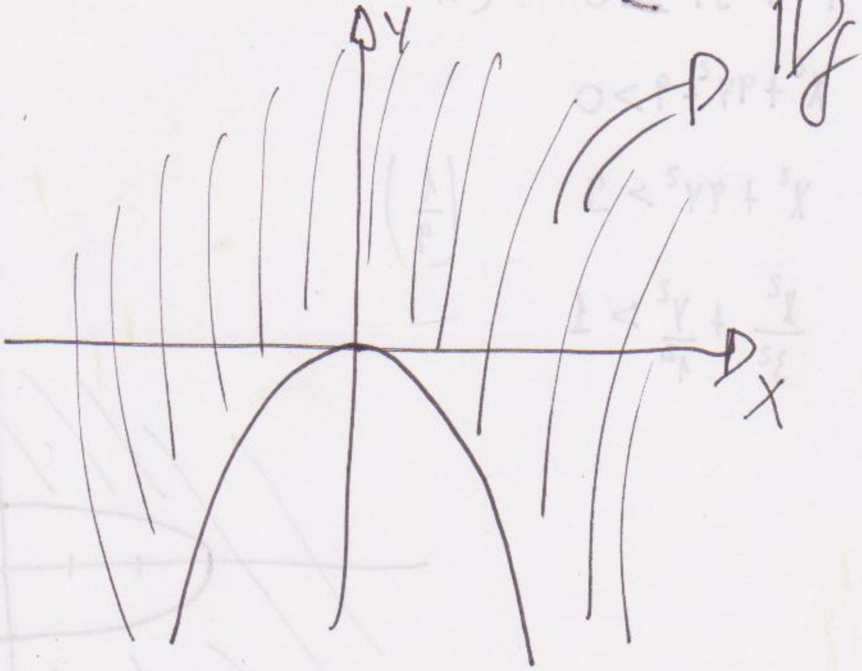


(7) $f(x,y) = xy\sqrt{x^2+y}$

<u>Solución</u>:

$x^2+y \geq 0$

$y \geq -x^2$



(8) $f(x,y) = 4x^2 + y^2$

$\mathbb{D}_f = \mathbb{R}^2$

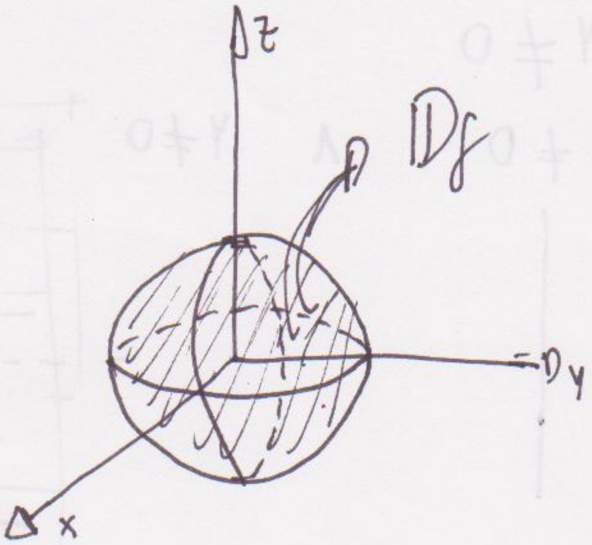
(9) $f(x,y,z) = \sqrt{1 - x^2 - y^2 - z^2}$

Solución:

$1 - x^2 - y^2 - z^2 \geq 0 \qquad (-1)$

$x^2 + y^2 + z^2 - 1 \leq 0$

$x^2 + y^2 + z^2 \leq 1^2$



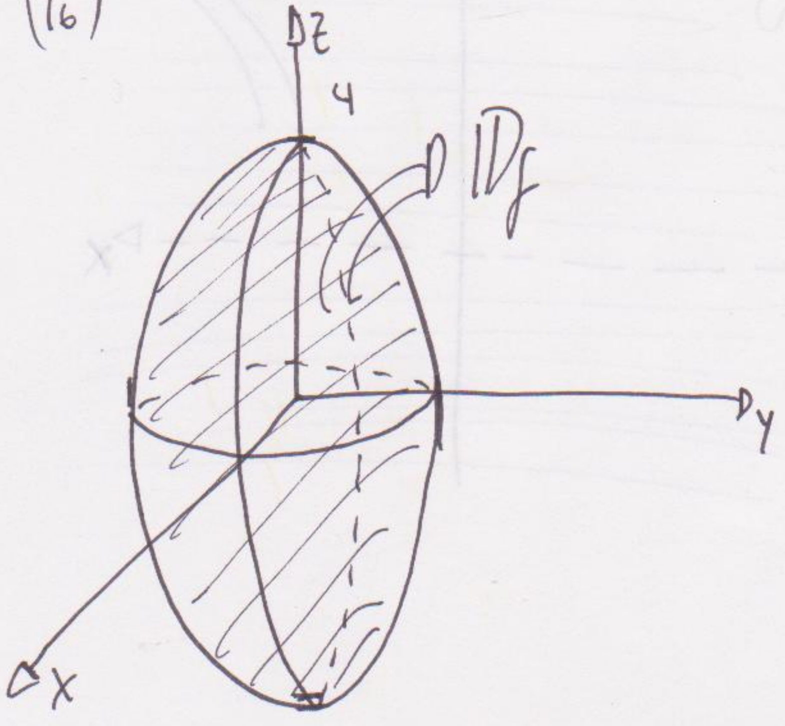
(10) $f(x,y,z) = \ln(16 - 4x^2 - 4y^2 - z^2)$

Solución

$16 - 4x^2 - 4y^2 - z^2 \geq 0 \qquad (-1)$

$4x^2 + 4y^2 + z^2 - 16 \leq 0 \qquad \left(\frac{1}{16}\right)$

$\frac{x^2}{2^2} + \frac{y^2}{2^2} + \frac{z^2}{4^2} \leq 1$

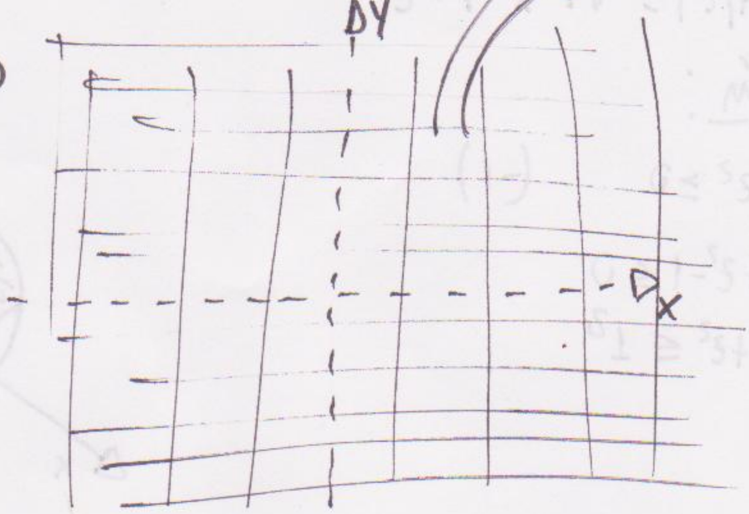


(11) $f(x,y) = \dfrac{x+y}{xy}$

Solución:

$xy \neq 0$

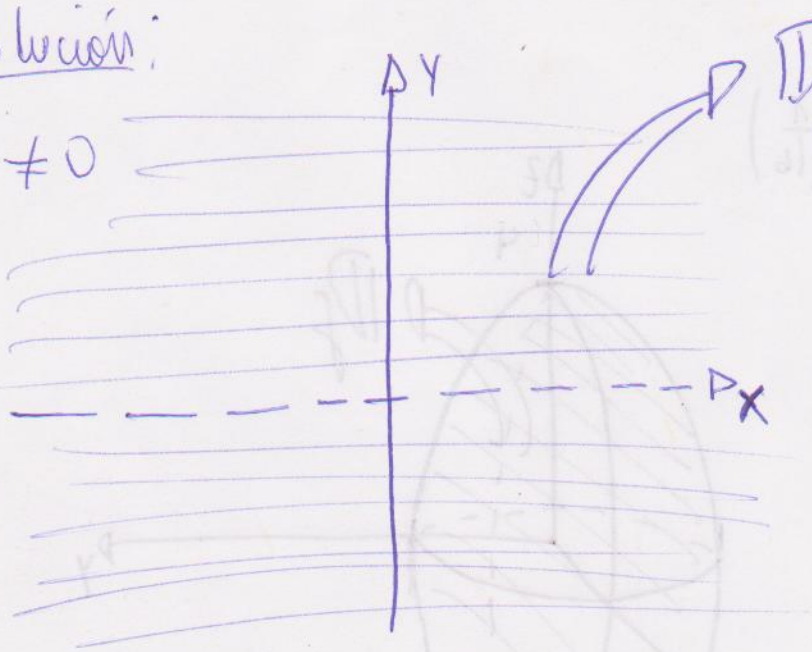
$x \neq 0 \quad \wedge \quad y \neq 0$



(12) $f(x,y) = e^{\frac{x}{y}}$

Solución:

$y \neq 0$



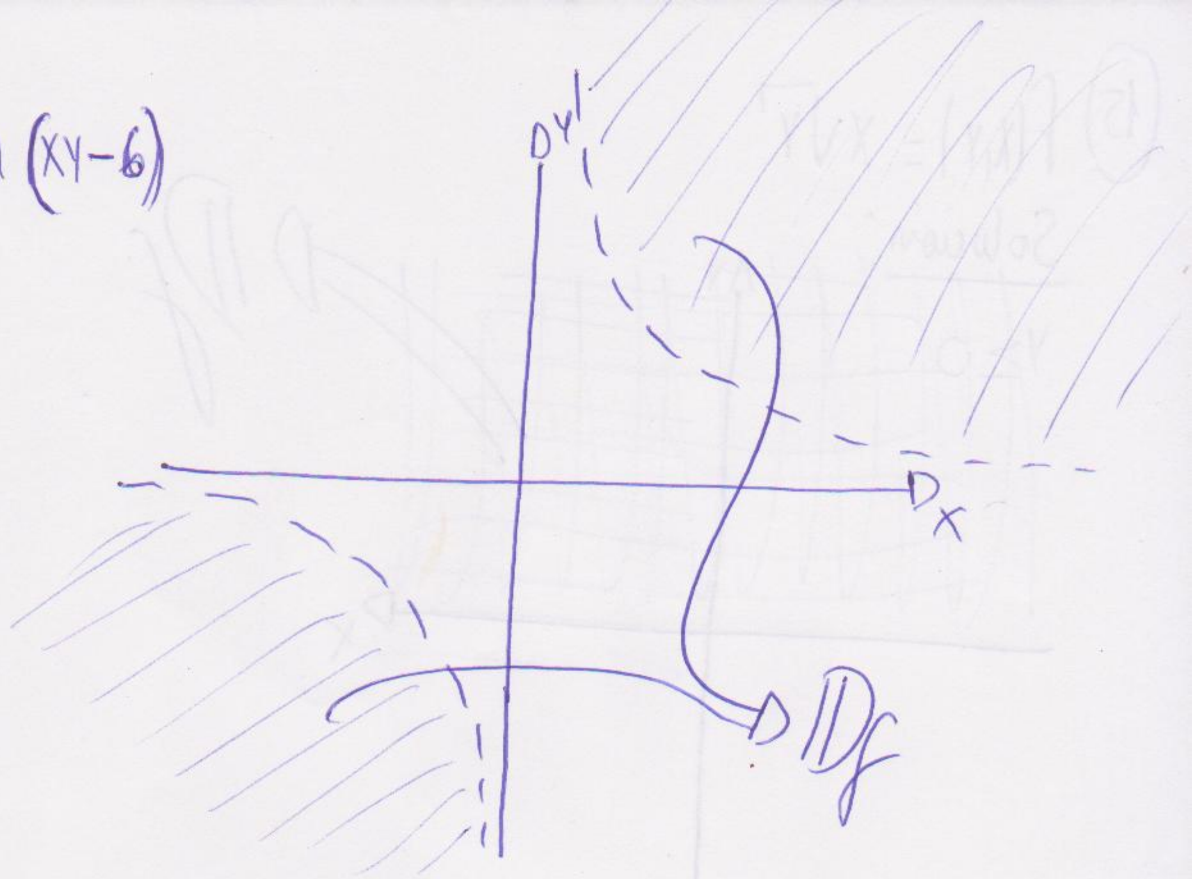
(13) $f(x,y) = \ln(xy-6)$

Solución:

$xy - 6 > 0$

$xy > 6$

$y > \frac{6}{x}$

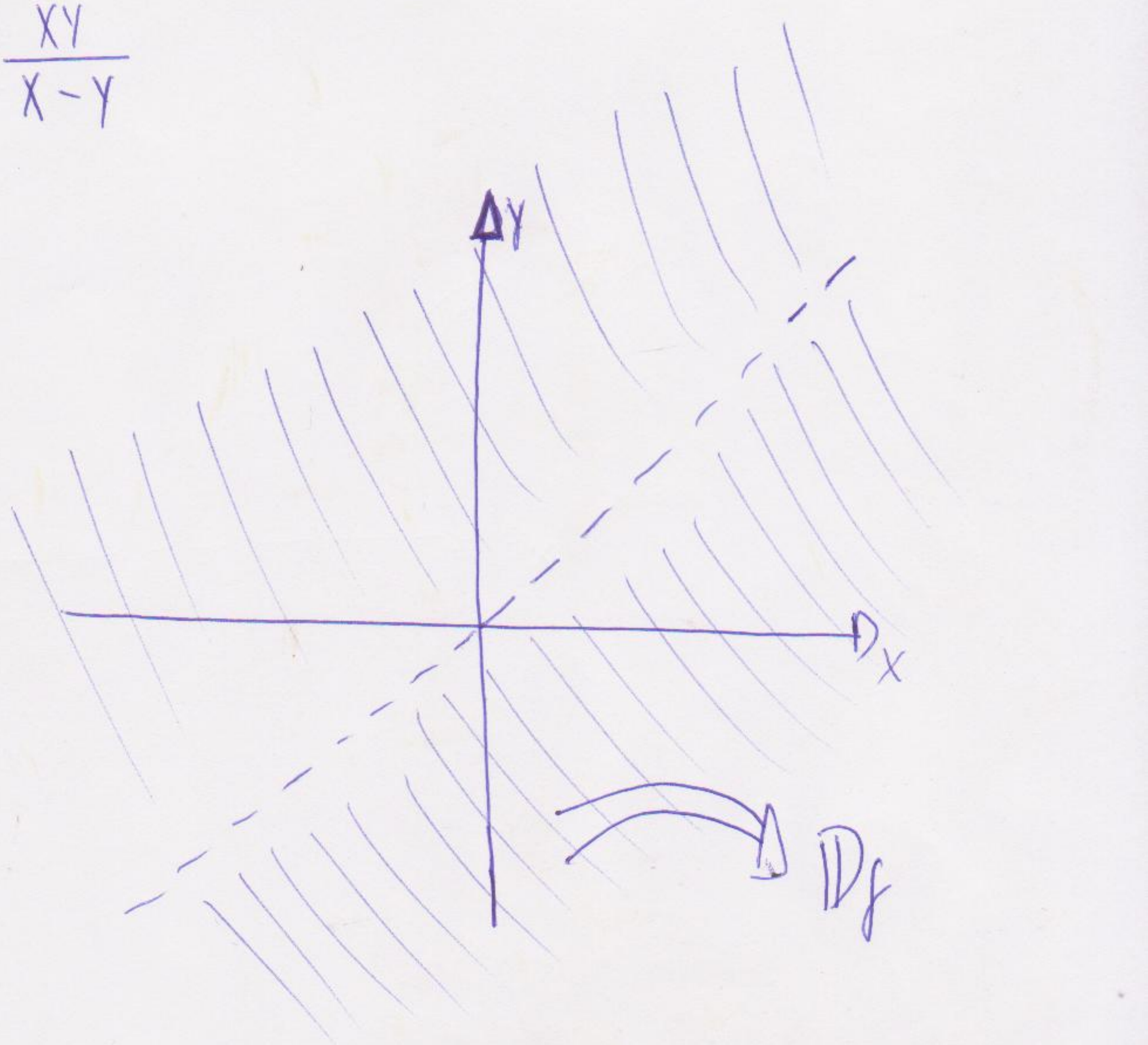


(14) $f(x,y) = \frac{xy}{x-y}$

Solución:

$x - y \neq 0$

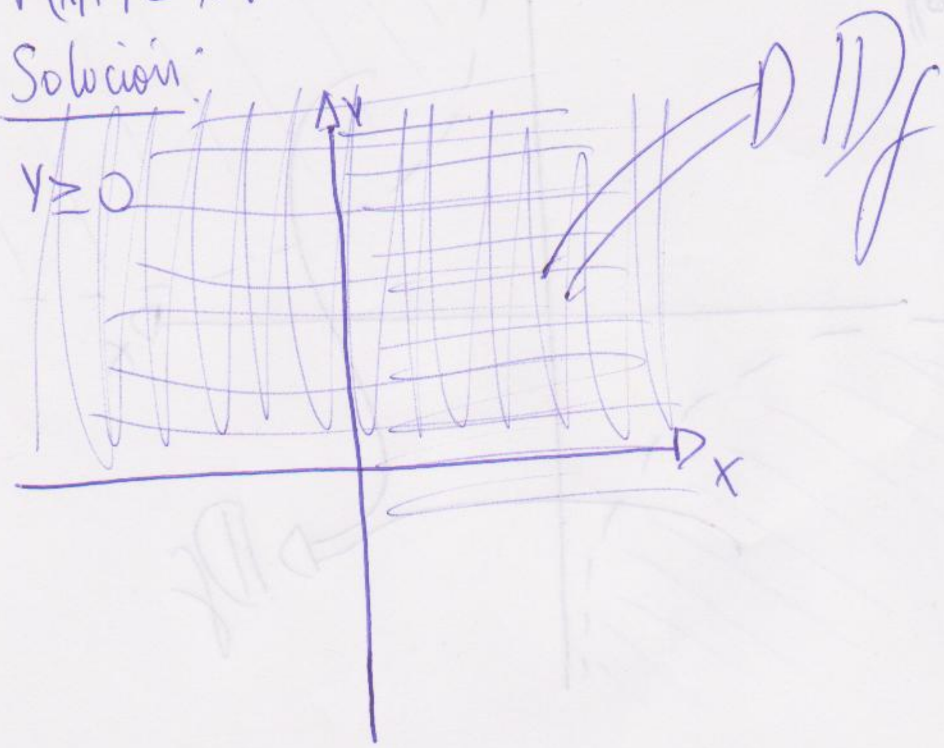
$y \neq x$



(15) $f(x,y) = x\sqrt{y}$

Solución:

$y \geq 0$



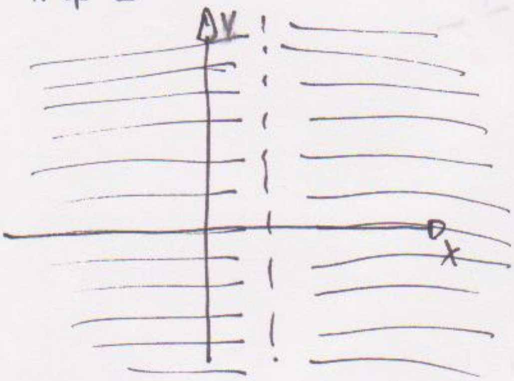
Practico n° 1

① $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Solución:

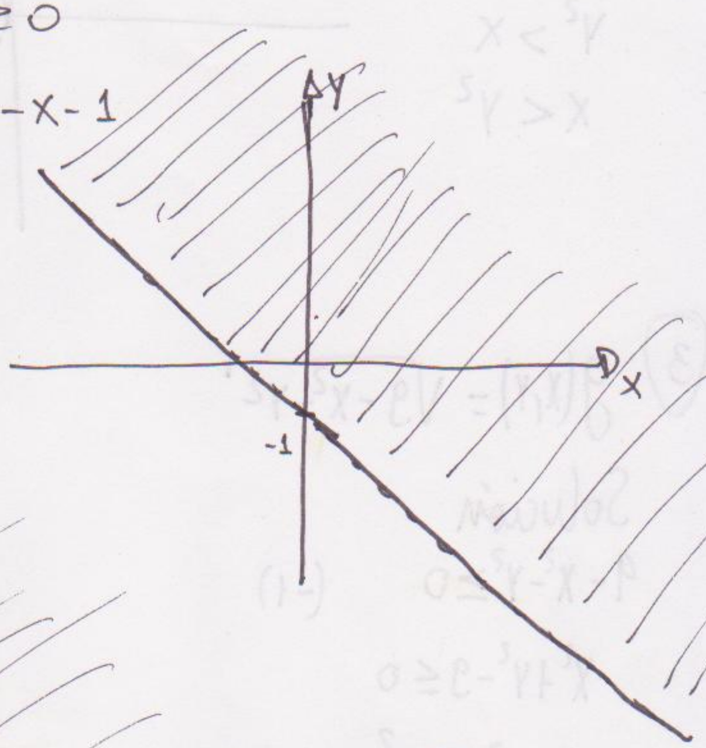
$x-1 \neq 0$

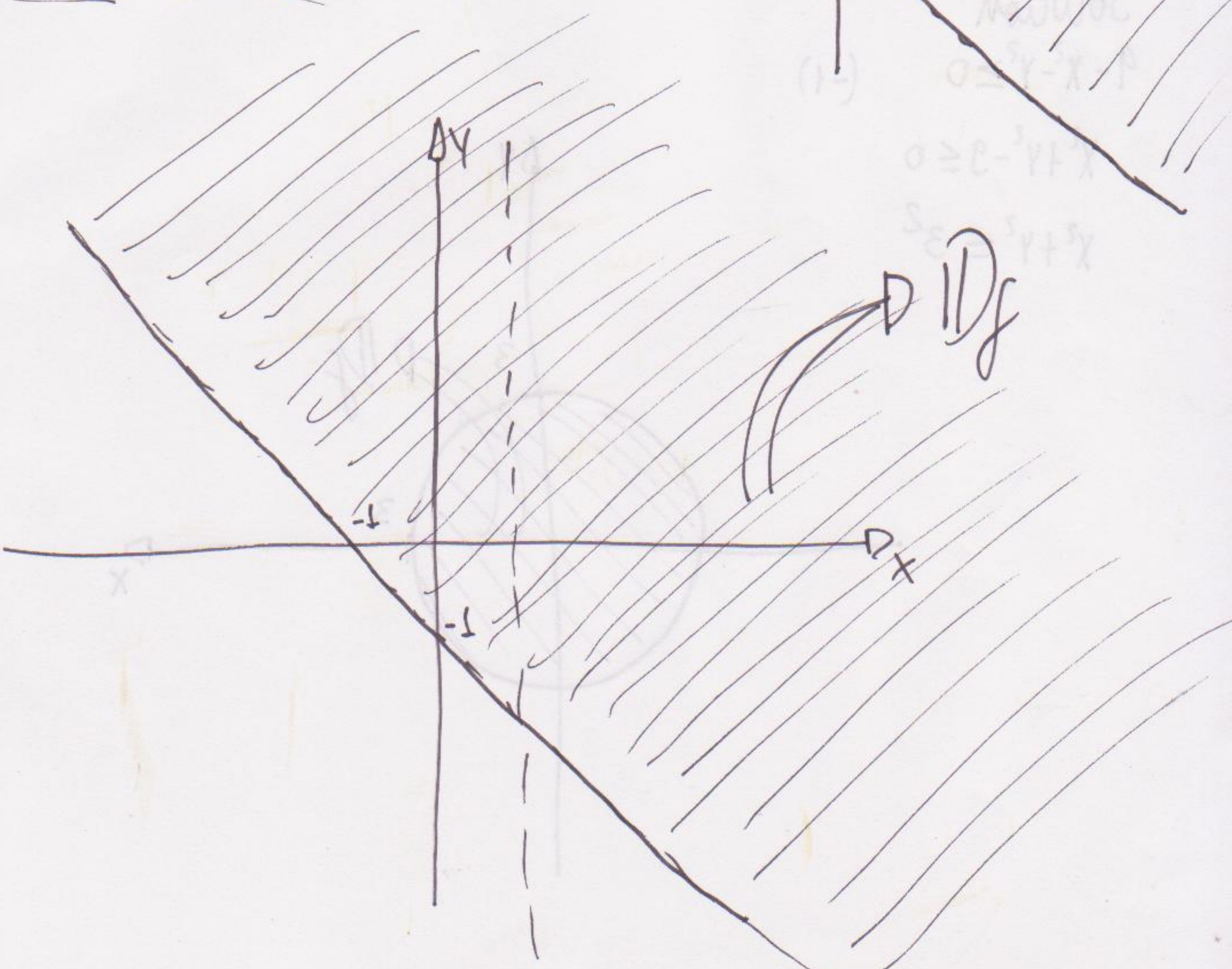
$x \neq 1$



$x+y+1 \geq 0$

$y \geq -x-1$





Practico nº 2

* localice el vertice y el foco, Ec. de la directriz y grafique.

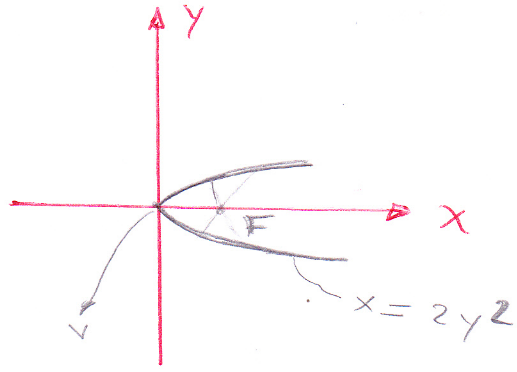
$x = 2y^2$

$V = (0,0)$

$y^2 = 4px$

$\frac{\cancel{x}}{2} = 4p\cancel{x}$

$\boxed{p = \frac{1}{8}}$



foco

$(p,0) = \left(\frac{1}{8}, 0\right)$

Directriz

$x = \frac{1}{8}$ #

$4y + x^2 = 0$

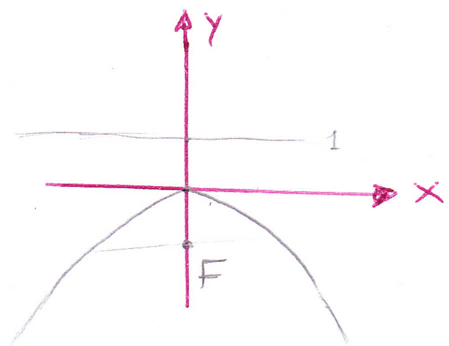
$x^2 = -4y$

$V(0,0)$

$x^2 = 4yp$

$-4\cancel{y} = 4\cancel{y}p$

$\boxed{p = -1}$



Foco

$(0,p) = (0,-1)$

Directriz

$\boxed{y = 1}$

$4x^2 = -y$

$y = -4x^2$

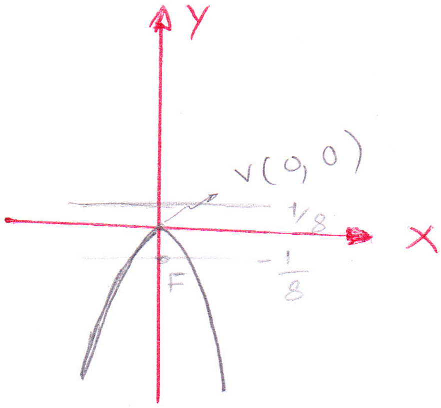
$x^2 = -\frac{y}{4}$

Foco

$x^2 = 4yp$

$-\frac{\cancel{y}}{4} = 4\cancel{y}p$

$\boxed{p = -\frac{1}{8}}$



Foco

$(0,p) = \left(0, -\frac{1}{8}\right)$

Directriz

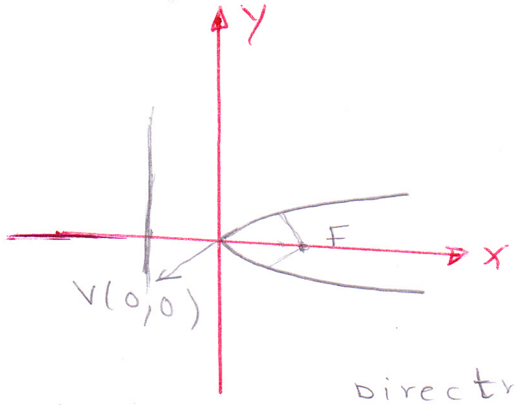
$y = \frac{1}{8}$

$y^2 = 12x$

$y^2 = 4xp$

$12\cancel{x} = 4\cancel{x}p$

$\boxed{p = 3}$



Foco

$(p,0) = (3,0)$

Directriz

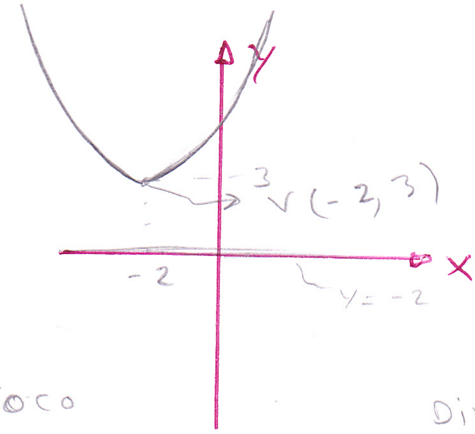
$\boxed{x = -3}$

$(x+2)^2 = 8(y-3)$

$(x-h)^2 = 4p(y-k)$

$8\cancel{(y-3)} = 4p\cancel{(y-3)}$

$\boxed{p = 2}$



Foco

$(0,p) = (0,2)$

Directriz

$\boxed{y = -2}$

$x^2 + 12x - y + 39 = 0$

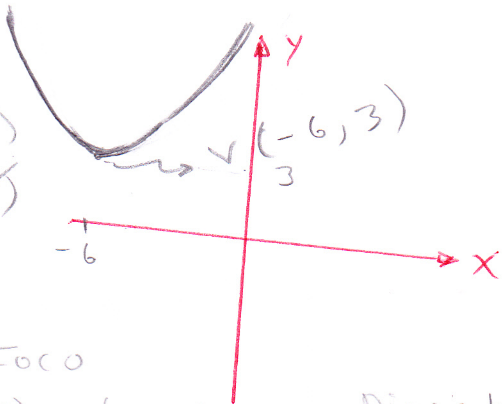
$x^2 + 12x + 36 - 36 = y - 39$

$(x+6)^2 = (y-3)$

$(x-h)^2 = 4p(y-k)$

$\cancel{(y-3)} = 4p\cancel{(y-3)}$

$\boxed{p = \frac{1}{4}}$



Foco

$(0,p) = \left(0, \frac{1}{4}\right)$

Directriz

$y = -\frac{1}{4}$

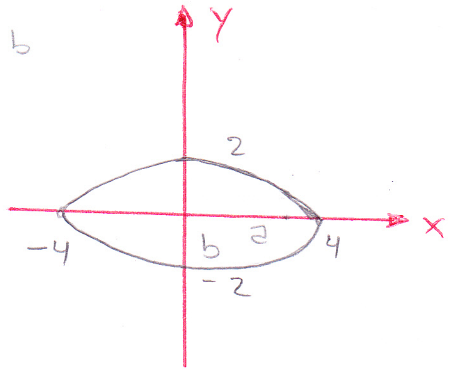
//

*Halle vertices y focos de la elipse y grafique.

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

si $a \geq b$

$V(\pm a, 0)$



$V(\pm 4, 0)$

$c^2 = a^2 - b^2$

Foco

$c^2 = 16 - 4 = \sqrt{12}$

los Focos $F(\pm c, 0)$

$(\pm\sqrt{12}, 0)$

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

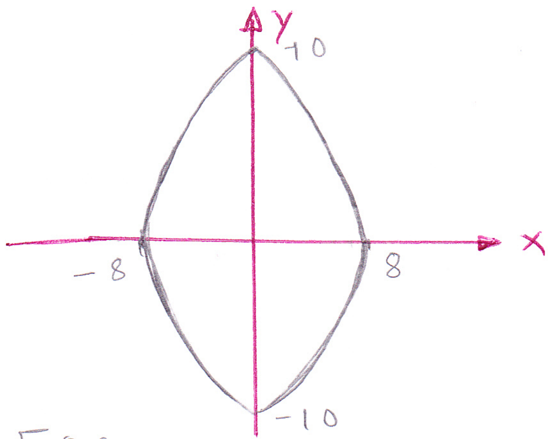
$V(0, \pm 10)$

Foco

$c^2 = b^2 - a^2$

$c^2 = 100 - 64$

$c = \pm 6$



Foco

$(0, \pm c) = (0, \pm 6)$

$25x^2 + 9y^2 = 225 \quad * \frac{1}{225}$

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

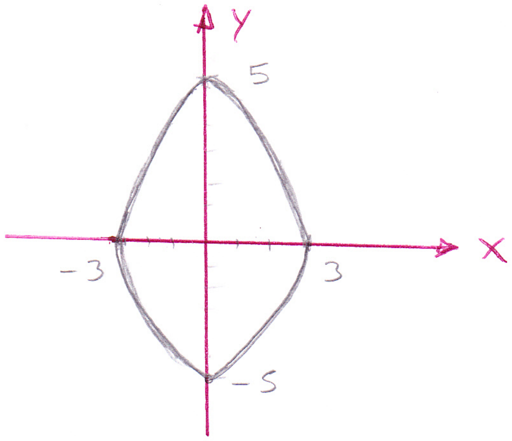
$V(0, \pm 5)$

Foco

$c^2 = b^2 - a^2$

$c^2 = 25 - 9$

$\boxed{c = \pm 4}$



Foco

$F(0, \pm 4)$

$4x^2 + 25y^2 = 25 \quad * \frac{1}{25}$

$\frac{x^2}{\frac{25}{4}} + y^2 = 1$

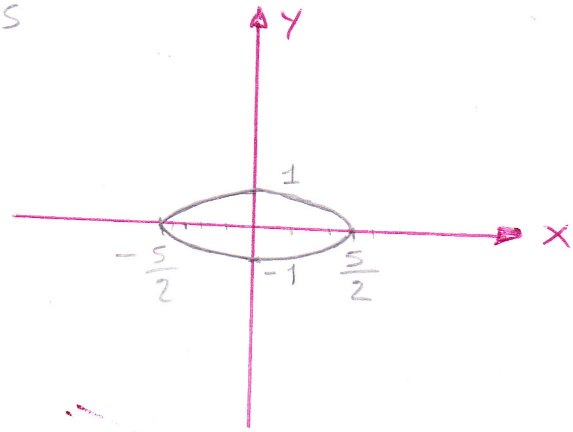
$V(\pm \frac{5}{2}, 0)$

Foco

$c^2 = a^2 - b^2$

$c = \frac{25}{4} - 1$

$c = \pm \frac{\sqrt{21}}{2}$



$F(\pm \frac{\sqrt{21}}{2}, 0)$

$$x^2+2y^2-6x+4y+7=0$$

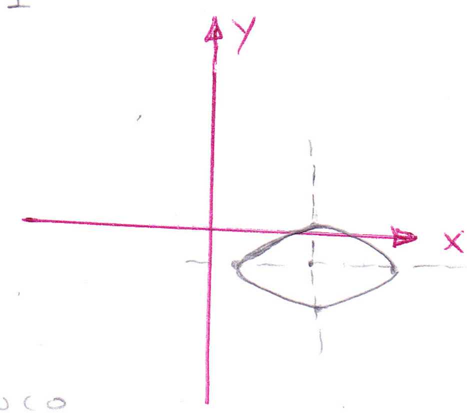
$$x^2-6x \quad +2y^2+4y+7=0$$

$$x^2-6x+9-9+2(y^2+2y+1-1)+7=0$$

$$(x-3)^2+2(y+1)^2-2+7-9=0$$

$$(x-3)^2+2(y+1)^2=4 \qquad * \frac{1}{4}$$

$$\frac{(x-3)^2}{4}+\frac{(y+1)^2}{2}=1$$



V (3, -1)

Foco

$$c^2=a^2-b^2$$

$$c^2=4-2$$

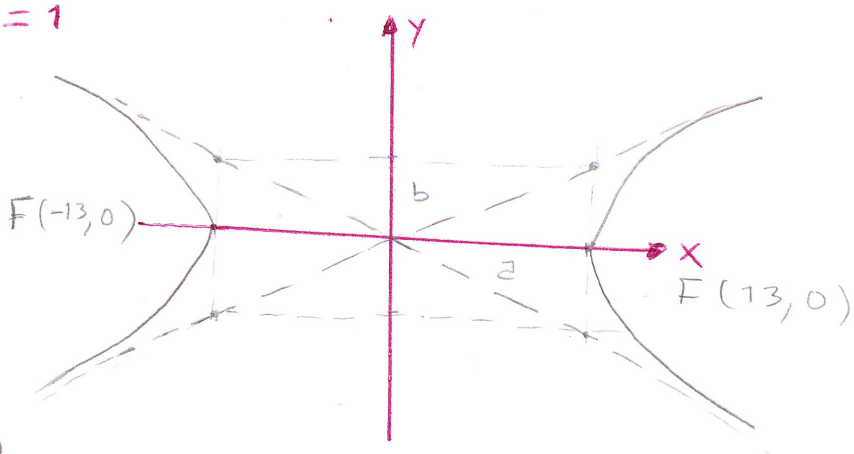
$$c=\pm\sqrt{2}$$

Foco

$F(\pm\sqrt{2}, 0)$

* Hallar vertice y foco de las Hipervolas.

$$\frac{x^2}{144}-\frac{y^2}{25}=1$$



Foco

$F(\pm c, 0)$

$$c^2=a^2+b^2$$

$$c^2=144+25$$

$$c=\pm 13$$

V $(\pm 12, 0)$

Ec. de asintotas.

$$y=\pm\left(\frac{b}{a}\right)x$$

$$y=\frac{25}{144}x \quad ; \quad y=-\frac{25}{144}x$$

\#

$$\frac{y^2}{16} - \frac{x^2}{36} = 1$$

Foco

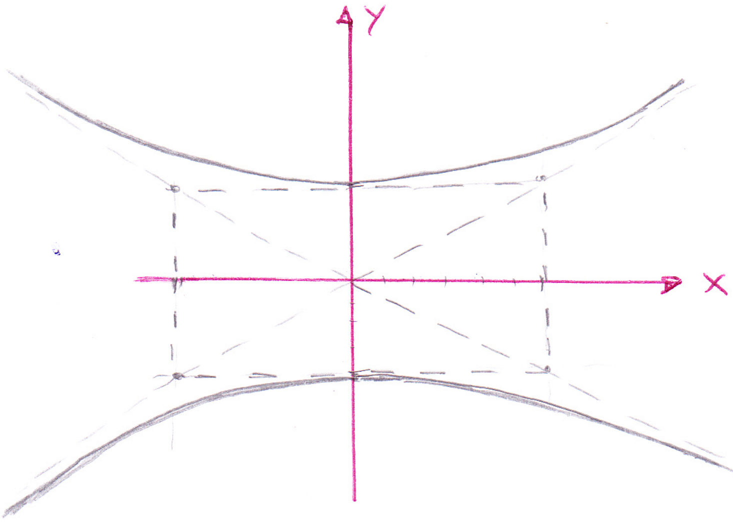
$c^2 = 16 + 36$

$c = \pm\sqrt{52}$

$F(0, \pm c)$

$F(0, \pm\sqrt{52})$

$V(0, \pm 4)$



asintotas

$Y = \frac{a}{b} x$

$Y = -\frac{16}{36} x \quad ; \quad Y = \frac{16}{36} x$

$Y = -\frac{4}{9} x \quad ; \quad Y = \frac{4}{9} x$ //

$$9y^2 - x^2 = 9 \quad * \frac{1}{9}$$

$y^2 - \frac{x^2}{9} = 1$

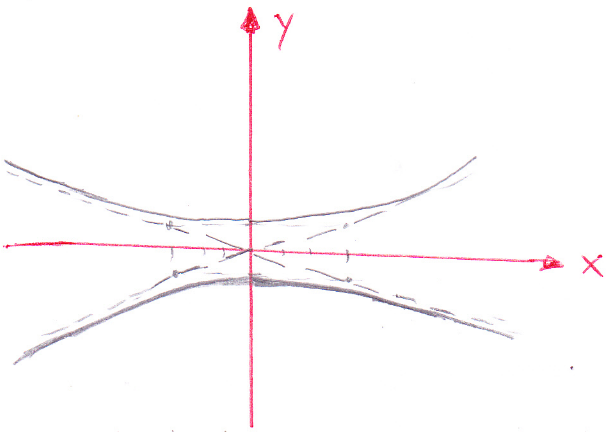
Foco

$c^2 = 1 + 9$

$c = \pm\sqrt{10}$

$F(0, \pm\sqrt{10})$

$V(0, \pm 1)$



Asintotas

$Y = -\frac{1}{9} x \quad , \quad Y = \frac{1}{9} x$ //

$x^2 - y^2 = 1$

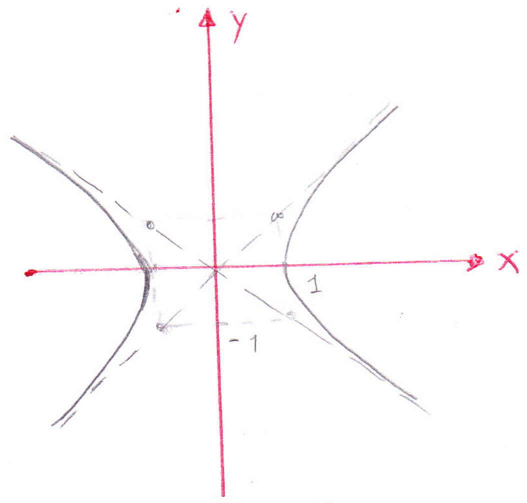
Foco

$c^2 = 1 + 1$

$c = \pm \sqrt{2}$

$F(\pm\sqrt{2}, 0)$

$V(\pm 1, 0)$



Asintota

$\boxed{Y = -X}$ $\boxed{Y = X}$

$16x^2 - 9y^2 + 64x - 90y = 305$

$16(x^2+4x) - 9(y^2+10y) = 305$

$16(x^2+4x+4-4) - 9(y^2+10y+25-25) = 305$

$(x+2)^2 - 9(y+5)^2 = 144 \quad * \frac{1}{144}$

$\frac{(x+2)^2}{144} - \frac{(y+5)^2}{16} = 1$

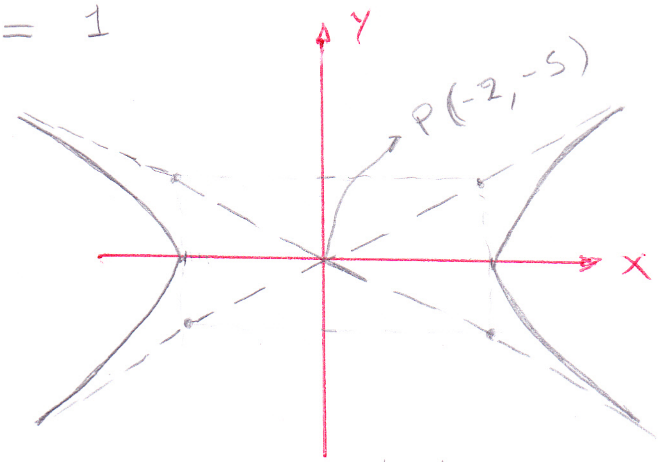
$V(-2, -5)$

Foco

$c^2 = 144 + 16$

$c = \pm\sqrt{160}$

$F(\pm\sqrt{160}, -5)$



Asintota

$Y = \frac{b}{a}(x+2)$, $\boxed{Y = \frac{1}{9}(x+2)}$

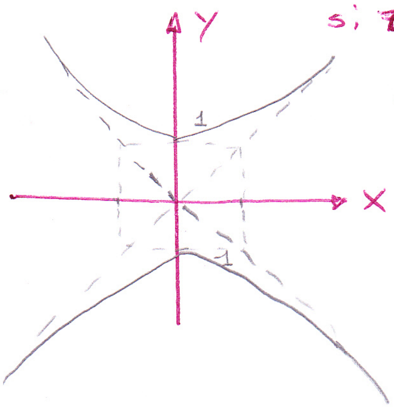
$\boxed{Y = -\frac{1}{9}(x+2)}$

Practico nº 3

* Trazas y superficies en el espacio.

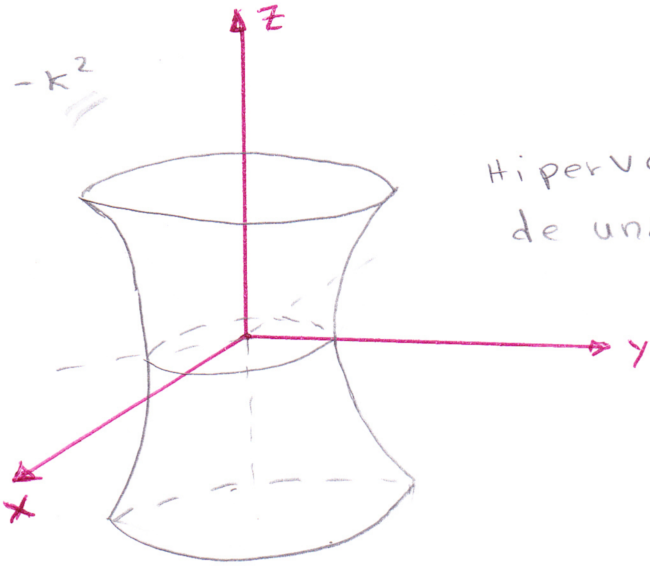
$\mathbb{R}$ $x=k$; $y=k$, $z=k$

1: $x^2-y^2+z^2=1$

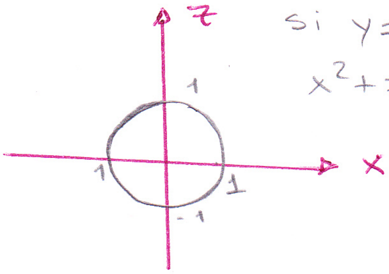


si $z=k$

$x^2-y^2=1-k^2$



Hipervoloide de una Hoja.



si $y=k$

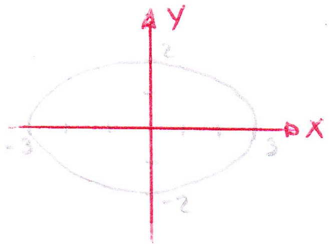
$x^2+z^2=1-k^2$

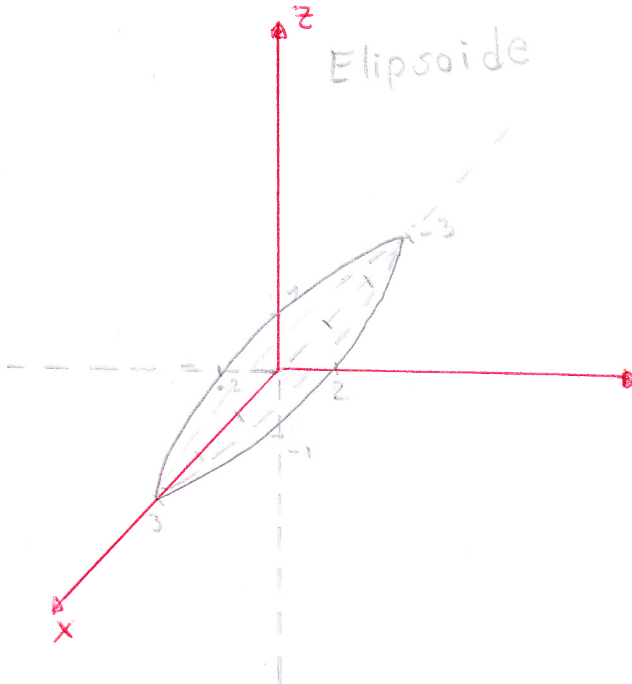
2: $4x^2+9y^2+36z^2=36$

si $z=k$

$4x^2+9y^2=36-36k^2$

$\frac{1}{9}x^2+\frac{1}{4}y^2=1$

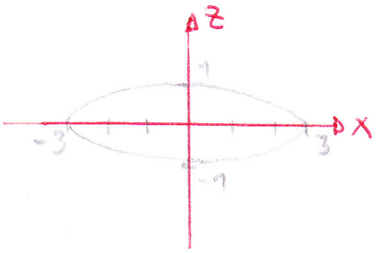




Elipsoide

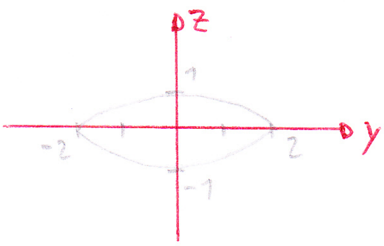
si $y=k$

$\frac{1}{9}x^2+1z^2=1$



si $x=k$

$\frac{1}{4}y^2+1z^2=1$

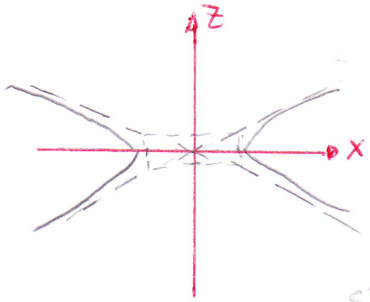


3.- $4z^2 - x^2 - y^2 = 1$

si $y = K$

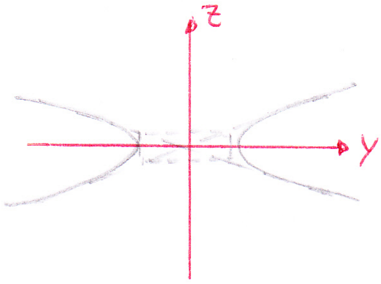
$4z^2 - x^2 = 1$

$\dfrac{z^2}{\frac{1}{4}} - x^2 = 1$



si $x = K$

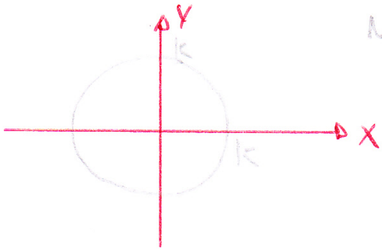
$\dfrac{z^2}{\frac{1}{4}} - y^2 = 1$



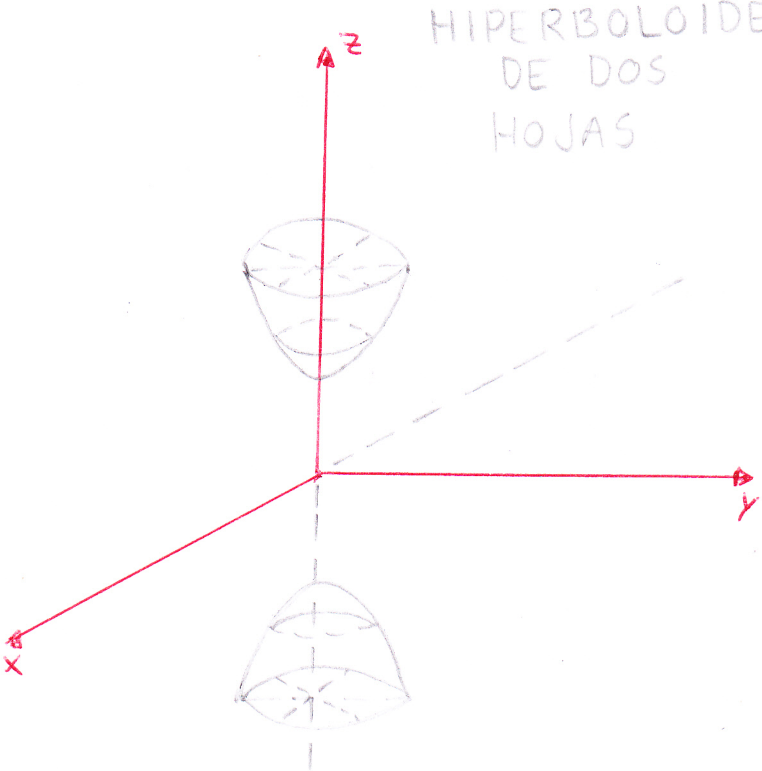
si $z = K$

NO $\nexists$ gráfica de

$-x^2 - y^2 = 1$

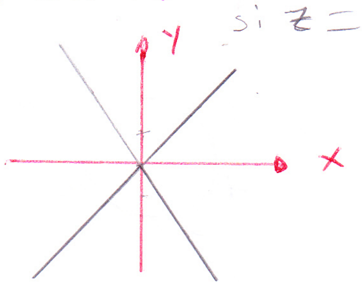


HIPERBOLOIDE DE DOS HOJAS

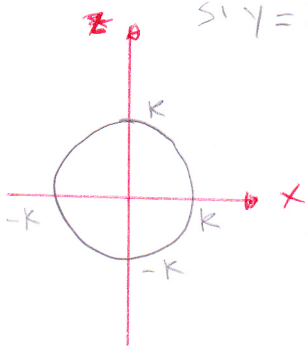


4.- $y^2 = x^2 + z^2$

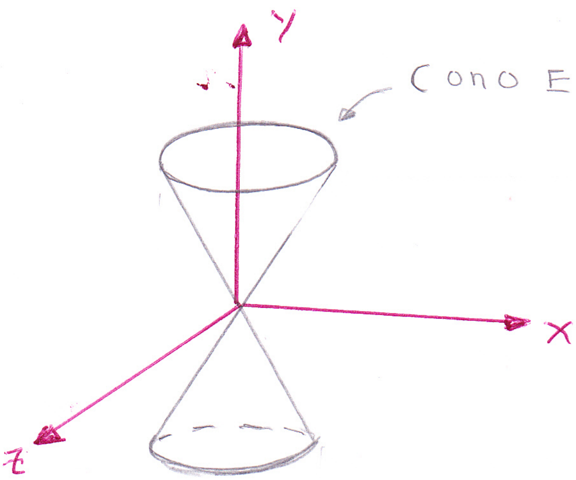
si $z = K$



si $y = K$



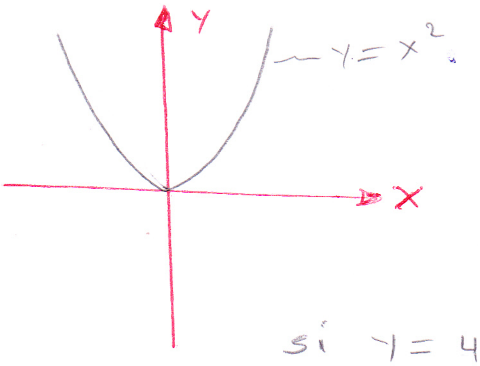
Cono Eliptic



5.- $x^2+4z^2-y=0$

si $z=0$

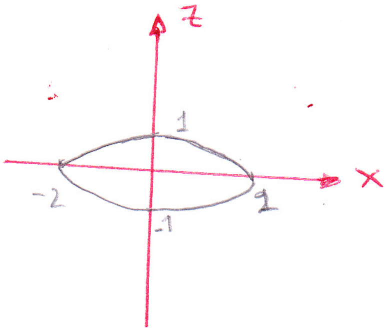
$y=x^2$



si $y=4$

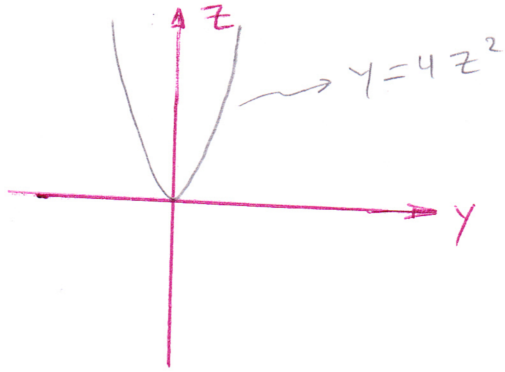
$x^2+4z^2=4 \quad / * \frac{1}{4}$

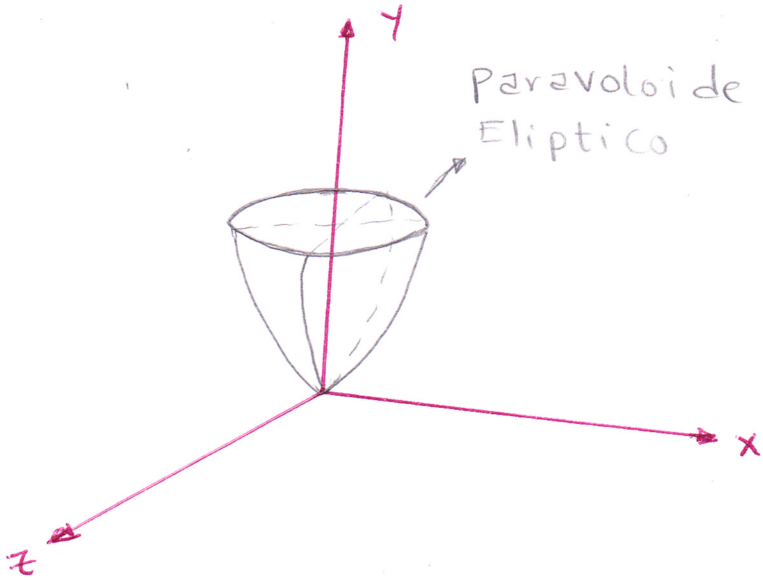
$\frac{x^2}{4}+z^2=1$



si si $x=0$

$y=4z^2$

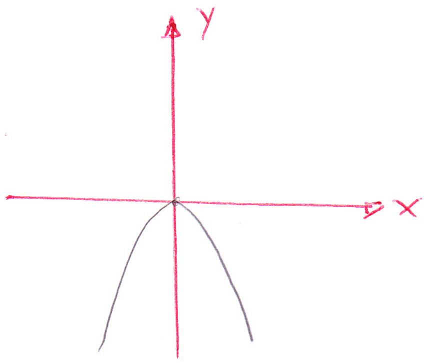




6. $y = z^2 - x^2$

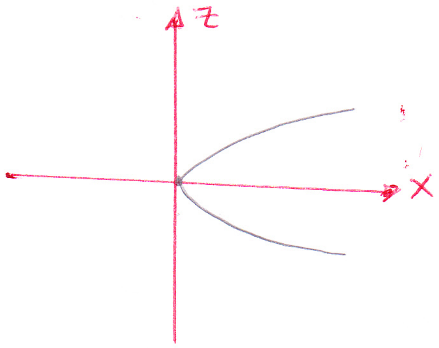
si $z = 0$

$y = -x^2$



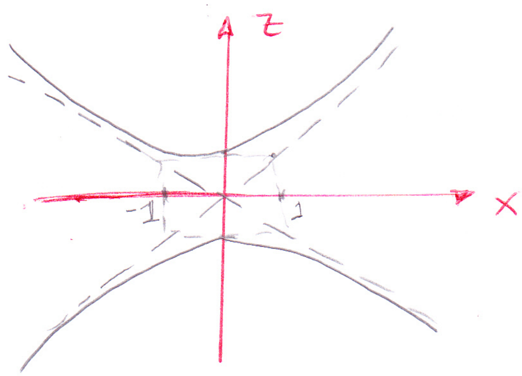
si $x = 0$

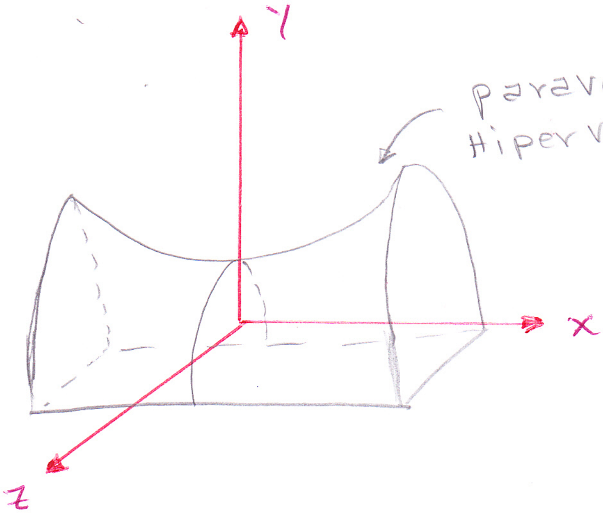
$y = z^2$



si $y = 1$

$z^2 - x^2 = 1$



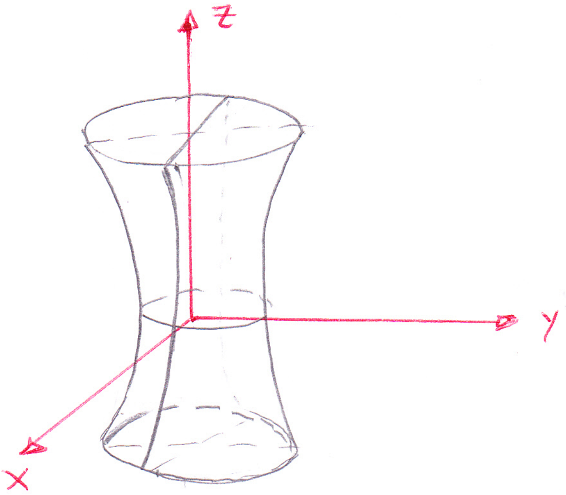


II)

1.- $z^2 = 3x^2 + 4y^2 - 12$

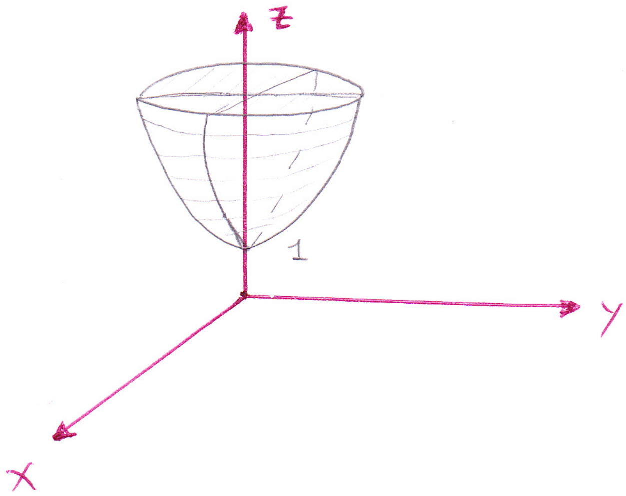
$3x^2 + 4y^2 - z^2 = 12 \quad /* \frac{1}{12}$

$\frac{x^2}{4} + \frac{y^2}{3} - \frac{z^2}{12} = 1 \Rightarrow$ Hipervoloide de una hoja



2.- $z = x^2 + y^2 + 1$

$x^2 + y^2 = z - 1 \rightarrow$ paravoloide Eliptico



3. $x^2+y^2-4z^2+4x-6y-8z=13$

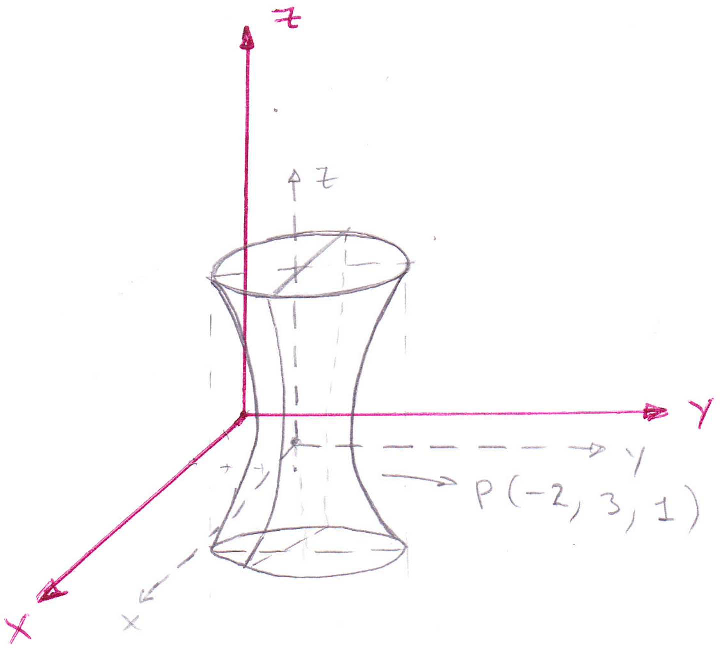
$x^2+4x+y^2-6y-4z^2-8z=13$

$x^2+4x+4-4+y^2-6y+9-9-4(z^2-2z+1-1)=13$

$(x+2)^2-\cancel{4}+(y-3)^2-9-4(z-1)^2+\cancel{4}=13$

$(x+2)^2+(y-3)^2-4(z-1)^2=22 \quad * \,/\frac{1}{22}$

$\frac{(x+2)^2}{22}+\frac{(y-3)^2}{22}-\frac{(z-1)^2}{\frac{11}{2}}=1 \Rightarrow$ Hipervoloide de una hoja



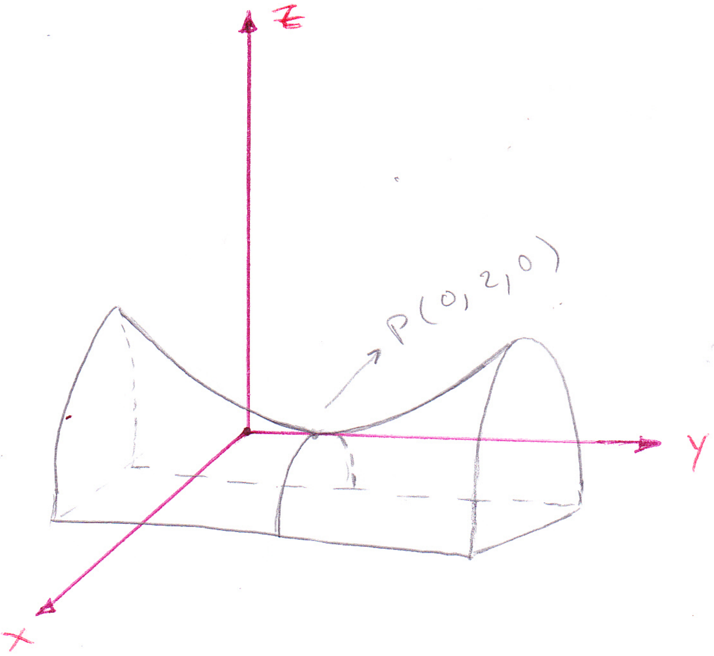
4 = $x^2 - y^2 + 4y + z = 4$

$$x^2 - (y^2 - 4y) + z = 4$$

$$x^2 - (y^2 - 4y + 4 - 4) + z = 4$$

$$x^2 - (y-2)^2 + \cancel{4} + z = \cancel{4}$$

$(y-2)^2 - x^2 = z \Rightarrow$ paravoloide Hipervolico



Practico nº 4

* De cartecianas a polares.

a) $(1,1) \rightarrow (r,\theta)$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{1+1}$$
$$r = \sqrt{2}$$
$$tg\,\theta = \frac{y}{x}$$
$$\theta = arctag(1)$$
$$\boxed{\theta = \frac{\pi}{4}}$$

$(1,1) \rightarrow \left(\sqrt{2}, \frac{\pi}{4}\right)$ #

b) $(2\sqrt{3}, -2) \rightarrow (r,\theta)$

$$r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$$
$$\boxed{r = 4}$$

$$tag\,\theta = \left(\frac{-2}{2\sqrt{3}}\right)$$
$$\theta = -\frac{\pi}{6}$$ #

$\therefore (2\sqrt{3}, -2) \rightarrow \left(4, -\frac{\pi}{6}\right)$ #

c) $(-1, -\sqrt{3})$

$$r = \sqrt{(1)^2 + (\sqrt{3})^2}$$
$$\boxed{r = 2}$$

$$tag\,\theta = \left(\frac{-\sqrt{3}}{-1}\right)$$
$$\boxed{\theta = \frac{\pi}{3}}$$

$\left(2, \frac{\pi}{3}\right)$ #

* De polar a cartesiana.

a) $r = 3\,\text{sen}\,\theta$

$x = r.\cos\theta$

$y = r.\,\text{sen}\theta$

$\text{sen}\theta = \dfrac{y}{r}$

$r = \dfrac{3y}{r} \Rightarrow r^2 = 3y$

si $x^2 + y^2 = r^2$

$$\boxed{x^2 + y^2 - 3y = 0}$$

//

b) $r.\cos\theta = 1$

$x = r\cos\theta$

$\cos\theta = \dfrac{x}{r}$

$\cancel{r}.\dfrac{x}{\cancel{r}} = 1$

$x = \dfrac{r^2}{r^2}(\cos^2\theta + \text{sen}\,\theta)$

$x^2 + y^2 - x - 1 = 0$ //

c) $r = \dfrac{1}{1 + 2\,\text{sen}\,\theta}$

$r = \sqrt{x^2 + y^2}$

$x = r.\cos\theta$

$y = r.\,\text{sen}\,\theta$

$\dfrac{y}{r} = \text{sen}\,\theta$

$r.(1 + 2\,\text{sen}\,\theta) = 1$

$r\left(1 + 2\left(\dfrac{y}{r}\right)\right) = 1$

$2y + r = 1$

$2y + \sqrt{x^2 + y^2} = 1$

$\left(\sqrt{x^2 + y^2}\right)^{\cancel{2}} = (1 - 2y)^2$

$x^2 + y^2 + 4y - 4y^2 - 1 = 0$

$$\boxed{x^2 - 3y^2 + 4y - 1 = 0}$$

//

* De Ec. cartesiana a Ec. polar.

a) $x^2+y^2=25$

$x^2+y^2=r^2$

$r^2\cos^2\theta + r^2 \cdot sen^2\theta = 25$

$r^2=25$

$\boxed{r=5}$

$x = r\cdot\cos\theta$

$y = r\cdot sen\theta$

$$\boxed{r = 5\cdot\cos\theta}$$

//

b) $x^2 = 4y$

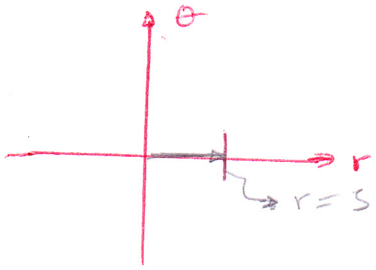
$r^{\cancel{2}}\cdot\cos^2\theta = 4\cdot\cancel{r}\cdot sen\theta$

$$\boxed{r = 4\,\frac{tg\theta}{\cos\theta}}$$

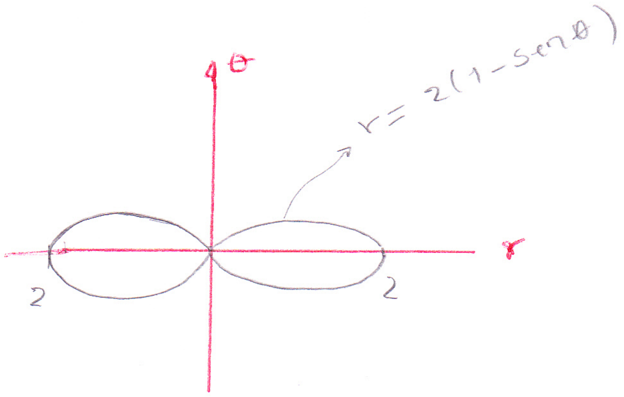
//

* graficar.

a) $r=5$

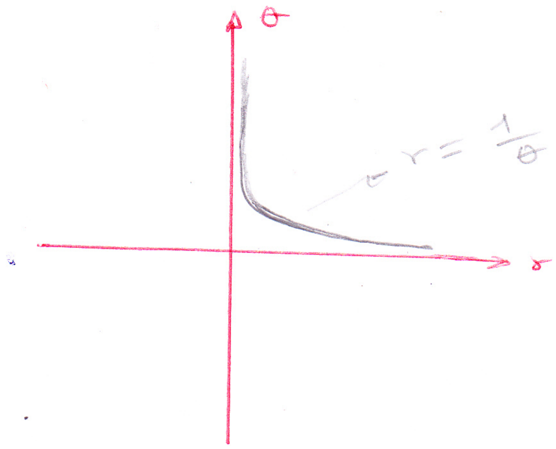


b) $r= 2(1- sen\theta)$

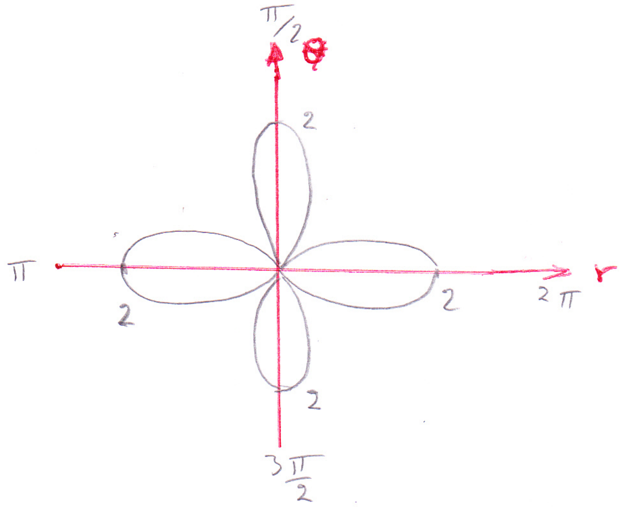


b) $r = \frac{1}{\theta}$

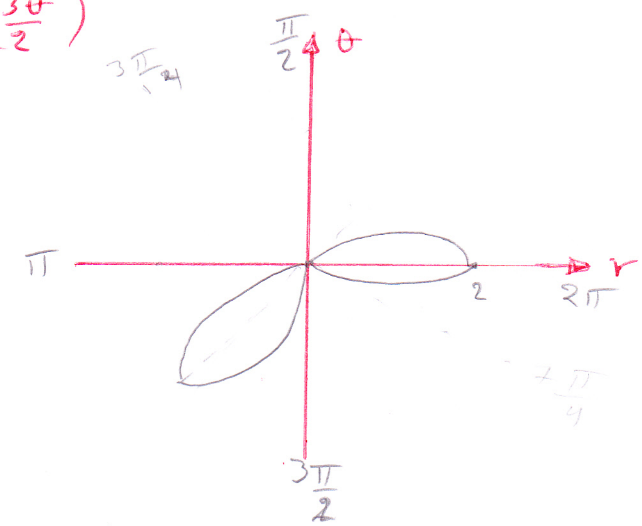
$\theta = \frac{1}{r}$



d) $r = 2\cos 4\theta$



e) $r = 2 \cdot \cos\left(\frac{3\theta}{2}\right)$



Practico nº 5

* Graficar y encontrar las coordenadas rectangulares de los sgt p. q estan en coordenadas silindricas.

$(3, \frac{\pi}{2}, 1) \rightarrow (r, \theta, z)$

$x = r \cos\theta \qquad y = r \cdot \text{sen}\,\theta \ ; \ z = z$

$x = 3 \cdot \frac{\sqrt{3}}{2}$

$x = \frac{3}{2}\sqrt{3}$

$y = \frac{\pi}{2} \cdot \frac{1}{2}$

$y = 90 \cdot \frac{1}{2}$

$\boxed{y = 45}$

$\therefore (\frac{3}{2}\sqrt{3}, 45, 1)$ #

$(3, 0, -6) \rightarrow (r, \theta, z) \rightarrow (x, y, z)$

$x = 3 \cdot \cos\theta$

$\boxed{x = 3}$

$y = 0 \cdot \text{sen}\,\theta$

$\boxed{y = 0}$

$\therefore (3, 0, -6)$ #

$(4, -\pi/3, 5) \rightarrow (x, y, z)$

$x = 4 \cdot \cos\theta$

$\boxed{x = 3{,}86}$

$y = -\frac{\pi}{3} \cdot \text{sen}\,\theta$

$y = -60 \cdot \text{sen}(15)$

$\boxed{y = -15{,}53}$

$\therefore (3{,}86, -15{,}53, 5)$

* De coor. rectangulares a silindricas

$(1,-1,4) \to (r,\theta,z)$

$r^2 = x^2+y^2$ $\quad tg\,\theta = \frac{y}{x}$

$r = \sqrt{2}$ $\quad tg\,\theta = \left(-\frac{1}{1}\right)$

$r = \sqrt{2}$ $\quad \theta = -\frac{\pi}{4}$

$\therefore \left(\sqrt{2}, -\frac{\pi}{4}, 4\right)$

$(-1,-\sqrt{3},2) \to (r,\theta,z)$

$r = \sqrt{1+3}$ $\quad tg\,\theta = \sqrt{3}$

$\boxed{r = 2}$ $\quad \boxed{\theta = \frac{\pi}{3}}$

$\therefore \left(2, \frac{\pi}{3}, 2\right)$ //

* de coo. rectangulares a esfericas.

$(-3,0,0) \to (\rho,\theta\ \phi)$ $\quad z = \rho \cdot \cos\theta$

$\rho = \sqrt{(3)^2+0^2+0^2}$ $\quad \phi = \frac{0}{3}$

$\boxed{\rho = 3}$ $\quad \boxed{\phi = 0}$

$0 = 3 . \operatorname{sen}\phi . \operatorname{sen}\theta$

$\boxed{\theta = 0}$

$\therefore (3,0,0)$

* de coor. esfericas a silindricas.

$(2,0,0) \to (r,\theta,0)$ $\quad r = \sqrt{2^2+0}$ $\quad \boxed{\theta = 0}$

$\boxed{r = 2}$

$\therefore (2,0,0)$

$\left(8/\pi/6, \pi/2\right) \to \left(r,\theta,\frac{\pi}{2}\right)$ $\quad \therefore \left(10, 2\pi, \frac{\pi}{2}\right)$ //

Practico Nº 6

* continuidad de funciones.

$$f(x,y) = \frac{x^2}{y-1}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{y-1} = \frac{(0)^2}{0-1} = 0 \quad \text{en } p(0,0)$$

$$f(x,y) = \frac{4x^2y+3y^2}{2x-y}$$

$$\lim_{(x,y)\to(0,0)} \frac{4x^2y+3y^2}{2x-y} = \frac{0}{0}$$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{4x^2y+3y^2}{2x-y}\right) = 0$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{4x^2y+3y^2}{2x-y}\right) = 0$$

= es continua.

$$f(x,y) = \ln(25-x^2-y^2)$$

$$\lim_{(x,y)\to(0,0)} \ln(25-x^2-y^2) = \ln(25)$$

$f(x,y) \to$ continua $p(0,0)$

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{xy}{\sqrt{x^2+y^2}}\right) = 0$$

$$=$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{xy}{\sqrt{x^2+y^2}}\right) = 0$$

$$f(x,y) = \frac{x}{\sqrt{4x^2+9y^2-36}}$$

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{4x^2+9y^2-36}} = 0$$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x}{\sqrt{4x^2+9y^2}}\right) = \infty$$

$$\neq$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{x}{\sqrt{4x^2+9y^2-36}}\right) = 0$$

$f(x,y)$ no es continua

$$\lim_{\substack{x \to 2 \\ y \to 3}} \left(3x^2 + xy - 2y^2\right) = 3(2)^2 + (2)(3) - 2(3)^2 = \boxed{0}$$

$$\lim_{\substack{x \to -1 \\ y \to 4}} \left(5x^2 - 2xy + y^2\right) = 5(-1)^2 - 2(-1)(4) + (4)^2 = \boxed{29}$$

$$\lim_{\substack{x \to 0 \\ y \to 1}} \frac{x^4 - (y-1)^4}{x^2 + (y-1)^2} = \frac{(0)^4 - (1-1)^4}{(0)^2 + (1-1)^2} = 0$$

$$\lim_{\substack{x \to 0 \\ y \to 1}} \frac{\cancel{\left(x^2 + (y-1)^2\right)}\left(x^2 - (y-1)^2\right)}{\cancel{\left(x^2 + (y-1)^2\right)}} = \lim_{\substack{x \to 0 \\ y \to 1}} x^2 - (y-1)^2$$

$$\lim_{\substack{x \to 0 \\ y \to 1}} (0)^2 - (1-1)^2 = \boxed{0}$$

$$\lim_{\substack{x \to -2 \\ y \to 4}} y^3 \cdot \sqrt{x^3 + 2y} = (4)^3 \sqrt{(-2)^3 + 2(4)} = \boxed{0}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \frac{0}{0}$$

$$\left.\begin{array}{l}\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^2-y^2}{x^2+y^2}\right) = \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}} = 1 \\ \lim_{y\to 0}\left(\lim_{x\to 0} \frac{x^2-y^2}{x^2+y^2}\right) = \lim_{y\to 0} \frac{-y^2}{y^2} = -1\end{array}\right] \neq$$

$S_1 : \left[f(x,y) \,/\, y = x \right]$

$$\lim_{x\to 0} \frac{x^2-x^2}{x^2+x^2} = 0$$

$S_2 :\ f(x,y) \,/\, y = \sqrt{x}$

$$\lim_{x\to 0} \frac{x^2-x}{x^2+x} = \lim_{x\to 0} \frac{\cancel{x}(x-1)}{\cancel{x}(x+1)} = -1$$

$S_3 : \left[f(x,y) \,/\, y = kx \right]$

$$\lim_{x\to 0} \frac{x^2-(kx)^2}{x^2+(kx)^2} = \frac{\cancel{x^2}(1-k^2)}{\cancel{x^2}(1+k^2)} = \frac{1-k^2}{1+k^2} \#$$

el. limite no $\exists$

$$\lim_{(x,y)\to(0,0)} \frac{x^4+3x^2y^2+2xy^3}{(x^2+y^2)^2}$$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^4+3x^2y^2+2xy^3}{(x^2+y^2)^2}\right) = \lim_{x\to 0} \frac{x^4}{x^4} = 1$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{x^4+3x^2y^2+2xy^3}{(x^2+y^2)^2}\right) = 0$$

$L_1 \neq L_2$

$L \to \nexists$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{x^2+y^2} = \frac{0}{0}$$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^2+y}{x^2+y^2}\right) = \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}} = 1$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{x^2+y}{x^2+y^2}\right) = \lim_{y\to 0} \frac{1}{y} = \infty$$

como

$L_1 \neq L_2$ el Lim $\nexists$

Practico Nº 7

En cada de los siguientes problemas Hallar. $f_1(x,y)$ - y - $f_2(x,y)$

* $f(x,y) = x^2 + 2xy^2 - 2x$

$f_1(x,y) = 2x - 2y^2 - 2$ #

$f_2(x,y) = -4xy$ #

* $f(x,y) = x^3y - 3x^2y^2 + 2xy$

$f_1(x,y) = 3x^2y - 6xy^2 + 2y$ #

$f_2(x,y) = x^3 - 6x^2y + 2x$ #

* $f(x,y) = \sqrt{x^2+y^2}$

$f_1(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$ #

$f_2(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$ #

* $f(x,y) = \ln(x^2+y^2)$

$f_1(x,y) = \dfrac{2x}{x^2+y^2}$ #

$f_2(x,y) = \dfrac{2y}{x^2+y^2}$ #

* $f(x,y) = \arctan\left(\frac{y}{x}\right)$

$f_1(x,y) = \dfrac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2}$ #

$f_2(x,y) = \dfrac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}$ #

* $f(x,y) = x.\arcsen(x-y)$; $x=1$; $y=2$

$f_1(x,y) = \arcsen(x-y) + \dfrac{x}{\sqrt{1-(x-y)^2}}$

$f_1(1,2) = \arcsen(1-2) + \dfrac{1}{\sqrt{1-(1-2)^2}} = -90 + \cancel{\dfrac{1}{0}}\ \exists$

$f_2(x,y) = \dfrac{x}{\sqrt{1-(x-y)^2}} = \dfrac{1}{\sqrt{1-(1-2)^2}} = \nexists$

* $f(x,z) = e^{\operatorname{sen} x} . \operatorname{tg} xz$; $x = \frac{\pi}{4}$; $z=1$

$f_1(x,z) = \cos x . e^{\operatorname{sen} x} . \operatorname{tg} xz + z . e^{\operatorname{sen} x} . \sec^2(xz)$

$f_1(\frac{\pi}{4}, 1) = \cos(\frac{\pi}{4}) . e^{\operatorname{sen}(\frac{\pi}{4})} . \tan(\frac{\pi}{4} . 1) + 1 . e^{\operatorname{sen}(\frac{\pi}{4})} . \sec^2(\frac{\pi}{4} . 1)$

$f_1(\frac{\pi}{4}, 1) = 6{,}330$ #

$f_2(x,z) = e^{\operatorname{sen} x} . \sec^2 xz . x$

$f_2(x,z) = e^{\operatorname{sen}(\frac{\pi}{4})} . \sec^2(\frac{\pi}{4} . 1)(\frac{\pi}{4})$

$f_2(\frac{\pi}{4}, 1) = 4{,}056$ #

* $f(x,y) = x^{xy}$; $x = y = 2$

$f_1(x,y) = x^{xy}[y . \ln x + \cancel{\frac{xy}{x}} . (1)]$

$f_1(2,2) = 2^4(2 . \ln 2 + 2) = 64(\ln 2 + 1)$ #

$f_2(x,y) = x^{xy} . \ln x . y$

$f_2(2,2) = 2^4 . \ln(2) . 2$

$f_2(2,2) = 32 \ln(2)$ #

* Hallar: $f_1(x,y,z)$; $f_2(x,y,z)$ y $f_3(x,y,z)$

$$f(x,y,z) = x^2y - 2x^2z + 3xyz - y^2z + 2xz^2$$

$$f_1(x,y,z) = 2xy - 4xz + 3yz + 2z^2 \;\#$$

$$f_2(x,y,z) = x^2 + 3xz - 2yz \;\#$$

$$f_3(x,y,z) = -2x^2 + 3xy - y^2 + 4xz \;\#$$

* $f(x,y,z) = \dfrac{xyz}{x^2+y^2+z^2}$

$$f_1(x,y,z) = \frac{yz(x^2+y^2+z^2) - xyz(2x)}{(x^2+y^2+z^2)^2}$$

$$f_2(x,y,z) = \frac{xz.(x^2+y^2+z^2) - xyz(2y)}{(x^2+y^2+z^2)^2}$$

$$f_3(x,y,z) = \frac{xy(x^2+y^2+z^2) - xyz(2z)}{(x^2+y^2+z^2)^2} \;\#$$

* $f(x,y,z) = e^{xyz} \cdot \text{sen}(xy) \cdot \cos(xz)$

$$f_1(x,y,z) = yze^{xyz} \cdot \text{sen}(x.y) \cdot \cos(xz) + ye^{xyz} \cdot \cos(xy)\cos(xz) - z.e^{xyz} \cdot \text{sen}(xy)\,\text{sen}(xz) \;\#$$

$$f_2(x,y,z) = xze^{xyz} \cdot \text{sen}(x.y)\cos(xz) + x.e^{xyz} \cdot \cos(xy)\cos(xz) \;\#$$

$$f_3(x,y,z) = xye^{xyz} \cdot \text{sen}(xy)\cos(xz) - xe^{xyz} \cdot \text{sen}(xy) \cdot \text{sen}(xz) \;\#$$

* $w = \ln\left(\dfrac{xy}{x^2+y^2}\right)$; $\dfrac{\partial w}{\partial x}$; $\dfrac{\partial w}{\partial y}$

$$w = \ln(xy) - \ln(x^2+y^2)$$

$$\frac{\partial w}{\partial x} = \frac{1}{x} - \frac{2x}{x^2+y^2} \quad \#$$

$$\frac{\partial w}{\partial y} = \frac{1}{y} - \frac{2y}{x^2+y^2} \quad \#$$

* $w = (r^2+s^2+t^2).\cos(r.s.t)$; $\dfrac{\partial w}{\partial r}$; $\dfrac{\partial w}{\partial t}$

$$\frac{\partial w}{\partial r} = 2r(\cos(r.s.t) - st(r^2+s^2+t^2)\,\mathrm{sen}(r.s.t)$$

$$\frac{\partial w}{\partial t} = 2t\cos(r.s.t) - r.s(r^2+s^2+t^2).\mathrm{sen}(rst) \quad \#$$

* $w = e^{\mathrm{sen}(y/x)}$; $\dfrac{\partial w}{\partial y}$; $\dfrac{\partial w}{\partial x}$

$$\frac{\partial w}{\partial x} = -\frac{y.\cos(y/x).e^{\mathrm{sen}(y/x)}}{x^2}$$

$$\frac{\partial w}{\partial y} = \frac{\cos(y/x).e^{\mathrm{sen}(y/x)}}{x} \quad \#$$

* $w = \sec(tu).\mathrm{arcsen}(t.v)$; $\dfrac{\partial w}{\partial t}$; $\dfrac{\partial w}{\partial u}$; $\dfrac{\partial w}{\partial v}$

$$\frac{\partial w}{\partial t} = u\sec(tu).\mathrm{tang}(tu).\mathrm{arcsen}(t.v) + \frac{v.\sec(tu)}{\sqrt{1-(tv)^2}} \quad \#$$

$$\frac{\partial w}{\partial u} = t.\sec(t.u).\mathrm{tg}(tu).\mathrm{arcsen}(t.v) \quad \#$$

$$\frac{\partial w}{\partial v} = \frac{t.\sec(tu)}{\sqrt{1-(tv)^2}} \quad \#$$

Practico # 8

1:- $f(x,y) = x^2+y^2;\ \begin{cases} x = s-2t \\ y = 2s+t \end{cases}$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial s} = 2x\cdot(1) + 2y(2) = 2s - \cancel{4t} + 8s + \cancel{4t}$$

$$\frac{\partial f}{\partial s} = 10s$$

\#

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial f}{\partial t} = 2x\cdot(-2) + 2y(1) = -\cancel{4s} + 8t + \cancel{4s} + 2t$$

$$\frac{\partial f}{\partial t} = 10t$$

\#

* $f(x,y) = x^2+y^2;\ \begin{cases} x = s^2-t^2 \\ y = 2st \end{cases}$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial s} = 2x(2s) + 2y(2t) = 4s^3 - 4st^2 + 8st^2$$

$$\frac{\partial f}{\partial s} = 4s^3 + 4st^2$$

\#

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial f}{\partial t} = 2x(-2t) + 2y\cdot(2s) = -4s^2t + 4t^3 + 8s^2t$$

$$\frac{\partial f}{\partial t} = 4t^3 + 4s^2t$$

\#

* $f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$; $\begin{cases} x = 2s-t \\ y = s+2t \end{cases}$

$$\frac{\partial f}{\partial x} = \frac{\sqrt{x^2+y^2} - \dfrac{x^2}{\sqrt{x^2+y^2}}}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$\frac{\partial f}{\partial x} = \frac{\dfrac{(x^2+y^2) - x^2}{\sqrt{x^2+y^2}}}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$\frac{\partial f}{\partial x} = \frac{y^2}{\left(\sqrt{x^2+y^2}\right)^3} \;\#$$

$$\frac{\partial f}{\partial y} = -\frac{x}{\left(\sqrt{x^2+y^2}\right)^2} \cdot \frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y} = -\frac{xy}{\left(\sqrt{(x^2+y^2)}\right)^3} \;\#$$

$$\frac{\partial x}{\partial s} = 2 \qquad \frac{\partial x}{\partial t} = -1$$

$$\frac{\partial y}{\partial s} = 1 \qquad \frac{\partial y}{\partial t} = 2$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial s} = \frac{(s+2t)^2}{\left(\sqrt{(2s-t)^2+(s+2t)^2}\right)^3}\cdot 2 - \frac{(2s-t)(s+2t)}{\left(\sqrt{(2s-t)^2+(s+2t)^2}\right)^3} \;\#$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial f}{\partial t} = -\frac{(s+2t)^2}{\left(\sqrt{(2s-t)^2+(s+2t)^2}\right)^3} - \frac{2(2s-t)(s+2t)}{\left(\sqrt{(2s-t)^2+(s+2t)^2}\right)^3} \;\#$$

* $f(x,y,z) = x^2+y^2+z^2+3xy-2xz+4$; $\begin{cases} x = 3s+t \\ y = 2s-t \\ z = s+2t \end{cases}$

$$\frac{\partial f}{\partial x} = 2x+3y-2z \qquad \frac{\partial x}{\partial s} = 3 \qquad \frac{\partial x}{\partial t} = 1$$

$$\frac{\partial f}{\partial y} = 2y+3x \qquad \frac{\partial y}{\partial s} = 2 \qquad \frac{\partial y}{\partial t} = 2$$

$$\frac{\partial f}{\partial z} = 2z-2x \qquad \frac{\partial z}{\partial s} = 1 \qquad \frac{\partial z}{\partial t} = 2$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial s}$$

$$\frac{\partial f}{\partial s} = 3(6s+2t+6s-3t-2s-4t)+2(4s-2t+6s+3t)+2s+4t-6s-2t$$

$$\frac{\partial f}{\partial s} = 30s-15t+20s+2t-4s+2t$$

$$\frac{\partial f}{\partial s} = 46s-11t \quad /\!/$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$\frac{\partial f}{\partial t} = (10s-5t)+(10s+t)(2)+(2t-4s)(2)$$

$$\frac{\partial f}{\partial t} = 22s+t \quad /\!/$$

* $f(x,y,z) = x^2 - y^2 + 2z^2$; $\begin{cases} x = r^2+1 \\ y = r^2-2r+1 \\ z = r^2-2 \end{cases}$

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial x}{\partial r} = 2r$$

$$\frac{\partial f}{\partial y} = -2y \qquad \frac{\partial y}{\partial r} = 2r-2$$

$$\frac{\partial f}{\partial z} = 4z \qquad \frac{\partial z}{\partial r} = 2r$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\frac{\partial f}{\partial r} = 2x\cdot(2r) + (-2y)(2r-2) + 4z(2r)$$

$$\frac{\partial f}{\partial r} = 4r(r^2+1) - 2(r^2-2r+1)(2r-2) + 8r(r^2-2) \quad //$$

* $f(x,y) = f(u) = u^3+2u^2-3u+1$; $u = r^2-s^2+t^2$

$$\frac{\partial f}{\partial u} = 3u^2+4u-3$$

$$\frac{\partial u}{\partial r} = 2r \qquad \frac{\partial u}{\partial t} = 2t$$

$$\frac{\partial u}{\partial s} = -2s$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial r}$$

$$\frac{\partial f}{\partial r} = (3u^2+4u-3)(2r)$$

$$\frac{\partial f}{\partial r} = \left[3(r^2-s^2+t^2)^2+4(r^2-s^2+t^2)-3\right]\cdot 2r \quad //$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial s}$$

$$\frac{\partial f}{\partial s} = -2s\left[3(r^2-s^2+t^2)^2+4(r^2-s^2+t^2)-3\right] \quad //$$

$$\frac{\partial f}{\partial t} = 2t\left[3(r^2-s^2+t^2)^2+4(r^2-s^2+t^2)-3\right] \quad //$$

2.-

a) $z = x^2 - y^2$; $\begin{cases} x = r.\cos\theta \\ y = r.\operatorname{sen}\theta \end{cases}$; $\frac{\partial z}{\partial r}$; $\frac{\partial z}{\partial \theta}$ donde $r = \sqrt{2}$
$\theta = \frac{\pi}{4}$

$$z = r^2.\cos^2\theta - r^2.\operatorname{sen}^2\theta$$

$$z = r^2(\cos^2\theta - \operatorname{sen}^2\theta)$$

$$\frac{\partial z}{\partial r} = 2r(\cos^2\theta - \operatorname{sen}^2\theta)$$

$$\frac{\partial z}{\partial r} = 2\sqrt{2}\left(\cos^2\left(\frac{\pi}{4}\right) - \operatorname{sen}^2\left(\frac{\pi}{4}\right)\right)$$

$$\frac{\partial z}{\partial r} = 2\sqrt{2}.\left(\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2\right)$$

$$\boxed{\frac{\partial z}{\partial r} = 0}$$

$$\frac{\partial z}{\partial \theta} = r^2(-2\cos\theta.\operatorname{sen}\theta - 2\operatorname{sen}\theta.\cos\theta)$$

$$\frac{\partial z}{\partial \theta} = r^2(-4.\cos\theta.\operatorname{sen}\theta)$$

$$\frac{\partial z}{\partial \theta} = \cancel{(\sqrt{2})^2}(-4.\cos\left(\frac{\pi}{4}\right).\operatorname{sen}\left(\frac{\pi}{4}\right))$$

$$\frac{\partial z}{\partial \theta} = \cancel{-4}.\frac{\sqrt{2}}{\cancel{2}}.\frac{\sqrt{2}}{\cancel{2}}$$

$$\boxed{\frac{\partial z}{\partial \theta} = -2}$$

b) $w = xy + yz + zx$; $\begin{cases} x = t.\cos t \\ y = t.\operatorname{sen} t \\ z = t \end{cases}$; $\frac{dw}{dt}$ donde $t = \frac{\pi}{4}$

$$\frac{\partial w}{\partial x} = y + z$$

$$\frac{\partial w}{\partial x} = t.\operatorname{sen} t + t = \frac{\pi}{4}.\operatorname{sen}\left(\frac{\pi}{4}\right) + \frac{\pi}{4} = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1\right)$$

$$\frac{\partial w}{\partial y} = x + z = t.\cos t + t = \frac{\pi}{4}.\left(\frac{\sqrt{2}}{2} + 1\right)$$

$$\frac{\partial w}{\partial z} = y + x = t.\operatorname{sen} t + t.\cos t = \frac{\pi}{4}(\sqrt{2})$$

$$\frac{\partial x}{\partial t} = \cos t - t\operatorname{sen} t = \cos\left(\frac{\pi}{4}\right) - \frac{\pi}{4}.\operatorname{sen}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}.\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\partial y}{\partial t} = \operatorname{sen} t + t.\cos t = \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right)$$

$$\frac{\partial z}{\partial t} = 1$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}.\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}.\frac{\partial z}{\partial t}$$

$$\frac{dw}{dt} = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1\right).\frac{\sqrt{2}}{2}\left(1 - \cancel{\frac{\pi}{4}}\right) + \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1\right).\frac{\sqrt{2}}{2}\left(1 + \cancel{\frac{\pi}{4}}\right) + \frac{\pi}{4}.\sqrt{2}\,(1)$$

$$\frac{dw}{dt} = \frac{\pi}{4}.\left(\frac{\sqrt{2}}{2} + 1\right).\sqrt{2} + \frac{\pi}{4}\sqrt{2}$$

$$\frac{dw}{dt} = \frac{\sqrt{2}}{2}.\frac{\pi}{4}. + \sqrt{2}\,\frac{\pi}{2}$$

$$\frac{dw}{dt} = \frac{5\sqrt{2}.\pi}{8}$$ #

c) $z = \dfrac{xy}{x^2+y^2}$; $\begin{cases} x = r.\cos\theta \\ y = r.\operatorname{sen}\theta \end{cases}$; $\dfrac{\partial z}{\partial r}, \dfrac{\partial z}{\partial \theta}$ donde; $r = 3$
$\theta = \dfrac{\pi}{6}$

$$z = \frac{r^2.\cos\theta.\operatorname{sen}\theta}{r^2.\cos^2\theta + r^2.\operatorname{sen}^2\theta}$$

$$z = \frac{\cancel{r^2}.\cos\theta.\operatorname{sen}\theta}{\cancel{r^2}(\underbrace{\cos^2\theta + \operatorname{sen}^2\theta}_{1})}$$

$$z = \cos\theta.\operatorname{sen}\theta$$

$$\frac{\partial z}{\partial r} = 0$$

$$\frac{\partial z}{\partial \theta} = -\operatorname{sen}^2\theta + \cos^2\theta$$

$$\frac{\partial z}{\partial \theta} = \cos^2\left(\frac{\pi}{6}\right) - \operatorname{sen}^2\left(\frac{\pi}{6}\right)$$

$$\frac{\partial z}{\partial \theta} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\frac{\partial z}{\partial \theta} = \frac{3}{4} - \frac{1}{4}$$

$$\frac{\partial z}{\partial \theta} = \frac{2}{4}$$

$$\boxed{\frac{\partial z}{\partial \theta} = \frac{1}{2}}$$

d) $w = x^3 + y^3 + z^3 - u^2 - v^2$; $\begin{cases} x = r^2 + s^2 + t^2 \\ y = r^2 + s^2 - t^2 \\ z = r^2 - s^2 - t^2 \\ u = r^2 + t^2 \\ v = r^2 - s^2 \end{cases}$ donde $r = 1$, $s = 0$, $t = -1$

$$\frac{\partial w}{\partial x} = 3x^2 = 3(r^2 + s^2 + t^2)^2$$

$$\frac{\partial w}{\partial x} = 3(1 + 0 + 1) = 6$$

$$\frac{\partial w}{\partial y} = 3y^2 = 3(r^2 + s^2 - t^2)^2$$

$$\frac{\partial w}{\partial y} = 3(1 + 0 - (-1)^2) = 0$$

$$\frac{\partial w}{\partial z} = 3z^2 = 3(r^2 - s^2 - t^2)$$

$$\frac{\partial w}{\partial z} = 3(1 - 0 - (-1)^2) = 0$$

$$\frac{\partial w}{\partial u} = -2u = -2(r^2 + t^2)$$

$$\frac{\partial w}{\partial u} = -2(1 + 1) = -4$$

$$\frac{\partial w}{\partial v} = -2v = -2(r^2 - s^2)$$

$$\frac{\partial w}{\partial v} = -2(1 - 0) = -2$$

$$\frac{\partial x}{\partial r} = 2r = 2$$

$$\frac{\partial y}{\partial r} = 2r = 2$$

$$\frac{\partial z}{\partial r} = 2r = 2$$

$$\frac{\partial u}{\partial r} = 2r = 2$$

$$\frac{\partial v}{\partial r} = 2r = 2$$

$$\frac{\partial x}{\partial s} = 2s = 0$$

$$\frac{\partial y}{\partial s} = 2s = 0$$

$$\frac{\partial z}{\partial s} = -2s = 0$$

$$\frac{\partial u}{\partial s} = 0$$

$$\frac{\partial v}{\partial s} = 0$$

$$\frac{\partial x}{\partial t} = 2t = -2 \qquad \frac{\partial z}{\partial t} = -2t = 2$$

$$\frac{\partial y}{\partial t} = -2t = 2 \qquad \frac{\partial u}{\partial t} = 2t = -2$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s} + \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial s} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial s}$$

$$\frac{\partial w}{\partial s} = 6.(0) + 0.0 + 0.0 + (-4).0 + (-2)(0)$$

$$\boxed{\frac{\partial w}{\partial s} = 0}$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t} + \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial t}$$

$$\frac{\partial w}{\partial t} = 6(-2) + \cancel{0(2)}^{0} + \cancel{0(2)}^{0} + (-4)(-2) + \cancel{(-2)(0)}^{0}$$

$$\boxed{\frac{\partial w}{\partial t} = -4}$$

Practico #9

1.- En un cierto instante; el radio de la base de un cilindro circular recto es de 10 pulg y la altura de 15 pulg. En ense instante el radio decrese a razón de 5 pulg/sg y la altura crece a razón de 4 pulg/seg. ¿conque rapides cambia el V?

Solusión

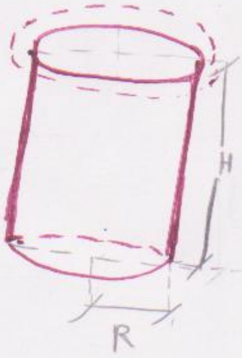
DATOS

$R = 10$ pulg

$H = 15$ pulg

$\frac{\partial R}{\partial t} = -5$ pul/seg

$\frac{\partial H}{\partial t} = 4$ pulg/sg



$$V = \pi R^2 H$$

$$V \begin{cases} R - t \\ H - t \end{cases}$$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial R} \cdot \frac{\partial R}{\partial t} + \frac{\partial V}{\partial H} \cdot \frac{\partial H}{\partial t}$$

$$\frac{\partial V}{\partial t} = 2\pi R H \cdot (-5) + \pi R^2 \cdot (4)$$

$$\frac{\partial V}{\partial t} = 2\pi (10)(15)(-5) + \pi (15)^2 (4)$$

$$\frac{\partial V}{\partial t} = -1500\pi + 900\pi$$

$$\boxed{\frac{\partial V}{\partial t} = -600\pi \text{ pulg}^3/\text{sg}}$$

//

2.- un gas satiface la ley $pv = RT$. En un cierto instante mientras el gas es comprimido; $V = 15 pies^3$, $P = 25 Lb/pulg^2$ "V" decrece a razón de $3 pies^3/min$ y "p" crece a razón de $\frac{2}{3} Lb/pulg^2/min$. Hallar $\frac{dT}{dt}$ respuesta en función de R.

$$T = \frac{P.V}{R}$$

$$\frac{dT}{dt} = \frac{\partial T}{\partial p} \cdot \frac{\partial p}{\partial t} + \frac{\partial T}{\partial v} \cdot \frac{\partial v}{\partial t}$$

$$\frac{dT}{dt} = \frac{V}{R}\left(\frac{2}{3}\right) + \frac{P}{R} \cdot (3)$$

$$\frac{dT}{dt} = \frac{2}{3} \cdot \frac{15}{R} + \frac{25}{R} \cdot 3$$

$$\boxed{\frac{dT}{dt} = \frac{85}{R}}$$

3- De un forma de forma cónica sale a razón de 0,5 $pies^3/min$; el tanque se dilata de manera que mientras permanece cónica, la distancia del vertice a la sup. de agua crece a razón de 0,2 pies/min. ¿con que rapides cambia la altura "h" del agua en el instante en que h=10 y el volumen del agua es de 75 pies³?

DATOS

$\frac{\partial v}{\partial t} = 0{,}5\ pies^3/mint.$

$\frac{\partial r}{\partial t} = 0{,}2\ pies/min$

$\frac{\partial h}{\partial t} = \ ?$

$h = 10$

$V = 75 pies^3$

$$V = \frac{1}{3}\pi r^2 h \Rightarrow 75 \cdot 3 = \pi r^2 \cdot 10$$

$$r^2 = \frac{22{,}5}{\pi}$$

$$\boxed{r = 2{,}68}$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial t} + \frac{\partial v}{\partial h}\cdot\frac{\partial h}{\partial t}$$

$$\frac{\partial v}{\partial t} = \frac{2}{3}\pi h r \cdot (0{,}2) + \frac{\pi r^2}{3}\cdot\frac{\partial h}{\partial t}$$

$$0{,}5 = \frac{2}{3}\cdot\pi\cdot(10)(2{,}68)(0{,}2) + (2{,}68)^2\cdot\frac{\pi}{3}\cdot\frac{\partial h}{\partial t}$$

$$7{,}5\,\frac{\partial h}{\partial t} = 0{,}5 - 11{,}22$$

$$\boxed{\frac{\partial h}{\partial t} = -1{,}43\ pies/min}$$

4.- Dado $f(x,y) = x^2 - y^2 + xy\ln(y/x)$, demostrar que $x \cdot \frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f$

$$f(x,y) = x^2 - y^2 + xy\ln y - xy\ln x$$

$$\frac{\partial f}{\partial x} = 2x + y\ln y - y\ln x - y$$

$$\frac{\partial f}{\partial y} = -2y + x\ln y + x - x\ln x$$

$$x(2x + y\ln y - y\ln x - y) + y(-2y + x\ln y + x - x\ln x) = 2f$$

$$2x^2 + xy\ln y - xy\ln x - \cancel{xy} - 2y^2 + xy\ln y + \cancel{xy} - xy\ln x = 2f$$

$$2x^2 - 2y^2 + 2xy\ln y - 2xy\ln x = 2f$$

$$2\underbrace{\left(x^2 - y^2 + xy\ln\left(\frac{y}{x}\right)\right)}_{f} = 2f$$

$$\boxed{2f = 2f}$$

Práctico Nº 10

- Hallar la ecuación del plano tangente y la de la recta normal.

1.- $z = 3x^2 - y^2 - 2 \; ; \; (-1,2,-3)$

$\frac{\partial z}{\partial x} = 6x = -6$

$\frac{\partial z}{\partial y} = -2y = -4$

EC Del plano tang

$\frac{\partial z}{\partial x}(x - x_0) + \frac{\partial z}{\partial y}(y - y_0) - (z - z_0) = 0$

$-6(x-(-1)) - 4(y-2) - (z-(-3)) = 0$

$-6x - 6 - 4y + 8 - z - 3 = 0$

$\boxed{-6x - 4y - z - 1 = 0}$

EC De la Recta Normal.

$\frac{x - x_0}{\frac{\partial z}{\partial x}} = \frac{y - y_0}{\frac{\partial z}{\partial y}} = \frac{z - z_0}{-1}$

$\boxed{\frac{x+1}{-6} = \frac{y-2}{-4} = \frac{z+3}{-1}}$

2.- $z = xy \; ; \; (2,-1,-2)$

$\frac{\partial z}{\partial x} = y = -1$

$\frac{\partial z}{\partial y} = x = 2$

EC. Del plano tang.

$-1(x-2) + 2(y+1) - (z+2) = 0$

$\boxed{-x + 2y - z + 2 = 0}$

EC. De la Rec. NORMal.

$\boxed{\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z+2}{-1}}$

3.- $z = x^2y^2$; $(-2, 2, 16)$

$$\frac{\partial z}{\partial x} = 2xy^2 = 2(-2)(2)^2 = -16$$

$$\frac{\partial z}{\partial y} = 2x^2y = 2(-2)^2(2) = 16$$

Ec. plano tang

$$-16(x+2) + 16(y-2) - (z-16) = 0$$

$$-16x - 32 + 16y - 32 - z + 16 = 0$$

$$\boxed{-16x + 16y - 48 = 0}$$

Ec. Recta NOR

$$\boxed{\frac{x+2}{-16} = \frac{y-2}{16} = \frac{z-16}{-1}}$$

4.- $z = e^x \cdot \text{sen}\, y$; $(1, \frac{\pi}{2}, e)$

$$\frac{\partial z}{\partial x} = e^x \cdot \text{sen}\, y = e^{(1)} \cdot \text{sen}\left(\frac{\pi}{2}\right) = e$$

$$\frac{\partial z}{\partial y} = e^x \cdot \cos y = e^{(1)} \cdot \cos\left(\frac{\pi}{2}\right) = 0$$

Ec. plano tang

$$e(x-1) + 0\left(y - \frac{\pi}{2}\right) - (z - e) = 0$$

$$\boxed{ex - z = 0}$$

Ec. Rect. NOR.

$$\boxed{\frac{x-1}{e} = \frac{y - \frac{\pi}{2}}{0} = \frac{z-e}{-1}}$$

5.- $z = e^{2x} \cdot \cos 3y \; ; \; (1, \frac{\pi}{3}, -e^2)$

$$\frac{\partial z}{\partial x} = 2e^{2x} \cdot \cos 3y = 2e^2 \cdot \cos(\pi) = -2e^2$$

$$\frac{\partial z}{\partial y} = -3e^{2x} \cdot \operatorname{sen} 3y = -3e^2 \cdot \operatorname{sen}(\pi) = 0$$

Ec. plano tang

$$-2e^2(x-1) + 0(y - \tfrac{\pi}{3}) - (z + e^2) = 0$$

$$\boxed{-2e^2 x - z - e^2 = 0}$$

Ec. Rect. NOR.

$$\boxed{\frac{x-1}{-2e^2} = \frac{y - \frac{\pi}{3}}{0} = \frac{z + e^2}{-1}}$$

6.- $z = \ln\sqrt{x^2+y^2} \; ; \; (-3, 4, \ln 5)$

$$\frac{\partial z}{\partial x} = \frac{x}{x^2+y^2} = \frac{-3}{9+16} = -\frac{3}{25}$$

$$\frac{\partial z}{\partial y} = \frac{y}{x^2+y^2} = \frac{4}{25}$$

Ec. plano tang

$$-\frac{3}{25}(x+3) + \frac{4}{25}(y-4) - (z - \ln 5) = 0$$

$$-3x - 9 + 4y - 16 - 25z - 25\ln 5 = 0$$

$$\boxed{-3x + 4y - 25z - 25 - 25\ln 5 = 0}$$

Ec. Rect. NOR.

$$\frac{x+3}{-\frac{3}{25}} = \frac{y-4}{\frac{4}{25}} = \frac{z - \ln 5}{-1}$$

$$\boxed{\frac{25(x+3)}{-3} = \frac{25(y-4)}{4} = \frac{z - \ln 5}{-1}}$$

7.- $x^2+2y^2+3z^2=6$; $(1,1,-1)$

$\frac{\partial f}{\partial x} = 2x$ $\qquad \frac{\partial z}{\partial x} = -\frac{2x}{6z} = -\frac{1}{-3} = \frac{1}{3}$

$\frac{\partial f}{\partial y} = 4y$ $\qquad \frac{\partial z}{\partial y} = -\frac{4y}{6z} = -\frac{2}{-3} = \frac{2}{3}$

$\frac{\partial f}{\partial z} = 6z$

Ec. plano tang.

$$\frac{1}{3}(x-1) + \frac{2}{3}(y-1) - (z+1) = 0$$

$$x-1+2y-2-3z+3=0$$

$$\boxed{x+2y-3z=0}$$

Ec. Rec.t. NOR.

$$\boxed{\frac{x-1}{\frac{1}{3}} = \frac{y-1}{\frac{2}{3}} = \frac{z+1}{-1}}$$

8.- $x^2+2y^2-3z^2=3$; $(2,1,-1)$

$x^2+2y^2-3z^2-3=0$

$\frac{\partial f}{\partial x} = 2x = 4$ $\qquad \frac{\partial z}{\partial x} = -\frac{4}{6} = -\frac{2}{3}$

$\frac{\partial f}{\partial y} = 4y = 4$ $\qquad \frac{\partial z}{\partial y} = -\frac{4}{6} = -\frac{2}{3}$

$\frac{\partial f}{\partial z} = -6z = 6$

Ec. plano tang

$$-\frac{2}{3}(x-2) - \frac{2}{3}(y-1) - (z+1) = 0$$

$$\boxed{-2x-2y-3z+3=0}$$

Ec Rect. NOR.

$$\boxed{\frac{x-2}{-\frac{2}{3}} = \frac{y-1}{-\frac{2}{3}} = \frac{z+1}{-1}}$$

9 = $x^2+3y^2-z^2=0$; $(2,-2,4)$

$\frac{\partial f}{\partial x} = 2x = 4$

$\frac{\partial f}{\partial y} = 6y = -12$

$\frac{\partial f}{\partial z} = -2z = -8$

$\frac{\partial z}{\partial x} = -\frac{4}{-12} = \frac{1}{3}$

$\frac{\partial z}{\partial y} = -\frac{12}{8} = \frac{3}{2}$

Ec. plano tang

$\frac{1}{3}(x-2) + \frac{3}{2}(y+2) - (z-4) = 0$

$2x-4+9y+18-6z+24=0$

$\boxed{2x+9y-6z+38=0}$

Ec. Rec. NOR.

$\boxed{\frac{x-2}{\frac{1}{3}} = \frac{y+2}{\frac{3}{2}} = \frac{z-4}{-1}}$

10 = $x^2+z^2=25$; $(4,-2,-3)$

$\frac{\partial f}{\partial x} = 2x = 8$

$\frac{\partial f}{\partial y} = 0$

$\frac{\partial f}{\partial z} = 2z$

Ec. plano tang.

$8(x-4) - (z+3) = 0$

$\boxed{8x-z-35=0}$

Ec. Rec. NOR.

$\boxed{\frac{x-4}{8} = \frac{z+3}{-1}}$

11.- $x^2+z^2+y^2=0 \quad (4,-2,-3)$

$\frac{\partial f}{\partial x} = 2x = 8$

$\frac{\partial f}{\partial y} = 2y = -4$

$\frac{\partial f}{\partial z} = 2z = -6$

$\frac{\partial z}{\partial x} = -\frac{8}{-6} = \frac{4}{3}$

$\frac{\partial z}{\partial y} = -\frac{4}{6} = -\frac{2}{3}$

Ec. plano tang.

$\frac{4}{3}(x-4) - \frac{2}{3}(y+2) - (z+3) = 0$

$\boxed{4x-2y-4z-32=0}$

Ec. Rec. NOR. mal

$\boxed{\frac{x-4}{\frac{4}{3}} = \frac{y+2}{-\frac{2}{3}} = \frac{z+3}{-1}}$

12.- $x^{1/2}+y^{1/2}+z^{1/2}=6 \; ; \; (4,1,9)$

$\sqrt{x}+\sqrt{y}+\sqrt{z}-6=0$

$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x}} = \frac{1}{4}$

$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{y}} = \frac{1}{2}$

$\frac{\partial f}{\partial z} = \frac{1}{2\sqrt{z}} = \frac{1}{6}$

$\frac{\partial z}{\partial x} = -\frac{\frac{1}{4}}{\frac{1}{6}} = -\frac{3}{2}$

$\frac{\partial z}{\partial y} = -\frac{\frac{1}{2}}{\frac{1}{6}} = -3$

Ec. Plano tang.

$-\frac{3}{2}(x-4) - 3(y-1) - (z-9) = 0$

$\boxed{-3x-6y-2z+36=0}$

Ec. Rec. NOR.

$\boxed{\frac{x-4}{-\frac{3}{2}} = \frac{y-1}{-3} = \frac{z-9}{-1}}$

14.- $y^{1/2} + z^{1/2} = 7$; $(3, 16, 9)$

$$y^{1/2} + z^{1/2} - 7 = 0$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{y}} \qquad \frac{\partial z}{\partial y} = -\frac{\frac{1}{2\sqrt{y}}}{\frac{1}{2\sqrt{z}}}$$

$$\frac{\partial f}{\partial z} = \frac{1}{2\sqrt{z}} \qquad \frac{\partial z}{\partial y} = -\frac{\sqrt{z}}{\sqrt{y}}$$

$$\frac{\partial z}{\partial y} = -\frac{3}{4}$$

Ec. plano tang.

$$-\frac{3}{4}(y-16) - (z-9) = 0$$

$$\boxed{-3y - 4z + 84 = 0}$$

Ec. Rect. NOR.

$$\boxed{\frac{y-16}{-\frac{3}{4}} = \frac{z-9}{-1}}$$

Práctico nº 11

* Hallar de $f(x,y)$ en el punto dado

1.- $f(x,y) = x^2 + y^2$; $(3,4)$

$$\frac{\partial f}{\partial x} = 2x = 6 \qquad \text{tang}\,\theta = \frac{8}{6}$$

$$\frac{\partial f}{\partial y} = 2y = 8 \qquad \theta = 48°48'$$

2.- $f(x,y) = x^3 + y^3 - 3x^2y - 3xy^2$; $(1,-2)$

$$\frac{\partial f}{\partial x} = 3x^2 - 6xy - 3y^2 = 3 - 6(-2) - 3(-2)^2 = 3$$

$$\frac{\partial f}{\partial y} = 3y^2 - 3x^2 - 6xy = 3(2)^2 - 3(1) - 6(1)(-2) = 21$$

$$\text{tang}\,\theta = \left(\frac{21}{3}\right)$$

$$\theta = 81°52'$$

3.- $f(x,y) = \text{arctang}\,(y/x)$; $(4,3)$

$$\frac{\partial f}{\partial x} = \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2} = \frac{-\frac{3}{16}}{1+\left(\frac{3}{4}\right)^2} = -\frac{0{,}1875}{1{,}56} = -0{,}12$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{\frac{1}{x}}{\frac{x^2+y^2}{x^2}} = \frac{x}{x^2+y^2} = \frac{4}{25} = 0{,}16$$

$$\text{tang}\,\theta = \frac{0{,}16}{0{,}12}$$

$$\theta = 53°7'$$

4.- $f(x,y) = \operatorname{sen} xy \,;\ (2, \pi/4)$

$$\frac{\partial f}{\partial x} = y\cos xy = \frac{\pi}{4}\cdot\cos\left(\frac{\pi}{2}\right) = 0 \qquad \operatorname{tang}\theta = 0°$$

$$\frac{\partial f}{\partial y} = x.\cos xy = 2.\cos\left(\frac{\pi}{2}\right) = 0$$

\#

5.- $f(x,y) = e^{x}.\cos y\,;\ (0, \frac{\pi}{3})$

$$\frac{\partial f}{\partial x} = e^{x}.\cos y = e^{0}.\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \qquad \operatorname{tang}\theta = \frac{1}{\sqrt{3}}$$

$$\frac{\partial f}{\partial y} = -e^{x}.\operatorname{sen} y = -e^{0}.\operatorname{sen}\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \qquad \theta = 30°$$

6.- $f(x,y) = (\operatorname{sen} x)^{xy}\,;\ (\pi/2, 0)$

$$\frac{\partial f}{\partial x} = (\operatorname{sen} x)^{xy}\left[y\ln(\operatorname{sen} x) + \frac{xy}{\operatorname{sen} x}.\cos x\right]$$

$$\frac{\partial f}{\partial x} = 1\left[0.\ln(1) + 0.\right] = 0$$

$$\frac{\partial f}{\partial y} = (\operatorname{sen} x)^{xy}\ln(\operatorname{sen} x).x = 1^{0}.\cancel{\ln(1)}^{0}. = 0$$

$$\operatorname{tang}\theta = 0$$

$$\boxed{\theta = 0°}$$

\#

7 - $f(x,y) = x^2+y^2-2x+3y$; $(2,-1)$

$\frac{\partial f}{\partial x} = 2x-2 = 2$ $\qquad$ $\tan g\,\theta = \frac{1}{2}$

$\frac{\partial f}{\partial y} = 2y+3 = 1$ $\qquad$ $\theta = 26°33'$

$d_\theta f(x,y) = 2.\cos\theta + (1).\operatorname{sen}\theta$ #

8 - $f(x,y) = \operatorname{arctang}(x/y)$; $(3,4)$

$\frac{\partial f}{\partial x} = \frac{\frac{1}{y}}{1+\left(\frac{x}{y}\right)^2} = \frac{y}{y^2+x^2} = \frac{4}{25}$ $\qquad$ $\tan g\,\theta = \frac{3}{4}$

$\theta = 36°52'$

$\frac{\partial f}{\partial y} = \frac{-\frac{x}{y^2}}{1+\left(\frac{x}{y}\right)^2} = -\frac{x}{y^2+x^2} = -\frac{3}{25}$

$d_\theta f(3,4) = \frac{4}{25}.\cos\theta - \frac{3}{25}.\operatorname{sen}\theta$

9 - $f(x,y) = e^x.\operatorname{sen} y$, $(0, \pi/2)$

$\frac{\partial f}{\partial x} = e^x.\operatorname{sen} y = e^0.\operatorname{sen}\left(\frac{\pi}{2}\right) = 1$ $\qquad$ $\tan g\,\theta = \left(\frac{0}{1}\right)$

$\frac{\partial f}{\partial y} = e^x.\cos y = e^0 \cos\left(\frac{\pi}{2}\right) = 0$ $\qquad$ $\theta = 0°$

$d_\theta f(0, \frac{\pi}{2}) = \cos\theta$ #

10 - $f(x,y) = (\operatorname{sen} y)^{xy}$; $(0, \pi/2)$

$\frac{\partial f}{\partial x} = (\operatorname{sen} y)^{xy}. \ln(\operatorname{sen} y).y = 1.\ln(0) = 0$ $\qquad$ $\tan g\,\theta = 0$

$\frac{\partial f}{\partial y} = (\operatorname{sen} y)^{xy}\left[xy.\ln(\operatorname{sen} y) + \frac{xy}{\operatorname{sen} y}.\cos y\right] = 0$ $\qquad$ $\theta = 0°$

$d_\theta f(0, \frac{\pi}{2}) = 0$ #

11 : $f(x,y,z) = x^2 + xy - xz + y^2 - z^2;\ (2,1,-2)$

$$\frac{\partial f}{\partial x} = 2x + y - z = 4 + 1 + 2 = 7$$

$$\frac{\partial f}{\partial y} = x + 2y = 2 + 2 = 4$$

$$\frac{\partial f}{\partial z} = -x - 2z = -2 - 2(-2) = 2$$

$$D_\theta f = 7.\cos\theta\, \hat{i} + 4.\operatorname{sen}\theta\, \hat{j} + 2\cos\theta\, \hat{k}$$

12 : $f(x,y,z) = x^2 y + xze^y - xye^z;\ (-2,3,0)$

$$\frac{\partial f}{\partial x} = 2xy + ze^y - ye^z = 2(-2)(3) + 0 - 3e^0 = -15$$

$$\frac{\partial f}{\partial y} = x^2 + xze^y - xe^z = 4 - 0 + 2 = 6$$

$$\frac{\partial f}{\partial z} = xe^y - xye^z = -2e^3 + 2.3\, e^0 = -2e^3 + 6$$

$$D_\theta f = -15\cos\theta\, \hat{i} + 6.\operatorname{sen}\theta\, \hat{j} - 34{,}17.\cos\theta\, \hat{k}$$

13 : $f(x,y,z) = \cos xy + \operatorname{sen} xz;\ (0,2,-1)$

$$\frac{\partial f}{\partial x} = -\operatorname{sen} xy \cdot y + z\cos xz = -1$$

$$\frac{\partial f}{\partial y} = -x.\operatorname{sen} xy = 0$$

$$D_\theta f = -1\cos\theta\, \hat{i}$$

$$\frac{\partial f}{\partial z} = x\cos xz = 0$$

14 : $f(x,y,z) = \ln(x+y+z) - xyz;\ (-1,2,1)$

$$\frac{\partial f}{\partial x} = \frac{1}{x+y+z} - yz = \frac{1}{2} - 2 = -\frac{3}{2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x+y+z} - xz = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\frac{\partial f}{\partial z} = \frac{1}{x+y+z} - xy = \frac{1}{2} + 2 = \frac{5}{2}$$

$$\boxed{D_\theta f = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + \frac{5}{2}\hat{k}}$$

Practico nº 12

* comprobar. $f_{1,2} = f_{2,1}$

1 - $f(x,y) = x^2 - 2xy - 3y^2$

$$\frac{\partial f}{\partial x} = 2x - 2y \qquad \frac{\partial f}{\partial y} = -2x - 6y.$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2 \qquad \frac{\partial^2 f}{\partial y \partial x} = -2$$

$$\boxed{-2 = -2}$$

2 - $f(x,y) = x^3 - x^2 y + 2xy^2$

$$\frac{\partial f}{\partial x} = 3x^2 - 2xy + 2y^2 \qquad \frac{\partial f}{\partial y} = -x^2 + 4xy$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2x + 4y \qquad \frac{\partial^2 f}{\partial y \partial x} = -2x + 4y$$

3 - $f(x,y) = e^{rs} \cdot \operatorname{sen} r \cdot \cos(s)$

$$\frac{\partial f}{\partial r} = s e^{rs} \cdot \operatorname{sen}(r) \cdot \cos(s) + e^{rs} \cdot \operatorname{sen}(r) \cos(s)$$

$$\frac{\partial^2 f}{\partial r \cdot \partial s} = e^{rs} \operatorname{sen}(r) \cdot \cos(s) + rse^{rs} \operatorname{sen}(r) \cdot \cos(s) - se^{rs} \operatorname{sen}(r) \operatorname{sen}(s) + re^{rs} \operatorname{sen} r \cdot \cos$$

$$\frac{\partial f}{\partial s} = re^{rs} \operatorname{sen} r \cdot \cos(s) - e^{rs} \cdot \operatorname{sen}(r) \cdot \operatorname{sen}(s)$$

$$\frac{\partial f}{\partial s \partial r} = e^{rs} \operatorname{sen} r \cos(s) + sre^{rs} \operatorname{sen} r \cdot \cos(s) - re^{rs} \cdot \cos r \cdot \cos(s) - se^{rs} \operatorname{sen}(r) \operatorname{sen}(s) - e^{rs} \cos($$

4 - $f(x,y) = \operatorname{arctang}(t/s)$

$$\frac{\partial f}{\partial t} = \frac{\frac{1}{s}}{1 + \left(\frac{t}{s}\right)^2} = \frac{s}{s^2 + t^2} \qquad \frac{\partial f}{\partial s} = \frac{-\frac{t}{s^2}}{1 + \left(\frac{t}{s}\right)^2} = -\frac{t}{s^2 + t^2}$$

$$\frac{\partial^2 f}{\partial t \partial s} = \frac{s^2 + t^2 - s(2s)}{(s^2 + t^2)^2} \qquad \frac{\partial^2 f}{\partial s \partial t} = \frac{-(s^2 + t^2) + t(2t)}{(s^2 + t^2)^2}$$

5: $u = \ln\sqrt{x^2+y^2+z^2}$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{-2xy}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{8xyz}{(x^2+y^2+z^2)^3}$$

$$\frac{\partial u}{\partial z} = \frac{z}{x^2+y^2+z^2}$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{-2zx}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^3 u}{\partial z \partial x \partial y} = \frac{8xyz}{(x^2+y^2+z^2)^3}$$

6: $u = x^3+y^3+z^3-3xyz$

$$\frac{\partial u}{\partial x} = 3x^2-3yz$$

$$\frac{\partial^2 u}{\partial x \partial y} = -3z$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = -3$$

$$\frac{\partial u}{\partial z} = 3z-3xy$$

$$\frac{\partial^2 u}{\partial z \partial y} = -3x$$

$$\frac{\partial^3 u}{\partial z \partial y \partial x} = -3$$

#

7: $u = \dfrac{e^{xy}}{\sqrt{x^2+z^2}}$

$$\frac{\partial u}{\partial x} = \frac{y e^{xy}\sqrt{x^2+z^2} - e^{xy}\frac{x}{\sqrt{x^2+z^2}}}{x^2+z^2}$$

$$\frac{\partial u}{\partial x} = \frac{\frac{y e^{xy}(x^2+z^2) - x e^{xy}}{\sqrt{x^2+z^2}}}{\left(\sqrt{x^2+z^2}\right)^2} = \frac{x^2 y e^{xy} + y z^2 e^{xy} - x e^{xy}}{\sqrt{x^2+z^2}}$$

$$\frac{\partial u}{\partial x \partial y} = \frac{x^2 e^{xy} + x^3 y e^{xy} + z^2 e^{xy} + yxz^2 e^{xy} - x^2 e^{xy}}{\sqrt{x^2+z^2}}$$

Dado $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ comprobar. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

$$\frac{\partial u}{\partial x} = \frac{-1}{\left(\sqrt{x^2+y^2+z^2}\right)^2} \cdot \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial u}{\partial x} = \frac{-x}{\sqrt{(x^2+y^2+z^2)^3}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{-\left(\sqrt{x^2+y^2+z^2}\right)^3 + \frac{3x}{\cancel{2}} \sqrt{x^2+y^2+z^2} \cdot \cancel{2}x}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{3x^2\sqrt{x^2+y^2+z^2} - \left(\sqrt{x^2+y^2+z^2}\right)^3}{(x^2+y^2+z^2)^2} \#$$

$$\frac{\partial u}{\partial y} = \frac{-y}{\sqrt{(x^2+y^2+z^2)^3}}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{-\sqrt{(x^2+y^2+z^2)^3} - \frac{3y}{\cancel{2}} \sqrt{x^2+y^2+z^2} \cdot \cancel{2}y}{(x^2+y^2+z^2)^2}.$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{3y^2\sqrt{x^2+y^2+z^2} - \left(\sqrt{x^2+y^2+z^2}\right)^3}{(x^2+y^2+z^2)^2} \#$$

$$\frac{\partial u}{\partial z} = \frac{-z}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{3z^2\sqrt{x^2+y^2+z^2} - \left(\sqrt{x^2+y^2+z^2}\right)^3}{(x^2+y^2+z^2)^2} \#$$

Demostrando

$$3x^2 + 3y^2 + 3z^2 - 3(x^2+y^2+z^2) = 0$$

$$\boxed{0 = 0}$$

\#

Practico $n^{\underline{o}}$ 13

1.- $f(x,y) = x^2 - xy + 2y^2$; $x = 2$; $h = -0{,}01$
$y = -1$; $k = 0{,}02$

$$df = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\cdot\Delta y$$

$$df = (2x-y).h + (-x+4y)k$$

$$df = 5.(-0{,}01) - 6(0{,}02)$$

$$\boxed{df = -0{,}07}$$

2.- $f(x,y) = 2x^2 + 3xy - y^2$; $x = 1$; $h = -0{,}02$
$y = 2$; $k = -0{,}01$

$$df = (4x+3y)h + (3x-2y)k$$

$$df = 10(-0{,}02) + 1(0{,}01)$$

$$\boxed{df = -0{,}19}$$

3.- $f(x,y) = \operatorname{sen} xy + \cos(x+y)$; $x = \pi/6$ $h = 2\pi$
$y = 0$ $k = 3\pi$

$$df = [y\cos xy - \operatorname{sen}(x+y)]h + [x.\operatorname{sen} xy - \operatorname{sen}(x+y)]k$$

$$df = -\operatorname{sen}\left(\frac{\pi}{6}\right)\cdot 2\pi - \operatorname{sen}\left(\frac{\pi}{6}\right)\cdot 3\pi$$

$$df = -\pi - \frac{3}{2}\pi = -\frac{5}{2}\pi$$

5.- $f(x,y) = x^3 - 3xy + y^3$; $x = -2$ $h = -0,03$
$y = 1$ $k = -0,02$

$df = (3x^2 - 3y)\,h + (-3x + 3y^2)\,k$

$df = 9.(-0,03) + 3(-0,02)$

$\boxed{df = -0,33}$ //

6.- $f(x,y) = x^2y - 2xy^2 + 3x$; $x = 1$ $h = 0,02$
$y = 1$ $k = 0,01$

$df = (2xy - 2y^2 + 3)\,h + (x^2 - 4xy)\,k$

$df = 3(0,02) - 3(0,01)$

$\boxed{df = 0,03}$ //

7.- $f(x,y,z) = x^2 - 2y^2 + z - xz = (2,-1,3)$; $h = 0,01$
$k = -0,02$
$L = 0,03$

$df = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\cdot\Delta z$

$df = (2x - z)\,h + (-4y)\,k + (1-x)\,L$

$df = 1(0,01) + 4(-0,02) - 0,03$

$df = -0,1$ //

8.- $f(x,y,z) = xy - xz + yz + 2x - 3y + 1$; $(x,y,z) = (2,0,-3)$

$(h,k,L) = (0{,}1, 0{,}2, 0{,}1)$

$df = (y-z+2)h + (x+z-3)k + (-x+y).L$

$df = 5.(0{,}1) - 4(0{,}2) - 2(0{,}1)$

$df = -0{,}5$ #

9.- $f(x,y,z) = x^2y - xyz + z^3$; $(x,y,z) = (1,2,-1)$; $(h,k,L) = (-0{,}02, 0{,}01, 0{,}02)$

$df = (2xy - yz)h + (x^2 - yz)k + (-xy + 3z^2)L$

$df = 6(-0{,}02) + 3(0{,}01) + 1(0{,}02)$

$df = -0{,}07$ #

10.- $f(x,y,z) = \operatorname{sen}(x+y) - \cos(x-z) + \operatorname{sen}(y+2z)$; $x = \pi/3$ $y = \pi/6$ $z = 0$ $h = \pi/4$ $k = \pi/2$ $L = 2\pi$

$df = [\cos(x+y) + \operatorname{sen}(x-z)]h + [\cos(x+y) + \cos(y+2z)]k + [\operatorname{sen}(x-z) + 2\operatorname{sen}(y+2z)]L$

$df = \left(0 + \frac{\sqrt{3}}{2}\right)\frac{\pi}{4} + \left(0 + \frac{\sqrt{3}}{2}\right)\frac{\pi}{2} + \left(-\frac{\sqrt{3}}{2} + 1\right)2\pi$

$df = \frac{\sqrt{3}.\pi}{8} + \frac{\sqrt{3}}{4}\pi - \sqrt{3}\pi + 2\pi$

$df = 2\pi - \frac{5}{8}\sqrt{3}.\pi$ #

Practico n° 14

* Hallar $\frac{dy}{dx}$

1.- $x^2+3xy-4y+2x-6y+7=0$

$$\frac{dy}{dx}=\frac{2x+3y+2x}{3x-4-6}$$

2.- $x^3+3x^2y-4xy^2+y^3-x^2+2y-1=0$

$$\frac{dy}{dx}=\frac{3x^2+6xy-4y^2-2x}{3x^2-8xy+3y^2+2y}$$

3.- $\ln(1+x^2+y^2)+e^{xy}=5$

$\ln(1+x^2+y^2)+e^{xy}-5=0$

$$\frac{dy}{dx}=\frac{\frac{2x}{1+x^2+y^2}+ye^{xy}}{\frac{2y}{1+x^2+y^2}+xe^{xy}} \#$$

4.- $x^3\cdot x-3x^2y^2+y^4-x^2y+2xy^2=3$

$x^4-3x^2y^2+y^4-x^2y+2xy^2-3=0$

$$\frac{dy}{dx}=\frac{4x^3-6xy^2-2xy+2y^2}{-6x^2y+4y^3-x^2+2y^2} \#$$

5.- $e^{xy}+\operatorname{sen} xy+1=0$

$$\frac{dy}{dx}=\frac{ye^{xy}+y\cos xy}{xe^{xy}+x\cdot\cos xy}$$

6 : $xe^y + ye^x + \text{sen}(x+y) - 2 = 0$

$$\frac{dy}{dx} = \frac{e^y + ye^x + \cos(x+y)}{xe^y + e^x + \cos(x+y)} \quad \#$$

7 : $\arctan(y/x) + (x^2+y^2)^{3/2} = 2$

$$\arctan\left(\frac{y}{x}\right) + (x^2+y^2)^{3/2} - 2 = 0$$

$$\frac{\partial f}{\partial x} = \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2} + \frac{3}{\cancel{2}}\sqrt{x^2+y^2} \cdot \cancel{2}x$$

$$\frac{\partial f}{\partial x} = \frac{-y}{x^2+y^2} + 3x\sqrt{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2} + 3y\sqrt{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{x}{x^2+y^2} + 3y\sqrt{x^2+y^2}$$

$$\frac{dy}{dx} = \frac{3x\sqrt{x^2+y^2} - \frac{y}{x^2+y^2}}{3y\sqrt{x^2+y^2} + \frac{x}{x^2+y^2}} \quad \#$$

8.- $x^2+y^2+w^2-3xy-4=0 \quad \frac{\partial w}{\partial y}$

$$\frac{\partial w}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial w}}$$

$$\frac{\partial f}{\partial y} = 2y-3x \qquad \frac{\partial w}{\partial y} = \frac{2y-3x}{2w} \#$$

$$\frac{\partial f}{\partial z} = 2w$$

9.- $x^3+3x^2w-y^2w+2yw^2-3w+2x=8 \quad \frac{\partial w}{\partial x}$

$$x^3+3x^2w-y^2w+2yw^2-3w+2x-8=0$$

$$\frac{\partial w}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial w}}$$

$$\frac{\partial f}{\partial x} = 3x^2+6xw+2 \qquad \frac{\partial w}{\partial x} = -\frac{3x^2+6xw+2}{3x^2-y^2+4yw-3} \#$$

$$\frac{\partial f}{\partial w} = 3x^2-y^2+4yw-3$$

10.- $e^{xy}+e^{yw}-e^{xw}+xyw=4 \ ; \quad \frac{\partial w}{\partial y}$

$$\frac{\partial f}{\partial y} = xe^{xy}+we^{yw}+xw$$

$$\frac{\partial f}{\partial w} = ye^{yw}-xe^{xw}+xy \qquad \frac{\partial w}{\partial y} = -\frac{xe^{xy}+we^{yw}+xw}{ye^{yw}-xe^{xw}+xy} \#$$

11.- $\text{sen}(xyw)+x^2+y^2+w^2=3 \ ; \quad \frac{\partial w}{\partial x}$

$$\text{sen}(xyw)+x^2+y^2+w^2-3=0$$

$$\frac{\partial f}{\partial x} = yw.\cos(xyw)+2x$$

$$\frac{\partial f}{\partial w} = xy.\cos(xyw)+2w$$

$$\frac{\partial w}{\partial y} = -\frac{yw.\cos(xyw)+2x}{xy.\cos(xyw+2w} \#$$

12 - $(w^2-y^2)(w^2+x^2)(x^2-y^2)=1$ $\frac{\partial w}{\partial y}$

$(w^2-y^2)(w^2+x^2)(x^2-y^2)-1=0$

$$\frac{\partial f}{\partial x}= 2x(w^2-y^2)(x^2-y^2)+2x(w^2-y^2)(w^2+x^2)$$

$$\frac{\partial f}{\partial y}= -2y(w^2+x^2)(x^2-y^2)-2y(w^2-y^2)(w^2+x^2)$$

$$\frac{\partial f}{\partial w}= 2w(w^2+x^2)(x^2-y^2)+2w(w^2-y^2)(x^2-y^2)$$

$$\frac{\partial w}{\partial y}=-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial w}}$$

$$\frac{\partial w}{\partial y}=\frac{2y(w^2+x^2)(x^2-y^2)+2y(w^2-y^2)(w^2+x^2)}{2w(w^2+x^2)(x^2-y^2)+2w(w^2-y^2)(x^2-y^2)}$$ #

Practico $N^{\underline{o}}_{11}$ 15

* Diferencial exacta.

1.- $\underbrace{(x^3+3x^2y)}_{P}dx+\underbrace{(x^3+y^3)}_{Q}dy$

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}\Rightarrow \text{exacta}$$

$$\left.\begin{aligned}\frac{\partial P}{\partial y}&=3x^2\\ \frac{\partial Q}{\partial x}&=3x^2\end{aligned}\right]=\text{exacta}$$

$$\int(x^3+3x^2y)\,dx+\int(x^3+y^3)\,dy=0$$

$$f(x,y)=\frac{x^4}{4}+x^3y+x^3y+\frac{y^4}{4}+c$$

$$\boxed{f(x,y)=\frac{x^4}{4}+x^3y+\frac{y^4}{4}+c}$$

//

2.- $\underbrace{(2x+3y)}_{P}dx+\underbrace{(3x+2y)}_{Q}dy$

$$\left.\begin{aligned}\frac{\partial P}{\partial y}&=3\\ \frac{\partial Q}{\partial x}&=3\end{aligned}\right]=\text{exacta}$$

$$\int(2x+3y)\,dx+\int(3x+2y)\,dy=0$$

$$f(x,y)=x^2+3xy+3xy+y+c$$

$$\boxed{f(x,y)=x^2+3xy+y+c}$$

//

3.- $\underbrace{\left(2y - \frac{1}{x}\right)}_{P} dx + \underbrace{\left(2x + \frac{1}{y}\right)}_{Q} dy$

$$\left.\begin{array}{l} \frac{\partial P}{\partial y} = 2 \\ \frac{\partial Q}{\partial x} = 2 \end{array}\right] = \qquad \int \left(2y - \frac{1}{x}\right) dx + \int \left(2x + \frac{1}{y}\right) dy = 0$$

$$f(x,y) = 2xy - \ln x + 2xy + \ln y + c$$

$$\boxed{f(x,y) = 2xy - \ln x + \ln y + c}$$

4.- $\underbrace{(x^2 + 2xy)}_{P} dx + \underbrace{(y^3 - x^2)}_{Q} dy$

$$\left.\begin{array}{l} \frac{\partial P}{\partial y} = 2x \\ \frac{\partial Q}{\partial x} = -2x \end{array}\right] \neq \text{NO es Exacta.}$$

5.- $\underbrace{x^2 \cdot \operatorname{sen} y}_{P} dx + \underbrace{x^2 \cdot \cos y}_{Q} dy$

$$\left.\begin{array}{l} \frac{\partial P}{\partial y} = x^2 \cdot \cos y \\ \frac{\partial Q}{\partial x} = 2x \cdot \cos y \end{array}\right] \neq \text{no es exacta.}$$

6.- $\underbrace{\frac{x^2+y^2}{2y^2}}_{P}dx - \underbrace{\frac{x^3}{3y^3}}_{Q}dy$

$$\frac{\partial P}{\partial y} = \frac{2y(2y^2)-(x^2+y^2)(4y)}{4y^4}$$

$$\frac{\partial P}{\partial y} = \frac{4\cancel{y^3}-\cancel{4}x^2\cancel{y}-4\cancel{y^3}}{\cancel{4}y^{\cancel{4}}}$$

$$\frac{\partial P}{\partial y} = -\frac{x^2}{y^3}$$

$$\frac{\partial Q}{\partial x} = -\frac{\cancel{3}x^2}{\cancel{3}y^3} = -\frac{x^2}{y^3}$$

= exacta

$$\int \frac{x^2+y^2}{2y^2}dx - \int \frac{x^3}{3y^3}dy = 0$$

$$f(x,y) = \frac{x^3}{6y^2} + \frac{x}{2} + \frac{x^3}{6y^2} + c$$

$$\boxed{f(x,y) = \frac{x}{2} + \frac{x^3}{6y^2} + c}$$

7.- $\underbrace{2xe^{x^2}\operatorname{sen}y\,dx}_{P} + \underbrace{e^{x^2}\cos y\,dy}_{Q}$

$$\frac{\partial P}{\partial y} = 2xe^{x^2}\cdot\cos y$$

$$\frac{\partial Q}{\partial x} = 2xe^{x^2}\cdot\cos y$$

= exacta

$$\int 2xe^{x^2}\operatorname{sen}y\,dx + \int e^{x^2}\cdot\cos y\,dy = 0$$

$$f(x,y) = e^{x^2}\cdot\operatorname{sen}y + e^{x^2}\cdot\operatorname{sen}y + c$$

$$\boxed{f(x,y) = e^{x^2}\cdot\operatorname{sen}y + c}$$

#

$10 :- \underbrace{(2x * \ln y)}_{P}\, dx + \underbrace{\frac{x^2}{y}}_{Q}\, dy$

$$\left.\begin{array}{l}\frac{\partial P}{\partial y} = \frac{2x}{y} \\ \frac{\partial Q}{\partial x} = \frac{2x}{y}\end{array}\right] = \qquad \int 2x.\ln y\, dx + \int \frac{x^2}{y}\, dy = 0$$

$$f(x,y) = x^2.\ln y + x^2 \ln y + c$$

$$\boxed{f(x,y) = x^2.\ln y + c}$$

$11 :- \underbrace{(x + \cos x.\tan g y)}_{P}\, dx + \underbrace{(y + \tan g x.\cos y)}_{Q}\, dy$

$$\left.\begin{array}{l}\frac{\partial P}{\partial y} = \cos x.\sec^2 y. \\ \frac{\partial Q}{\partial x} = \sec^2 x.\cos y\end{array}\right] \neq \text{no es exacta}$$

$12 :- \underbrace{\frac{1}{y} \cdot e^{2x/y}}_{P}\, dx - \underbrace{\frac{1}{y^3} e^{2x/y}(y+2x)}_{Q}\, dy$

$$\frac{\partial P}{\partial y} = -\frac{e^{2x/y}}{y^2} - \frac{2x}{y^3}.e^{2x/y} \qquad \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$$

$$\frac{\partial Q}{\partial x} = -\frac{2x}{y^4} e^{2x/y}(y+2x) - \frac{e^{2x/y}}{y^3}.2$$

$$\frac{\partial Q}{\partial x} = -\frac{2x}{y^3} e^{2x/y} - \frac{4x}{y^4} e^{2x/y} - \frac{2e^{2x/y}}{y^3}$$

13.- $\underbrace{(3x^2 \cdot \ln y - x^3)}_{P}\,dx + \underbrace{\frac{3x^2}{y}}_{Q}\,dy$

$$\left.\begin{aligned}\frac{\partial P}{\partial y} &= \frac{3x^2}{y} \\ \frac{\partial Q}{\partial x} &= \frac{6x}{y}\end{aligned}\right] \neq \text{ no es exacta}$$

14.- $\underbrace{\frac{x\,dx}{\sqrt{x^2+y^2}}}_{P} + \underbrace{\left(\frac{y}{\sqrt{x^2+y^2}} - 2\right)}_{Q} dy$

$$\frac{\partial P}{\partial y} = \frac{-x}{(\sqrt{x^2+y^2})^2} \cdot \frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial Q}{\partial x} = \frac{-y}{(\sqrt{x^2+y^2})^2} \cdot \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial P}{\partial y} = -\frac{xy}{(\sqrt{x^2+y^2})^3}$$

$$\frac{\partial Q}{\partial x} = -\frac{xy}{(\sqrt{x^2+y^2})^3}$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$\int \frac{x\,dx}{\sqrt{x^2+y^2}} + \int\left(\frac{y}{\sqrt{x^2+y^2}} - 2\right) dy = 0$$

$$f(x,y) = \sqrt{x^2+y^2} + \sqrt{x^2+y^2} - 2y + C$$

$$\boxed{f(x,y) = \sqrt{x^2+y^2} - 2y + c}$$

15.- $\underbrace{(2x+y+3z)}_{P}dx + \underbrace{(3y+2z-x)}_{Q}dy + \underbrace{(2x+3y-z)}_{R}dz$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ $\quad \left.\begin{array}{l}\frac{\partial P}{\partial y} = 1 \\ \frac{\partial Q}{\partial x} = -1\end{array}\right] \neq$ no es Exacta

$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$

$\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$

16.- $\underbrace{(2xy+z^2)}_{P}dx + \underbrace{(2yz+x^2)}_{Q}dy + \underbrace{(2xz+y^2)}_{R}dz$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ $\quad \frac{\partial P}{\partial y} = 2x \quad \frac{\partial P}{\partial z} = 2z \quad \frac{\partial Q}{\partial z} = 2y$

$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$ $\quad \frac{\partial Q}{\partial x} = 2x \quad \frac{\partial R}{\partial x} = 2z \quad \frac{\partial R}{\partial y} = 2y$

$\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$

Exacta

$$\int(2xy+z^2)dx + \int(2yz+x^2)dy + \int(2xz+y^2)dz = 0$$

$$f(x,y,z) = x^2y + z^2x + y^2z + x^2y + xz^2 + y^2z + c$$

$$\boxed{f(x,y,z) = x^2y + y^2z + xz^2 + c}$$

#

17.- $\underbrace{(e^x \cdot \operatorname{sen} y \cos z)}_{P}dx + \underbrace{(e^x \cdot \cos y \cdot \cos z)}_{Q}dy - \underbrace{(e^x \cdot \operatorname{sen} y \cdot \operatorname{sen} z)}_{R}dz$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad \frac{\partial P}{\partial y} = e^x \cdot \cos y \cdot \cos z \qquad \frac{\partial P}{\partial z} = -e^x \cdot \operatorname{sen} y \cdot \operatorname{sen} z$$

$$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \qquad \frac{\partial Q}{\partial x} = e^x \cdot \cos y \cdot \cos z \qquad \frac{\partial R}{\partial x} = -e^x \cdot \operatorname{sen} y \cdot \operatorname{sen} z$$

$$\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y} \qquad \frac{\partial Q}{\partial z} = -e^x \cos y \cdot \operatorname{sen} z$$

Exacta

$$\frac{\partial R}{\partial y} = -e^x \cos y \cdot \operatorname{sen} z$$

$$\int (e^x \cdot \operatorname{sen} y \cdot \cos z)\,dx + \int (e^x \cdot \cos y \cdot \cos z)\,dy - \int (e^x \cdot \operatorname{sen} y \cdot \operatorname{sen} z)\,dz = 0$$

$$f(x,y,z) = e^x \cdot \operatorname{sen} y \cos z + e^x \cdot \operatorname{sen} y \cdot \cos z + e^x \cdot \operatorname{sen} y \cos z + c$$

$$\boxed{f(x,y,z) = e^x \cdot \operatorname{sen} y \cdot \cos z + c}$$

18.- $\underbrace{\left(\frac{1}{y^2} - \frac{y}{x^2 z} - \frac{z}{x^2 y}\right)}_{P}dx + \underbrace{\left(\frac{1}{xz} - \frac{x}{y^2 z} - \frac{z}{xy^2}\right)}_{Q}dy + \underbrace{\left(\frac{1}{xy} - \frac{x}{yz^2} - \frac{y}{xz^2}\right)}_{R}dx$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad \frac{\partial P}{\partial y} = -\frac{2}{y^3} - \frac{1}{x^2 z} + \frac{z}{x^2 y^2}$$

$$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \qquad \frac{\partial Q}{\partial x} = -\frac{1}{zx^2} - \frac{1}{y^2 z} + \frac{z}{y^2 x^2}$$

no es exacta

Practico n° 16

* maximos y minimos relativos.

1.- $f(x,y) = x^2+2y^2-4x+4y-3$

$\frac{\partial f}{\partial x} = 2x-4 = 0 \Rightarrow \boxed{x=2}$ $\quad P(2,1)$

$\frac{\partial f}{\partial y} = 4y-4 = 0 \Rightarrow \boxed{y=1}$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\quad \Delta = (2).(4) - (0)^2$

$\frac{\partial^2 f}{\partial y^2} = 4$ $\quad \boxed{\Delta = 8}$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

como

$\frac{\partial^2 f}{\partial x^2} > 0 \wedge \Delta > 0$ hay un minimo en $P(2,1)$

$f_{mimo}(2,1) = 4-2-8+4-3$

$\boxed{f_{mim} = -5}$

2.- $f(x,y) = x^2+2xy+3y^2+2x+10y+9$

$\frac{\partial f}{\partial x} = 2x+2y+2 = 0$ $\quad 2x+2y=-1$

$\frac{\partial f}{\partial y} = 2x+6y+10 = 0$ $\quad 2x+6y=-10 \quad /(-1)$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\quad \frac{\partial^2 f}{\partial x \partial y} = 2$

$\frac{\partial^2 f}{\partial y^2} = 6$

$\cancel{-2x}-2y = 1$

$\cancel{2x}+6y=-10$

$4y=-9$

$\boxed{y=-\frac{9}{4}}$

$2x+2(-\frac{9}{4}) = -1$

$2x = \frac{9}{2}-1$

$\boxed{x = \frac{7}{4}}$

$\Delta = (2)(6)-(2)^2$

$\boxed{\Delta = 8}$

hay un minimo en $P(-\frac{9}{4}, \frac{7}{4})$

3.- $f(x,y) = y^3 + x^2 - 6xy + 3x + 6y - 7$

$\frac{\partial f}{\partial x} = 2x - 6y + 3 = 0$ $\qquad 2x - 6y = -3 \quad * 3$

$\frac{\partial f}{\partial y} = 3y^2 - 6x + 6 = 0$ $\qquad 3y^2 - 6x = -6$

$\cancel{6x} - 18y = -9$

$3y^2 - \cancel{6x} = -6$

$3y^2 - 18y = -15 \quad * \frac{1}{3}$

$y^2 - 6y + 5 = 0$

$(y-5)(y-1) = 0$

$y_1 = 5 \qquad y_2 = 1$

y_1

$2x = -3 + 6(5)$

$x_1 = \frac{27}{2}$

y_2

$2x = -3 + 6(1)$

$\boxed{x_2 = \frac{3}{2}}$

$P_1(\frac{27}{2}, 5)$

$P_2(\frac{3}{2}, 1)$

$\frac{\partial^2 f}{\partial x^2} = 2$

$\frac{\partial^2 f}{\partial y^2} = 6y$

$\frac{\partial^2 f}{\partial x \partial y} = -6$

$\Delta = 2(6y) - (-6)^2$

Para P_1

$\Delta = 12(5) - 36$

$\boxed{\Delta = 24}$

para P_2

$\Delta = 12(1) - 36$

$\boxed{\Delta = -24}$

$\therefore$ para P_1 $\frac{\partial^2 f}{\partial x^2} > 0 \wedge \Delta > 0 \rightarrow$ hay un minimo

$\therefore$ para P_2 $\frac{\partial^2 f}{\partial x^2} > 0 \wedge \Delta < 0$ hay punto de silla

4.- $f(x,y) = 3x^2y + x^2 - 6x - 3y - 2$

$\frac{\partial f}{\partial x} = 6xy + 2x - 6 = 0 \qquad 6xy + 2x = 6$

$\frac{\partial f}{\partial y} = 3x^2 - 3 = 0 \qquad 3x^2 - 3 = 0$

$x = \pm\sqrt{1}$

$x_1 = 1 \wedge x_2 = -1$

para y_1

$6y = 4$

$\boxed{y_1 = \frac{2}{3}}$

para y_2

$-6y = 8$

$\boxed{y_2 = -\frac{4}{3}}$

$\frac{\partial f}{\partial x^2} = 6y + 2$

$\frac{\partial f}{\partial y^2} = 0$

$\frac{\partial^2 f}{\partial x \partial y} = 6x$

$P_1(1, \frac{2}{3}) \quad P_2(-1, -\frac{4}{3})$

$\Delta = (6y)(0) - (6x)^2$

para P_1

$\frac{\partial^2 f}{\partial x^2} = 6 \wedge \Delta = -36$

como $\frac{\partial^2 f}{\partial x^2} > 0 \wedge \Delta < 0 \rightarrow$ hay un punto de silla

para P_2

$\frac{\partial^2 f}{\partial x^2} = -6 \wedge \Delta = -36$

como $\frac{\partial^2 f}{\partial x^2} < 0 \wedge \Delta < 0$

Hay un maximo en $P_2(-1, -\frac{4}{3} \ldots)$

5.- $f(x,y) = \operatorname{sen} x + \operatorname{sen} y + \operatorname{sen}(x+y)$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \cos x + \cos(x+y) = 0 \\ \frac{\partial f}{\partial y} &= \cos y + \cos(x+y) = 0 \end{aligned}\right\}$$

$\cos x + \cancel{\cos(x+y)} = 0$

$-\cos y - \cancel{\cos(x+y)} = 0 \quad *(-1)$

$\cos x = \cos y$

$\boxed{x = y}$

$\cos x + \cos 2x = 0$

$\cos x + 2\cos^2 x - 1 = 0$

$\underset{a}{2\cos^2 x} + \underset{b}{\cos x} \underset{c}{- 1} = 0$

$x_1 = \frac{\pi}{3} \wedge x_2 = \pi$

$P_1(\frac{\pi}{3}, \frac{\pi}{3}) \wedge P_2(\pi, \pi)$

$\frac{\partial^2 f}{\partial x^2} = -\operatorname{sen} x - \operatorname{sen}(x+y) = -\cancel{2}\frac{\sqrt{3}}{\cancel{2}} = -\sqrt{3}$

$\frac{\partial^2 f}{\partial y^2} = -\operatorname{sen} y - \operatorname{sen}(x+y) = -\sqrt{3}$

$\frac{\partial^2 f}{\partial x \partial y} = -\operatorname{sen}(x+y) = \frac{-\sqrt{3}}{2}$

$\Delta = (-\sqrt{3})(-\sqrt{3}) - \left(\frac{-\sqrt{3}}{2}\right)^2$

$\boxed{\Delta = \frac{9}{2}}$

para P_1

hay un punto de silla.

6.- $f(x,y) = 8^{2/3} - x^{2/3} - y^{2/3}$

$\frac{\partial f}{\partial x} = -\frac{2}{3}x^{-1/3} = 0 \quad x = 0$

$\frac{\partial f}{\partial y} = -\frac{2}{3}y^{-1/3} = 0 \quad y = 0$

$P(0,0)$

$\frac{\partial^2 f}{\partial x^2} = 0 \qquad \boxed{\Delta = 0}$ no existe información

7.- $f(x,y,z) = x^2+y^2-2z^2+3x+y-z-2$

$\frac{\partial f}{\partial x} = 2x+3 = 0 \quad x = -\frac{3}{2}$

$\frac{\partial f}{\partial y} = 2y+1 = 0 \Rightarrow y = -\frac{1}{2}$

$\frac{\partial f}{\partial z} = -4z-1=0 \Rightarrow z = \frac{1}{4}$

$P_{crit}\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{4}\right)$

8.- $f(x,y,z) = x^2+y^2+z^2-t^2-2xy+4xz+3xt-2yt+4x-5y-3$

$\frac{\partial f}{\partial x} = 2x-2y+4z+3t+4 = 0$

$\frac{\partial f}{\partial y} = 2y-2x-2t-5 = 0$

$\frac{\partial f}{\partial z} = 2z+4x = 0$

$\frac{\partial f}{\partial t} = -2t+3x-2y = 0$

$$\left[\begin{array}{cccc|c} 2 & -2 & 4 & 3 & -4 \\ -2 & 2 & 0 & -2 & 5 \\ 4 & 0 & 2 & 0 & 0 \\ 3 & -2 & 0 & -2 & 0 \end{array}\right] \propto \left[\begin{array}{cccc|c} 1 & -1 & 2 & \frac{3}{2} & -2 \\ -2 & 2 & 0 & -2 & 5 \\ 4 & 0 & 2 & 0 & 0 \\ 3 & -2 & 0 & -2 & 0 \end{array}\right]$$

$M_1\left(\frac{1}{2}\right)$

$A_{21}(2)$

$A_{3,1}(-4)$

$A_{4,1}(-3)$

$P_{crit}(-1, 2, 0, 3)$

\#

Practico nº 17

*** Multiplicadores de lagrange:**

1.- Hallar el minimo de $f(x,y,z) = x^2+y^2+z^2$ con la condición de $2x+4y-6z+5=0$

$$F(x,y,z,\lambda) = x^2+y^2+z^2-\lambda(2x+4y-6z+5)$$

$$\frac{\partial f}{\partial x} = 2x-\lambda(2) \Rightarrow \lambda_1 = x$$

$$\frac{\partial f}{\partial y} = 2y-\lambda(4) \Rightarrow \lambda_2 = \frac{y}{2}$$

$$\frac{\partial f}{\partial z} = 2z-\lambda(-6) \Rightarrow \lambda_3 = -\frac{z}{3}$$

$$\frac{\partial f}{\partial \lambda} = 2x+4y-6z+5=0$$

$\lambda_1 = \lambda_2 = \lambda_3$

$x = \frac{y}{2} \rightarrow y = 2x$

$x = -\frac{z}{3} \rightarrow z = -3x$

para "x"

$$2x+4(2x)-6(-3x) = -5$$

$$2x+8x+18x = -5$$

$\boxed{x = -\frac{5}{28}}$ $\boxed{y = -\frac{5}{14}}$ $\boxed{z = \frac{15}{28}}$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$\Delta = 4-0$

$\boxed{\Delta = 4}$ hay minimo en

$P\left(-\frac{5}{28}, -\frac{5}{14}, \frac{15}{28}\right)$

2.- $f(x,y,z) = (3x^2+2y^2+4z^2)$ condición $2x+4y-6z+5=0$

$F(x,y,z,\lambda) = 3x^2+2y^2+4z^2-\lambda(2x+4y-6z+5)$

$\frac{\partial f}{\partial x} = 6x-\lambda(2) \Rightarrow \lambda_1 = 3x$

$\frac{\partial f}{\partial y} = 4y-\lambda(4) \Rightarrow \lambda_2 = y$

$\frac{\partial f}{\partial z} = 8z-\lambda(-6) \Rightarrow \lambda_3 = -\frac{4z}{3}$

$\frac{\partial f}{\partial \lambda} = 2x+4y-6z=5$

$3x = y$

$x = \frac{y}{3}$

$3x = -\frac{4z}{3}$ $\Big\}$ $z = -\frac{9}{4}x$

$x = -\frac{4}{9}z$

para "x"

$2x+4(3x)-6\left(-\frac{9}{4}x\right)=5$

$2x+12x+\frac{27}{2}x=5$

$\frac{55}{2}x=5$

$\boxed{x=\frac{2}{11}}$ $\boxed{y=\frac{6}{11}}$ $\boxed{z=-\frac{9}{22}}$

$\frac{\partial^2 f}{\partial x^2} = 6$ hay un minimo en $P\left(\frac{2}{11}, \frac{6}{11}, -\frac{9}{22}\right)$

$\frac{\partial^2 f}{\partial y^2} = 4$

3.- $f(x,y,z) = x^2+y^2+z^2$ si $x+2y+z-1=0$

$x-y+2z=6$

si $\bar{N}_1 \times \bar{N}_2 = \bar{n} \Rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{i} - 1\hat{j} - 3\hat{k}$

$\bar{n} = (5,-1,-3)$

$f(x,y,z,h) = x^2+y^2+z^2 - h(5x-y-3z-6)$

$\frac{\partial f}{\partial x} = 2x - h(5) \Rightarrow h_1 = \frac{2}{5}x$

$\frac{\partial f}{\partial y} = 2y - h(-1) \Rightarrow h_2 = -2y$

$\frac{\partial f}{\partial z} = 2z - h(-3) \Rightarrow h_3 = -\frac{2}{3}z$

$\frac{\partial f}{\partial h} = 5x-y-3z-6=0$

$h_1 = h_2 = h_3$

$\frac{\cancel{2}}{5}x = -\cancel{2}y$

$x = -5y$

$y = -\frac{x}{5}$

$\frac{\cancel{2}}{5}x = -\frac{\cancel{2}}{3}z$

$x = -\frac{5}{3}z$

$z = -\frac{3}{5}x$

$5x - \left(\frac{-x}{5}\right) - 3\left(\frac{-3}{5}x\right) = 6$

$\boxed{x = \frac{6}{7}}$ $\boxed{y = -\frac{6}{35}}$

$\boxed{z = -\frac{18}{35}}$

$\frac{\partial^2 f}{\partial x^2} = 2 \rightarrow$ Exite un minimo //

5.- $f(x,y,z) = x^3 + y^3 + z^3 \rightarrow x+y+z=4$

$$x+y+z-4=0$$

$$f(x,y,z,\lambda) = x^3+y^3+z^3-\lambda(x+y+z-4)$$

$$\frac{\partial f}{\partial x} = 3x^2-\lambda(1) \Rightarrow \lambda_1 = 3x^2$$

$$\frac{\partial f}{\partial y} = 3y^2-\lambda(1) \Rightarrow \lambda_2 = 3y^2$$

$$\frac{\partial f}{\partial z} = 3z^2-\lambda(1) \Rightarrow \lambda_3 = 3z^2$$

$$\frac{\partial f}{\partial \lambda} = x+y+z-4 = 0$$

$\lambda_1 = \lambda_2 = \lambda_3$

$\cancel{3}x^2 = \cancel{3}y^2$

$\boxed{x = y}$

$\boxed{x = z}$

$$x+x+x=4$$

$\boxed{x = \frac{4}{3}}$ $\boxed{y = \frac{4}{3}}$ $\boxed{z = \frac{4}{3}}$

$$\frac{\partial^2 f}{\partial x^2} = 6x = \boxed{8}$$

$$\frac{\partial^2 f}{\partial y^2} = 6y = 8$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$\Delta = 64 - 0$

$\boxed{\Delta = 64}$

hay un minimo en $P(\frac{\pi}{3}, \frac{4}{3}, 4/3)$

6.- $f(x,y,z,t) = x^2+y^2+z^2+t^2 \rightarrow 3x+2y-z+t=2$

$$F(x,y,z,t,\lambda) = x^2+y^2+z^2+t^2-\lambda(3x+2y-z+t-2)$$

$$\frac{\partial f}{\partial x} = 2x-\lambda(3) \Rightarrow \lambda_1 = \frac{2x}{3}$$

$$\frac{\partial f}{\partial y} = 2y-\lambda(2) \Rightarrow \lambda_2 = y$$

$$\frac{\partial h}{\partial z} = 2z-\lambda(-1) \Rightarrow \lambda_3 = -2z$$

$$\frac{\partial h}{\partial t} = 2t-\lambda(1) \Rightarrow \lambda_4 = 2t$$

$$\frac{\partial f}{\partial t} = 3x+2y-z+t-2 = 0$$

$x = \frac{3}{2}y$

$y = \frac{2}{3}x$

$\frac{2x}{3} = -2z$

$\boxed{z = -\frac{x}{3}}$

$\frac{\cancel{2}x}{3} = \cancel{2}t$

$\boxed{t = \frac{1}{3}x}$

$$3x+2\cdot\left(\frac{2}{3}x\right)-\left(\frac{-x}{3}\right)+\frac{1}{3}x = 2$$

$$\frac{18x}{\cancel{3}}+\frac{2}{\cancel{3}}x = 2$$

$\boxed{x = -\frac{3}{10}}$ $\boxed{y = -\frac{1}{5}}$ $\boxed{z = \frac{1}{10}}$

$\boxed{t = -\frac{1}{10}}$

$$\frac{\partial^2 f}{\partial x^2} = 2 \rightarrow \text{Existe un minimo}$$

Practico N° 18

* Integrales

1.- $\int_1^4 \int_2^5 (x^2 - y^2 + xy - 3)\,dx\,dy$

$$= \int_1^4 \left(\frac{x^3}{3} - xy^2 + \frac{x^2y}{2} - 3x \right)\Big|_2^5 dy = \int_1^4 \left[\frac{125}{3} - 5y^2 + \frac{25y}{2} - 3(5)\right] - \left[\frac{8}{3} - 2y^2 + 2y - 6\right] d$$

$$= \int_1^4 \left(39 - 3y^2 + \frac{21}{2}y - 9\right) dy = \left(30y - y^3 + \frac{21y^2}{4}\right)\Big|_1^2$$

$$= 30(2) - (2)^3 + \frac{21(2)^2}{4} - \left(30(1) - (1)^3 + \frac{21(1)}{4}\right) = 27 + \frac{63}{4}$$

$$= \frac{171}{4}$$

#

2.- $\int_0^2 \int_{-3}^2 (x^3 + 2x^2y - y^3 + xy)\,dy\,dx$

$$= \int_0^2 \left(x^3y + \frac{\cancel{2}x^2y^2}{\cancel{2}} - \frac{y^4}{4} + \frac{xy^2}{2}\right)\Big|_{-3}^2 dx$$

$$= \int_0^2 \left[2x^3 + 4x^2 - 4 + 2x - \left(-3x^3 - 9x^2 - \frac{81}{4} + \frac{9}{2}x\right)\right] dx$$

$$= \int_0^2 \left(5x^3 + 13x^2 - \frac{5}{2}x + \frac{65}{4}\right) dx = \left(\frac{5x^4}{4} + 13\frac{x^3}{3} - \frac{5x^2}{4} + \frac{65}{4}x\right)\Big|_0^2$$

$$= \left(20 + \frac{104}{3} - 5 + \frac{65}{2}\right) - 0$$

$$= \frac{493}{6} = 82,16$$

#

$$3.- \int_1^4 \int_{\sqrt{x}}^{x^2} (x^2+2xy-3y^2)\,dy\,dx$$

$$=\int_1^4 \left(x^2y+\cancel{2}x\frac{y^2}{\cancel{2}}-\cancel{3}\frac{y^3}{\cancel{3}}\right)\Bigg|_{\sqrt{x}}^{x^2} dx$$

$$=\int_1^4 \left[\left(x^4+x^5-x^6\right)-\left(x^{5/2}+x^2-x^{3/2}\right)\right] dx$$

$$=\int_1^4 \left(x^4+x^5-x^6-x^{5/2}-x^2+x^{3/2}\right) dx$$

$$=\left(\frac{x^5}{5}+\frac{x^6}{6}-\frac{x^7}{7}-\frac{2x^{7/2}}{7}-\frac{x^3}{3}+\frac{2x^{5/2}}{5}\right)\Bigg|_1^4$$

$$=\left(\frac{(4)^5}{5}+\frac{(4)^6}{6}-\frac{(4)^7}{7}-\frac{2(2)^7}{7}-\frac{(4)^3}{3}+\frac{2(2)^5}{5}\right)-\left(\frac{1}{5}+\frac{1}{6}-\frac{1}{7}-\frac{2}{7}-\frac{1}{3}+\frac{2}{5}\right)$$

$$=217+\frac{4095}{6}-\frac{16643}{7}-21=-\frac{20987}{14}$$

$$\boxed{=-1499{,}01}$$

#

$$4.- \int_0^1 \int_{x^3}^{x^2} (x^2 - xy)\, dy\, dx$$

$$\int_0^1 \left(x^2y - \frac{xy^2}{2}\right)\Big|_{x^3}^{x^2} dx = \int_0^1 \left[x^4 - \frac{x^3}{2} - \left(x^5 - \frac{x^7}{2}\right)\right] dx$$

$$= \int_0^1 \left(x^4 - \frac{x^3}{2} - x^5 + \frac{x^7}{2}\right) dx = \left(\frac{x^5}{5} - \frac{x^4}{8} - \frac{x^6}{6} + \frac{x^8}{16}\right)\Big|_0^1$$

$$= \frac{1}{5} - \frac{1}{8} - \frac{1}{6} + \frac{1}{16} - 0$$

$$= -\frac{7}{240}$$

$$5.- \int_2^3 \int_1^2 (x^2y + xy^2)\, dx\, dy$$

$$= \int_2^3 \left(\frac{x^3}{3}y + \frac{x^2}{2}y^2\right)\Big|_1^2 dy = \int_2^3 \left(\frac{8}{3}y + 2y^2 - y - y^2\right) dy$$

$$= \int_2^3 \left(\frac{5}{3}y + y^2\right) dy = \left(\frac{5}{6}y^2 + \frac{y^3}{3}\right)_2^3$$

$$= \frac{5}{6}(9) + \frac{27}{3} - \left(\frac{5}{6}(4) + \frac{8}{3}\right)$$

$$= \frac{15}{2} + 9 - \frac{10}{3} - \frac{8}{3} = \frac{33}{2} - \frac{18}{3}$$

$$\boxed{= \frac{21}{2}}$$

$$6.- \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} y\,dy\,dx$$

$$= \int_{-2}^{2} \frac{y^2}{2} \Big|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx = \frac{1}{2}\int_{-2}^{2} [(4-x^2)-(4-x^2)]\,dx = 0$$

$$\boxed{I = 0}$$

$$7.- \int_{-3}^{3} \int_{-\sqrt{8-2y^2}}^{\sqrt{8-2y^2}} x\,dx\,dy$$

$$= \int_{-3}^{3} \frac{x^2}{2} \Big|_{-\sqrt{8-2y^2}}^{\sqrt{8-2y^2}} dy = \frac{1}{2}\int_{-3}^{3} [(8-2y^2)-(8-2y^2)]\,dy = \boxed{0}$$

$$8.- \int_{-3}^{3} \int_{x^2}^{18-x^2} x y^3\,dy\,dx$$

$$= \int_{-3}^{3} x\,\frac{y^4}{4} \Big|_{x^2}^{18-x^2} dx = \int_{-3}^{3} \left[\frac{x(18-x^2)^4}{4} - \frac{x^9}{4}\right] dx$$

$$= \left(-\frac{(18-x^2)^5}{10} - \frac{x^{10}}{40}\right)\Big|_{-3}^{3} = -\frac{(18-(3)^2)^5}{10} - \frac{(3)^{10}}{40} - \left[-\frac{(18-9)^5}{10} - \frac{(-3)^{10}}{40}\right]$$

$$= -\cancel{\frac{(9)^5}{10}} - \cancel{\frac{(3)^{10}}{40}} + \cancel{\frac{(9)^5}{10}} + \cancel{\frac{(3)^{10}}{40}} = \boxed{0}$$

$$9.- \int_{0}^{2} \int_{x^2}^{2x^2} x.\cos y\,dy\,dx = \int_{0}^{2} x.\operatorname{sen} y \Big|_{x^2}^{2x^2} dx$$

$$= \int_{0}^{2} [x.\operatorname{sen}(2x^2) - x\operatorname{sen} x^2]\,dx = \left[-\frac{1}{4}\cos(2x^2) + \frac{1}{2}\cos x^2\right]\Big|_{0}^{2}$$

$$= -\frac{1}{4}.\cos(8) + \frac{1}{2}\cos(4) - \left(-\frac{1}{4} + \frac{1}{2}\right) = \frac{\cos(4)}{2} - \frac{\cos(8)}{4} - \frac{1}{4}$$

10.- $\int_1^2 \int_{x^3}^{4x^3} \frac{1}{y}\, dy\, dx$

$$= \int_1^2 \ln y \Big|_{x^3}^{4x^3} dx = \int_1^2 \left(\ln 4x^3 - \ln x^3\right) dx$$

$$= \int_1^2 \ln\left(\frac{4\cancel{x^3}}{\cancel{x^3}}\right) dx = \int_1^2 \ln(4)\, dx = \ln(4)\, x \Big|_1^2$$

$$= 2\ln(4) - \ln(4) = \boxed{\ln(4)}$$

#

11.- $\iint_R (x^2 - y^2)\, dA$ F: $0 \leq y \leq 2$
$y^2 \leq x \leq 4$

$$A = \int_0^2 \int_{y^2}^4 (x^2 - y^2)\, dx\, dy = \int_0^2 \left(\frac{x^3}{3} - xy^2\right)\Big|_{y^2}^4 dy$$

$$A = \int_0^2 \left[\frac{64}{3} - 4y^2 - \left(\frac{y^6}{3} - y^4\right)\right] dy$$

$$A = \int_0^2 \left(y^4 - \frac{y^6}{3} - 4y^2 + \frac{64}{3}\right) dy = \left[\frac{y^5}{5} - \frac{y^7}{21} - \frac{4}{3}y^3 + \frac{64}{3}y\right]\Big|_0^2$$

$$A = \frac{(2)^5}{5} - \frac{(2)^7}{21} - \frac{4(2)^3}{3} + \frac{64(2)}{3} - 0$$

$$A = \frac{32}{5} - \frac{128}{21} - \frac{32}{3} + \frac{128}{3} = \frac{32}{105} + 32$$

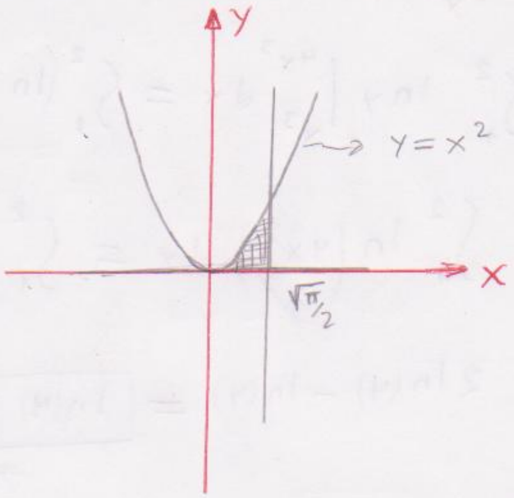
$$\boxed{A = 32{,}30 \text{ u.A}}$$

#

12.- $\iint\limits_F x\cdot\cos y\, dA$ $\quad y = x^2$

$y = 0$

$x = \sqrt{\pi/2}$



$$A = \int_0^{\sqrt{\frac{\pi}{2}}}\int_0^{x^2} x\cdot\cos y\, dy\, dx$$

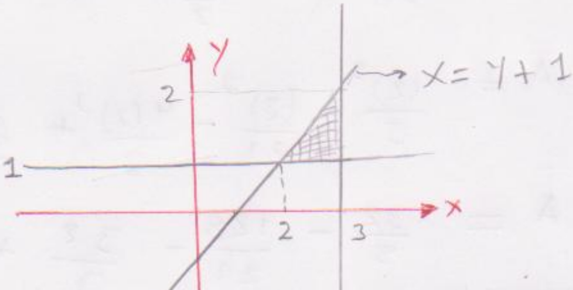
$$A = \int_0^{\sqrt{\pi/2}} x\cdot \operatorname{sen} y\Big|_0^{x^2} dx = \int_0^{\sqrt{\pi/2}} \left(x\cdot \operatorname{sen} x^2 - 1\right) dx$$

$$A = \left(-\frac{1}{2}\cos x^2 - x\right)\Big|_0^{\sqrt{\frac{\pi}{2}}} = -\frac{1}{2}\cdot \overset{0}{\cancel{\cos\left(\frac{\pi}{2}\right)}} - \sqrt{\frac{\pi}{2}} - \left(-\frac{1}{2}\right)$$

$$\boxed{A = \frac{1}{2} - \sqrt{\frac{\pi}{2}}}$$

14.- $\iint\limits_F \ln y\, dA$: $\quad y = 1 \qquad x = 3$

$y = x - 1$



$$A = \int_1^2\int_{y+1}^{3} \ln y\, dx\, dy$$

$$A = \int_2^3 x\ln y\Big|_{y+1}^{3} dy$$

$$A = \int_2^3 \ln y\,(3 - y - 1)\, dy = \int_2^3 (2\ln y - y\ln y)\, dy$$

$$A = \left(2(y\cdot\ln y - y) - \frac{y^2}{2}\cdot\ln y - \frac{y^2}{4}\right)\Big|_2^3 = 6\ln 3 - 6 - \frac{9}{2}\cdot\ln 3 - \frac{9}{4} - (4\ln 2 - 4 - 2\ln 2 - 2)$$

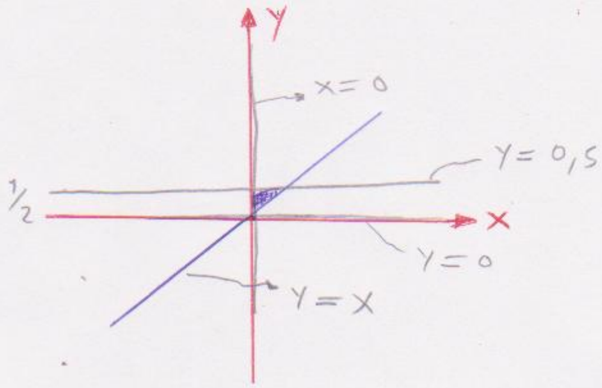
$$\boxed{A = 3{,}23 \text{ u.A}}$$

$15.- \iint_F \frac{x}{\sqrt{1-y^2}}\, dA$ $\quad x=0$

$y=0$

$y=\frac{1}{2}$

$y=x$



$$A=\int_0^{1/2}\int_0^{y}\frac{1}{\sqrt{1-y^2}}\,dx\,dy$$

$$A=\int_0^{1/2}\frac{x}{\sqrt{1-y^2}}\Big|_0^{y}dy=\int_0^{1/2}\frac{y}{\sqrt{1-y^2}}\,dy$$

$$A=-\sqrt{1-y^2}\Big|_0^{1/2}=-\sqrt{1-\tfrac{1}{2}}-\left(-\sqrt{1-0}\right)$$

$$A=-\frac{1}{\sqrt{2}}+1$$

$$\boxed{A=\frac{\sqrt{2}-1}{\sqrt{2}}\ \text{u.A}}$$

#

Práctico nº 19

* Integrales Triples

1.- $\int_0^1\int_0^x\int_0^{x-y} x\,dz\,dy\,dx$

$$I=\int_0^1\int_0^x xz\Big|_0^{x-y}dy\,dx=\int_0^1\int_0^x x(x-y)\,dy\,dx=\int_0^1\int_0^x(x^2-xy)\,dy\,dx$$

$$I=\int_0^1\left(x^2y-\frac{xy^2}{2}\right)\Big|_0^x dy=\int_0^1\left(x^3-\frac{x^3}{2}\right)dy=\int_0^1\frac{1}{2}x^3dx=\frac{1}{2}-0$$

$$\boxed{I=\frac{1}{2}}$$

\#

2.- $\int_{-1}^{1}\int_0^{1-y^2}\int_{-\sqrt{x}}^{\sqrt{2x}} 2y^2\sqrt{x}\,dz\,dx\,dy$

$$I=\int_{-1}^{1}\int_0^{1-y^2}2y^2\sqrt{x}\cdot z\Big|_{-\sqrt{x}}^{\sqrt{x}}dx\,dy$$

$$I=\int_{-1}^{1}\int_0^{1-y^2}2y^2\sqrt{x}(\sqrt{x}+\sqrt{x})\,dx\,dy=\int_{-1}^{1}\int_0^{1-y^2}4xy^2\,dx\,dy$$

$$I=\int_{-1}^{1}2x^2y^2\Big|_0^{1-y^2}dy=\int_{-1}^{1}2(1-y^2)^2y^2dy=\int_{-1}^{1}2y^2(1-2y^2+y^4)\,dy$$

$$I=\int_{-1}^{1}(2y^2-4y^4+2y^6)\,dy=\left(\frac{2}{3}y^3-\frac{4}{5}y^5+\frac{2}{7}y^7\right)\Big|_{-1}^{1}$$

$$I=\left(\frac{2}{3}-\frac{4}{5}+\frac{2}{7}\right)-\left(-\frac{2}{3}+\frac{4}{5}-\frac{2}{7}\right)=\frac{4}{3}-\frac{8}{5}+\frac{4}{7}=\frac{32}{105}$$

$$\boxed{I=0{,}305}$$

\#

$3.- \displaystyle\int_0^1\int_{y^2}^{\sqrt{y}}\int_0^{y+z} xy\,dx\,dz\,dy$

$$I=\int_0^1\int_{y^2}^{\sqrt{y}} y\frac{x^2}{2}\Big|_0^{y+z} dz\,dy=\int_0^1\int_{y^2}^{\sqrt{y}} y(y+z)\,dz\,dy$$

$$I=\int_0^1\int_{y^2}^{\sqrt{y}}\left(y^2+yz\right)dz\,dy=\int_0^1\left(y^2z+y\frac{z^2}{2}\right)\Big|_{y^2}^{\sqrt{y}}dy$$

$$I=\int_0^1\left[y^2\sqrt{y}+\frac{y.y}{2}-\left(y^4+\frac{y^5}{2}\right)\right]dy=\int_0^1\left(y^{3/2}+\frac{y^2}{2}-y^4-\frac{y^5}{2}\right)dy$$

$$I=\left(\frac{2}{5}y^{5/2}+\frac{y^3}{6}-\frac{y^5}{5}-\frac{y^6}{12}\right)\Big|_0^1=\frac{2}{5}+\frac{1}{6}-\frac{1}{5}-\frac{1}{12}-(0)$$

$$I=\frac{1}{5}+\frac{1}{12}=\frac{17}{60}$$

$$\boxed{I=0,283}\ \#$$

$4.- \displaystyle\int_0^4\int_0^{\sqrt{16-x^2}}\int_0^{\sqrt{16-x^2-y^2}}(x+y+z)\,dz\,dy\,dx$

$$=\int_0^4\int_0^{\sqrt{16-x^2}}\left(xz+yz+\frac{z^2}{2}\right)\Big|_0^{\sqrt{16-x^2-y^2}}dy\,dx=\int_0^4\int_0^{\sqrt{16-x^2}}\left[x\sqrt{16-x^2}+y\sqrt{16-x^2}+\frac{(16-x^2)}{2}\right]dy\,dx$$

$$=\int_0^4\left(x\sqrt{16-x^2}+\frac{x^3}{2}+16x^2\right)dx=\left[(16-x^2)^{3/2}+\frac{x^4}{8}+\frac{16(x^3)}{3}\right]\Big|_0^4$$

$$I=\cancelto{0}{(16-16)^{3/2}}+\frac{(4)^4}{8}+\frac{16(4)^3}{3}=32+341,33$$

$$\boxed{I=373,33\ uA}\ \#$$

5.- $\int_0^2 \int_0^{\sqrt{4-z^2}} \int_0^{2-z} z\,dx\,dy\,dz$

$$I = \int_0^2 \int_0^{\sqrt{4-z^2}} zx \Big|_0^{2-z} dy\,dz$$

$$I = \int_0^2 \int_0^{\sqrt{4-z^2}} z(2-z)\,dy = \int_0^2 \int_0^{\sqrt{4-z^2}} \left(2z - \frac{z^2}{2}\right) dy\,dz$$

$$I = \int_0^2 \left(2zy - \frac{z^2 y}{2}\right)\Big|_0^{\sqrt{4-z^2}} dz = \int_0^2 \left(2z\sqrt{4-z^2} - \sqrt{(4-z^2)} \cdot z^2\right) dz$$

$$I = \left[\frac{4}{3}(4-z^2)^{3/2} - \frac{z}{4}\sqrt{4-z^2} + \frac{5}{16} z\right]\Big|_0^2$$

$$\boxed{I = \frac{32}{3} \text{ .u.v}}$$

6.- $\int_0^1 \int_0^x \int_0^y \frac{1+\sqrt[3]{z}}{\sqrt{z}}\,dz\,dy\,dx$

$$I = \int_0^1 \int_0^x \left(\sqrt{z} + \frac{6}{5} z^{5/6}\right)\Big|_0^y dy\,dx = \int_0^1 \int_0^x \left(\sqrt{y} + \frac{6}{5} y^{5/6}\right) dy\,dx$$

$$I = \int_0^1 \left(\frac{2}{3} y^{3/2} + \frac{36}{55} y^{11/6}\right)\Big|_0^x = \int_0^1 \left(\frac{2}{3} x^{3/2} + \frac{36}{55} x^{11/6}\right) dx = 0$$

$$I = \left(\frac{4}{15} x^{5/2} + \frac{216}{935} x^{17/6}\right)\Big|_0^1 = \frac{4}{15} + \frac{216}{935} - 0 = 0,266 + 0,23$$

$$\boxed{I = 0,49}$$

$$\iiint_S f(x,y,z)\,dv$$

7: $z=0,\ y=0$; $f(x,y,z) = x$

$y = x$

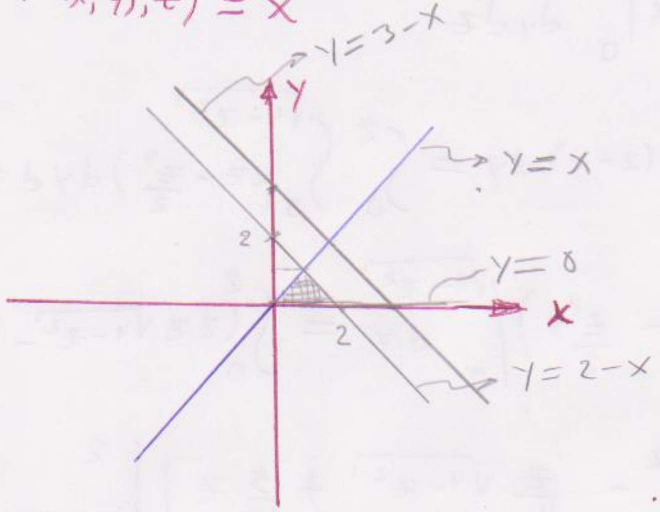
$x+y=2$

$x+y+z=3$

$y = 2-x$

$y = 3-x$

$y = x$



$$V = \int_0^1 \int_y^{2-y} \int_0^{3-x-y} x \, dz\,dx\,dy$$

$$V = \int_0^1 \int_y^{2-y} xz \Big|_0^{3-x-y} dx\,dy$$

$$V = \int_0^1 \int_y^{2-y} (3x - x^2 - xy)\,dx\,dy$$

$$V = \int_0^1 \left(\frac{3x^2}{2} - \frac{x^3}{3} - \frac{x^2 y}{2}\right)\Bigg|_y^{2-y} dy$$

$$V = \int_0^1 \left\{\frac{3}{2}(2-y)^2 - \frac{(2-y)^3}{3} - \frac{(2-y)^2}{2}y - \left[\frac{3}{2}y^2 - \frac{y^3}{3} - \frac{y^3}{2}\right]\right\} dy$$

$$V = \int_0^1 \frac{3}{2}(4-4y+y^2) - \frac{(8-6y\cdot4+6y^2-y^3)}{3} - \frac{(4-4y+y^2)}{2}y - \cdots$$

$$-\cdots\left[\frac{3y^2}{2} - \frac{y^3}{3} - \frac{y^3}{2}\right] dy$$

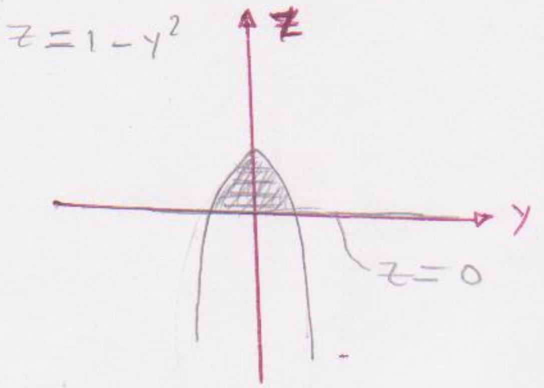
$$V = \int_0^1 \left(6 - 6y + 3y^2 - \frac{8-24y+6y^2-y^3}{3} - \frac{4y-4y^2+y^3}{2} - \frac{3y^2}{2} + \frac{5}{6}y^3\right) dy$$

$$V = 6 - 3 + 1 - \frac{8}{3} + \frac{12}{3} - \frac{2}{3} + \frac{1}{12} - 1 + \frac{2}{3} - \frac{1}{8} - \frac{1}{2} + \frac{5}{24} = 7 - \frac{7}{3}$$

$$\boxed{V = \frac{14}{3}\ uv}$$

8.- $z = 0\ ,\ x^2 + z = 1 \qquad f(x,y,z) = z^2$

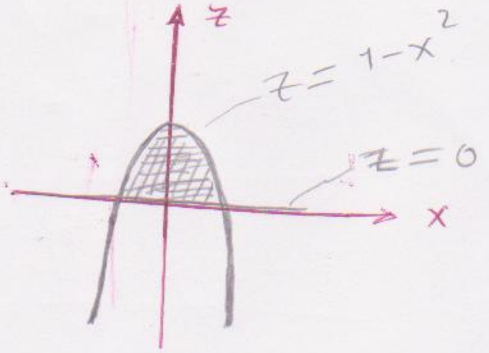
$y^2 + z = 1$



$$V = 2\int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-x^2} z^2\, dz\, dx\, dy$$

$$V = 2\int_{-1}^{1}\int_{0}^{1} (1-x^2)^3\, dx\, dy$$

$$V = 2\int_{-1}^{1}\int_{0}^{1} (1 - 3x^2 + 3x^4 + x^6)\, dx\, dy$$



$$V = 2\int_{-1}^{1} \left(x - x^3 + \frac{3x^5}{5} + \frac{x^7}{7}\right)\Big|_{0}^{1} dy$$

$$V = 2\int_{-1}^{1} \left(1 - 1 + \frac{3}{5} + \frac{1}{7}\right) dy$$

$$V = \frac{52}{35}\int_{-1}^{1} dy = \frac{52}{35} y\Big|_{-1}^{1}$$

$$\boxed{V = 2{,}97 \text{ u.v}}$$

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