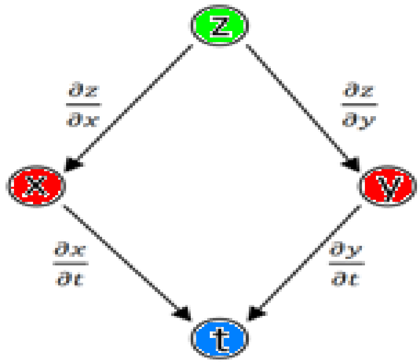
Derivadas de funciones compuestas y su aplicación

Definición

Regla de la cadena

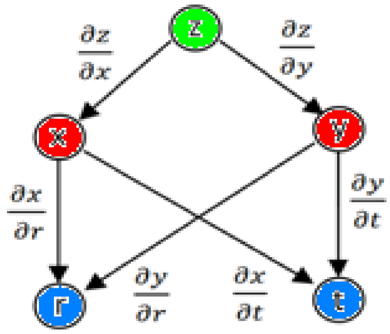
1. *variable dependiente*
2. *variables intermedias*
3. *variables independientes*

I. $z = f(x, y) \quad ; \quad x = x(t) \quad ; \quad y = y(t)$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

II. $z = f(x,y)$; $x = x(r,t)$; $y = y(r,t)$



$$\frac{z\partial}{\partial r} = \frac{\partial z}{\partial x} * \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} * \frac{\partial y}{\partial r}$$

$$\frac{z\partial}{\partial t} = \frac{\partial z}{\partial x} * \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} * \frac{\partial y}{\partial t}$$

NOTA.

Tipo I

1. *Dada la función* $z = x^2+y^2 + xy$ si $x = sen\ t$; $y = e^t$ Determinar $\frac{\partial z}{\partial t}$

- Variable independiente: t
- Variable dependiente: z
- Variable intermediaria: x, y

Regla de la Cadena

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial x} = 2x + y \qquad \frac{\partial x}{\partial t} = \cos t$$

$$\frac{\partial z}{\partial y} = 2y + x \qquad \frac{\partial y}{\partial t} = e^t$$

Reemplazando en RC

$$\frac{\partial z}{\partial t} = (2x + y)\cos t + (2y + x)e^t$$

2. Función $w = xe^{y/z}$, $x = t^2$, $y = 1 - t$, $z = 1 + 2t$ dterminar $\frac{\partial w}{\partial t}$

- Variable independiente: t
- Variable dependiente: w
- Variable intermediaria: x, y, z

Regla de la Cadena

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial x} = e^{\frac{y}{z}} \qquad \frac{\partial x}{\partial t} = 2t$$

$$\frac{\partial w}{\partial y} = xe^{\frac{y}{z}}\frac{1}{z} \qquad \frac{\partial y}{\partial t} = -1$$

$$\frac{\partial w}{\partial z} = xe^{\frac{y}{z}}\left(-\frac{y}{z^2}\right) \qquad \frac{\partial z}{\partial t} = 2$$

Reemplazando en RC

$$\frac{\partial w}{\partial t} = e^{\frac{y}{z}}2t + xe^t\frac{1}{z}(-1) + \left(-\frac{y}{z^2}xe^{\frac{y}{z}}\right)2$$

$$\frac{\partial w}{\partial t}=2te^{\frac{y}{z}}-\frac{xe^{\frac{y}{z}}}{z}-2\frac{y}{z^2}xe^{\frac{y}{z}}$$

$$\frac{\partial w}{\partial t}=e^{\frac{y}{z}}\left[2t-\frac{x}{z}-\frac{2xy}{z^2}\right]$$

3. Función $z=e^r\cos\theta$, $r=st$, $\theta=\sqrt{s^2+t^2}$ determinar $\frac{\partial z}{\partial s}$ $\frac{\partial z}{\partial t}$

- Variable independiente: t, s
- Variable dependiente: z
- Variable intermediaria: r, θ

Regla de la Cadena 1

$$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial r}\frac{\partial r}{\partial s}+\frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s}$$

$$\frac{\partial z}{\partial r}=\cos\theta\, e^r \qquad \frac{\partial r}{\partial s}=t$$

$$\frac{\partial z}{\partial \theta}=e^r(-\,sen\,\theta) \qquad \frac{\partial \theta}{\partial s}=\frac{2s}{2\sqrt{s^2+t^2}}$$

Reemplazando en RC

$$\frac{\partial z}{\partial s}=\cos\theta\, e^r t+e^r(-\,sen\,\theta)\frac{s}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial s}=e^r\left(t\cos\theta-\frac{s}{\sqrt{s^2+t^2}}sen\,\theta\right)$$

Regla de la Cadena 2

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial r}\frac{\partial r}{\partial t}+\frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t}$$

$$\frac{\partial z}{\partial r}=\cos\theta\, e^r \qquad \frac{\partial r}{\partial t}=s$$

$$\frac{\partial z}{\partial \theta}=e^r(-sen\,\theta) \qquad \frac{\partial \theta}{\partial t}=\frac{2t}{2\sqrt{s^2+t^2}}$$

Reemplazando en RC

$$\frac{\partial z}{\partial t}=\cos\theta\, e^r s+e^r(-\,sen\,\theta)\frac{t}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial t}=e^r\left(s\cos\theta-\frac{t}{\sqrt{s^2+t^2}}sen\,\theta\right)$$

Tipo II

1. Si $w = f(x,y)$, donde $x = rcos\theta$ y $y = r\ sen\ \theta$, demuestre que :

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left[\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r}\left(\frac{\partial w}{\partial \theta}\right)^2\right]$$

Primer paso: Identificar las variables

Variable Intermediaria: x, y

Variable Independiente: r, θ

Variable Dependiente: z

Segundo paso: Aplicar la Regla de la cadena

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} * \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} * Cos\theta + \frac{\partial w}{\partial y} * Sen\theta \qquad (1)$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} * \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} * \frac{\partial y}{\partial \theta}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} * (-rSen\theta) + \frac{\partial w}{\partial y}(rCos\theta) \qquad (2)$$

Tercer paso: (5) y (6) reemplazando en la ecuación diferencial

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left[\left(\frac{\partial w}{\partial x}cos\theta + \frac{\partial w}{\partial y}sen\theta\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial x}(-rsen\theta + \frac{\partial w}{\partial y}(rcos\theta)\right)^2\right]$$

$$\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2=\left(\frac{\partial w}{\partial x}\right)^2(cos\theta)^2+2\frac{\partial w}{\partial x}cos\theta\frac{\partial w}{\partial x}sen\theta+\left(\frac{\partial w}{\partial y}\right)^2(sen\theta)^2$$

$$+\frac{1}{r^2}\left[\left(\frac{\partial w}{\partial y}\right)^2r^2(cos\theta)^2-2\frac{\partial w}{\partial x}rcos\theta\frac{\partial w}{\partial x}rsen\theta+\left(\frac{\partial w}{\partial x}\right)^2r^2(sen\theta)^2\right]$$

$$\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2=\left(\frac{\partial w}{\partial x}\right)^2[(cos\theta)^2+(sen\theta)^2]+\left(\frac{\partial w}{\partial y}\right)^2[(cos\theta)^2+(sen\theta)^2]$$

$$\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2=\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2 \qquad \text{Demostrado}$$

2. Si $u=f(x,y)$, donde $x=e^s\cos t$ y $y=e^s\,sen\,t$, demuestre que :

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=e^{-2s}\left[\frac{\partial^2u}{\partial s^2}+\frac{\partial^2u}{\partial t^2}\right]$$

Var: Dependiente: u
Var: Intermediaria: x,y
Var: independiente: s,t

Donde:

$$\frac{\partial x}{\partial s}=e^s\cos t \qquad \frac{\partial y}{\partial s}=e^s sen\ t$$

$$\frac{\partial^2x}{\partial s^2}=e^s\cos t \qquad \frac{\partial^2y}{\partial s^2}=e^s sen\ t$$

$$\frac{\partial x}{\partial t}=-e^s sen\ t \qquad \frac{\partial y}{\partial t}=e^s\cos t$$

$$\frac{\partial^2x}{\partial t^2}=-e^s\cos t \qquad \frac{\partial^2y}{\partial t^2}=-e^s sen\ t$$

1.) Aplicando la Regla de la Cadena para $\frac{\partial u}{\partial s}$

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$$

2.) Determinando la Segunda Derivada

$$\frac{\partial^2 u}{\partial s^2} = \frac{\partial \left[\frac{\partial u}{\partial x}\frac{\partial x}{\partial s}\right]}{\partial s} + \frac{\partial \left[\frac{\partial u}{\partial y}\frac{\partial y}{\partial s}\right]}{\partial s}$$

$$\frac{\partial^2 u}{\partial s^2} = \frac{\partial \left[\frac{\partial u}{\partial x}\right]}{\partial s}\left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial u}{\partial x}\right)\frac{\partial \left[\frac{\partial x}{\partial s}\right]}{\partial s} + \frac{\partial \left[\frac{\partial u}{\partial y}\right]}{\partial s}\left(\frac{\partial y}{\partial s}\right) + \left(\frac{\partial u}{\partial y}\right)\frac{\partial \left[\frac{\partial y}{\partial s}\right]}{\partial s}$$

$$\frac{\partial^2 u}{\partial s^2} = \frac{\partial^2 u}{\partial x\partial s} * \frac{\partial x}{\partial s} + \frac{\partial u}{\partial x} * \frac{\partial^2 x}{\partial s^2} + \frac{\partial^2 u}{\partial y\partial s} * \frac{\partial y}{\partial s} + \frac{\partial u}{\partial y} * \frac{\partial^2 y}{\partial s^2}$$

$$\frac{\partial^2 u}{\partial s^2} = \frac{\partial^2 u}{\partial x\partial s}(e^s \cos t) + \frac{\partial u}{\partial x}(e^s \cos t\ ) + \frac{\partial^2 u}{\partial y\partial s}(e^s sen\ t) + \frac{\partial u}{\partial y}(e^s sen\ t)$$

$$\frac{\partial^2 u}{\partial s^2} = e^s \cos t\left(\frac{\partial^2 u}{\partial x\partial s} + \frac{\partial u}{\partial x}\right) + e^s sen\ t\left(\frac{\partial^2 u}{\partial y\partial s} + \frac{\partial u}{\partial y}\right)$$

3.) Aplicando la Regla de la Cadena para $\frac{\partial u}{ts}$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

4.) Determinando la Segunda Derivada

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial \left[\frac{\partial u}{\partial x}\frac{\partial x}{\partial t}\right]}{\partial t} + \frac{\partial \left[\frac{\partial u}{\partial y}\frac{\partial y}{\partial t}\right]}{\partial t}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial \left[\frac{\partial u}{\partial x}\right]}{\partial t}\left(\frac{\partial x}{\partial t}\right) + \left(\frac{\partial u}{\partial x}\right)\frac{\partial \left[\frac{\partial x}{\partial t}\right]}{\partial t} + \frac{\partial \left[\frac{\partial u}{\partial y}\right]}{\partial t}\left(\frac{\partial y}{\partial t}\right) + \left(\frac{\partial u}{\partial y}\right)\frac{\partial \left[\frac{\partial y}{\partial t}\right]}{\partial t}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x\partial t} * \frac{\partial x}{\partial t} + \frac{\partial u}{\partial x} * \frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 u}{\partial y\partial t} * \frac{\partial y}{\partial t} + \frac{\partial u}{\partial y} * \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x\partial t}(-e^s sen\ t) + \frac{\partial u}{\partial x}(-e^s \cos t\ ) + \frac{\partial^2 u}{\partial y\partial t}(e^s \cos t) + \frac{\partial u}{\partial y}(-e^s sen\ t)$$

$$\frac{\partial^2 u}{\partial t^2} = -e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t}\right) - e^s \cos t\left(\frac{\partial u}{\partial x}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t}\right) - e^s sen\, t\left(\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial^2 u}{\partial t^2} = -e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial u}{\partial y}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial u}{\partial x}\right)$$

5.) Demostrando

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left[e^s \cos t\left(\frac{\partial^2 u}{\partial x \partial s} + \frac{\partial u}{\partial x}\right) + e^s sen\, t\left(\frac{\partial^2 u}{\partial y \partial s} + \frac{\partial u}{\partial y}\right) + \left(-e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial u}{\partial y}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial u}{\partial x}\right)\right)\right]$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left[e^s \cos t\left(\frac{\partial^2 u}{\partial x \partial s} + \frac{\partial u}{\partial x}\right) + e^s sen\, t\left(\frac{\partial^2 u}{\partial y \partial s} + \frac{\partial u}{\partial y}\right) - e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial u}{\partial y}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial u}{\partial x}\right)\right]$$

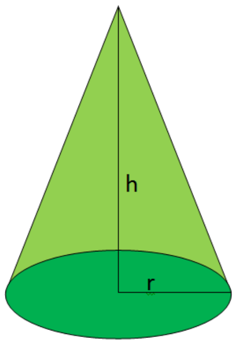
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{e^{2s}}\left[e^s \cos t\left(\frac{\partial^2 u}{\partial x \partial s}\right) + e^s sen\, t\left(\frac{\partial^2 u}{\partial y \partial s}\right) - e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t}\right)\right]$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{e^{2s}}\left[e^s \cos t\left(\frac{\partial^2 u}{\partial x \partial s}\right) + e^s sen\, t\left(\frac{\partial^2 u}{\partial y \partial s}\right) - e^s sen\, t\left(\frac{\partial^2 u}{\partial x \partial t}\right) + e^s \cos t\left(\frac{\partial^2 u}{\partial y \partial t}\right)\right]$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Tipo III Aplicación de la regla de la cadena

Problema 1 ***La altura de un cono circular recto crece a razón de 40 cm/min y el radio disminuye a razón de 15 cm/min. Calcule la razón de cambio del volumen en el instante en que la altura es de 200 cm y el radio es 60 cm.***



1º PAS O: Esquema y datos:

$$h = 200cm \; ; \; r = 60cm;$$

$$\frac{\partial h}{\partial t} = 40cm/min; \; = \frac{\partial r}{\partial t} - 15cm/min$$

2º PASO: Identificar la función y sus variables:

$$v = \frac{\pi r^2 h}{3}$$

v : *Variable dependiente*

$(h.r)$: *Variables intermedias*

t : *Variable independiente*

3º PASO: Aplicación de la regla de la cadena para calcular la tasa de variación de volumen:

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial v}{\partial r}\frac{\partial r}{\partial t}$$

$$\frac{\partial v}{\partial t} = \frac{\pi r^2}{3}\frac{\partial h}{\partial t} + \frac{2\pi r h}{3}\frac{\partial r}{\partial t}$$

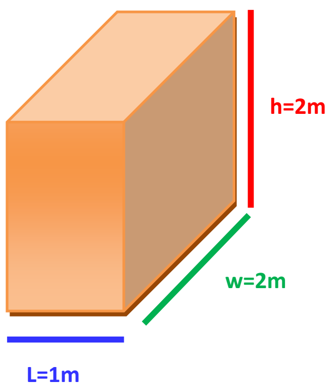
$$\frac{\partial v}{\partial t} = \frac{\pi 60^2}{3}(40) + \frac{2\pi(60)(200)}{3}(-15) = -7200\pi$$

CONCLUSION: *El volumen del recipiente disminuye a razón de* $7200\pi cm^3$ *cuando la altura crece y el radio disminuye*

Problema 2 ***La longitud ℓ, ancho w y la altura h de una caja cambia con el tiempo. En un cierto instante, las dimensiones son $\ell = 1\ m$ y $w = h = 2\ m$, y ℓ y w se incrementan a razón de 2m/s, en tanto que h disminuye a razón de 3m/s. Encuentre en ese instante las razones a las cuales las siguientes magnitudes cambian.***

a).- El volumen
b).- El área superficial
c).- La longitud de la diagonal

1º PASO: Esquema y datos:



2º PASO: Identificar la función y sus variables:

$\ell = 1m$	$\dfrac{\partial \ell}{\partial t} = 2\,{}^{m}/_{s}$	Variable Intermediaria: ℓ, w, h
$w = h = 2m$	$\dfrac{\partial w}{\partial t} = 2\,{}^{m}/_{s}$	Variable Independiente: t
	$\dfrac{\partial h}{\partial t} = -3\,{}^{m}/_{s}$	Variable Dependiente: V

a) Función Volumen : $V = \ell w h$

$$\frac{\partial V}{\partial \ell} = wh \qquad \frac{\partial V}{\partial w} = \ell h \qquad \frac{\partial V}{\partial h} = \ell w$$

$$\frac{\partial V}{\partial \ell} = (2m)(2m) \qquad \frac{\partial V}{\partial w} = (1m)(2m) \qquad \frac{\partial V}{\partial h} = (1m)(2m)$$

$\frac{\partial V}{\partial \ell} = 4m^2$ $\frac{\partial V}{\partial w} = 2m^2$ $\frac{\partial V}{\partial h} = 2m^2$

3º PASO: Aplicación de la regla de la cadena para calcular la tasa de variación de volumen:

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial \ell}\frac{\partial \ell}{\partial t} + \frac{\partial V}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial V}{\partial h}\frac{\partial h}{\partial t}$$

$$\frac{\partial V}{\partial t} = (4m^2)(2\,{}^{m}/_{s}) + (2m^2)(2\,{}^{m}/_{s}) + (2m^2)(-3\,{}^{m}/_{s})$$

$$\frac{\partial V}{\partial t} = 6\frac{m^3}{s}$$

CONCLUSION: *La Razón con la que aumenta el Volumen es de* $6\frac{m^3}{s}$

b) Función Área: $\boldsymbol{A = 2\ell w + 2\ell h + 2wh}$

$\frac{\partial A}{\partial \ell} = 2w + 2h$ $\frac{\partial A}{\partial w} = 2\ell + 2h$ $\frac{\partial A}{\partial h} = 2\ell + 2w$

$\frac{\partial A}{\partial \ell} = 2(2) + 2(2)$ $\frac{\partial A}{\partial w} = 2(1) + 2(2)$ $\frac{\partial A}{\partial h} = 2(1) +$ $2(2)$

$\frac{\partial A}{\partial \ell} = 8$ $\frac{\partial A}{\partial w} = 6$ $\frac{\partial A}{\partial h} = 6$

3º PASO: Aplicación de la regla de la cadena

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial \ell}\frac{\partial \ell}{\partial t} + \frac{\partial A}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial A}{\partial h}\frac{\partial h}{\partial t}$$

$$\frac{\partial A}{\partial t} = (4)(2) + (6)(2) + (6)(-3)$$

$$\frac{\partial A}{\partial t} = 10\frac{m^2}{s}$$

CONCLUSION: *La razón con la que el área aumenta es de* $10\frac{m^2}{s}$

c) Función Diagonal: $\boldsymbol{D = \sqrt{\ell^2+w^2+h^2}}$

$$\frac{\partial D}{\partial \ell} = \frac{\ell}{\sqrt{\ell^2+w^2+h^2}} \qquad \frac{\partial D}{\partial w} = \frac{w}{\sqrt{\ell^2+w^2+h^2}} \qquad \frac{\partial D}{\partial h} = \frac{h}{\sqrt{\ell^2+w^2+h^2}}$$

$$\frac{\partial D}{\partial \ell} = \frac{1}{\sqrt{1+2^2+2^2}} \qquad \frac{\partial D}{\partial w} = \frac{2}{\sqrt{1+2^2+2^2}} \qquad \frac{\partial D}{\partial h} = \frac{2}{\sqrt{1+2^2+2^2}}$$

$$\frac{\partial D}{\partial \ell} = \frac{1}{3} \qquad \frac{\partial D}{\partial w} = \frac{2}{3} \qquad \frac{\partial D}{\partial h} = \frac{2}{3}$$

3º PASO: Aplicación de la regla de la cadena

$$\frac{\partial D}{\partial t} = \frac{\partial D}{\partial \ell}\frac{\partial \ell}{\partial t} + \frac{\partial D}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial D}{\partial h}\frac{\partial h}{\partial t}$$

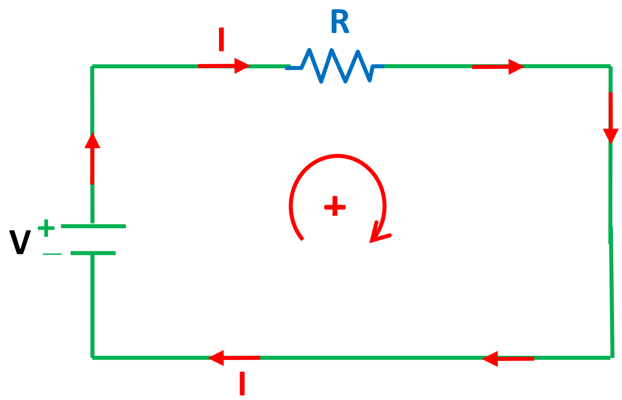
$$\frac{\partial D}{\partial t} = \left(\frac{1}{3}\right)(2) + \left(\frac{2}{3}\right)(2) + \left(\frac{2}{3}\right)(-3)$$

$$\frac{\partial D}{\partial t} = 0$$

CONCLUSION: *La diagonal no sufre cambio su razón es 0*

Problema 3 *El voltaje V en un circuito eléctrico simple disminuye con lentitud a medida que la batería se gasta. La resistencia R se incrementa lentamente cuando el resistor se calienta. Mediante la ley de Ohm, $V = IR$, determine cómo cambia la corriente I en el momento en que* $R = 400\Omega,\ I = 0.08\ A,\ \frac{dV}{dt} = -0.01\ {}^{V}/_{s}$ *y* $\frac{dR}{dt} = 0.03\ {}^{\Omega}/_{s}$

1º PASO: Esquema y datos:



2º PASO: Identificar la función y sus variables:

$R = 400\Omega$ $\qquad \frac{\partial V}{\partial t} = -0.01\ {}^{V}/_{s}$ $\qquad$ Variable Intermediaria: R, I

$I = 0.08$ A $\qquad \frac{\partial R}{\partial t} = 0.03\ {}^{\Omega}/_{s}$ $\qquad$ Variable Independiente: t

Variable Dependiente: V

Función: $V = IR$

3º PASO: Realizar las derivadas parciales

$$\frac{\partial V}{\partial I} = R \qquad \frac{\partial V}{\partial R} = I$$

$$\frac{\partial V}{\partial I} = 400\Omega \qquad \frac{\partial V}{\partial R} = 0.08\ A$$

4º PASO: Aplicación de la regla de la cadena

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial I}\frac{\partial I}{\partial t} + \frac{\partial V}{\partial R}\frac{\partial R}{\partial t}$$

Despejando de RC y Reemplazando

$$\frac{\partial V}{\partial I}\frac{\partial I}{\partial t}=\frac{\partial V}{\partial t}-\frac{\partial V}{\partial R}\frac{\partial R}{\partial t}$$

$$\frac{\partial I}{\partial t}=\frac{\frac{\partial V}{\partial t}-\frac{\partial V}{\partial R}\frac{\partial R}{\partial t}}{\frac{\partial V}{\partial I}}$$

$$\frac{\partial I}{\partial t}=\frac{-0.01-(0.08)(0.03)}{400}$$

$$\frac{\partial I}{\partial t}=-0.00031\,\frac{A}{s}$$

CONCLUSION: *La corriente I respecto al tiempo disminuye a razón de* $0.00031\,\frac{A}{s}$