

Introduction of K-Map (Karnaugh Map)

In many digital circuits and practical problems, we need to find expression with minimum variables. We can minimize Boolean expressions of 3, 4 variables very easily using K-map without using any Boolean algebra theorems. K-map can take two forms Sum of Product (SOP) and Product of Sum (POS) according to the need of problem. K-map is table like representation, but it gives more information than TRUTH TABLE. We fill grid of K-map with 0's and 1's then solves it by making groups.

Steps to solve expression using K-map-

1. Select K-map according to the number of variables.
2. Identify minterms or maxterms as given in problem.
3. For SOP put 1's in blocks of K-map respective to the minterms.
4. For POS put 0's in blocks of K-map respective to the maxterms.
5. Make rectangular groups containing total terms in power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.
6. From the groups made in step 5 find the product terms and sum them up for SOP form.
- 7.

SOP FORM :

1. K-map of 3 variables –

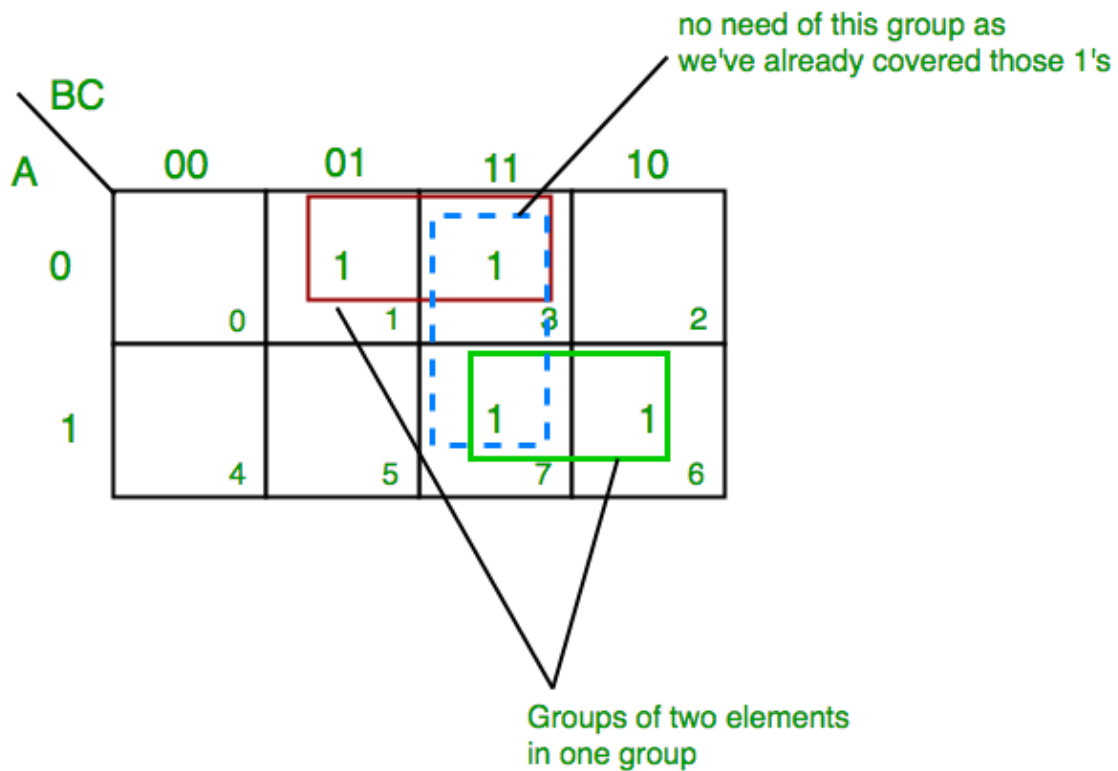
| | | | | | |
|---|------|--------|-------|------|-------|
| | | BC | | | |
| | | B'C' | B'C | BC | BC' |
| | | 00 | 01 | 11 | 10 |
| A | A' 0 | A'B'C' | A'B'C | A'BC | A'BC' |
| | | 0 | 1 | 3 | 2 |
| | A 1 | AB'C' | AB'C | ABC | ABC' |
| | | 4 | 5 | 7 | 6 |

SOP(MINTERMS)

8 Blocks = 1
 4 Blocks = 1 variable term
 2 Blocks = 2 variable term
 1 Block = 3 variable term

K-map SOP form for 3 variables

$$Z = \sum A,B,C(1,3,6,7)$$



From **red** group we get product term—
 $A'C$

From **green** group we get product term—
 AB

Summing these product terms we get- **Final expression** ($A'C + AB$)

2. K-map for 4 variables –

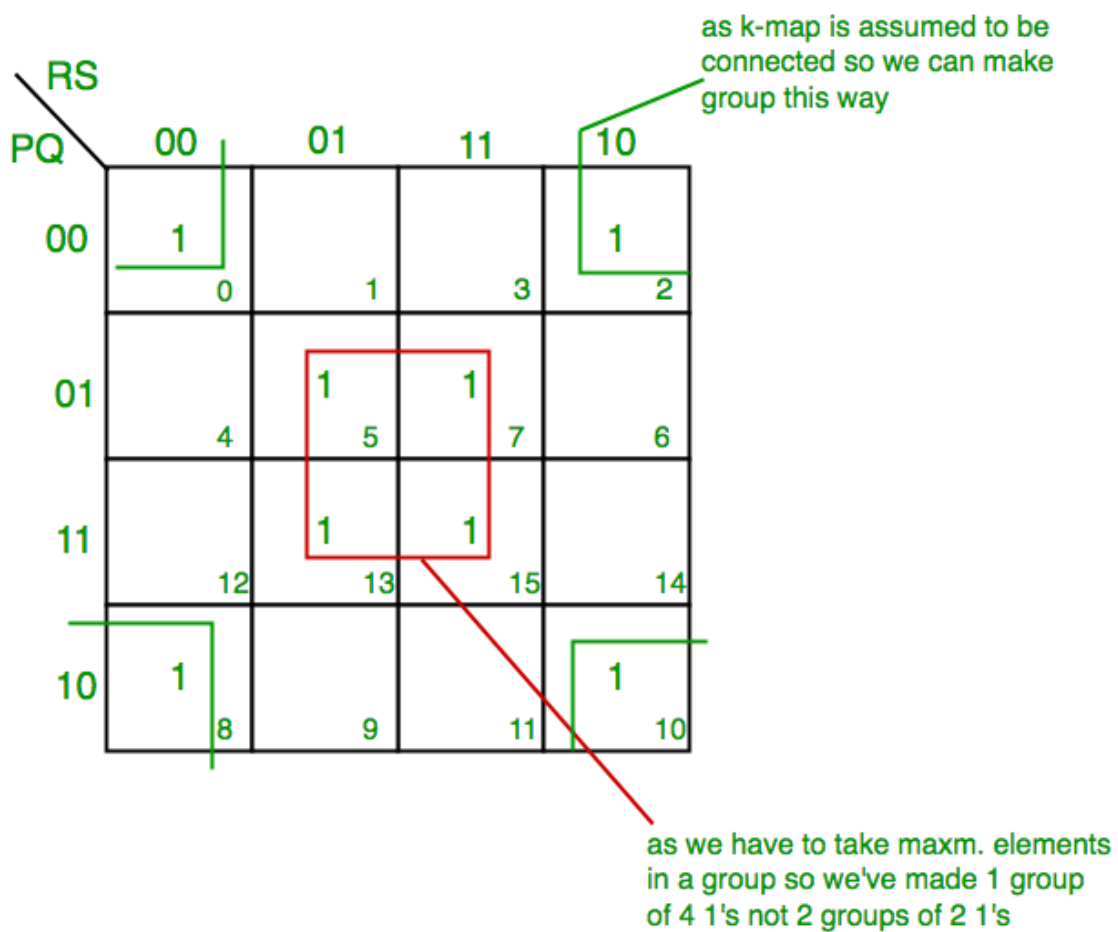
| | | CD | | | |
|----|------|----------|---------|--------|---------|
| | | C'D' | C'D | CD | CD' |
| AB | A'B' | A'B'C'D' | A'B'C'D | A'B'CD | A'B'CD' |
| | A'B | A'BC'D' | A'BC'D | A'BCD | A'BCD' |
| AB | AB | ABC'D' | ABC'D | ABCD | ABCD' |
| | AB' | AB'C'D' | AB'C'D | AB'CD | AB'CD' |

SOP(MINTERMS)

16 Blocks = 1
 8 Blocks = 1 variable term
 4 Blocks = 2 variable term
 2 Blocks = 3 variable term
 1 Block = 4 variable term

K-map 4 variable SOP form

$$F(P,Q,R,S) = \sum(0,2,5,7,8,10,13,15)$$



From **red** group we get product term—
QS

From **green** group we get product term—
Q'S'

Summing these product terms we get- **Final expression (QS+Q'S')**

POS FORM :

1. K-map of 3 variables –

| | | | | | |
|---|---|-------------|--------------|---------------|--------------|
| | | BC | | | |
| | | B+C 00 | B+C' 01 | B'+C' 11 | B'+C 10 |
| A | 0 | A+B+C 0 | A+B+C' 1 | A+B'+C' 3 | A+B'+C 2 |
| | 1 | A'+B+C 4 | A'+B+C' 5 | A'+B'+C' 7 | A'+B'+C 6 |

POS (MAXTERMS)

8 Blocks = 0
 4 Blocks = 1 variable term
 2 Blocks = 2 variable term
 1 Block = 3 variable term

K-map 3 variable POS form

$$F(A,B,C)=\pi(0,3,6,7)$$

| | | | | | |
|---|---|----|----|----|----|
| | | BC | | | |
| | | 00 | 01 | 11 | 10 |
| A | 0 | 0 | | 0 | |
| | 1 | | | 0 | 0 |

From **red** group we find terms
A B

Taking complement of these two

A' B'

Now **sum** up them

$$(A' + B')$$

From **brown** group we find terms

$$B \ C$$

Taking complement of these two terms

$$B' \ C'$$

Now sum up them

$$(B' + C')$$

From **yellow** group we find terms

$$A' B' C'$$

Taking complement of these two

$$A \ B \ C$$

Now **sum** up them

$$(A + B + C)$$

We will take product of these three terms : **Final expression –**

$$(A' + B') (B' + C') (A + B + C)$$

2. K-map of 4 variables –

| | | CD | C+D | C+D' | C'+D' | C'+D |
|--------|-------|----|-----|------|-------|------|
| AB | | 00 | 01 | 11 | 10 | |
| | A + B | 00 | 01 | 11 | 10 | |
| A + B' | 01 | 01 | 11 | 10 | 00 | |
| | A'+B' | 11 | 10 | 00 | 01 | |
| A'+B | 11 | 10 | 01 | 00 | 11 | |
| | A'+B | 10 | 01 | 00 | 11 | |

POS(MAXTERMS)

POS(MAXTERMS)

16 Blocks = 0

8 Blocks = 1 variable term

4 Blocks = 2 variable term

2 Blocks = 3 variable term

1 Block = 4 variable term

K-map 4 variable POS form

$$F(A,B,C,D)=\pi(3,5,7,8,10,11,12,13)$$

| | | CD | | | |
|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| AB | 00 | 0 | 1 | 0 | 3 |
| | 01 | 4 | 0 | 0 | 7 |
| | 11 | 0 | 0 | | |
| | 10 | 0 | | 0 | 0 |

From **green** group we find terms
 $C' D B$

Taking their complement and summing them
 $(C+D'+B')$

From **red** group we find terms
 $C D A'$

Taking their complement and summing them
 $(C'+D'+A)$

From **blue** group we find terms
 $A C' D'$

Taking their complement and summing them
 $(A'+C+D)$

From **brown** group we find terms
 $A B' C$

Taking their complement and summing them
 $(A'+B+C')$

Finally, we express these as product –

$$(C+D'+B').(C'+D'+A).(A'+C+D).(A'+B+C')$$