Intro to AI and ML

Matrix Project

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Question

Two sides of a rhombus are along the lines, x - y + 1= 0 and 7x - y - 5 =0. If its diagonals intersect at (-1,-2), then find all its vertices.

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Question in Matrix form

Two sides of a rhombus are along the lines

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{x} + 1 = 0$$

$$\begin{bmatrix} 7 & -1 \end{bmatrix} \mathbf{x} - 5 = 0$$

If its diagonals intersect at $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$, then find all its vertices.

Solution

We have equations of two lines in matrix form. We can find one vertex by finding point of intersection of both the lines.

Vertex **A** from intersection of lines **L1** and **L2**.

$$\mathrm{n}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \textit{and} \quad n_2 = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} 1 & -1 \\ 7 & -1 \end{bmatrix} \quad \textit{and} \quad |N^T| = 6$$

$$N^{-T} = \begin{bmatrix} -1/6 & 1/6 \\ -7/6 & 1/6 \end{bmatrix}$$

$$p = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$\mathbf{A} = \mathrm{N}^{-T} p = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solution

Also given, mid-point O $= \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

Since we have **A** and **O**, we can find opposite vertex of A which is **C**.

C = 2*O - A $= \begin{bmatrix} -3 \\ -6 \end{bmatrix}$

We know that diagonals of rhombus are perpendicular to each other. So, director vector of **AC** is normal vector of **BD**.

n = A - C $= \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

We can find equation of BD using mid-point O.

p = (n.T)O = -20

BD: $\begin{bmatrix} 4 & 8 \end{bmatrix} \mathbf{x} + 20 = 0$

We can find other two vertices B and D using point of intersection of lines **AB,BD** and **AD,BD** respectively like we did between L1 and L2.

B $= \begin{bmatrix} -7/3 \\ -4/3 \end{bmatrix}$ and **D** $= \begin{bmatrix} 1/3 \\ -8/3 \end{bmatrix}$

Graph