19/07

Let $Ax = b$ be a system of linear eq^n where A is an $m \times n$ matrix & b is a $m \times 1$ col vector and x is an $n \times 1$ col. vector unknowns. WOF is false?

A) The system has a sol^n if and only if, both A and the augmented matrix $[A\, b]$ have the same rank. (T)

B) If $m < n$ & b is a zero vector, then the system has infinitely many sol^n (T)

C) If $m = n$ & b is a non-zero vector, then the system has a unique sol^n

D) The system will have only a trivial sol^n when $m = n$, b is the zero vector ~~(rank)~~ rank$(A) = n$ (T)

$Ax = b$

$\rho(A:b) = \rho(A) = n \rightarrow$ unique

$\rho(A:b) = \rho(A) = r \longrightarrow \infty$ many.

A) $\left[\begin{array}{ccc|c} 0 & & & \\ & 0 & & \\ & & & 0 \end{array}\right]$ ↑ pivot

$\rho(A) = 2$

$\rho(A:b) = 3$

ek sol^n exist karega.

B) $m < n$

$\left[\begin{array}{ccc|c} 0 & & & 0 \\ & 0 & & 0 \end{array}\right]$

LI, LD, B

Eg: $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{array}{c} \quad A \qquad\qquad b \\ \left[\begin{array}{ccc|c} 1 & 3 & 4 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \end{array}$

$v_1 \neq v_2 \neq v_3$

$Ax = b = 0$

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x + 3y + 4z = 0$

$-2y - 2z = 0$

$z = -1$ → free variable

x, y, z

$3 - 2 = 1$ → free variable

3 → scaling factor, 2 → vectors

$\boxed{z = -1}$

$\Rightarrow -2y + 2 = 0$

$\boxed{y = 1}$

$x + 3 - 4 = 0$

$\boxed{x = +1}$

col = 3

LI col = 2

$3 - 2 = 1$ → # LD col

C) $m = n$ $\rho(A) = 2$

$\left[\begin{array}{ccc|c} 0 & - & - & - \\ - & 0 & - & - \\ - & - & - & - \end{array}\right]$ $m = 3$, $n = 3$

echelon form

∞ many sol^n

UNIQUE SOL^n $\rho(A:b) = \rho(A) = n = 3$

echelon form

$\left[\begin{array}{ccc|c} 0 & - & - & - \\ - & 0 & - & - \\ - & - & - & 0 \end{array}\right]$ $\rho(A) = 2$, $\rho(A:b) = 3$

No sol^n

$\left[\begin{array}{ccc|c} 0 & - & - & - \\ - & 0 & - & - \\ - & - & 0 & - \end{array}\right]$ → Unique sol^n

D) $\rho(A) = n$

$\left[\begin{array}{ccc|c} 0 & - & - & 0 \\ - & 0 & - & 0 \\ - & - & 0 & 0 \end{array}\right]$

LI

jitne col hai utni rank toh sare vectors LI honge

$\{v_1\ v_2\ v_3\}$ $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$

$\forall\ \alpha_i = 0$

⇒ sare scaling factor 0.

GATE 2016

Q. Consider the system, each consisting of m linear eq^n in n variable

I) if $m < n$, then all such system have a sol^n (F)

II) If $m > n$, then none of these system has a sol^n (F)

III) If $m = n$ then there exists a system exits which has a sol^n (T)

WOF is CORRECT

a) I, II & III are true

b) Only II & III are true

c) Only III is true

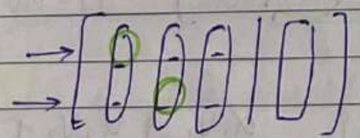
d) None of them

m linear eq.

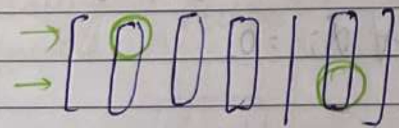
n vari

I) $\begin{bmatrix} a_1 & \cdots & a_n \\ \vdots & & \\ a_{m} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ $m \times n$

m ≶ n rows col

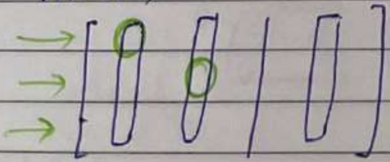


∓ 1 sme free.



No sol^n

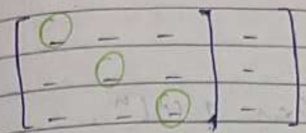
II) $m > n$,



Infinitely many sol^n

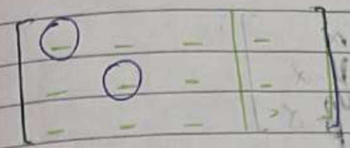
no sol^n ek case

III $m = n$,



Unique sol^n

or



Infinitely many sol^n

or

No sol^n

Q. IIT JAM

Suppose $\alpha, \beta, \gamma \in \mathbb{R}$ Consider the given system of linear eq^n

$$x + y + z = \alpha$$
$$x + \beta y + z = \gamma$$
$$x + y + \alpha z = \beta$$

If the system has atleast one sol^n then WOTF statements are True.

a) if $\alpha = 1$ then $\gamma = 1$

b) if $\beta = 1$ then $\gamma = \alpha$

c) if $\beta \neq 1$ then $\alpha = 1$

d) if $\gamma = 1$ then $\alpha = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 1 & \beta & 1 & \gamma \\ 1 & 1 & \alpha & \beta \end{array}\right]$$

$$Ax = b$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 0 & \beta-1 & 0 & \gamma \\ 0 & 0 & \alpha-1 & \beta \end{array}\right]$$

If the given eq^n has 1 sol^n.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 0 & \beta-1 & 0 & \gamma-\alpha \\ 0 & 0 & \alpha-1 & \beta-\alpha \end{array}\right]$$

$\beta - 1 = 1$ $\quad$ $\alpha - 1 = 1$

$\beta = 2$ $\quad$ $\alpha = 2$

~~a)~~ $\alpha = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & \beta-1 & 0 & \gamma-1 \\ 0 & 0 & 0 & \beta-1 \end{array}\right] \Rightarrow \text{Infinite sol}^n$$

$\alpha = 1$ then $\gamma = 1$ for infinite many sol^n

& $\beta = 1$

~~b)~~ if $\beta = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \gamma-1 \\ 0 & 0 & \alpha-1 & \beta-1 \end{array}\right] \Rightarrow \text{Infinite sol}^n$$

pivot isse bhi ban jayega

$\gamma = \alpha$.

c) if $\beta \neq 1$ then $\alpha = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & \beta-1 & 0 & \gamma-1 \\ 0 & 0 & 0 & \beta-1 \end{array}\right] \quad \text{Infinity}$$

$\beta \neq 1$ the $\gamma = 1$. $\Rightarrow$ for infinite sol^n.

...pivot b ne chahiye ...sol^n

D) if $\gamma = 1$ then $\alpha = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & \beta-1 & 0 & \gamma-1=0 \\ 0 & 0 & \alpha-1 & \beta-1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & \beta-1 & 0 & 0 \\ 0 & 0 & 0 & \beta-1 \end{array}\right] = \text{if } \beta \neq 1 \text{ No sol}^n.$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \alpha \\ 0 & \beta-1 & 0 & \gamma-\alpha \\ 0 & 0 & \alpha-1 & \beta-\alpha \end{array}\right]$$

$\alpha - 1 = 0$

$\gamma - \alpha = \gamma - 1 = 0$

$\beta = 2$

$\beta = 1$

isko pivot nhi banana hai

$\alpha = 1, \gamma = 1, \beta = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Q JEE ADV 2023

Let α, β, and γ be real no's consider & the following system of linear eq^n.

$$x + 2y + z = 7$$
$$x + \alpha z = 11$$
$$2x + 3y + \beta z = \gamma$$

List I	List II
P) if $\beta = \frac{1}{2}(7\alpha - 3)$ & $\gamma = 28$ then system	① No sol^n

Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ &
$\gamma \neq 28$ then the system has

R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where
$\alpha = 1$ & $\gamma \neq 28$
then the system has

(2) Unique sol^n

(3) Infinite many sol^n

R.E.V

Options

A) $P \to 3$; $Q \to 1$; $R \to 2$

B) $P \to 2$; $Q \to 1$; $R \to 3$

C) $P \to 3$; $Q \to 2$; $R \to 1$

D) $P \to 1$; $Q \to 1$; $R \to 2$.

Q. $\{\ln x, \ln x^2, \ln x^3\}$ is this set LI or LD

& $\{\ln x, \ln x^2, \ln x^3\}$

$\alpha \ln x + \beta 2 \ln x + \gamma 3 \ln x = 0$

$(\alpha + 2\beta + 3\gamma) \ln x = 0$

$\ln x \neq 0$ so $\alpha + 2\beta + 3\gamma = 0$

$\alpha = 1, \beta = 1, \gamma = -1$

means there exist non-zero scalar
hence LD

REVISION

LI iff all $\alpha_i = 0$ is the only soln.