LEC - 13

23/07

Q $W = \{(x_1, x_2, x_3 \cdots x_n) \mid x_1 + x_2 + x_3 \cdots x_n = 0\}$

W is a sub space of $\mathbb{R}^n(\mathbb{R})$

$\dim(W) = ?$

dim(sub-space) = dim(vector-space) − # LI restriction eg $V = \mathbb{R}^3(\mathbb{R})$

$W = \{ (x, y, z) \mid x + y + z = 0 \} \in \mathbb{R}^3$

$\boxed{x = -(y+z)}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad a = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \qquad a+b = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \in W$

$\Downarrow$

$\begin{bmatrix} -(y+z) \\ y \\ z \end{bmatrix} = \begin{bmatrix} -y \\ y \\ 0 \end{bmatrix} + \begin{bmatrix} -z \\ 0 \\ z \end{bmatrix}$

$= y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ ⇒ yeh do vector ki help se basis me koi bhi vector ko laa skte.

$\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\boxed{c_1 = -1 \qquad c_2 = 1}$

$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} +1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

dim (W) = dim (V) − # LI restriction

LI restriction jitne pivot element

$x + y + z = 0$

pivot → $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Given question.

$x_1 = -(x_2 + x_3 \cdots x_n)$

pivot

$\begin{matrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots \\ 0 & 0 \\ 0 & 0 \\ \vdots \end{matrix}$ # LI restriction = n−1

dim(W) = dim(V) − # LI restriction

$= n - (n-1)$

=

Q: $W = \{ (x, y, z) \mid x + y + z = 0 \ \& \ x + 2y + 3z = 0 \}$

dim(W) = ?

$x + y + z = 0 \qquad\qquad x + 2y + 3z = 0$

$x = -(y+z) \longrightarrow -(y+z) + 2y + 3z = 0$

$y + 2z = 0$

$y = -2z$

$\boxed{y = -2z}$

$x = -y - z$

$= -(-2z) - z$

$x = 2z - z = z$

$\boxed{x = z}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ -2z \\ z \end{bmatrix} = z \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

iss ek vector ke help se koi bhi vector desive kr skte hai.

$x + y + z = 0$ & $x + 2y + 3z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

· Echelon form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\#$ LI ~~[illegible]~~ $r^n = 2.$

$\dim(W) = 3 (1 - 2) = 1$