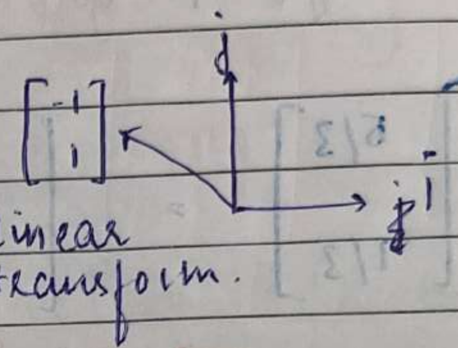
A^{-1}

EIGEN VALUES AND EIGEN VECTOR

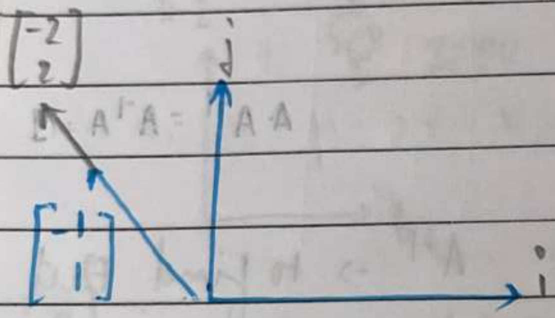
01/08

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$\hat{i}$ lands $\quad$ $\hat{j}$ lands $\quad$ $\vec{v}$ $\quad$ $\vec{w}$

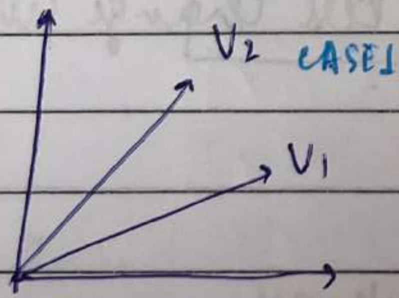


linear transform.

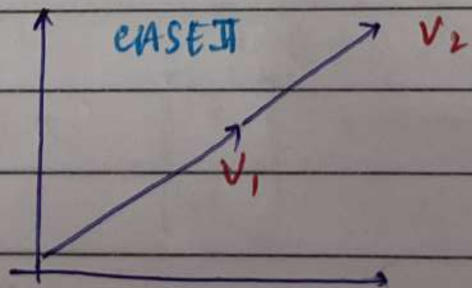


ags humlog his matrix se multiple kare toh linear transformation $\vec{w}$ is transformed form of $\vec{v}$

$\vec{w}$ is nothing but scaled version of $\vec{v}$



$T(V_1) \longrightarrow V_2$



$T(V_1) \longrightarrow V_2$

$A \cdot \vec{v} = \lambda \vec{v}$

is known as eigen value & vector.

A x

n x n

CASE 1 :

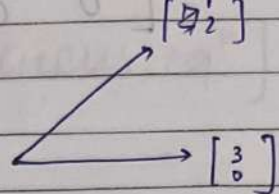
CASE 2 :

Eg = A

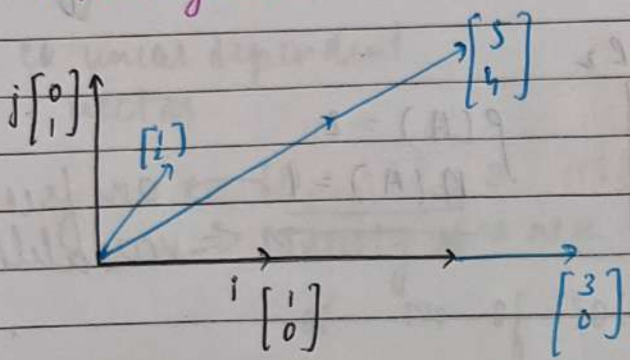
Scalable value. → Eigen value.

eg. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$



Pusana system.



$AX = 0$ [homogenous]

$n \times n$

CASE 1: $\rho(A) = n$ $\quad X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is the only sol^n

CASE 2: $\rho(A) = r < n$ $\quad$ Rank nullity theorem:

RANK + NULLITY = #COLUMNS

$r + \text{nullity}(A) = n$

$\text{nullity}(A) = n - r$ ↳ # of sol^n is atleast $(n - r)$

$(n-r)$ free variable

Eg: $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & 1 \\ 1 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & -1 \end{bmatrix}$

$R_2 \leftarrow \frac{R_2}{3}$ $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow$ $R_3 \leftarrow R_3 - R_2$ $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Row echelon form

Reduce Echelon form

R_1 & $R_1 - 2R_2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\rho(A) = 2$, $n(A) = 1 \rightarrow$ one free variable.

$\rho + n = 3$

$x + y = 0 \Rightarrow \boxed{x = -y}$ → x ki value y ke term me La skte

$z = 0$

$\begin{bmatrix} -y \\ y \\ 0 \end{bmatrix} = y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -100 \\ 100 \\ 0 \end{bmatrix} \begin{bmatrix} -250 \\ 250 \\ 0 \end{bmatrix}$

$k \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ $y \in \mathbb{R}$

$\boxed{A\vec{x} = \lambda \vec{x}}$ ← EIGEN VECTOR

E·VEC EIGENVALUE

$A\vec{x} = \lambda(I\vec{x})$

$A\vec{x} = (\lambda I)\vec{x}$

$A\vec{x} - (\lambda I)\vec{x} = 0$

$\boxed{(A - \lambda I)\vec{x} = 0}$

$\underbrace{(A - \lambda I)}_{M}\vec{x} = 0$

$M\vec{x} = 0$ [Homogeneous eqⁿ]

$|A - \lambda I| = 0$ $\vec{x} \neq 0$

ek linear dependent vectors

$|A - \lambda I| = 0 = |M|$ kissi bhi ~~det~~ matrix ka det 0 hai toh

$\Rightarrow$ Nullity $\neq$ NZ

$\propto$ No. of solⁿ

det (A) = 0

$\rho(A) \neq n$ $n(A) \neq 0$ → free variable

infinite solⁿ

$\underbrace{(A - \lambda I)}_{M}\vec{x} = 0$

$M\vec{x} = 0$

EIGEN VECTORS

$|A - \lambda I| = 0$

Solⁿ corresponding to this is eigen vectors.

Eg. $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ $A\vec{x} = \lambda\vec{x}$

$(A - \lambda I)\vec{x} = 0$

$A - \lambda I$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $|A - \lambda I| = 0$

$$= \begin{bmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix} \rightarrow \begin{vmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$\boxed{(3-\lambda)(2-\lambda) = 0}$$

char eq^n $\lambda_1 = 2$, $\lambda_2 = 3$

$$6 - 3\lambda - 2\lambda + \lambda^2 = 0$$

$$\boxed{\lambda^2 - 5\lambda + 6 = 0}$$

↳ char eq^n in terms of λ

let root corresponding to this character eq^n be λ_1 & λ_2

$\lambda_1 = 2$ $\lambda_2 = 3$.

2×2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ 2 degree $\rightarrow ax^2 + bx + c = 0$

$$\lambda^2 - (5)\lambda + 6 = 0$$

(5): TRACE OF THE MATRIX — sum of the principal diagonal elements

6: $|A|$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0$$

$$ad - a\lambda - d\lambda + \lambda^2 - bc = 0$$

$$\cancel{\lambda^2 - a\lambda - d\lambda}$$

$$\lambda^2 - (a+d)\lambda + (ad - bc) = 0$$

↳ CHARACTERISTIC

$$\boxed{\lambda^2 - (Tr)\lambda + det = 0}$$

$$\lambda^2 - (a+d)\lambda + (ad-bc) = 0 \begin{cases} \rightarrow \lambda_1 \\ \rightarrow \lambda_2 \end{cases}$$

$$\lambda_1 + \lambda_2 = \frac{-b}{a}$$

$$\lambda_1 + \lambda_2 = -(-(a+d))$$

$$\boxed{\lambda_1 + \lambda_2 = a+d}$$

$$\lambda_1 \lambda_2 = (ad - bc) = det(A)$$

$ax^2 + bx + c = 0$
sum of roots $= \frac{-b}{a}$
prod of roots $= \frac{c}{a}$

- sum of
- prod of eigen value = det of matrix.

$$\lambda^2 - (Tr)\lambda + det(A) = 0$$

$$\lambda_1 + \lambda_2 = Trace(A)$$

$$\lambda_1 \lambda_2 = det(A)$$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ $\boxed{\lambda_1 + \lambda_2 = 5}$

$$\lambda_1 \lambda_2 = 6$$

$$(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2$$

$$= 25 - 24$$

$$\boxed{\lambda_1 - \lambda_2 = \pm 1}$$

$\lambda_1 + \lambda_2 = 5$ $\lambda_1 + \lambda_2 = 5$

$\lambda_1 - \lambda_2 = 1$ $\lambda_1 - \lambda_2 = -1$

$$2\lambda_1 = 6$$

$$\lambda_1 = 3 , \lambda_2 = 2$$

2×2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$(a-\lambda)(d-\lambda) - bc = 0$

$ad - a\lambda - d\lambda + \lambda^2 - bc = 0$

$\lambda^2 - (a+d)\lambda + (ad-bc) = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b = -(a+d)$

$c = (ad-bc)$

$a = 1$

$\lambda = \dfrac{-(-(a+d)) \pm \sqrt{(-(a+d))^2 - 4 \times 1 \times (ad-bc)}}{2 \times 1}$

$= \dfrac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$

$= \dfrac{a+d}{2} \pm \sqrt{\dfrac{(a+d)^2 - 4(ad-bc)}{4}}$

$= \dfrac{a+d}{2} \pm \sqrt{\dfrac{(a+d)^2}{4} - (ad-bc)}$

$= \dfrac{\text{Trace}}{2} \pm \sqrt{\left(\dfrac{\text{trace}}{2}\right)^2 - \text{Det}}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Trace = a+d

Det = ad-bc