10/07

BASIS

Basis is a minimal vectors needed to span a given space.

$\mathbb{R}^n$ $\quad V_k \in \mathbb{R}^n \quad \underbrace{\overbrace{\{ v_1 \cdots v_n \}}^{n} \cup V_k}_{n+1}$

↓

n LI vectors

$= \{ v_1, \cdots v_n, V_k \}$ ← use again

$=$ LINEARLY INDEPENDANT

A vector space has infinitely many basis

Eg. $\mathbb{R}^2$ $\quad \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$ ← no of elements same.

- Linearly independent
- Any where in space
- ★ Minimal vectors.

Q. i) Let $v_i \in \mathbb{R}^n$ and $S = \{ v_i \mid v_i \in \mathbb{R}^n ; 1 \text{ to } 10 \}$ & this set S of vectors is LI then every subset of S is LI. [True / False.] ★

agr teen vector LI for do bhi ~~to~~ LI.

ii) Every superset of ~~*~~S where superset contains few addition vectors $v_k \in \mathbb{R}^n$ is LD. True / False

iii) Let $S = \{v_i \mid v_i \in \mathbb{R}^n\}$, $i = 1$ to $10\}$ this set of vectors is LD then every
a) superset of S is LD. True / False.

$v_k \in \mathbb{R}^n$

$v_k = \alpha_1 v_1 + \alpha_2 v_2 - - - + 0 \neq v_k \quad \alpha_k = 0.$

b) every subset is LD. False.

iv) A set of n vector from $\mathbb{R}^m$ is LD if $n > m$ True / False.

$\mathbb{R}^2$ me agr $\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)$

linearly dependant.

DOT PRODUCT ~~AND~~ / INNER PRODUCT

let us consider 2 vector u & v

$u = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad v = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

$u \cdot v := a_1 b_1 + a_2 b_2 + a_3 b_3 - - - a_n b_n$

* u.v is a scalar quantity therefore it belongs to field.

ORTHOGONAL : VECTORS Have 90° angle b/w them

Eg. $u = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix} \quad u \cdot v = 6 - 3 \neq 8$

$= 6 - 3 + 8 = 11$

★ if $u \cdot v = 0$ then we say that both of these vectors are ORTHOGONAL

Eg. $u = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad v = \begin{bmatrix} 6 \\ k \\ -8 \\ 2 \end{bmatrix}$ u & v are orthogonal find value of k

$u \cdot v = 0$

$6 + 2k - 24 + 8 = 0$

$2k = 10$

$k = 5.$

NORM OF A VECTOR.

har element power 2 se raise kar rhe. P = 2

Euclidean norm

$v_i \in \mathbb{R}^n$

$\|v_i\| = \sqrt{a_1^2 + a_2^2 + - - - a_n^2}$

$u \cdot v = \|u\| \cdot \|v\| \cos\theta$

ECHELON FORM

$v_i \in \mathbb{R}^n$ & $i = 1$ to 10

$\{v_1, v_2, \dots v_{10}\}$

$$v_1 = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \rightarrow \begin{bmatrix} | & | & | & | & & | \\ v_1 & v_2 & v_3 & v_4 & \cdots & v_{10} \\ | & | & | & | & & | \end{bmatrix}_{n \times 10}$$

A matrix 'A' is called an echelon matrix echelon form is the following 2 condition are satisfied.

1. All zero rows if any will be at last/bottom of the matrix.
2. Each leading non-zero entry in a row is to the right of the leading non-zero ~~row~~ entry in the preceding row.

$$\text{RHS} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

First ~~to~~ non-zero entry = leading non-zero

Eg
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 0 & 8 & 9 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$
not in echelon.

not in echelon
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 3 & 6 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
echelon form

1, 2, 5 pivot element

PIVOT ELEMENT

If a given matrix satisfies echelon form then the leading non-zero element in every row is known as the pivot element.

ROW REDUCTION

with the help of elementary opsⁿ we will transform matrix A to it echelon form B.

HOW TO CONVERT A GIVEN MATRIX INTO ECHELON FORM

1. $R_i \rightarrow kR_i$
2. $R_i \leftrightarrow R_j$
3. $R_i \rightarrow R_i + kR_j$

} Elementary opsⁿ.

Eg.
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad v_i \in \mathbb{R}^2$$

$R_2 \rightarrow R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$
pivot element ~~=~~ 1, -2

Eg.
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$
pivot element 1.

chalitye.

CONCEPT OF LI / LD IN EF

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ is called basis vector $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

if given set of vectors are LI then no. of pivot elements are equal to ~~no. of rows~~ dimension

if given set of vectors are LD then no of element always less than no. of rows.

Q. $A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$

$V_i \in \mathbb{R}^4$

$R_1 \longleftrightarrow R_4$

$A = \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix}$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$

$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & 6 \\ 0 & 5 & 10 & -15 & -15 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix}$

$R_3 \to R_3 - \frac{5}{2} R_2$

$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix}$ $10 - \frac{5}{2} \times 4$ $-15 - \left(\frac{5}{2} \times -6\right)$

$R_3 \longleftrightarrow R_4$

$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & 6 \\ 0 & -3 & -6 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 + \left(\frac{3}{2}\right) R_2$ $4 + \left(\frac{3}{2}\right) \times -6$

$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & 6 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $-3 + \left(\frac{3 \times 2}{2}\right)$ $-6 + \frac{3}{2} \times 4$ $9 + \frac{3}{2} \times 6$ $4 + \frac{3}{2} \times -6$ $4 - 6$

$R_2 \to \frac{R_2}{2}$ $\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & +3 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

* Row ~~Reduced~~ echelon form is not unique. (need not be unique)
* Reduced echelon form is unique.
* In reduced echelon form every pivot element is 1.
* All elements above & below pivot element in a column is 0.

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 3 & 5 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + -3R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

edhr space ka dimension = 2

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

yeh ek plane pr lie kar rhe

pivot elements.

3rd dimension = 0.

LI vectors = 2.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$: Reduced Row Echelon form.

→ Pivot element is 1.
→ All elements above & below pivot element are 0.
→ Reduced echelon form is unique.

RANK $\rho(A)$

★ Rank of a matrix A is the dimension of vector space generated or spanned by its column

OR

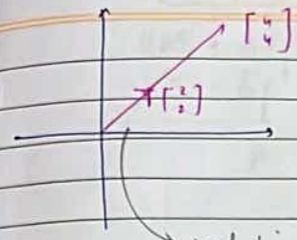
The no. of linearly independent columns of a matrix A is known as rank of it's matrix

OR

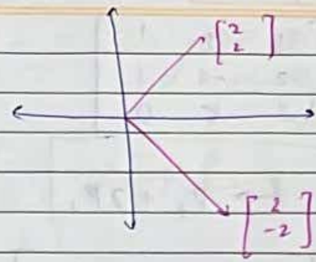
For a given matrix A in echelon form the no of pivot elements is equal to the rank of ~~its~~ ~~me~~ that matrix.

NOTE

Rank of a matrix do not change after applying row reduction.



→ yeh line ka dimension = 1

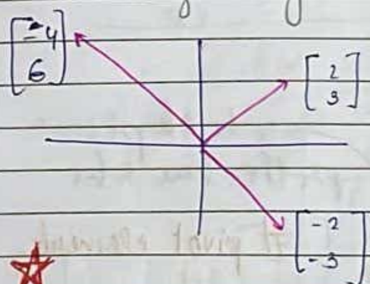


$\mathbb{R}^2$ ↳ dimension = 2

$$\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 2 & 2 \\ 0 & -4 \end{bmatrix}$$

yeh dono ek line pr lie karenge. isyle unka representation line ke through hoga toh unka dimension = 1.



Dimension = 2.

∵ because $v_i \in \mathbb{R}^2$

★ If a given matrix is in echelon form then $\rho(A)$ = # LI vectors / columns
= # pivot element
= Dim of space spanned by the col. of A.

Q. $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ # no of LI vectors

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 3 & 5 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + -3R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

yeh ek plane pr lie kar rhe

edhr space ka dimension = 2

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

pivot elements.

3rd dimension = 0.

LI vectors = 2.