$v_1 \in \mathbb{F}$ $\quad \alpha = -1$

$v_2 = -v_1$

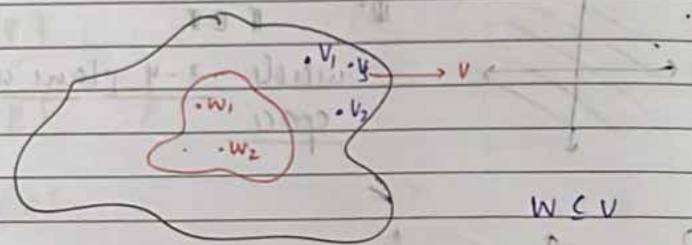
$v_1 + v_2 \in V(\mathbb{F})$

$v_1 - v_1 \in V(\mathbb{F})$

$0 \in V(\mathbb{F})$

22/07 VECTOR SUBSPACE

Let $V(\mathbb{F})$ be a vector space over field $\mathbb{F}$ then a non-empty set $W \subseteq V$ is known as vector-subspace if $W(\mathbb{F})$ is a vector space.



$W \subseteq V$

1) $w_1 + w_2 \in W \ \forall \ w_1, w_2 \in W$

2) $\alpha \cdot w \in W \ \forall \ \alpha \in \mathbb{F}, \ w \in W$

this given is a subspace for any corresponding vector $\boxed{\alpha w_1 + w_2 \in W} \in \forall \ \alpha \in \mathbb{F}, \ w_1, w_2 \in W$

eg $\mathbb{R}^2(\mathbb{R})$ is a vector space

(1) WOTF is a sub-space of $\mathbb{R}^2(\mathbb{R})$

i) $W_1 = \{(x, y) \in \mathbb{R}^2 ; x^2 + y^2 = 0\}$

$w_a = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W_1 \quad w_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W_1$

$w_a + w_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W_1$

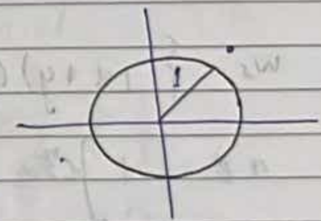
$5 \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in W_1$

W_1 is a sub-space of $\mathbb{R}^2(\mathbb{R})$

(2.) $W_2 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$

$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in W_2$

$1^2 + 0^2 = 1$



$v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in W_2$

$v_1 + v_2 \notin W_2 \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W_2$

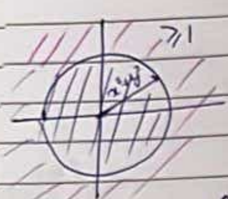
(3) $W_3 = \{(x, y) \in \mathbb{R}^2 ; x^2 + y^2 \leq 1\}$

$a = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in W_3 \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in W_3$

$a + b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W_3$

W_3 is not a sub-space of $\mathbb{R}^2(\mathbb{R})$

4. $W_4 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x^2+y^2 \geq 1 \right\}$



$0^2 + 0^2 \neq 1$

No it is not.

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not present in subspace

$a = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$ $b = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$

$a+b = \begin{bmatrix} +1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ +1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

5. $W_5 = \{ (x,y) \in \mathbb{R}^2 \;;\; x^2 - y^2 = 0 \}$

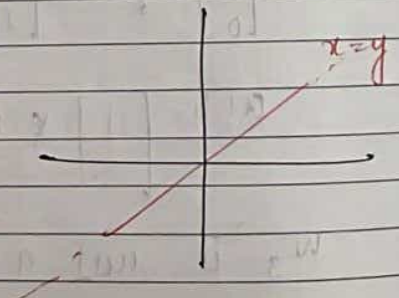
$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in W_5$ $b = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \in W_5$

$a+b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $2^2 - 0 = 4 \neq 0$

6. $W_6 = \{ (x,y) \in \mathbb{R}^2 \;,\; x - y = 0 \}$

$x = y$

$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$



$a+b = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \in W_6$

It is a subspace

NOTE:

any line passing through the origin is a sub-space of $\mathbb{R}^2(\mathbb{R})$

7. $W_7 = \{ (x,y) \in \mathbb{R}^2 \;;\; x = 3y \}$

$y = mx$ $y = \frac{x}{3} = \frac{1}{3}x$

W_7 is a sub space of $\mathbb{R}^2(\mathbb{R})$

$\mathbb{R}^2(\mathbb{R})$

↳ 0 vector

↳ line passing through origin

↳ $\mathbb{R}^2(\mathbb{R})$

8. $W_8 = \{ (x,y) \in \mathbb{R}^2 \;;\; xy = 0 \}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W_8$

W_8 is not a sub-space.

9. $W_9 = \{ (x,y) \in \mathbb{R}^2 \;,\; x + y = 0 \}$

$x = -y$

$a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$a + b = \begin{bmatrix} 3 \\ -3 \end{bmatrix} \in W_9$

W_9 is a sub space.

$W_{10} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid xy \geq 0 \right\}$ $\mathbb{R}^2(\mathbb{R})$

$a = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

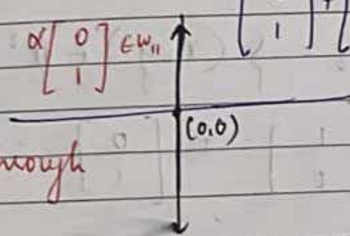
$a+b = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$2 * (-1) = -2 \not\geq 0$

W_{10} is not a subspace of $\mathbb{R}^2(\mathbb{R})$

$W_{11} = \{ (x, y) \in \mathbb{R}^2 \mid x = 0 \}$

$x = 0 \rightarrow y = \text{axis}$ $\alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in W_{11}$

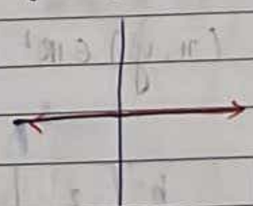


$\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \in W_{11}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \in \mathbb{R}^2$ passing through origin

$W_{12} = \{ (x, y) \in \mathbb{R}^2 \mid y = 0 \}$

$y = 0$, eq^n of x axis

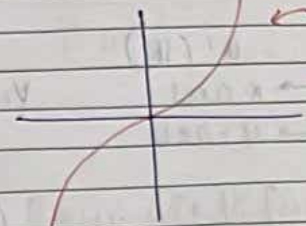


Agr line passing through origin hua toh to always voh ek sub-space hoga $\mathbb{R}^2(\mathbb{R})$ ka.

W_{11} & W_{12} subspace hai.

$W_{13} = \{ (x, y) \in \mathbb{R}^2 \mid y^3 = x \}$

$y = \sqrt[3]{x}$



← curve passing through origin not line

$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$a+b = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \notin W_{13}$

Not a subspace of $\mathbb{R}^2(\mathbb{R})$

RESULT FOR $\mathbb{R}^3(\mathbb{R})$

1. 0 vector is always a sub space of $\mathbb{R}^3(\mathbb{R})$
2. line through origin is always a sub-space of $\mathbb{R}^3(\mathbb{R})$
3. plane through origin is always a sub-space of $\mathbb{R}^3(\mathbb{R})$
4. $\mathbb{R}^3(\mathbb{R})$ itself

IMPORTANT RESULT

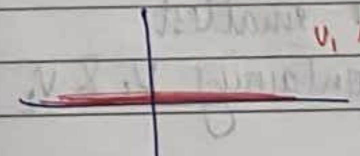
1. Let $V_1(\mathbb{F})$ & $V_2(\mathbb{F}) \subset V(\mathbb{F})$ be sub-space then, $V_1 \cap V_2$ is always a sub-space & if it is the LARGEST SUB-SPACE IN V_1 & V_2

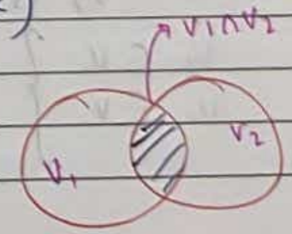
eg. $\mathbb{R}^2(\mathbb{R})$ is a vector space

$V_1 \longrightarrow x$ axis

$V_2 \longrightarrow x\text{-}y$ plane $= \mathbb{R}^2(\mathbb{R})$

$V_1 \cap V_2 = x$ axis





- If V_1 & V_2 are sub-space of $V(F)$ then $V_1 \cup V_2$ need not be a sub-space.

$V(F) = \mathbb{R}^2(\mathbb{R})$

$V_1 \rightarrow$ x axis

$V_2 \rightarrow$ y-axis

$V_1 \cup V_2$

① $V_1 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdots \right\}$

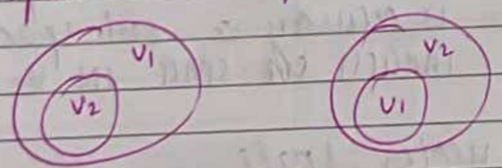
$V_2 = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdots \right\}$

$V_1 \cup V_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdots \right\}$

- $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ not present in $V_1 \cup V_2$
- $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \notin V_1 \cup V_2$

② $\left. \begin{array}{l} V_1 = \text{x axis} \\ V_2 = \mathbb{R}^2(\mathbb{R}) \end{array} \right\} \mathbb{R}^2(\mathbb{R})$

Subspace tab hi possible hai jab union ke case



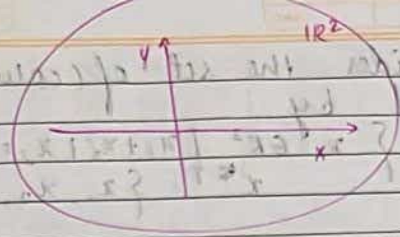
* $V_1 + V_2$ is always a sub space of $V(F)$

$\left. \begin{array}{l} V_1 \subseteq V \\ V_2 \subseteq V \end{array} \right\}$ $V_1 + V_2$ is the smallest sub-space containing V_1 & V_2

$V_1 \rightarrow$ x-axis

$V_2 \rightarrow$ y axis

$V_1 + V_2$

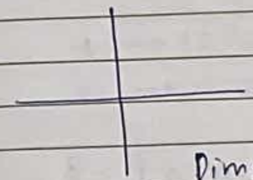


DIMENSION

No. of vectors in a Basis of that sub space.

eg. $\mathbb{R}^2(\mathbb{R})$ is a V.S

W_1 = x-axis W_1 is a sub-space of $\mathbb{R}^2(\mathbb{R})$

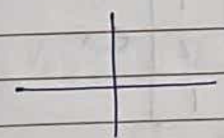


$\begin{bmatrix} x \\ 0 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

$Dim(W_1) = 1$

$\{0\}$ is a sub-space of $\mathbb{R}^2(\mathbb{R})$,

HOW MANY VECTORS ARE NEEDED TO REACH $\{0\}$ = 0



$\{\}$, ϕ

$\dim[\{0\}] = 0$

$\mathbb{R}^3(\mathbb{R})$

* W_1 = line through origin in a sub-space.
  $\dim(W_1) = 1$

W_2 : plane through origin
$\dim(W_2) = 2.$

① Consider the set of (column) vectors defined by

$X : \{ x \in R^3 \mid x_1 + x_2 + x_3 = 0, \text{ where } x^T = \{x_1, x_2, x_3\}^T \}$

$\dim(X) = 2$

WOF is TRUE?

A) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a basis for the subspace X.

B) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a linearly independent set, but does not span X & therefore is not basis of X ✗

C) X is not a subspace of R^3. ✗

D) None of the above.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in x \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$$

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \qquad a + b = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \in X$$

scale. $10 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix}$

$$a = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 &= -(x_2 + x_3) \end{aligned}$$

$$\Rightarrow \quad a = \begin{pmatrix} -x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix}$$

$$a = \begin{bmatrix} -(x_2+x_3) \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Q. If v_1 & v_2 are 4-dimensional subspace of a 6-dimensional vector space v, then the smallest possible dimension of $v_1 \cap v_2$ is ___

$v_1 \rightarrow 4$ vector — min rhena chahiye uske basis me

$v_2 \rightarrow 4$ vector

$v = \{a_1, a_2, a_3, a_4, a_5, a_6\}$

$v_1 = \{a_1\ a_2\ a_3\ a_4\}$

$v_2 = \{a_1\ a_2\ a_3\ a_4\}$

$v_1 \cap v_2 = \{a_1\ a_2\}$

$v_2 = \{a_1\ a_2\ a_5\ a_6\}$ = 2.

2.