21/07

$$\begin{array}{cccc} u & v & w & y \\ \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ C_1 & C_2 & C_3 & C_4 \end{array}$$

Linearly Independent = C_1 & C_3

Linearly Dependant = C_2 & C_4

Scaling factors corresponding to pivot = u & w
v & y = free

SCALING FACTOR

CORRESPONDING TO PIVOT

CORRESPONDING TO 2 D. [FREE VARIABLE]

IMP: To find solⁿ we assign arbitary value to free variable.

$$Ax = 0$$

$$u + 3v - y = 0 \Rightarrow u = y - 3v$$

$$w + y = 0 \Rightarrow w = -y$$

jo pivot wale variable hai unko free wale variable se ~~derive~~ terms of ~~krna~~ likhna hai

Solⁿ vector

vector me likhna

$$\begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix} = \begin{bmatrix} y-3v \\ v \\ -y \\ y \end{bmatrix} = \cancel{\begin{bmatrix} -3v \\ v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} y \\ 0 \\ \\ \end{bmatrix}}$$

$$\begin{bmatrix} -3v + y \\ v + 0 \\ 0 - y \\ 0 + y \end{bmatrix} = v \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

LI.

ab yeh dono vector LI through aapn ~~given dimension me koi bhi vector space store kr skte~~

NULL SPACE
The space spanned by linear combination of the vector which are being scaled by free variable.

RANK - NULLITY THEOREM

RANK - # linearly independent columns
- # pivot

NULLITY = $A_{m \times n}$ n columns

$r(A) = r \rightarrow r$ LI vectors

$(n - r) \rightarrow$ free variable

corresponding col = Nullity

$r + (n - r) = n$

$\rho(A)$ + Nullity(A) = # of columns

Jitne bhi free variable hai, utne hi vector scale kar sake honge, us dim ko null space.

★ **Nullity is dim of null space**

Ex.

$$\text{SPAN}\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\} \quad v_i \in \mathbb{R}^2$$

$$c_1\begin{bmatrix}1\\0\end{bmatrix}+c_2\begin{bmatrix}0\\1\end{bmatrix} \rightarrow \mathbb{R}^2 \Rightarrow \left\{\begin{bmatrix}x\\y\end{bmatrix} \middle| \begin{array}{l} x\in\mathbb{R}\\ y\in\mathbb{R}\end{array}\right\}$$

$Ax = 0$ $(n > m)$

$m \times n$ → variable

m → eq^n

more variables than eq^n

★ $\rho(A) = m$ **IMP** Gate 2024

nullity $= n-m$

⇒ $(n-m)$ ~~i.e.~~ atleast $n-m$ free variable

⇒ atleast $(n-m)$ LI vectors.

Q. The column of A are independent exactly when $N(A) = \{\text{zero vector}\}$ True.

$N(A)$ → Null space of A.

∴ True because no free variable.

If in null space there is only one vector & that vector is zero vector than each vector is LI

$$a_1v_1 + a_2v_2 + \cdots a_nv_n = 0 \quad \begin{bmatrix}0\\0\\\vdots\\0\end{bmatrix}$$

$\forall a_i = 0$

VECTOR SPACE AND SUBSPACE

$$v_1 = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \qquad 5v_1 = 5\begin{bmatrix}1\\2\\3\\4\end{bmatrix} \qquad 5 \in \text{Field}.$$

FIELD: It is a set denoted by F

$\mathbb{R}$ → 1, 2

$1+2 = 3 \in \mathbb{R}$

$1-2 = -1 \in \mathbb{R}$

$1 \times 2 = 2 \in \mathbb{R}$

$1/2 = 0.5 \in \mathbb{R}$

$\alpha \in \mathbb{F}$

VECTOR SPACE

Let V be an non-empty set & F be a field then V together with "2" operation.

1. '+' : $V + V \rightarrow V$ [BINARY OPERATION]
   $v_1, v_2 \in V(\mathbb{F})$
   then $v_1 + v_2 \in V(\mathbb{F})$

2. SCALAR MULTIPLICATION
   $v \in V$ and $\alpha \in \mathbb{F}$
   then $\alpha \cdot v \in V(\mathbb{F})$ $\forall\, v \in V(\mathbb{F})$, $\alpha \neq 0 \in \mathbb{F}$

$V(\mathbb{F})$ — set of jaha se vectors aate hai; $\mathbb{F}$ → scalar

If $V(\mathbb{F})$ is a vector space then $V \supseteq \mathbb{F}$

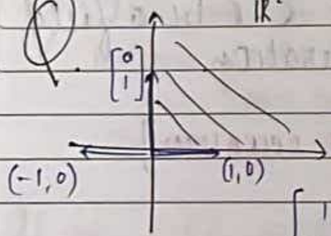
$\mathbb{R}^2(\mathbb{C})$ → Real, complex

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \in \mathbb{R}^2$

$i \in \mathbb{C} \Rightarrow i\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} i \\ 2i \end{bmatrix} \notin \mathbb{R}^2$

1. $\mathbb{R}^n(\mathbb{R})$ 1
2. $\mathbb{C}(\mathbb{R})$ 1
3. $\mathbb{C}^n(\mathbb{R})$ 1
4. $\mathbb{R}^n(\mathbb{C})$ 0
5. $\mathbb{R}^n(\mathbb{R})$ 1

Q. $\mathbb{R}^2$, $\mathbb{F} = \mathbb{R}$



is 1st QUADRANT is VECTOR SPACE OR NOT

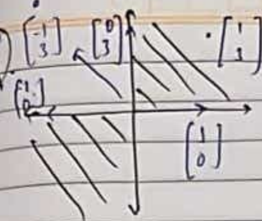
$\begin{bmatrix} x \\ y \end{bmatrix}$ $x \geq 0$, $y \geq 0$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{R}^2$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in \mathbb{R}^2$

$-1 \in \mathbb{R}$ $\quad -1\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ Not a vector space

Q. $\mathbb{R}^2$, $\mathbb{F} \in \mathbb{R}$



IS THIS VECTOR SPACE

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

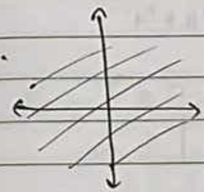
$\alpha = -1 \quad -1\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ VECTOR SPACE NHI AAYA. $\notin$ I OR III Quadrant.

$\mathbb{R}^2(\mathbb{R})$

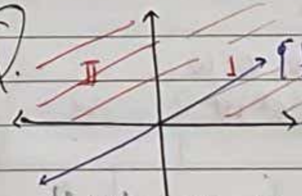
↳ $\mathbb{R}$ over $\mathbb{R}^2$ space ka jab tak subpart lenge tab tak valid vector space nhi aayega.

Q. $\mathbb{R}^2$, $\mathbb{F} \in \mathbb{R}$



whole x-y plane is vector space

Q.



No.

↳ yaha pe jo vector aayega, toh scale nhi kr paayega.

If $V(\mathbb{F})$ is a vector space then $0 \in V(\mathbb{F})$

Zero vector existence

→ Zero vector is always present in the vector space

$v_1 \in \mathbb{F}$ $\quad \alpha = -1$

$v_2 = -v_1$

$v_1 + v_2 \in V(\mathbb{F})$

$v_1 - v_1 \in V(\mathbb{F})$

$0 \in V(\mathbb{F})$