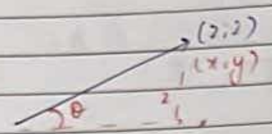
Assuming

08/07

VECTORS

ACCORDING TO PHYSICS

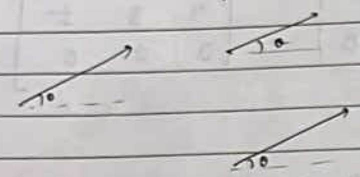
vector → direction & magnitude



magnitude $= \sqrt{(2)^2+(2)^2}$
$= \sqrt{4+4} = \sqrt{8}$
$= 2\sqrt{2}$

↳ This vector can be shifted as it is any where in plane.

- This called linear transformation



ACCORDING TO CSE

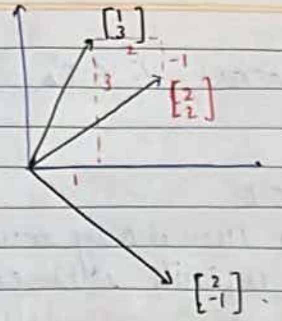
VECTOR - vector is an ordered list

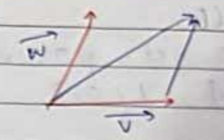
$$\begin{array}{c} \text{Area} \\ \text{Price} \end{array}\begin{bmatrix} 200\,ft^2 \\ 50\ \text{lakh} \end{bmatrix} \quad \begin{bmatrix} 150\,ft^2 \\ 27\ \text{lk} \end{bmatrix} \neq \begin{bmatrix} 300\,ft^2 \\ 70\ \text{lk} \end{bmatrix}$$

- If in one vector first ~~col~~ row represents area then whole of the vectors will be representing area.

$\begin{bmatrix} 200\,ft^2 \\ 50\ \text{lakh} \end{bmatrix}$ ← column matrix

VECTOR REPRESENTATION.





DEFINITION

Vector is an element of $\mathbb{R}^n$ (Real no. space) where n is the dimension of the vector

eg. V_i is a vector in $\mathbb{R}^n$

$$\boxed{V_i \in \mathbb{R}^n \quad \text{or} \quad V_i = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix},\ a_i \in \mathbb{R}}$$

$\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \cdots \mathbb{R}}_{n}$

↳ cartesian product of n real no.

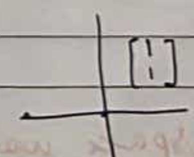
Eg. $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$

$V_i \in \mathbb{R}^3 \Rightarrow$ Dimension $= 3$.

1D - point 2, 3, 5.2235

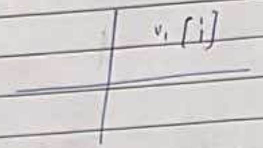
2D : $n = 2$ Dimension $= 2$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 2.57 \\ 3.57 \end{bmatrix}$

UNCOUNTABELY INFINITE ELEMENTS.



* SPACE

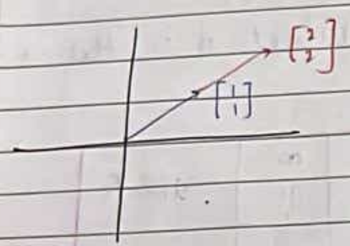
set of all possible vectors $v_i \in \mathbb{R}^n$ is called space



$v_i \in \mathbb{R}^1$

↳ How many ~~z~~ vectors

→ infinite elements.



$v_1 = \begin{bmatrix}1\\1\end{bmatrix} \in \mathbb{R}^2$ $v_i \in \mathbb{R}^2(\mathbb{R})$

$2v_1 = \begin{bmatrix}2\\2\end{bmatrix}$ FIELD

* The scalars in vector multiplication or vector used for scaling a vector ~~to~~ comes from $\mathbb{R}$ is called FIELD.

ADDITION OF TWO VECTORS

- we add corresponding elements of 2 vectors.
- For addition of 2 vector the dimension of 2 vector must be same.

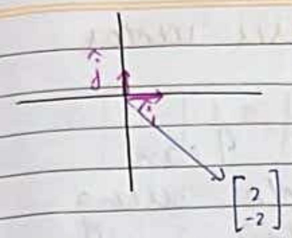
$v_1 \in \mathbb{R}^2$, $v_2 \in \mathbb{R}^3$

Both these vector cannot be added

$\begin{bmatrix}1\\2\end{bmatrix} \in \mathbb{R}^2$ $\begin{bmatrix}1\\2\\0\end{bmatrix} \in \mathbb{R}^3$

yeh bhi real nahi hai

Same space wale vectors ka addition.



$i \in \mathbb{R}^2$

$\begin{bmatrix}1\\0\end{bmatrix} \in \mathbb{R}^2$

$j \in \mathbb{R}^2$

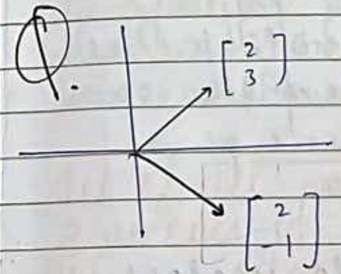
$\begin{bmatrix}0\\1\end{bmatrix} \in \mathbb{R}^2$

$\begin{bmatrix}2\\3\end{bmatrix}$ $\begin{bmatrix}2\\-1\end{bmatrix}$ $\begin{bmatrix}10\\3\end{bmatrix}$

given vector can be written in the form of unit vectors or represented by unit vectors

$2\begin{bmatrix}1\\0\end{bmatrix} - 2\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

Q.



Can both these vector define $\begin{bmatrix}-10\\3\end{bmatrix}$

$a\begin{bmatrix}2\\3\end{bmatrix} + b\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}10\\3\end{bmatrix}$

$2a + 2b = 10$

$3a - b = 3$

$\left\{ \left\{ \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} \right\}, \left\{ \begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} \right\} \right\}$

Basis vector

$a\begin{bmatrix}2\\3\end{bmatrix} + b\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$

$\begin{bmatrix}2a\\3a\end{bmatrix} + \begin{bmatrix}2b\\-b\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$

$2a + 2b = x$

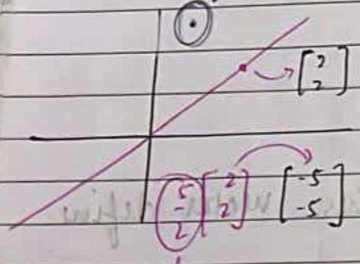
$3a - b = y$

Basis Vector

* There are infinite no. of possible basis vector for a given space $\mathbb{R}^n$.

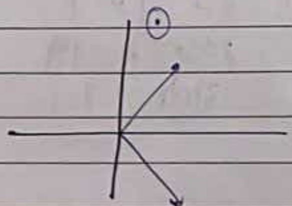
* Q given any 2 vectors in $\mathbb{R}^2$, I can represent any target vector using these 2 vectors. ~~True~~/False

any 2 se nhi pauch skte, specific rules follow karna jaruri hai



any 2 vector toh ley but ek random point pe nhi pauch payenge kyuki scale karne ke baad bhi voh same line pe hi aayega.

$\frac{5}{2} \in \mathbb{R}$ Field.



Aese wale do vectors agr hue toh target vector pe pauch skte

LINEAR COMBINATION OF VECTOR

For any given set of vectors $v_i \in \mathbb{R}^n$ we can always write these vectors in the form of $\sum_{i=1}^{n} \alpha_i v_i$

Linear combination of n vectors $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 \cdots \alpha_n v_n$
where $\alpha_i \in \mathbb{R}$ and known as scalar

$v_i \in \mathbb{R}^2$ $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

$\alpha_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$\left.\begin{matrix} \alpha_1 1 + \alpha_2 2 \\ \alpha_1 0 + \alpha_2 5 \end{matrix}\right\}$ linear combination

↳ VECTOR ADDITION

$\mathbb{R}^2 \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ $= 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

yeh do vectors ke liye jitne bhi possible linear combn hai uska set banana hai Span of Vector.

Set of all possible linear combination of 2 vectors v_1 & v_2 is called span of these vectors.

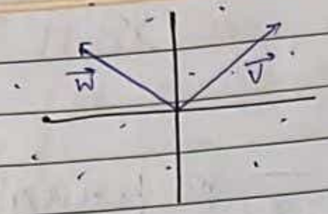
$\alpha v_1 + \beta v_2$ → linear combination.

$\mathbb{R}^2$ $y = mx$ where $m = 2$ $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} \cdots \right\}$

↳ what is span of $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$

all possible combination that are mapped in

linearly dependent vectors.



$\vec{V} \in \mathbb{R}^2$

$\vec{W} \in \mathbb{R}^2$

Span of $\vec{v}$ & $\vec{w}$ will be ~~the~~ $\mathbb{R}^2$ itself.

LINEARLY DEPENDANT VECTORS.

In a given set of vectors if one of the vectors can be represented as a linear combination of other vector then we say that given vector is linearly dependant

Eg $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix} \right\}$ $V \in \mathbb{R}^3$

$2V_1 + V_2 = V_3$ linearly dependant

$2V_1 + 0V_2 - 0V_3$ in dep

LINEARLY INDEPENDANT VECTORS

let us assume, we have set of n vectors each of them having m dimension

$S = \{ V_1, V_2, V_3 --- V_n \}$ $V_i \in \mathbb{R}^m$

dimension = m

where $V_i = \begin{bmatrix} a_1 \\ a_2 \\ a_m \end{bmatrix}$ $a_i \in \mathbb{R}$

we say that this set of vectors are linearly independant if and only if

$\alpha_1 V_1 + \alpha_2 V_2 - - \alpha_n V_n = 0 \quad \forall \alpha_i = 0$

Eg. $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\} \Rightarrow$ linearly dependant vector

to make these vector linearly independant.

$\alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 0$

$\alpha = 0$ & $\beta = 0$ iske alwa koi or option nhi aana are imp. condition otherwise vector will be dependant

$\alpha = 2 \quad \beta = 1$

Eg. $2a + 2b = 0$

IMP CONCEPT

$3a - b = 0$

yaha pr 0 tabhi ayega jab $a = 0$ $b = 0$

$\therefore$ means linearly independant vector.

$a \begin{bmatrix} 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 3 \\ -1 \end{bmatrix} = 0$