04/07/24

LINEAR ALGEBRA

RATIONAL NO : $\frac{p}{q}$, $q \neq 0$, p & q ~~&~~ coprime

REAL NO - $-\infty$ to ∞ , that can be represented in no. line.

MATRICES

Matrix is nothing but a 2-D array of no.'s

any no. K $\in \mathbb{R}$, $\in \mathbb{N}$, $\in \mathbb{Q}$ } belongs to
$\in \mathbb{C}$ * complex no.

REPRESENTATION : $\boxed{A = [a_{ij}]}$ $\longrightarrow$ $\begin{array}{cc} & \text{COLUMN} \\ \begin{bmatrix} 7 & 8 \\ 9 & 1 \\ 7 & 10 \end{bmatrix}_{3\times 2} & \rightarrow \text{ROW} \end{array}$

ROWS COLUMNS

~~a1~~ $\boxed{a_{21} = 9}$

ORDER

* A matrix having m rows & n columns is called an matrix of order $\underline{m \times n}$
* where, a_{ij} is an element lying in the i^{th} row & j^{th} column.

no. of elements in $m \times n$ matrices $\longrightarrow$ mn elements

Q. ~~no~~ # elements = ~~8~~ in matrix = 8

2×4 $\quad$ 8×1

4×2 $\quad$ 1×8

Q. Construct a 2×2 matrix $A=[a_{ij}]$ where $a_{ij}=|i-j|$

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11}=|1-1|=0$
$a_{12}=|1-2|=1$
$a_{21}=|2-1|=1$

$$=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Q. The no. of all possible matrices of order 3×3 where $a_{ij}\in\{0,1\}$

$$a_{ij}=\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3} = 2^9 = 2^{3\times3}$$

har place ke liye do possibility rhegi isliye 2 ki power

Q. $A=[a_{ij}]_{n\times n}$

$A_{n\times n}$ = $\begin{cases} a_{ij}=0 & \text{if } i=j \quad (n) \\ a_{ij}=k & \text{if } i>j \quad \left(\frac{n^2-n}{2}\right) \\ a_{ij}\in\{1,2,3\} & \text{otherwise} \end{cases}$

Total possible matrices

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & a_{2n} \\ a_{31} & & & a_{3n} \\ \vdots & & & \vdots \\ a_{n1} & & & a_{nn} \end{bmatrix}$$

$= \cancel{\frac{n}{3} + \frac{n}{3^2} + 3^n = 6}$

$$A_{3\times3}=\begin{bmatrix} 0 & 1,2,3 & 1,2,3 \\ k & 0 & 1,2,3 \\ k & k & 0 \end{bmatrix}_{3\times3} = \begin{cases} 3^3 + 3 + 3 \\ = 3^3+6 \end{cases}$$

$= 3^n + n + \cancel{\ }$

$$A_{4\times4}=\begin{bmatrix} 0 & - & - & - \\ k & 0 & - & - \\ k & k & 0 & - \\ k & k & k & 0 \end{bmatrix}$$

$(n^2-n) \bmod 2 = 0$! → by default yeh cheez 2 se divide hogi

$n(n-1)$
odd → toh yeh part even
even → toh yeh part odd

\# elements $= n\times n = n^2$

- Diagonal elements $= n$
- Non-diagonal elements $= n^2-n$
- then a_{ij} ~~other~~ otherwise condition $=\frac{n^2-n}{2}$

Total possible matrices $=3^{\frac{n^2-n}{2}}$

TYPES OF MATRIX

COLUMN MATRIX.

A matrix is said to be a column matrix if it has only 1 column.

Gen $A=[a_{ij}]_{m\times1}$

ROW MATRIX

A matrix is said to be row matrix if it has only 1 row.

Gen $A = [a_{ij}]_{1 \times n}$

SQUARE MATRIX

matrix in which no of rows = no of columns

Gen $A[a_{ij}]_{n \times n}$

FAT MATRIX AND THIN MATRIX

FAT

- no. of columns are more than rows.

THIN

→ no of rows are more than columns

DIAGONAL MATRIX

It is a sq. matrix where all non-diagonal ~~matrix~~ element are zero.

Gen $B = [a_{ij}]_{n \times n}$

where, $b_{ij} = 0$ if $i \neq j$

Ex. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3 \times 3}$

SCALAR MATRIX.

A diagonal matrix is said to be scalar matrix if it's diagonal ~~matrix~~ elements are equal.

$$B = [b_{ij}]_{n \times n} \begin{cases} b_{ij} = k & \text{if } i = j \\ b_{ij} = 0 & \text{if } i \neq j \end{cases}$$

eg. $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

- Every scalar matrix is a diagonal matrix.
- But not every diagonal matrix is scalar matrix

SCALAR $\longrightarrow$ DIAGONAL

$\times$ NOT POSSIBLE

IDENTITY → diagonal

A sq. matrix is said to be identity matrix if every diagonal element is 1 & & non-diagonal ~~mat~~ element is 0.

- Every diagonal matrix is sq. matrix

Gen $b_{ij} = 1$ if $i = j$

$b_{ij} = 0$ if $i \neq j$.

Every identity matrix is sq. matrix vice versa not true.

Ex $[4]_{1 \times 1}$ is it a diagonal matrix.

yes.

It is also scalar matrix

$[1]_{1 \times 1}$ identity matrix.

Zero matrix [0]

ZERO MATRIX.
A matrix is said to be zero matrix if all elements are 0.

Gen ~~Bij~~ $B = [b_{ij}]$

$b_{ij} = 0 \ \forall\ i, j$

Ex. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$

[0] zero matrix or NULL matrix.

↳ ~~but not~~ diagonal ~~the~~ matrix.

$\boxed{0 \neq [0]}$

bij element   B matrix

LOW TRIANGULAR MATRIX.
All the elements below diagonal are non-zero & elements above diagonal are zero. It is a sq. matrix.

$$\begin{bmatrix} k & 0 & 0 & 0 \\ 1 & a & 0 & 0 \\ 2 & 3 & b & 0 \\ 4 & 5 & 6 & c \end{bmatrix}$$ → LOWER TRIANGULAR MATRIX.

gen, $L = [l_{ij}]$

$l_{ij} = 0$ if $i < j$

EQUALITY OF MATRIX.

$A = [a_{ij}]$ $B = [b_{ij}]$

Two matrices A & B are said to be equal if

1. They are of same order.
2. Every element in A is equal to corresponding element in B.

→ $a_{ij} = b_{ij} \ \forall\ i, j$

Ex $\begin{bmatrix} a & 2 \\ 3 & b \end{bmatrix}_{2\times2} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}$ Matrix equal then

then $a = 1$, $b = 4$
$a_{11} = b_{11}$ $a_{22} = b_{22}$

Eg. $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$

Find a, b, c, d

$2a + b = 4$ — (1) $5c - d = 11$ — (3) ×3
$a - 2b = -3$ — (2) $4c + 3d = 24$ — (4)

$2a + b = 4$
$\cancel{2a} - 4b = -6$
(−) (+) (+)

$5b = 10$
$\boxed{b = 2}$

$a = -3 + 2 \times 2$
$\boxed{a = 1}$

$15c + 4c = 33 + 24$
$19c = 57$
$c = \frac{57}{19} = 3$

$\boxed{c = 3}$

$\cancel{\frac{5 \times 35}{9} - d = 11}$

$\cancel{d = \frac{5 \times 35}{9} - 11}$

$5 \times 3 - d = 11$
$-d = 11 - 15 =$
$\boxed{d = +4}$

$\cancel{\frac{5 \times 35 - 99}{9} = \frac{175 - 99}{9} =}$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

OPERATION ON MATRICES

1. ADDITION OF MATRICES

Two matrices can only be added if they are of same order.

For eg. $A = [a_{ij}]_{m \times n}$
$B = [b_{ij}]_{m \times n}$

$$A + B = C$$

$$a_{ij} + b_{ij} = c_{ij} \quad \forall i, j$$

Gate notation.

Matrix Addition Pseudo Code

1. start
2. matrixA[m][n];
   matrix B[m][n]
   matrix C[m][n]
3. Read r, c, A[][], & B[][]
4. i=0, j=0
5. if i < r
   if j < c
   C[i][j] = A[i][j] + B[i][j]
   j = j+1
   i = i+1
6. Addition of matrix C
7. stop

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3} \quad B = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{3\times 3}$$

arithmetic operations = n^2

$\therefore$ Complexity of Matrix Addition = $O(n^2)$

MULTIPLICATION OF A MATRIX

BY SCALAR.

we define multiplication of matrix by scalar

if $A_{m\times n} = [a_{ij}]$ & k is a scalar.

gen ka_{ij} $\forall i,j \longrightarrow [ka_{ij}]_{m\times n}$

$= k \cdot A_{m\times n}$