$= K \cdot A_{m \times n}$

05/07

PROPERTIES OF MATRIX ADDITION.

1. COMMUTATIVE PROPERTY

$$[A]_{m \times n} + [B]_{m \times n} = [C]_{m \times n}$$

$$a_{ij} + b_{ij} = c_{ij}$$

$$\boxed{A + B = B + A}$$

2. ASSOCIATIVE PROPERTY

$$[a_{ij}]_{m \times n} + \left([b_{ij}]_{m \times n} + [c_{ij}]_{m \times n}\right) = \left([a_{ij}]_{m \times n} + [b_{ij}]_{m \times n}\right) + [c_{ij}]_{m \times n}$$

ADDITIVE IDENTITY

$[a_{ij}]_{m\times n} + ? = [a_{ij}]_{m\times n}$

$? = [0]_{m\times n}$

ADDITIVE INVERSE

$[a_{ij}]_{m\times n} + ? = [0]_{m\times n}$

$? = \boxed{[-a_{ij}]}$

Ex. $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

PROPERTY OF SCALAR MULTIPLICATION OF A MATRIX

1. $(k+l)[a_{ij}]_{m\times n} = k[a_{ij}]_{m\times n} + l[a_{ij}]_{m\times n}$
2. $k\left[[a_{ij}]_{m\times n} + [b_{ij}]_{m\times n}\right] = k[a_{ij}]_{m\times n} + k[b_{ij}]_{m\times n}$

Q. Find x & y

$x + y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ $\quad x - y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$2x = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$

$x = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

$y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

MULTIPLICATION OF MATRICES.

$A = [a_{ij}]_{m\times n}$ $\quad B = [b_{ij}]_{n\times p}$

column vector

$A \times B = \begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 50 & 40 \end{bmatrix}$

hr ek column ko scale karenge

$5\begin{bmatrix} 2 \\ 8 \end{bmatrix} + 50\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 260 \\ 540 \end{bmatrix}$

$4\begin{bmatrix} 2 \\ 8 \end{bmatrix} + 40\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 208 \\ 432 \end{bmatrix}$

jab A no of rows(n) equal nhi hoga no. of column B(n) then hum log scaling nhi kar payenge.

$AB = \begin{bmatrix} 260 & 208 \\ 540 & 432 \end{bmatrix}$

$[a_{ij}]_{m\times n}$ $\quad [b_{ij}]_{n\times p}$

$m\times \boxed{n \quad n} \times p$ = Resultant matrix $m \times p$

The product of matrices A ($[a_{ij}]_{m\times n}$) & B ($[b_{ij}]_{n\times p}$) is a matrix C ($[c_{ij}]_{m\times p}$) of order $m \times p$.

★ $O(n^{2.80})$ = named brought the complexity of matrix multiplication as $O(n^{2.80})$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times2}$$

a_{11}, a_{12}, a_{13}; b_{11}, b_{21}, b_{31}

$m\times n$ $n\times p$

$(mp)n$ — n times multiple

multiplication

$$\begin{bmatrix} c_{11}{:}3 & 3 \\ 3 & 3 \\ 3 & 3 \end{bmatrix}_{3\times2}$$

$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$

$c_{12} = a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32}$

c_{11} me humlog dekh rhe hai ki a ka i^{th} or b ka j^{th} same hai

TOTAL MULTIPLICATION $= (mp)\,n \approx O(n^3)$

(mp) = # elements; n = n time

TOTAL SUMMATION $= (m\,p)(n-1)$

(m p) = # elements; (n-1) = (n-1) times

Q. Matrix A is of order 20×30 & matrix B is of order 30×10 find how many multiplication & additions are needed to perform $A\cdot B$

$$[A]_{20\times30} \quad [B]_{30\times10}$$

\# Multiplication $= (20\times10)\,30$
$= (200)\,30 = 6000$

\# Addition $= (20\times10)\,29$
$= 200\times29 = 5800$

$$\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times3} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_{3\times3}$$

CODE

```
for( i=1; i<=3; i++ )
{
    for( j=1; j<=3; j++ )
    {
        int sum = 0;
        for( k=1; k<=3; k++ )
        {
            sum = sum + a[i][k] . b[k][j];
        { c[i][j] = sum;
        }
    }
}
```

ych do loop elements ke liye

ych multiple karke sum store karne ke liye

$c_{11} = a_{11}.b_{11} + a_{12}b_{21} + a_{13}b_{31}$ (i k k j)

IMP. if AB is possible then B.A may be or may not be possible.

- Matrix Multiplication is not commutative
- Matrix Multiplication is Associative.
- Multiplicative Identity for Matrix Multiplication

$$AI = IA = A$$

ONLY EXIST IF A matrix IS SQUARE MATRIX.

Q) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}_{2\times 2}$ find $B \neq 0$ such that $AB = 0$

$AB = 0$

$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} a+2c & b+2b \\ 3a+6c & 3b+6d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$a+2c = 0$ $\quad$ $b+2b = 0$ $\times 3$

$3a+6c = 0$ $\quad$ $3b+6d = 0$

here, we do not have unique sol^n there can be more than one sol^n

$a = -2c$ $\quad$ $b = -2d$

$c = 1$ $\quad$ $d = 1$

$a = -2$ $\quad$ $b = -2$

$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\det A = 0$ $\quad$ $\det B = 0$

SINGULAR MATRIX.

Matrix having determinant $= 0$, then the matrix is called as singular matrix.

If $A \cdot B = 0$ when $|A| = 0$ & $|B| = 0$

TRANSPOSE OF A MATRIX.

$A = [a_{ij}]_{m\times n}$ is represented as A^T or A'

$A^T = [a_{ji}]_{n\times m}$

Ex $\begin{bmatrix} 2 & 3 \\ 5 & 7 \\ 1 & 8 \end{bmatrix} \xrightarrow{\text{TRANSPOSE}} \begin{bmatrix} 2 & 5 & 1 \\ 3 & 7 & 8 \end{bmatrix}$

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(AB)^T = B^T A^T$
4. $(kA)^T = k(A^T)$

k is a const.

SYMMETRIC AND SKEW SYMMETRIC MATRICES

SYMMETRIC when $A^T = A$ is only for sq matrix

$[a_{ij}]^T_{n\times n} = [a_{ji}]_{n\times n}$

SKEW SYMMETRIC when $A^T = -A$ is only for sq matrix.

$[a_{ij}]^T_{n\times n} = -[a_{ji}]_{n\times n}$

- $[a_{ii}] = -[a_{ii}]$

$2[a_{ii}] = 0$ $\Rightarrow$ diagonal elements in skew symmetric $[a_{ii}] = 0$

THEOREM 1.

For any sq. matrix A if ~~A is symmetric~~

i) $A + A^T$ is always symmetric, $a_{ij} \in \mathbb{R}$

ii) $A - A^T$ is skew symmetric

$$(A + A^T)^T = A^T + (A^T)^T \quad \text{prop 1}$$
$$= A^T + A \quad \text{prop 2}$$
$$= A + A^T \quad \text{commutative}$$

$\therefore (A + A^T)$ is symmetric matrix.

$$(A - A^T)^T = \cancel{\{A + (-A^T)\}^T} = A^T - (A^T)^T$$
$$= \cancel{A^T + (-A^T)^T} = A^T - A$$
$$= \cancel{A^T - A} = -(A - A^T)$$

$\therefore (A - A^T)$ is skew symmetric.

THEOREM 2.

Any sq. matrix A can be represented as sum of symmetric & skew symmetric matrix.

$$\frac{1}{2}\left(A + A^T\right) + \left(A - A^T\right) = \frac{2A}{2} = A.$$

$$\boxed{\frac{1}{2}(A + A^T) + (A - A^T) = \frac{2A}{2} = A}$$

Q. If A & B are symmetric matrices then Symmetric matrix of ~~some~~ same order then $AB - BA$ is

~~A)~~ SKEW SYMMETRIC

B) SYMMETRIC

C) IDENTITY

D) ZERO

$$\cancel{AB - BA = A(B - B)}$$
$$\cancel{= A \times 0}$$

$$(AB - BA)^T = (AB)^T - (BA)^T$$
$$= \cancel{A^T B^T - B^T A^T} \quad B^T A^T - A^T B^T$$
$$= \cancel{AB} = BA - AB$$
$$= -(AB + BA)$$

Skew symmetric.

Q. If matrix A is both symmetric & skew symmetric then. A is.

A) DIAGONAL

B) SKEW SYMMETRIC

C) SYMMETRIC

~~D)~~ NULL MATRIX

$A^T = A \quad \& \quad A^T = -A$

$\cancel{A} \quad A = -A \Rightarrow$ Null matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{90°?} \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

Hint : Transpose.