12/07

$\begin{cases} 2x + y = 2 \\ x - y = 3 \end{cases}$ $\quad \begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$

$A \; x = b$ → TARGET VECTOR

$\begin{bmatrix}2 & 1\\1 & -1\end{bmatrix} \xrightarrow{\text{ECHELON}} \begin{bmatrix}2 & 1\\0 & -3\end{bmatrix}$

A → SET OF VECTORS OR SET OF COEFFICIENT OF GIVEN VARIABLE MATRIX

$\begin{bmatrix}2 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$

- $Ax = b$ has a solⁿ iff $b \in \text{span}(v_1 \cdots v_n)$ where $v_i \in \mathbb{R}^m$

$A_{m \times n}$ → # vectors (n); m ↓ Dim of a vector or # rows

OR.

- $b \in$ L.C of $\{v_1, v_2 \cdots v_n\}$

OR

- $b \in$ Linearly dependent on $\{v_1, v_2 \cdots v_n\}$

AUGMENTED MATRIX $[A : b]$

$Ax = b$

$A \cup b = [A \; b]$

A → n columns; $[A \; b]$ → n+1 columns.

Q. $\left[\begin{array}{cc|c}2 & 1 & 2\\1 & -1 & 3\end{array}\right]$

$R_2 \to R_2 - 2R_1$ $\qquad 3 - 2\times2 = 3 + -4$

$\left[\begin{array}{cc|c}2 & 1 & 2\\0 & -3 & -1\end{array}\right]$

Q. $2x + 4y = 6$
$4x + 8y = 10$

$\begin{bmatrix}2\\4\end{bmatrix} \begin{bmatrix}4\\8\end{bmatrix} \begin{bmatrix}6\\10\end{bmatrix}$ ← Target

$\begin{bmatrix}2 & 4\\4 & 8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\10\end{bmatrix}$

Augmented matrix → $[A : b]$

$\left[\begin{array}{cc|c}2 & 4 & 6\\4 & 8 & 10\end{array}\right]$

humlog do vector ke help se 3rd ko derive kar sake the or voh dono LI the

$R_2 \to R_2 - 2R_1$

$\left[\begin{array}{cc|c}2 & 4 & 6\\0 & 0 & -2\end{array}\right]$
$\quad v_1 \quad v_2 \quad v_3$

but yeh case me v_1 or v_T ko bol skte hai ki yeh log hi LI ban gye

★ jo pivot element hai voh target me chale gya hai mtlb

∴ The given set of eqⁿ have no solⁿ

★ Vectors containing pivot element are LI in echelon form.

$Ax = b$

$b = 0$ ↙ HOMOGENOUS SYSTEM

$b \neq 0$ ↘ NON-HOMOGENOUS SYSTEM

HOMOGENOUS

$AX = 0$

If a given linear system is homogenous system then it has atleast one sol^n that exists or ~~two~~ sol^n exists.
[i.e all variable 0]

Q $A = [a_{ij}]_{m \times n}$

$f(A) = n$

discuss about the ~~sol^n~~ nature of sol^n.

$V_i \in \mathbb{R}^m$

$$\left[\begin{array}{ccc|c} | & | & | & \\ V_1 & V_2 & V_n & \begin{matrix}0\\ \vdots \\ 0\end{matrix} \\ | & | & | & \end{array}\right]$$

This implies that the n vectors are linearly independent.

CONCEPT $\alpha V_1 + \beta V_2 + \gamma V_3 = 0$
$\forall \alpha, \beta, \gamma = 0$

for the system of linear eq^n we have unique sol^n i.e all variable value = 0.

Q $A = [a_{ij}]_{m \times n}$

$f(A) = m$

Rank of matrix is less than equal min of (m, n).

$f(A) \leq \min(m, n)$

$f(A) = m \quad [m = \min(m, n)]$

$\Rightarrow n \geq m$

These will ~~be~~ always be some dependent ~~matrix~~ vector

First vector ka first element $= a_{11}$

$$\left[\begin{array}{ccc} a_{11} & a_{21} & a_{n1} \\ \vdots & \vdots & \vdots \\ a_{1m} & a_{2m} & a_{nm} \end{array}\right]$$

$$\left[\begin{array}{cccc} 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \\ 4 & 7 & 10 & 14 \end{array}\right] \quad \begin{matrix} R_2 \to R_2 - \frac{3}{2}R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\left[\begin{array}{cccc} 2 & 5 & 8 & 10 \\ 0 & & & \\ 0 & -3 & -6 & \end{array}\right]$$

Q. $\begin{bmatrix} 2 & 4 & 8 \\ 3 & 5 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \rightarrow R_2 - \frac{3}{2} R_1$

$A = \begin{bmatrix} 2 & 4 & 8 \\ 0 & -1 & -1 \end{bmatrix}$

$\rho(A) = 2.$

$2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 4 \\ 5 \end{bmatrix} - 1\begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

ags scaling factor pta chl gya toh

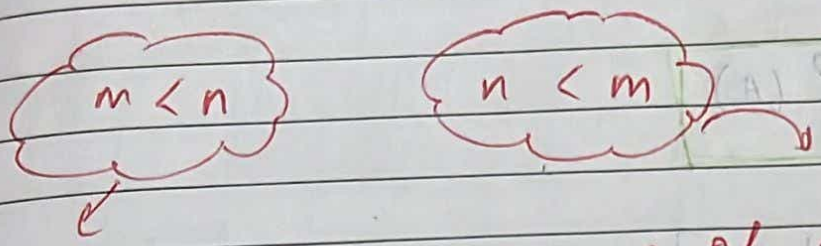
$4\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix} - 2\begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Infinite many sol^n — .

if $\rho(A) =$ # col^n, unique sol^n

$\rho(A) =$ # rows, infinite many sol^n

$\rho(A) <$ # rows, infinite many sol^n



no. of vector space ke dimension se jayad hai

no. of vector space ke dimension se km hai.

$Ax = 0$

Trivial sol^n

UNIQUE SOL^N

$\rho(A) = n$

INFINITELY MANY SOL^N

$\rho(A) = m$

$Ax = b$ where $b = 0$

INFINITELY MANY SOL^N

$\Rightarrow$

$\rho(A) = s < m$