11/07

Q1. Reduce the matrix in echelon form

$$A = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 2 & -3 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1, \quad R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & +1 & -5 & -10 \\ 0 & +6 & -10 & -24 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 6 \times R_2$

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 20 & 36 \end{bmatrix}$$

$6 + \frac{6}{7}$

$= 6$

$-10 - 6$

$-10 +$

-24

-24

(2) find the Rank of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 + \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

$2 + \frac{1}{3} \times -6$

CONCEPT

★ $A = [a_{ij}]_{m \times n}$

$\rho(A) \le \min(m, n)$ — rows, col.

if $n > m$, $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \begin{bmatrix} g \\ h \end{bmatrix} \right\}$ max of $\mathbb{R}^m$ hi span kar skti hai

$n - m$ min dependent

if $n < m$,

$m = 3$ $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix} \right\}$ isme $\mathbb{R}^2$ span kariga.

for eg.

$n > m$, jayad no of vector, baki dependent no jayegaa

Max rank of a given matrix A is less than equal to min of no of rows & col.

Q. $\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \right\}$ does this span $\mathbb{R}^3$

No

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\rho(A) = 2.$

It can span any vector in $\mathbb{R}^2$ not in $\mathbb{R}^3$

★ ONLY A ZERO MATRIX HAS A RANK ZERO

★ If there is a given non-zero matrix then min rank of that matrix will be 1, in any condition.

agr given hai non-zero toh min ek column linear independant exist karega hi.

$rank(A + B) \le rank(A) + rank(B)$

$rank(AB) \le \min(\rho(A), \rho(B))$

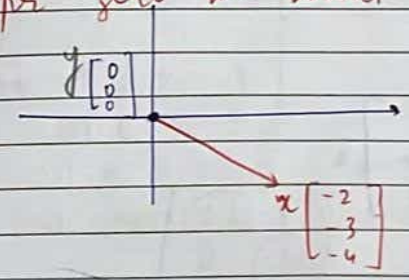
good questions.

Q. If $\vec{z} \in span(\vec{x}, \vec{y})$ then $\{\vec{x}, \vec{y}, \vec{z}\}$ is linearly dependant

True / False.

Q. If $\vec{x}, \vec{y} \in \mathbb{R}^3$ and $\vec{x}$ is not a multiple of $\vec{y}$ then $\{\vec{x}, \vec{y}\}$ is LI True / False

linear dependant tab hoga jab scale karke vector lao pe the hai ki nhi, but yaha zero se scale kar skte pr zero ka nhi



$y \begin{bmatrix}0\\0\\0\end{bmatrix}$ $x \begin{bmatrix}-2\\-3\\-4\end{bmatrix}$

$2 \cdot x = y$

$0 \cdot x = \begin{bmatrix}0\\0\\0\end{bmatrix}$

Q. Does $\left\{ \begin{bmatrix}1\\2\\3\end{bmatrix} \begin{bmatrix}3\\5\\7\end{bmatrix} \begin{bmatrix}0\\1\\2\end{bmatrix} \right\}$ spans $\mathbb{R}^3$?

No.

$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$ $R_3 \to R_3 + 2R_2$

$\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = 2.$

$v_i \in \mathbb{R}^3$ $\{v_1 v_2 v_3\}$ v_2 is LD on v_1 & v_3 therefore max span $\mathbb{R}^2$
$\downarrow$
LI

(1) $\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$

$\vec{a_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\vec{a_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\vec{a_3} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

LD / LI

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \rho(A) = 2.$$

Linear dependant.

$\vec{a_2} = \vec{a_1} + \vec{a_3}$

$\vec{a_1} = \vec{a_2} - \vec{a_3}$

SYSTEM OF LINEAR EQUATION

$2x + 3y = 5$ — (1) $\times 2$
$4x + 5y = 4$ — (2)

$4x + 6y = 10$
$\underline{-4x - 5y = -4}$
$y = 6$

$4x + 5y = 4$
$4x = 4 - 5 \times 6$
$4x = -26$
$x = -\frac{26}{4}$

$$x \begin{bmatrix} 2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$A \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \quad \rho(A) = 2$$

LI vectors = 2

Eg. $2x + 4y = 6$
$4x + 8y = 12$

$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$Ax = b$ → Target vector

$$x \begin{bmatrix} 2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$\cancel{4x + 8y = 12}$
$\cancel{4x + 8y = 12}$
$0 = 0$

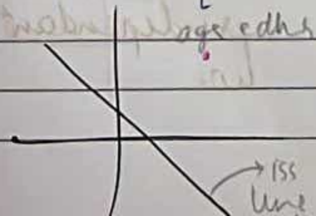
$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix} \quad = 1 \text{ LI vector}$$

agr edhe hota point toh derivative leke

$y = mx + c$ → intercept
m → Slope.



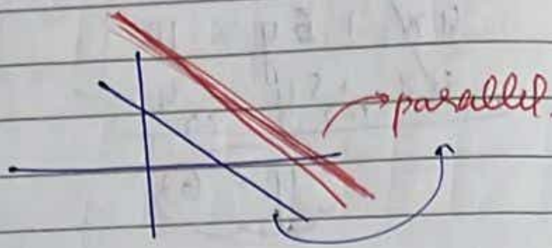
→ 1st line

1 0 a b

IN A SYSTEM OF LINEAR EQ^n

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ for infinite many sol^n

Eg. $2x + 4y = 6$
$4x + 8y = 10$



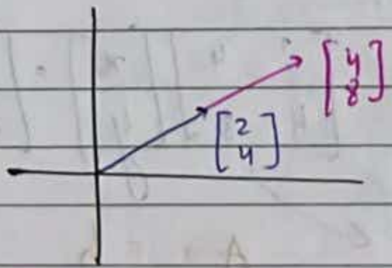
$2x + 4y = 6$
$y = \frac{-2x}{4} + \frac{6}{4}$

slope $= \frac{-2}{4}$

$4x + 8y = 10$
$y = -\frac{4}{\cancel{8}8}x + \frac{10}{8}$

slope $= \frac{-2}{\cancel{8}4}$

whenever the slope is same the lines will be parallel, that means no intersection.



In FORM OF VECTOR.

$\begin{bmatrix} 6 \\ 10 \end{bmatrix}$ cannot derive.

$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

given,

$A_1 x + B_1 y = C_1$
$A_2 x + B_2 y = C_2$

1. UNIQUE SOL^n : $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$ ∴ ek vector dusre se independant ho.

(2) INFINITELY MANY SOL^n

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

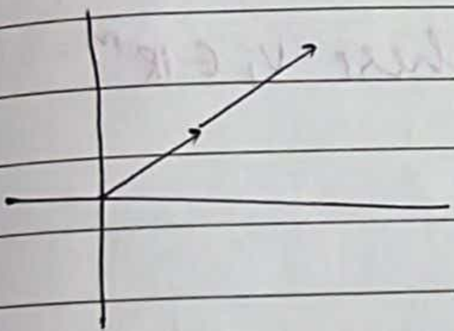
ek vector dusre pe dependant. hai

(3) NO SOL^n

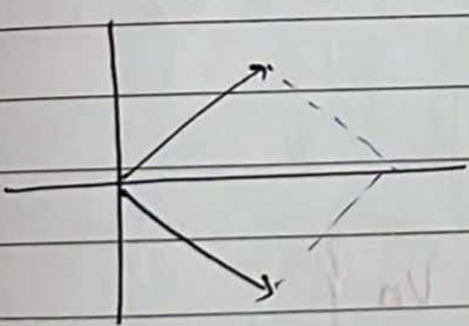
$$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$$

ek vector dusre pe dependant hai but dono ke help se scale nhi kar skte.

DETERMINANT



$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ det $= 4 - 4 = 0$



$\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$ det $= -2 - 2 = -4$.

det is nothing but area covered by given vectors.