09/07

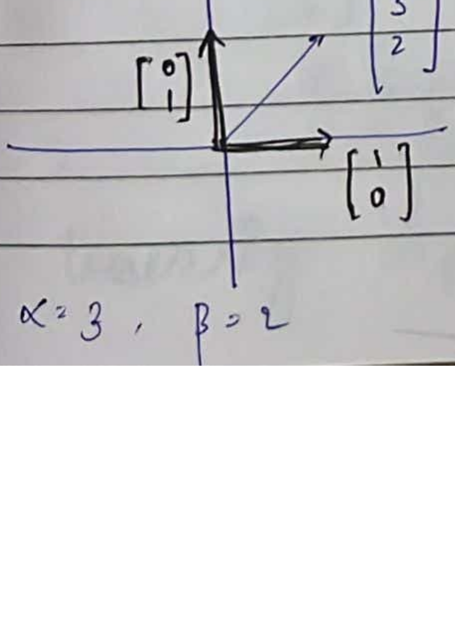
REPRESENTING A VECTOR.

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right\}$$

This can also be represented as

$$\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; \begin{matrix} x \in \mathbb{R} \\ y \in \mathbb{R} \end{matrix} \right\}$$

VECTOR SET $\in \mathbb{R}^2$



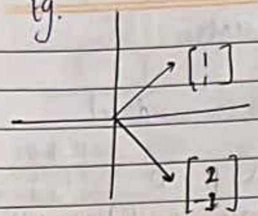
GENERAL EQ^{n}

$$\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$\alpha = 3, \quad \beta = 2$

eg.



$$\alpha\begin{bmatrix}1\\1\end{bmatrix}+\beta\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix}\in \mathbb{R}^2$$

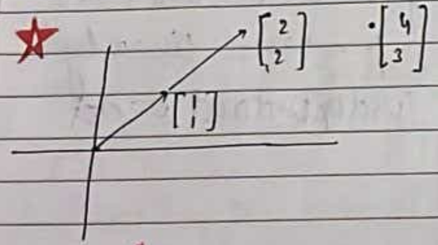
$$\alpha+2\beta=a$$
$$\alpha-\beta=b$$

let's take a vector $\begin{bmatrix}16\\-24\end{bmatrix}$

$\begin{bmatrix}16\\-24\end{bmatrix}$

$$\alpha+2\beta=a$$
$$\alpha-\beta=b$$
$$\alpha=-\frac{38}{3}\quad \beta=\frac{40}{3}$$

★



NO sol^n

$$\alpha\begin{bmatrix}1\\1\end{bmatrix}+\beta\begin{bmatrix}2\\2\end{bmatrix}=\begin{bmatrix}4\\3\end{bmatrix}$$

$$\begin{bmatrix}\alpha\\\alpha\end{bmatrix}+\begin{bmatrix}2\beta\\2\beta\end{bmatrix}=\begin{bmatrix}4\\3\end{bmatrix}$$

$$\alpha+2\beta=4$$
$$\alpha+2\beta=3$$

SPAN

set of all possible linear combination for given vectors / given set of vectors.

$$V_1=\left\{\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\right\}\qquad V_2=\left\{\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix} \\ \end{bmatrix}\right\}$$

Span $S(V_1)\in \mathbb{R}^2$

LINEAR DEPENDANT VECTOR

Depandancy of vector

Q. Set of 3 vectors where $V_i\in\mathbb{R}^2$ & $i=1,2,3$ where $V_i=\begin{bmatrix}i+1\\i+2\end{bmatrix}$ whether it is linearly dependant or not.

$$V_1=\begin{bmatrix}2\\3\end{bmatrix}\quad V_2=\begin{bmatrix}3\\4\end{bmatrix}\quad V_3=\begin{bmatrix}4\\5\end{bmatrix}$$

$$\alpha V_1+\beta V_2=X$$
$$\alpha V_1+\beta V_2=4$$

$$\rightarrow\quad 2\alpha+3\beta=4$$
$$3\alpha+4\beta=5$$
$$\alpha=-1,\ \beta=2$$

Hence we can write V_3 in form of V_1 & V_2

$$\left\{V_i \;\middle|\; \begin{matrix}V_i\in\mathbb{R}^n\\ i=1\text{ to }n\end{matrix}\right\}\qquad \{V_1, V_2, V_3\cdots V_k\cdots V_n\}$$

$$\alpha_1V_1+\alpha_2V_2+\cdots\alpha_{k-1}V_{k-1}$$

Q. $V_i\in\mathbb{R}^2$

$$S=\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\2\end{bmatrix},\begin{bmatrix}3\\4\end{bmatrix}\right\}$$

$$\alpha\begin{bmatrix}2\\2\end{bmatrix}+\beta\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}2\\2\end{bmatrix}+0\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

linearly dependant.

- For any given 2 vectors v_i & v_j where $v_i, v_j \in \mathbb{R}^2$, both of these vectors are linearly dependent iff $v_i = \alpha v_j$ or v_i, v_j lie on same line.

Q. $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\}$

Any 2 independent vectors can be used derive 3rd vector in $\mathbb{R}^2$.

For a given space $\mathbb{R}^n$ if we have $(n+1)$ or more than $n+1$ vector then this set of vector is always linearly dependent.

isme bolna yeh chah rha hai ki max agr 2 linear independent vectors hai toh agr usse jayad vector aaye toh phir voh set linear dependent ho jayega

Imp. In a set of n vectors, if there exist a zero vector, then this set is also linearly dependent.

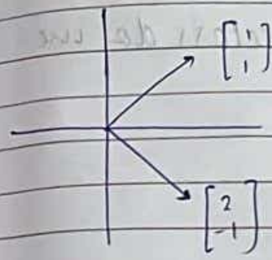
$v_i \in \mathbb{R}^m$

$\{ v_1, v_2, v_3 \ldots 0 \}$

Eg. $v_i \in \mathbb{R}^3$ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\alpha = 0, \beta = 0$ $\quad 0 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

LINEAR INDEPENDANCE OF VECTOR.



$\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

No such α exist.

n vectors of m dimension $v_i \in \mathbb{R}^m$

$\alpha_1 v_1 + \alpha_2 v_2 + \ldots \alpha_n v_n = 0 \quad \forall \alpha_i = 0$.

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow \alpha_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\alpha_1 + 3\alpha_2 = 0$

$2\alpha_1 + 4\alpha_2 = 0$

$\alpha_1 = 0 \quad \alpha_2 = 0$

$\therefore$ the vectors are independant.

Eg $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \Rightarrow \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\alpha + 2\beta = 0$

$\alpha + 2\beta = 0$

multiple sol^n

$\therefore$ dependant.

BASIS OF VECTOR:-

The basis of vector space is a set of linearly independant vectors that spans the full space

OR

★ Given set of vectors
i) SET OF VECTORS must be LI (linearly indep.)
ii) SET OF VECTORS SPANS THE WHOLE SPACE.

Q. To span $\mathbb{R}^n$ how many min. [LI] vectors do we need?

$v_i \in \mathbb{R}^n$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \qquad \text{min} = n \text{ (LI)}$$

if $v_i \in \mathbb{R}^2$ then min = 2 vector (LI)

Q. In $\mathbb{R}^2$ space two vectors are linearly dependant if one of them can be obtain by scaling the other one. If not then these two vectors are linearly independant.
Statement is TRUE.

DEGREE OF FREEDOM = n

mtlab yeh n vector (LI) unki help se other vector ko derive kar skte ha