caling
~~pta~~ $\cancel{d}$
gya toh

18/07

$Ax = b$ where $b = 0$ homogeneous.

$$\begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} b \end{bmatrix}$$

$\alpha v_1 + \beta v_2 + \gamma v_3 = b \rightarrow$ LD

$Ax = b$; $b \neq 0$ Non Homogeneous system.

coeff variable Target

AUGMENTED MATRIX : $[A : b]$

NO SOL^n

$\boxed{\rho(A:b) \neq \rho(A)}$

Eg. $\left[\begin{array}{cc|c} 0 & & \\ & 0 & \\ & & 0 \\ & & \cancel{0} \end{array}\right]$ $\rho(A) = 2$

$\rho(A:b) = 3$

$Ax = b$ $A_{m \times n}$

$\rho(A:B) \neq \rho(A)$ NO SOL^n [Inconsistent]

$\rho(A:B) = \rho(A)$ [Consistent]

- $\rho(A:B = \rho(A) = r$ $(r < m)$ → Infinitely many sol^n
- $\rho(A:B) = \rho(A) = n$ All col. are LI Unique sol^n
- $\rho(A:B) = m$ $\dim(v_i) = m$, $v_i \in \mathbb{R}^m$ → Infinitely many sol^n

Q $Ax = b$

$$\begin{bmatrix} 1 & 4 & 7 & 8 \\ 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 7 & 8 \\ 0 & -3 & -6 & -6 \\ 0 & -6 & -12 & -12 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\begin{bmatrix} 1 & 4 & 7 & 8 \\ 0 & -3 & -6 & -6 \\ 0 & 0 & 0 & \neq 0 \end{bmatrix} \quad R_3 \to R_3 + 2R_2$$

$\rho(A) = 2$

Augmented matrix.

$$\left[\begin{array}{cccc|c} 1 & 4 & 7 & 8 & 5 \\ 0 & -3 & -6 & -6 & \\ 0 & 0 & 0 & & \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$\rho(A:b) = \rho(A)$

$$\begin{array}{cccc} \multicolumn{2}{c}{LI} & & \\ \begin{bmatrix} 1 & 4 & 7 & 8 \\ 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = 2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8-1 \\ 10-2 \\ 12-3 \end{bmatrix}$$

Eg. $\begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 6 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 19 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 2 & 5 & 7 & 12 \\ 3 & 6 & 13 & 19 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 0 & -3 & -5 & -8 \\ 0 & -6 & -15 & -11 \end{array}\right]$$

$-15 - 10 =$

$R_2 \longrightarrow R_3 + 2R_2$

$-15 + 2 \times -5 = -5$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 0 & -3 & -5 & -8 \\ 0 & 0 & 5 & -11 \end{array}\right]$$

$-11 - 8 \times 2$

-27

$\rho(A:b) = \rho(A) = 3.$

$$\begin{bmatrix} 1 & 4 & 6 \\ 0 & -3 & -5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \\ 5 \end{bmatrix}$$

$x + 4y + 6z = 10$

$-3y - 5z = -8$

$5z = 5 \quad \Rightarrow z = 1$

$z = 1$

$-3y = -8 + 5$

$-3y = -3$

$y = 1$

$x = 10 - 6z - 4y$

$= 10 - 6 \times 1 - 4 \times 1$

$x = 10 - 10 = 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$x + 4y + 6z = 10$

$2x + 5y + 7z = 12$

$3x + 6y + 13z = 19$

$x = 0, \ y = 1, \ z = 1$

$0 + 6 + 13 = 19$ True.

Q. Determine λ & μ such that the eq^n

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

i) No sol^n
ii) A unique sol^n
iii) Infinitely many sol^n

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & \lambda-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

i) No sol^n $\quad Ax = b$ ← Pivot

for no sol^n $\lambda - 3 = 0$
$\mu - 10 \neq 0$

$\Rightarrow \lambda = 3 \quad \cancel{\mu = 10} \quad \& \quad \mu \neq 10$

ii)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

UNIQUE SOL^n
All col are LI or $\rho(A:b) = \rho(A) = n$

for the concept of unique sol^n there is no necessity for μ's value.

$\Rightarrow \lambda - 3 \neq 0 \qquad \mu - 10 = 0$ we can or cannot apply this condition.
$\lambda \neq 3 \quad$ & $\quad \mu \in \mathbb{R}$.

iii)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

Iyen dono ko pivot banne se rokna hai

INFINITELY MANY SOL^n
$\rho(A:b)$ & $\rho(A)$ should be less than dim

$\lambda - 3 = 0 \qquad \mu - 10 = 0$
$\lambda = 3 \qquad \mu = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Taking $z = 1$

$x + y + z = 6 \qquad x + 2 + 1 = 6 \Rightarrow x = 3$
$y + 2z = 4 \qquad y + 2 = 4 \Rightarrow y = 2$
$0z = 0$

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$z = 2$, $y = 0$
$x =$

Q2. consider the system of simaltaneous eq^n

$2x - 2y - 2z = a_1$

$-2x - 3z + 2y = a_2 \Rightarrow -2x + 2y - 3z = a_2$

$4x - 4y + 5z = a_3$

find the condition on a_1, a_2, a_3 for the system to have no sol^n

$A \; x = b$

$$\begin{bmatrix} 2 & -2 & -2 \\ -2 & 2 & 3 \\ 4 & -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -2 & -2 & a_1 \\ -2 & +2 & 3 & a_2 \\ 4 & -4 & 5 & a_3 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 2 & -2 & -2 & a_1 \\ 0 & \cancel{-4}0 & \cancel{-5} & a_2 + a_1 \\ 0 & \cancel{-8}0 & 9 & a_3 - 2a_1 \end{array}\right]$$

$\cancel{R_2 \to R_3 - R_3 + 2R_2}$

$$\begin{bmatrix} 2 & \cancel{-2} & -2 \\ 0 & \cancel{-4} & 1 \\ & & \end{bmatrix}$$

$R_3 \longrightarrow R_3 + 9/5\,(R_2)$

$$\left[\begin{array}{ccc|c} 2 & -2 & -2 & a_1 \\ 0 & 0 & -5 & a_2 + a_1 \\ 0 & 0 & 0 & a_3 - 2a_1 + \frac{9}{5}(a_1 + a_2) \end{array}\right]$$

$$a_3 - 2a_1 + \frac{9}{5}(a_1 + a_2) \neq 0$$

$$5a_3 - 10a_1 + 9a_1 + 9a_2 \neq 0$$

$$5a_3 + 9a_2 - a_1 \neq 0$$