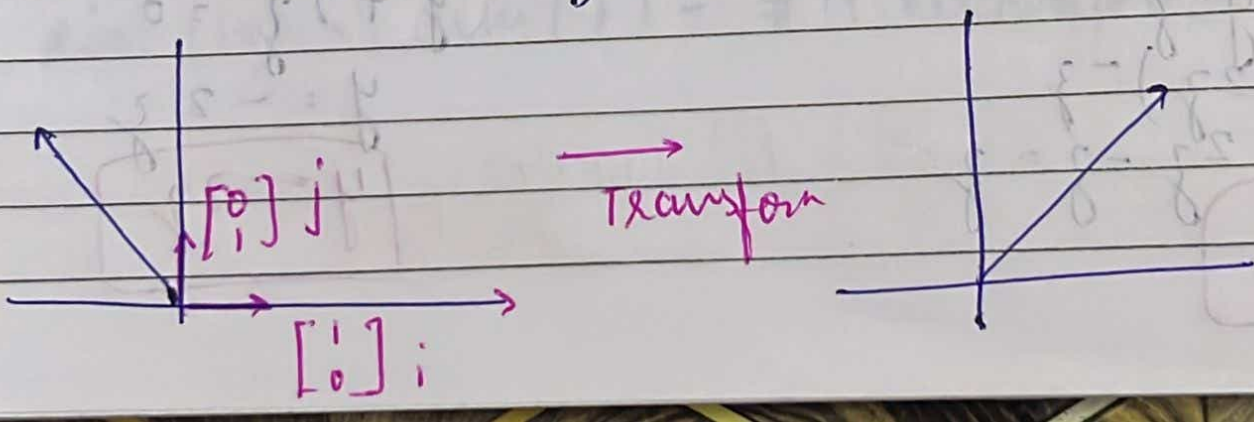
24/07

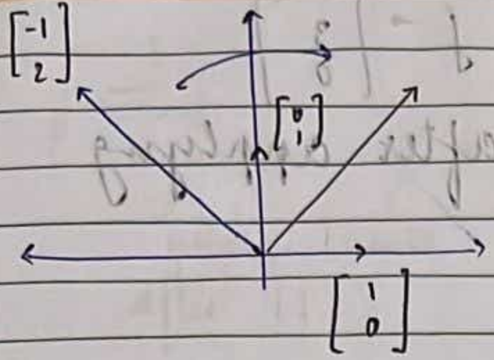
LINEAR TRANSFORMATION

It is a fun^n which takes vectors as i/p & o/ps a vector

$$v \longrightarrow \boxed{f} \longrightarrow v'$$

$$T(x) = y$$

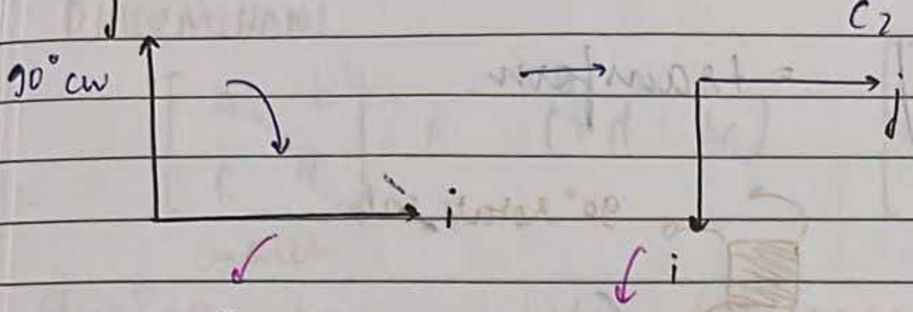




$$\begin{bmatrix}-1\\2\end{bmatrix} = c_1\begin{bmatrix}1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{bmatrix}-1\\2\end{bmatrix} = c_1\begin{bmatrix}1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\end{bmatrix}$$

$$\left.\begin{aligned} c_1 &= -1\\ c_2 &= 2\end{aligned}\right\}$$



$$i = \begin{bmatrix}1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}0\\-1\end{bmatrix}$$

$$j = \begin{bmatrix}0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}1\\0\end{bmatrix}$$

$$c_1\begin{bmatrix}0\\-1\end{bmatrix} + c_2\begin{bmatrix}1\\0\end{bmatrix}$$

$c_1 = -1$

$c_2 = 2$

$$-1\begin{bmatrix}0\\-1\end{bmatrix} + 2\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\1+0\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$

$$\begin{bmatrix}-1\\2\end{bmatrix} \xrightarrow[\text{90° cw}]{\text{linear transf}} \begin{bmatrix}2\\1\end{bmatrix}$$

eg. $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$ 2x2 2x1 = 2x1 vector

Transf. of i Transf. of j

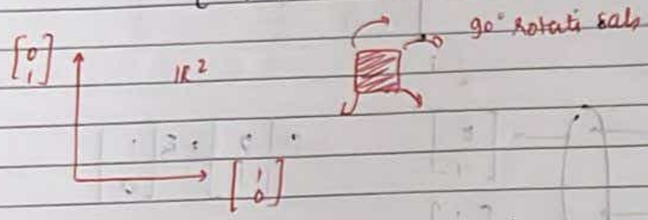
when we do LT ~~that~~ the origin remains the same (fixed).

eg. $v = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ & $i \rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $j \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

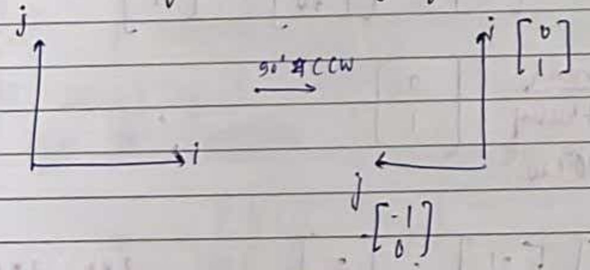
what will be the vector v after applying this transformation

$$\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \text{transform.}$$

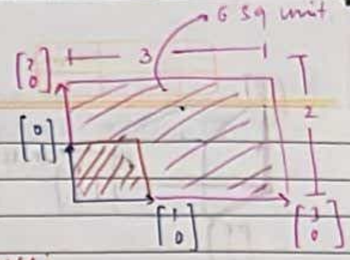


1. If there is a vector $v = \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$ in the first quadrant. what will be the resultant vector after rotating it 90° cw



$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$

Basis ka area kitne factors se scale hua voh det hai



$i \rightarrow i'$
$j \rightarrow j'$
$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

where i lands after LT

this is where j lands after LT

DETERMINANT

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (ad - bc) \qquad \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = 6 - 0 = 6.$$

area.

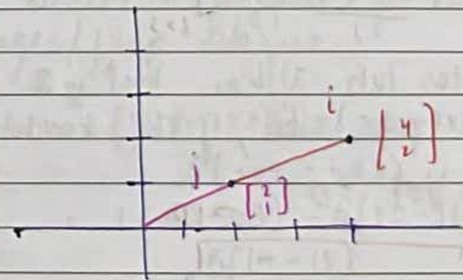
In 2D, basis vector ka area kitne factor se scale hua hai voh det hai

DETERMINANT OF A TRANSFORMATION

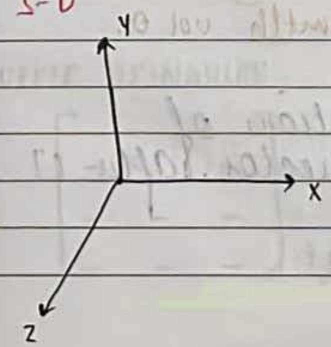
After the LT ~~to~~ of i, j the scaled area (in $\mathbb{R}^2$) is known as det

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} = 0$$

$i \rightarrow i'$
$j \rightarrow j'$



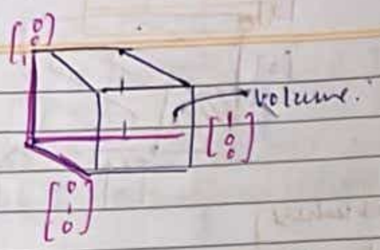
3-D

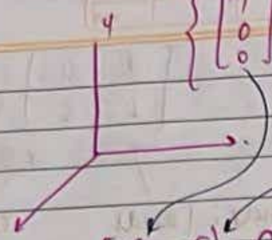


$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

i j k

$v_i \in \mathbb{R}^3$

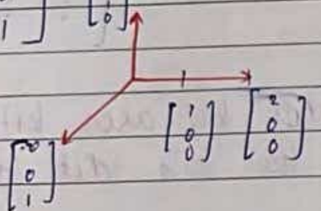




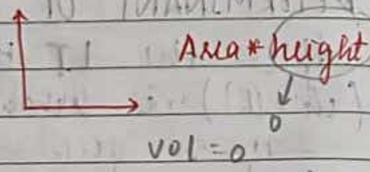
$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$$

$$\underbrace{\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\end{bmatrix}}_{3\times3}$$

eg
$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \end{array}$$
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



eg
$$\begin{array}{ccc} C_1 & C_2 & C_3 \\ \end{array}$$
$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 5 & 7 \\ 3 & 7 & 10 \end{bmatrix}_{3\times3}$$
LI LD



Vector toh 3 hai but $\mathbb{R}^2$ wale ko hi span kar paa rhe, jiske karn height = 0
$\Rightarrow$ Vol = 0

$\{v_1, v_2, \ldots, 0\}$ $\Rightarrow$ ek 0 hone se sab LD ho gye mtlb vol 0.

Sq. matrix is the representation of standard basis of a vector after LT
$\hookrightarrow$ iske sq matrix ka hi det

Sq. matrix is collection of st basis vectors after LT

★ DETERMINANTS

Det is only calculated for sq. matrix

SYMBOL : for a given sq. matrix A we write det (A) or |A|

Det for a 2x2 matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad |A| = (ad - bc)$$

eg
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad |A| = 4 - 6 = -2$$

area can be negative
— we jo original orientation tha woh change / flip hogya.

$$3\times3 \quad \begin{bmatrix} 1 & 4 & 5 \\ 2 & 5 & 7 \\ 3 & 7 & 10 \end{bmatrix}$$

Det = $1(5\times10 - 7\times7) - 4(2\times10 - 7\times3) + 5(2\times7 - 5\times3)$
$= 1(50-49) - 4(20-21) + 5(14-15)$
$= 1 + 4 - 5 = 0$

a_{11}
$a_{12} = (-1)^{1+2} = (-1)^3 = -1$

UPPER TRINGULAR.

$$\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}$$

RESULT

If we have any of these following matrices then det of is equal to product of diagonal elements.

→ Diagonal matrix
→ UTM
→ LTM

$|A|$ = pro. of diagonal elements.

eg: $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ $\quad |A| = xyz$

Q Let U be a sq. matrix of order n st.

$$[U_{ij}]_{n\times n} = \begin{cases} 0 & \text{if } i>j \\ i+j & \text{if } i=j \\ K & \text{otherwise} \end{cases}$$

det = (U)

$$\begin{bmatrix} i+j & 0 & \cdots & 0 \\ K & i+j & & \\ K & K & \ddots & \\ K & K & \cdots\ K & i+j \end{bmatrix}$$

10×10

$$\begin{array}{c} \begin{array}{cccc} a_1 & \cdots & a_{10} & \end{array} \\ \begin{bmatrix} 2 & K & K & \cdots & K \\ 0 & 4 & K & K \cdots & \\ 0 & 0 & 6 & K \cdots & K \\ 0 & 90 & 0 & \cdots & \\ \vdots & & & & 2n \end{bmatrix} \end{array}$$

$2\times 4\times 6 \cdots (2n) = 2^n(1\times 2\times 3 \cdots n)$ → har ek-term ke liye common.

$= 2^n\, n!$