03/08

$2\times2 = m \pm \sqrt{m^2 - p}$

where m mean of trace & p : product.

A) $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ $\lambda_1 = 2$, $\lambda_2 = 1$

$m = \frac{3}{2}$ $p = 0 - (-2)$

$= \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2}$

$= \frac{3}{2} \pm \sqrt{\frac{1}{4}} = \frac{3}{2} + \frac{1}{2}$

$= \frac{3}{2} - \frac{1}{2} =$

~~B) $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ $\lambda_1 =$ $\lambda_2 =$~~

$-4ac$

$\lambda_1 = 2,$
$\lambda_2 = 1$

B) $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ $\lambda_1 = ?$ $\lambda_2 = ?$

$m = \frac{7}{2}$ $p = 10 - (4) = 6$

$= \frac{7}{2} \pm \sqrt{\frac{49}{4} - 6} = \frac{7}{2} \pm \sqrt{\frac{49 - 24}{4}} = \frac{7}{2} \pm \sqrt{\frac{25}{4}}$

$= \frac{7}{2} \pm \frac{5}{2}$

$\lambda_1 = 6$ $\lambda_2 = 1.$

CONCEPT $n\times n$ $MX = 0$

~~$|M| = 0$~~

$|M| = 0 \longrightarrow \rho(M) < n$

If we have det = 0 then we can say that the rank will be less than the no. of columns.

$$A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix} \qquad \lambda_1 = -1 \qquad \lambda_2 = 2$$

$\lambda^2 - (\text{Trace})\lambda + \det = 0$

$m = 1 \quad p = 2.$

$\lambda^2 + \lambda + (-2) = 0$

$\lambda^2 + \lambda - 2 = 0 \Rightarrow \lambda^2 - \lambda + 2\lambda - 2$

$\lambda(\lambda - 1) - 2(\lambda + 1) = 0 \Rightarrow \underline{\lambda = -1 \quad \lambda = +2.}$

STEPS

1. Find λ
2. $A - \lambda I$

$\boxed{(A - \lambda I)}x = 0$, $(A-\lambda I) \to M$, $Mx = 0$

$$\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix} = M$$

① $\boxed{\lambda = -1}$

$$\begin{bmatrix} 5 & -5 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

rank 1

$\rho(A - \lambda I) = 1$
$\eta(A - \lambda I) = 1$

$\boxed{\text{nullity} \rightarrow \text{free variable.}}$

$5x - 5y = 0 \Rightarrow 5p - 5q = 0$
$x - y = 0 \qquad p - q = 0$
$\boxed{p = q.}$

$$\vec{x} = \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} q \\ q \end{bmatrix} = q\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigen vector.

$4-\lambda$

valid generate kar rkta hai

$AX = \lambda X$

$$\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigen vector

$q = 2 \qquad \begin{bmatrix} 2 \\ 2 \end{bmatrix} \in \mathbb{R}^2$

$q \in \mathbb{R}$

for this given eigen values -1 we have ∞ eigen values.

② $\lambda = 2$

$$\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

∞ sol^n kyuki nullity 1

$MX = 0 \longrightarrow$ Homogeneous. $\to$ full rank nhi

$\lambda = 2$

$$\begin{bmatrix} 2 & -5 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Rank : 1
$\rho(A - \lambda I) = 1$
$\eta(A - \lambda I) = 1$

$2p - 5q = 0$
$2p = 5q$
$\boxed{p = \frac{5}{2}q}$

$$X = \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 5/2\, q \\ q \end{bmatrix}$$

$AX = \lambda X$
$(A - \lambda I)X = 0$

$$x = \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \frac{5}{2}q \\ q \end{bmatrix} = q\begin{bmatrix} 5/2 \\ 1 \end{bmatrix}$$

$q=1$ $\begin{bmatrix} 5/2 \\ 1 \end{bmatrix}$

$q=2$ $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

ek eigen value ke liye ∞ eigen vectors honge.

Q. The homogeneous system $(A-\lambda I)X=0$ has infinite solⁿ corresponding to a λ such that $|A-\lambda I|=0$ True / False.

Q. If x is solⁿ corresponding to λ_1 s.t. $(A-\lambda_1 I)x=0$ & kX_1 is also a solⁿ to where $k \in \mathbb{R}$ True / False

Q. If A is a 3×3 matrix & it has 3 eigen values $\lambda_1, \lambda_2, \lambda_3$ & eigen vectors corresponding to λ_1 is $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

WOTF is/are True

~~A)~~ $v_1 = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix}$ is also an eigen vector corresponding to λ_1

B) $v_2 = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$ is also an eigen vector corresponding to λ_2

koi detail di nhi hai

C) $v_3 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is never an eigen vector corres. to λ_3

→ kuch diya nhi

D) $AX=0$ will always have non-trivial solⁿ. → homogenous

agr $v_1, v_2, v_3 = [\,]$ hue toh

if yeh $(A-\lambda I)X=0$ → NON-TRIVIAL SOLⁿ toh True.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \rho(A)=2$$

$\boxed{AX=0}$ iske baare me aapn comment nhi kr skte

$(A-\lambda I)=0$ yeh hamesha non-trivial solⁿ hoga. in homogenous.

Eg. $A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ → $\lambda_1=1$, $\lambda_2=6$ } distinct

$\det(A)=6$

give one eigen vector correspond to λ_1

$$\begin{bmatrix} 2-\lambda & -1 \\ -4 & 5-\lambda \end{bmatrix} = \begin{bmatrix} 2-1 & -1 \\ -4 & 5-1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 1$

$\eta(A) = 1$ → Free variable

$x-y=0$

$\boxed{x=y}$

$c\begin{bmatrix}1\\1\end{bmatrix}$ where $c \in \mathbb{R}$

$\lambda = 6$

$$\begin{bmatrix}2-\lambda & -1\\ -4 & 5-\lambda\end{bmatrix} = \begin{bmatrix}2-6 & -1\\ -4 & 5-6\end{bmatrix} = \begin{bmatrix}-4 & -1\\ -4 & -1\end{bmatrix}$$

$\rho(A) = 1$

$\eta(A) = 1$ ↳ free variable

$-4x - y = 0$

$-4x = y$

$x = \frac{-1}{4}y$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-\frac{1}{4}y\\y\end{bmatrix} = y\begin{bmatrix}-1/4\\1\end{bmatrix}$$

$y \in \mathbb{R}$

$$\left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1/4\\1\end{bmatrix}\right\}$$

If we have distinct eigen value then the eigen vectors corresponding to them forms a Linear Independent set.

Q. Let S is a set of eigen vectors corresponding to n distinct eigen values & $|S| = 2n$ then the set S is LI

True / False ✓

$n \times n \longrightarrow n$ distinct eigen values $\longrightarrow$ n eigen vectors LI

$\{v_1 \longrightarrow v_{2n}\}$

$\{v_1 - v_n\} \rightarrow$ LI.

↳ isme n vectors LI but question me diya hai ki 2n vectors LI hai jo true nhi n ke baad aage ko vector aayga voh LD ho jayega.

Q. $\begin{bmatrix}5 & 3\\3 & 5\end{bmatrix}$ eigen values & eigen vectors

$\begin{bmatrix}5 & 3\\3 & 5\end{bmatrix}$ ∴ Char. eq^n

$\lambda^2 - (10)\lambda + (25-9) = 0$

$\lambda^2 - 10\lambda + 16 = 0$

$\lambda^2 - 8\lambda - 2\lambda + 16 = 0$

$\lambda(\lambda - 8) - 2(\lambda - 8) = 0$

$(\lambda - 8)(\lambda - 2) = 0$

$\lambda_1 = +8$

$\lambda_2 = +2$

$\lambda_1 = +8$

$$\begin{bmatrix}5-\lambda & 3\\3 & 5-\lambda\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 3\\3 & -3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$-3x + 3y = 0$

$-\cancel{3}x = -\cancel{3}y$

$x = y$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}y\\y\end{bmatrix} = y\begin{bmatrix}1\\1\end{bmatrix}$$

$\lambda_2 = 2.$

$$\begin{bmatrix}5-\lambda & 3\\3 & 5-\lambda\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}3 & 3\\3 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$3x + 3y = 0$

$\cancel{3}x = -\cancel{3}y$

$x = -y$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-y\\y\end{bmatrix} = y\begin{bmatrix}-1\\1\end{bmatrix}$$

$Ax = \lambda x.$

$$\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

to check $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an eigen vector corresponding to $\lambda = 2$.

$$\boxed{\{ V_1 \; V_2 \} \longrightarrow LI}$$

$$\boxed{V_1^T \cdot V_2 = 0}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}^T = \underset{1\times 2}{\begin{bmatrix} 1 & -1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

- V_1 & V_2 are orthogonal vectors ~~= 0~~
- If A is a symmetric matrix & eigen values ~~of~~ are distinct then the eigen vectors ~~are~~ are mutually orthogonal to each other.