$|\mathrm{adj}(\mathrm{adj}($

29/07

Q. $A_{n \times n}$ and $|A| = 4$ where $n = 3$

$B = 4 \cdot A$ then $|\mathrm{adj}(\mathrm{adj}\, B)| = ?$

$|\cancel{\mathrm{adj}}\, A| = 4 \Rightarrow |B| = 4 \cdot |A| = K^n |A|$

$|B| = \cancel{4 \times 4 = 16}\; 4^4$

$|\mathrm{adj}(\mathrm{adj}\, B)| = |B^{(n-1)^2}|$

$= |B^{(3-1)^2}|$

$= |B^4|$

$= \cancel{|B^4|}$

$= \cancel{16^4}\,(4^4)^{4}$

$= (4^{16})$

$|A|$

29/07 Q A is a 6×6 matrix and B is obtained as follows $V_b = 2V_a$, $\forall V_a \in A$ | V_a is a column corresponding to [A]

Also $|B| = 3$ find $|adj(adj(adj A))| = ?$

$n = 6$ $\quad V_B = 2V_A$

$|B| = 2^6 |A|$

$|A| = \frac{|B|}{2^6}$

$$|adj(adj(adj\,A)))| = |A|^{(n-1)^3}$$
$$= \left(\frac{3}{2^6}\right)^{(6-1)^3}$$
$$= \left(\frac{3}{2^6}\right)^{5^3}$$
$$= \left(\frac{3}{2^6}\right)^{125}$$

SQUARE SUB-MATRIX.

$$[A] = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 0 \\ 3 & 7 & 3 \end{bmatrix}_{3\times 3}$$

★ Sub matrix corresponding to a given sq matrix can be rectangular

Delete R_1 $\begin{bmatrix} 2 & 5 & 0 \\ 3 & 7 & 3 \end{bmatrix}$ Delete R_2 $\begin{bmatrix} 1 & 4 & 6 \\ 3 & 7 & 3 \end{bmatrix}$

3×3
$\hookrightarrow$ 3×2 $\quad 2\times 1$ $\quad 1\times 2$
3×1 $\quad 2\times 2$ $\quad 1\times 3$

$A \longrightarrow 3\times 3$

possible order of sq. sub matrix

$\rightarrow 3\times 3$
$\hookrightarrow 2\times 2$
$\rightarrow 1\times 1$

How many 2×2 sub matrix are possible

$$\begin{matrix} R_1 \rightarrow \\ R_2 \rightarrow \\ R_3 \rightarrow \end{matrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \longrightarrow \begin{bmatrix} & \\ & \end{bmatrix} \begin{matrix} \leftrightarrow R_1 \\ \leftarrow R_2 \end{matrix}$$

$\uparrow C_1 \quad \uparrow C_2 \quad \uparrow C_3 \qquad\qquad \uparrow C_1 \quad \uparrow C_2$

col $\leftarrow {}^3C_2 \times {}^3C_2 \rightarrow$ Row

$\left.\begin{matrix} R_1 - R_2 \\ R_1 - R_3 \\ R_2 - R_3 \end{matrix}\right\}$ 3 $= 9$ possible.

3 $\begin{cases} C_1 - C_2 \\ C_1 - C_3 \\ C_2 - C_3 \end{cases}$

Eg. $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ $\quad C_1 - C_2 \rightarrow R_1 R_2,\ R_1 R_3,\ R_2 - R_3$

$\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$ $\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$ $\begin{bmatrix} 4 & 7 \\ 3 & 9 \end{bmatrix}$

$C_2 - C_3 \rightarrow R_1 R_2,\ R_1 R_3,\ R_2 R_3$

$\begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$ $\begin{bmatrix} 5 & 8 \\ 6 & 9 \end{bmatrix}$ $\begin{bmatrix} 4 & 7 \\ 6 & 9 \end{bmatrix}$

$C_1 - C_3 \rightarrow R_1 R_2,\ R_1 R_3,\ R_2 R_3$

$\begin{bmatrix} 1 & 7 \\ 3 & 9 \end{bmatrix}$ $\begin{bmatrix} 1 & 7 \\ 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 2 & 8 \\ 3 & 9 \end{bmatrix}$

If any given matrix has m rows & n col. then total possible sq. sub-matrix of order r is ${}^{m}C_{r} \cdot {}^{n}C_{r}$

Eg $A_{100 \times 50}$ find the total # sub-matrix of order 30×30

${}^{100}C_{30} * {}^{50}C_{30}$

Q. ~~what~~ if matrix $A_{5 \times 6}$ is given, what is the order of highest possible sq. sub matrix.

5×5 = order

highest possible sq sub matrix $\rightarrow {}^{6}C_{5} \times {}^{5}C_{5} = 6$

eg. $[3 \times 2] \rightarrow \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

- $\rho(A) \leq \min(m, n)$

$\leftarrow$ ~~min~~

Sq $\longrightarrow$ Rank

$\downarrow$

det

$A = 4 \times 4$ $\det(A) = 0$

$\rho(A) \neq 4$

$4 \times 4 \rightarrow$ 1 sq sub matrix 4×4

$3 \times 3 \rightarrow {}^{4}C_{3} \cdot {}^{4}C_{3} = 16$

RANK : Rank is the highest order non-zero minor corresponding to a given matrix.

Eg. $A_{4 \times 4} = \begin{bmatrix} a & m & x & p \\ b & n & y & q \\ c & o & z & r \\ d & p & w & s \end{bmatrix}_{4 \times 4}$ $\det(A) = 0$, $\rho(A) \neq 4$

$3 \times 3 \rightarrow$ 16 possible.

$\exists$ a minor / det sub matrix $|M|_{3 \times 3} \neq 0$ means $\rho(A) = 3$.

eg. $|A|_{6 \times 5} \rightarrow 5 \times 5$ sub matrix SM; $|SM| = 0$ then $\rho(A) = 5$ T/F

* a 5×5 sub matrix SM such that $|SM| = 0$
$\rho(A) = 5$ T/F can't be said

* a 5×5 sub-matrix SM such that
$|SM| \neq 0$
$\rho(A) = 5$ T/F

Q. for $\forall$ 5×5 sq matrix. SM
$|SM| = 0$
then max possible rank is 4 True

Q. for any given row matrix / column matrix A $\rho(A)$ is 0 or 1 T/F

eg $\rho(A) = 0$ or 1

$A = [1 \ 2 \ 3 \ 4]$ $[0 \ 0 \ 0 \ 0]$

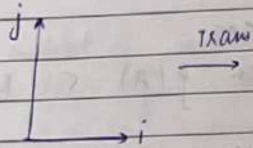
$\rho(A) = 1$ $[1] \ [2] \ [3] \ [4]$ $\rho(A) = 0$

Q. A is a row matrix of order $1\times n$ & is a col matrix of order $n\times 1$

$f(AB) = 0$ or 1 — T/F

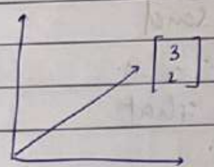
$AB = [\ a\ ]_{1\times 1}$ $[\ 0\ ]_{1\times 1}$

EIGEN VALUES AND EIGEN VECTORS



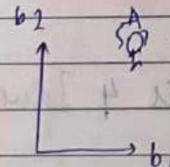
$\xrightarrow{\text{Tran}}$ $\begin{bmatrix} \ & \ \end{bmatrix}$ — $\mathbb{R}^2 = \infty$ basis vector.

aise set honge do ki;



$\left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{LI}, \ - - - \right\}$

$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

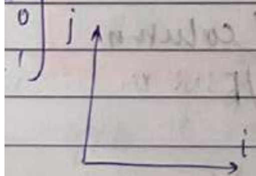


JADU

$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ $\begin{bmatrix} -1 \\ 2 \end{bmatrix} = -1\vec{b_1} + 2\vec{b_2}$

isko inke form me dikhega.

NORMAL (A)



$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \end{bmatrix}$??

transform karke kidhar jayega.

aapne system ke woh vector nikal sakte.

$x\hat{i} + y\hat{j}$

$x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$i \to b_1$

$j \to b_2$

$-1\underset{b_1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} + 2\underset{b_2}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ OR $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

$\begin{bmatrix} | & | \\ b_1 & b_2 \\ | & | \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix}$ Transformation of Basis Vector

Jadu ke jand gole me — mere system me kese dikhega

JADU's WORLD $\begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix}$ MY WORLD

$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ — MY WORLD

$\vec{A}\cdot\vec{x} = \vec{y} \to$ my world

LHS se Basis Vector

$\vec{A^{-1}}\cdot\vec{A}\cdot X = A^{-1}\cdot\vec{y}$

$A\cdot A^{-1} = A^{-1}A = I$

$$\boxed{\vec{X} = A^{-1}\cdot\vec{y}}$$

$A^{-1} \to$ to find out ki basis vector mere system se uske system kese change ho rhe.

$A = \{$

Eg. $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|}$

$\text{Adj } A = \frac{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}}{|A|}$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$

$|A| = 2 + 1 = 3$

$y = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\underbrace{\frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}}_{X}.$