$AX = 0$

$AX = b$ where $b = 0$

Unique Solution (Trivial Solution) $\rho(A) = n$

Infinitely many solution $\rho(A) = m$

Infinitely many solution $\rho(A) = r < m$

* $AX = b$; $b \neq 0$ } "Non - Homogeneous System"

A → coefficient

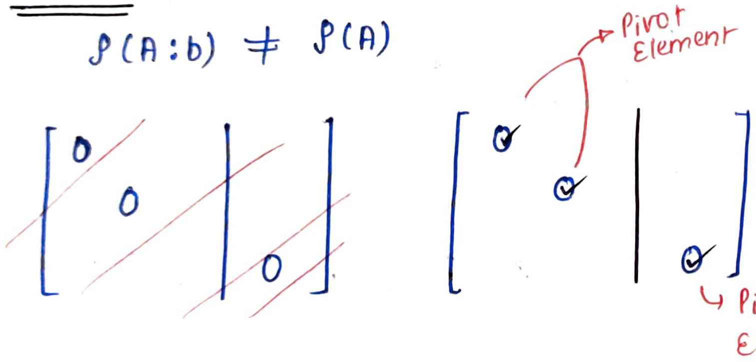
X → variable

b → Target

Augmented matrix → $[A : b]$

<u>No Solution</u> :-

$\rho(A:b) \neq \rho(A)$



$\rho(A) = 2$

$\rho(A:b) = 3$

$AX = b$

$A_{m \times n}$ → n columns, 'n' no. of vectors; ↓ m rows

$\rho(A:B) \neq \rho(A)$ No Solution [Inconsistent]

$\rho(A:B) = \rho(A)$ [Consistent]

$\rho(A:B) = \rho(A) = r$ $(r < m)$ [Infinitely many solutions]

$\rho(A:B) = \rho(A) = n$ → All columns are L.I. [Unique Solution]

$\rho(A:B) = m$ where $\dim(v_i) = m$, $v_i \in \mathbb{R}^m$ [Infinitely many solutions]

Eg: $Ax = b$

$$\begin{bmatrix} 1 & 4 & 7 & 8 \\ 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

$$\hookrightarrow \quad x\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + z\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} + w\begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 7 & 8 & 5 \\ 2 & 5 & 8 & 10 & 7 \\ 3 & 6 & 9 & 12 & 9 \end{array}\right]$$

Forming Echleon form

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

L.I.

$$\left[\begin{array}{cccc|c} 1 & 4 & 7 & 8 & 5 \\ 0 & -3 & -6 & -6 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A:B) = 2 = \rho(A)$

$\hookrightarrow$ [Consistent]

$$\rightarrow \begin{bmatrix} 1 & 4 & 7 & 8 \\ 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

L.I. vectors.

$$\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = 2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8-1 \\ 10-2 \\ 12-3 \end{bmatrix}$$

Eg: $\underbrace{\begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 6 & 13 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 10 \\ 12 \\ 19 \end{bmatrix}}_{b}$. Is Given system consistent or not ??

Sol^n:

$[A : b]$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 2 & 5 & 7 & 12 \\ 3 & 6 & 13 & 19 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 - 3R_1]{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 0 & -3 & -5 & -8 \\ 0 & -6 & -5 & -11 \end{array}\right].$$

$R_3 \leftarrow \cancel{2R_2}\ R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 10 \\ 0 & -3 & -5 & -8 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

$\rho(A) = 3 = \rho(A : B)$.

$$\begin{bmatrix} 1 & 4 & 6 \\ 0 & -3 & -5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \\ 5 \end{bmatrix}$$

$\Rightarrow \quad x + 4y + 6z = 10$

$-3y - 5z = -8$

$5z = 5$

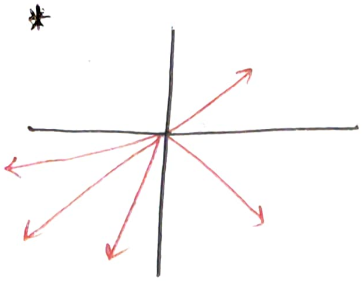
$\hookrightarrow \boxed{z = 1}, \boxed{y = 1}$ and $\boxed{x = 0}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\left.\begin{array}{l} x + 4y + 6z = 10 \\ 2x + 5y + 7z = 12 \\ 3x + 6y + 13z = 19 \end{array}\right\}$ Put $x = 0$, $y = 1$ and $z = 1$ to check whether it satisfies or not.

$\hookrightarrow 0 + 6 + 13 = 19$

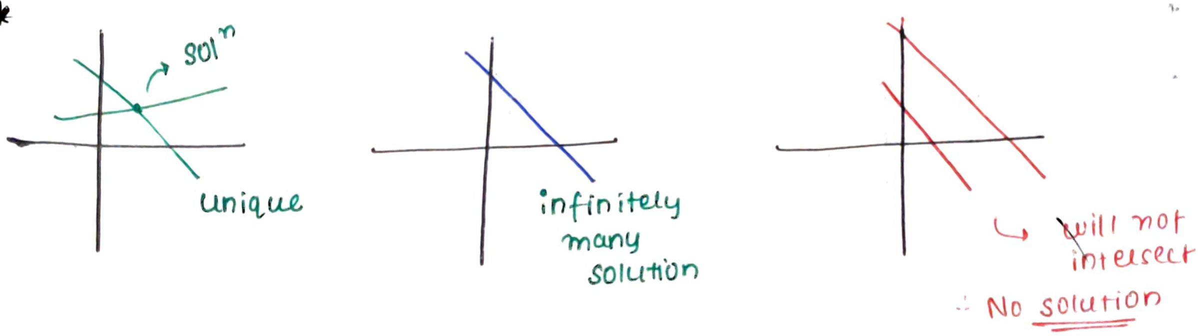
All are satisfying

* 

$v_i \in \mathbb{R}^2$

$$\begin{bmatrix} | & | & | & | & | \\ v_1 & v_2 & v_3 & v_4 & v_5 \\ | & | & | & | & | \end{bmatrix}$$

Max^m Rank = 2 → no. of LI vectors

* 

Ques: Determine λ and μ such that the equation :

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu.$$

(i) No Solution (ii) A unique sol^n (iii) ∞ly many solution.

Sol^n: ~~(i) No Solution~~ :-

$Ax = b$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$[A:b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \leftarrow R_2 - R_3$
$R_3 \leftarrow R_3 - R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(i) for No Solution :-

So, $\lambda = 3$
$\mu \neq 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$\lambda = 3$

B mein Pivot element bana denge.

So, $\mu \neq 10$

(ii) for Unique Solution :- 3 columns hain toh ~~there~~ the rank should be 3.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$\lambda \neq 3$ $\quad \cancel{\mu \neq 10}$

μ can be anything.

$\mu \in \mathbb{R}$

$\lambda \neq 3$ and $\mu \in \mathbb{R}$.

So that $\rho(A) = \rho(A:b)$.

(iii) for ∞ly many Solution :-

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$\lambda = 3$ $\quad \mu = 10$

$\rho(A:b) = \rho(A) = 2 <$ no. of columns.

$\lambda = 3$ and $\mu = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow$ $x + y + z = 6$
$y + 2z = 4$
$0z = 0$

if $z = 1$, then
$y + 2 = 4 \Rightarrow y = 2$
$x + 2 + 1 = 6$
$\Rightarrow x = 3$

$\therefore \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ One possible sol^n

if $z = 2$, then
$y + 4 = 4$
$\Rightarrow y = 0$
and $x + 0 + 2 = 6$
$x = 4$

$\therefore \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ Another possible sol^n

Similarly, we can have ∞ly many sol^n

Ques: Consider the system of simultaneous equation

$$+2x - 2y - 2z = a_1$$
$$-2x - 3z + 2y = a_2$$
$$4x - 4y + 5z = a_3$$

Iske hisaab se matrix nahi banana hain

"Always check the order"

Find the condition on a_1, a_2 and a_3 for the system to have no solution.

Sol^n: NOTE

$$+2x - 2y - 2z = a_1$$
$$-2x + 2y - 3z = a_2$$
$$4x - 4y + 5z = a_3$$

$Ax = b$

$$\left[\begin{array}{ccc|c} +2 & -2 & -2 & a_1 \\ -2 & 2 & -3 & a_2 \\ 4 & -4 & 5 & a_3 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$

$R_3 \leftarrow R_3 - 2R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & -2 & a_1 \\ 0 & 0 & -5 & a_1 + a_2 \\ 0 & 0 & 9 & a_3 - 2a_1 \end{array}\right]$$

$R_3 \leftarrow R_3 + \frac{9}{5} R_2$

$$\left[\begin{array}{ccc|c} 2 & -2 & -2 & a_1 \\ 0 & 0 & -5 & a_1 + a_2 \\ 0 & 0 & 0 & a_3 - 2a_1 + \frac{9}{5}(a_1 + a_2) \end{array}\right]$$

$a_3 - 2a_1 + \frac{9}{5}(a_1 + a_2)$

$$a_3 - 2a_1 + \frac{9}{5}(a_1 + a_2) \neq 0$$

$$\Rightarrow a_3 - 2a_1 + \frac{9a_1}{5} + \frac{9a_2}{5} \neq 0$$

$$\Rightarrow -\frac{a_1}{5} + \frac{9a_2}{5} + a_3 \neq 0$$

$$\Rightarrow \boxed{5a_3 + 9a_2 - a_1 \neq 0}$$

"Condition for no solution"