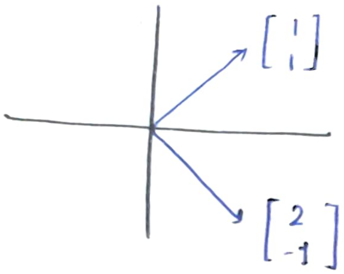


Target vector $= \begin{bmatrix} 16 \\ -24 \end{bmatrix}$

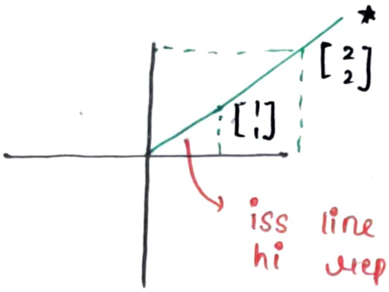
Can we represent target vector using linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$?

Sol^n: write in general form, linear combination

$$\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 16 \\ -24 \end{bmatrix}$$

$$\alpha + 2\beta = 16$$
$$\alpha - \beta = -24$$

if there exist value of α & $\beta \in \mathbb{R}$, then yes we can represent $\begin{bmatrix} 16 \\ -24 \end{bmatrix}$ using $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$



iss line waale ko
hi represent kar skte

Target vector $= \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Can we represent target $\vec{v}$ using $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$?

$$\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \Rightarrow \begin{aligned} \alpha + 2\beta &= 4 \\ \alpha + 2\beta &= 3 \end{aligned}$$

No solution exists for α and β. Therefore, we can't ~~expr~~ represent vector $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ But any vector lying in the span of vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ i.e. line can be represented using $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 0$$

if only $\alpha_i = 0$ could satisfy this equation then linearly independent, but with $\alpha = -2$ equation is satisfied. Hence, linearly dependent.

* $\{ v_i \mid v_i \in \mathbb{R}^n \}$
  $i = 1$ to n

v_i would have dimension "n"

$\{ v_1, v_2, v_3 \dots v_k \dots v_n \}$.

if $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots + \alpha_{k-1} v_{k-1} +$
$\alpha_{k+1} v_{k+1} + \alpha_{k+2} v_{k+2} \dots + \alpha_n v_n = v_k$

i.e. if $\alpha_1 v_1 + \alpha_2 v_2 \dots$, can represent v_k then there set of vector are linearly dependent.

* $V_i \in \mathbb{R}^2$ and $S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$

$$\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow \begin{aligned} \alpha + 2\beta &= 3 \\ \alpha + 2\beta &= 4 \end{aligned}$$

No solution exists, seems like Independent, but.

$$\alpha \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\alpha = 0$, $\beta = 1/2$ $\therefore$ Linearly Dependent.

* For any given 2 vectors, V_i and V_j where V_i & $V_j \in \mathbb{R}^2$, Both of these vectors are linearly dependent if and only if $V_i = \alpha \cdot V_j$ or V_i & V_j will lie on the same line.

* For a given space $\mathbb{R}^n$, if we have $(n+1)$ or more vectors then this set of vectors is always linearly dependent.

Eg : $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \rightarrow$ independent as dimension $= \mathbb{R}^2$ & # vectors also $= 2$

Eg : $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 9 & 6 \end{bmatrix} \rightarrow$ dependent as ~~vector~~ dimension $= \mathbb{R}^2$ and # vectors $= 3$

* In a set of "n" vectors if there exists a zero vector then this set of is also linearly dependent.

Eg: $\begin{array}{c} \begin{matrix} V_1 & V_2 & V_3 \end{matrix} \\ \begin{bmatrix} 2 & 5 & 0 \\ 9 & 12 & 0 \end{bmatrix} \end{array}$ here $\alpha V_1 + \alpha V_2$ can always derive V_3

i.e. when $\alpha = 0$ & $\beta = 0$, $V_1 + V_2 = V_3$

$\therefore$ a set of vectors having a zero vector is always Linearly Dependent.

* <u>Independence</u> :-

$\alpha_1 V_1 + \alpha_2 V_2 = V_3 \longrightarrow (1)$

as linear combination of V_1 & $V_2 = V_3$ $\therefore$ (1) is dependent,

Rel^n b/w $\alpha V_1 + \beta V_2 = 0$ & this 0

$\alpha_1 V_1 + \alpha_2 V_2 = V_3$

$\alpha_1 V_1 + \alpha_2 V_2 - V_3 = 0$

$\hookrightarrow \alpha_1 = 0$ $\hookrightarrow \alpha_2 = 0$ $\hookrightarrow \alpha_3 = -1$

$\therefore$ For a set of vector to be independent $\alpha_i = 0$, here $\alpha_i = -1$ $\therefore$ Not independent

To check whether $\begin{bmatrix}1\\2\end{bmatrix}$ & $\begin{bmatrix}3\\4\end{bmatrix}$ are linearly dependent or independent

if $\alpha \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$

or $\alpha \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

No such value of α exists which may scale $\vec{v_1}$ & $\vec{v_2}$ to each other.

$\therefore$ Linearly independent

Trivially; $\vec{v_1}$ & $\vec{v_2} \in \mathbb{R}^2$
$\therefore$ we have 2 vectors. } independent

* **BASIS of SPACE :-**

The basis of a vector space is a set of linearly independent vectors that spans the full space.

Eg: $\vec{v} = \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$ $\Rightarrow$ Both vectors are linearly independent
$S(\vec{v}) = \mathbb{R}^2$

* **Condition for Basis of Space :-**

(i) Vector should be linearly independent.
(ii) L.I. vectors should span the full space.

Ques: In $\mathbb{R}^2$ space, two vectors are linearly dependent if one of them can be obtained by scaling the other one, if not then these two vectors are linearly independent. $\Rightarrow$ True

Eg: $\vec{v} = \left\{ \begin{bmatrix}1\\1\end{bmatrix} \begin{bmatrix}2\\2\end{bmatrix} \right\}$. $\alpha \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$

$\boxed{\alpha = 2}$

we were able to scale $\begin{bmatrix}1\\1\end{bmatrix}$ to obtain $\begin{bmatrix}2\\2\end{bmatrix}$

$\therefore \left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix} \right\}$ are linearly dependent.

Ques: To span $\mathbb{R}^n$ how many minimum Linearly Independent vectors do we need?

Ans. $\boxed{n}$