Ques: If A and B are symmetric matrices, ~~of~~ of same order, then $AB - BA$ is :

(a) Skew Symmetric ✓ (b) Symmetric (c) Identity Matrix (d) Zero Matrix

Sol^n: $(AB-BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T$

$= BA - AB$

$= -(AB - BA)$

$\therefore$ Skew Symmetric Matrix

Given : $A^T = A$, $B^T = B$

Ques: If matrix A is both symmetric & skew-symmetric, then :

(a) Diagonal (b) Skew Symmetric (c) Symmetric (d) Null ✓

Sol^n: A → symmetric $\Rightarrow A^T = A$

A → Skew Symmetric $\Rightarrow A^T = -A$

only possible for Null Matrix

* Approach for rotating a matrix by 90° :-

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{90^\circ} \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

a_{12}, a_{13} → a_{23}, a_{33}

Another method :

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \xrightarrow{\text{Reverse each array}} \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

* Approach for Transpose of a Matrix :-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

swap

swap

Diagonal elements remains same

* Code :

```
for(int i=1 ; i<=n ; i++)
{
    for (int j=i+1 ; j<=n ; j++)
    {
        swap (a[i][j], a[j][i]);
    }
}
```

This Algorithm is only for square Matrix

$$\# \text{ swaps} = \frac{n^2 - n}{2}$$

Time Complexity $= O(n^2)$

Space Complexity $= O(1)$.

Eg :

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1\,100} \\ a_{21} & \cdots & & a_{2\,100} \\ \vdots & & & \vdots \\ a_{100\,1} & \cdots & a_{100} & a_{100} \end{bmatrix}$$

$$\# \text{ swaps} = \frac{n^2 - n}{2} = \frac{100^2 - 100}{2} = \frac{100(100-1)}{2} = 50 \times 99 = \boxed{4950}$$

- **ELEMENTARY OPERATIONS :-**

① Interchange of any 2 rows or columns :-

$\boxed{R_i \longleftrightarrow R_j}$ or $\boxed{C_i \longleftrightarrow C_j}$

Eg :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{R_1 \longleftrightarrow R_2} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

Eg :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{C_1 \longleftrightarrow C_2} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

② Multiplication of any row / column by a non-zero k

$$\boxed{\begin{array}{l} R_i \longrightarrow kR_i \\ C_i \longrightarrow kC_i \end{array}}$$

③ For any row r_i, we can write it as :
or column c_i

$$\boxed{\begin{array}{l} r_i \longrightarrow r_i + k r_j \\ C_i \longrightarrow C_i + k C_j \end{array}}$$

Eg: $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ $\quad R_2 \rightarrow R_2 - 2R_1$

Solⁿ: $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$

Eg: Verify $\boxed{IA = A}$

Solⁿ: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

Perform this matrix on Identity

Perform same operation in I, then multiply to get result.

Always hit/multiply from LHS.

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

Q. ⇒ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 5 \\ 8 & 6 & 10 \end{bmatrix}$ $\quad R_3 \longrightarrow R_3 - 2R_2$ $\quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

Perform on I

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

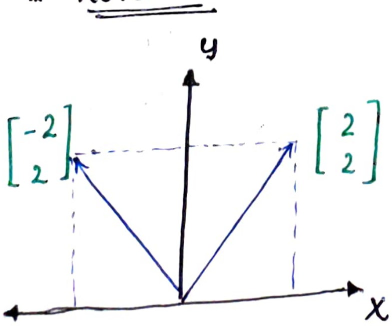
$\begin{array}{ccc} 0 & 0 & 1 \\ -\,0 & 2 & 0 \\ \hline 0 & -2 & 1 \end{array}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 5 \\ 8 & 6 & 10 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

* Rotations :-

$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ → x-axis, → y-axis.



$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\Rightarrow 2\hat{i} + 2\hat{j} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$2\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$