- $A_1 x + B_1 y = C_1$
  $A_2 x + B_2 y = C_2$

① unique solution

$$\boxed{\frac{A_1}{A_2} \neq \frac{B_1}{B_2}}$$

② ∞ely many Solutions

$$\boxed{\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}}$$

③ No Solution

$$\boxed{\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}}$$

Ques: $2x + y = 2$
$x - y = 3$ $\Big\} \rightarrow Ax = b.$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$$

Echleon form

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Unique Solution

* $Ax = b$ has a solution if and only if $b \in span(v_1 \dots v_n)$ where $v_i \in \mathbb{R}^m$

OR

$b \in$ Linear Combination of $\{v_1, v_2 \dots v_n\}$

OR

b is linearly dependent on $\{v_1, v_2 \dots v_n\}$.

$A_{m \times n}$

$n \rightarrow$ # Vectors

$m \rightarrow$ Dimension of a vector or # Rows

- **AUGMENTED MATRIX** :-

$Ax = b$

$[A : b]$

$A \cup b = [A : b]$

$A \rightarrow$ n columns

$[A : b] \rightarrow$ n+1 columns

Eg: $2x + y = 2$
$x - y = 3$

$$\left[\begin{array}{cc|c} 2 & 1 & 2 \\ 1 & -1 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 2 & 1 & 2 \\ 0 & -3 & -2 \end{array}\right]$$

Unique Solution

Q: $2x + 4y = 6$
$4x + 8y = 10$

$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix}$ → Target vector

$\Downarrow$

$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Aug. Matrix → $[A : B]$

$\left[\begin{array}{cc|c} 2 & 4 & 6 \\ 4 & 8 & 10 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 2 & 4 & 6 \\ 0 & 0 & -2 \end{array}\right]$

No Solution Exist

Kabhi bhi pivot element original matrix ke bahar nahi jaana chahiye, in this case, we can see pivot element (-2) is outside the original matrix, so. ⇒ No Solution.

Jis bhi column mein pivot element aa raha hain woh Linearly Independent Vector hota hain.

"Echleon form ke inside"

* $\boxed{Ax = b}$

$b = 0$ → Homogeneous System

$b \neq 0$ → Non-homogeneous System

• HOMOGENEOUS SYSTEM of EQUATIONS :-

$\boxed{Ax = 0}$

eg: $2x + 3y = 0$
$2x + 5y = 0$

"~~One~~ Solution will always exist for the Homogeneous system of equations."
[i.e. No. of variables = 0]

Ques: $A = [a_{ij}]_{m \times n}$
$\rho(A) = n$

Discuss about the nature of solution.

UNIQUE SOLUTION.

Sol^n: $V_i \in \mathbb{R}^m$

$\left[\begin{array}{ccccc|c} | & | & | & & | & 0 \\ v_1 & v_2 & v_3 & \cdots & v_n & \vdots \\ | & | & | & & | & 0 \end{array}\right]$ ⇒ Unique Solution (All variables = 0)

can also say "Trivial Solution"

$\underline{\underline{Q.}} \Rightarrow A = [a_{ij}]_{m \times n}$ $\quad v_i \in \mathbb{R}^m$

$\rho(A) = m$ $\rightarrow$ ∞ely many solutions

$\Rightarrow \boxed{n \geq m}$

$\rho(A) \leq \min(m, n)$

$\rho(A) = m \leq \min(m, n).$

$$\begin{bmatrix} a_{11} & a_{21} & & a_{n1} \\ \vdots & \vdots & \text{---} & \vdots \\ a_{1m} & a_{2m} & & a_{nm} \end{bmatrix}$$

There will always be some dependent vectors !!

$$\begin{bmatrix} 2 & 5 & 8 & 10 \\ 3 & 6 & 9 & 12 \\ 4 & 7 & 10 & 14 \end{bmatrix}$$

$\underline{Eg}$:

$$\begin{bmatrix} 2 & 4 & 8 \\ 3 & 5 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\downarrow$ Echleon form.

$$\begin{bmatrix} 2 & 4 & 8 \\ 0 & -1 & -1 \end{bmatrix}$$

$$2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$$2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 4 \\ 5 \end{bmatrix} - 1\begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\times 2$

$$4\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix} - 2\begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\times 3$

$\times 4$

Here, we can see there are many possibilities for x any y

So, ∞ely many solutions.

$\boxed{AX = 0}$

$AX = b$
where $b = 0$

Unique Solution
(Trivial Solution)
$\rho(A) = n$

Infinitely many solution
$\rho(A) = m$

Infinitely many solution
$\rho(A) = r < m$