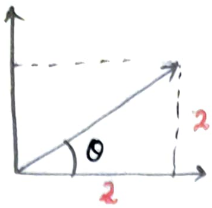
• VECTORS :-

(in terms of Physics) : → which has both magnitude & direction.



Magnitude $= 2\sqrt{2}$

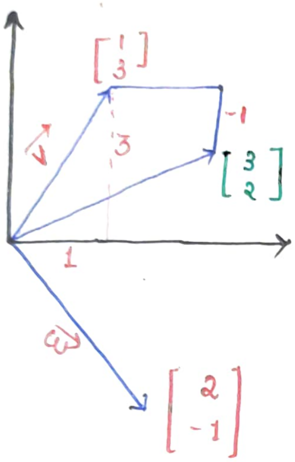
Direction $(\theta) = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{2}{2}\right| = \tan^{-1}(1)$

$= 45°$ from x-axis . .

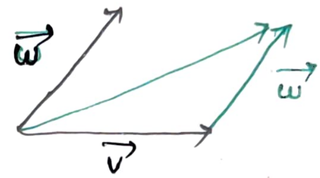
(in terms of CS) : → Vector is an ordered list of elements.

$$\begin{array}{c} \text{Area} \\ \text{Price} \end{array} \begin{bmatrix} 1200\ ft^2 \\ 50\ lac \end{bmatrix} \neq \begin{bmatrix} 25\ lac \\ 2500\ ft^2 \end{bmatrix}$$

Vector Addition :-



$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$



* Def^n: Vector is an element of $\mathbb{R}^n$ where n is the dimension of the vector.

Real No

↳ Space → {Set of all possible vectors V_i where $V_i \in \mathbb{R}^n$}

Eg : V_i is a ~~in~~ vector in R^n

$V_i \in \mathbb{R}^n$ or $V_i = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \ a_i \in \mathbb{R}$

$\mathbb{R} \rightarrow$ Real No.

$\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \mathbb{R} \cdots\cdots \mathbb{R}}_{n}$ (Cartesian Product)

Eg : $\mathbb{R}^2 \longrightarrow \mathbb{R} \times \mathbb{R}$

$\{(x, y) \mid x \in \mathbb{R},\ y \in \mathbb{R}\}$

$\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$

element

n = 1 : 1D vector

Eg : 2, 3, 5.2235

n = 2 :

Dimension = 2

Eg : $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2.57 \\ 3.57 \end{bmatrix}$

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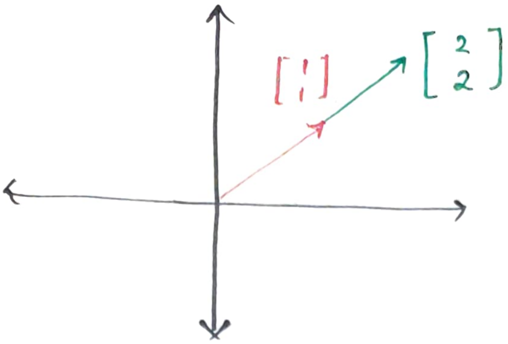

$v_1 = \begin{bmatrix}1\\1\end{bmatrix} \in \mathbb{R}^2$

$v_i \in \mathbb{R}^2(\mathbb{R})$ ↳ Field.

* The scalers used for vector scaling always comes from this Field.

$c \cdot v_1$ where $c \in \mathbb{R}$ (where R is the field).

* Addition of 2 vectors :-

→ For addition of 2 vectors, dimension of both the vector must be same.

Eg : $v_1 \in \mathbb{R}^2$, $v_2 \in \mathbb{R}^3$ So we can't add these vectors.

$\begin{bmatrix}1\\2\end{bmatrix} \in \mathbb{R}^2$

$\begin{bmatrix}1\\2\\0\end{bmatrix} \in \mathbb{R}^3$

Dimension different.

zero can also be a element.

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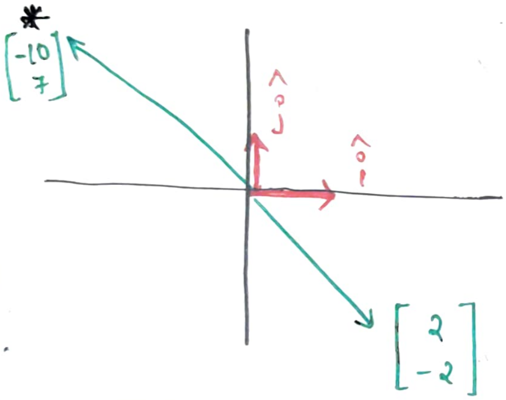

$\hat{i} \in \mathbb{R}^2$

$\begin{bmatrix}1\\0\end{bmatrix} \in \mathbb{R}^2$

$2\begin{bmatrix}1\\0\end{bmatrix} - 2\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

$\hat{j} \in \mathbb{R}^2$

$-10\begin{bmatrix}1\\0\end{bmatrix} + 7\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-10\\7\end{bmatrix}$

$\hat{j} = \begin{bmatrix}0\\1\end{bmatrix}$

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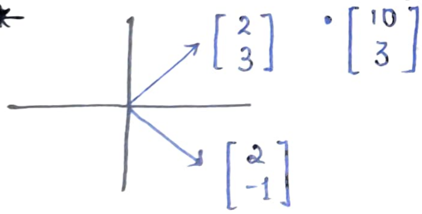


$a\begin{bmatrix}2\\3\end{bmatrix} + b\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}10\\3\end{bmatrix}$

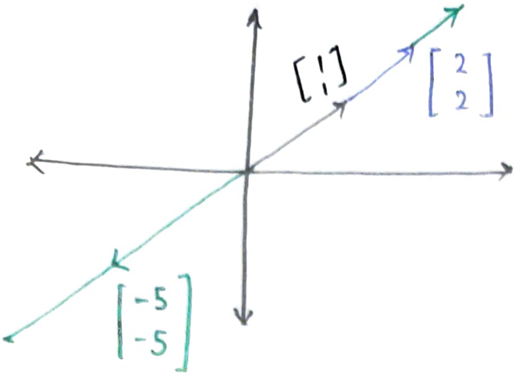
$2\begin{bmatrix}2\\3\end{bmatrix} + 3\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}10\\3\end{bmatrix}$

$\left\{ \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}, \cancel{\left\{\begin{bmatrix}2\\3\end{bmatrix}\right\}} \left\{\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}\right\} \cdots \right\}$ → Basis Vector {we can write any vector in form of Basis vector}.

- There are ∞ possible Basis Vectors for a given Space $\mathbb{R}^n$. $\mathbb{R}^n \rightarrow \infty$

- Given any two vectors in $\mathbb{R}^2$, I can represent any target vector using these two vectors. False

Explain-ation ⇒



$-\frac{5}{2}\begin{bmatrix}2\\2\end{bmatrix} \quad \begin{bmatrix}-5\\-5\end{bmatrix}$

We cannot represent using these vectors because both are in same line [Linear Dependent]

* Linear Combination of Vectors :-

For any given set of vectors $V_i \in \mathbb{R}^n$, we can always write these vectors in the form of $\underbrace{\alpha_1 V_1 + \alpha_2 V_2 + \dots \alpha_n V_n}_{\text{Linear Combination of } n\text{-vectors.}}$ where $\alpha_i \in \mathbb{R}$ and known as scalar.

Sabhi vectors ko scale karke combine karna !!

Eg : $V_i \in \mathbb{R}^2$

$$\left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix} \right\}$$

$$\left.\alpha_1 \begin{bmatrix}1\\0\end{bmatrix} + \alpha_2 \begin{bmatrix}2\\5\end{bmatrix}\right\} \rightarrow \text{Linear Combination}$$

* SPAN :-

Eg: $V_i \in \mathbb{R}^2$

$$\left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}.$$

$$2\begin{bmatrix}1\\0\end{bmatrix} + 5\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\5\end{bmatrix}$$

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$$

Def^n:-

The set of all possible linear combinations of two vectors v_1 and v_2 is known as Span of these vectors.

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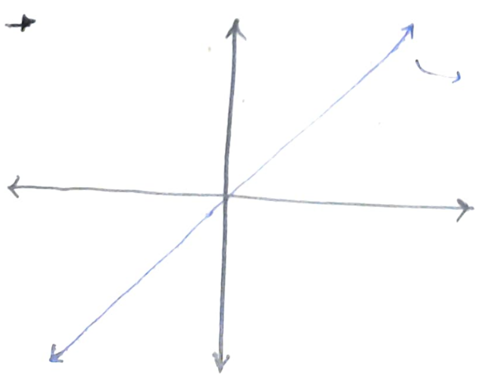


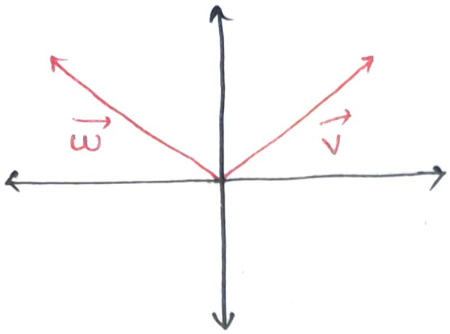
$y = mx$

$m = 2 \quad \left\{ \begin{bmatrix}1\\2\end{bmatrix} \begin{bmatrix}2\\4\end{bmatrix} \begin{bmatrix}3\\6\end{bmatrix} \dots \right.$

What is the span of $\left\{ \begin{bmatrix}1\\2\end{bmatrix} \begin{bmatrix}3\\6\end{bmatrix} \right\}$?

Line : $\boxed{y = 2x}$ → SPAN.

Eg:



$\vec{v} \in \mathbb{R}^2$

$\vec{w} \in \mathbb{R}^2$

SPAN : $\mathbb{R}^2$ (Space).

* **Linearly Dependent Vectors** : In a given set of vectors, if one of the vector can be represented as a linear combination of other vectors, then we say that the ~~line~~ vector is linearly dependent.

Eg: $\left\{ \underset{v_1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}, \underset{v_2}{\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}}, \underset{v_3}{\begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}} \right\}$ $v \in \mathbb{R}^3$

$2v_1 + v_2 = v_3.$ $\therefore$ Linearly dependent

* **Linearly Independent Vectors** : Let us assume, we have a set of n-vector, each of them having m dimensions, we say that these set of vectors are linearly independent if and only if :

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots \alpha_n v_n = 0$$
$$\forall \alpha_i = 0$$

$S = \{v_1, v_2, v_3 \dots v_n\}$ $v_i \in \mathbb{R}^m$ m → dimension

OR

$v_i = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ where $a_i \in \mathbb{R}$.

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots \alpha_n v_n = 0$$
$$\forall \alpha_i = 0$$

& ~~Agar~~ Sirf $\alpha_i = 0$ ~~ki value~~ pe 0 hona chahiye agar α_i ki kisi aur value pe 0 hora hain toh linearly dependent.