Que: Reduce the matrix in Echleon form

(1) $A = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 2 & -3 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix}$

(2) Find the Rank of the Matrix

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Sol$^n$: (i)

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 2 & -3 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 3R_1]{R_2 \leftarrow R_2 - 2R_1 \ \&} \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & -5 & -10 \\ 0 & 6 & \cancel{-7} \ -10 & -24 \end{bmatrix}$$

~~$R_3 - 6R_2 \rightarrow R_3$~~

$R_3 \leftarrow R_3 - 6R_2$

Rank = Pivot Elements = 3

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 20 & 36 \end{bmatrix}$$

This is Required Echleon form.

Sol$^n$: (ii)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_3 \leftarrow \cancel{R_3}\ 3R_3 + R_2$

Rank = Pivot Elements = 2

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

Echleon form

Rank of the matrix = 2

NOTE : $A = [a_{ij}]_{m \times n}$ $\quad V_i \in \mathbb{R}^m$

$\rho(A) \leq \min(m, n)$ OR Maximum rank of any matrix A is less than equal to minimum of no. of rows or columns.

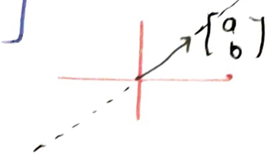
$n > m$

$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \begin{bmatrix} g \\ h \end{bmatrix} \right\}$$

$n < m$ $\quad m = 3$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

$\mathbb{R}^2$



Ques : $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ Does this span $\mathbb{R}^3$ ?

Solⁿ : $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ Rank = 2 → Maximum 2 dimension ke space ko span kar paayega ($\mathbb{R}^2$)

It can span any vector in $\mathbb{R}^2$

So, This does not span $\mathbb{R}^3$.

* Only a zero matrix has rank '0'.
* Minimum rank of a non-zero matrix is '1'

→ Rank $(A+B) \leq$ rank$(A)$ + rank$(B)$

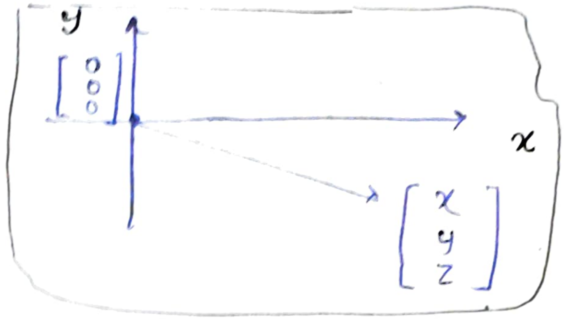
→ Rank $(AB) \leq$ ~~rank~~ $\min(\rho(A), \rho(B))$

Ques : If $\vec{z} \in$ span $(\vec{x}, \vec{y})$ then $\{\vec{x}, \vec{y}, \vec{z}\}$ is linearly dependent. ⇒ True

$\vec{z}$ vector $\vec{x}, \vec{y}$ ke span pe aa raha hain toh we can say it can be derived by $\vec{x}$ & $\vec{y}$, so that's why linear dependent.

Ques: If $\vec{x}, \vec{y} \in \mathbb{R}^3$ and $\vec{x}$ is not a multiple of $\vec{y}$, then $\{\vec{x}, \vec{y}\}$ is LI

→ False




Ques: Does $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$ spans $\mathbb{R}^3$?

Solⁿ:

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ 3 & 7 & 2 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 3R_1]{R_2 \leftarrow R_2 - 2R_1} \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Echleon form

Rank = 2

So, it does not span $\mathbb{R}^3$

Ques: $\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$

$\vec{a_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{a_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{a_3} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ Linearly Dependent / Linear Independent.

Solⁿ:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Echleon form

Rank = 2 = # LI vectors

# • SYSTEM of LINEAR EQUATIONS :-

Eg : $2x + 3y = 5 \rightarrow (1)$

$4x + 5y = 4 \rightarrow (2)$

Substitution Method :

$4x + 6y = 10$

$4x + 5y = 4$

$y = 6$ and $4x + 30 = 4$

$4x = -26$

$x = \frac{-26}{4}$

$$x\begin{bmatrix}2\\4\end{bmatrix} + y\begin{bmatrix}3\\5\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$$ Target Vector.

$$\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix} \xrightarrow{\text{Echleon form}} \begin{bmatrix}2 & 3\\0 & -1\end{bmatrix}$$

Unique Solution

These 2 linear independent vectors se hum $\begin{bmatrix}5\\4\end{bmatrix}$ create vectors ko laa skte hain.

Ques : $2x + 4y = 6$

$4x + 8y = 12.$

$Ax = b$ → Target Vector.

$$\Rightarrow \begin{bmatrix}2 & 4\\4 & 8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\12\end{bmatrix}$$

$y = mx + c$ (m → slope, c → intercept.)

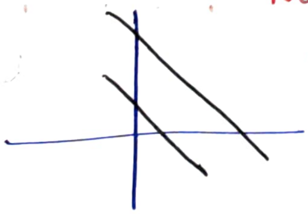
$$x\begin{bmatrix}2\\4\end{bmatrix} + y\begin{bmatrix}4\\8\end{bmatrix} = \begin{bmatrix}6\\12\end{bmatrix}$$ $\infty$ many solutions.

Ques : $2x + 4y = 6$

$4x + 8y = 10.$

$$\Rightarrow \begin{bmatrix}2 & 4\\4 & 8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\10\end{bmatrix}$$

No solution



$4y = -2x + 6$

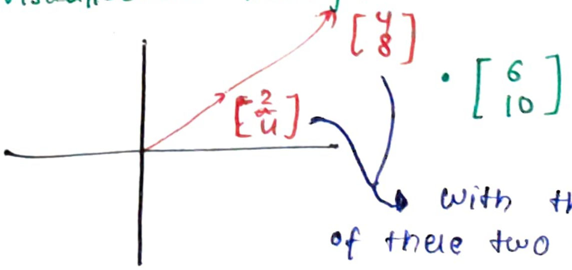
$y = \frac{-2}{4}x + \frac{6}{4}$

$2x + 4y = 6$

$4x + 8y = 10.$

$m = \frac{-2}{4}$

Visualisation through Vector.




with the help of these two vectors, we can never derive $\begin{bmatrix}6\\10\end{bmatrix}$, so No solution.

- $A_1 x + B_1 y = C_1$
  $A_2 x + B_2 y = C_2$

① unique solution

$$\boxed{\frac{A_1}{A_2} \neq \frac{B_1}{B_2}}$$

② ∞ely many Solutions

$$\boxed{\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}}$$

③ No Solution

$$\boxed{\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}}$$