MATRICES

⇒ Matrix is a 2D array of numbers.

⇒ Representation : $A = [a_{ij}]$ (Rows, Columns)

Eg.
$$\begin{bmatrix} 7 & 8 \\ 9 & 1 \\ 7 & 10 \end{bmatrix}$$

↓ Column → Row

where number $K \in R$, $\in N$, $\in Q$

$\boxed{a_{21} = 9}$

⇒ Order of a matrix : A matrix having 'm' rows & 'n' columns, is called as matrix of order "$m \times n$"

where a_{ij} is an element lying in the i^{th} row & j^{th} column.

⇒ No. of elements in matrix having order $m \times n$: $\boxed{mn}$

Ques : No. of elements in matrix is 8.

Ans : Possible order of matrix : 1×8, 8×1, 2×4, 4×2

Ques : Construct a 2×2 matrix $A = [a_{ij}]$ where $a_{ij} = |i-j|$

where $1 \le i \le n$, $1 \le j \le n$.

Ans :

$$\begin{bmatrix} \overset{11}{0} & \overset{12}{1} \\ \overset{21}{1} & \overset{22}{0} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$a_{12} = |1-2| = 1$

$a_{21} = |2-1| = 1$

Ques : The number of all possible matrices of order 3×3 where $a_{ij} \in \{0, 1\}$

Sol^n :

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$$

2 possibilities are there for each element ∴ 2^9 matrices possible

Ques : $A_{n\times n}$ or $A = [a_{ij}]_{n\times n}$

$$\begin{cases} a_{ij} = 0 \ ; \text{ if } i = j \\ a_{ij} = k \ ; \text{ if } i > j, \ k \text{ is a const} \\ a_{ij} \in \{1, 2, 3\} \ ; \text{ otherwise} \end{cases}$$

Total Possible matrices.

$$\begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 0 & - & - \\ k & 0 & - \\ k & k & 0 \end{bmatrix}$$

→ 3 Possibilities

Sol^n : $n \times n$

↓

n^2 elements

$n \rightarrow$ Diagonal entries

∴ $(n^2 - n)$ = Non-Diagonal Entries.

So, Total Possible matrices = $\boxed{3^{(n^2-n)/2}}$

• TYPES of MATRICES :-

(1.) Column Matrix : A matrix is said to be a column matrix if it has only one column.

THIN MATRIX ($i > j$)

$\boxed{A = [a_{ij}]_{m \times 1}}$

(2.) Row Matrix : A matrix is said to be a row matrix if it has only one row.

FAT MATRIX ($i < j$)

$\boxed{A = [a_{ij}]_{1 \times m}}$

(3.) Square Matrix : A matrix in which no. of rows = no. of columns.

$\boxed{A = [a_{ij}]_{n \times n}}$

(4.) Diagonal Matrix : Diagonal matrix is a square matrix where all non-diagonal elements are zero.

$\boxed{B = [b_{ij}]_{n \times n}}$

$b_{ij} = 0 \;;\; \text{if } i \neq j$

(5.) Scalar Matrix : A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal.

$$\boxed{B = [b_{ij}]_{n \times n} = \begin{cases} b_{ij} = k \;, & \text{if } i = j \\ b_{ij} = 0 \;; & \text{if } i \neq j \end{cases}}$$

Eg: $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}_{2 \times 2}$

Every Scalar Matrix is a diagonal matrix.

(6.) Identity Matrix : A square matrix is said to be identity matrix if all the diagonal elements are 1 and non-diagonal elements are 0.

$$\boxed{\begin{matrix} b_{ij} = 1 \;; & \text{if } i = j \\ b_{ij} = 0 \;; & \text{if } i \neq j \end{matrix}}$$

Eg : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$

Q. => $[4]_{1 \times 1}$ → Diagonal Matrix, Scalar Matrix

Q. => $[1]_{1 \times 1}$ → Identity Matrix.

(7.) Zero Matrix : A matrix is said to be zero matrix if all the elements are zero

$$\boxed{\begin{matrix} B = [b_{ij}] \\ b_{ij} = 0 \;\; \forall \, i, j \end{matrix}}$$

Eg : $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2 \times 3}$

Q. $[0] \longrightarrow$ Zero / Null Matrix ✓

$\quad\searrow$ Diagonal Matrix ✗

NOTE: $0 \neq [0]_{1\times1}$

(8) Lower Angular Matrix : Matrix in which the elements below the diagonal are non-zero & the elements above the diagonal are zero, is called Lower Angular Matrix.

$$\boxed{\begin{aligned} B = b_{ij} \quad \text{where} \quad & b_{ij} \neq 0 \ ; \ i > j \\ & b_{ij} = 0 \ ; \ i < j \end{aligned}}$$

(9.) Upper Angular Matrix : Matrix in which the elements above the diagonal are non-zero & the elements below the diagonal are zero, is called Upper Angular Matrix.

$$\boxed{\begin{aligned} B = b_{ij} \quad \text{where} \quad & b_{ij} \neq 0 \ ; \ i < j \\ & b_{ij} = 0 \ ; \ i > j \end{aligned}}$$

- EQUALITY of MATRICES :-

⇒ Two matrices A and B are said to be equal ~~(i) if they are of same order~~ (i) if they are of same order.

(ii) every element in a is equal to corresponding element of b.

$A = [a_{ij}]_{m\times n}$

$B = [b_{ij}]_{m\times n}$ ⟶ (i) Same Order

(ii) $a_{ij} = b_{ij} \ \forall \ i, j$

Eg: $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ Find a, b, c, d.

Soln: $2a + b = 4$ $\qquad$ $5c - d = 11$

$a - 2b = -3$ $\qquad$ $4c + 3d = 24$

Solving, we get $\qquad$ Solving, we get

$\boxed{\begin{aligned} a &= 1 \\ b &= 2 \end{aligned}}$ $\qquad$ $\boxed{\begin{aligned} c &= 3 \\ d &= 4 \end{aligned}}$

• **OPERATIONS on MATRICES :-**

① **Addition of Matrices :-**

→ Two matrices can only be added if they are of the same order.

$$\left.\begin{aligned} A &= [a_{ij}]_{m\times n} \\ B &= [b_{ij}]_{m\times n} \end{aligned}\right\} \Rightarrow A + B = C_{m\times n} \quad \boxed{c_{ij} = a_{ij} + b_{ij} \quad \forall\ i,j}$$

NOTE : Complexity of Matrix Addition & Matrix Subtraction → $\boxed{O(n^2)}$

Eg : $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times3} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}_{3\times3}$

↳ Here we have to perform (3×3) times Arithmetic oper (Add^n).

② **Multiplication of a Matrix by Scalar :-**

We define it as follows :-

if $A_{m\times n} = [a_{ij}]_{m\times n}$

and k is a scalar, then

$$\boxed{k\cdot a_{ij}\ \forall\ i,j \longrightarrow k\cdot A_{m\times n} \text{ OR } k\,[a_{ij}]\ \forall\ i,j \text{ OR } [k\cdot a_{ij}]}$$