Problem 4: Support Vector Machines (20 points)

Given 10 points in Table 1, along with their classes and their Lagrangian multipliers (α'), answer the following questions:

(1) What is the equation of the SVM hyperplane $h(x)$? Draw the hyperplane with the 10 points.

(2) What is the distance of $x($ from the hyperplane? Is it within the margin of the classifier?

(3) Classify the point $z = (3,3))$ using $h(x)$ from above.

Answer:

Problem 4: Support Vector Machine

(1) The values having zero in α_i are not support vectors

→ Only non-zero values are considered.

Data	x_{i1}	x_{i2}	y	α_i
x_1	4	2.9	1	0.414
x_4	2.5	1	-1	0.018
x_7	3.5	4	1	0.018
x_9	2	2.1	-1	0.414

WKT,

$$w = \sum_{i=1}^{N} \alpha_i y_i x_i$$

$$w = \left(0.444 \cdot \begin{bmatrix} 4 \\ 2.9 \end{bmatrix} \cdot 1 \right) + \left(0.018 \cdot \begin{bmatrix} 2.5 \\ 1 \end{bmatrix} \cdot -1 \right) +$$

$$\left(0.018 \cdot \begin{bmatrix} 3.5 \\ 4 \end{bmatrix} \cdot 1 \right) + \left(0.444 \cdot \begin{bmatrix} 2 \\ 2.1 \end{bmatrix} \cdot -1 \right)$$

$$= \begin{bmatrix} 1.656 \\ 1.2006 \end{bmatrix} - \begin{bmatrix} 0.045 \\ 0.018 \end{bmatrix} + \begin{bmatrix} 0.063 \\ 0.072 \end{bmatrix} - \begin{bmatrix} 0.828 \\ 0.8694 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.846 \\ 0.3852 \end{bmatrix}$$

The table is,

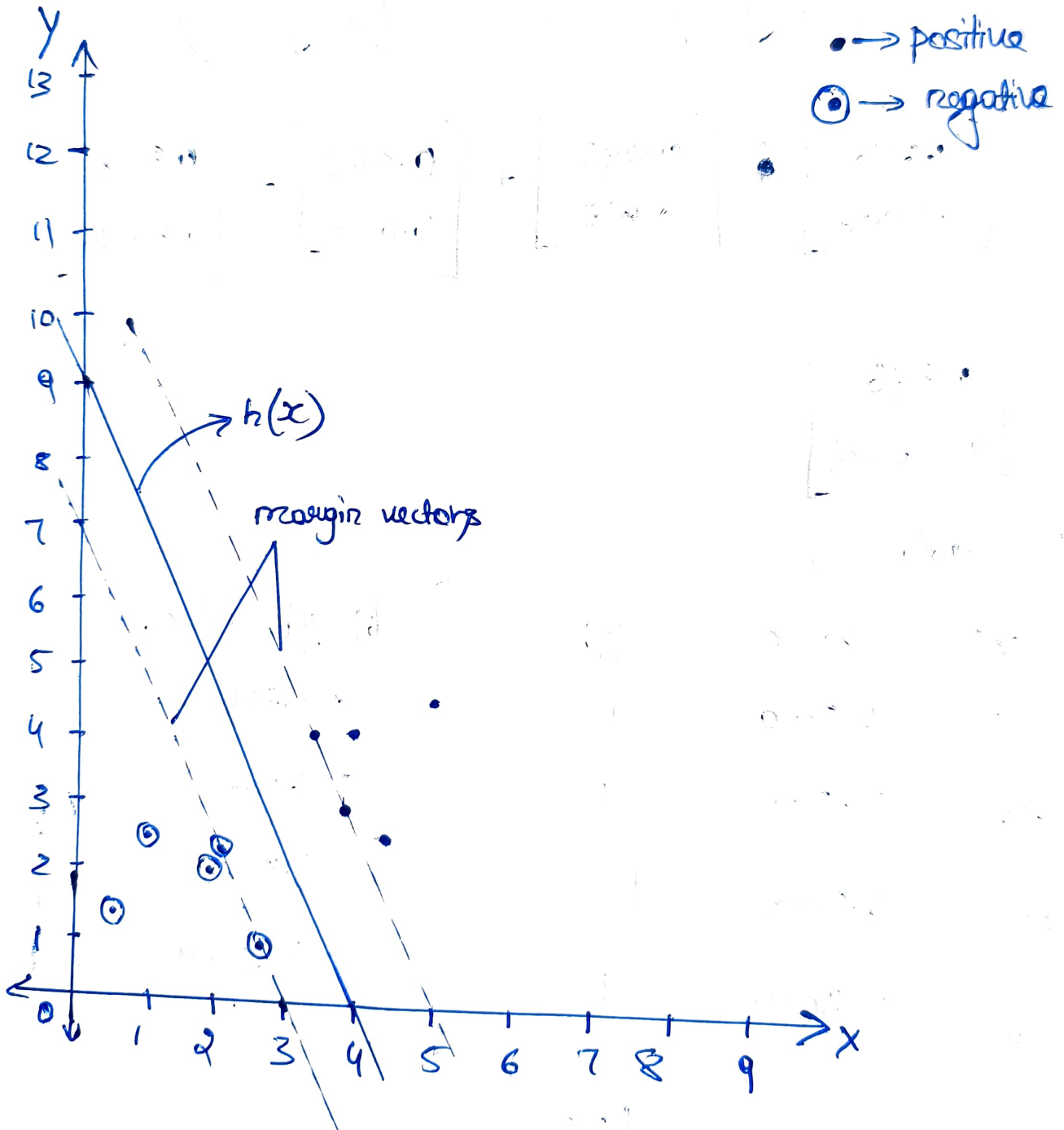| x_i	$w^T x$	y_i	$b_i = y_i - w^T x$
x_1	4.5010	1	-3.501
x_4	2.5002	-1	-3.5002
x_7	4.5018	1	-3.5018
x_9	2.5009	-1	-3.5009

$$\therefore b = \text{average} \{b_i\}$$

$$= \underline{-3.5009}$$

The optimal equation of SVM hyperplane, is $\omega^T x + b = 0$

$h(x) = \begin{bmatrix} 0.846 & 0.3852 \end{bmatrix} x + (-3.5009) = 0$

$\therefore h(x) = 0.846\, x_1 + 0.3852\, x_2 - 3.5009 = 0$

(2) Distance of x_6 from hyperplane,

$$= \frac{w^T x + b}{\|w\|}$$

Here, the selected datapoint is x_6

$$\therefore x_6 = (1.9, 1.9)$$

Now, substituting all values,

$$\leq \left| \frac{1.9 \times 0.846 + 1.9 \times 0.3852 - 3.5009}{\sqrt{(0.846)^2 + (0.3852)^2}} \right|$$

$$\leq \frac{1.1617}{\sqrt{0.864}} \leq 1.2498$$

Margin of the classifier, $= \frac{1}{\|w\|} \quad \leq \quad \frac{1}{\sqrt{0.864}} \leq 1.0758$

Since, $1.2498 > 1.0758$,

$\therefore x_6$ is farther from hyperplane and it lies outside of the margin of the classifier.

(3) $z = (3,3)$

let's compute the value of $h(x)$ for the given input point

$$h(x) = \begin{pmatrix} 0.846 \\ 0.385 \end{pmatrix}^T \begin{pmatrix} 3 \\ 3 \end{pmatrix} - 3.5009$$

$$= 2.538 + 1.155 - 3.5009$$

$$\boxed{h(x) = 0.1921}$$

$\therefore$ The value is positive, thus the datapoint $z = (3,3)$ belongs to the positive class label.