Problem-3 : Adaboost

(1) and (2)

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	1	1	1	-1	-1	-1	-1	-1	1	1

→ Now, H1 : if $X \leq 0.35 \rightarrow Y = 1$, else $Y = -1$

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.10
Y	1	1	1	-1	-1	-1	-1	-1	1	-1
$S_{(0)}$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
H1	1	1	1	-1	-1	-1	-1	-1	-1	-1

$$err_1 = 2 \times 0.1 = 0.2$$

$$\alpha_1 = \frac{1}{2} \ln\left(\frac{1-err_1}{err_1}\right) = \frac{1}{2} \times \log\left(\frac{1-0.2}{0.2}\right) = 0.3010$$

- For those classified correctly

$$D_{(t)}(i) = \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$= 0.1 \times e^{(-1) \times 0.3} = 0.07408$$

- For those classified incorrectly,

$$D_{(t)}(i) = (0.1) \times e^{0.3 \times 1} = 0.1349$$

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	1	1	1	-1	-1	-1	-1	-1	1	1
S_{t0}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
H1	1	1	1	-1	-1	-1	-1	-1	-1	-1
S_{t+1}	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.13	0.13

After 1st iteration, ID = 9, 10 will be re-weighted because they are classified wrong according to the hypothesis H1.

$\Rightarrow$ H2 : if $X \leq 0.75 \Rightarrow Y = -1$, else $Y = 1$

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	1	1	1	-1	-1	-1	-1	-1	1	1
S_{t0}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
H2	-1	-1	-1	-1	-1	-1	-1	1	1	1

$$err_2 = 0.1 \times 4 = 0.4$$

$$\alpha_2 = \frac{1}{2} \times \log\left(\frac{1 - err_2}{err_2}\right) = \frac{1}{2} \times \log\left(\frac{1-0.4}{0.4}\right) = 0.0880$$

For those classified correctly,

$$D_{(t)}(i) = (0.1) \times e^{0.088 \times (-1)} = 0.09157$$

For those classified incorrectly,

$$D_{(t)}(i) = (0.1) \times e^{0.088 \times (1)} = 0.1091$$

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	1	1	1	-1	-1	-1	-1	-1	1	1
H_2	-1	-1	-1	-1	-1	-1	-1	1	1	1
S_{round}	0.109	0.109	0.109	0.091	0.091	0.091	0.091	0.109	0.091	0.091

⇒ After 1st iteration ID = 1, 2, 3, 8 will be re-weighted because they are classified wrong according to the hypothesis H_2.

⇒ H_3: if $X \leq 0.3$ or $X \geq 0.95 \rightarrow Y = 1$, else $Y = -1$

ID	1	2	3	4	5	6	7	8	9	10
X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	1	1	1	-1	-1	-1	-1	-1	1	1
H_3	1	1	1	-1	-1	-1	-1	-1	-1	1
S_{ci}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

$$err_3 = 0.1 \times 1 = 0.1$$

$$\alpha_3 = \frac{1}{2} \times \log\left(\frac{1 - err_3}{err_3}\right) = \frac{1}{2} \log\left(\frac{1 - 0.1}{0.1}\right) = 0.4771$$

For those correctly classified,

$$D_{(t)}^{(i)} = (0.1) \times e^{0.4771 \times (-1)} = 0.0620$$

For those incorrectly classified,

$$D_{(t)}^{(i)} = (0.1) \times e^{0.4771} = 0.1611$$

ID	1	2	3	4	5	6	7	8	9	10
x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	1	1	-1	-1	-1	-1	-1	1	1
H_3	1	1	1	-1	-1	-1	-1	-1	-1	1
ζ_{new}	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.1611	0.062

⇒ After 1st iteration, ID = 9 will be reweighted because they are classified wrong according to the hypothesis H_3.