Non-Entanglement based resources - Discord and Coherence

Team: 8

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Quantum Discord

Measure of the 'Quantumness' of correlations

Correlations between parts of a given system can be classic or quantum, and both of them may coexist for a given system.
Entanglement was proposed as a manifestation of quantum correlations.Entanglement was thought to be the sole resource for quantum information processing.
Separable (i.e. not entangled) states were considered insufficient to implement quantum information processing.
That belief has changed since Ollivier and Zurek as well as Henderson and Vedral independently introduced a new measure of non-classical correlations named 'discord'.

Separable and Entangled states

A bipartite state ρ^{ab} shared by two parties a and b is separable if it can be represented as $\rho^{ab} = \sum_i p_i \rho_i^a \otimes \rho_i^b$ where, ρ and ρ^b are families of density matrices for parties a and b, respectively.
Otherwise, ρ^{ab} is called entangled.
However, entanglement is *not sufficient* to encapsulate all quantum correlations as there is significant amount of work been done even in absence or near absence of entanglement.
Separable states do not have a purely classical nature.

Mutual Information

Mutual information is a way to measure correlation. It quantifies the amount of information obtained about one random process by observing the other random process. Mutual information is defined using relative entropy. In case of classical information, we use Shannon entropy.

$$H = -\sum_i p(i) log_2(p(i))$$

The mutual Information is given by

$$\begin{aligned} I(A:B) &= H(A) + H(B) - H(A,B) \\ &= H(A) - H(A|B) \end{aligned}$$

However, in a quantum system,

$$H(A) + H(B) - H(A,B) \neq H(A) - H(A|B)$$

Quantum Mutual Information

In quantum information theory, the Von Neumann entropy gives the information content of a density matrix.

$$\begin{aligned} S(\rho) &= -Tr(\rho log_2(\rho)) \\ &= -\sum_i \lambda_i log_2(\lambda_i) \end{aligned}$$

Here,

$$I(\rho_A : \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_A, \rho_B)$$

$$J(\rho_A : \rho_B) = S(\rho_B) - S(\rho_B|\rho_A)$$

Note that $J(\rho_A : \rho_B) \neq J(\rho_B : \rho_A)$

Discord

Quantum discord is a measure of nonclassical correlations between two subsystems of a quantum system. It includes correlations that are due to quantum physical effects but do not necessarily involve quantum entanglement.
In classical information theory :

$$I(A : B) = J(A : B)$$

In quantum information theory :

$$I(A : B) \neq J(A : B)$$

The difference between the two expressions defines the basis-dependent quantum discord.

$$D(B|A) = I(A : B) - J(A : B)$$

Discord

The notation J represents the part of the correlations that can be attributed to classical correlations and varies in dependence on the chosen eigenbasis.
Thus, quantum discord that reflect the purely non-classical correlations independently of basis is given by

$$D(B|A) = I(A:B) - max_{\Pi_i^a} J(A:B)$$

Where Π_i^a are measurement operators corresponding to a von Neumann measurement on the subsystem A, i.e., orthogonal projectors with rank one.

Some Key Points

- Discord is zero if and only if local measurements can not disturb the quantum system.
- Discord is equivalent to entanglement for pure states. The value of Discord in pure state coincides with the von Neumann entropy of entanglement.
- Discord is not symmetric.
  i.e.
  $$D(B|A) \neq D(A|B)$$

Example of Discord

Consider the following state

$$\rho_{AB} = \frac{1}{2}(|00\rangle\langle 00| + |1+\rangle\langle 1+|)$$

It is easy to see that the state is not entangled.
Note that performing a measurement on A in the 0,1 basis reveals the state of B as well.
i.e.

$$D(B|A) = 0$$

However,
Measuring B does not reveal all information about A. Hence,

$$D(A|B) > 0$$

Types of Discord

- **Change in Mutual Information:**
- **Geometric discord:**
  $D_{geo}(\rho) = min_{\chi}||\rho - \chi||^2$
  Can be seen as the minimal geometric distance between the given state and all states with zero discord.
- **Thermal Discord:** Change in Entropy

Each of these definitions of discord is useful in different contexts. However they capture the same quantity- The 'quantumness' of a system.

Discord as a Resource

While Entanglement is still the primary resource used in quantum information procession, Quantum discord has been seen as a possible basis for the performance in terms of quantum computation ascribed to certain mixed-state quantum systems. Quantum discord is more resilient to dissipative environments than is quantum entanglement. In situations where noise may be so strong to prevent the distribution and distillation of entanglement, discord can be used as a resource. One such example is Quantum cryptography.

Quantum Discord: Use Case

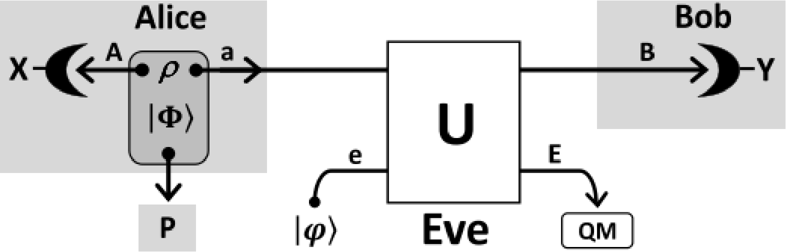
Quantum Cryptography

Quantum discord as a resource for quantum cryptography

Quantum discord is an important resource necessary for quantum key distribution (QKD). Quantum discord and its geometric formulation relate to the concept of non-orthogonality, which is essential for quantum cryptography. Quantum discord plays a more significant role in device dependent quantum cryptography, where the presence of detection noise is very strong such that it prevents the distribution and distillation of entanglement, but still a secure key can be extracted due to presence of non-zero discord. In order to implement QKD, quantum discord must be non-zero.

Device-dependent QKD protocols

Any QKD protocol can be recast into a measurement-based scheme, where Alice sends Bob part of a bipartite state, then subject to local detections. Here a device-dependent QKD protocol is taken for representing the use of quantum discord.



Device-dependent QKD protocols (continued . . .)

Alice prepares two systems a and A in her private space resulting in a mixed state ρ_{Aa} Now, this state is purified into to a 3-partite state with the ancillary system P being inaccessible to Alice, Bob or Eve.

Device-dependent QKD protocols (continued . . .)

This system accounts for the trusted noise in Alice's side. Then, system b is sent to Bob, who gets the output B after the channel. Bob's output B is assumed to be affected by other local trusted noise in Bob's private space (denoted as P as before).

Device-dependent QKD protocols (continued . . .)

Finally, from the shared state ρ_{AB}, Alice and Bob extract two correlated variables by applying suitable measurements. On the output data, they perform error correction and privacy amplification with the help of one-way classical communication (CC), which can be either forward (direct reconciliation), or backward (reverse reconciliation).

Proof for necessity of non-zero discord

Quantum-classical state prepared by Alice is,

$$\rho_{Aa} = \sum_k p_k \rho_A(k) \otimes |k\rangle \langle k| \; where, |k\rangle \; are \; all \; orthogonal$$

The classical part of this state is perfectly clone-able by Eve due to this orthogonality.Now the joint state of Alice, Bob and Eve can be represented as,

$$\rho_{ABE} = \sum_k p_k \rho_A(k) \otimes |k\rangle_B \langle k| \otimes |k\rangle_E \langle k|$$

Proof for necessity of non-zero discord (continued . . .)

As the above state is symmetric to perturbations of B and E, Eve can decode the information in variable A of Alice with the same accuracy as Bob. We can also see that secure communication in the presence of eavesdropper also fails in the reverse direction.

Proof for necessity of non-zero discord (continued ...)

After measurement by Bob, the joint state of the system is,

$$\rho_{AE|y} = \sum_k p_{k|y} \rho_A(k) \otimes |k\rangle_E \langle k| \, here, p_{k|y} = \langle k| M_y |k\rangle$$

The Eve makes measurement using POVM to get K $= \{k, p_{k|y}\}$ and Alice recovers variable X with the distribution $Tr[Mx\rho_A(k)]$. Since $Y \rightarrow K \rightarrow X$ form a Markov chain, the data processing inequality states that $I(Y, K) \geq I(Y, X)$. Therefore, Eve recovers more information than Alice.
Thus, non-zero discord, that is $D(A : B) > 0$, is essential to device dependent cryptography in the above scenario.

Conclusion

Essentially, the same features of traditional cryptography, the information encoded in non-orthogonal states, is manifested as non-zero discord in a bipartite setting and this enables secure communication.
It is important to notice that entanglement can be completely absent and yet cryptography is successful due to quantum discord.
It was shown in that any prepare, and measure protocol based on non-orthogonal quantum states can be recast into an entanglement free device dependent with nonzero discord as a necessary condition for it to be successful.

Quantum Coherence

Intro to resource theory

In a practical sense, formal resource theory seeks to characterise what tasks, or transformations of a physical system, an experimenter may do when specific constraints on the accessible states and dynamics exist.

These constraints may be due to various reasons such as

- fundamental conservation laws such as say energy conservation
- constraints due to the practical difficulty of executing certain operations for example, the restrictions imposed on the "Local Operations and Classical Communication (LOCC)" which form the basis to the resource theory of entanglement.

What are states and resources in resource theory?

A quantum resource theory is defined once we specify the following:

- The set of **free states** - those which are considered freely available at no cost
- The set of **free operations** - any quantum operations which respect some given experimental restriction

What are states and resources in resource theory?

States that lie outside the set of free states are known as **resources**.

It is to be noted that we assume the set of free states to be closed under all free operations. Because, otherwise, resource states would be created for free.

Intuitive sense of state transformations

A basic question to ask in the context of resource theory is whether one state can be transformed into another by using free operations.

Usually, we take the set of free operations to be **LOCC** (Local operations and classical communication). That is, those that can be performed by two distant individuals who have full quantum control over their state but can only send each other classical information.

What is Quantum Coherence?

- Quantum coherence and quantum entanglement are both rooted in the superposition principle – the phenomenon in which a single quantum state simultaneously consists of multiple quantum states
- The concept of quantum coherence is that all objects exhibit wave-like qualities.

What is Quantum Coherence?

- If the wave-like character of an object is split in two, the two waves may coherently interfere with one other, resulting in a single state that is a superposition of the two states.
- This superposition ceases to exists when this state experiences de-coherence
- Any quantum state on measurement gives decohered result.

Quantum coherence as a resource

Incoherent operators

For Coherence, we fix a reference basis first in which we formulate the resource theory. We can fix the basis as per our needs for example as the energy eigenbasis.

Given a d-dimensional Hilbert space $\mathcal{H}$ (with d assumed finite) we denote its reference orthonormal basis by $\{|i\rangle\}_{i=0,\ldots,d-1}$.
The density matrices that are diagonal in this specific basis are called **incoherent**.

These states are accessible free of charge, and form the set $\mathcal{I} \subset \mathcal{B}(\mathcal{H})$ where $\mathcal{B}(\mathcal{H})$ denotes the set of all bounded trace class operators on $\mathcal{H}$ (trace of the compact operator is defined, finite and independent of the basis).

Hence, all incoherent density operators $\rho \in \rangle$ are of the form

$$\rho = \sum_{i=0}^{d-1} p_i \left|i\right\rangle \left\langle i\right| \tag{1}$$

with probabilities p_i.

Observe that any density matrix would have elements on only the main diagonal. So, any density matrix that has elements on the off-diagonal elements lie outside the set $\mathcal{I}$ of incoherent states and hence has a resource content.

Maximally incoherent operations (MIO)

- This is the largest class of incoherence preserving operations.
- It is defined as any trace preserving, completely positive and non-selective quantum operations $\Lambda : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ such that,

$$\Lambda[\mathcal{I}] \subseteq \mathcal{I} \tag{2}$$

- If any MIO, Λ can be implemented by using an incoherent state σ and a global incoherent unitary U (a unitary that is diagonal in our required basis), according to the below equation,

$$\Lambda[\rho] = Tr_E[U(\rho \otimes \sigma)U^\dagger] \tag{3}$$

(provision of an ancillary environment state σ, unitary operation U followed by tracing out of the environment) then we say that the operation has *free dilation*

Incoherent operations (IO)

- A smaller and more relevant class of free operations for the theory of coherence is that of incoherent operations (IO)
- They are characterized as the set of trace preserving completely positive maps $\Lambda : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H})$ **admitting a set of Kraus operators** $[K_n]$ such that $\sum_n K_n^\dagger K_n = 1$ (trace preservation) and, for all n and $\rho \in \mathcal{I}$,

$$\frac{K_n \rho K_n^\dagger}{Tr[K_n \rho K_n^\dagger]} \in \mathcal{I}$$

Incoherent operations (IO)

- The following definition of IO ensures that coherence can never be formed from an incoherent input state in any of the potential outcomes of such an operation. (Because *rho* is an incoherent state and the Kraus operators preserve the trace, the output would be an incoherent state as well.)
- It is to be noted that this class of operations does not admit a free dilation in general

Other incoherent operations

The emphasis in the previous two definitions was on ensuring that incoherent processes did not yield coherence. We can, however, add further constraints.

This gives us various other classes of incoherent operations such as, the *strictly incoherent operations (SIO)*, *Energy preserving operations (EPO)*, *Genuinely incoherent operations (GIO)* and so on.

Ref: Quantum coherence as a resource - Alexander Streltsov, Gerardo Adesso, Martin B. Plenio

The canonical example for a maximally coherent state (for a d-dimensional space) as,

$$|\Psi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle \tag{4}$$

- It allows for the generation of all other d-dimensional quantum states via free operations.
- It is to be noted that it being the maximally coherent state is independent of any particular coherence quantifiers
- Since coherence is basis dependent, not all frameworks involving the set of incoherent operations allow for the existence of a maximally coherent state.

It has been proven that the state $\rho_{max} = \sum_{n=1}^{d} p_n \left| n_+ \right\rangle \left\langle n_+ \right|$, where $\{|n+\rangle\}$ denotes a mutually unbiased basis with respect to the incoherent basis $\{|i\rangle\}$ (that is, $|\left\langle i | n_+ \right\rangle|^2 = \frac{1}{d}$), is a **universally maximal coherent mixed state** with respect to any *coherence monotone* (defined further ahead) under the set MIO.

- A $|\phi\rangle$ pure state can be converted into another pure state $|\psi\rangle$ via IO and SIO if and only if majorization condition holds.
- The majorization relation for density matrices $\rho > \sigma$ means their spectra $spec(\rho) = (p_1 \geq p_2 \geq \cdots \geq p_d)$ and $spec(\sigma) = (q_1 \geq q_2 \geq \cdots \geq q_d)$ fullfill the relation

$$\sum_{i=1}^{t} p_i > \sum_{j=1}^{t} q_j \ \forall \ t < d \tag{5}$$

Postulates for coherence monotones and measures

Nonnegativity :

$$C(\rho) \geq 0 \tag{6}$$

- Coherence of any coherent state is strictly greater than 0
- Coherence measure of any incoherent state is $= 0$

Monotonicity :

- Coherence will not increase on applying any Incoherent Operators

$$C(\Lambda[\rho]) \leq C(\rho) \tag{7}$$

for any incoherent Operation Λ

Postulates for coherence monotones and measures

Strong Monotonicity :

Under selective incoherent operations, Cohorence does not rise on average.

$$\sum_i q_i C(\sigma_i) \leq C(\rho) \tag{8}$$

where probabilities $q_i = Tr[K_i \rho K_i^\dagger]$

Post-measurement states $\sigma_i = K_i \rho K_i^\dagger / q_i$

Incoherent Kraus operators K_i

Convexity: :

C is a convex function of the state

$$\sum p_i C(\rho_i) \geq C\left(\sum_i p_i \rho_i\right) \tag{9}$$

Postulates for coherence monotones and measures

- For any function C to be seen as meaningful resource quantifier for any coherence based task, it should atleast satisfy Nonnegativity and Monotonicity.
- Strong monotonicity quantifies the intuition that coherence should not rise under incoherent measurements.
- A quantity C which satisfies condition *Nonnegativity* and either condition *Monotonicity or Strong monotonicity(or both)*is called a **coherence monotone.**
- Finally Conherence measures is a quantity C which satisfies all four consitions.

Analogies with quantum entanglement as a resource

- The maximally entangled state and the maximally coherent state are analogous and share similar properties.
- The majorization condition of coherence has an analogous part in quantum entanglement theory
- The LOCC framework from quantum entanglement theory has an analogous LICC (Local incoherent operations and classical communication) framework

Conclusion

In this presentation, we have learnt about quantum coherence and its importance. Specifically, if the experimenter is limited to quantum operations that cannot cause coherence (free operations or incoherent operations), then coherence might be seen as a resource, similar to entanglement but even more fundamental.

Quantum Coherence: Use Case

Abstract

Coherent phenomena in molecular chromophores interacting with a dissipative environment is addressed. We defined coherence by the phenomena of decoherence which collapses the system to pointer states. Coherent irreducible phenomena takes place in a time window before the system collapses. We describe a computational model: The Stochastic Surrogate Hamiltonian that can deal with such complex quantum systems.

Abstract

The conditions for coherent control are analyzed. A prerequisite for coherent phenomena is the ability to perform coherent control using shaped light sources. We show that weak field coherent control is enabled by interaction with the environment.

Introduction

Can quantum coherent phenomena have a significant role in the dynamics of a large system at room temperature? Only a positive answer can support the claims which are at the base of quantum biology. Coherence is a manifestation of quantum phenomena which has no classical analogue.

Introduction

This statement is elusive and much effort has been invested for its clarification. The naive idea that quantum phenomena can only be described by employing a superposition state is basis dependent. One can always find a basis which diagonalizes the state. The consequence is that coherence can be defined only relative to a privileged basis set.

Decoherence Process

Decoherence is relevant if its timescale matches the timescale of the free unitary molecular dynamics. Very fast decoherence leads to a classical-like dynamical evolution. Slow decoherence can be approximated by pure quantum unitary dynamics. Decoherence is defined as a dynamical process which generates loss of purity: $P(t) < P(0)$, where purity is defined as $P = tr\{\hat{\rho}^2\}$.

Decoherence Process

Unitary evolution preserves the eigenvalues of $\hat{p}$and will therefore preserve the trace of any function of $\hat{p}$, which includes $f(\hat{p}) = \hat{p}^2$ and $f(\hat{p}) = -\hat{p}\ln\hat{p}$. As a result, the purity as well as the von Neumann entropy $S_{VN} = -tr\{\hat{p}\ln\hat{p}\}$ are constant under unitary evolution. A change in purity requires a non unitary evolution of a system coupled to the environment. The most studied model is of a system bath combination initially uncorrelated $\hat{p}_{S\ B}(0) = \hat{p}_S \otimes \hat{p}_B$

Surrogate Hamilton

We consider a molecular system coupled to a radiation field. The molecular system is subject to dissipative forces due to coupling to a primary bath. In turn the primary bath is subject to interactions with a secondary bath: $\hat{H}_T = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}$

Surrogate Hamilton

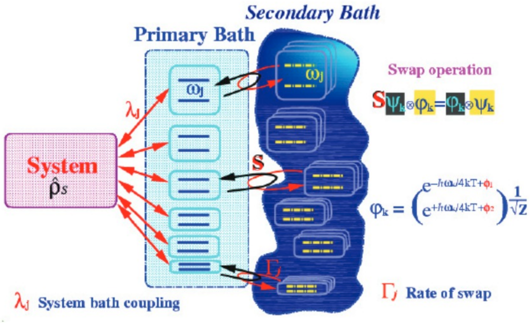


Figure: Flowchart of energy currents between the primary system, the primary bath and and the secondary bath. The system and the primary bath are coupled via the Hamiltonian interaction represented by the interaction λ_j. the primary bath and the secondary bath interact via the swap operation $\hat{S}$