Assignment 3 - 2014

1. Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for all $a < x < b$. Prove that $f(b) < g(b)$.

2. For the function $f(x) = 2x^3 - 15x^2 + 36x - 20$.
(a) Find where $f(x)$ is increasing and decreasing.
(b) Find the relative minima and relative maxima of $f(x)$.
(c) Find where $f(x)$ is convex and concave.
(d) Find the inflection points of $f(x)$.

3. Suppose that f is defined everywhere on R and that all its derivatives are continuous. Let $f'''(x) < 0$ for all $x \in R$. Show that f has at most two critical points.

4. Let $f(x) = \begin{cases} x \ \ sin \ \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
Check if f is differentiable at $x = 0$.

5. Show that the function $f(x) = \begin{cases} x^2 \ \ sin \ \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at all $x \in R$ Also show that the function $f'(x)$ is not continuous at $x = 0$ (Thus, a function that is differentiable at every point of R need not have a continuous derivative $f'(x)$).

6. A continues function f with the domain $[-7, 4]$ satisfies $f(-7) = 3$ and $f'(x) = 0$ for all $x \in [-7, 4]$.Show that f is a constant function.

7. Find the maximum values of $f(x) = x^a(1 - x)^b, 0 \leq x \leq 1$ where a and b are positive numbers,