

Assignment- 2

Suprio Dubey

2013036

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Answer1

If two ordinary dice with faces 1...6 are thrown the probability of getting any face for the first dice is $\frac{1}{6}$ and similarly the probability of getting any face for the other dice is $\frac{1}{6}$. There are 36 possible outcomes of the additive dice roll. So, the probability of picking up a sum of two dice faces is $\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$. The minimum possible sum is 2, and the maximum is 12. So, let us see the probability of all the combinations.

Sum	Combinations	Probability
2	[1,1]	$\frac{1}{36}$
3	[1,2],[2,1]	$\frac{2}{36}$
4	[1,3],[3,1],[2,2]	$\frac{3}{36}$
5	[1,4],[4,1],[2,3],[3,2]	$\frac{4}{36}$
6	[1,5],[5,1],[2,4],[4,2],[3,3]	$\frac{5}{36}$
7	[1,6],[6,1],[2,5],[5,2],[3,4],[4,3]	$\frac{6}{36}$
8	[2,6],[6,2],[3,5],[5,3],[4,4]	$\frac{5}{36}$
9	[3,6],[6,3],[4,5],[5,4]	$\frac{4}{36}$
10	[4,6],[6,4],[5,5]	$\frac{3}{36}$
11	[5,6],[6,5]	$\frac{2}{36}$
12	[6,6]	$\frac{1}{36}$

Table 1: Probability Distribution of Sum of two dices

2 dice with 10000 throws has a mean 6.9711 and variance 5.7250647900000001.

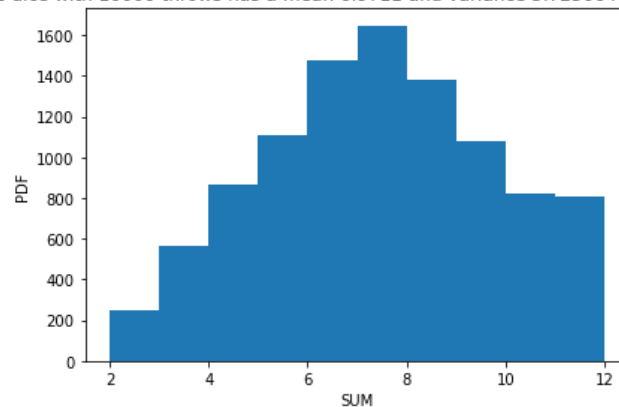


Figure 1:

Difference	Combinations	Probabilty
0	[1,1],[2,2],[3,3],[4,4],[5,5],[6,6]	$\frac{6}{36}$
1	[1,2],[2,1],[2,3],[3,2],[3,4],[4,3],[5,4],[4,5],[5,6],[6,5]	$\frac{10}{36}$
2	[1,3],[3,1],[2,4],[4,2],[3,5],[5,3],[6,4],[4,6]	$\frac{8}{36}$
3	[1,4],[4,1],[2,5],[5,2],[6,3],[3,6]	$\frac{6}{36}$
4	[1,5],[5,1],[2,6],[6,2]	$\frac{4}{36}$
5	[1,6],[6,1]	$\frac{2}{36}$

Table 2: Probability Distribution of absolute diff of two dices

The distribution of absolute difference

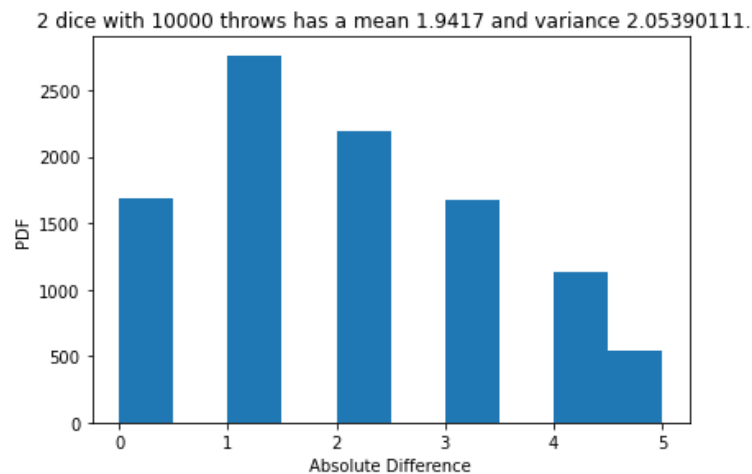


Figure 2:

ANSWER 2

The value of one die has a mean of $(1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$ and a variance of $35/12 = 2.9166$. For 100 throws the mean will be $100 * 3.5 = 350$ and the variance will be $100 * 2.9166 \approx 292$. In accordance to the Central-Limit theorem the probability distribution is approximately Gaussian with mean 350 and variance approximately 292.

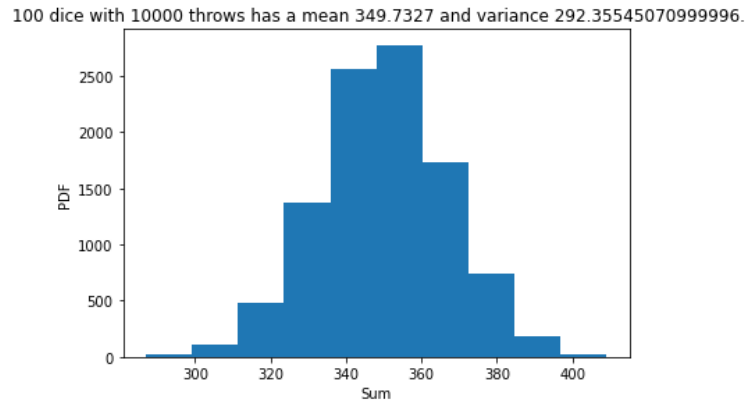


Figure 3:

Answer 3

In this question we need to choose two cubical dice with the numbers 0,1,2,3,4,5,6 such that on throwing the dices we have a uniform probability distribution i.e. the probability of the sum of dices are equal. We need a uniform distribution from 1-12 which limits the condition of the sum of dices to be 0. So we can't have [0,0]. So, one of the dices has 0 and the other won't have zero but 1 for sure. The second thing we notice is the max sum is 12 so we have only one condition [6,6]. This means both dices have 6. So, the unique solution will be a dice 1,2,3,4,5,6 and the other will be 6,6,6,0,0,0 the distribution we will have is:

Sum	Combinations	Probability
1	[0,1]	$\frac{3}{36}$
2	[0,2]	$\frac{3}{36}$
3	[0,3]	$\frac{3}{36}$
4	[0,4]	$\frac{3}{36}$
5	[0,5]	$\frac{3}{36}$
6	[0,6]	$\frac{3}{36}$
7	[6,1]	$\frac{3}{36}$
8	[6,2]	$\frac{3}{36}$
9	[6,3]	$\frac{3}{36}$
10	[6,4]	$\frac{3}{36}$
11	[6,5]	$\frac{3}{36}$
12	[6,6]	$\frac{3}{36}$

Table 3: Probability Distribution of absolute diff of two dices

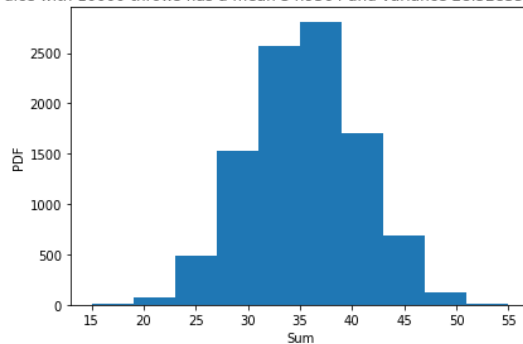
Answer 4

It is not possible to label hundred dices by just six numbers to give unique combinations. So, we can consider other integers. One such possibility would be to use labeling that is completely different from one another and also their combinations. The possible solution can be, as mentioned in book 3 is, $label = (0, 1, 2, 3, 4, 5) * 6^r$

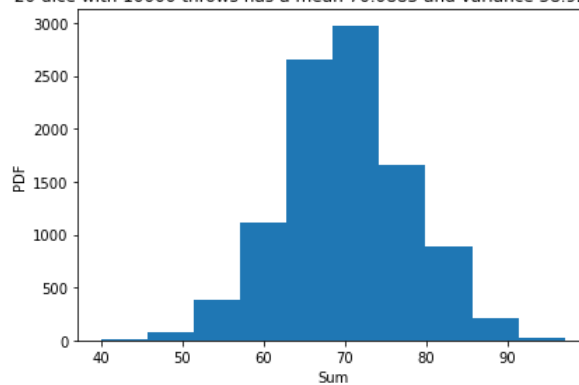
Answer 5

The distribution looks like a Gaussian when the Number of dice is = 2, 3, 4, 10, 20. Here, I have shown for Number of dice = 10 and 20

10 dice with 10000 throws has a mean 34.9364 and variance 28.528355039999994.



20 dice with 10000 throws has a mean 70.0883 and variance 58.92190311.

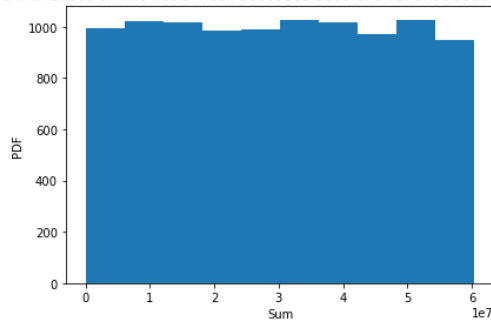


Answer 6

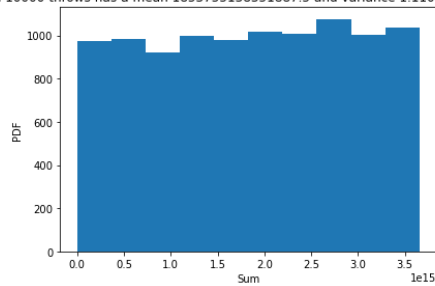
The distribution does not look like a Gaussian when the Number of dice is = 2, 3, 4, 10, 20. Here, I have shown for Number of dice = 10 and 20.

.The reason might be that the variance is very large and increases exponentially on increasing the number of dices so it doesn't converge.

10 dice with 10000 throws has a mean 30069333.3999 and variance 302262425571547.9.



20 dice with 10000 throws has a mean 1853755138351887.5 and variance 1.1103392129189871e+30.



Answer 4 Python Code for the simulation of the problem

```
1 #!/usr/bin/env python3
2 # -*- coding: utf-8 -*-
3 """
4 Created on Thu Oct 27 15:48:03 2022
5
6 @author: draco
7 """
8
9 import numpy as np
10 import matplotlib.pyplot as plt
11 import random
12 from scipy.stats import norm
13
14
15
16 print("call dicethrow with !st variable input as the number of
17       throws, second as no. of dice and the mode.The default mode is
18       sum. In case of abs diff the number mode != sum and to be noted
19       that it only works when the number of dice is two \n" )
20
21 def dicethrow(no_throws,no_dice,mode ='add'):
22
23     dice_sum_outcome = []# the elements in number of sum should be
24     the sum of the dices.
25
26     for i in range (1,no_throws+1):
27
28         dice_face = []
29
30         for j in range(1,no_dice+1):
31
32             dice = random.randint(1,6)
33
34             dice_face.append(dice)
35
36         if mode == 'add':
37             out_add = sum(dice_face)
38
39         else:
40             ##### only in case of two dice #####
41
42             out_add = abs(dice_face[0]-dice_face[1])
43             dice_sum_outcome.append(out_add)
44
45     return dice_sum_outcome
```

```
42
43
44 def diceinteg(no_throws, no_dice):
45     Dice=np.zeros((6 , no_dice))
46     for r in range (no_dice) :
47         Dice[:,r]=np.array ( [ 0 , 1 , 2 , 3 , 4 , 5 ] )*6**r
48
49     output_sum = np.zeros(no_throws)
50     for i in range (no_throws):
51
52         for j in range(no_dice):
53             dice_output = random.choice(Dice[:,j])
54             output_sum[i] += dice_output
55
56     dice_sum_outcome = output_sum
57     return (dice_sum_outcome)
58
59 no_dice=2
60 no_throws=10000
61 dice_sum_output = dicethrow(no_throws,no_dice)
62 #dice_sum_output = diceinteg(no_throws,no_dice)
63
64
65 mu = np.mean(dice_sum_output)
66 sigma = np.var(dice_sum_output)
67 print(mu,sigma)
68
69 ##### Plot #####
70
71 plt.hist(dice_sum_output)
72 plt.ylabel("PDF")
73 plt . xlabel("SUM")
74 plt.tight_layout()
75 plt.title(" %s dice with %s throws has a mean %s and variance %s."
76           %(no_dice,no_throws,mu,sigma))
76 plt.show()
```