Bauhaus-Universität Weimar

Modeling Complex Systems

Research Project

Chair of Applied Mathematics / Faculty of Civil Engineering / Bauhaus-University Weimar

Submitted By

Supritha Gubbi Channanjappa

Matrikelnummer: 125805

First Professor	:	Prof. Dr. Michael Schönlein
Second Professor	:	Prof. Dr. Björn Rüffer

March 2025

DECLARATION OF AUTHORSHIP

I hereby certify that the work embodied in the Project is my own work, conducted under normal supervision. I confirm that the project contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference has been made. I give consent to the final version of my project being made available worldwide when deposited in the University's digital repository, subject to applicable copyright law and any approved embargo.

Weimar, March 2025

Supritha Gubbi Channanjappa

CONTENTS

LIST OF FIGURES

1 INTRODUCTION

Our project explains the interplay between the canadian lynx and the rabbits showing a periodic behavior. We want to describe the evolution of two interacting populations for the time period of 100 years with length of such a cycle is 9-10 years.

Rabbits (Lepus americanus) are the primary food for the Canadian lynx (Lynx canadensis). When rabbits are abundant, Lynx will eat rabbits about two every three days almost to the complete exclusion of other foods. As a consequence, the population dynamics of the two species are closely linked[2]

This study explores the dynamics of predator-prey relationships using a mathematical model combined with an agent-based simulation. The classical Lotka-Volterra model provides a set of differential equations that describe the dynamics of predator and prey populations in a homogeneous environment. In this project, we will extend this classical framework by incorporating spatial heterogeneity through randomness in the predation rate and modeling interactions in a discrete environment. A unique creek area is introduced as a non-hunting area of prey to the predation process to better capture real-world variability. This hybrid approach allows us to explore how spatial constraints and randomness impact population fluctuations and spatial distributions within the predator-prey system. The findings contribute to ecological modeling and wildlife management by integrating randomness in predation and spatial constraints in an agent-based framework. we have used literature "Introduction to the modeling and analysis of complex systems by Sayama, Hiroki in 2015"[3] in our project.

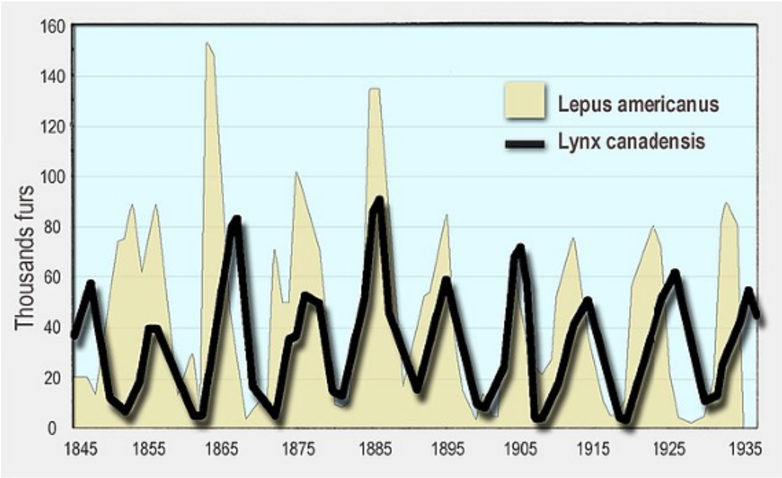
1.0.1 *The key behaviors and rules for our simulation*

RABBITS:

Rabbits reproduce at a certain rate if there is enough food.They move randomly and have a certain chance of being eaten if a lynx is in the same space.

LYNNX:

Lynx move randomly looking for rabbits.They hunt rabbits to gain energy; if they eat a rabbit, their energy increases, and if they don't eat, their energy decreases.They Reproduce at a certain rate if they do enough predation of rabbits.Habitat or other natural reasons are also considered for lynx birth rate. Lynx will die if there is no enough food sources.



(Source: Wikipedia.)

Figure 1.1: Rabbibts and Lynx Population over the period of 100 years

1.0.2 *About Modeling Complex Systems*

Our Project aims for two different modelling approaches Ordinary Differential Equations(ODE) and Agent Based Modeling (ABM). The ODE approach of using discrete equation methods instead of classic Lotka-Volterra Model , provides an analytical framework for studying the lifecycle of rabbits and lynx over the period of 100 years. On the other hand we have used ABMs which allows more detailed spatial simulation showing the movement of rabbits and lynx across the region and their interaction based on local conditions , offering insights into behaviour of population dynamics.Together, these approaches enable a more comprehensive exploration of complex ecological systems, balancing theoretical clarity with spatial and discrete aspects of real world ecological process.

In our project we have introduced randomness to the predation rate and the natural birth rate of lynx where as the previous studies have used the constant parameters for predation and birth rate of Lynx. In ODE no individual rabbbit, lynx is modelled instead we have used entire population.

In our Agent based Model we have showed an environmental space a creek region where lynxes cannot hunt the rabbits and when the lynx is in that grid there is Zero hunting or interaction between rabbits and lynxes. We tried to incorporate live interactions of Lynx and rabbits in our ABM.

2 METHODOLOGY

2.1 MODELLING PREDATOR-PREY SYSTEMS IN PYTHON

In this project we will look at a mathematical model used for predator-prey systems. The model we used is the classic Lotka-Volterra model and extended it introducing the randomness to the birth rate and death rates of predator prey and we have implemented it in the discrete environment. Along with ODE we have also implemented Agent Based Model showing the real time interaction of predator prey.

The Lotka-Volterra model is a mathematical model that describes the interaction between predator and prey populations. It is based on differential equations that represent how predator and prey populations change over time[1]

We will do this by first building a deterministic, idealized model for predator-prey systems. Toward the end of the project, we will then introduce a discrete equations and random hunting rates and randomness in of predator birth rates. so that the model more accurately captures the noise found in real-world scenarios.

2.2 THE LOTKA-VOLTERRA EQUATIONS

$\delta x/\delta t = \alpha x - \beta xy$
$\delta y/\delta t = \beta xy - \gamma y$

2.3 DEFINING THE EQUATIONS

From the above equation, the parameters can be described of as the following:

$\delta x/\delta t =$ growth rate of rabbit population

$\delta y/\delta t =$ growth rate of lynnx population

x= rabbit (prey) population

y = lynnx (predator) population

$\alpha =$ growth rate of rabbits

$\beta =$ death rate of rabbits due to predatation from lynx

γ = death rate of lynx

From the above equations, we can conclude that the Lotka-Volterra model makes several assumptions about the dynamics of predator-prey systems. **Some of which are as follows:**

The rabbit population is provided with limitless food.

The rate of change of populations is proportional to their size.

The lynxes eat only rabbits and are completely dependent on them as a source of food.

The lynxes have unlimited appetite.

2.4 DISCRETE-TIME EQUATIONS WITH RANDOMNESS

In our project we have extended the classic Lotka-Volterra predator-prey model by introducing randomness and implementing it in a discrete-time framework.

The traditional Lotka-Volterra model is continuous, described by differential equations. We have adapted this to a discrete-time model, which updates populations in discrete steps. This approach is particularly useful for simulations and computational modeling.

2.4.1 *The discrete equations are:*

$$\text{nextx} = x + \left(a \cdot x \cdot \left(1 - \frac{x}{K}\right) - \mathcal{N}(0.06, 0.02) \cdot x \cdot y\right) \cdot \Delta t$$
$$\text{nexty} = y + (-c \cdot y + \mathcal{N}(0.06, 0.02) \cdot x \cdot y \cdot \mathcal{N}(0.3, 0.9)) \cdot \Delta t$$

Where:

x and y represent the current populations of rabbits and lynxes, respectively.

a is the intrinsic growth rate of the prey.

K is the carrying capacity of the prey.

c is the natural death rate of the predators.

δt is the discrete time step.

2.4.2 *Incorporating Randomness*

In real-world ecosystems, many unpredictable factors influence birth and death rates, such as environmental changes, the availability of food,natural calamities and random events. To capture this inherent variability, we've introduced randomness into the model:

2.4.3 *Predation Rate Variability:*

The term
`np.random.normal (std_mean,std_dev)` introduces randomness to the predation rate, simulating fluctuations in hunting or prey availability.

2.4.4 *Predator Birth Rate Variability:*

The additional `np.random.normal (std_mean,std_dev)` term in the predator equation also accounts for variability in lynx birth rate/ reproduction, which could be due to factors like health, availability of mates, or other environmental conditions.

By sampling from a normal distribution with specified means and standard deviations, the model reflects the probabilistic nature of these interactions, making the simulation more realistic.

2.5 AGENT-BASED MODEL (ABM)

The ABM used in this study simulates predator-prey interactions in a 100×100 grid-based environment, where rabbits and lynx move and interact based on probabilistic rules. The model incorporates spatial constraints such as a non-hunting area, where the lynx cannot hunt rabbits, introducing a heterogeneous landscape for the simulation.

2.5.1 *Grid-Based Environment*

The environment is a 100×100 spatial grid, where:

Each unit represents a discrete space that can be occupied by agents. Rabbits and lynx move within the grid according to predefined rules. The creek area 40×40 to 60×60 acts as a safe zone for rabbits, where lynx cannot hunt.

2.5.1.1 *Agent Representation*

Each rabbit is an agent represented by a coordinate (x, y) in the grid. Each lynx is an agent also represented by a coordinate (x, y), moving within the spatial grid to hunt rabbits.

2.5.1.2 *Initialization*

A random number of rabbits and lynx are placed at random locations in the grid. Their population sizes are set based on the initial conditions of the Lotka-Volterra model. The creek area is defined as a 20×20 region where lynx are restricted from hunting.

2.5.1.3 *Movement and Interactions Randomized Movement:*

Both rabbits and lynx move randomly within the grid. Lynx preferentially move toward areas with higher rabbit density (except in the creek).
Predation Mechanism: When a lynx agent moves into the same grid cell as a rabbit, the rabbit has a probability of being killed or hunted. Predation does not occur in the creek area, acting as a refuge for rabbits.
Reproduction and Mortality: Rabbits reproduce based on a logistic growth model, considering the carrying capacity (K). Lynx reproduction is dependent on successful predation events. Lynx die if they cannot find enough food.

2.5.1.4 *Time Step Evolution*

The simulation updates at each time step (t = 0.01 years), where:

Movement Rules are applied to both rabbits and lynx.

Predation occurs if lynx encounter rabbits (outside the creek area).

Reproduction and Mortality events are computed based on population equations. Environmental constraints (such as the creek) are enforced.

2.5.1.5 *Visualization and Analysis*

Population Trends: Time series plots show the predator and prey populations over time.
Phase-Space Diagrams: These illustrate the relationship between the rabbits and lynxes populations.
Scatter Plot: Show the distribution of rabbits and lynx over the grid.

3 IMPLEMENTATION

The implementation of the predator-prey model is based on an agent-based simulation of rabbits (prey) and lynx (predators) in a spatial environment. The model incorporates population dynamics, random predation rates, and a creek area that acts as a non-hunting zone for predators. This section describes the step-by-step approach taken to develop the simulation using Python, NumPy, and Matplotlib.

3.0.1 *Programming Environment*

Language: Python
Libraries Used:
matplotlib.pyplot: For plotting population graphs and visualizing spatial positions.
numpy: For handling numerical operations, including population updates and random movements.
matplotlib.animation: For live animation of agents (rabbits and lynx).

3.1 MODEL PARAMETERS & INITIALIZATION

The model defines key parameters affecting population growth, predation, and movement:
Rabbit Dynamics
Birth rate:a=0.8
Carrying capacity: K=500
Death rate: randomly varies
Lynx Dynamics
Natural birth rate: varies randomly
Natural death rate: c=0.6
Predation efficiency: varies randomly
Simulation Environment
Grid Size: 100×100 spatial units
Creek Area (Non-Hunting Zone): 20×20 region where lynx cannot hunt.
Initialization Function initialize(): Sets the initial populations of rabbits and lynx. Randomly assigns spatial coordinates to each agent.Defines the non-hunting zone using coordinate ranges.

3.1.1 *Population Update Mechanism*

The predator-prey interactions follow Lotka-Volterra dynamics with additional randomness for realism.

Rabbit Update Equation:

$$\text{nextx} = x + \left(a \cdot x \cdot \left(1 - \frac{x}{K}\right) - \mathcal{N}(0.06, 0.02) \cdot x \cdot y\right) \cdot \Delta t$$

Lynx Update Equation:

$$\text{nexty} = y + (-c \cdot y + \mathcal{N}(0.06, 0.02) \cdot x \cdot y \cdot \mathcal{N}(0.3, 0.9)) \cdot \Delta t$$

This equation ensures that rabbit reproduction follows a logistic growth pattern while predation reduces their numbers.
If the lynx is inside the creek area, its population remains unchanged. Otherwise, it follows the modified Lotka-Volterra equation with stochastic elements. Update Function (update()) Calculates new population values. Prevents lynx from hunting in the creek. Updates the simulation time step.

3.1.2 *Visualization & Results:*

The model generates multiple visualizations
Population Dynamics Over Time

A time-series plot :
This shows how the rabbit and lynx populations fluctuate. Use `fill_between()` to differentiate between two species. The x-axis represents years, while the y-axis shows population size.

Phase Space Plot (Chaos Analysis) :
A phase plot maps rabbit population against lynx population to observe chaotic behaviors. Start and end points are marked to track system evolution.

Spatial Agent-Based Representation :
Displays the positions of rabbits (green) and lynx (red) on a 100×100 grid.The non-hunting area is colored brown to indicate a safe zone. Uses `scatter()`to mark the location of agents.

Animated Simulation :
The matplotlib.animation module creates a live simulation where agents move dynamically.The animation updates predator-prey interactions in real time.

3.2 CONCLUSION

This simulation demonstrates how agent-based modeling can be used to study predator-prey dynamics. The introduction of a creek as a non-hunting zone alters population cycles, adding environmental constraints to the classical Lotka-Volterra equations and making it a discrete model. The animation provides an interactive way to visualize the behavior of both rabbits and lynx over time.

4 RESULTS AND GRAPHS

The simulation results of the predator-prey model show dynamics between rabbit and lynx populations. The population trajectories exhibit oscillatory behavior, characterized by periodic peaks and troughs, indicating the interdependent relationship between rabbits and lynxes. Notably, fluctuations in the rabbit population precede corresponding changes in the lynx population, highlighting the predator's(lynx) reliance on prey(rabbit) availability.

The introduction of elements, such as random hunting rates and birth rates, adds variability to the model, capturing the unpredictability observed in natural ecosystems. These findings align with established ecological theories, demonstrating that predator-prey interactions are influenced by both deterministic factors and random environmental variations.

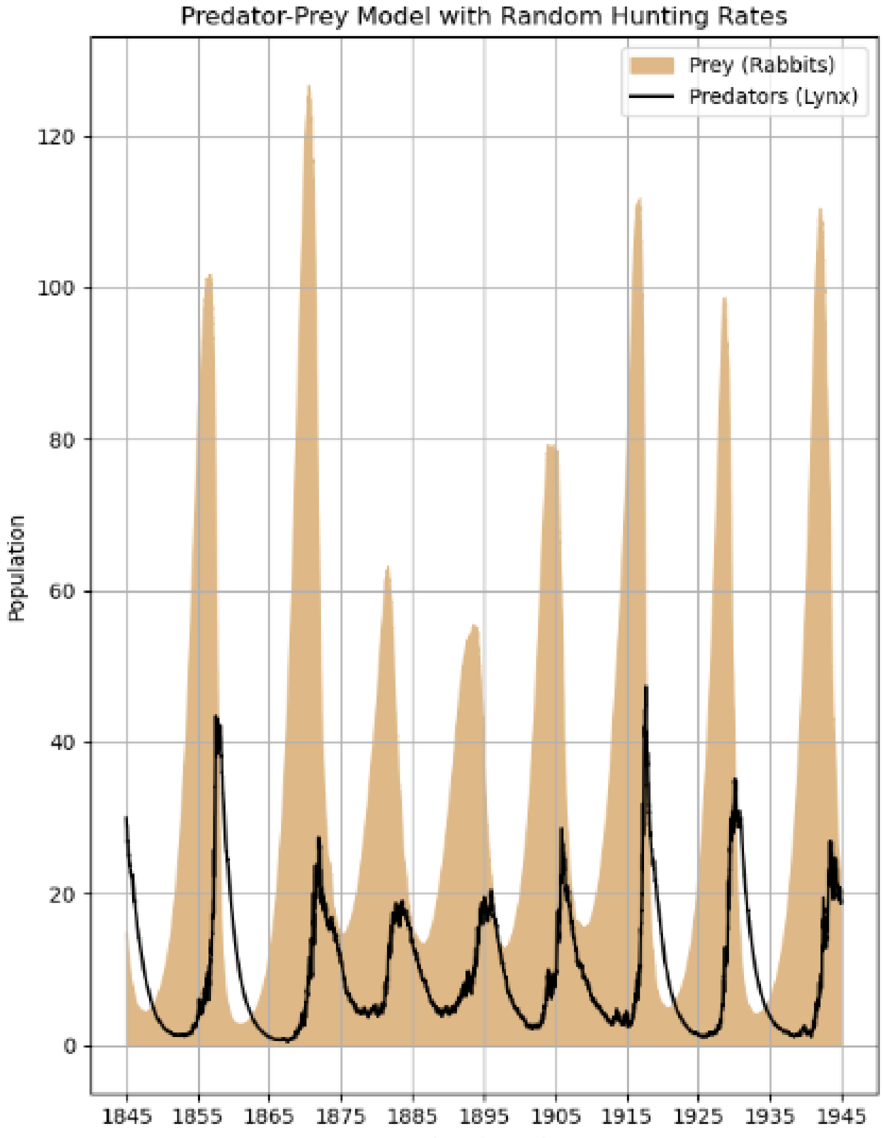


Figure 4.1: PRedator Prey Population

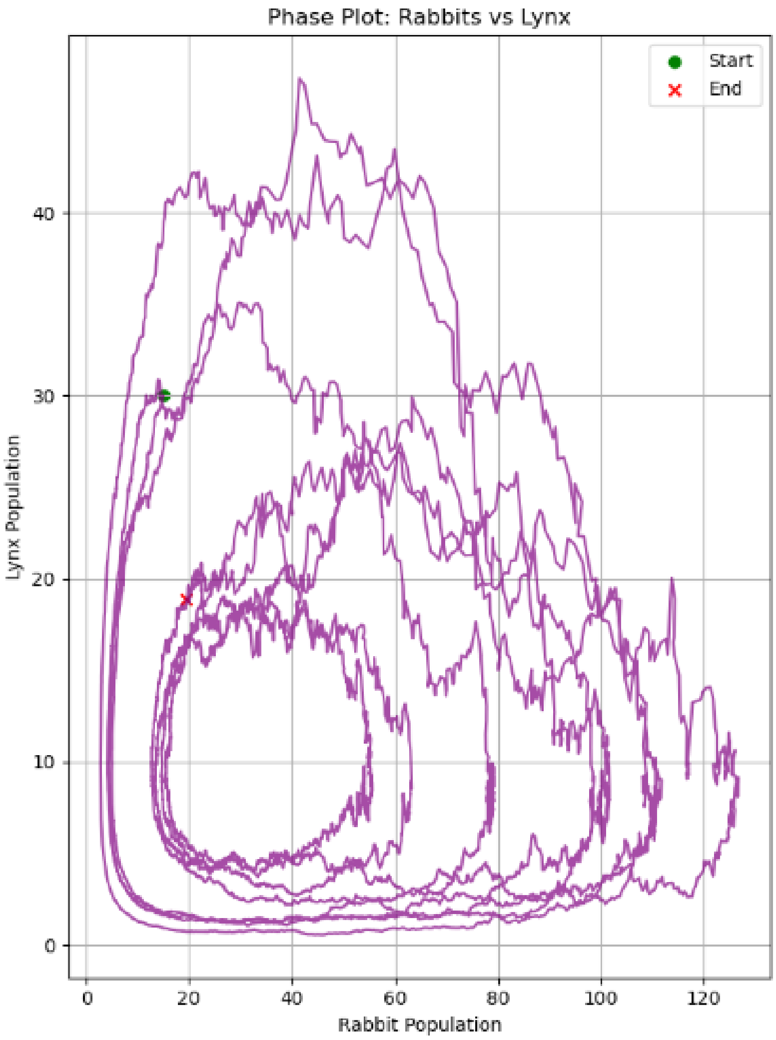


Figure 4.2: Phase Plot of the Rabbits and Lynx population

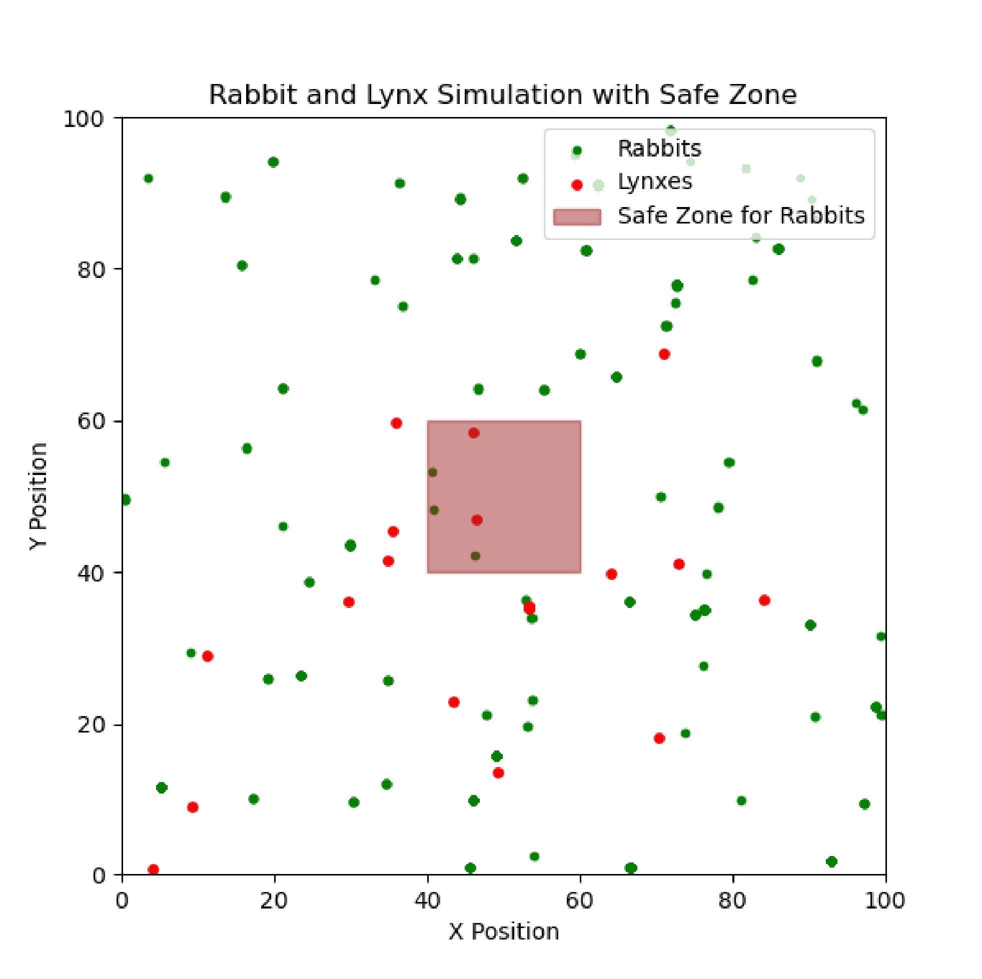


Figure 4.3: Agent Based Model of Predator Prey Interaction

[1] Dobromir T. Dimitrov and Hristo V. Kojouharov. "Complete mathematical analysis of predator–prey models with linear prey growth and Beddington–DeAngelis functional response." In: *Applied Mathematics and Computation* 162.2 (2005), pp. 523–538. ISSN: 0096-3003. DOI: https://doi.org/10.1016/j.amc.2003.12.106. URL: https://www.sciencedirect.com/science/article/pii/S0096300304001122.

[2] *Hare and Lynx Population Dynamics*. URL: https://jckantor.github.io/cbe30338-book/notebooks/02.08-Hare-and-Lynx.html.

[3] *Introduction to the modeling and analysis of complex systems by Sayama, Hiroki in 2015*.