COMP6245 Foundations of Machine Learning – Lab 3

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1. Class Boundaries and Posterior Probabilities

The below given data has been used to generate the two class classification problems in the two dimensions.

For Figure 1 and Figure 4, parameters are m1= $\begin{bmatrix}0\\3\end{bmatrix}$, m2 = $\begin{bmatrix}3\\2.5\end{bmatrix}$, c1=c2= $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$, P1=P2=0.5.

For Figure 2 and Figure 5, parameters are m1= $\begin{bmatrix}0\\3\end{bmatrix}$, m2 = $\begin{bmatrix}3\\2.5\end{bmatrix}$, c1=c2= $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$, P1=0.7, P2=0.3.

For Figure 3 and Figure 6, parameters are m1= $\begin{bmatrix}0\\3\end{bmatrix}$, m2 = $\begin{bmatrix}3\\2.5\end{bmatrix}$, c1=$\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$, c2=$\begin{bmatrix}1.5 & 0\\0 & 1.5\end{bmatrix}$, P1=P2=0.5.

The distributions are shown in Figure 1, Figure 2, Figure3 and Figure 4 and the posterior probability contour is shown in Figure 4, Figure 5 and Figure 6.

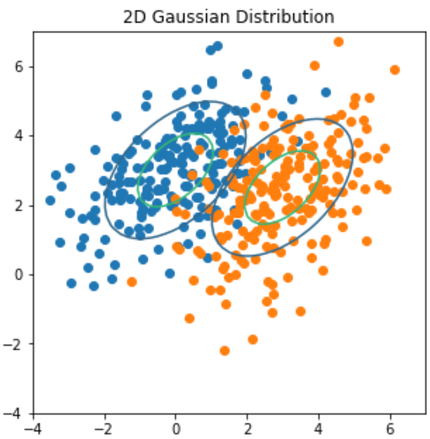

Figure 1: Gaussian Distribution 1

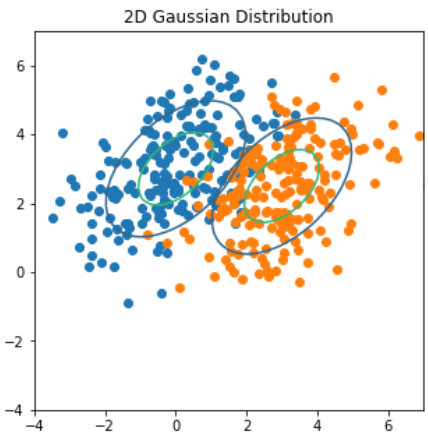

Figure 2: Gaussian Distribution 2

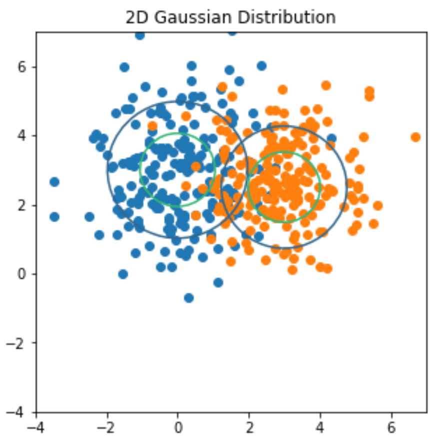

Figure 3: Gaussian Distribution 3

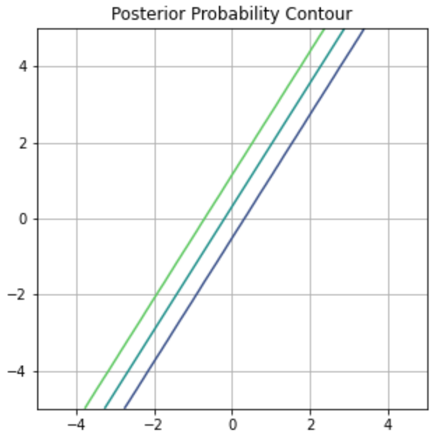

Figure 4: Posterior Distribution 4

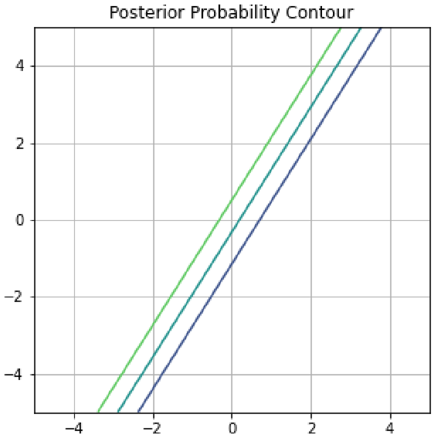

Figure 5: Posterior Distribution 5

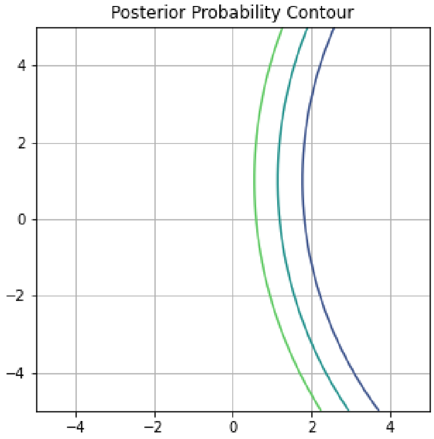

Figure 6: Posterior Distribution 6

When the mean and covariance matrices are same, the distribution contour is consistent. When P1 and P2 are changed, it doesn't affect the contour. Even in the case of posterior probability, the contour lines are linear and doesn't get affected when the probabilities are changes. In Figure 6, we can see that the posterior probability contour has a curve. In Figure 3, one of the contours is bigger than the other because they have different covariance matrices. Hence, if we change the probabilities, it doesn't affect the contour whereas if we take two different covariance matrices it changes the contour. So, there is similar variation for both the class boundaries and posterior probability.

2. Fisher's LDA and ROC Curve

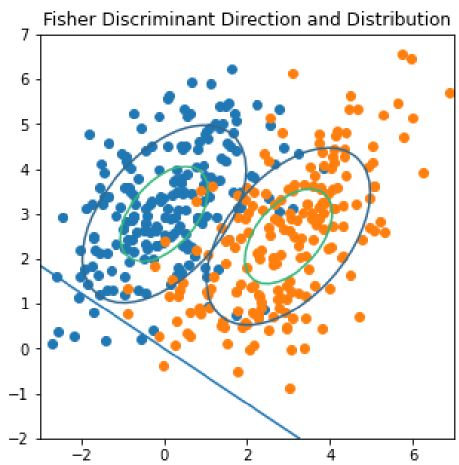

Figure 7: Fisher Discriminant Direction and Distribution

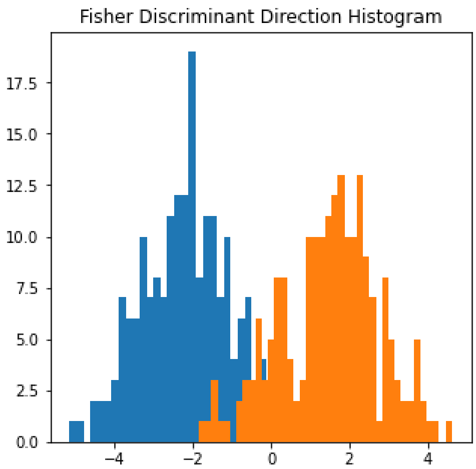

Figure 8: Fisher Discriminant Direction Histogram

In Figure 7, contours of the two densities are plotted using a scatter plot. Also, the fisher discriminant direction is plotted in the same graph using the formula $w_f = (C1 + C2)^{-1} (m1 - m2)$. In Figure 8, histogram for the fisher discriminant is plotted. It is evident from the graph that, the orange distribution is overlapping the blue distribution.

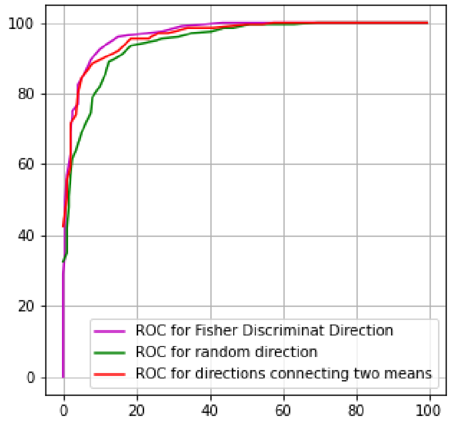

Figure 9: ROC curve for the Fisher Discriminant, Random Direction and directions connecting two means

In Figure 9, graph is plotted consisting of three ROC curves for Fisher Discriminant, random direction and directions connecting the two means. The x-axis is the True positives, and the y-axis is the False Positives. It is seen that when the true positive is higher, false positive is low.

```
In [100]:  #AUC for Fisher Discriminat Direction
          print("AUC for Fisher Discriminat Direction: " ,np.trapz(ROC[:,0],ROC[:,1]) )

          #AUC for random direction
          print("AUC for random direction: " ,np.trapz(ROC3[:,0],ROC3[:,1]) )

           #AUC for directions connecting two means
          print("AUC for directions connecting two means: " ,np.trapz(ROC2[:,0],ROC2[:,1]) )

          AUC for Fisher Discriminat Direction:  -9647.625
          AUC for random direction:  -9395.5
          AUC for directions connecting two means:  -9578.875
```

Figure 10: AUC for the three different directions

The area under the ROC curve for Fisher's is -9647.635, for random direction it is -9395.5 and for directions of two means, it is -9578.875. From the above observation it is evident that Fisher Discriminant Direction has a good accuracy as compared to the other two directions.

3. Mahalanobis Distance

Mahalanobis is used to measure the distance between a point and its distribution. When we have a data that is widely distributed/dispersed, we cannot measure accurately the distance between the two points. Generally, we use the Euclidean Distance to measure the distance between two points. But in case of Mahalanobis, we do not consider the co-ordinates of the two points we consider the distance between a point and the distribution.

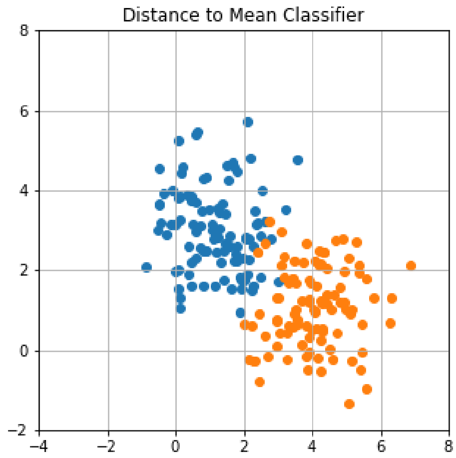

Figure 11: Normal Distance to Mean Classifier

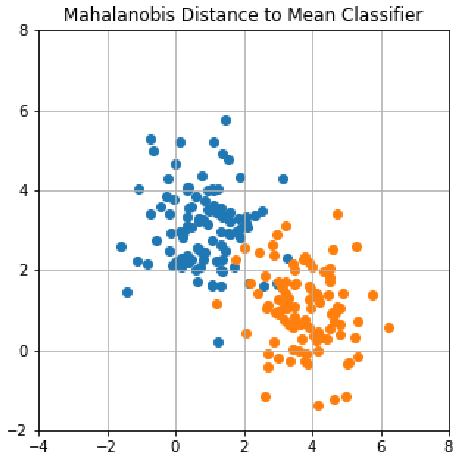

Figure 12: Mahalanobis Distance to Mean Classifier