classmate

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Math Basics / Foundations : ~~Identify Efficiency~~

1. $\sum_{i=1}^{n-1} 1$

$\Rightarrow$ $(n-1-1+1) = n-1$ [Upperbond - Lowerbound + 1]

Σ - Samation

2. $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1$

$\Rightarrow \sum_{i=0}^{n-2} = (n-1)-i-1+1) = \sum_{i=0}^{n-2} n-1-i$

$\Rightarrow \sum_{i=0}^{n-2} n-1 - \sum_{i=0}^{n-2} i$

$\Rightarrow n-1 \sum_{i=0}^{n-2} 1 - \sum_{i=0}^{n-2} i \rightarrow$ sum of n natural numbers $= \frac{n(n+1)}{2}$

$= (n-1)[n-2-0+1] - \frac{(n-2)(n-1)}{2}$

$= (n-1)(n-1) - \frac{(n-1)(n-2)}{2}$

$= \frac{2(n-1)(n-1) - (n-1)(n-2)}{2}$

$= \frac{(n-1)}{2}[2n-2-n+2] = \frac{n-1(n)}{2}$

$= \frac{n^2}{2} - \frac{n}{2} \approx \frac{n^2}{2} \approx n^2$

3. $\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$

$\Rightarrow \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n-1-0+1 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n$

$= \sum_{i=0}^{n-1} n \sum_{j=0}^{n-1} 1 = \sum_{i=0}^{n-1} n(n-1-0+1) = \sum_{i=0}^{n-1} n^2$

$= n^2 \sum_{i=0}^{n-1} 1 = \{n^2(n-1-0+1] \Rightarrow n^3$

4. Important Summation Formulas:

i) $\sum_{i=1}^{n} i = 1 + 2 + \dots + n = \frac{n(n+1)}{2} \approx \frac{n^2}{2}$

ii) $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \dots n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3}$

iii) $\sum_{i=1}^{n} i^k = 1^k + 2^k + \dots n^k = \frac{1}{k+1} n^{k+1}$

iv) $\sum_{i=0}^{n} a^i = 1 + a + \dots + a^n = \frac{a^{n+1}-1}{a-1} \quad a \neq 1$

v) $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

vi) $\sum_{i=1}^{n} i.2^i = 1*2 + 2*2^2 + \dots n*2^n$
$= (n-1)2^{n+1} + 2$

vii) $\sum_{i=1}^{n} \lg i = n \lg n$

viii) $\sum_{i=1}^{n} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \dots \frac{1}{n} = \ln n + \gamma$

$\gamma = 0.5772$, Euler's constant n^{th} Harmonic number.

Orders of Growth :

$\Theta(n)$ Indicates in the average case as we keep increasing the input, the time taken increases in the linear order.