Control Systems

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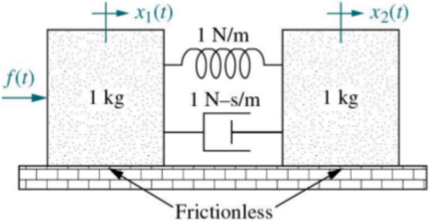
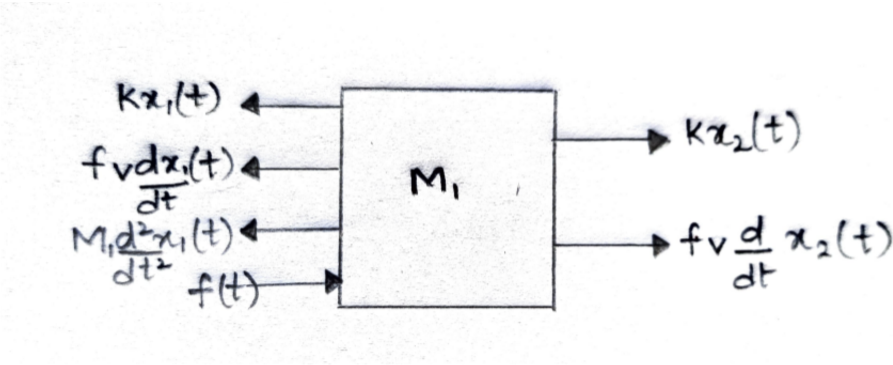
Problem Statement

Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translation mechanical network shown in the figure



The forces acting on the left side block are $kx_1(t), M_1\frac{d^2(x_1(t))}{dt^2}, f_v\frac{dx_1(t)}{dt}$ towards left and f(t),$kx_2(t), f_v\frac{dx_2(t)}{dt}$ towards right. The free body diagram of left block is given below



The equation of forces acting on the block in time domain is

$$f(t) = kx_1(t) + f_v\frac{dx_1(t)}{dt} + M_1\frac{d^2(x_1(t))}{dt^2} - kx_2(t) - f_v\frac{dx_2(t)}{dt} \quad (3.1)$$

We know the Laplace transform of first derivative is

$$\mathscr{L}\{f'(t)\} = sF(s) - f(0) \quad (3.2)$$

And Laplce transform of second derivative is

$$\mathscr{L}\{f"(t)\} = s^2F(s) - sf(0) - f'(0) \quad (3.3)$$

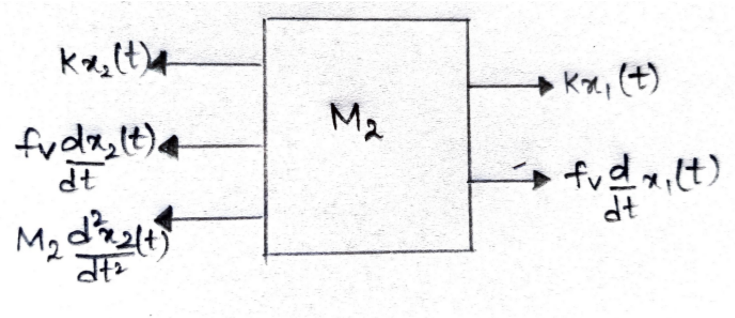
Transforming the equation into Laplace domain
We get

$$F(s) = kX_1(s) + f_v sX_1(s) + M_1 s^2 X_1(s) - kX_2(s) - f_v sX_2(s) \quad (3.4)$$

Substitue k=f_v=M_1=1.
As $x_1(t)$,$x_2(t)$ are 0 at t=0 then $x_1(0)$,$x_2(0)$,$\frac{dx_1(0)}{dt}$ are also 0.
Then,

$$F(s) = (s^2 + s + 1)X_1(s) - (s + 1)X_2(s) \quad (3.5)$$

The forces acting on the right side block are $kx_2(t)$,$M_2\frac{d^2(x_2(t))}{dt^2}$,$f_v\frac{dx_2(t)}{dt}$ towards left and $kx_1(t)$,$f_v\frac{dx_1(t)}{dt}$ towards right. The free body diagram of right block is below



The equation of forces acting on the block in time domain is

$$0 = kx_2(t) + f_v \frac{dx_2(t)}{dt} + M_1 \frac{d^2(x_2(t))}{dt^2} - kx_1(t) - f_v \frac{dx_1(t)}{dt} \quad (3.6)$$

We know the Laplace transform of first derivative is

$$\mathscr{L}\{f'(t)\} = sF(s) - f(0) \quad (3.7)$$

And Laplce transform of second derivative is

$$\mathscr{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0) \quad (3.8)$$

Transforming the equation into Laplace domain
We get

$$0 = kX_2(s) + f_v sX_2(s) + M_2 s^2 X_1(s) - kX_1(s) - f_v sX_1(s) \quad (3.9)$$

Substitue k=f_v=M_2=1.
As $x_1(t)$,$x_2(t)$ are 0 at t=0 then $x_1(0)$,$x_2(0)$,$\frac{dx_2(0)}{dt}$ are also 0.
Then,

$$0 = -(s+1)X_1(s) + (s^2+s+1)X_2(s) \quad (3.10)$$

From the equations 3.5 and 3.10

$$\begin{pmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & s^2+s+1 \end{pmatrix} \begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{3.11}$$

Let us consider

$$\begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = X(s) \tag{3.12}$$

Then,

$$\begin{pmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & s^2+s+1 \end{pmatrix} X(s) = \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{3.13}$$

$$X(s) = \begin{pmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & s^2+s+1 \end{pmatrix}^{-1} \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{3.14}$$

$$X(s) = \frac{\begin{pmatrix} s^2+s+1 & s+1 \\ s+1 & s^2+s+1 \end{pmatrix}}{(s^2+s+1)^2 - (s+1)^2} \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{3.15}$$

$$\begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \frac{\begin{pmatrix} (s^2+s+1)F(s) \\ (s+1)F(s) \end{pmatrix}}{s^4+2s^3+2s^2} \tag{3.16}$$

$$X_2(s) = \frac{(s+1)F(s)}{s^2(s^2+2s+2)} \tag{3.17}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^2(s^2+2s+2)} \tag{4.1}$$

Therefore the transfer function is

$$G(s) = \frac{s+1}{s^2(s^2+2s+2)} \tag{4.2}$$