Advertising Strategies for Mobile Platforms With "Apps"

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***Abstract*—Mobile platforms are prevalently used in the age of social media. They serve a two-sided market to connect users and app developers. In this system, members simultaneously sell products, publish advertisements (ads), and advertise. This paper aims to explore advertising strategies for the mobile platform incorporating their roles as sellers, ad publishers, and advertisers. Specifically, we develop a game-theoretical model which captures the relationship among the works of the platform owner and app developers in a dynamic setting. Our analysis shows that when the platform displays apps' ads, the owner may be better off to participate in apps' advertising under certain conditions, rather than always charging them aggressively as suggested by convention wisdom. Surprisingly, although the negative impacts evoked by multiple apps' entry deserve much attention, it is unnecessary for app developers to take into account when making advertising decisions. We further find that the coordinating bilateral participation in advertising is a new mechanism to improve the profitability. Unlike the result proposed in the previous literature, the mechanism here will cause free-riding among app developers when they participate in the platform's advertising. Furthermore, when accompanied with the revenue sharing policy, the relatively low participation rates could absolutely eliminate the system inefficiency.**

***Index Terms*—Advertising, app, platform, Stackelberg differential game, system coordination.**

I. Introduction

PLATFORMS, such as Apple's iPhone operating system (iOS) and Google's Android, are prevalently used nowadays. They provide a basic environment for apps to reach users [60]. Many software developers create apps which can function on the platform [33]. For instance, Apple's iOS, a mobile platform, is shared by all the apps in the Apple's App Store, and the apps, such as "Wechat," offer users various services (e.g., games, news, and online shopping). The platform here serves as a two-sided market, which connects users and app developers. To cater for users' requirements on the variety of apps, the platform owner usually encourages various third-party apps' entry by "opening" the market to them [12], [22]. Then those apps, certified by the owner, become compatible with the platform [35].

The revenues of the platform owner and her third-party app developers come mainly from user purchases, transaction-based charges, and displaying ads [21]. First, the platform owner's and app developers' sales revenues are generated when a user buys a service or a product from the platform and the apps, respectively. Second, when the purchase occurs in the app, the platform owner usually shares a portion of the app developer's sales revenue. This is called transaction-based charges and the platform owner's share is her transaction revenue. For instance, Apple obtains 30% of its apps' revenue on sales and in-app purchase.[1] When more users purchase in apps, the platform owner will get more transaction revenue. Third, an advertising revenue is earned from displaying the ads. In practice, the platform owner receives the advertisers' advertising requirement and then delivers their ads in either the platform or her third-party apps. When the ads are shown in an app, the platform owner will pay a part of her advertising revenue to the app developer [33]. Otherwise, she takes all the advertising revenue. An example is that an app, integrating into iAd Workbench, an Apple's advertising service, can get 70% of the ad revenue generated while presenting advertisers' ads to its users.[2] Since advertisers hope that their ads could reach more audiences, a platform with a large number of users attracts more potential advertisers [33] and thereby generates more advertising revenue.

Since the number of users significantly affects the revenue, advertising plays a crucial role owing to its efficiency in attracting users. However, in the mobile platform, the decision process on advertising policies is challenging. On one hand, advertising is not only a way of promotion but also an income source since the platform and apps play as both the advertiser and ad publisher. On the other hand, the platform owner's

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[1] *Source:* https://developer.apple.com/programs/; https://developer.apple.com/in-app-purchase/. Accessed September 20, 2015.

[2] *Source:* https://developer.apple.com/iad/monetize/. Accessed September 20, 2015.

advertising investment and app developers' advertising investment interact. This is because apps' advertising affects the platform owner's transaction revenue via attracting app users, and the platform's advertising influences app developers' sales revenue through enlarging their potential market. The revenues of the platform owner and that of app developers also directly affect platform advertising investment and app advertising investment decisions, respectively. The purpose of this paper is to explore advertising strategies for a mobile platform when system members simultaneously play as sellers, ad publishers, and advertisers. Prior studies normally consider them as sellers and ad publishers but ignore their role as advertisers (e.g., [30] and [33]). Although the policy on advertising in many contexts has been studied, our views on the policy may significantly change when members sell products, display ads, and advertise at the same time.

Why is it relevant to consider all the three roles? Convention wisdom suggests that ad publishers should announce a high price in a monopoly setting to maximize their profits. This is true when displaying ads is their sole function. However, in a mobile platform, other functions such as selling products and advertising products are also presented, so ad publishers should make a decision with caution. For example, when app developers advertise in the platform, different platforms make different decisions. "360 platform" usually promotes its apps for free.[3] But Apple charges app advertisers aggressively and makes the charge as a revenue source. Actually, although a high participation in apps' advertising will increase the platform owner's advertising cost, it could increase her transaction revenue because it stimulates more app advertising and eventually leads to more app users. In contrast, a high charge for apps' advertising can increase the platform owner's advertising revenue, but it may discourage app developers from raising advertising investments, resulting in the owner's lower transaction revenue. Under such a circumstance, the platform owner faces a tradeoff between participating in apps' advertising versus overcharging it. But when app developers advertise in other apps, the tradeoff becomes different. While this kind of ads, called the in-app advertisement, could generate advertising revenue for those apps which display in-app ads, these ads may result in apps' user loss since they usually distract users' attention and cause user annoyance [30]. Faced with the opposite impact, many app developers cannot appropriately adjust their advertising policies accordingly. Some third party apps of Apple, which integrate into iAd Workbench, complain that their total revenues remain more or less the same before and after displaying in-app ads [33]. In this condition, app developers should decide whether to highlight or mitigate the contrary impact. Thus, failing to capture any of the three roles may yield insufficient guidance for a mobile platform.

By taking into account the three roles, many questions regarding the advertising strategies may arise: when the platform displays app ads, should the platform owner focus on the advertising revenue or the transaction revenue? How should developers decide their advertising investments when their apps display in-app ads? Should app developers offer the platform owner an advertising subsidy? The subsidy will indirectly attract more app users, since it undoubtedly requires the platform to increase advertising investment and thereby enlarges the apps' potential market. However, the subsidy is also a kind of cost for app developers. Based on the twofold impact, does this subsidy always reduce the apps' profitability? Could it alleviate the system inefficiency?

To rigorously address the above questions, we develop a dynamic model for a mobile platform to analyze the interaction among the multiple roles of system members. Specifically, the platform owner, as a Stackelberg leader, grants third-party apps access to her platform and shares their sales revenue. To draw the attention of unaware users, the platform owner invests in various advertising programs outside the platform. The app developers, on the contrary, would usually like to advertise either in the platform or in other apps. The reason is twofold. First, many users connected to the platform and apps are their target users. Second, the platform owner often provides developers an advertising subsidy if they advertise in the platform or in her apps. We start with the analysis of the platform having a single app, and then discuss the strategies in the face of two apps' entry.

Our result reveals that rather than always charging app developers aggressively, it would be better for the platform owner to participate in apps' advertising when her platform displays apps' ads and gets a high share of apps' sales revenue. Additionally, tempting multiple apps' entry is another way to improve the profitability, while our analysis shows that its negative impacts, such as user loss caused by app competition and in-app ads, deserve much attention. Surprisingly, we find that it is unnecessary for app developers to take into account the negative impact when making advertising decisions. Our further analysis develops a new mechanism to raise the profits for the mobile platform, that is, the coordinating bilateral participation in advertising. Even though previous literature has proposed it in some contexts (e.g., [69] and [70]), we find that in our considered system it will evoke free-riding among app developers when they participate in the platform's advertising. This has not been identified or formally clarified before. Moreover, when the bilateral participation policy is combined with the revenue sharing policy, the relatively low participation rates, instead of spending high participation rates, are enough to absolutely eliminate the system inefficiency. The interpretation of this result is that revenue sharing has participated in the task to alleviate the inefficiency. In this paper, we propose particular strategies and provide impactful implications for system members regarding how to deal with the complex interaction among their works.

The remainder is organized as follows. Section II reviews the related literature. Section III develops the basic model with a single app and introduces a bilateral participation contract to restore system efficiency. We then extend the model to the scenario with two apps in Section IV. Finally, Section V concludes and provides limitations of our analysis. All proofs are placed in the Appendix in the supplementary material.

II. LITERATURE REVIEW

Research on platforms was extensively conducted (see [7]). Most of them focused on platform designs (e.g., [23], [42], [45], and [60]), platform governance (e.g., [12], [21], and [61]), platform performance investment (e.g., [3] and [32]), platform encroachment [28], platform operation mechanisms [8], [24], and components which are compatible with the platform, including their

[3] *Source:* http://dev.360.cn/. Accessed September 20, 2015.

strategies [35], and user preference [30]. Recent literature specified the platform as a two-sided market which provides an interface between software applications and users [54] and mainly investigates pricing structure, promotional activities, the optimal level of openness [25], and innovative investments [12]. In such a two-sided market, choosing an appropriate price allocation, which identifies the side of a profit center and the side of a loss center, is a critical issue for the platform owner. This pricing strategy, called one-sided pricing, depends on the platform governance structure [52]. Economides and Katsamakas [24] extended the literature by introducing a two-sided pricing strategy, which intends that the platform owner charges both sides an amount of money, and showed that the two-sided pricing strategy always outperforms the one-sided pricing strategy. However, Albuquerque *et al.* [2] found that the impact of pricing promotion on the platform’s profitability is limited, while some other promotional activities have broader effects through empirically measuring the value of promotional activities, including pricing promotion, advertising, and content creator referrals. Although both [2] and this paper emphasize the role the platform owner’s advertising investment, the objectives are different. Albuquerque *et al.* [2] focused on the comparative value of advertising investment and aimed to offer insights to optimally allocate resources across marketing tools, while we are more interested in investigating the optimal level of investment in advertising to maximize the profit of the platform owner and that of app developers, developing a dynamic two-sided market model and capturing the cross-side network effects in the platform. In addition, unlike the platform discussed in [2] which does not permit in-app ads, the platform in this paper delivers in-app ads, and their impact on advertising investments is explored.

Numerous studies on online advertising market in an information system have been developed to address issues related to online advertisement slot [1], [17], [20], [66], [68], the impact of online advertisements on consumer behaviors [10], [31], [67] and charging schemes of online advertising (e.g., [4], [16], [48], and [50]). Some studies investigate how an ad publisher should display the optimal level of ads to maximize the profit (e.g., [6], [14], [43], and [53]), which are closely related to this paper. Specifically, Kumar and Sethi [43] solved a firm’s optimization problem when it collects revenue from displaying ads and providing contents. Casadesus-Masanell and Zhu [14] proposed an incumbent’s optimal strategy in terms of the mix of pricing and the number of ads that it carries in the presence of an ad-sponsored entrant, which gives users a free access to ad-supported products. Most of the related papers consider general online advertising, while our interest lies in the online advertising in a platform and its apps. In this sense, the related studies are [33] and [71]. Zhang and Sarvary [71] studied a horizontal differentiation between two platforms or two apps which compete for consumers by setting the level of advertising that they displayed and offering the user-generated content, but the authors did not consider the relationship between the platform and apps. Hao *et al.* [33] emphasized this relationship and focused on the in-app advertising program and agency pricing for the apps in a mobile platform. Although both [33] and this paper discuss in-app ads and the revenue sharing scheme between the platform owner and app developers, the model settings are distinctly different. Hao *et al.* [33] only characterized the platform and the apps as sellers and ad publishers in a static setting, while they did not consider its other role, that is, advertisers. In contrast, we capture all the above roles, including sellers, ad publishers, and advertisers, and explore their advertising investments in a dynamic case, incorporating the impact of in-app ads and revenue sharing, which is also not studied in other related papers.

Since we specify conditions where the platform owner participates in the apps advertising, this paper is also related to the dynamic cooperative advertising literature. This research stream is inspired by [36] who argued that the manufacturer who adopts global advertising is willing to offer an advertising subsidy to the retailer who introduces the local advertising because the subsidy could motivate the local advertising investment and thereby immediate sales. To characterize the distinction between the effects of global and local advertising, the literature on dynamic advertising model has been developed. These mainly consist of two streams. The first stream follows the time-dependent demand model proposed by Nerlove and Arrow [49], in which the demand is indirectly affected by advertising via product goodwill to investigate various issues, including firms’ advertising decision, pricing decision, and quality decision in a monopoly setting (e.g., [26], [27], and [44]), in a horizontal competition setting (e.g., [9], [15], [47], and [63]), in a supply chain [38], [39], and in some especial contexts, such as a franchise system [56], brand competition between the national brand and the store brand [40], [41], reference pricing [70]. Among them, Jørgensen *et al.* [38] first proposed the dynamic cooperative advertising model [5] and demonstrated that this cooperative scheme could achieve the coordination. The other stream builds upon [62] and its extension [55], in which advertising directly and continually affects sales beyond the current period but the effect diminishes over time. In addition to studying the above issues, this body of work also considers the marketing communications [51] and uncertain pricing decisions [34]. This paper differs from the above literature in that we develop a dynamic cooperative advertising model in a distinct context, i.e., a mobile platform which serves as a two-sided market. In this setting, the platform plays as an advertiser and an ad publisher. Hence, when app developers advertise in the platform, in addition to choosing to participate in the apps’ advertising, the platform owner can choose to aggressively charge app developers for displaying their ads to collect advertising revenue. We explore the strategy on advertising investments and find that the cooperative advertising could align members’ incentives in such a two-sided market, while it could not coordinate the system even along with other cooperative mechanisms such as the revenue sharing policy.

The literature on supply chain coordination focuses on various mechanisms which can enhance the channel efficiency by aligning independent firms’ incentives [18]. Supply contracts which can coordinate supply chains include the revenue-sharing contract [13], quantity discount [64], pull price discount [29], two-part tariffs [37], sales rebate contract [59], and bilateral participation contract [69]. Most studies discuss the problem in a static setting and do not consider the system dynamics. The few papers (e.g., [46] and [57]) that examine coordination under a dynamic setting typically follow a discrete time model. In this paper, we utilize a bilateral participation contact in a continuous differential game. One related study is [70]. It considered a bilateral participation contract in a continuous dynamic setting with a manufacturer and a retailer and assumed that the participation rate is independent of time. Unlike [70], we apply the contract in a mobile platform, explore the impact of the revenue sharing policy on the coordinating conditions and assume firms' participation rates depend on time and state. Besides, we further extend the literature by exploring the bilateral participation contract in the condition with more than two members and find the existence of free-riding between two members when they jointly participate in other member's advertising.

III. Benchmark: Single App Case

We consider a system consisting of a platform and a third-party app. With the platform's permission, the app is exposed to the platform users. The platform owner's revenue is divided into three categories: 1) sales revenue; 2) transaction revenue; and 3) advertising revenue. The first is generated when a user purchases from the platform. The second is obtained when users purchase the app's products or services via the platform, and then the platform owner takes a share of the app developer's sales revenue. The third comes from the platform displaying the app's ads. However, the app developer gets revenue only when a user purchase happens in his app.

We characterize two different types of advertisements in our model, i.e., the platform's advertisement outside the system and the app's advertisement inside the platform. The platform owner advertises in traditional media (e.g., TVs and newspapers), portal sites or social media sites to attract more unaware users. The app developer advertises in the platform to enlarge his market. It is an efficient promotion, since many platform users are his potential users and he could utilize the incentive policy on advertising offered by the platform owner.

Denote the platform's advertising effort by $u_P(t)$ and the app's advertising effort by $u_A(t)$. Following previous studies such as [19], [55], and [70], we assume that the advertising cost takes a quadratic form, that is:

$$C_P(t) = \frac{1}{2}u_P^2(t) \text{ and } C_A(t) = \frac{1}{2}u_A^2(t) \tag{1}$$

which implies an increasing marginal advertising cost.

The advertising cost is undertaken as follows. With regard to the platform's advertisements, the platform owner pays the expenditure $C_P(t)$ to the third-party advertisement agency. With regard to the app's advertisements, because the platform designs and delivers the app's advertising to platform users, the app advertising cost $C_A(t)$ first occurs in the platform side and then is proportionally paid by the app developer. Denote the proportion as $\phi(t) \geq 0$ and we define it as the platform owner's charge on the app developer's payment rate. When $0 \leq \phi(t) < 1$, the platform owner undertakes a part of the app's advertising cost to foster the app's development. When $\phi(t) = 1$, the app developer pays all the advertising cost. When $\phi(t) > 1$, the platform owner overcharges the app developer and takes the advertising fee as a source of revenue.

Those two types of advertisements can increase or keep the number of platform users and app users. Denote the number of the platform users and that of the app users by $x(t)$ and $y(t)$, respectively. Following [43], we assume

$$\dot{x}(t) = \alpha u_P(t) - \delta x(t), x(0) = x_0 > 0 \tag{2}$$

$$\dot{y}(t) = \beta x(t) + \gamma u_A(t) - \delta y(t), y(0) = y_0 > 0 \tag{3}$$

where α, β, γ, and δ are positive parameters. The constants α and γ, respectively, measure the effectiveness of the platform's advertising and that of the app's advertising. Note that in our model the app's ads cannot increase the platform's user growth $\dot{x}(t)$. It is because the app developer advertises inside the platform rather than outside the system. These inside ads cannot reach those users who are outside the system, that is, the platform unaware users. But they can negatively affect $\dot{x}(t)$ by distracting the attention of the platform users and causing their complaints. We assume that the platform's scale is so large that the negative impact can be ignored. The term $\beta x(t)$ characterizes the positive impact of the number of platform users on the app's user growth $\dot{y}(t)$. A loss of existing users influences the user growth negatively and it is captured by the decay coefficient δ. Motivated by the reality that an app tends to join the platform with a giant user base, we assume that the number of platform's initial users is much larger than that of the app's initial users, that is, $x_0 \gg y_0$.

Let p_P and p_A denote the marginal sales revenue of the platform and that of the app, respectively. We specify the platform owner's sharing rate of the app developer's sales revenue as λ, which is exogenous and satisfies $0 \leq \lambda \leq 1$. The instantaneous profit of the platform owner is

$$\pi_P(t) = p_P x(t) + \lambda p_A y(t) + \frac{1}{2}\phi(t)u_A^2(t) - \frac{1}{2}u_P^2(t) - \frac{1}{2}u_A^2(t) \tag{4}$$

and that of the app developer is

$$\pi_A(t) = (1-\lambda)p_A y(t) - \frac{1}{2}\phi(t)u_A^2(t). \tag{5}$$

The system's instantaneous profit is

$$\pi_I(t) = p_P x(t) + p_A y(t) - \frac{1}{2}u_P^2(t) - \frac{1}{2}u_A^2(t). \tag{6}$$

We consider an infinite horizon with a positive discount rate ρ. When the platform owner and the app developer integrate together as a single firm, the members aim to maximize the current value of the system profit by setting the appropriate advertising efforts, $u_P(t)$ and $u_A(t)$, that is

$$V_I(x,y) = \max_{u_P(t)\geq 0, u_A(t)\geq 0} \int_0^\infty e^{-\rho t}\left[p_P x(t) + p_A y(t) - \frac{1}{2}u_P^2(t) - \frac{1}{2}u_A^2(t)\right]dt \tag{7}$$

subject to (2) and (3). When the two members make decisions independently, they aim to maximize the current value of their own profits. Their optimization problems are

$$V_P(x,y) = \max_{u_P(t)\geq 0, \phi(t)\geq 0} \int_0^\infty e^{-\rho t}\left[p_P x(t) + \lambda p_A y(t) - \frac{1}{2}u_A^2(t) - \frac{1}{2}u_P^2(t) + \frac{1}{2}\phi(t)u_A^2(t)\right]dt \tag{8}$$

and

$$V_A(x,y) = \max_{u_A(t)\geq 0} \int_0^\infty e^{-\rho t}\left[(1-\lambda)p_A y(t) - \frac{1}{2}\phi(t)u_A^2(t)\right]dt \tag{9}$$

subject to (2) and (3). We argue that system members decide advertising policies according to the system state $(x(t), y(t))$, so decision variables can be denoted as $u_P(x(t), y(t))$,

TABLE I
NOTATIONS

t	Time t, $t \geq 0$
$x(t)$, $y(t)$	The number for platform users and app users at time t, respectively
$u_P(t)$, $u_A(t)$	Advertising efforts for the platform owner and the app developer at time t, respectively
$\phi(t)$	The app developer's payment rate at time t
α	The platform's advertising effectiveness on its user growth
γ	The app's advertising effectiveness on its user growth
β	The platform's user base effectiveness on the app's user growth
δ	User growth decay parameter
ρ	Discount rate
λ	The platform owner's sharing rate of the app developer's sales revenue
p_P, p_A	Marginal sales revenue for the platform and the app, respectively
π_P, π_A, π_I	Instantaneous profits for the platform owner, the app developer and the integrated system, respectively
V_P, V_A, V_I	Value functions for the platform owner, the app developer and the integrated system, respectively

$\phi(x(t), y(t))$ and $u_A(x(t), y(t))$. For convenience, we denote them as $u_P(x, y)$, $\phi(x, y)$, and $u_A(x, y)$, with an abuse of notation.

We restrict our analysis to the sufficient condition of "$\beta \leq (\delta/2)$ and $\alpha > \gamma\sqrt{([\delta(\rho+\delta)]/\beta^2\lambda)}$" throughout this section. It ensures that our model is consistent with the real world practice that the number of app users is no larger than that of platform users.

We summarize all notations in Table I.

A. Integrated System

When the platform and the app act as an integrated system [58], [65], their optimization problem is given by (7). The Hamilton–Jacobi–Bellman (HJB) equation is

$$\rho V_I = \max_{u_P \geq 0, u_A \geq 0}\bigg[p_P x + p_A y - \frac{1}{2}u_P^2 - \frac{1}{2}u_A^2 + V_{Ix}(\alpha u_P - \delta x) + V_{Iy}(\beta x + \gamma u_A - \delta y)\bigg]. \quad (10)$$

In (10), $V_{Ix} = \partial V_I/\partial x$ and $V_{Iy} = \partial V_I/\partial y$, which can be interpreted as the impact of an increase in the number of platform users and app users on future profits. Solving (10) yields the optimal advertising strategies for the integrated system.

Proposition 1: In the integrated system, the platform owner's and app developer's optimal advertising efforts are

$$u_{IP}^* = \frac{\alpha p_P}{\rho+\delta} + \frac{\alpha\beta p_A}{(\rho+\delta)^2} \quad (11)$$

$$u_{IA}^* = \frac{\gamma p_A}{\rho+\delta}. \quad (12)$$

All proofs are relegated to the Appendix in the online supplementary material. When confronted with: 1) a high platform advertising effectiveness α and a high platform's marginal sales revenue p_P and 2) a high effectiveness of platform user base on the app's user growth β and a high app's marginal sales revenue p_A, the platform owner tends to invest more in advertising u_{IP}. The former happens because a high effectiveness α and a high marginal revenue p_P increase the system's revenue by enhancing the platform owner's revenue, while the latter occurs since they raise the system's revenue by increasing the app developer's revenue. In this situation, members act as an integrated firm, so an increase in system's revenue induces the platform owner to invest more in advertising. Similarly, the app developer is willing to increase the advertising effort u_{IA} with its high advertising effectiveness γ and marginal sales revenue p_A. Additionally, a high discount rate ρ and a high decay rate δ, resulting in a decrease in the system's revenue, would force members to lower the advertising efforts.

Proposition 2: In the integrated system, the optimal trajectories of the number of platform users and app users are

$$x(t) = (x_0 - x_{SS})e^{-\delta t} + x_{SS} \quad (13)$$

$$y(t) = (y_0 - y_{SS})e^{-\delta t} + y_{SS} \quad (14)$$

where

$$x_{SS} = \frac{\alpha u_{IP}^*}{\delta} = \frac{\alpha^2 p_P}{\delta(\rho+\delta)} + \frac{\alpha^2\beta p_A}{\delta(\rho+\delta)^2}$$

$$y_{SS} = \frac{\beta x_{SS}}{\delta} + \frac{\gamma u_{IA}^*}{\delta} = \frac{\beta}{\delta}\left[\frac{\alpha^2 p_P}{\delta(\rho+\delta)} + \frac{\alpha^2\beta p_A}{\delta(\rho+\delta)^2}\right] + \frac{\gamma^2 p_A}{\delta(\rho+\delta)}.$$

Generally, these optimal trajectories converge to the steady states, i.e., x_{SS} and y_{SS}. It is straightforward to show that the number of app users $y(t)$ increases with the platform's advertising effectiveness α and the app's advertising effectiveness γ, whereas the number of platform users $x(t)$ is only positively related to the platform's advertising effectiveness rather than the app's. This happens because we focus on the situation, where the platform owner places the platform's ads outside the system but displays the app's ads in the platform, and these app ads cannot help attract unaware platform users. We find that the number of platform users is increasing with the effectiveness of the platform user base β, since a higher β induces the platform owner to invest more in advertising, as shown in Proposition 1. By combining (13) and (14), we can specify the optimal trajectory of the number of app users as

$$y(t) = \frac{\beta}{\delta}\left[x(t) - x_0 e^{-\delta t}\right] + y_0 e^{-\delta t} + \frac{p_A\gamma^2\left(1 - e^{-\delta t}\right)}{\delta(\rho+\delta)}.$$

It indicates that the number of platform users $x(t)$ and the effectiveness of the platform's user base β could increase the number of app users $y(t)$. This is consistent with the phenomenon that the size of the platform's user base and its impact on an app's user growth play a significant role in the app's entry decision [11].

B. Decentralized System

When the platform owner and the app developer make their decisions independently, the problems they faced are described by (8) and (9). We consider that the platform is the Stackelberg leader and the app is the follower. The platform owner first declares her advertising effort $u_P(x, y)$ and the app developer's payment rate $\phi(x, y)$. Based on these data, the app developer decides his advertising effort $u_A(x, y)$.

The HJB equations for (8) and (9) are

$$\rho V_P = \max_{u_P \geq 0, \phi \geq 0} \left[p_P x + (1-\lambda) p_A y + \frac{1}{2}\phi u_A^2 - \frac{1}{2}u_P^2 - \frac{1}{2}u_A^2 + V_{Px}(\alpha u_P - \delta x) + V_{Py}(\beta x + \gamma u_A - \delta y) \right] \tag{15}$$

$$\rho V_A = \max_{u_A \geq 0} \left[(1-\lambda) p_A y - \frac{1}{2}\phi u_A^2 + V_{Ax}(\alpha u_P - \delta x) + V_{Ay}(\beta x + \gamma u_A - \delta y) \right] \tag{16}$$

where $V_{Px} = \partial V_P/\partial x$, $V_{Py} = \partial V_P/\partial y$, $V_{Ax} = \partial V_A/\partial x$, and $V_{Ay} = \partial V_A/\partial y$. We solve (15) and (16) and then obtain the stationary Stackelberg equilibrium (ϕ^*, u_P^*, u_A^*) as summarized in Proposition 3.

Proposition 3 (Stationary Stackelberg Equilibrium): In the decentralized system, the platform owner's equilibrium advertising effort is

$$u_P^* = \frac{\alpha[p_P(\rho+\delta) + p_A\beta\lambda]}{(\rho+\delta)^2} = \frac{\alpha p_P}{\rho+\delta} + \frac{\alpha\beta\lambda p_A}{(\rho+\delta)^2} \tag{17}$$

her equilibrium charge on the app developer's payment rate is

$$\phi^* = \frac{2(1-\lambda)}{1+\lambda} \tag{18}$$

and the app developer's equilibrium advertising effort is

$$u_A^* = \frac{\gamma(1+\lambda)p_A}{2(\rho+\delta)}. \tag{19}$$

Proposition 3 demonstrates that the platform prefers to claim a higher payment rate, when facing a lower sharing rate of the app developer's sales revenue λ. Specifically, we have $\phi \geq 1$ if $\lambda \in [0, (1/3)]$ and $0 \leq \phi < 1$ if $\lambda \in ([1/3], 1]$. When the revenue sharing rate is lower than $\hat{\lambda} = (1/3)$, the platform owner will overcharge the app developer, i.e., $\phi > 1$, to compensate for the lower profitability evoked by her low share of the app developer's sales revenue. In contrast, when the revenue sharing rate exceeds $\hat{\lambda}$, we have $\phi < 1$, implying that the platform owner will participate in the app's advertising. This result is consistent with the reality. For example, Apple does not provide its third party apps with the advertising subsidy but only takes 30% of their sales revenue. In contrast, "360 platform" often promotes its third-party apps for free but it claims that the apps must pay it 50% of their sales revenue.[4] Moreover, the equilibrium payment rate here is independent of the members' marginal sales revenues, p_P and p_A. This result differs from the one reported in the cooperative advertising literature (e.g., [34], [40], and [56]), where the participation rate depends on the ratio of the profit margins of channel members.

Similar to the integrated system, with a high effectiveness of platform user base β and/or a high app marginal sales revenue p_A, the platform owner is also willing to increase her advertising effort u_P, but the reason is different. In this system, her objective is to maximize the platform revenue rather than the system revenue. The high effectiveness of platform user base β and/or the high app marginal revenue p_A here would increase the platform's revenue via the revenue sharing policy. The platform owner can utilize this to invest more in advertising. With the same reason, her advertising effort u_P increases as the revenue sharing λ increases.

A surprising result is that the app developer's equilibrium advertising effort u_A is monotonically increasing in the platform owner's sharing rate λ. In other words, the more the platform owner shares the app developer's sales revenue, the more advertising the app developer invests. This is counterintuitive, since a firm is more likely to invest less in advertising when its marginal revenue is lower. The reason behind is that in the considered system the platform owner tends to charge the app developer a low payment rate ϕ when she takes a high share of the app developer's sales revenue, which eventually stimulates the app developer to invest more in advertising. Moreover, compared with the centralized system, we find that advertising efforts are lower when members behave as a decentralized system.

Proposition 4: In the decentralized system, the equilibrium numbers of platform users and app users evolve over time as

$$x(t) = (x_0 - x_{SS})e^{-\delta t} + x_{SS} \tag{20}$$

$$y(t) = (y_0 - y_{SS})e^{-\delta t} + y_{SS} \tag{21}$$

where

$$x_{SS} = \frac{\alpha u_P^*}{\delta} = \frac{\alpha^2 p_P}{\delta(\rho+\delta)} + \frac{\alpha^2\beta\lambda p_A}{\delta(\rho+\delta)^2}$$

$$y_{SS} = \frac{\beta x_{SS} + \gamma u_A^*}{\delta} = \frac{\beta}{\delta}\left[\frac{\alpha^2 p_P}{\delta(\rho+\delta)} + \frac{\alpha^2\beta\lambda p_A}{\delta(\rho+\delta)^2}\right] + \frac{\gamma^2(1+\lambda)p_A}{2\delta(\rho+\delta)}.$$

Different from the integrated system, the number of system users here depends not only on advertising effectiveness, α and γ, but also on the revenue sharing rate λ. This comes from the decentralized system's feature in which the members independently maximize their own profits. Additionally, compared with the integrated system, the decentralized system possesses fewer users because of the lower advertising investments.

Inserting these solutions into system value functions, we find that the current value of the decentralized system is lower than that of the integrated system. This implies that there exists a system inefficiency in the decentralized case.

C. Bilateral Participation Contract

In this section, we utilize a bilateral participation contract to coordinate the system. In this contract, the platform owner not only shares a part of the app developer's advertising expenditure, but also asks the app developer to participate in the platform's advertising. Denote the platform owner's and the app developer's participation rates as ψ and φ, respectively. Then the bilateral participation contract can be designated as (ψ, φ). Note that with the app's payment rate ϕ, $0 \leq \phi < 1$, the platform owner participates in the app's advertising, so $1-\phi$ is just the platform owner's participation rate ψ. For any given participation contract (ψ, φ), the optimization problems of the platform owner and the app developer are

$$V_{BP}(x, y) = \max_{u_P(t) \geq 0} \int_0^\infty e^{-\rho t}\left[p_P x(t) + \lambda p_A y(t) - \frac{1}{2}(1-\varphi)u_P^2(t) - \frac{1}{2}\psi u_A^2(t)\right]dt \tag{22}$$

[4] *Source:* http://it.sohu.com/20140305/n396098722.shtml. Accessed September 22, 2015.

and

$$V_{BA}(x, y) = \max_{u_A(t)\geq 0} \int_0^{\infty} e^{-\rho t}\bigg[(1-\lambda)p_A y(t) - \frac{1}{2}(1-\psi)u_A^2(t) - \frac{1}{2}\varphi u_P^2(t)\bigg]dt \quad (23)$$

subject to (2) and (3), respectively. We next verify that there exists a contract (ψ, φ), which can coordinate the dynamic system, in Proposition 5.

Proposition 5: The bilateral participation contract (ψ, φ) can coordinate the system when

$$\psi = \lambda \quad (24)$$

$$\varphi = \frac{(1-\lambda)\beta p_A}{(\rho+\delta)p_P + \beta p_A}. \quad (25)$$

Proposition 5 illustrates that there is a unique bilateral participation contract that can coordinate the system. Specifically, in the coordinating conditions, the platform owner's participation rate ψ equals her sharing rate on the app developer's sales revenue λ. This means that the platform owner obtains a proportion of the app developer's sales revenue but meanwhile undertakes the same proportion of the app developer's advertising expenditure. The similar result has also been obtained in [34], where He *et al.* considered the pricing and cooperative advertising problem under a supply chain consisting of a single manufacturer and a single retailer. Additionally, from (25) we find that φ decreases in λ, which implies that the more the platform owner shares the app developer's sales revenue, the less willing the app developer is to share the platform owner's advertising cost. Note that here $\psi+\varphi < 1$. This is contrary to the result in [69] and [70], which revealed that in the bilateral participation contract, the sum of the manufacturer's and the retailer's participation rates equals one. This difference stems from the fact that we consider both the bilateral participation and the revenue sharing. The revenue sharing policy makes the participation rates in terms of advertising cost lower, because revenue sharing can also alleviate the double marginalization effect [13], [34].

For the comparison between members' current values with and without the bilateral participation contract, we conduct a numerical analysis. The result is shown in Fig. 1. We find that the contract can increase the system efficiency, the ratio of the decentralized system's current value to the centralized system's, by more than 12.91%. Moreover, the two players significantly benefit from the coordinating contract and the benefit slightly increases with a higher effectiveness of the platform's advertising α.

IV. Extension: Two Apps Case

In this section, we extend our model to the scenario in which two apps are involved in the platform. For convenience, we denote the two apps as Apps 1 and 2. Similar to those in the previous section, denote the platform owner's advertising effort by $u_P(t)$, and App i developer's advertising effort by $u_{Ai}(t)$, $i = 1, 2$. The platform's user growth is then

$$\dot{x}(t) = \alpha u_P(t) - \delta x(t), x(0) = x_0 > 0 \quad (26)$$

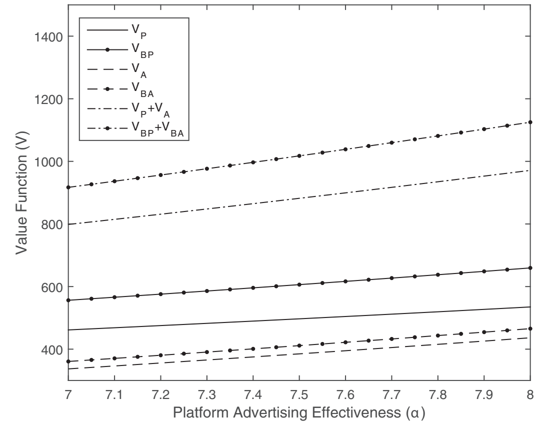

Fig. 1. Value function comparison between cases with and without the bilateral participation contract.
Notes: Fig. 1. is generated based on $p_P = 3$, $p_A = 7$, $\beta = 0.34$, $\gamma = 1$, $\rho = 0.7$, $\delta = 0.7$, $\lambda = 0.2$, $x_0 = 100$, and $y_0 = 1$.

and the App i's user growth is assumed to be

$$\dot{y}_i(t) = \beta_i x(t) + \gamma_i u_{Ai}(t) - \eta_i u_{Aj}(t) - \delta y_i(t)$$
$$y_i(0) = y_{i0} > 0, \ i, \ j \in \{1, 2\} \text{ and } i \neq j. \quad (27)$$

In (26) and (27), the parameters α, β_i, γ_i, and δ have the same implications as those in the benchmark model. An exception is that we add a new term $-\eta_i u_{Aj}(t)$ in (27). This new term could capture the impact of the apps' advertising under two situations. The first situation is the competition between the two apps in attracting users. The second situation is that despite there is no competition between the two apps, the platform may deliver one app's advertisements in the other app. These advertisements in an app, called in-app advertisements, may cause user annoyance since they usually distract users' attention [30]. Under such a condition, this term implies that this kind of ads may negatively affect the user growth. This assumption is commonly used in previous studies, such as [43]. It is worthy to note that no matter whether the apps' ads are displayed by the platform or other apps, they cannot reach the audiences who are not connected to the platform, that is, the platform's unaware users. Thus, these ads cannot increase the platform's user growth. Similar to the benchmark, we ignore their negative impact on the platform.

As for the advertising cost and the advertising process, we use the following assumptions. First, similar to Section III, the advertising cost is quadratic to the advertising effort, i.e., $C_P(t) = (1/2)u_P^2(t)$ and $C_{Ai}(t) = (1/2)u_{Ai}^2(t)$. Second, the platform's advertisements are delivered outside the platform. But an app's ads could be displayed either in the platform or in the other app. For the former case, the platform owner undertakes App i developer's advertising cost $C_{Ai}(t)$ and then charges him $\phi_i(t)C_{Ai}(t)$. Here, $\phi_i(t) \geq 0$ has the same implication as that in Section III. As for the latter case, in practice many apps utilize the platform's advertising tools to select audience. Taking Apple as an example, it provides the iAd Workbench to all apps, allowing them to present their ads in other apps. In return, the app, which presents these ads to its users, can obtain a part of the advertising revenue. Following such a practice, when the platform delivers the ads of App i

in App j, the platform owner initially undertakes the app's advertising cost $C_{Ai}(t)$, charges App i developer an amount of $\phi_i(t)C_{Ai}(t)$ and then pays $\theta_j\phi_i(t)C_{Ai}(t)$ to App j developer. Here, $0 \leq \theta_j < 1$ is an exogenous parameter indicating the platform owner's advertising payment rate for App j developer.

When the sales revenue sharing policy is the same as that in Section III, the instantaneous profit of the three members under the above two conditions can be unified by (28) and (29), that is

$$\pi_P(t) = p_P x(t) - \frac{1}{2}u_P^2(t) + \sum_{i=1}^{2}\Bigg[\lambda_i p_{Ai} y_i(t) + \frac{1}{2}(1-\theta_{-i})\phi_i(t)u_{Ai}^2(t) - \frac{1}{2}u_{Ai}^2(t)\Bigg] \tag{28}$$

and

$$\pi_{Ai}(t) = (1-\lambda_i)p_{Ai}y_i(t) + \frac{1}{2}\theta_i\phi_j(t)u_{Aj}^2(t) - \frac{1}{2}\phi_i(t)u_{Ai}^2(t),\ i,j \in \{1,2\} \text{ and } i \neq j \tag{29}$$

where p_{Ai} is App i developer's marginal sales revenue, and $0 \leq \lambda_i \leq 1$, $i \in \{1, 2\}$, is the platform owner's sharing rate of the App i developer's sales revenue.

It is worthy to note that when we set $\theta_1 = \theta_2 = 0$, the above functions capture the first situation, where the platform displays apps' ads, and when we let $0 < \theta_i \leq 1$, they characterize the second situation, where the apps' ads are delivered in other apps.

Then, the problem faced by the platform owner and App i developer are

$$V_P(x,y) = \max_{u_P(t)\geq 0, \phi_i(t)\geq 0} \int_0^{\infty} e^{-\rho t} \times \Bigg[p_P x(t) - \frac{1}{2}u_P^2(t) + \sum_{i=1}^{2}\Bigg[\lambda_i p_{Ai} y_i(t) - \frac{1}{2}u_{Ai}^2(t) + \frac{1}{2}(1-\theta_{-i})\phi_i(t)u_{Ai}^2(t)\Bigg]\Bigg]dt \tag{30}$$

and

$$V_{Ai}(x,y) = \max_{u_{Ai}(t)\geq 0} \int_0^{\infty} e^{-\rho t}\Bigg[(1-\lambda_i)p_{Ai}y_i(t) + \frac{1}{2}\theta_i\phi_j(t)u_{Aj}^2(t) - \frac{1}{2}\phi_i(t)u_{Ai}^2(t)\Bigg]dt \quad i,\ j \in \{1,2\} \text{ and } i \neq j \tag{31}$$

subject to (26) and (27). To ensure that the app users are no more in number than the platform users, we restrict our discussion with the following conditions, i.e., $2(\beta_1 + \beta_2) < \delta$ and $\alpha > \sqrt{[\delta(\rho+\delta)(\gamma_i^2 + \eta_i^2)]/[(\beta_1+\beta_2)\beta_i\lambda_i]}$, $i \in \{1, 2\}$.

A. Decentralized System

Suppose that the decisions of the three members follow a Stackelberg–Nash game. As the Stackelberg game leader, the platform owner first announces her advertising policies, $u_P(x, y)$, $\phi_1(x, y)$, and $\phi_2(x, y)$. Then, the two apps, as the followers, determine their advertising efforts $u_{A1}(x, y)$ and $u_{A2}(x, y)$ independently and simultaneously. Utilizing the differential game theory, we have Proposition 6.

Proposition 6: When $\eta_j < [(p_{Ai}\gamma_i[1 + \lambda_i - \theta_j(1 - \lambda_i)])/2\lambda_j p_{Aj}]$ holds, under the Stackelberg–Nash game, the platform owner's equilibrium advertising effort is

$$u_P^* = \frac{\alpha p_P}{\rho+\delta} + \frac{\alpha\sum_{i=1}^{2}\beta_i\lambda_i p_{Ai}}{(\rho+\delta)^2} \tag{32}$$

her equilibrium charge on App i developer's advertising payment rate is

$$\phi_i^* = \frac{2(1-\lambda_i)}{(1+\lambda_i) - \theta_j(1-\lambda_i) - \dfrac{2\lambda_j p_{Aj}\eta_j}{p_{Ai}\gamma_i}} \quad i,\ j \in \{1,2\} \text{ and } i \neq j \tag{33}$$

and App i developer's equilibrium advertising effort is

$$u_{Ai}^* = \frac{p_{Ai}\gamma_i(1+\lambda_i)}{2(\rho+\delta)} - \frac{p_{Ai}\gamma_i\theta_j(1-\lambda_i)}{2(\rho+\delta)} - \frac{\lambda_j p_{Aj}\eta_j}{\rho+\delta} \quad i,\ j \in \{1,2\} \text{ and } i \neq j. \tag{34}$$

The condition $\eta_j < [(p_{Ai}\gamma_i[1 + \lambda_i - \theta_j(1 - \lambda_i)])/2\lambda_j p_{Aj}]$ is to guarantee that the advertising efforts of the two apps are positive. Proposition 6 demonstrates the following managerial insights.

First, comparing (32) with (17), we find that the platform owner's equilibrium advertising effort has the same structure as that in the single app scenario. This structure implies that the platform owner will invest more in advertising when more apps join the platform.

Second, (33) illustrates that the platform owner's charge on app developers' advertising expenditure is related to her share of both apps' sales revenue. Furthermore, the more she shares with App j developer, the higher she will charge App i developer. This happens because when faced by a higher portion of App j developer's sales revenue λ_j, the platform owner will decrease her charge on App j's advertising ϕ_j. To offset the lower profitability caused by a smaller ϕ_j, she tends to charge App i developer more, i.e., a higher ϕ_i.

Third, when multiple apps join the platform, their payment rates are also affected by θ and η. The parameter θ reflects the platform's cost when it delivers app ads in other apps, and the parameter η characterizes the negative effect of apps' advertising on other apps. Specifically, when $\theta_j = 0$ and $\eta_j = 0$, (33) becomes the same as (18). Differentiating (33) with respect to θ_j and η_j, we have $(\partial\phi_i^*/\partial\theta_j) > 0$ and $(\partial\phi_i^*/\partial\eta_j) > 0$. The former implies that the platform owner will charge an app developer a higher advertising fee if she has to transfer a higher portion of this fee to the other app developer. The latter means that the platform owner tends to increase the charge on App i developer's payment rate ϕ_i when App i's advertising significantly reduces the other app's user growth, which is a way to decrease the negative impact of app competition or in-app ads. As expected, from (34) we find that when faced with high θ_j and η_j, App i developer prefers to decrease his advertising efforts because of the platform owner's higher charge ϕ_i. But this charge ϕ_i is independent of App j's negative advertising effectiveness η_i. This happens because the platform owner sets App i developer's payment rate only according to the negative effectiveness of App i rather than that of both apps. This suggests app developers not to take into account other apps' negative impact, such as the negative influence of competition or in-app ads, when making advertising decisions, because the platform owner will help restrict these adverse impacts.

Fourth, we have $0 < \phi_i < 1$ when $[(1+\theta_j)/(3+\theta_j)] < \lambda_i < 1$ and $\eta_j < [(p_{Ai}\gamma_i[3\lambda_i - 1 + \theta_j(\lambda_i - 1)])/2\lambda_j p_{Aj}]$ hold simultaneously, and $\phi_i \geq 1$ otherwise. This implies that the platform owner will subsidize an app's advertising only when she takes a large portion from the app's sales revenue and the negative impact of this app's advertising is low; otherwise, she should charge this app much to get advertising revenue.

Proposition 7: When each app displays the other app's ads, the platform's and App i's equilibrium user trajectories are

$$x(t) = (x_0 - x_{SS})e^{-\delta t} + x_{SS} \tag{35}$$

$$y_i(t) = (y_0 - y_{SSi})e^{-\delta t} + y_{SSi},\; i, j \in \{1, 2\} \text{ and } i \neq j \tag{36}$$

where

$$\begin{aligned} x_{SS} &= \frac{\alpha u_P^*}{\delta} = \frac{\alpha^2 p_P}{\delta(\rho+\delta)} + \frac{\alpha^2 \sum_{i=1}^{2} p_{Ai}\beta_i\lambda_i}{\delta(\rho+\delta)^2} \\ y_{SSi} &= \frac{1}{\delta}\left(\beta_i x_{SS} + \gamma_i u_{Ai}^* - \eta_i u_{Aj}^*\right) \\ &= \frac{1}{\delta(\rho+\delta)}\Bigg\{\frac{p_P\alpha^2\beta_i}{\rho+\delta} + p_{Ai}\Bigg[\frac{\alpha^2\beta_i\lambda_i}{\rho+\delta} + \frac{\gamma_i^2}{2} \\ &\qquad \times \left(1+\lambda_i - \theta_j(1-\lambda_i)\right) + \eta_i^2\lambda_i\Bigg] \\ &\qquad + p_{Aj}\Bigg[\frac{\alpha^2\beta_i\beta_j\lambda_i}{\rho+\delta} - \frac{\gamma_j\eta_i}{2}\left(1+\lambda_j - \theta_i\left(1-\lambda_j\right)\right) \\ &\qquad - \gamma_i\eta_j\lambda_j\Bigg]\Bigg\}. \end{aligned}$$

Proposition 7 demonstrates that the impact of more apps' entry is twofold. First, it will cause competition or stimulate the emergence of in-app ads. These can result in system loss in terms of the number of app users, since the number of app users $y_i(t)$ and $y_j(t)$ decrease with both apps' negative effectiveness η_i and η_j. Second, multiple apps' entry can enlarge the platform's user base via encouraging the platform owner to increase her advertising investment. This suggests that the platform owner should attract more apps, but decreasing the negative influence of app competition and in-app ads is also necessary. This finding is supported by some empirical observations. For example, Apple refuses to grant those apps with some competing functions access to its platform.[5] To lower the app user loss because of in-app ads, not only does Apple provide an excellent advertising service, iAd, to help design high-quality app ads, but also spends little on promoting its iAd and charges advertisers much to keep the number of in-app ads low.[6] Furthermore, when the platform's effect is sufficiently high, i.e., $\alpha^2\beta_i\beta_j > [(\rho+\delta)/\lambda_i][(\gamma_j\eta_i/2)(1+\lambda_j - \theta_i(1-\lambda_j)) + \gamma_i\eta_j\lambda_j]$, the number of App i's users increases as App j's marginal revenue p_{Aj}. This implies that when the platform's influence of its advertising and/or user base is so high that it can offset the negative impact of app competition or in-app ads, apps will absolutely benefit from other apps' entry, such as obtaining advertising revenue.

[5] *Source:* https://developer.apple.com/app-store/review/guidelines/. Accessed July 3, 2016.

[6] *Source:* http://www.pingwest.com/iad-failure/. Accessed January 18, 2016.

Thus, it is of significance for app developers to choose a platform with proficient advertising skills and a great amount of their target users [11].

B. Bilateral Participation Contract

Similar to the benchmark scenario, we utilize the bilateral participation contract to restore the system efficiency. In this contract, each app developer participates in the platform's advertising and the platform owner participates in both apps' advertising. For any given contract (ψ_i, φ_i), $i \in \{1, 2\}$, the optimization problem of the platform owner and that of App i developer are

$$\begin{aligned} V_{BP}(x, y) = \max_{u_P(t)\geq 0} \int_0^\infty e^{-\rho t}\Bigg[p_P x(t) - \frac{(1-\varphi_1-\varphi_2)}{2}u_P^2(t) \\ + \sum_{i=1}^{2}\Bigg[\lambda_i p_{Ai} y_i(t) - \frac{u_{Ai}^2(t)}{2} \\ + \frac{(1-\theta_{-i})(1-\psi_i)u_{Ai}^2(t)}{2}\Bigg]\Bigg]dt \end{aligned} \tag{37}$$

and

$$\begin{aligned} V_{BAi}(x, y) = \max_{u_{Ai}(t)\geq 0} \int_0^\infty e^{-\rho t}\Bigg[(1-\lambda_i)p_{Ai}y_i(t) \\ - \frac{1}{2}(1-\psi_i)u_{Ai}^2(t) - \frac{1}{2}\varphi_i u_P^2(t) \\ + \frac{1}{2}\theta_i(1-\psi_j)u_{Aj}^2(t)\Bigg]dt \\ i, j \in \{1, 2\} \text{ and } i \neq j \end{aligned} \tag{38}$$

subject to (26) and (27), respectively.

Proposition 8: Given the condition $(\eta_2/\lambda_1\gamma_1) < (p_{A1}/p_{A2}) < (\lambda_2\gamma_2/\eta_1)$ holds, the bilateral participation contract (ψ_i, φ_i), $i \in \{1, 2\}$, can coordinate the system when

$$\psi_i = \frac{\lambda_i p_{Ai}\gamma_i - p_{Aj}\eta_j}{p_{Ai}\gamma_i - p_{Aj}\eta_j},\; i, j \in \{1, 2\} \text{ and } i \neq j \tag{39}$$

$$\varphi_1 + \varphi_2 = \frac{(1-\lambda_1)\beta_1 p_{A1} + (1-\lambda_2)\beta_2 p_{A2}}{p_P(\rho+\delta) + p_{A1}\beta_1 + p_{A2}\beta_2}. \tag{40}$$

Proposition 8 points out that in the setting with two apps, where there exists app competition and/or in-app ads, the bilateral participation contract can also coordinate the system. However, when compared with the coordinating conditions in the benchmark, the conditions here reveal different findings.

First, while (24) shows that in the scenario with single app, the platform owner's participation rate on the app's advertising should be equivalent to her sharing rate of the app's sales revenue, (39) illustrates that when facing multiple apps, the platform owner should consider the negative impact of app competition or in-app ads and thus decrease her participation rate. When the negative impact is negligible, i.e., $\eta_j = 0$, the participation rate given by (39) is the same as that in (24).

Second, since $0 < \lambda_i < 1$, $i \in \{1, 2\}$, it is obvious that $0 < \varphi_1 + \varphi_2 < 1$. Thus, (40) suggests that the platform owner will set her advertising effort equal to the optimal level in the integrated system if and only if the summation of two

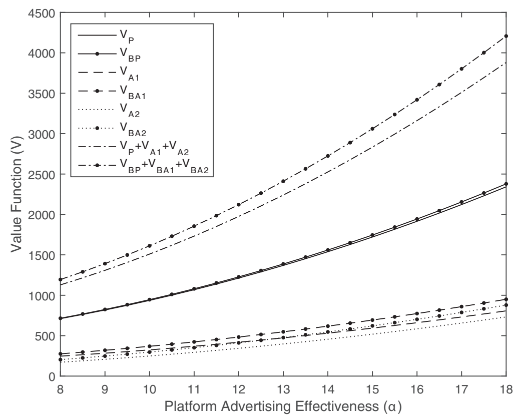

Fig. 2. Value function comparison between cases with and without the bilateral participation contract.
Notes: Fig. 2. is generated based on $p_P = 3$, $p_{A1} = 4.8$, $p_{A2} = 6$, $\beta_1 = 0.18$, $\beta_2 = 0.16$, $\gamma_1 = 2$, $\gamma_2 = 1.9$, $\eta_1 = 0.9$, $\eta_2 = 1$, $\rho = 0.7$, $\delta = 0.7$, $\theta_1 = 0.7$, $\theta_2 = 0.7$, $\lambda_1 = 0.5$, $\lambda_2 = 0.4$, $x_0 = 100$, $y_{01} = 40$, and $y_{02} = 10$.

apps' participation rates is the same as the right hand of (40). This evokes the free-riding effect between app developers on participating in the platform's advertising. An app developer's high participation rate will lead to the other app developer's low one. Furthermore, we find that (40) has the same structure as (25). This structure implies that our model can be extended to the case of multiple apps and the main conclusions will be robust.

In Fig. 2, we report a numerical study to evaluate the performance of the bilateral participation contract in the two apps' case. We characterize App i's participation rate as $([(1-\lambda_i)\beta_i p_{Ai}]/[p_P(\rho+\delta)+p_{A1}\beta_1+p_{A2}\beta_2])$. The contract can increase the system efficiency by more than 5.5% and the improvement becomes larger as the platform advertising effectiveness α increases. With respect to system members, we find that the platform owner benefits little with low α but app developers always benefit from the coordinating contract. As α becomes high, their benefits slightly rise. Therefore, if both app developers undertake a reasonable share of the platform's advertising cost, the contract is feasible, particularly when the platform advertising effectiveness is high. The above findings provide a novel insight that for mobile platform members the bilateral participation in advertising expenditure is another way to improve profitability, and choosing satisfying participation rates is of importance to achieve the improvement.

V. Conclusion

Advertising plays a momentous role for a mobile platform which serves as a two-sided market connecting users and app developers. Not only does it attract users but also generates advertising revenue. Besides, the platform owner's advertising policy interacts with app developers' advertising policy. Thus, it is difficult for system members to make advertising decisions. To propose valuable suggestions, this paper explores advertising strategies for this system and develops a game-theoretical model that characterizes members' roles as sellers, ad publishers, and advertisers, which are only separately captured in the previous literature. Under this new framework, we analytically generate some relevant insights that had not been previously clarified.

Convention wisdom suggests that ad publishers should charge advertisers aggressively. Our analysis indicates that when the ad publishers have another income source, which is dependent of advertisers' revenue, overcharging advertisers is not always advisable. In the considered system, we suggest that the platform owner who presents app ads to platform users participate in apps' advertising when she obtains a high share of app developers' sales revenue; otherwise, she should overcharge them for displaying apps' ads. Looking at the insights from a different perspective, our results imply that the platform owner's high share is not evil for app developers, since in this condition the platform owner tends to provide them with advertising subsidies, which will offset apps' low marginal sales revenue caused by the high share. Then app developers could use the subsidy to increase the advertising efforts. This finding is contrary to common wisdom that suggests a firm with a low marginal revenue reduce the advertising investment. The implication here is that the marginal revenue and incentive policies are equally significant in a decision-making process.

Empirical observations reveal that it is popular for the platform owner to attract more apps' entry, since it can generate much transaction revenue and advertising revenue. However, we find that multiple entries will result in app user loss because of the aggressive competition among apps and increasing user annoyance caused by in-app ads. In reality, several defensive strategies have been developed to alleviate the negative impact, such as granting those apps whose functions are less similar to the existing functions in the platform the access right and keeping the number of in-app ads low. Moreover, we demonstrate that to mitigate the negative influence it is efficient for apps to enter a platform with proficient advertising skills and many target users, but counterintuitively app developers need not consider the negative impact in the process of deciding advertising investments because the platform owner will lower this impact via adjusting the charge for displaying apps' advertising.

Our further analysis indicates that the coordinating bilateral participation in advertising is a new mechanism to enhance the profitability. Past research has shown that the coordinating conditions should clarify every participation rate (e.g., [69] and [70]). However, we find that when this contract involves more than two members, such as our studied system with a platform and several apps, it is adequate to specify the summation of participation rates instead of specifying every one, but this specification will give rise to free-riding among members on participating in others' advertising. Thus, to guarantee the enhancement of the profits, it is crucial to find the satisfying participation rates. Unexpectedly, we find that when accompanied with the revenue sharing policy, the relatively low participation rates could remedy system inefficiency by aligning the incentives of independent system members, because revenue sharing could also mitigate the inefficiency.

To ensure that investigations are tractable and are with potentials to derive valuable insights, we characterize a basic case which is, however, limited in several respects. For example, our model, which ignores the impact of apps' ads on the platform, only considers that the advertising revenue is from app developers, but does not capture the advertising revenue from firms which are not involved in the platform. Relaxing any of these assumptions could possibly lead to some interesting findings. Moreover, to focus on advertising strategies, we isolate other possible issues which could be considered in the future. This paper does not explore the decision on pricing and revenue sharing by assuming that the selling price, that is, the marginal sales revenue, and the sales revenue sharing rate are exogenous. In real market, to improve profitability, it is a significant way to dynamically adjust pricing and revenue sharing policy according to the user base. Hence, it would be desirable to endogenize these decisions in the future study. Another direction for the further research could be to investigate the competition among platforms. Many studies have shown that users choose a platform to enjoy its functions. Thus, to attract more users and prevent user migration, a particular question would be whether platforms should become more similar or different in terms of functions. In the presence of platform competition, app developers also should make tradeoffs between joining one platform versus several platforms. Further studies deserve to be conducted on these questions.

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