

1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.

Given that seasons are equally likely and all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays

Total outcomes: Each person is allotted a season out of 4. Hence 48 possibilities

Number of Outcomes that one or more season has no student having their birthday:

Using Inclusion and Exclusion,  $4C1 \cdot 3^6 - 4C2 \cdot 2^6 + 4C3 \cdot 1^6$  [i.e. Exclude 1 season - Exclude 2 season + exclude 3 season, also note that we can't exclude all the 4 seasons]

Probability that one or more season has no student having their birthday:

$$(4C1 \cdot 3^6 - 4C2 \cdot 2^6 + 4C3 \cdot 1^6) / 48 = 0.377$$

Required probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays:

$$1 - 0.377 = 0.623$$

$$= 4P(A1) - 6P(A1 \cap A2) + 4P(A1 \cap A2 \cap A3). \text{ We have } P(A1) = (3/4)^7$$

2. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?

There are two general ways that Alice can have class every day: either she has 2 days with 2 classes and 3 days with 1 class,

or she has 1 day with 3 classes, and has 1 class on each of the other 4 days. The number of possibilities for the former is:

$$\binom{52}{2} \binom{62}{2} \binom{26}{3} \binom{52}{1} \binom{62}{1} \binom{26}{1} \binom{62}{1} \text{ (choose the 2 days when she has 2 classes, and then select 2 classes on those days and 1 class for the other days).}$$

The number of possibilities for the latter is:

$$(51)(63)64(51)(63)64$$

So the probability is:  $(52)(62)263+(51)(63)64(307)(52)(62)263+(51)(63)64(307)$  which is close to 30.2%.