lab_05_solution

September 16, 2021

1 Lab 5

Optimization Using Newton's Method

In this lab, we'll explore an alternative to gradient descent for nonlinear optimization problems: Newton's method.

1.1 Newton's method in one dimension

Consider the problem of finding the *roots* $\mathbf{x}$ of a nonlinear function $f : \mathbb{R}^N \rightarrow \mathbb{R}$. A root of f is a point $\mathbf{x}$ that satisfies $f(\mathbf{x}) = 0$.

In one dimension, Newton's method for finding zeroes works as follows:

1. Pick an initial guess x_0
2. Let $x_{i+1} = x_i + \frac{f(x_i)}{f'(x_i)}$
3. If not converged, go to #2.

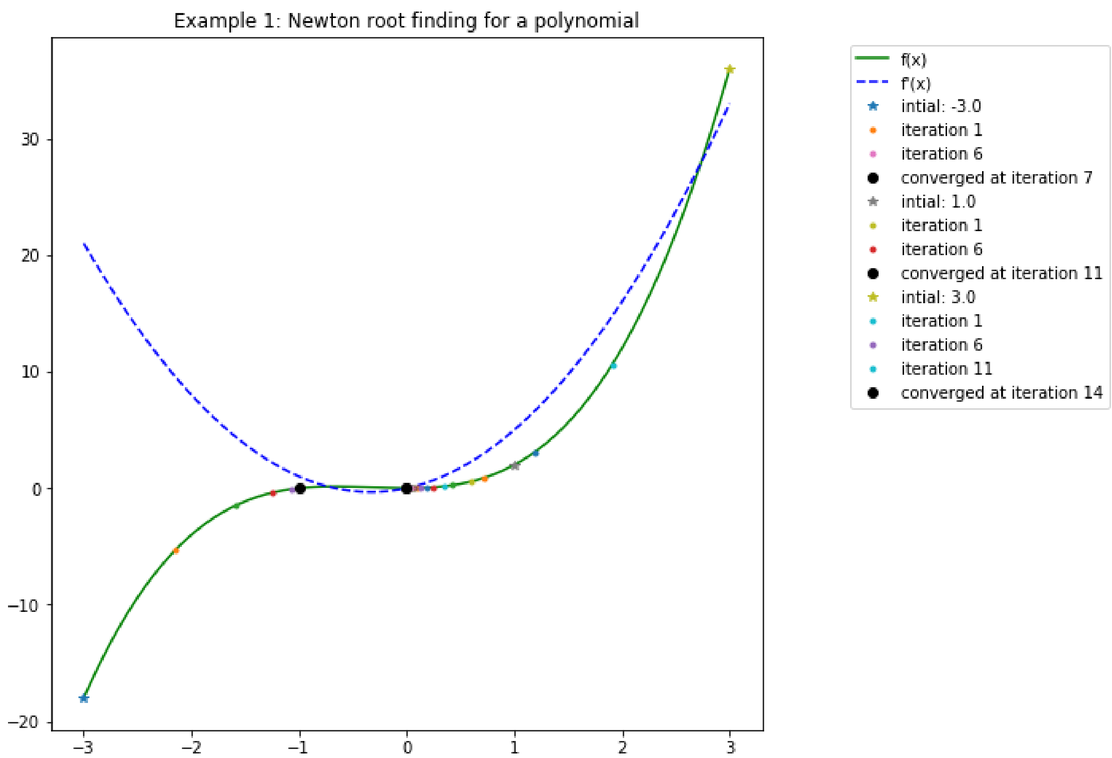
Convergence occurs when $|f(x_i)| < \epsilon_1$ or when $|f(x_{i+1}) - f(x_i)| < \epsilon_2$.

Let's see how this works in practice.

[17]:
```
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
import pandas as pd
```

[18]:
```
# Example 1: Root finding for cubic polynomial

n = 200

x = np.linspace(-3, 3, n)

def fx(x, p):
    f_x = np.polyval(p, x)
    return f_x

# Create the polynomial f(x) = x^3 + x^2

p = np.poly1d([1, 1, 0, 0])
```

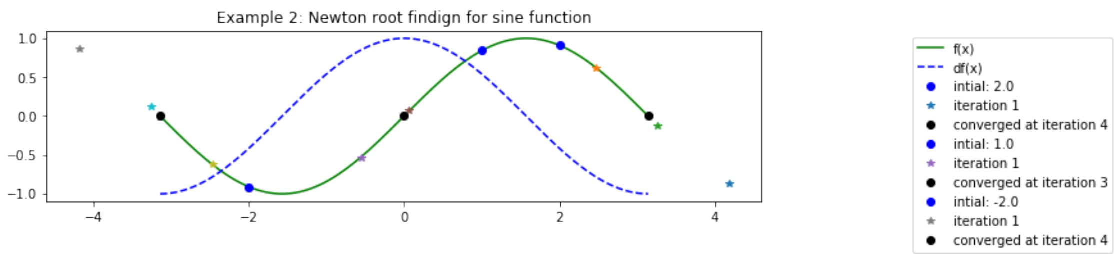
```python
# Derivative of a polynomial
# This is a convenient method to obtain p_d = np.poly1d([3, 2, 0])

p_d = np.polyder(p)

# Get values for f(x) and f'(x) for graphing purposes

y = fx(x, p)
y_d = fx(x,p_d)

# Try three possible guesses for x0

x0_arr = [-3.0, 1.0, 3.0]
max_iter = 30
threshold = 0.001
roots = []

fig1 = plt.figure(figsize=(8,8))
ax = plt.axes()
plt.plot(x, y, 'g-', label='f(x)')
plt.plot(x, y_d, 'b--', label="f\'(x)")

for x0 in x0_arr:
    # Plot initial data point
    plt.plot(x0, fx(x0,p), '*', label='intial: ' + str(x0))
    i = 0
    while i < max_iter:
        # x1 = x0 - f(x0)/f'(x0)
        x1 = x0 - fx(x0, p) / fx(x0, p_d)
        # Check for delta (x) less than threshold
        if np.abs(x0 - x1) <= threshold:
            roots.append(round(x1,4))
            break;
        # Plot current root after every 5 iterations
        if i % 5 == 0:
            plt.plot(x1, fx(x1, p), '.', label='iteration '+ str(i+1))
        else:
            plt.plot(x1, fx(x1, p), '.')
        x0 = x1
        i = i + 1
    plt.plot(x1, fx(x1, p), 'ko', label='converged at iteration '+ str(i+1))

plt.legend(bbox_to_anchor=(1.5, 1.0), loc ='upper right')
plt.title('Example 1: Newton root finding for a polynomial')

plt.show()
```



[19]:

```
# Example 2: Root finding for sine function

n = 200

x = np.linspace(-np.pi, np.pi, n)

def fx(x):
    f_x = np.sin(x)
    return f_x

def fx_d(x):
    return np.cos(x)

# Get f(x) and f'(x) for plotting

y = fx(x)
y_d = fx_d(x)

# Consider three possible starting points

x0_arr = [2.0, 1.0, -2.0]
max_iter = 30
i = 0
```

```
threshold = 0.01
roots = []

fig1 = plt.figure(figsize=(10,10))
ax = plt.axes()
ax.set_aspect(aspect = 'equal', adjustable = 'box')
plt.plot(x, y, 'g-', label='f(x)')
plt.plot(x, y_d, 'b--', label='df(x)')

for x0 in x0_arr:
    plt.plot(x0, fx(x0), 'bo', label='intial: ' + str(x0))
    i = 0;
    while i < max_iter:
        x1 = x0 - fx(x0) / fx_d(x0)
        if np.abs(x0 - x1) <= threshold:
            roots.append(x1)
            plt.plot(x1,fx(x1),'ko',label='converged at iteration '+ str(i))
            break;
        if i % 5 == 0:
            plt.plot(x1, fx(x1), '*', label='iteration '+ str(i+1))
        else:
            plt.plot(x1, fx(x1), '*')
        x0 = x1
        i = i + 1

plt.legend(bbox_to_anchor=(1.5, 1.0), loc ='upper right')
plt.title('Example 2: Newton root findign for sine function')

plt.show()

print('Roots: %f, %f, %f' % (roots[0], roots[1], roots[2]))
```




```
Roots: 3.141593, 0.000000, -3.141593
```

1.2 Newton's method for optimization

Now, consider the problem of minimizing a scalar function $J : \mathbb{R}^n \mapsto \mathbb{R}$. We would like to find

$$\theta^* = \text{argmin}_\theta J(\theta)$$

We already know gradient descent:

$$\theta^{(i+1)} \leftarrow \theta^{(i)} - \alpha \nabla_J(\theta^{(i)}).$$

But Newton's method gives us a potentially faster way to find θ^* as a zero of the system of equations

$$\nabla_J(\theta^*) = \mathbf{0}.$$

In one dimension, to find the zero of $f'(x)$, obviously, we would apply Newton's method to $f'(x)$, obtaining the iteration

$$x_{i+1} = x_i - f'(x_i)/f''(x_i).$$

The multivariate extension of Newton's optimization method is

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathtt{H}_f(\mathbf{x}_i)\nabla_f(\mathbf{x}_i),$$

where $\mathtt{H}_f(\mathbf{x})$ is the *Hessian* of f evaluated at $\mathbf{x}$:

$$\mathtt{H}_f(\mathbf{x}) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 x_n} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n x_1} & \frac{\partial^2 f}{\partial x_n x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

This means, for the minimization of $J(\theta)$, we would obtain the update rule

$$\theta^{(i+1)} \leftarrow \theta^{(i)} - \mathtt{H}_J(\theta^{(i)})\nabla_J(\theta^{(i)}).$$

1.3 Application to logistic regression

Let's create some difficult sample data as follows:

Class 1: Two features x_1 and x_2 jointly distributed as a two-dimensional spherical Gaussian with parameters

$$\mu = \begin{bmatrix} x_{1c} \\ x_{2c} \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}.$$

Class 2: Two features x_1 and x_2 in which the data are generated by first sampling an angle θ according to a uniform distribution, sampling a distance d according to a one-dimensional Gaussian with a mean of $(3\sigma_1)^2$ and a variance of $(\frac{1}{2}\sigma_1)^2$, then outputting the point

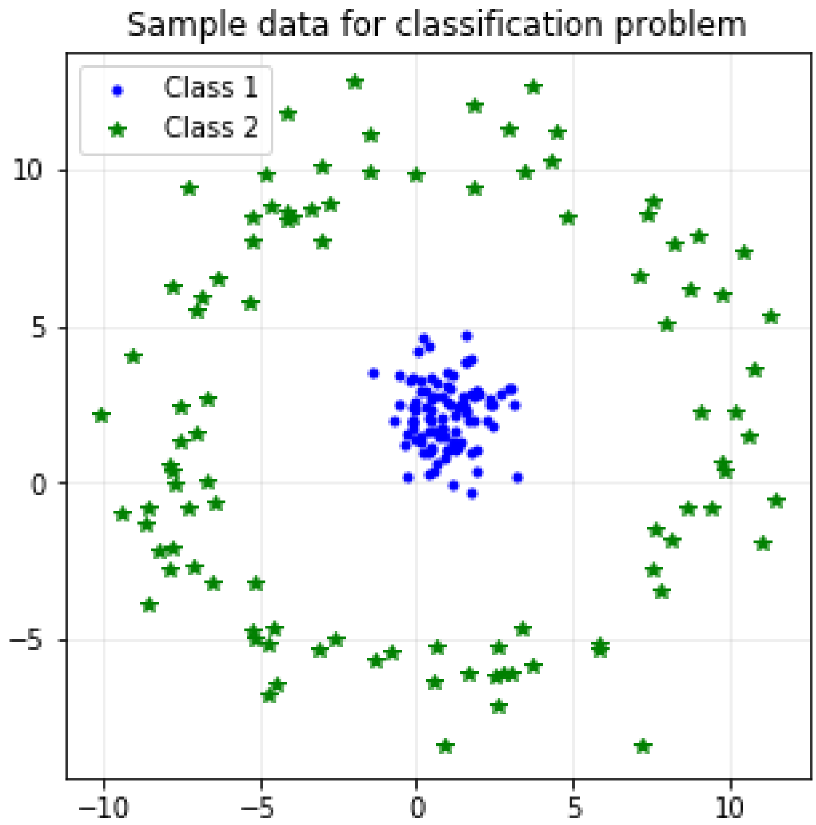
$$\mathbf{x} = \begin{bmatrix} x_{1c} + d\cos\theta \\ x_{2c} + d\sin\theta \end{bmatrix}$$

.

Generate 100 samples for each of the classes.

[20]:
```
# Generate data for class 1

mu_1 = np.array([1.0, 2.0])
sigma_1 = 1
num_sample = 100
cov_mat = np.matrix([[sigma_1,0],[0,sigma_1]])
X1 = np.random.multivariate_normal(mean= mu_1, cov=cov_mat, size = num_sample)

# Generate data for class 2

angle = np.random.uniform(0, 2*np.pi, num_sample)
d =  np.random.normal(np.square(3*sigma_1),np.square(.5*sigma_1),num_sample)
X2 = np.array([X1[:,0] + d*np.cos(angle), X1[:,1] + d*np.sin(angle)]).T

# Combine X1 and X2 into single dataset

X = np.concatenate([X1, X2],axis = 0)
y = np.append(np.zeros(num_sample),np.ones(num_sample))
```

[21]:
```
# Plot the data
fig1 = plt.figure(figsize=(5,5))
ax = plt.axes()
plt.title('Sample data for classification problem')
plt.grid(axis='both', alpha=.25)
plt.plot(X1[:,0],X1[:,1],'b.', label = 'Class 1')
plt.plot(X2[:,0],X2[:,1],'g*', label = 'Class 2')
plt.legend(loc=2)
plt.axis('equal')
plt.show()
```



[22]:
```
# Split data into training and test datasets

idx = np.arange(0,len(X),1)
np.random.shuffle(idx)
idx_train = idx[0:int(.8*len(X))]
idx_test = idx[len(idx_train):len(idx)]

X_train = X[idx_train]
X_test = X[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]
```

[23]:
```
# Normalization of data
def normalization(X):
    """
    Take in numpy array of X values and return normalize X values,
    the mean and standard deviation of each feature
    """
    mean=np.mean(X,axis=0)
    std=np.std(X,axis=0)
```

```
    X_norm = (X - mean)/std

    return X_norm

XX = normalization(X)
X_train_norm = XX[idx_train]
X_test_norm = XX[idx_test]
```

[24]:

```
# define class for logistic regression: batch gradient descent
class Logistic_BGD:

    def __init__(self):
        pass

#     def softmax(self, z):
#         z -= np.max(z)
#         return np.exp(z) / np.sum(np.exp(z))

#     def h(self, X, theta):
#         return self.softmax(np.dot(X,theta))

    def fx(self, X, theta):
        return X@theta

    def h(self, X, theta):
        # sigmoid function
        z = self.fx(X, theta)
        return 1 / (1 + np.exp(-1*z))

    def gradient(self, X, y, y_pred):
        m = len(y)
        return 1/m * np.dot(X.T,(y_pred - y))

    def costFunc(self, theta, X, y):
        m = len(y)
        y_pred = self.h(X, theta)
        error = (y * np.log(y_pred)) + ((1-y)*np.log(1-y_pred))
        cost = -1/m * sum(error)
        grad = self.gradient(X, y, y_pred)
        return cost, grad

    def gradientDescent(self, X, y, theta, alpha, num_iters):
#         X = np.insert(X, 0, 1, axis=1)
        m = len(y)
        J_history = []
        theta_history = []
```

```
        for i in range(num_iters):
            cost, grad = self.costFunc(theta,X,y)
            theta = theta - alpha * grad
            J_history.append(cost)
            theta_history.append(theta)
        J_min_index = np.argmin(J_history)
        return theta_history[J_min_index] , J_history

    def predict(self,X, theta):
        labels=[]
        for i in range(X.shape[0]):
            y1=self.h(X[i], theta)
            if y1 >=  0.5:
                labels.append(1)
            else:
                labels.append(0)

        labels=np.asarray(labels)
        return labels

    def checkAccuracy(self,predicted,y):
        predicted=predicted.tolist()
        y = y.tolist()
        correct=0
        for i in range(0,len(predicted)):
            if y[i]==predicted[i]:
                correct+=1
        return (float(correct)/len(predicted))*100
```

1.4 In lab exercises

1. Verify that the gradient descent solution given above is correct. Plot the optimal decision boundary you obtain.
2. Write a new class that uses Newton's method for the optmization rather than simple gradient descent.
3. Verify that you obtain a similar solution with Newton's method. Plot the optimal decision boundary you obtain.
4. Compare the number of iterations required for gradient descent vs. Newton's method. Do you observe other issues with Newton's method such as a singular or nearly singular Hessian matrix?

[25]:
```
# We define a class for logistic regression (batch gradient descent)
class Logistic_NWT:

    def __init__(self):
        pass
```

```
    def fx(self, X, theta):
        return X @ theta

    def h(self, X, theta):
        # sigmoid function
        z = self.fx(X, theta)
        return 1 / (1 + np.exp(-1*z))

    def gradient(self, X, y, y_pred):
        m = len(y)
        return 1/m * np.dot(X.T,(y_pred - y))

    def hessian(self, xi, theta):
        '''
        Ref : https://stats.stackexchange.com/questions/68391/
↪hessian-of-logistic-function

        X is X_i one row of X.
        '''
        y_pred = self.h(xi,theta) # sigmoid output (prediction)
        xi = xi.reshape(-1,1) # convert from 1d array to 2d (matrix)
        return xi @ xi.T * (y_pred) * (1-y_pred)

    def costFunc(self, theta, X, y):
        m = len(y)
        y_pred = self.h(X, theta)
        error = (y * np.log(y_pred)) + ((1-y)*np.log(1-y_pred))
        cost = -1/m * sum(error)
        grad = self.gradient(X, y, y_pred)
        return cost, grad

    def newtonMethod(self, X, y, theta, num_iters):
        m = len(y)
        J_history = []
        theta_history = []

        # see error
        found_sigular_matrix = False
        error_msg = None
        for i in range(num_iters):
            hessian_mat = np.zeros((X.shape[1], X.shape[1]))

            # compute hessian matrix of x_i
            for i in range(X.shape[0]):
                hmat_xi = self.hessian(X[i], theta)
```

```
                # sum of all matrixs (by element position)
                hessian_mat += hmat_xi
            cost, grad = self.costFunc(theta, X,y)

            # replace gradient descent with newton method
            # follow the formula above.
            try :
                theta = theta - np.linalg.inv(hessian_mat) @ grad
            except Exception as e :
                error_msg = e
                found_sigular_matrix = True
                # leverages SVD to approximate initial matrix.
                theta = theta - np.linalg.pinv(hessian_mat) @ grad
            J_history.append(cost)
            theta_history.append(theta)
        if found_sigular_matrix :
            print('Found error while compute newton method error ! ')
            print(error_msg)
        J_min_index = np.argmin(J_history)
        return theta_history[J_min_index] , J_history

    def predict(self,X, theta):
        labels=[]
        for i in range(X.shape[0]):
            y1=self.h(X[i], theta)
            if y1 >=  0.5:
                labels.append(1)
            else:
                labels.append(0)

        labels=np.asarray(labels)
        return labels

    def checkAccuracy(self,predicted,y):
        predicted=predicted.tolist()
        y = y.tolist()
        correct=0
        for i in range(0,len(predicted)):
            if y[i]==predicted[i]:
                correct+=1
        return (float(correct)/len(predicted))*100
```

[26]:
```
def boundary_points(X, theta):
    theta = theta.reshape(-1,1)
    v_orthogonal = np.array([[theta[1,0]],[theta[2,0]]])
    v_ortho_length = np.sqrt(v_orthogonal.T @ v_orthogonal)
    dist_ortho = theta[0,0] / v_ortho_length
```

```
    v_orthogonal = v_orthogonal / v_ortho_length
    v_parallel = np.array([[-v_orthogonal[1,0]],[v_orthogonal[0,0]]])
    projections = X @ v_parallel
    proj_1 = min(projections)
    proj_2 = max(projections)
    point_1 = proj_1 * v_parallel - dist_ortho * v_orthogonal
    point_2 = proj_2 * v_parallel - dist_ortho * v_orthogonal
    return point_1, point_2
```

[27]:
```
# create X to equal to polynomial function
x_df = pd.DataFrame(X, columns=['X0', 'X1'])

x_df['y'] = y

x_df['X0'] = normalization(x_df.X0)
x_df['X1'] = normalization(x_df.X1)
linX = x_df[['X0','X1']].values
linX = np.insert(linX, 0, 1, axis=1)

X_train = linX[idx_train]
X_test = linX[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]
```

Getting optimal theta sing Logistic Batch Gradient Descent:

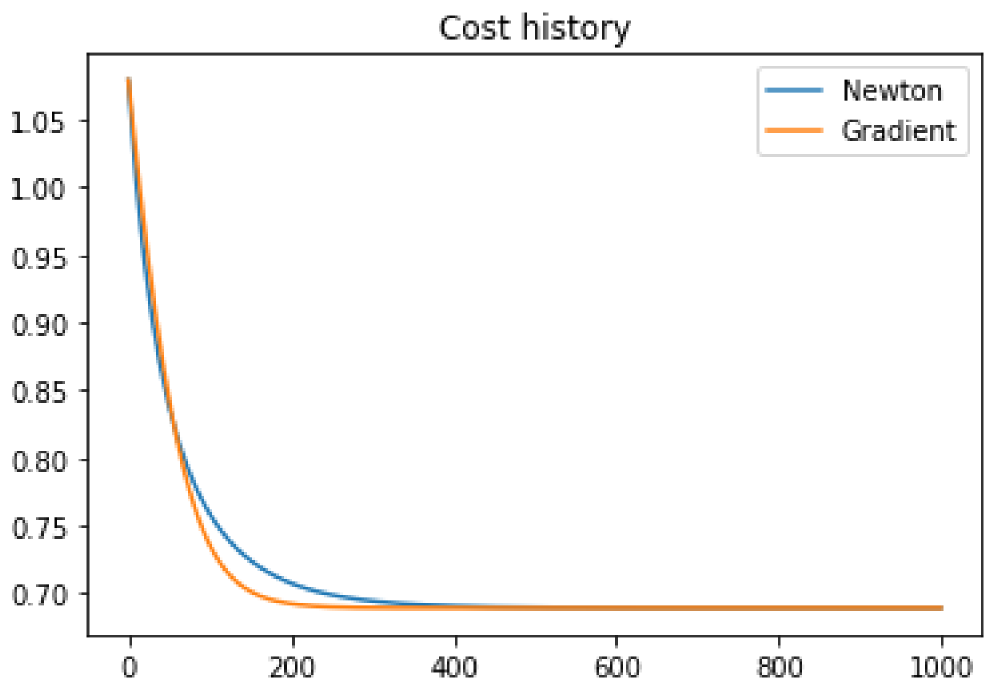
[28]:
```
alpha = 0.05
iterations = 1000
init_theta = np.ones(X_train.shape[1])

lgx = Logistic_BGD()
theta_x, j_hist_x = lgx.gradientDescent(X_train, y_train, init_theta, alpha,␣
 ↪iterations)
```

Getting optimal theta using Newton's Method

[29]:
```
iterations = 1000
lnx = Logistic_NWT()
theta_xn, j_hist_xn = lnx.newtonMethod(X_train, y_train, init_theta, iterations)
```

[30]:
```
plt.title('Cost history')
plt.plot(j_hist_xn, label='Newton')
plt.plot(j_hist_x, label='Gradient')

plt.legend()
plt.show()
```

```
print('Minimum Cost for each method')
print('Gradient :', np.min(j_hist_x))
print('Newton :', np.min(j_hist_xn))
```

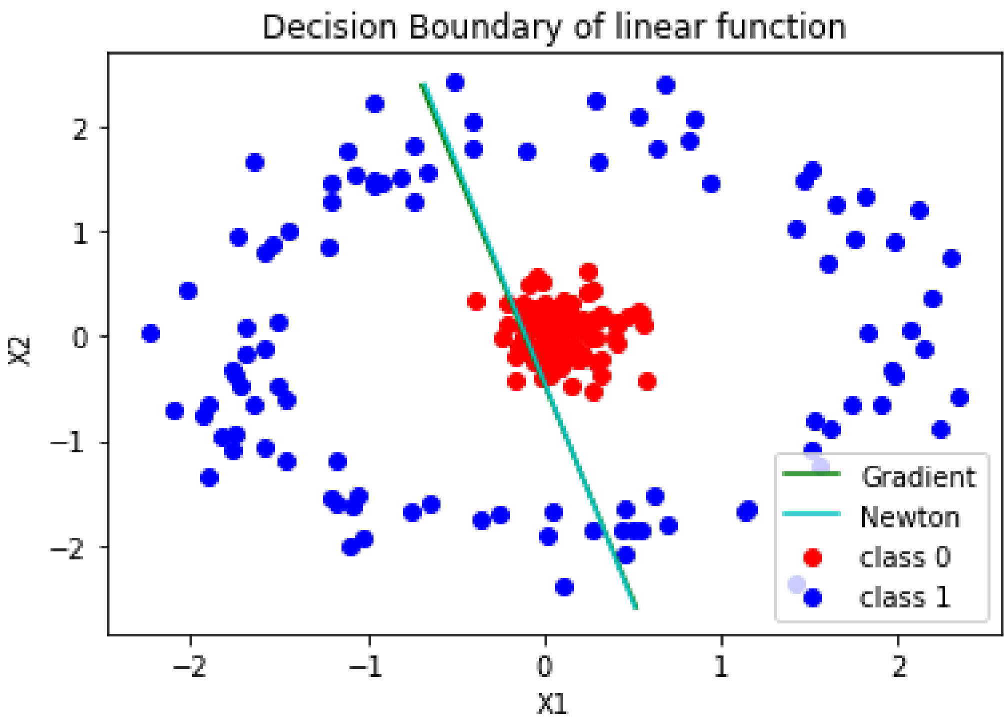



```
Minimum Cost for each method
Gradient : 0.6889016406221803
Newton : 0.6889024628224568
```

[31]:

```
# Plotting decision boundary for the above classes from solution obtained using␣
↪batch gradient descent and Newton's method
y0_df = x_df[x_df.y == 0]
y1_df = x_df[x_df.y == 1]


point_1, point_2 = boundary_points(linX[:,1:], theta_x)
point_1n, point_2n = boundary_points(linX[:,1:], theta_xn)

plt.title('Decision Boundary of linear function')
plt.scatter(y0_df.X0, y0_df.X1, c='r', label='class 0')
plt.scatter(y1_df.X0, y1_df.X1, c='b', label='class 1')
plt.legend()
plt.xlabel('X1')
plt.ylabel('X2')
# plot the boundaries for both methods
```

```
plt.plot([point_1[0,0], point_2[0,0]],[point_1[1,0], point_2[1,0]], 'g-',␣
↪label='Gradient')
plt.plot([point_1n[0,0], point_2n[0,0]],[point_1n[1,0], point_2n[1,0]], 'c-',␣
↪label='Newton')
plt.legend(loc=0)
plt.show()
```



[32]:
```
yg_pred = lgx.predict(X_test, theta_x)
yn_pred = lnx.predict(X_test, theta_xn)
g_acc = lgx.checkAccuracy(yg_pred, y_test)
n_acc = lnx.checkAccuracy(yn_pred, y_test)

print("Test Accuracy")
print('Gradient Accuracy : ', g_acc)
print('Newton Accuracy : ', n_acc)
```

```
Test Accuracy
Gradient Accuracy :  72.5
Newton Accuracy :  72.5
```

1.5 Take-home exercises

1. Perform a *polar transformation* on the data above to obtain a linearly separable dataset.

2. Verify that you obtain good classification accuracy for logistic regression with GD or Netwon's method after the polar transformation
3. Apply Newton's method to the dataset you used for the take home exercises in Lab 03.

1.6 The report

Write a brief report covering your experiments (both in lab and take home) and send as a Jupyter notebook to the TAs, Manish and Abhishek before the next lab.

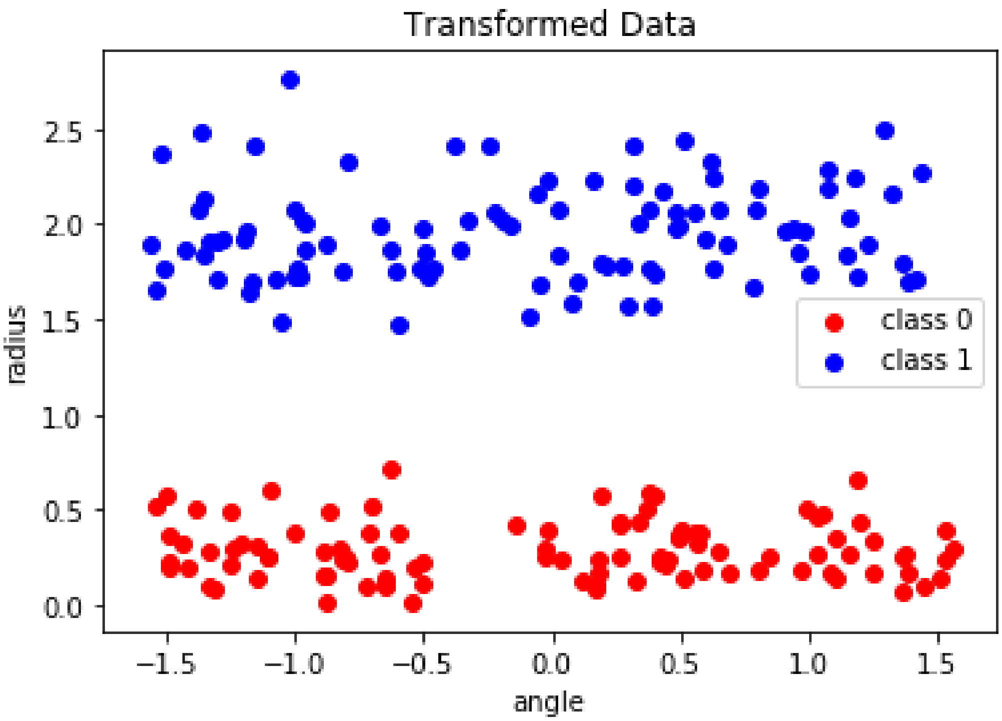
In your solution, be sure to follow instructions.

1.6.1 Performing Polar Transformations

[33]:
```
# Prepare data
# Convert X to angle and radius
df = pd.DataFrame(X, columns=['X0', 'X1'])
df['angles'] = np.arctan(df.X1 / df.X0)
df['radius'] = np.sqrt(df.X0 ** 2 + df.X1 ** 2)
df['y'] = y

newX = df[['angles', 'radius']].values
newX = np.insert(newX, 0, 1, axis=1)
X_train = newX[idx_train]
X_test = newX[idx_test]
y_train = y[idx_train]
y_test = y[idx_test]
```

[34]:
```
y0_df = df[df.y == 0]
y1_df = df[df.y == 1]

plt.title('Transformed Data')
plt.scatter(y0_df.angles, y0_df.radius, c='r', label='class 0')
plt.scatter(y1_df.angles, y1_df.radius, c='b', label='class 1')
plt.legend()
plt.xlabel('angle')
plt.ylabel('radius')
```

[34]:
```
Text(0, 0.5, 'radius')
```

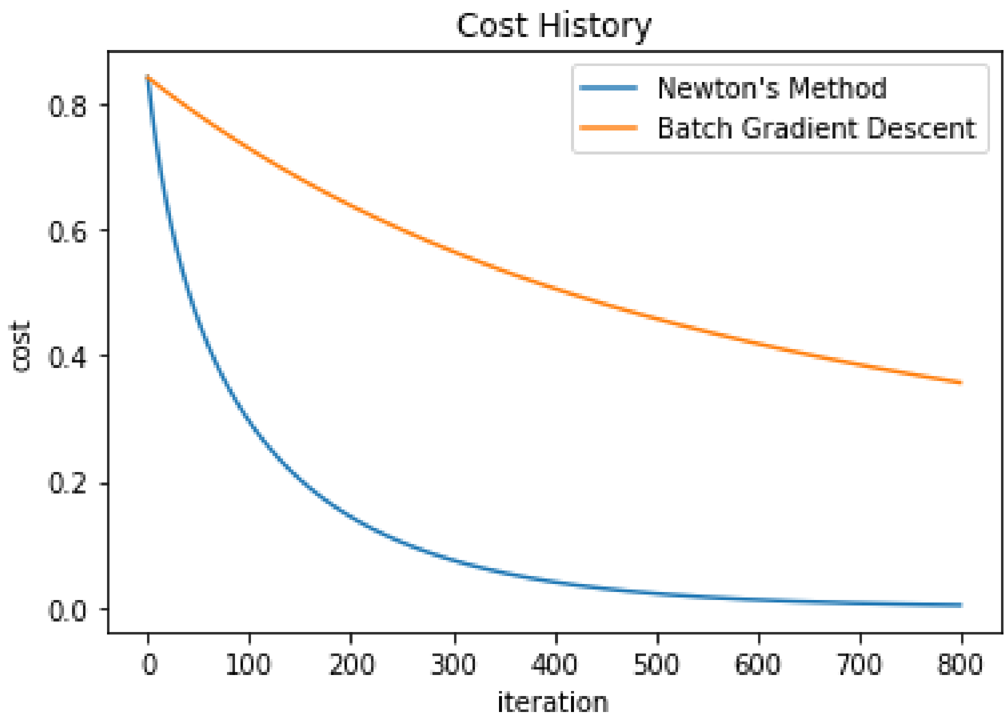


[35]:
```
alpha = 0.01
iterations = 10000
init_theta = np.ones(newX.shape[1])

lg = Logistic_BGD()
theta, j_hist = lg.gradientDescent(X_train, y_train, init_theta, alpha,␣
↪iterations)
```

[36]:
```
iterations = 800
init_theta = np.ones(newX.shape[1])

ln = Logistic_NWT()
theta_n, j_hist_n = ln.newtonMethod(X_train, y_train, init_theta, iterations)
```

[37]:
```
plt.plot(j_hist_n, label='Newton\'s Method')
plt.plot(j_hist[:iterations], label='Batch Gradient Descent')
plt.title('Cost History')
plt.xlabel('iteration')
plt.ylabel('cost')
plt.legend()
plt.show()

print('Minimum Cost for each method from polar transformation')
```

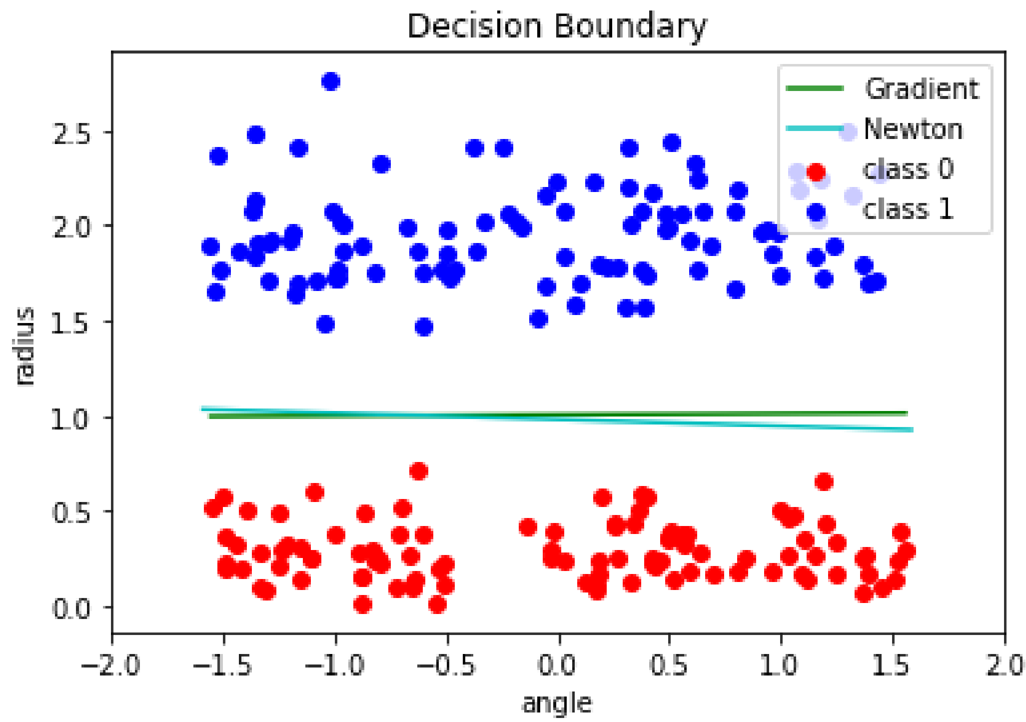
```
print('Gradient Descent:', np.min(j_hist))
print('Newton\'s Method :', np.min(j_hist_n))
```




```
Minimum Cost for each method from polar transformation
Gradient Descent: 0.0449652208110658
Newton's Method : 0.0036222957878202957
```

[38]:
```
y0_df = df[df.y == 0]
y1_df = df[df.y == 1]



point_1, point_2 = boundary_points(newX[:,1:], theta)
point_1n, point_2n = boundary_points(newX[:,1:], theta_n)

plt.title('Decision Boundary')
plt.scatter(y0_df.angles, y0_df.radius, c='r', label='class 0')
plt.scatter(y1_df.angles, y1_df.radius, c='b', label='class 1')
plt.legend()
plt.xlabel('angle')
plt.ylabel('radius')
# plot the boundaries for both methods
plt.plot([point_1[0,0], point_2[0,0]],[point_1[1,0], point_2[1,0]], 'g-',␣
 ↪label='Gradient')
```

```
plt.plot([point_1n[0,0], point_2n[0,0]],[point_1n[1,0], point_2n[1,0]], 'c-',␣
↪label='Newton')
plt.legend(loc=0)
# plt.ylim(0,14)
plt.xlim(-2,2)
plt.show()
```



[39]:
```
yg_pred = lg.predict(X_train, theta)
yn_pred = ln.predict(X_train, theta_n)
g_acc = lg.checkAccuracy(yg_pred, y_train)
n_acc = lg.checkAccuracy(yn_pred, y_train)

print("Train accuracy for polar transformation")
print('Gradient Accuracy : ', g_acc)
print('Newton Accuracy : ', n_acc)

yg_pred = lg.predict(X_test, theta)
yn_pred = ln.predict(X_test, theta_n)
g_acc = lg.checkAccuracy(yg_pred, y_test)
n_acc = lg.checkAccuracy(yn_pred, y_test)

print("Test accuracy for polar transformation")
print('Gradient Accuracy : ', g_acc)
```

```python
print('Newton Accuracy : ', n_acc)
```

```
Train accuracy for polar transformation
Gradient Accuracy :  100.0
Newton Accuracy :  100.0
Test accuracy for polar transformation
Gradient Accuracy :  100.0
Newton Accuracy :  100.0
```

Here we can see, we get the perfect accuracy while performing polar transoformation in our dataset.

1.6.2 Using Newton method in Lab 3- Loan prediction dataset

[40]:
```python
# Import the data

data_train = pd.read_csv('train_LoanPrediction.csv')
data_test = pd.read_csv('test_LoanPrediction.csv')

# Start to explore the data

print('Training data shape', data_train.shape)
print('Test data shape', data_test.shape)

print('Training data:\n', data_train)
```

```
Training data shape (614, 13)
Test data shape (367, 12)
Training data:
      Loan_ID  Gender Married Dependents     Education Self_Employed  \
0    LP001002    Male      No          0      Graduate            No
1    LP001003    Male     Yes          1      Graduate            No
2    LP001005    Male     Yes          0      Graduate           Yes
3    LP001006    Male     Yes          0  Not Graduate            No
4    LP001008    Male      No          0      Graduate            No
..        …     …     …        …           …           …
609  LP002978  Female      No          0      Graduate            No
610  LP002979    Male     Yes         3+      Graduate            No
611  LP002983    Male     Yes          1      Graduate            No
612  LP002984    Male     Yes          2      Graduate            No
613  LP002990  Female      No          0      Graduate           Yes

     ApplicantIncome  CoapplicantIncome  LoanAmount  Loan_Amount_Term  \
0               5849                0.0         NaN             360.0
1               4583             1508.0       128.0             360.0
2               3000                0.0        66.0             360.0
3               2583             2358.0       120.0             360.0
4               6000                0.0       141.0             360.0
..               …                …         …               …
```

```
609             2900                0.0        71.0               360.0
610             4106                0.0        40.0               180.0
611             8072              240.0       253.0               360.0
612             7583                0.0       187.0               360.0
613             4583                0.0       133.0               360.0

     Credit_History Property_Area Loan_Status
0               1.0         Urban           Y
1               1.0         Rural           N
2               1.0         Urban           Y
3               1.0         Urban           Y
4               1.0         Urban           Y
..              …             …         …
609             1.0         Rural           Y
610             1.0         Rural           Y
611             1.0         Urban           Y
612             1.0         Urban           Y
613             0.0     Semiurban           N

[614 rows x 13 columns]
```

[41]:

```
# Now checking for the missing values of the train and test datasets

print('Missing values for train data:\n------------------------\n', data_train.
 ↪isnull().sum())
print('Missing values for test data \n ------------------------\n', data_test.
 ↪isnull().sum())
```

```
Missing values for train data:
------------------------
 Loan_ID              0
Gender              13
Married              3
Dependents          15
Education            0
Self_Employed       32
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount          22
Loan_Amount_Term    14
Credit_History      50
Property_Area        0
Loan_Status          0
dtype: int64
Missing values for test data
 ------------------------
 Loan_ID              0
Gender              11
```

```
Married              0
Dependents          10
Education            0
Self_Employed       23
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount           5
Loan_Amount_Term     6
Credit_History      29
Property_Area        0
dtype: int64
```

[42]:

```
# Filling the missing values

def fill_martial_status(data, yes_num_train, no_num_train):
    data['Married'].fillna('Yes', inplace = True, limit = yes_num_train)
    data['Married'].fillna('No', inplace = True, limit = no_num_train)

fill_martial_status(data_train, 2, 1)
fill_martial_status(data_test, 2, 1)

def fill_dependent_status(num_0_train, num_1_train, num_2_train, num_3_train,␣
 ↪num_0_test, num_1_test, num_2_test, num_3_test):
    data_train['Dependents'].fillna('0', inplace=True, limit = num_0_train)
    data_train['Dependents'].fillna('1', inplace=True, limit = num_1_train)
    data_train['Dependents'].fillna('2', inplace=True, limit = num_2_train)
    data_train['Dependents'].fillna('3+', inplace=True, limit = num_3_train)
    data_test['Dependents'].fillna('0', inplace=True, limit = num_0_test)
    data_test['Dependents'].fillna('1', inplace=True, limit = num_1_test)
    data_test['Dependents'].fillna('2', inplace=True, limit = num_2_test)
    data_test['Dependents'].fillna('3+', inplace=True, limit = num_3_test)

fill_dependent_status(9, 2, 2, 2, 5, 2, 2, 1)

data_train['Dependents'].replace('3+', '4', inplace = True)
data_test['Dependents'].replace('3+', '4', inplace = True)

loan_amount_mean = np.mean(data_train["LoanAmount"])
data_train['LoanAmount'].fillna(loan_amount_mean, inplace=True, limit = 22)
data_test['LoanAmount'].fillna(loan_amount_mean, inplace=True, limit = 5)

# filling Gender by mode
data_train.Gender.fillna(data_train.Gender.mode()[0], inplace=True)
data_test.Gender.fillna(data_test.Gender.mode()[0], inplace=True)

# filling Self Employed by mode
```

```
data_train.Self_Employed.fillna(data_train.Self_Employed.mode()[0] ,␣
 ↪inplace=True)
data_test.Self_Employed.fillna(data_test.Self_Employed.mode()[0] , inplace=True)

# filling loan amount term by mean
data_train.Loan_Amount_Term.fillna(data_train.Loan_Amount_Term.mean() ,␣
 ↪inplace=True)
data_test.Loan_Amount_Term.fillna(data_test.Loan_Amount_Term.mean() ,␣
 ↪inplace=True)

# filling credit history by mode
data_train.Credit_History.fillna(data_train.Credit_History.mode()[0]  ,␣
 ↪inplace=True)
data_test.Credit_History.fillna(data_test.Credit_History.mode()[0]  ,␣
 ↪inplace=True)
```

[43]:
```
#Now checking for the missing values of the train and test datasets

print('Missing values for train data:\n------------------------\n', data_train.
 ↪isnull().sum())
print('Missing values for test data \n ------------------------\n', data_test.
 ↪isnull().sum())
```

```
Missing values for train data:
------------------------
 Loan_ID              0
Gender               0
Married              0
Dependents           0
Education            0
Self_Employed        0
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount           0
Loan_Amount_Term     0
Credit_History       0
Property_Area        0
Loan_Status          0
dtype: int64
Missing values for test data
 ------------------------
 Loan_ID              0
Gender               0
Married              0
Dependents           0
Education            0
Self_Employed        0
```

```
ApplicantIncome      0
CoapplicantIncome    0
LoanAmount           0
Loan_Amount_Term     0
Credit_History       0
Property_Area        0
dtype: int64
```

[44]:

```
data_test = data_test.iloc[0:,1:]
data_train = data_train.iloc[0:,1:]

print(data_test.info())
print(data_train.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 367 entries, 0 to 366
Data columns (total 11 columns):
 #   Column             Non-Null Count  Dtype
---  ------             --------------  -----
 0   Gender             367 non-null    object
 1   Married            367 non-null    object
 2   Dependents         367 non-null    object
 3   Education          367 non-null    object
 4   Self_Employed      367 non-null    object
 5   ApplicantIncome    367 non-null    int64
 6   CoapplicantIncome  367 non-null    int64
 7   LoanAmount         367 non-null    float64
 8   Loan_Amount_Term   367 non-null    float64
 9   Credit_History     367 non-null    float64
 10  Property_Area      367 non-null    object
dtypes: float64(3), int64(2), object(6)
memory usage: 31.7+ KB
None
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 614 entries, 0 to 613
Data columns (total 12 columns):
 #   Column             Non-Null Count  Dtype
---  ------             --------------  -----
 0   Gender             614 non-null    object
 1   Married            614 non-null    object
 2   Dependents         614 non-null    object
 3   Education          614 non-null    object
 4   Self_Employed      614 non-null    object
 5   ApplicantIncome    614 non-null    int64
 6   CoapplicantIncome  614 non-null    float64
 7   LoanAmount         614 non-null    float64
 8   Loan_Amount_Term   614 non-null    float64
 9   Credit_History     614 non-null    float64
```

```
 10  Property_Area      614 non-null    object
 11  Loan_Status        614 non-null    object
dtypes: float64(4), int64(1), object(7)
memory usage: 57.7+ KB
None
```

[45]:

```
#Converting categorical into 0 and 1

data_train['Gender'].replace('Male', 0, inplace = True)
data_train['Gender'].replace('Female', 1, inplace = True)
data_train['Married'].replace('No', 0, inplace = True)
data_train['Married'].replace('Yes', 1, inplace = True)
data_train['Education'].replace('Not Graduate', 0, inplace = True)
data_train['Education'].replace('Graduate', 1, inplace = True)
data_train['Self_Employed'].replace('No', 0, inplace = True)
data_train['Self_Employed'].replace('Yes', 1, inplace = True)
data_train['Property_Area'].replace('Urban', 0, inplace = True)
data_train['Property_Area'].replace('Semiurban', 1, inplace = True)
data_train['Property_Area'].replace('Rural', 2, inplace = True)
data_train['Loan_Status'].replace('N', 0, inplace = True)
data_train['Loan_Status'].replace('Y', 1, inplace = True)


data_test['Gender'].replace('Male', 0, inplace = True)
data_test['Gender'].replace('Female', 1, inplace = True)
data_test['Married'].replace('No', 0, inplace = True)
data_test['Married'].replace('Yes', 1, inplace = True)
data_test['Education'].replace('Not Graduate', 0, inplace = True)
data_test['Education'].replace('Graduate', 1, inplace = True)
data_test['Self_Employed'].replace('No', 0, inplace = True)
data_test['Self_Employed'].replace('Yes', 1, inplace = True)
data_test['Property_Area'].replace('Urban', 0, inplace = True)
data_test['Property_Area'].replace('Semiurban', 1, inplace = True)
data_test['Property_Area'].replace('Rural', 2, inplace = True)

print(data_train.info())
print(data_test.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 614 entries, 0 to 613
Data columns (total 12 columns):
 #   Column             Non-Null Count  Dtype
---  ------             --------------  -----
 0   Gender             614 non-null    int64
 1   Married            614 non-null    int64
 2   Dependents         614 non-null    object
 3   Education          614 non-null    int64
 4   Self_Employed      614 non-null    int64
```

```
 5   ApplicantIncome    614 non-null    int64
 6   CoapplicantIncome  614 non-null    float64
 7   LoanAmount         614 non-null    float64
 8   Loan_Amount_Term   614 non-null    float64
 9   Credit_History     614 non-null    float64
 10  Property_Area      614 non-null    int64
 11  Loan_Status        614 non-null    int64
dtypes: float64(4), int64(7), object(1)
memory usage: 57.7+ KB
None
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 367 entries, 0 to 366
Data columns (total 11 columns):
 #   Column             Non-Null Count  Dtype
---  ------             --------------  -----
 0   Gender             367 non-null    int64
 1   Married            367 non-null    int64
 2   Dependents         367 non-null    object
 3   Education          367 non-null    int64
 4   Self_Employed      367 non-null    int64
 5   ApplicantIncome    367 non-null    int64
 6   CoapplicantIncome  367 non-null    int64
 7   LoanAmount         367 non-null    float64
 8   Loan_Amount_Term   367 non-null    float64
 9   Credit_History     367 non-null    float64
 10  Property_Area      367 non-null    int64
dtypes: float64(3), int64(7), object(1)
memory usage: 31.7+ KB
None
```

[46]:
```
print(data_train.dtypes)
data_train['Dependents'] = data_train['Dependents'].astype(str).astype(int)
data_train['Loan_Amount_Term'] = data_train['Loan_Amount_Term'].astype(int)
data_train['Credit_History'] = data_train['Credit_History'].astype(int)
data_train['CoapplicantIncome'] = data_train['CoapplicantIncome'].astype(int)
data_train['LoanAmount'] = data_train['LoanAmount'].astype(int)
print(data_train.dtypes)
```

```
Gender                 int64
Married                int64
Dependents            object
Education              int64
Self_Employed          int64
ApplicantIncome        int64
CoapplicantIncome    float64
LoanAmount           float64
Loan_Amount_Term     float64
Credit_History       float64
```

```
Property_Area          int64
Loan_Status            int64
dtype: object
Gender                int64
Married               int64
Dependents            int32
Education             int64
Self_Employed         int64
ApplicantIncome       int64
CoapplicantIncome     int32
LoanAmount            int32
Loan_Amount_Term      int32
Credit_History        int32
Property_Area         int64
Loan_Status           int64
dtype: object
```

[47]:
```
data_train
```

[47]:
```
     Gender  Married  Dependents  Education  Self_Employed  ApplicantIncome  \
0         0        0           0          1              0             5849
1         0        1           1          1              0             4583
2         0        1           0          1              1             3000
3         0        1           0          0              0             2583
4         0        0           0          1              0             6000
..      ...      ...         ...        ...            ...              ...
609       1        0           0          1              0             2900
610       0        1           4          1              0             4106
611       0        1           1          1              0             8072
612       0        1           2          1              0             7583
613       1        0           0          1              1             4583

     CoapplicantIncome  LoanAmount  Loan_Amount_Term  Credit_History  \
0                    0         146               360               1
1                 1508         128               360               1
2                    0          66               360               1
3                 2358         120               360               1
4                    0         141               360               1
..                 ...         ...               ...             ...
609                  0          71               360               1
610                  0          40               180               1
611                240         253               360               1
612                  0         187               360               1
613                  0         133               360               0

     Property_Area  Loan_Status
0                0            1
```

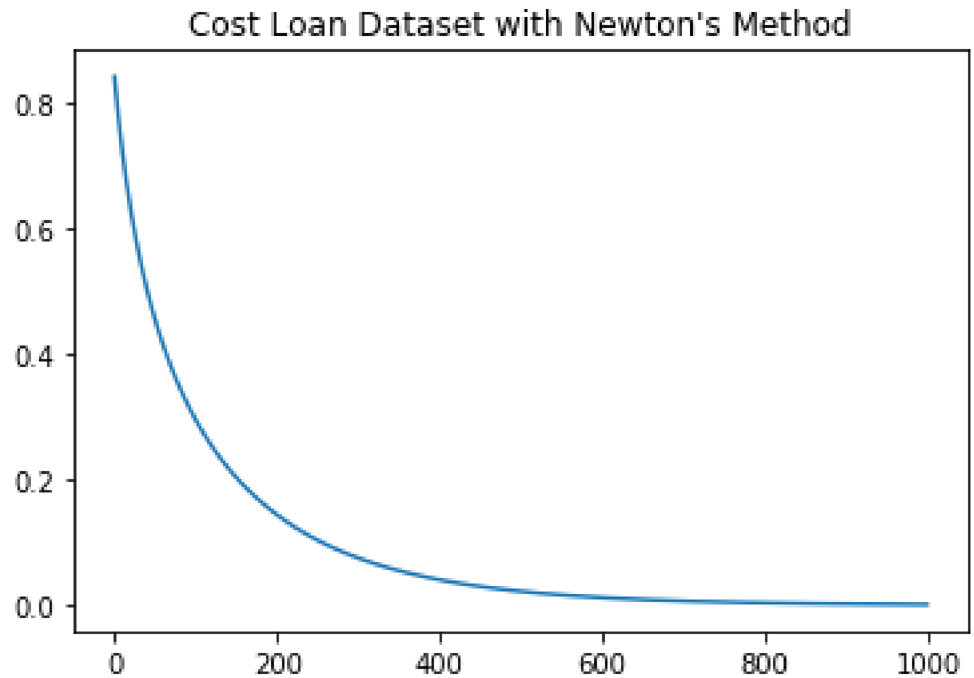
```
1                 2            0
2                 0            1
3                 0            1
4                 0            1
..              …           …
609               2            1
610               2            1
611               0            1
612               0            1
613               1            0

[614 rows x 12 columns]
```

[48]:

```
# apply newton method with Lab 3 dataset
iterations = 1000
init_theta = np.ones(X_train.shape[1])

ln_lab = Logistic_NWT()
theta_nlab, j_hist_nlab = ln_lab.newtonMethod(X_train, y_train, init_theta,␣
 ↪iterations)
```

[49]:

```
plt.title('Cost Loan Dataset with Newton\'s Method')
plt.plot(j_hist_nlab)
plt.show()

yn_pred = ln_lab.predict(X_train, theta_nlab)
n_acc = ln_lab.checkAccuracy(yn_pred, y_train)
print("Train Set Accuracy From Lab3 dataset")
print('Newton Accuracy : ', n_acc)

yn_pred = ln_lab.predict(X_test, theta_nlab)
n_acc = ln_lab.checkAccuracy(yn_pred, y_test)

print("Test Set Accuracy From Lab3 dataset")
print('Newton Accuracy : ', n_acc)
```




```
Train Set Accuracy From Lab3 dataset
Newton Accuracy :  100.0
Test Set Accuracy From Lab3 dataset
Newton Accuracy :  100.0
```

[ ]:

[ ]: