Example (i)

(i) In the following table, use the Newton-Gregory Forward Interpolation formula to find
(a) f(2.4) (b) f(8.7).

x	2	4	6	8	10
f(x)	9.68	10.96	12.32	13.76	15.28

<u>Solution</u> Form a difference table and note that all differences > 2 are zero.

x	**y=f(x)**	Δy	$\Delta^2 y$
2	9.68		
		1.28	
4	10.96		0.08
		1.36	
6	12.32		0.08
		1.44	
8	13.76		0.08
		1.52	
10	15.28		

(a) x = 2.4; x = 2; h = 2; k = 0.2

we get $f(2.4) \cong 9.68 + \frac{2.4-2}{2} \times 1.28 + \frac{(2.4-2)(2.4-4)}{4} \times \frac{0.08}{2}$

so $f(2.4) \cong 9.68 + 0.2 \times 1.28 + 0.1 \times (-1.6) \times 0.04 =$ 9.9296

(b) x = 8.7; x = 2; h = 2; k = 3.35

we get $f(8.7) \cong 9.68 + 3.35 \times 1.25 + 3.35 \times 2.35 \times 0.04 = 14.2829$

Example (ii)

In the following table of e^x use the Newton-Gregory formula of forward interpolation to calculate
(a) $e^{0.12}$, (b) $e^{2.00}$.

x	0.1	0.6	1.1	1.6	2.1
$\mathbf{e^x}$	1.1052	1.8221	3.0042	4.9530	8.1662

<u>Solution</u>: Form a difference table.

Note that in this case there is no difference column that is constant. This is to be expected since e^x cannot be represented by a polynomial function of finite degree.

x	**y=e^{x}**	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.1052				
		0.7169			
0.6	1.8221		0.4652		
		1.1821		0.3015	
1.1	3.0042		0.7667		0.1962
		1.9488		0.4977	
1.6	4.9530		1.2644		
		3.2132			
2.1	8.1662				

(a) With x = 0.12, x_o = 0.1, h = 0.5, k = 0.04

$$\therefore \quad e^{0.12} \cong 1.1052 + 0.04 \times 0.7169 + 0.04 \times (-0.96)\frac{0.4652}{2}$$
$$+0.04 \times (-0.96) \times (-1.96)\frac{0.3015}{6} + 0.04 \times (-0.96) \times (-1.96) \times (-2.96)\frac{0.1962}{24}$$

$\therefore$ $e^{0.12}$=1.1269 (correct value to 5 d.p. is 1.12750)

(b) x = 2, x_o=0.1, h = 0.5, k = 3.8

$$e^2 \cong 1.1052 + 2.72422 + 2.47486 + 0.96239 + 0.12525$$

$\therefore e^2 \cong 7.3919$ (to 4 dp). (correct value 7.3891 to 4 dp)

In Example (i) the interpolation formula is identical with f(x), which is a quadratic function, and the results for f(2.4) and f(8.7) will therefore be correct to the number of decimal places retained.
In example (ii) the function e^x is replaced by a 4th degree polynomial which takes the value of e^x at the five given entries. Because the successive difference decrease, higher differences are relatively small and the value of the estimate converges. From direct calculation it turns out that the error in the estimate for $e^{0.12}$ is about 0.05 percent and for $e^{2.00}$ it is about 0.04 percent.
It is important to note that interpolation may not always yield a valid result. Consider the following example: