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Geometric Brownian Motion in Stock Prices

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Abstract. Financial instability estimates the changes of the cost of a monetary instrument. It is a proportion of properties of the Stock prices stability. Fractal investigations are used to assess the money related instability. Forecasting of stock prices acts as an important challenge based on the Random Walk theory. This paper deals with comparison of two years 2013 -2014 and 2017(Jun to Nov) of stock prices. Explain the instability by the method of Box-Counting technique to find the Fractal dimensions of the Geometric Brownian Motion based on the Random Walk defective value. This creates the possibility that Fractal measurement is related with the monetary unpredictability. Its an essential instrument for both money related investigators and Financial specialists.

Keywords: Stock prices, Financial Volatility, Fractals, Fractal Dimension, Geometric Brownian Motion, Random Walk.

AMS Classification 2000: 28A80, 60J60, 97M40, 97M99.

I. INTRODUCTION

1.1 STOCK PRICE

The stock price is the highest amount someone pays to sell the stock, or the minimum amount to buy the stocks [10]. Each shares are allocated a monetary value or price, when traded in public, companies issue shares of stock to the investors. Stock prices fluctuates depending on different factors [2].

1.2 FINANCIAL VOLATILITY

Unexpected events resulting in huge price movements with varying variable, consequently financial market creates an unexpected behaviour that may confuse the investors. This leads to financial volatility and an unexpected price may return [3]

^{**} Corresponding Author



1.3 FRACTALS

Fractals was coined by Benoit Mandelbrot in 1975 and as derived from the Latin word fractus meaning *broken or fractured*. He was a father of Fractals [13]. It is endless models. It is immensely complex models that are self – alike during various levels. Examples are trees, rivers, coastlines, sea shells, Cantor set, Sierpinski Triangle, Von Koch curve, mountains etc.

1.4 RANDOM WALK

Random walk is a mathematical process and are referred as a stochastic otherwise random method to explain a route, which has a series of random steps on few mathematical spaces as like the integers [1].

Azme Khamis et.al explained Monetary Data using Geometric Brownian Motion based on Wiener process. But in this paper it explains the costs of the stock with Geometric Brownian Motion based Random Walk and Box Counting Dimension [3].

II.METHODS

2.1 FRACTAL DIMENSION

A Fractal dimension is a ratio which provides a statistical indicator of complexity comparing the model (exactly speaking, a Fractal model) gets changed through the scale on which it is calculated. The Hausdorff dimension that are much more broadly appropriate [12].

For example, Hausdorff dimension and the box counting dimension is described for any set and in these four example may be shown to equal the similarity dimension and Hausdorff and Box counting along with the methods used for their computation very roughly, a dimension provide a explanation of how much space a set occupies [12].It is a quantity of the importance of the abnormality of a set when looked upon at very small scales. A dimension has more details about the physical parameters of a set.

The Hausdorff measurement which characterizes the fractal measurement D as a partial quantity. This method does not finds an efficient solution and hence box counting method is preferred.

$$D \approx \frac{\log(N(s))}{\log(\frac{1}{s})} \quad (1)$$

The Hausdorff method expresses that the component of an article is connected with the quantity $N(s)$ of circles of size s expected to over to question in a D_E -dimensional Euclidean space.

$$N(s) \sim \frac{1}{s} D_E \quad (2)$$

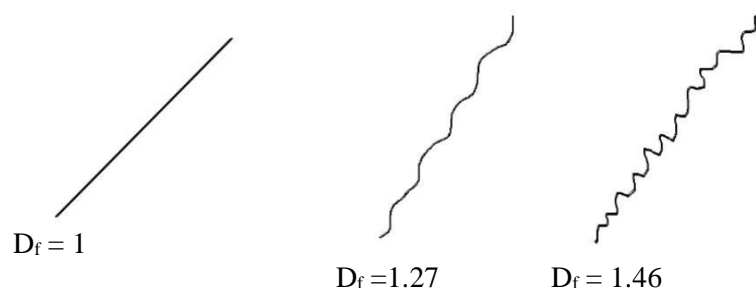


Figure 1. Fractal dimension of the different Random Walk

2.2 FRACTAL DIMENSION AND FINANCIAL VOLATILITY

Nowadays, there are different endeavours of utilizing fractal investigation in monetary market [3, 4]. Fractal measurement turned out to be an essential method for two reasons: As per observation of Mandelbrot, stocks exchange costs are fractal objects, second on the grounds that the fractal measurement is a proportion of the level of the pictures contour complication [11]. The random walk method recommends the alterations in stock prices, which has the similar distribution and not dependent on any other factor. Hence the previous movement or stock price or market will be used to forecast its future movement [8]. Profit values are plotted in fig 2. Mean values for the 2013 – 2014 years are calculated and the defective values present in Table1.

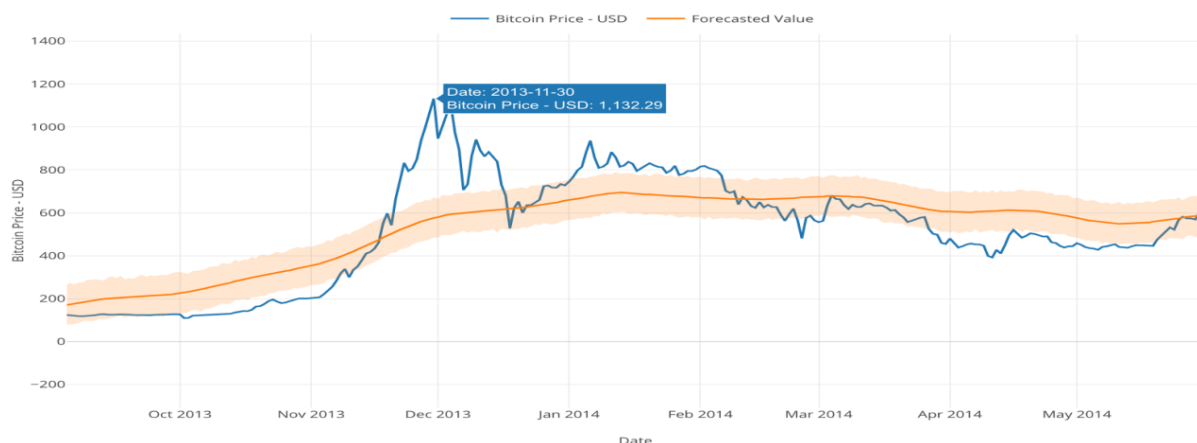


Figure 2. Forecast of Amazon Stock Prices in 2013-2014

Table 1. Actual and Forecast of the defective value in the Year 2013 – 2014.

Defective Values									
Month	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Mean
Actual	150	200	950	720	800	560	500	450	541.25
Forecast	210	380	590	650	700	710	600	590	553.75

The forecasting method is applied and then compared with the stock buying and selling for years. The model forecasts increase and decrease of the stocks on each day. If it forecasts as decrease, stocks should not be bought and if it forecasts as increase, buying stocks would give more profit to buyers [6]. The forecast and actual model for the year 2017 is shown in fig 3. The actual and forecast defective values present in Table 2.

The random walk theory is a financial theory which describes that the cost of the stock market evolves as per random walk and hence prediction is not possible [7] it is steady with the efficient market hypothesis. The random walk plot, the lower values are referred as defective Based on the mean value Poisson distribution is obtained.



Figure 3. Company Actual Stock Price in 2017

Table 2 Actual and Forecast of the defective value in the Year 2017.

Defective Values							
Month	Jun	Jul	Aug	Sep	Oct	Nov	Mean
Actual	220	300	210	400	320	310	293.3
Forecast	210	380	390	395	592	850	469.5

2.3 GEOMETRIC BROWNIAN MOTION

Geometric Brownian motion are referred as (exponential Brownian motion) is a continuous-time stochastic process in which the logarithm of the randomly changing quantity results a Brownian motion with drift [5].

Geometric Brownian Motion $X(t)$ is the solution of a stochastic differential equation

$$dX(t) = \mu X(t)dt + \sigma X(t)dw(t) \quad (3)$$

With initial value $(0) = x_0$,

The relative change is a grouping of a deterministic proportional development period alike to inflation or interest rate growth and a commonly distributed random transforms [10].

$$\frac{dX}{X} = rdt + \sigma dw \quad (4)$$

Random variable X is said to include the lognormal distribution (with Parameters r and σ)

The variance of Geometric Brownian Motion cannot be calculated by applying the normal distribution, along with the general formula

$$Var[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2 \quad (5)$$

This description is especially near toward the above equation, hence towards simulate the stock prices in this model. SDE or Stochastic Differential Equation of S_t (a stochastic process) is preferred.

The Stochastic Differential Equation implies that

$$S_t = \mu S_t t + \sigma S_t W_t \quad (6)$$

Where, S_t - stochastic process

μ - Percentage drift

σ -Percentage of volatility

W_t - Weiner's process or Brownian Motion

Let simulate the stock records from the equation by increasing interpretation.

$$S_t = S_{t-1} * ((\mu - (\frac{\sigma^2}{2})) * t + \sigma w_t) \quad (7)$$

Where, S_t .Stock price at time t

S_{t-1} .Stock price at time t-1

μ -Mean

σ - Mean daily volatility

t - Time interval of the step

w_t -Random normal noise.

2.4. RANDOM WALK

The random walk is executed in the stock market. A random walker on a straight road (which is located with a point on the real line R) takes a step of length δ at time intervals of τ , moving forwards or backwards with similar probability $\frac{1}{2}$ with the direction of the steps all independent [1].

Let $X_\tau(t)$ indicates the location of the walker at time t. Then, given the position $X_\tau(k\tau)$ at time $k\tau$, the position $X_\tau((k+1)\tau)$ at time $(k+1)\tau$ is equal to be $X_\tau(k\tau) + \delta$ or $X_\tau(k\tau) - \delta$. Considering that the walker begins at the origin at time 0, then for $t > 0$, the position at time t is expressed by the random variable [1].

$$X_\tau(t) = \delta(Y_1 + \dots + Y_{[t/\tau]}) \quad (8)$$

Where Y_1, Y_2, \dots are independent random variables, each taking the value of +1 with probability $\frac{1}{2}$ and -1 with probability $\frac{1}{2}$. Here $[t/\tau]$ denote the major integer less than or equal to t/τ . We normalise the step length by the parameter $\delta = \sqrt{\tau}$ so that

$$X_\tau(t) = \sqrt{\tau}(Y_1 + \dots + Y_{[t/\tau]}) \quad (9)$$

Then the mean and variance of the walker's positions at time t are

$$E(X_\tau(t)) = \sqrt{\tau}(E(Y_1 + \dots + Y_{[t/\tau]})) = 0 \text{ and} \quad (10)$$

$$\text{var}(X_\tau(t)) = \tau(\text{var}(Y_1) + \dots + \text{var}(Y_{[t/\tau]})) = \tau [t/\tau] \sim t \quad (11)$$

If t is large compared with τ . Thus over a large number of random walks the average position at time t is 0 and the typical distance of the walker from the origin will be around \sqrt{t} , the standard deviations of $X_\tau(t)$. Random walk finds the defective value of the stock prices for the annual period of 2013-2014 and 2017.

2.5 POISSON DISTRIBUTION

Poisson distribution is a discrete probability distribution and are most commonly in statistical work [9]. Poisson distribution are used in cases, in which the chance of any single incident even though a success is small. Poisson distribution is applied to calculate the drift of the defective value.

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad (12)$$

Where $X=0, 1, 2, 3, 4, \dots$

$e = 2.7183$ (the base of natural Logarithms)

$\lambda =$ Mean.

III. RESULTS

Prediction of stock prices plays an important role in share market, the stock price prediction based on Random walk based theory is analysed. This theory effectively analysis the forecasting of stock prices. Two years of stock prices was compared all together to find the instability. Geometric Brownian Motion method is also used to analyze the costs of the stocks. The defectives values are found for various month and shown in table 1 and table 2. The Actual and forecast defective value in the year 2013 - 2014 are tabulated in table 1. Similarly, the actual and forecast defective value in the year 2017 (Jan to Nov) are tabulated in table 2. From the Poisson distribution minimum and maximum defective value can be found.

CASE 1: THE ACTUAL DEFECTIVE FOR THE YEAR 2013-2014

Random walk for defective value is ranging from 150 to 200. Its dimension can be calculated by Box counting method and its D_f is 1.7784 and is shown in the table 3 and fig 3a.

Random walk for the defective value ranges from 200 to 950. Its Box Counting dimension D_f is 1.9077 and is shown in the table 3 and fig 3b.

In the table 3, fig 3c shows the Random walk of the defective value is ranging from 950 to 720 and its dimension D_f is 1.9603.

Random walk for defective value is ranging from 720 to 800. Its dimension can be calculated its D_f is 1.9101 and is shown in the table 3 and fig 3d.

In the table 3, fig 3e shows the random walk of the defective value is ranging from 800 to 560 and its dimension D_f is 1.8376.

Random walk for defective value is ranging from 560 to 500. Its dimension can be calculated by Box counting method and its D_f is 1.9043 and is shown in the table 3 and fig 3f.

In the table 3, fig 3g shows the Random walk of the defective value ranges from 500 to 450 and its dimension D_f is 1.962.

Mean value is obtained and Poisson distribution is calculated by the equation (12). The maximum defective value is 4.6933×10^{-233} and minimum defective value is 2.3414×10^{-220} .

CASE 2: THE FORECAST DEFECTIVE FOR THE YEAR 2013-2014

Random walk for defective value is ranging from 210 to 380. Its dimension can be calculated by Box counting method and its D_f is 1.868 and is shown in the table 4 and fig 4a.

In the table 4, fig 4b shown the Random Walk for the defective value ranges from 380 to 590 and its dimension D_f is 1.8419.

Random walk for defective value is ranging from 590 to 650. Its dimension can be calculated by Box counting method and its D_f is 1.8794 and is shown in the table 4 and fig 4c.

In the table 4, fig 4d shows the Random walk of the defective value is ranging from 650 to 700 and its dimension D_f is 1.9294.

Random walk for defective value is ranging from 700 to 710. Its dimension can be calculated by Box counting method and its D_f is 1.953 and is shown in the table 4 and fig 4e.

In the table 4, fig 4f shows the Random walk of the defective value is ranging from 710 to 600 and its dimension D_f is 1.9069.

Random walk for the defective value ranges from 600 to 590. Its dimension can be calculated by Box counting method and its D_f is 1.8537 and is shown in the table 4 and fig 4g.

Mean value is obtained and Poisson distribution is calculated by the equation (12). The maximum defective value is 1.7895×10^{-238} and minimum defective value is 1.2942×10^{-227} .

CASE 1: THE ACTUAL DEFECTIVE FOR THE YEAR 2017

Random walk for defective value is ranging from 220 to 300. Its dimension can be calculated by Box counting method and its D_f is 1.8111 and is shown in the table 5 and fig 5a.

In the table 5, fig 5b shows the Random walk of the defective value ranges from 300 to 210 and its dimension D_f is 1.8006.

Random walk for defective value is ranging from 210 to 400. Its dimension can be calculated by Box counting method and its D_f is 1.8797 and is shown in the table 5 and fig 5c.

In the table 5, fig 5d shows the Random walk of the defective value is ranging from 400 to 320 and its dimension D_f is 1.7932.

Random walk for defective value is ranging from 320 to 310. Its dimension can be calculated by Box counting method and its D_f is 1.7215 and is shown in the table 5 and fig 5e.

Mean value is obtained and Poisson distribution is calculated by the equation (12). The maximum defective value is 1.2267×10^{-125} and minimum defective value is 75649×10^{-118} .

CASE 2: THE FORECAST DEFECTIVE FOR THE YEAR 2017

Random walk for defective value is ranging from 210 to 380. Its dimension can be calculated by Box counting method and its D_f is 1.868 and is shown in the table 6 and fig 6a.

In the table 6, fig 6b shows the Random walk of the defective value ranges from 380 to 390 and its dimension D_f is 1.8399.

Random walk for defective value is ranging from 390 to 395. Its dimension can be calculated by Box counting method and its D_f is 1.8517 and is shown in the table 6 and fig 6c.

In the table 6, fig 6d shows the Random walk of the defective value is ranging from 395 to 592 and its dimension D_f is 1.8537.

Random walk for defective value is ranging from 592 to 850. Its dimension can be calculated by Box counting method and its D_f is 1.8732 and is shown in the table 6 and fig 6e.

Mean value is obtained and Poisson distribution is calculated by the equation (12). The maximum defective value is 5.8936×10^{-202} and minimum defective value is 2.3864×10^{-193} .

IV. CONCLUSIONS

In this paper, Stock prices can be analysed by Geometric Brownian motion based on Random Walk and its dimension was calculated by Box Counting Method. The dimension of the Random Walk gives defective value of the Stock prices is non-integer. Hence it is fractal set. This value cannot be

predicted. This creates the possibility that the fractal measurement is related with the monetary unpredictability.

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Table 3. Box Counting Dimension of the Actual defective value in the Year 2013-2014.

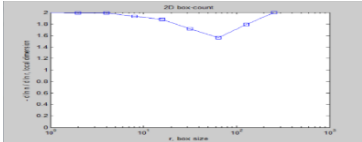
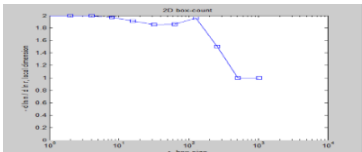
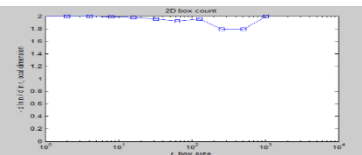
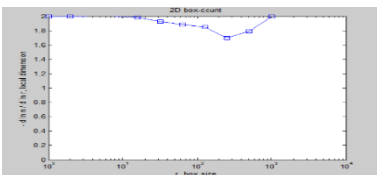
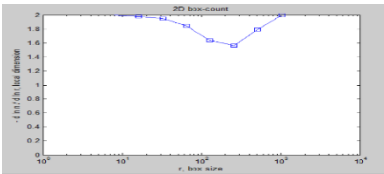
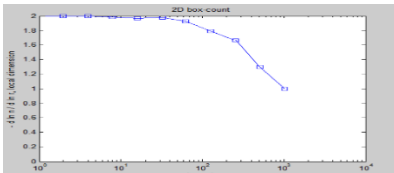
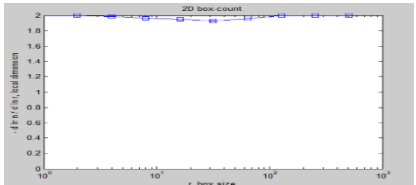
S.No	Range/Month	D _f	Figure
1	150-200/ Oct-Nov	1.7784	 <p>Figure 3a</p>
2	200-950/ Nov-Dec	1.9077	 <p>Figure 3b</p>
3	950-720/ Dec-Jan	1.9603	 <p>Figure 3c</p>
4	720-800/ Jan-Feb	1.9101	 <p>Figure 3d</p>
5	800-560/ Feb-Mar	1.8376	 <p>Figure 3e</p>
6	560-500/ Mar-Apr	1.9043	 <p>Figure 3f</p>
7	500-450/ Apr-May	1.962	 <p>Figure 3g</p>

Table 4. Box Counting Dimension of the Forecast defective value in the Year 2013-2014.

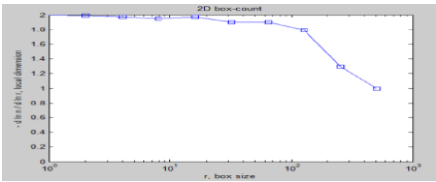
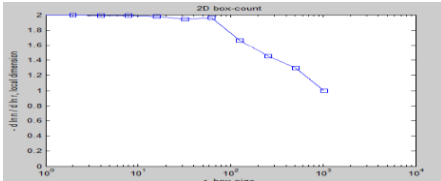
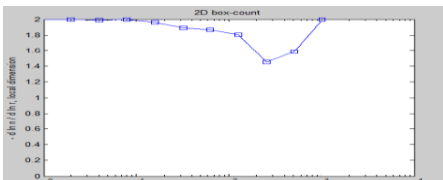
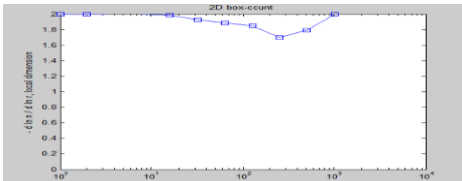
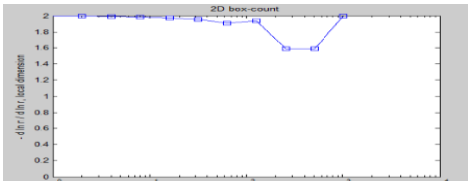
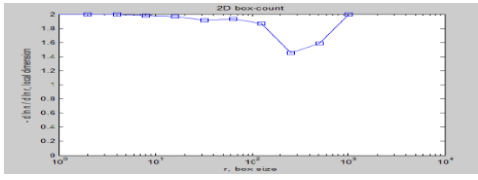
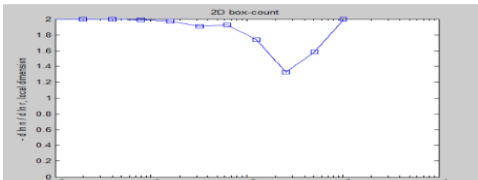
S.No	Range/month	D_f	Figures
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3	590-650/ Dec-Jan	1.8794	 <p>Figure 4c</p>
4	650-700/ Jan-Feb	1.9294	 <p>Figure 4d</p>
5	700-710/ Feb-Mar	1.953	 <p>Figure 4e</p>
6	710-600/ Mar-Apr	1.9069	 <p>Figure 4f</p>
7	600-590/ Apr-May	1.8537	 <p>Figure 4g</p>

Table 5. Box Counting Dimension of the Actual defective value in the 2017.

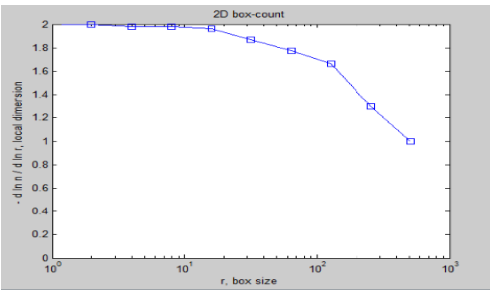
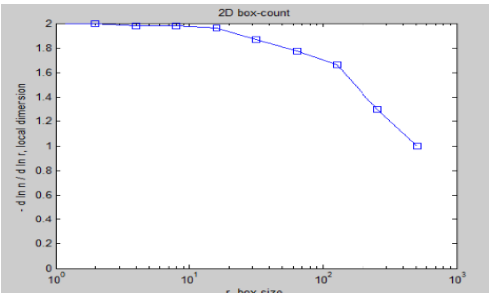
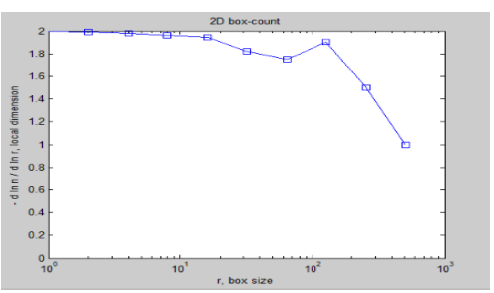
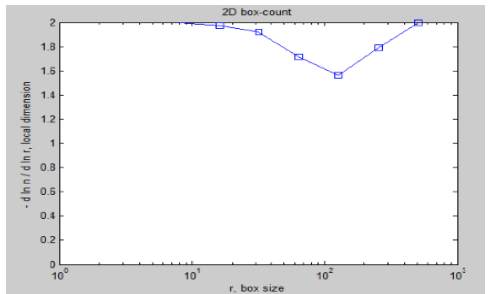
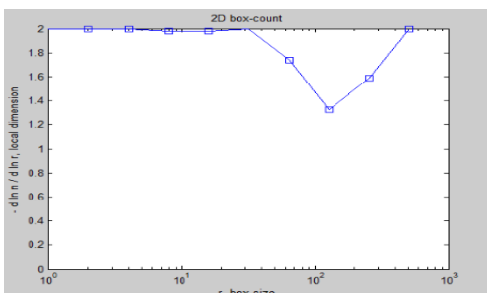
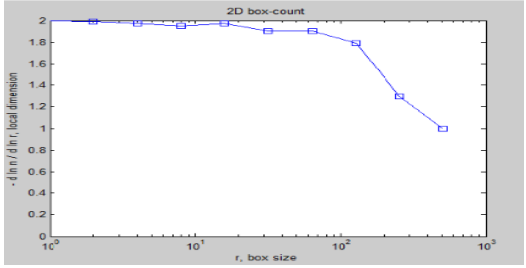
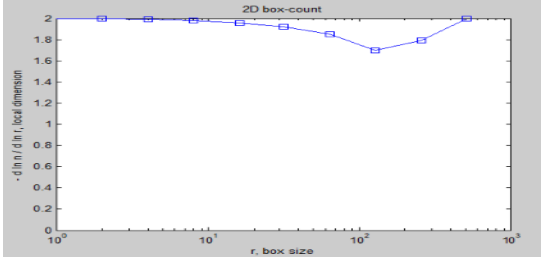
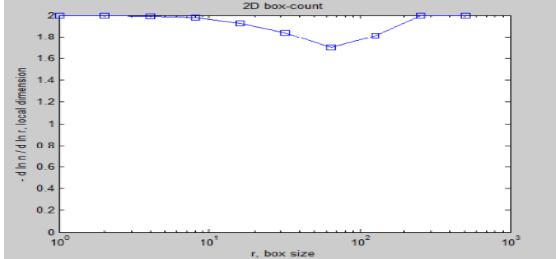
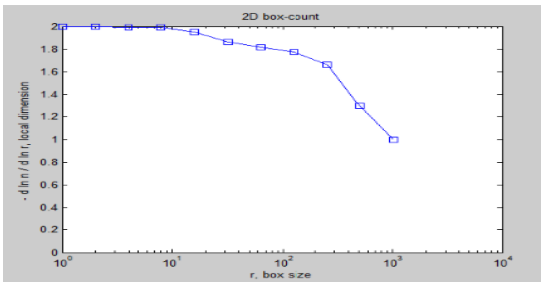
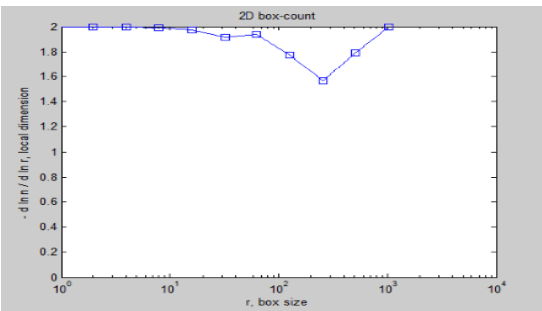
S.No	Range/Month	D_f	Figure
1	220-300/ Jun-July	1.8111	 <p>Figure 5a</p>
2	300-210/ July-Aug	1.8006	 <p>Figure 5b</p>
3	210-400/ Aug- sep	1.8797	 <p>Figure 5c</p>
4	400-320/ Sep-Oct	1.7932	 <p>Figure 5d</p>
5	320-310/ Oct-Nov	1.7215	 <p>Figure 5f</p>

Table 6. Box Counting Dimension of the Forecast defective value in the Year 2017.

S.No	Range/Month	D_f	Figure
1	210-380/ Jun-July	1.868	 <p>Figure 6a</p>
2	380-390/ July-Aug	1.8399	 <p>Figure 6b</p>
3	390-395/ Aug-sep	1.8517	 <p>Figure 6c</p>
4	395-592/ Sep-Oct	1.8537	 <p>Figure 6d</p>
5	592-850/ Oct-Nov	1.8732	 <p>Figure 6e</p>