Workshop 3

COMP90051 Statistical Machine Learning

Semester 1, 2023

Learning outcomes

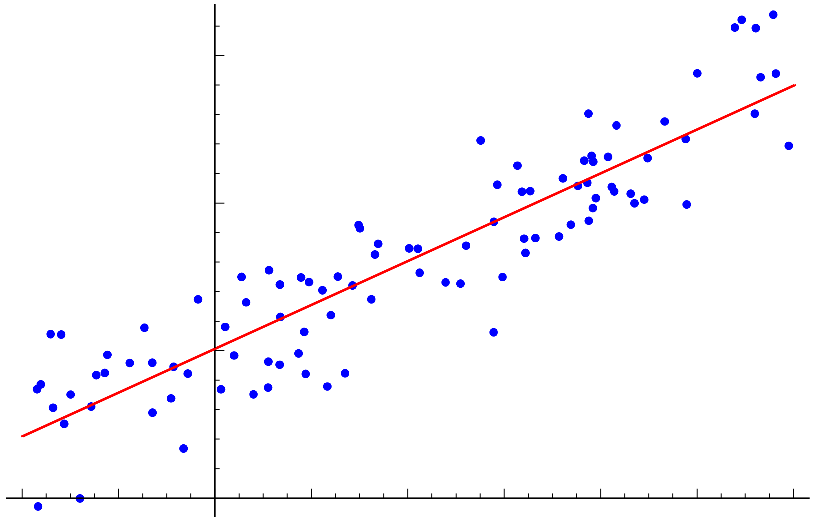
At the end of this workshop you should:

- be able to implement linear regression and logistic regression
- be able to explain how the optimisation problems for linear regression and logistic regression differ
- be able to implement gradient descent
- Optional: IRLS algorithm

Linear regression

Assume the response y is a *linear* function of the features $\mathbf{x} = [x_1, \ldots, x_m]^{\mathrm{T}}$:

$$y = w_0 + \sum_{i=1}^{m} w_i \cdot x_i$$



Write this more compactly as $y = \mathbf{x}^{\mathrm{T}}\mathbf{w}$ by redefining $\mathbf{x} = [x_0, x_1, \ldots, x_m]^{\mathrm{T}}$ with $x_0 = 1$ and defining $\mathbf{w} = [w_0, \ldots, w_m]^{\mathrm{T}}$

If we encodes noise: $y = \mathbf{x}^{\mathrm{T}}\mathbf{w} + \boldsymbol{\varepsilon}$

Question: How do we choose the weights?

Solving linear regression

Decision theoretic view

Make decision that minimises the empirical risk

$$\hat{R} = \frac{1}{n}\sum_{i=1}^{n} L(y_i, \hat{y}_i)$$

and choose the square loss $L(y, \hat{y}) = (\hat{y} - y)^2$.

Optimal decision for $\mathbf{w}$ minimises the sum-squared error.

Probabilistic view

Assume

$$y|\mathbf{x}, \mathbf{w} \sim \mathcal{N}(\mathbf{x}^{\mathrm{T}}\mathbf{w}; \sigma^2)$$

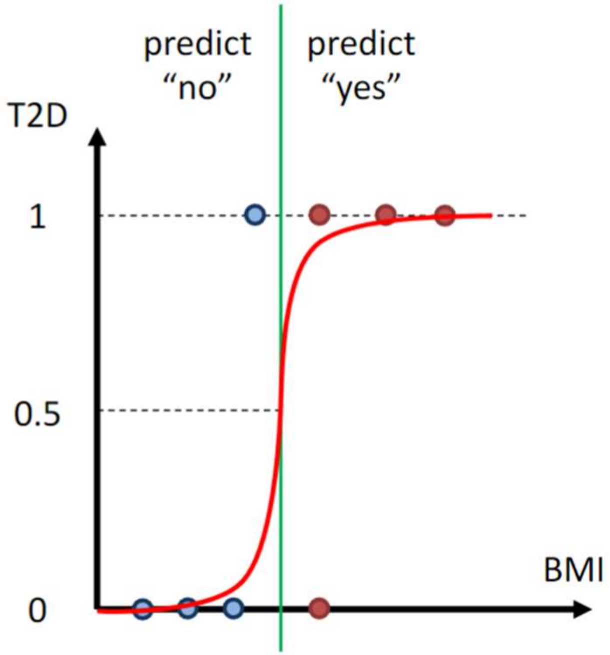
Can write down the likelihood for the observations

$$L(w|\boldsymbol{X}, \boldsymbol{Y}) = \prod_{i=1}^{n} p(y_i|\mathbf{x}_i, \mathbf{w}, \sigma)$$

MLE for $\mathbf{w}$ minimises the sum-squared error.

Logistic regression

- Logistic regression is a **linear binary (could be extend to multi-class) classifier** for **classification** task
- Linear regression: gives a continuous value of **output y** for a given input X.
- Logistic regression: gives a continuous value of **P(Y=1)** for a given input X, which is later converted to Y=0 or Y=1 based on a threshold value.



Solving logistic regression

Logistic regression optimisation problem:

$$\mathbf{w}^{\star} \in \arg\min_{\mathbf{w}} \frac{1}{n}\sum_{i=1}^{n} \ell(y_i, \mu_i)$$

where $\mu_i = \frac{1}{1+e^{-\mathbf{x}_i^{\top}\mathbf{w}}}$ and $\ell(y, \mu) = -y \log \mu - (1-y)\log(1-\mu)$

Unfortunately, no closed form solution, need to use optimization techniques:

* Gradient Descent: easy to compute, slow to converge
* IRLS (optional): hard to compute, quick to converge

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