

Homework #6

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Problem 4

Write an algorithm that takes an integer n as input and determines the total number of solutions to the n -Queens problem.

Solution

The n -queens problem involves placing n queens on a $n \times n$ chessboard such that no two queens threaten each other.

Two queens do not threaten each other if:

- if 2 queens are in the same row
- if 2 queens are in the same column
- if 2 queens are in the same diagonal

We need two separate functions- one to place the queen in a particular position and another to determine all combinations of the solutions.

Algorithm: Determine if a queen can be placed at a particular position

Input: k (int)- represents the k th queen

Output: Returns 1 if the queen can be placed and 0 if it cannot

Algorithm 1 Place(k, i)

```
1: for  $j = 1$  to  $k - 1$  do
2:   if  $x[j] = i$  or  $|j - k| = |x[j] - i|$  then
3:     return 0
4:   end if
5: end for
6: return 1
```

Algorithm: Function to determine the number of solutions to the n -queens problem

Input: n (int)- represents the number of queens and the size of the chessboard

Output: Number of solutions

Algorithm 2 NQueens(n)

```

1:  $k \leftarrow 1$ 
2:  $x[k] \leftarrow 0$ 
3:  $solution\_count \leftarrow 0$  ▷ Initialize counter for number of solutions
4: while  $k > 0$  do
5:    $x[k] \leftarrow x[k] + 1$ 
6:   while  $x[k] \leq n$  and not PLACE( $k, x[k]$ ) do
7:      $x[k] \leftarrow x[k] + 1$ 
8:   end while
9:   if  $x[k] \leq n$  then
10:    if  $k == n$  then
11:       $solution\_count \leftarrow solution\_count + 1$  ▷ Increment solution count
12:    else
13:       $k \leftarrow k + 1$ 
14:       $x[k] \leftarrow 0$ 
15:    end if
16:  else
17:     $k \leftarrow k - 1$ 
18:  end if
19: end while
20: return  $solution\_count$  ▷ Return the total number of solutions found

```

Problem 10

Find at least two instances of the n -Queens problem that have no solutions.

Solution

Problem

State the problem here.

Solution