1 Electrostatics

Electric Charge

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

Types of Charge

I. **Positive charge:** It is the deficiency of electrons as compared to proton.

II. **Negative charge:** It is the excess of electrons as compared to proton.

SI unit of charge: ampere × second i.e. Coulomb, Dimension: [A T]

Practical units of charge are ampere × hour (= 3600 C) and faraday (= 96500 C)

Specific Properties of Charge

- **Charge is a Scalar Quantity:** It represents excess or deficiency of electrons.

- **Charge is Transferable**

  If a charged body is put in contact with an another body, then charge can be transferred to another body.

- **Charge is Always Associated with Mass**

  Charge cannot exist without mass though mass can exist without charge.
  - So, the presence of charge itself is a convincing proof of existence of mass.
  - In charging, the mass of a body changes.
  - When body is given positive charge, its mass decreases.
  - When body is given negative charge, its mass increases.

- **Charge is Quantised**

  The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus, charge q of a body is always given by

  $q = ne$ $\qquad$ n = positive integer or negative integer

  The quantum of charge is the charge that an electron or proton carries.

  Note: Charge on a proton = (–) charge on an electron = 1.6×10^{-19} C

- **Charge Is Conserved**

  In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.

- **Charge Is Invariant**

  Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.

- **Attraction – Repulsion**

  Similar charges repel each other while dissimilar attract

Methods of Charging

- **Friction**

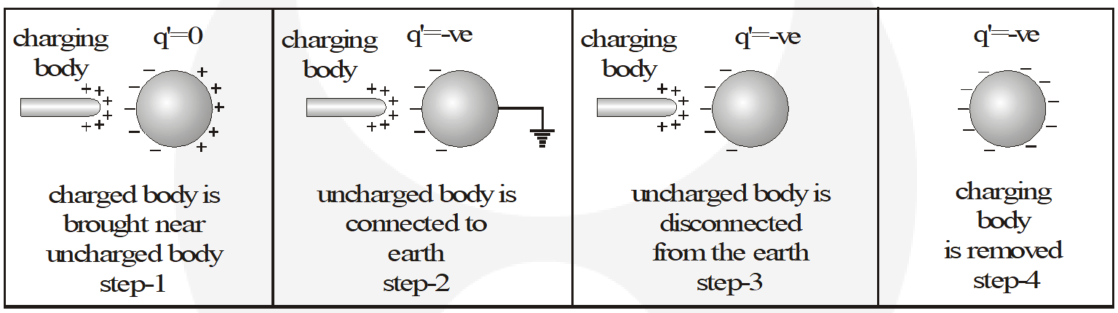
  If we rub one body with other body, electrons are transferred from one body to the other.

Positive charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic objects
Dry hair	Comb
Flannel or cat skin	Ebonite rod
Note : Clouds become charged by friction	

- **Electrostatic Induction**

  If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

  Charging a body by induction (in four successive steps)



Some important facts associated with induction-

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non-conducting) and not in particles.

- **Conduction**

  The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

  - The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
  - The charge gained by the uncharged body is always lesser than initial charge presents on the charged body.
  - Flow of charge depends upon the potential difference of both bodies. [No potential difference $\Rightarrow$ No conduction].
  - Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.

Key Points

- Charge differs from mass in the following sense.
  (i) In SI units, charge is a derived physical quantity while mass is fundamental quantity.
  (ii) Charge is always conserved but mass is not (Note: Mass can be converted into energy $E = mc^2$
  (iii) The quanta of charge is electronic charge while that of mass it is yet not clear.
  (iv) For a moving charged body mass increases while charge remains constant.
- True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body and also between two similarly charged bodies.
- For a non-relativistic (i.e. v << c) charged particle, specific charge $\frac{q}{m}$ = constant

Example:

When a piece of polythene is rubbed with wool, a charge of -2×10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to polythene.

Solution:

From Q = ne, So, the number of electrons transferred $n = \frac{Q}{e} = \frac{2\times10^{-7}}{1.6\times10^{-19}} = 1.25 \times 10^{12}$

Now mass of transferred electrons = n × mass of one electron = $1.25 \times 10^{12} \times 9.1 \times 10^{-31}$
$= 11.38 \times 10^{-19}$ kg

Example:

10^{12} α – particles (Nuclei of helium) per second falls on a neutral sphere, calculate time in which sphere gets charged by $2\mu C$.

Solution:

Number of α – particles falling in t second = $10^{12}t$

Charge on α – particle = +2e, So charge incident in time t = $(10^{12}t).(2e)$

$\because$ Given charge is $2\ \mu C$

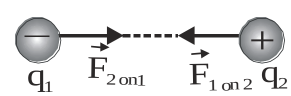
$\therefore\ 2 \times 10^{-6} = (10^{12}t).(2e) \Rightarrow t = \frac{10^{-18}}{1.6\times10^{-19}} = 6.25s$

Coulomb's Law

The electrostatic force of interaction between two **static point electric charges** is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

If two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law.

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

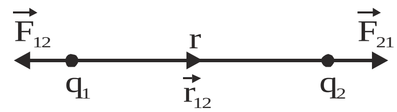


$F_e = \frac{kq_1q_2}{r^2}$ where $k = \frac{1}{4\pi \epsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$, coulomb's constant or electrostatic force constant.

Coulomb's Law in Vector Form

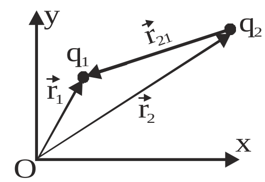
$\vec{F}_{12}$ = force on q_1 due to q_2 = $\frac{kq_1q_2}{r^2}$

$\vec{F}_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)



Coulomb's Law in Terms of Position Vector

$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$



Principle of Superposition

The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...$

Note: Nuclear force is many body interactions, so principle of superposition is not valid in case of nuclear force.

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

$F = \frac{kq_0q_1}{r_1^2}\hat{r}_{10} + \frac{kq_0q_2}{r_2^2}\hat{r}_{20} + \cdots + \frac{kq_0q_i}{r_i^2}\hat{r}_{i0} + \ldots \frac{kq_0q_n}{r_n^2}\hat{r}_{n0}$ in vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_{i0}$

Some Important Points Regarding Coulomb's Law and Electric Force

- The force is conservative, i.e., work done in moving a point charge once round a closed path under the action of Coulomb's force is zero.
- The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

  $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ and $F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$. So $\frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K$,
- Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.
- The law expresses the force between two point charges at rest. This law is not applicable for bodies of finite size

  Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.
- Electric force between two charges does not depend on neighboring charges.

Example:

If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Solution:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{r^2} \text{ ....(i) Again, } F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2} \text{ or } F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$$

So, the force will remain the same.

Example:

A particle of mass m carrying charge '$+q_1$' is revolving around a fixed charge '$-q_2$' in a circular path of radius r. Calculate the period of revolution.

Solution:

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$$

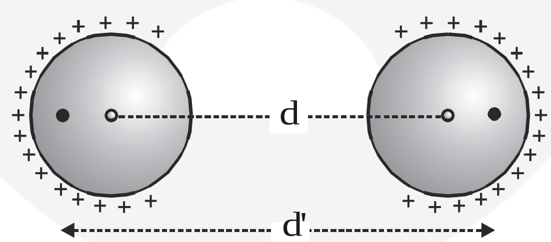
$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2} \text{ or } T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

Example:

The force of repulsion between two point charges is F, when these are at a distance of 1 m. Now the point charges are replaced by conducting spheres of radii 25 cm having the charge same as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in two cases.

Solution:

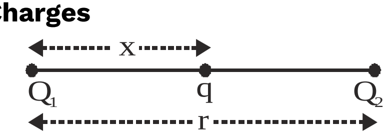
In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (d' > d) so force of repulsion decreases as $F \propto \frac{1}{d^2}$



Equilibrium of Charged Particles

In equilibrium net electric force on every charged particle is zero. The equilibrium of a charged particle, under the action of Colombian forces alone can never be stable.

- **Equilibrium of Three Point Charges**

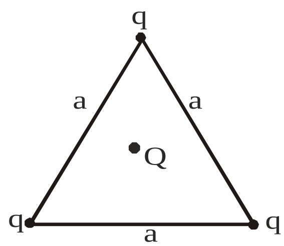


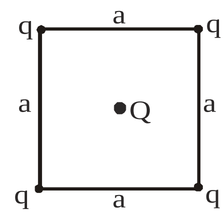
(i) Two charges must be of like nature as $F_q = \frac{KQ_1q}{x^2} + \frac{KQ_2q}{(r-x)^2} = 0$

(ii) Third charge should be of unlike nature as $F_{Q_1} = \frac{KQ_1Q_2}{r^2} + \frac{KQ_1q}{x^2} = 0$

- **Equilibrium of Symmetric Geometrical Point Charged System**

  Value of Q at centre for which system to be in state of equilibrium

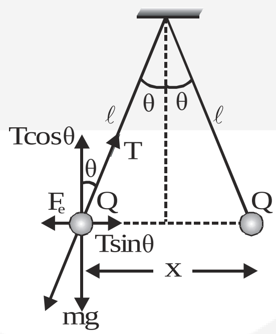




(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$ (ii) For square $Q = \frac{-q(2\sqrt{2}+1)}{4}$

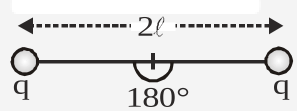
- **Equilibrium of Suspended Point Charge System**

  For equilibrium position



$T\cos\theta = mg$ and $T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$

If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) then $T = F_e = \frac{kq^2}{4\ell^2}$



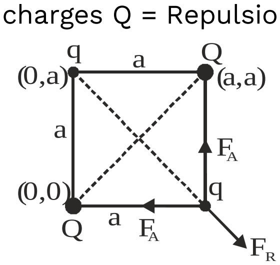
Example:

For the system shown in figure find Q for which resultant force on q is zero.

Solution:

For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

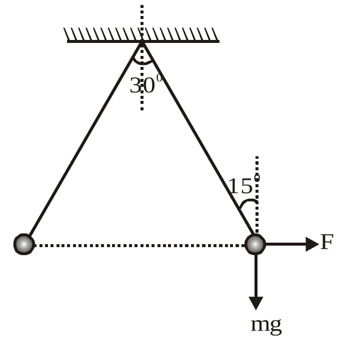


$\sqrt{2}\ F_A = F_R \Rightarrow \sqrt{2}\ \frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2}$

$\Rightarrow q = 2\sqrt{2}\ Q$ Hence $q = -2\sqrt{2}\ Q$

Example:

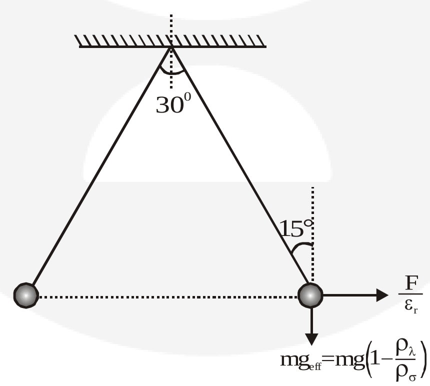
Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.



Solution:

When set up shown in figure is in air, we have tan 15° = $\frac{F}{mg}$ When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\frac{F}{\varepsilon_r}$ and the effective gravitational force will become mg$\left(1-\frac{\rho_\ell}{\rho_s}\right)$ Thus we have tan 15° =

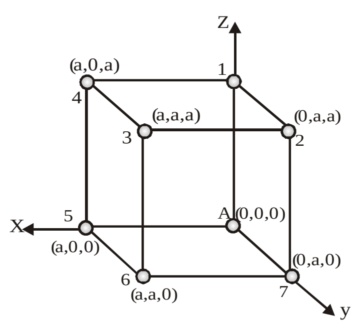
$$\frac{F}{mg \in_r \left(1-\frac{\rho_\ell}{\rho_s}\right)} = \frac{F}{mg} \Rightarrow \in_r = \frac{1}{1-\frac{\rho_\ell}{\rho_s}} = 2$$



Example:

Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Solution:



The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law

$$\vec{F} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

From the figure the different forces acting on A are given as

$$\vec{F}_{A_1} = \frac{kq^2(-a\hat{k})}{a^3}$$

$$\vec{F}_{A_2} = \frac{kq^2(-a\hat{j} - a\hat{k})}{(\sqrt{2}a)^3},\ \vec{F}_{A_3} = \frac{kq^2(-a\hat{i} - a\hat{j} - a\hat{k})}{(\sqrt{3}a)^3};\ \vec{F}_{A_4} = \frac{kq^2(-a\hat{i} - a\hat{k})}{(\sqrt{2}a)^3}$$

$$\vec{F}_{A_5} = \frac{kq^2(-a\hat{i})}{a^3},\ \vec{F}_{A_6} = \frac{kq^2(-a\hat{i} - a\hat{j})}{(\sqrt{2}a)^3},\ \vec{F}_{A_7} = \frac{kq^2(-a\hat{j})}{a^3}$$

The net force experienced by A can be given as

$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)(\hat{i} + \hat{j} + \hat{k})\right]$$

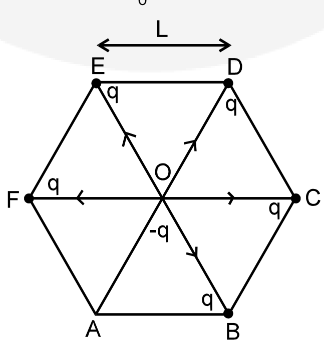
Example:

Five point charges, each of value +q are placed on five vertices of a regular hexagon of side Lm. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?

Solution:

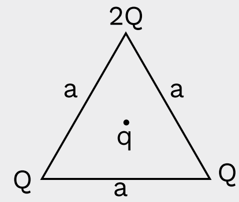
If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O will be zero (as the forces due to individual charges will balance each other). Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$



Concept Builders-1

Q.1 Find force on charge q placed at the centre of the equilateral triangle as shown.



Q.2 Two charges Q and 4Q are placed at a separation d, Find the position of a third charge q between them such that it does not experience any force.

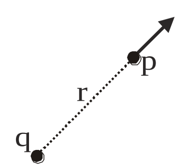
Electric Field

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge. Electric field at a point is characterized either by a vector function of position $\vec{E}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric intensity, potential and lines of force are different ways of describing the same field.

Intensity of Electric Field Due to Point Charge

Electric field intensity is defined as force on unit test charge.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}$$



Note: Test charge (q_0) is a fictitious charge and its value is kept small so that it does not change the original charge configuration.

Properties of Electric Field Intensity

(i) It is a vector quantity. Its direction is the same as the force experienced by positive test charge.

(ii) Electric field due to positive charge is always away from it while due to negative charge always towards it.

(iii) Its unit is Newton/coulomb

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Force on a point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

$\vec{F} = q\vec{E}$

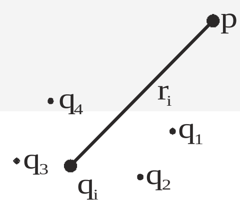
(vi) It obeys the superposition principle that is the field intensity point due to charge distribution is vector sum of the field intensities due to individual charges

Key Points

- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric, electric field decreases and becomes $\frac{1}{\epsilon_r}$ times of its value in free space.
- Test charge is always a positive charge. $\vec{E} = \frac{\vec{F}_{test}}{\text{test charge}}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ at centre = zero.

Electric Field Intensities Due to Various Charge Distributions

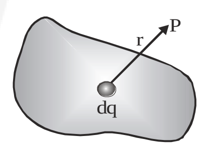
Due to discrete distribution of charge



By principle of superposition intensity of electric field due to ith charge $\vec{E}_{ip} = \frac{kq}{r_i^3}\vec{r}_i$

$\therefore$ Net electric field due to whole distribution of charge $\vec{E}_p = \sum_{i=1} \vec{E}_i$

Continuous Distribution of Charge



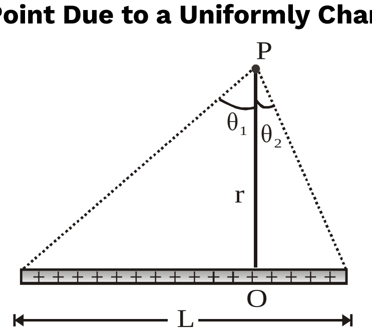
Treating a small element as particle $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^3}\vec{r}$

Due to linear charge distribution $E = k\int_{\ell} \frac{\lambda d\ell}{r^2}$ [λ = charge per unit length]

Due to surface charge distribution $E = k\int_{s} \frac{\sigma ds}{r^2}$ [σ = charge per unit area]

Due to volume charge distribution $E = k\int_{v} \frac{\rho dv}{r^2}$ [ρ = charge per unit volume]

Electric Field Strength at a General Point Due to a Uniformly Charged Rod



As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, we consider an element on rod of length dx at a distance x from point O as shown in figure. Now if dE be the electric field at P due to the element, then

$dE = \dfrac{kdq}{\left(x^2 + r^2\right)}$ Here $dq = \dfrac{Q}{L}dx$

$dE_x = dE \sin\theta = \left[\dfrac{kdq}{\left(x^2+r^2\right)}\right] \sin\theta = \dfrac{kQ\sin\theta}{L\left(x^2+r^2\right)}dx$

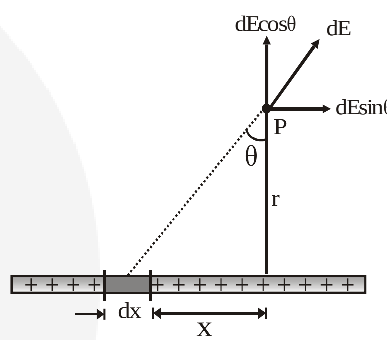
Here we have $x = r \tan\theta$ and $dx = r \sec^2\theta d\theta$

Thus $dE_x = \dfrac{kQ}{L} \dfrac{r\sec^2\theta\, d\theta}{r^2\sec^2\theta}\sin\theta = \dfrac{kQ}{Lr} \sin\theta d\theta$

$\Rightarrow E_x = \int dE_x = \dfrac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\sin\theta\, d\theta = \dfrac{kQ}{Lr}\left[-\cos\theta\right]_{-\theta_2}^{+\theta_1} = \dfrac{kQ}{Lr}[\cos\theta_2 - \cos\theta_1]$

Similarly, electric field strength at point P due to dq in y−direction is

$dE_y = dE\cos\theta = \dfrac{kQdx}{L\left(r^2+x^2\right)} \times \cos\theta$



Again we have $x = r \tan\theta$ and $dx = r \sec^2\theta d\theta$. Thus we have

$dE_y = \dfrac{kQ}{L}\cos\theta \times \dfrac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \dfrac{kQ}{Lr}\cos\theta\, d\theta$

$\Rightarrow E_y = \int dE_y = \dfrac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta\, d\theta = \dfrac{kQ}{Lr}\left[+\sin\theta\right]_{-\theta_2}^{+\theta_1}$

$= \dfrac{kQ}{Lr}[\sin\theta_1 + \sin\theta_2]$

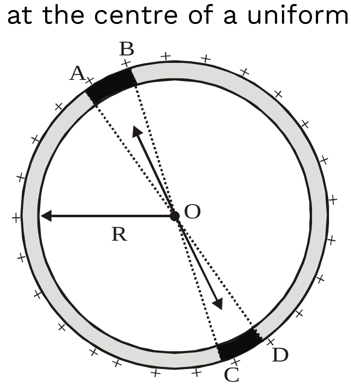
Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

in x−direction: $E_x = \dfrac{kQ}{Lr}(\cos\theta_1 - \cos\theta_2)$ and in y−direction $E_y = \dfrac{kQ}{Lr}(\sin\theta_1 + \sin\theta_2)$

Electric Field Due to a Uniformly Charged Ring

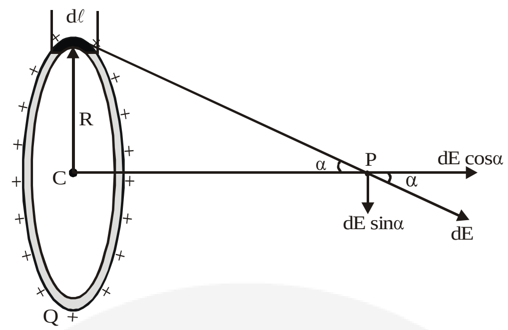
- **Case – I: At its centre**

  Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is $E_0 = 0$



- **Case II: At a point on the axis of Ring**

  Here we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length $d\ell$ on ring as shown. The charge on this elemental section is $dq = \frac{Q}{2\pi R} d\ell$ [Q= total charge of ring]



Due to the element $d\ell$, electric field strength dE at point P can be given as $dE = \frac{Kdq}{\left(R^2 + x^2\right)}$

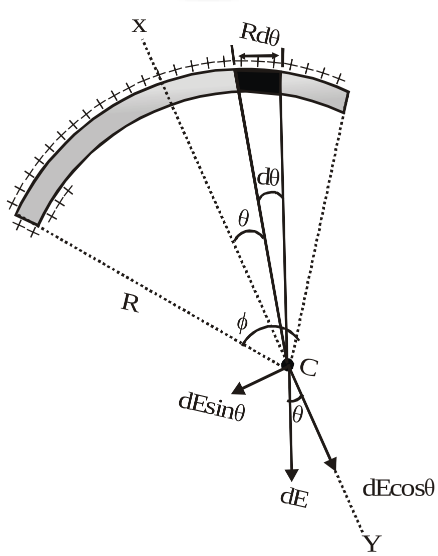
The component of this field strength dE sinα which is normal to the axis of ring will be cancelled out due to the ring section opposite to $d\ell$. The component of electric field strength along the axis of ring dE cosα due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \oint dE \cos\alpha = \oint \frac{kdq}{\left(R^2 + x^2\right)} \times \frac{x}{\sqrt{R^2 + x^2}} = \oint \frac{kQx}{2\pi R\left(R^2 + x^2\right)} d\ell = \frac{kQx}{2\pi R\left(R^2 + x^2\right)^{3/2}} \oint d\ell$$

$$= \frac{kQx}{2\pi R\left(R^2 + x^2\right)^{3/2}} \left[2\pi R\right] = \frac{kQx}{\left(R^2 + x^2\right)^{3/2}}$$

Electric Field Strength Due to a Charged Circular arc at its Centre

Figure shows a circular arc of radius R which subtend an angle ϕ at its centre. To find electric field strength at C, we consider a polar segment on arc of angular width $d\theta$ at an angle θ from the angular bisector XY as shown.



The length of elemental segment is $Rd\theta$, the charge on this element $d\ell$ is $dq = \left(\frac{Q}{\phi}\right).d\theta$

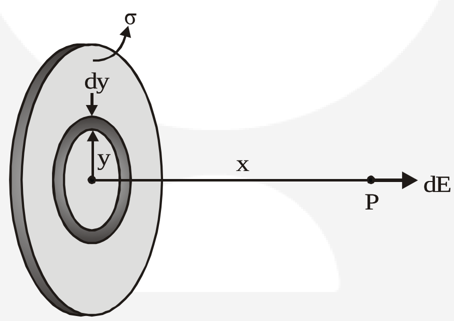
Due to this dq, electric field at centre of arc C is given as $dE = \frac{kdq}{R^2}$

Now electric field component due to this segment $dE\sin\theta$ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector which can be calculated by integrating $dE\cos\theta$ within limits from $-\frac{\phi}{2}$ to $\frac{\phi}{2}$. Hence net electric field strength at centre C is $E_C = \int dE\cos\theta$

$$= \int_{-\phi/2}^{+\phi/2} \frac{kQ}{\phi R^2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\int_{-\phi/2}^{+\phi/2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\left[\sin\theta\right]_{-\phi/2}^{+\phi/2} = \frac{kQ}{\phi R^2}\left[\sin\frac{\phi}{2}+\sin\frac{\phi}{2}\right] = \frac{2kQ\sin\left(\frac{\phi}{2}\right)}{\phi R^2}$$

Electric Field Strength Due to a Uniformly Surface Charged Disc

If there is a disc of radius R, charged on its surface with surface charge density σ, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.



To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. The charge on this elemental ring $dq = \sigma.2\pi y dy$ [Area of elemental ring $ds = 2\pi y\ dy$]

Now we know that electric field strength due to a ring of radius R, charge Q, at a distance x from its centre

on its axis can be given as $E = \frac{kQx}{\left(x^2+R^2\right)^{3/2}}$

Here due to the elemental ring electric field strength dE at point P can be given as

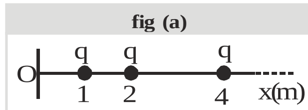
$$dE = \frac{kdqx}{\left(x^2+y^2\right)^{3/2}} = \frac{k\sigma 2\pi y\ dy\ x}{\left(x^2+y^2\right)^{3/2}}$$

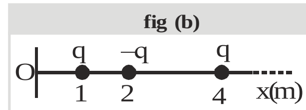
Net electric field at point P due to this disc is given by integrating above expression from 0 to R as

$$E = \int dE = \int_0^R \frac{k\sigma 2\pi xydy}{\left(x^2+y^2\right)^{3/2}} = k\sigma\pi x\int_0^R \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}} = 2k\sigma\pi x\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

Example:

Calculate the electric field at origin due to infinite number of charges as shown in figures below.





Solution:

(a) $E_0 = kq\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+----\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3}$ [$\because S_\infty = \frac{a}{1-r}$, a = 1 and r = $\frac{1}{4}$]

(b) $E_0 = kq\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}-----\right] = \frac{kq.1}{\left(1-\left(-1/4\right)\right)} = \frac{4kq}{5}$

Example:

A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.

Solution:

Let mass of the particle = m, Charge on particle = q

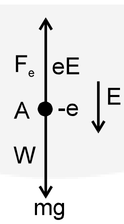
Intensity of electric field in between plates = E, Initially mg = qE

After reversing the field ma = mg + qE $\Rightarrow$ ma = 2mg

$\therefore$ Acceleration of particle $\Rightarrow$ a = 2g

Example:

Calculate the electric field intensity E which would be just sufficient to balance the weight of an electron. If this electric field is produced by a second electron located below the first one what would be the distance between them? [Given: e = 1.6 × 10^{-19} C, m = 9.1 × 10^{-31} kg and g = 9.8 m/s^2]



(A)

Solution:

As force on a charge e in an electric field E

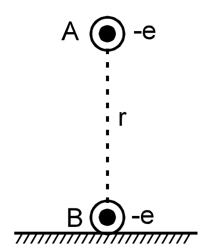
F_e = eE

So according to given problem

F_e = W $\Rightarrow$ eE = mg

$\Rightarrow E = \frac{mg}{e} = \frac{9.1\times10^{-31}\times9.8}{1.6\times10^{-19}}$ = 5.57 × 10^{-11} $\frac{V}{m}$

As this intensity E is produced by another electron B, located at a distance r below A.



(B)

$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \Rightarrow r = \sqrt{\frac{e}{4\pi\varepsilon_0 E}}$ So, $r = \left[\frac{9\times10^9\times1.6\times10^{-19}}{5.57\times10^{-11}}\right] \simeq 5$ m

Example:

A block having mass m = 4 kg and charge q = 50 μC is connected to a spring having a force constant k = 100 N/m. The block lies on a frictionless horizontal track and a uniform electric field E = 5 × 10^5 V/m acts on the system as shown in figure. The block is released from rest when the spring is unscratched (at x = 0)

(a) By what maximum amount does the spring expand?

(b) What is the equilibrium position of the block?

(c) Show that the block's motion is simple harmonic and determine the amplitude and time period of the motion.

Solution:

(a) As x increases, electric force qE will accelerate the block while elastic force in the spring kx will oppose the motion. The block will move away from its initial position x = 0 till it comes to rest, i.e., work done by the electric force is equal to the energy stored in the spring.

So, if x_{max} is maximum stretch of the spring.

$$\frac{1}{2}kx_{max}^2 = (qE)x_{max} \Rightarrow x_{max} = \frac{2qE}{k} \Rightarrow x_{max} = \frac{2\times(50\times10^{-6})\times(5\times10^5)}{100} = 0.5 \text{ m}$$

(b) In equilibrium position F_R = 0, so if x_0 is the stretch of the spring in equilibrium position

$$kx_0 = qE \Rightarrow x_0 = (qE/k) = \frac{1}{2}\, x_{max} = 0.25 \text{ m}$$

(c) If the displacement of the block from equilibrium position (x_0) is x, restoring force will be

$F = k(x \pm x_0) \mp qE = kx$ [ax $kx_0 = qE$]

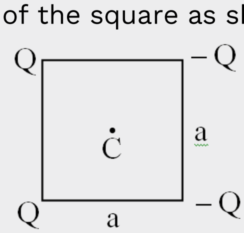
and as the restoring force is linear the motion will be simple harmonic with time period

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{100}} = 0.4\pi \text{ s}$$

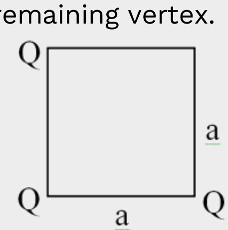
and amplitude = $x_{max} - x_0$ = 0.5 – 0.25 = 0.25 m

Concept Builders-2

Q.1 Find Electric field at the centre C of the square as shown:

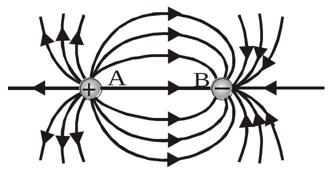


Q.2 Three equal charges (Q) are placed on three vertices of a square of side 'a' as shown. Find the magnitude of electric field at the remaining vertex.



Electric Lines of Force

Electric lines of electrostatic field have following properties



$q_A > q_B$

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_0)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_0.

(v) Lines of force ends or starts normally at the surface of a conductor.

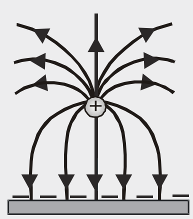
(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.

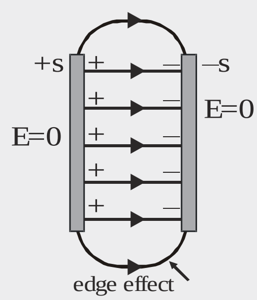
(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

Key Points

- Lines of force starts from (+ve) charge and ends on (–ve) charge.
- Lines of force start and end normally on the surface of a conductor.



fixed point charge near infinite metal plate



- The lines of force never intersect each other due to superposition principle.
- The property that electric lines of force contract longitudinally leads to explain attraction between opposite charges.
- The property that electric lines of force exert lateral pressure on each other leads to explain repulsion between like charges.

Electric Flux (ϕ)

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as $\phi_E = \int \vec{E} \cdot d\vec{A}$

This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d\vec{A}$ and the scalar quantity $\vec{E} \cdot d\vec{A}$ is to be evaluated for each element and summed over the entire surface.

Important Points About Electric Flux

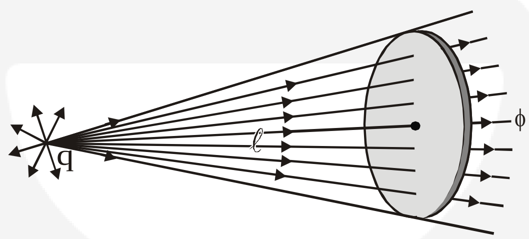
(i) It is a scalar quantity

(ii) **Units** (V–m) and N – m^2/C **Dimensions:** $[ML^3T^{-3}A^{-1}]$

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

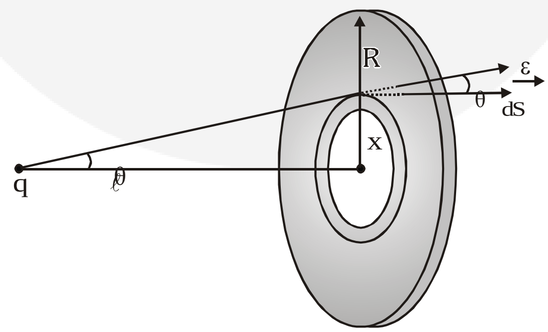
Electric Flux Through a Circular Disc

Figure shows a point charge q placed at a distance ℓ from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge q. We know a point charge q originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through the disc surface.



To calculate this flux, we consider on elemental ring an disc surface of radius x and width dx as shown. Area of this ring (strip) is dS = $2\pi x$ dx. The electric field due to q at this elemental ring is given as $E = \dfrac{kq}{\left(x^2 + \ell^2\right)}$.

If $d\phi$ is the flux passing through this elemental ring, then



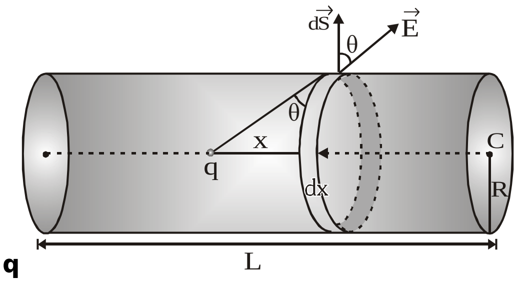
$$d\phi = EdS\cos\theta = \frac{kq}{\left(x^2 + \ell^2\right)} \times 2\pi x\, dx\, \frac{\ell}{\sqrt{x^2 + \ell^2}} = \frac{2\pi kq\ell\, x\, dx}{\left(\ell^2 + x^2\right)^{3/2}}$$

$$\phi = \int d\phi = \int_0^R \frac{q\ell}{2\in_0} \frac{x\, dx}{\left(\ell^2 + x^2\right)^{3/2}} = \frac{q\ell}{2\in_0} \int_0^R \frac{x\, dx}{\left(\ell^2 + x^2\right)^{3/2}} = \frac{q\ell}{2\in_0}\left[-\frac{1}{\sqrt{\ell^2 + x^2}}\right]_0^R = \frac{q\ell}{2\in_0}\left[\frac{1}{\ell} - \frac{1}{\sqrt{\ell^2 + x^2}}\right]$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

Electric Flux Through the Lateral Surface of a Cylinder Due to a Point Charge

Figure shows a cylindrical surface of length L and radius R. On its axis at its centre a point charge q is placed. Here we wish to find the flux coming out from the lateral surface of this cylinder due to the point charge q. For this we consider an elemental strip of width dx on the surface of cylinder as shown. The area of this strip is dS = 2πR.dx



q

The electric field due to the point charge on the strip can be given as $E = \dfrac{kq}{(x^2 + R^2)}$. If dϕ is the electric flux through the strip, then

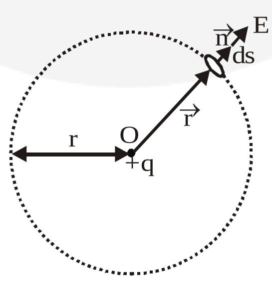
$$d\phi = ES\cos\theta = \frac{Kq}{(x^2+R^2)} \times 2\pi\, Rdx \times \frac{R}{\sqrt{x^2+R^2}} = 2\pi\, KqR^2 \times \frac{dx}{(x^2+R^2)^{3/2}}$$

Total flux through the lateral surface of cylinder $\phi = \int d\phi = \dfrac{qR^2}{2\in_0}\displaystyle\int_{-L/2}^{+L/2}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{q\in_0 \ell}{\sqrt{\ell^2+4R^2}}$

This situation can also be easily handled by using the concepts of Gauss's law.

Gauss's Law

It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface i.e., $\int_s \vec{E}.\vec{ds} = \dfrac{q}{\in_0}$



Regarding Gauss's Law it is Worth Noting that

(i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on charges present inside gaussian surface.

(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.

(iv) Electric field intensity at the gaussian surface is due to all the charges present (inside as well as out side)

(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive.

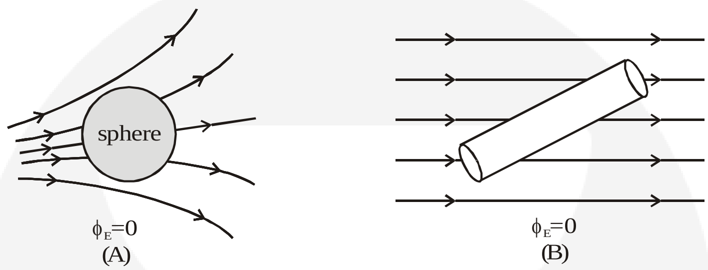
(vi) In a gaussian surface ϕ = 0 does not employ E = 0 but E = 0 employs ϕ = 0.

(vii) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice–versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian–surface] of radius r with point charge q at its centre as shown in figure. By Coulomb's law intensity at a point P on the surface will be, $\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\vec{r}$ And hence electric flux linked with area $\overrightarrow{ds} \Rightarrow \vec{E}.\overrightarrow{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}.\overrightarrow{ds}$

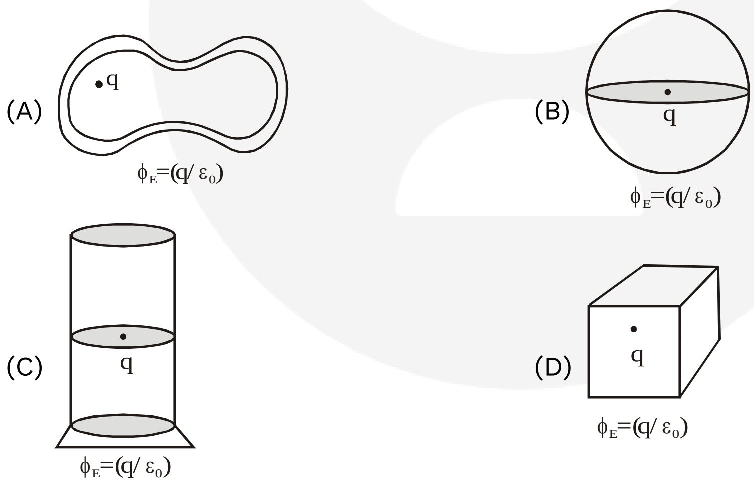
Here direction of $\vec{r}$ and $\overrightarrow{ds}$ are same, i.e., $\oint_S \vec{E}.\overrightarrow{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\oint_S dS = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(4\pi r^2\right)$, $\oint_S \vec{E}.\overrightarrow{ds} = \frac{q}{\epsilon_0}$

Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.

(viii) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non–uniform) total flux linked with it will be zero.



(b) If a closed body encloses a charge q, then total flux

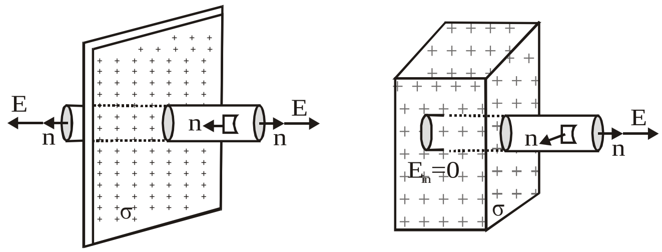


linked with the body will be $\int_S \vec{E}.\overrightarrow{ds} = \frac{q}{\epsilon_0}$

From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it. [figure]

Note: So in case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}\left(\phi_E\right) = \left(\frac{q}{2\varepsilon_0}\right)$ and the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\left(\frac{\phi_E}{n}\right) = \left(\frac{q}{n\varepsilon_0}\right)$

(ix) Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian– surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density σ. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross–section S as shown in figure.



The charge enclosed by the Gaussian–surface = σS and the flux linked with the pill box = ES + 0 + ES = 2ES (as E is parallel to curved surface and perpendicular to plane faces)

So, from Gauss's law, $\phi_E = \frac{1}{\varepsilon_0}$ (q), $2ES = \frac{1}{\varepsilon_0}(\sigma S) \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

(x) If $\vec{E} = 0$, $\phi = \oint \vec{E}.\overrightarrow{ds} = 0$, so q = 0 but if q = 0, $\oint \vec{E}.\overrightarrow{ds} = 0$ So $\vec{E}$ may or may not be zero.

If a dipole is enclosed by a closed surface then, q =0, so $\oint \vec{E}.\overrightarrow{ds} = 0$, but $\vec{E} \neq 0$

Note: If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B) E_{in}= 0. So ϕ_E = ES and hence in this case $E = \frac{\sigma}{\epsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius R is, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ with $q = 4\pi R^2\sigma$

So, for a point close to the surface of conductor, $E = \frac{1}{4\pi\varepsilon_0 R^2} \times \left(4\pi R^2\sigma\right) = \frac{\sigma}{\varepsilon_0}$

Example:

If a point charge q is placed at the centre of a cube.
What is the flux linked (a) with the cube? (b) with each face of the cube?

Solution:

(a) According to Gauss's law flux linked with a closed body is ($1/\varepsilon_0$) times the charge enclosed and here the closed body cube is enclosing a charge q so, $\phi_T = \frac{1}{\varepsilon_0}$ (q)

(b) Now as cube is a symmetrical body with 6–faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \frac{1}{6}\left(\phi_T\right) = \frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Example:

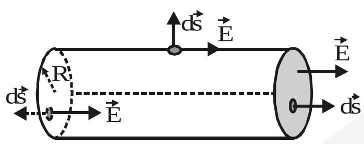
If a point charge q is placed at one corner of a cube, what is the flux linked with the cube?

Solution:

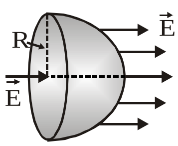
In this case by placing three cubes at three sides of given cube and four cubes above, the charge will be in the centre. So, the flux linked with each cube will be one–eight of the flux $\frac{q}{\varepsilon_0}$.

$\therefore$ Flux associated with given cube = $\frac{q}{8\varepsilon_0}$

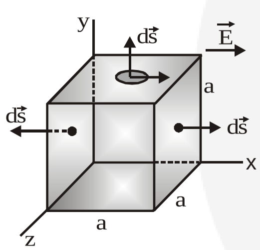
Flux Calculation Using Gauss Law



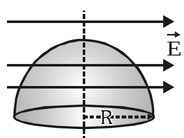
$\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E \Rightarrow \phi_{total} = 0$



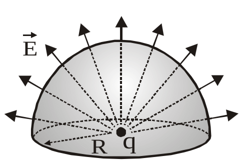
$\phi_{in} = \phi_{circular} = -\pi R^2 E$ and $\phi_{out} = \phi_{curved} = \pi R^2 E \Rightarrow \phi_{total} = 0$



$\phi_{in} = -a^2 E$ and $\phi_{out} = a^2 E \Rightarrow \phi_{total} = 0$



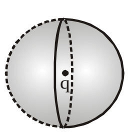
$\phi_{in} = -\frac{1}{2}\pi R^2 E$ and $\phi_{out} = \frac{1}{2}\pi R^2 E \Rightarrow \phi_{total} = 0$

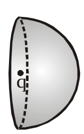


$E = \frac{kq}{R^2}$

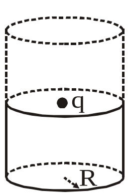
$\phi = 2\pi R^2 \times \frac{q}{4\pi \epsilon_0 R^2} = \frac{q}{2 \epsilon_0}$

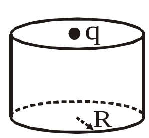
Note: here electric field is radial



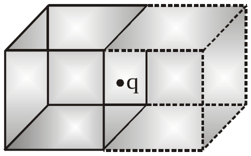


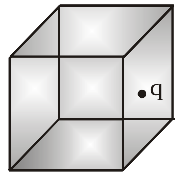
$\phi_{hemisphere} = \frac{q}{2 \epsilon_0}$



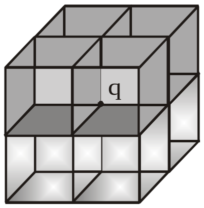


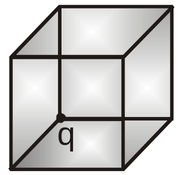
$\phi_{cylinder} = \frac{q}{2 \epsilon_0}$



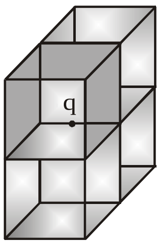


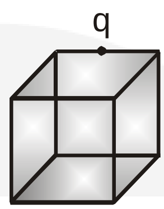
$\phi_{cube} = \frac{q}{2 \in_0}$





$\phi = \frac{q}{8 \in_0}$

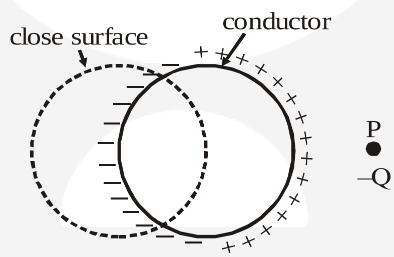




$\phi = \frac{q}{4 \in_0}$

Example:

As shown in figure a closed surface intersects a spherical conductor. If a negative charge is placed at point P. What is the nature of the electric flux coming out of the closed surface?



Solution:

Point charge Q induces charge on conductor as shown in figure.
Net charge enclosed by closed surface is negative so flux is negative.

Example:

Consider $\vec{E} = 3 \times 10^3 \hat{i}$ (N/C) then what is the flux through the square of 10 cm side, if the normal of its plane makes 60° angle with the X axis.

Solution:

$\phi = ES\cos\theta = 3 \times 10^3 \times [10 \times 10^{-2}]^2 \times \cos 60° = 3 \times 10^3 \times 10^{-2} \times ½ = 15 \text{ Nm}^2/\text{C}$

Example:

Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density λ at a distance half of the radius from its axis.

Solution:

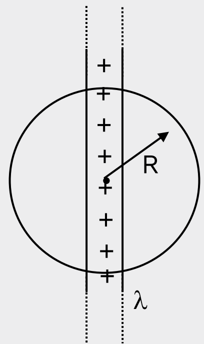
$r = \frac{R}{2}$ point will be inside so $E = \frac{2k\lambda r}{R^2} = \frac{2k\lambda}{R^2}\left(\frac{R}{2}\right) = \frac{\lambda}{4\pi \in_0 R}$

Concept Builders-3

Q.1 If a point charge q is placed at the centre of a face of a cube. Find the flux through the remaining faces of the cube.

Q.2 Find the flux through a spherical surface of radius R in a uniform Electric field E.

Q.3 Find the flux of Electric field through the spherical surface, where an infinitely long line charge (linear charge density λ) passes along the diameter of the sphere as shown:



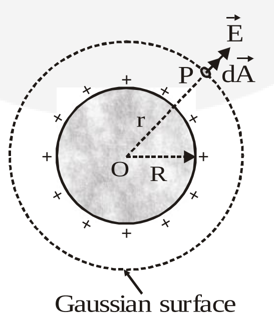
Electric Field Due to Solid Conducting or Hollow Sphere

- **For Outside Point (r > R)**

  Using Gauss's theorem $\int \vec{E}.d\vec{s} = \frac{\Sigma q}{\epsilon_0}$

  $\because$ At every point on the Gaussian surface $\vec{E} \parallel d\vec{s}$; $\vec{E}.d\vec{s}$ = E ds cos 0°= E ds

  $\therefore \int E.ds = \frac{\Sigma q}{\epsilon_0}$ [E is constant over the gaussian surface] $\Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E_P = \frac{q}{4\pi \epsilon_0 r^2}$



- **For Surface Point r = R**

  $E_S = \frac{q}{4\pi \epsilon_0 R^2}$

- **For Inside Point (r < R)**

  Because charge inside the conducting sphere or hollow is zero.

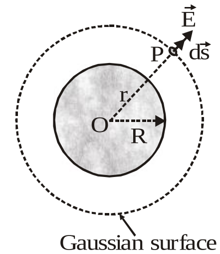
  (i.e. $\Sigma q = 0$) So $\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\epsilon_0} = 0 \Rightarrow E_{in} = 0$

Electric Field Due to Solid Non-Conducting Sphere

- **Outside (r > R)**

  From Gauss's theorem

  $$\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\epsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E_P = \frac{q}{4\pi \epsilon_0 r^2}$$



- **At Surface (r = R)**

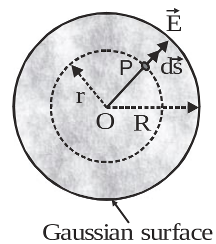
  $E_S = \dfrac{q}{4\pi \epsilon_0 R^2}$ Put r = R

- **Inside (r < R)**

  From Gauss's theorem $\oint_s \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\epsilon_0}$

  Where Σq charge contained within Gaussian surface of radius r

  $$E(4\pi r^2) = \frac{\Sigma q}{\epsilon_0} \Rightarrow E = \frac{\Sigma q}{4\pi r^2 \epsilon_0} \quad ...(i)$$



  As the sphere is uniformly charged, the volume charge density (charge/volume) ρ is constant throughout the sphere $\rho = \dfrac{q}{\frac{4}{3}\pi R^3}$

  ⇒ charge enclosed in gaussian surface

  $$\Sigma q = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \Rightarrow \sum q = \frac{qr^3}{R^3}$$

  put this value in equation (i) $E_{in} = \dfrac{1}{4\pi \epsilon_0}\dfrac{qr}{R^3}$

Electric Field Due to An Infinite Line Distribution of Charge

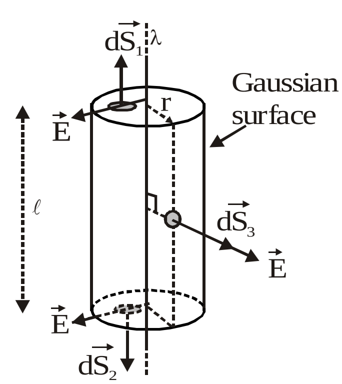
- Let a wire of infinite length is uniformly charged having a constant linear charge density λ.

  P is the point where electric field is to be calculated.

  Let us draw a coaxial Gaussian cylindrical surface of length ℓ.

  From Gauss's theorem

  $$\int_{s_1} \vec{E}.d\vec{S}_1 + \int_{s_2} \vec{E}.d\vec{S}_2 + \int_{s_3} \vec{E}.d\vec{S}_3 = \frac{q}{\epsilon_0}$$



  $\vec{E} \perp d\vec{S}_1$ so $\vec{E}.d\vec{S}_1 = 0$ and $\vec{E} \perp d\vec{S}_2$ so $\vec{E}.d\vec{S}_2 = 0$

  $$E \times 2\pi r\ell = \frac{q}{\epsilon_0} zd \ [\because \vec{E} \parallel d\vec{S}_3]$$

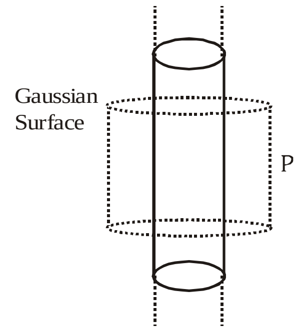
  Charge enclosed in the Gaussian surface q = λℓ.

  So $E \times 2\pi r\ell = \dfrac{\lambda\ell}{\epsilon_0} \Rightarrow E = \dfrac{\lambda}{2\pi \epsilon_0 r}$ or $E = \dfrac{2k\lambda}{r}$ where k = $\dfrac{1}{4\pi \epsilon_0}$

- Charged cylindrical nonconductor of infinite length

  Electric field at outside point $\vec{E}_A = \frac{2k\lambda}{r}\hat{r}$ $r > R$

  Electric field at inside point $\vec{E}_B = \frac{\lambda\vec{r}}{2\pi \in_0 R^2}$ $r < R$

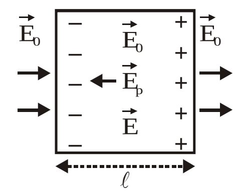


Dielectric in Electric Field

Let $\vec{E}_0$ be the applied field, Due to polarisation, electric field is $\vec{E}_p$.

The resultant field is $\vec{E}$. For homogeneous and isotropic dielectric, the direction of $\vec{E}_p$ is opposite to the direction of $\vec{E}_0$.

So, Resultant field is $E = E_0 - E_P$



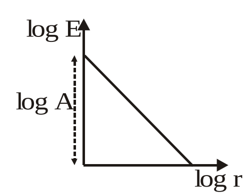
Key Points

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if non-zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >>> R$

Example:

For infinite line distribution of charge draw the curve between log E and log r.

Solution:



$\because\ E = \frac{\lambda}{2\pi \in_0 r} = \frac{A}{r}$ where $A = \frac{\lambda}{2\pi \in_0} = \text{constant}$

take log on both side log E = log A – log r

Example:

A point charge of 0.009 μC is placed at origin.

Calculate intensity of electric field due to this point charge at point $(\sqrt{2},\sqrt{7},0)$.

Solution:

$$\vec{E} = \frac{q\vec{r}}{4\pi \in_0 r^3}\ ;\text{ where } \vec{r} = x\hat{i} + y\hat{j} = \sqrt{2}\hat{i} + \sqrt{7}\hat{j},$$

$$\vec{E} = \frac{9\times 10^5 \times 9\times 10^{-9}(\sqrt{2}\,\hat{i} + \sqrt{7}\,\hat{j})}{(3)^3} = \left(3\sqrt{2}\,\hat{i} + 3\sqrt{7}\,\hat{j}\right)NC^{-1}$$

Electrostatic Potential Energy

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system

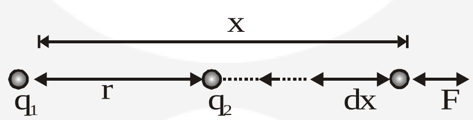
(ii) Self energy of a charged object

- **Electrostatic Interaction Energy**

  Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.

- **Interaction Energy of a System of Two Charged Particles**

  Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1.



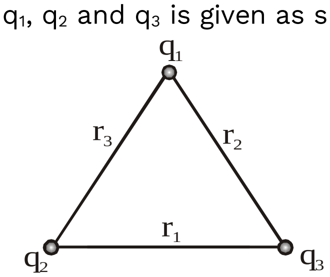
It can be calculated as $W = \int_{\infty}^{r} \vec{F}.\overrightarrow{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx$ [–ve sign shows that x is decreasing]

$W = \frac{kq_1q_2}{r}$ = U [interaction energy]

If the two charges here are of opposite sign, the potential energy will be negative as $U = -\frac{kq_1q_2}{r}$

- **Interaction Energy for a System of Charged Particles**

  When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energies of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure.



The total interaction energy of this system can be given as $U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$

Electric Potential

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as $V = \frac{U}{q_0}$ joule/coulomb

Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly, we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

Example:

A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity, if work done against electrostatic forces is −40μJ then potential at that point is?

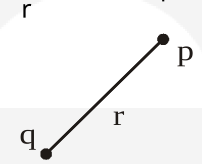
Solution:

$$V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20V$$

Note: Always remember to put sign of W and q.

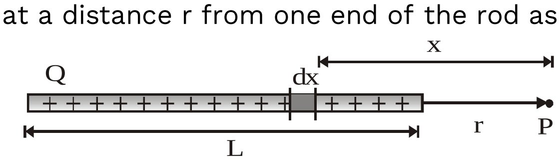
- **Electric Potential Due to a Point Charge in its Surrounding**

The potential at a point P at a distance r from the charge q $V_P = \frac{U}{q_0}$. Where U is the potential energy of charge q_0 at point p, $U = \frac{kqq_0}{r}$. Thus, potential at point P is $V_P = \frac{kq}{r}$



- **Electric Potential Due to a Charge Rod**

Figure shows a rod of length L, uniformly charged with a charge Q. Due to this we'll find electric potential at a point P at a distance r from one end of the rod as shown in figure.



For this we consider an element of width dx at a distance x from the point P.

Charge on this element is $dQ = \frac{Q}{L}dx$

The potential dV due to this element at point P can be given by using the result of a point charge as

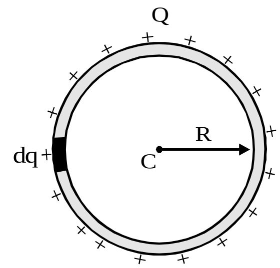
$$dV = \frac{kdq}{x} = \frac{kQ}{Lx}dx$$

Net electric potential at point P: $V = \int dV = \int_r^{r+L} \frac{kQ}{Lx}dx = \frac{kQ}{L}\ln\left(\frac{r+L}{r}\right)$

- **Electric Potential Due to a Charged Ring**

Case – I: At its centre

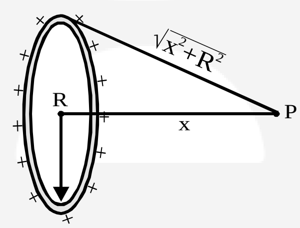
To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as $dV = \frac{kdq}{R}$. Total potential at C is $V = \int dV = \int \frac{kdq}{R} = \frac{kQ}{R}$.



As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all dq's is added as being a scalar quantity, we can directly say that the total electric potential at ring centre is $\frac{kQ}{R}$. Here we can also state that even if charge Q is non–uniformly distributed on ring, the electric potential C will remain same.

Case II: At a Point on Axis of Ring

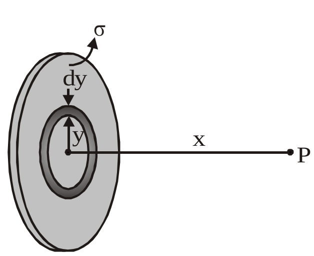
We find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2 + R^2}$ from the point P, thus the potential at P can be given as $V_P = \frac{kQ}{\sqrt{R^2 + x^2}}$



- **Electric Potential Due to a Uniformly Charged Disc**

Figure shows a uniformly disc of radius R with surface charge density ρ coul/m². To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is dq = σ2πy dy. Due to this ring, the electric potential at point P can be given as

$$dV = \frac{kdq}{\sqrt{x^2 + y^2}} = \frac{k.\sigma.2\pi y\ dy}{\sqrt{x^2 + y^2}}$$

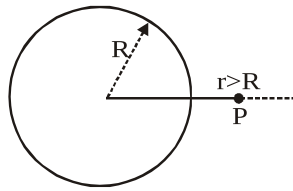


Net electric potential at Point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\epsilon_0} \cdot \frac{y\ dy}{\sqrt{x^2 + y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2 + y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

Electric Potential Due to Hollow or Conducting Sphere

Case - I At Outside Sphere



According to definition of electric potential, electric potential at point P

$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r} = -\int_{\infty}^{r} \frac{q}{4\pi \epsilon_0 r^2} dr \left[\because \quad E_{out} = \frac{q}{4\pi \epsilon_0 r^2}\right]; \; V = -\frac{q}{4\pi \epsilon_0}\int_{\infty}^{r} \frac{1}{r^2} dr = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\pi \epsilon_0 r}$$

Case - II At Surface

$$V = \int_{\infty}^{R} \vec{E}.d\vec{r} = -\int_{\infty}^{R} \frac{q}{4\pi \epsilon_0 r^2} dr \left[\because \quad E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right]; \; V = -\frac{q}{4\pi \epsilon_0}\int_{\infty}^{R} \left(\frac{1}{r^2}\right) dr = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R} \Rightarrow V = \frac{q}{4\pi \epsilon_0 R}$$

Case - III Inside the Surface

$\because$ Inside the surface $E = -\frac{dV}{dr} = 0 \Rightarrow$ V = constant so $V = \frac{q}{4\pi \epsilon_0 R}$

Electric Potential Due to Solid Non-Conducting Sphere

Case-I	**At Outside Sphere**	Same as conducting sphere.
Case-II	**At Surface**	Same as conducting sphere.
Case-III	**Inside the Sphere**	

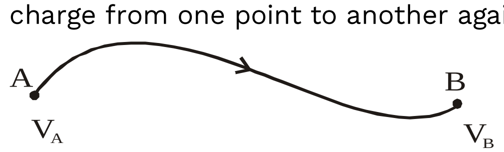
$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r} \qquad \Rightarrow V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right]$$

$$V = -\left[\int_{\infty}^{R}\left(\frac{kq}{r^2}\right)dr + \int_{R}^{r}\left(\frac{kqr}{R^3}\right)dr\right] \qquad \Rightarrow V = -\left[kq\left(-\frac{1}{r}\right)_{\infty}^{R} + \frac{kq}{R^3}\left(\frac{r^2}{2}\right)_{R}^{r}\right]$$

$$V = -kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \qquad \Rightarrow V = \frac{kq}{2R^3}\left[3R^2 - r^2\right]$$

Potential Difference Between Two Points in Electric Field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another against the electric forces.



If a unit +ve charge is displaced from a point A to B as shown work required can be given as

$V_B - V_A = -\int_A^B \vec{E}.\vec{dx}$

If a charge q is shifted from point A to B, work done against electric forces can be given as

$W = q(V_B - V_A)$

If in a situation work done by electric forces is asked, we use $W = q(V_A - V_B)$

If $V_B < V_A$, then charges must have tendency to move toward B (low potential point) it implies that electric forces carry the charge from high potential to low potential points. Hence, we can say that in the direction of electric field always electric potential decreases.

Example:

1μC charge is shifted from A to B and it is found that work done by external force is 80μJ against electrostic forces, find $V_A - V_B$

Solution:

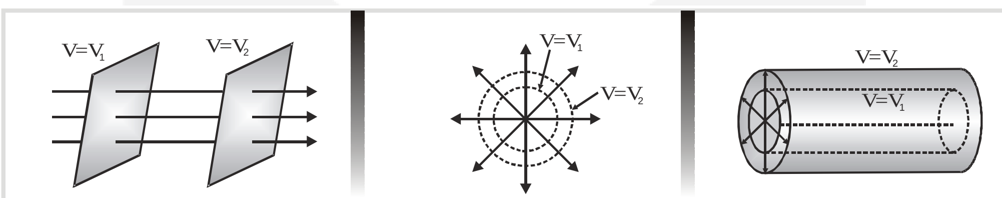
$W_{AB} = q(V_B - V_A)$

$80\ \mu J = 1\mu C\ (V_B - V_A) \Rightarrow V_A - V_B = -80\ V$

Equipotential Surfaces

For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.

- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd)
- Equipotential surfaces are always perpendicular to direction of electric field.
- If a charge is moved from one point to the other over an equipotential surface then work done $W_{AB} = -U_{AB} = q\ (V_B - V_A) = 0\ [\because V_B = V_A]$
- Shapes of equipotential surfaces



for uniform electric field equipotential surfaces are parallel plane | for a point charge, spherical conductor equipotential surfaces are spherical | for a line distribution of charge equipotential surfaces are cylinderical

- The intensity of electric field along an equipotential surface is always zero.

Electric Potential Gradient

The maximum rate of change of potential at right angles to an equipotential surface in an electric field is defined as potential gradient. $\vec{E} = -\vec{\nabla}V = -\text{grad } V$

Note: Potential is a scalar quantity but the gradient of potential is a vector quantity

In cartesian co–ordinates $\vec{\nabla}V = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

Example:

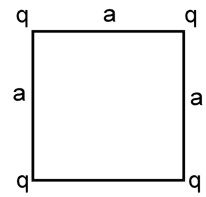
If $V = -5x + 3y + \sqrt{15}\,z$ then find magnitude of electric field at point (x,y,z).

Solution:

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] = -(-5\hat{i} + 3\hat{j} + \sqrt{15}\hat{k}) \Rightarrow |\vec{E}| = \sqrt{25+9+15} = \sqrt{46} = 7 \text{ unit}$$

Example:

The four charges q each are placed at the corners of a square of side a. Find the potential energy of one of the charges



The electric potential of a point A due to charges B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

$\therefore$ Potential energy of the charge at A is $PE = qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Example:

A proton moves with a speed of 7.45×10^5 m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons.

Given: $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{N-m^2}{C^2}$; $m_p = 1.67 \times 10^{-27}$ kg and $e = 1.6 \times 10^{-19}$ C

Solution:

As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So, if v is the common velocity of each particle at closest approach, by 'conservation of momentum'.

$$mu = mv + mv \Rightarrow v = \frac{1}{2}u$$

And by conservation of energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

So, $r = \frac{4e^2}{4\pi\varepsilon_0 mu^2}$ [as $v = \frac{u}{2}$]

And hence substituting the given data, $r = 9 \times 10^9 \times \frac{4\times(1.6\times10^{-19})^2}{1.67\times10^{-27}\times(7.45\times10^3)^2} = 10^{-12}$ m

Concept Builders-4

Q.1 Four-point charges q are placed at the corners of a square of side 'a'. Find the potential energy of the system

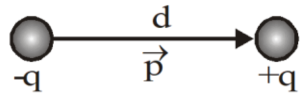
Q.2 Two identical particles of mass m and charge q are released at a separation r from each other. Find their velocities, when they become separated by large distance.

Q.3 If $V = 7x^2y + 2y - z^3x$, then find electric field at point (x,y,z).

Electric Dipole

A system of two equal and opposite charges separated by a certain distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as

$$\vec{p} = q\vec{d}$$



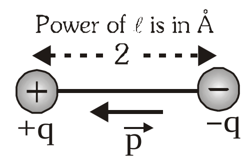
In some molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipole. Atom is non – polar because in it the centres of positive and negative charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

- **Dipole Moment**

  Dipole moment $\vec{p} = q\vec{d}$

  (i) Vector quantity, directed from negative to positive charge

  (ii) **Dimension:** [LTA], **Units:** coulomb × metre (or C-m)



Example:

A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, – 0.15m) and B; (0, 0, + 0.15 m) respectively. What is the total charge and electric dipole moment of the system?

Solution:

Total charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric diople moment,

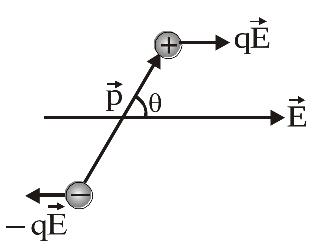
p = Mangitude of either charge × separation between charges

$= 2.5 \times 10^{-7}[0.15 + 0.15]$ C m $= 7.5 \times 10^{-8}$ C m. The direction of dipole moment is from B to A.

Dipole Placed in Uniform Electric Field

Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qE in opposite direction as shown.

$\vec{F}_{net} = \left[q\vec{E} + (-q)\vec{E}\right] = \vec{0}$



Thus, we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of actions of forces = qE × d sinθ = pE sinθ

$\vec{\tau} = \vec{r} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$

Work done in Rotation of a Dipole in Electric field

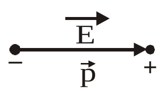
When a dipole is placed in an electric field at an angle θ, the torque on it due to electric field is $\tau = pE \sin\theta$

Work done in rotating an electric dipole from θ_1 to θ_2 [uniform field]

$$dW = \tau d\theta \text{ so } W = \int dW = \int \tau d\theta \text{ and } W_{\theta_1 \to \theta_2} = W = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta = pE (\cos\theta_1 - \cos\theta_2)$$

e.g. $W_{0 \to 180} = pE [1-(-1)] = 2\,pE$ $\qquad W_{0 \to 90} = pE (1 - 0) = pE$

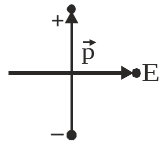
If a dipole is rotated from field direction ($\theta = 0°$) to θ then $W = pE (1 - \cos\theta)$



$\theta = 0$

τ = minimum = 0

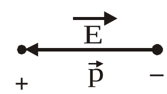
W = minimum = 0



$\theta = 90°$

τ = maximum = pE

W = pE



$\theta = 180°$

τ = minimum = 0

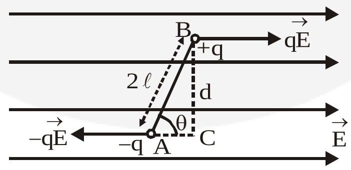
W = maximum = 2pE

Electrostatic Potential Energy

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e., $W_{90° \to \theta} = \int_{90°}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta = -\vec{p}.\vec{E}$, $\vec{E}$ is a conservative field so whatever work is done in rotating a dipole from θ_1 to θ_2 is just equal to change in electrostatic potential energy $W_{\theta_1 \to \theta_2} = U_{\theta_2} - U_{\theta_1} = pE (\cos\theta_1 - \cos\theta_2)$

Work Done in Rotating an Electric Dipole in an Electric Field

Suppose at any instant, the dipole makes an angle θ with the electric field. The torque acting on dipole. $\tau = qEd = (q\, 2\ell \sin\theta)\, E = pE \sin\theta$ The work done in rotating dipole from θ_1 to θ_2



$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$W = pE (\cos\theta_1 - \cos\theta_2) = U_2 - U_1$ ($\because U = -pE\cos\theta$)

Force on an Electric Dipole in Non–Uniform Electric Field

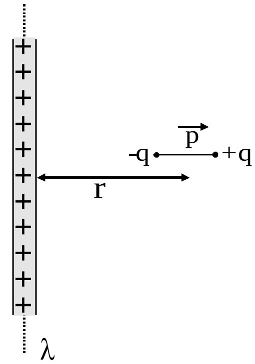
If in a non–uniform electric field dipole is placed at a point where electric field is E, the interaction energy of dipole at this point $U = -\vec{p}.\vec{E}$. Now the force on dipole due to electric field

$$F = -\frac{\Delta U}{\Delta r}$$

If dipole is placed in the direction of electric field then $F = -p\dfrac{dE}{dx}$

Example:

Calculate force on a dipole in the surrounding of a long-charged wire as shown in the figure.



Solution:

In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $E = \frac{2k\lambda}{r}$

Thus, force on dipole is $F = -\, p.\frac{dE}{dr} = -\, p\left[-\frac{2k\lambda}{r^2}\right] = \frac{2kp\lambda}{r^2}$

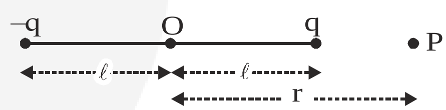
Here –ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

Electric Potential Due to Dipole

- **At Axial Point**

Electric potential due to +q charge $V_1 = \frac{kq}{(r-\ell)}$

Electric potential due to –q charge $V_2 = \frac{-kq}{(r+\ell)}$



Net electric potential $V = V_1 + V_2 = \frac{kq}{(r-\ell)} + \frac{-kq}{(r+\ell)} = \frac{kq \times 2\ell}{(r^2-\ell^2)} = \frac{kp}{r^2-\ell^2}$

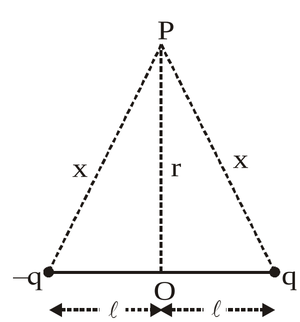
If $r >>> \ell \Rightarrow V = \frac{kp}{r^2}$

- **At Equitorial Point**

Electric potential of P due to +q charge $V_1 = \frac{kq}{x}$

Electric potential of P due to –q charge $V_2 = -\frac{kq}{x}$

Net potential $V = V_1 + V_2 = \frac{kq}{x} - \frac{kq}{x} = 0 \therefore V = 0$



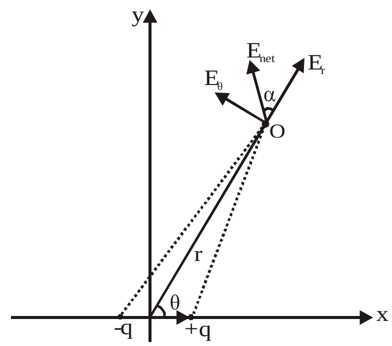
- **At General Point**

$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3}$ $\vec{p} = q\vec{d}$ electric dipole moment

Electric Field Due to an Electric Dipole

Figure shows an electric dipole placed on x–axis at origin. Here we wish to find the electric field and potential at a point O having coordinates (r, θ). Due to the positive charge of dipole electric field at O is in radially outward direction and due to the negative charge, it is radially inward as shown in figure.

$$E_r = -\frac{\partial V}{\partial r} = \frac{2kp\cos\theta}{r^3} \quad \text{and} \quad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{kp\sin\theta}{r^3}$$



Thus, net electric field at point O,

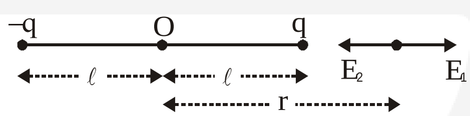
$$E_{net} = \sqrt{E_r^2 + E_\theta^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

If the direction of E_{net} is at an angle α from radial direction, then

$$\alpha = \tan^{-1}\frac{E_\theta}{E_r} = \left(\frac{1}{2}\tan\theta\right)$$

Thus, the inclination of net electric field at point O is (θ+α)

- **At A Point on the Axis of a Dipole**



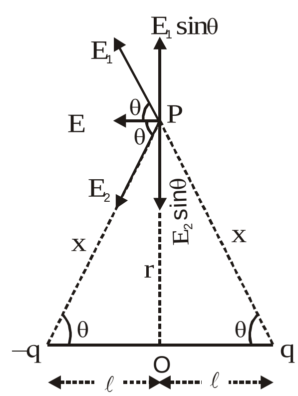
Electric field due to +q charge $E_1 = \frac{kq}{(r-\ell)^2}$

Electric field due to –q charge $E_2 = \frac{kq}{(r+\ell)^2}$

Net electric field $E = E_1 - E_2 = \frac{kq}{(r-\ell)^2} - \frac{kq}{(r+\ell)^2} = \frac{kq \times 4r\ell}{(r^2-\ell^2)^2}$ [$\because$ p = q × 2ℓ = Dipole moment]

$E = \frac{2kpr}{(r^2-\ell^2)^2}$ If r >>>ℓ then $E = \frac{2kp}{r^3}$

- **At a Point on Equitorial Line of Dipole**



Electric field due to +q charge $E_1 = \frac{kq}{x^2}$;

Electric field due to –q charge $E_2 = \frac{kq}{x^2}$

Vertical component of E_1 and E_2 will cancel each other and horizontal components will be added So net electric field at P

$E = E_1\cos\theta + E_2\cos\theta$ [$\because E_1 = E_2$]

$E = 2E_1\cos\theta = \frac{2kq}{x^2}\cos\theta \because \cos\theta = \frac{\ell}{x}$ and $x = \sqrt{r^2+\ell^2}$

$E = \frac{2kq\ell}{x^3} = \frac{2kq\ell}{(r^2+\ell^2)^{3/2}} = \frac{kp}{(r^2+\ell^2)^{3/2}}$ If r > > > ℓ then $E = \frac{kp}{r^3}$ or $\vec{E} = \frac{-k\vec{p}}{r^3}$

Key Points

- For a dipole, potential is zero at equatorial position, while at any finite point $E \neq 0$
- In a uniform $\vec{E}$, dipole may feel a torque but not a force.
- If a dipole placed in a field $\vec{E}$ (Non-Uniform) generated by a point charge, then torque on dipole may be zero, but $F \neq 0$

Distribution	Point charge	Dipole
Potential proportional to	r^{-1}	r^{-2}
E proportional to	r^{-2}	r^{-3}

Force between	Point charge	Dipole and point charge	Dipole-dipole
Proportional to	r^{-2}	r^{-3}	r^{-4}

Example:

A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant $r = \sqrt{5}$ m from origin. Find the position vector of the point in first quadrant.

Solution:

$\because |E_P| = |V_P|$

$\therefore \dfrac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \dfrac{kp\cos\theta}{r^2} \Rightarrow 1+3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

Position vector $\vec{r}$ of point P is $\vec{r} = \dfrac{\sqrt{5}}{2}(\hat{i}+\hat{j})$

Example:

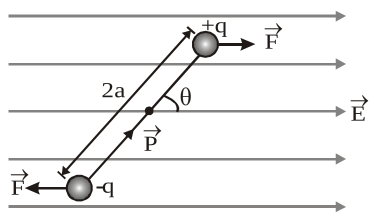
Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I for small amplitudes about its equilibrium position in a uniform electric field strength E is $\dfrac{1}{2\pi}\sqrt{\left(\dfrac{pE}{I}\right)}$

Solution:

Let an electric dipole (charge q and –q at a distance 2a apart) placed in a uniform external electric field of strength E.

Restoring torque on dipole $\tau = -\,pE\sin\theta = -pE\,\theta$ (as θ is small)

Here – ve sign shows the restoring tendency of torque.



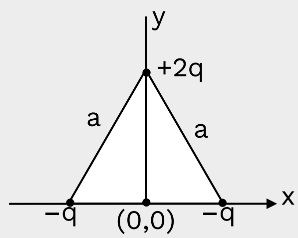
$\because \tau = I\alpha$ angular acceleration $= \alpha = \dfrac{\tau}{I} = \dfrac{PE}{I}\theta$

For SHM $a = -\omega^2\theta$ comparing we get $\omega = \sqrt{\dfrac{pE}{I}}$

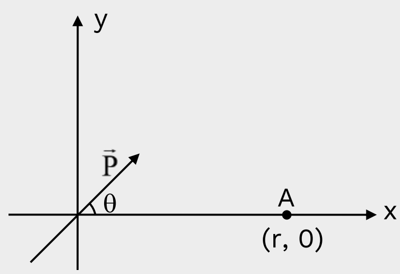
Thus, frequency of oscillations of dipole $n = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\left(\dfrac{pE}{I}\right)}$

Concept Builders-5

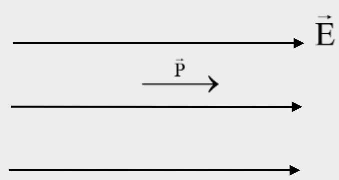
Q.1 Three charges are placed at the vertices of an equilateral triangle as shown. Find the dipole moment of the system.



Q.2 Find the Electric field due to the dipole ($\vec{p}$) at a point 'A' as shown in figure:



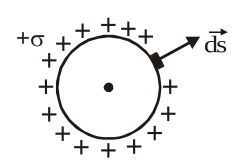
Q.3 A dipole ($\vec{p}$) is placed in a uniform electric field aligned parallel to it. Find work required to be done in rotating the dipole by 180^0.



Electrostatic Pressure

Force due to electrostatic pressure is directed normally outwards to the surface.

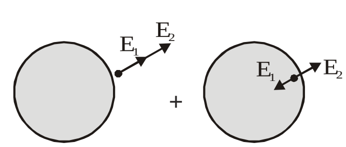
Force on small element ds of charged conductor



$$dF = (\text{Charge on ds}) \times \text{Electric field} = (\sigma\, ds)\ \frac{\sigma}{2\epsilon_0} = \frac{\sigma^2}{2\epsilon_0} ds$$

Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$

Just outside $E = E_1 + E_2 = 2E_2 \Rightarrow E_2 = \dfrac{\sigma}{2\epsilon_0}$



(E_1 is field due to point charge on the surface and
E_2 is field due to rest of the sphere).

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$P_{\text{eleectrostatic}} = \frac{dF}{dS} = \frac{\sigma^2}{2\epsilon_0}$$

Equilibrium of Liquid Charged Surfaces (Soap Bubble)

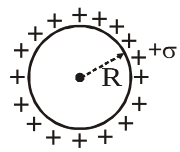
Pressures (forces) act on a charged soap bubble, due to

(i) Surface tension P_T (inward)

(ii) Air outside the bubble P_o (inward)

(iii) Electrostatic pressure P_e (outward)

(iv) Air inside the bubble P_i (outward) in state of equilibrium inward pressure = outward pressure
$P_T + P_o = P_i + P_e$



Excess pressure of air inside the bubble $(P_{ex}) = P_i - P_o = P_T - P_e$

but $P_T = \dfrac{4T}{r}$ and $P_e = \dfrac{\sigma^2}{2\epsilon_0} \Rightarrow P_{ex} = \dfrac{4T}{r} - \dfrac{\sigma^2}{2\epsilon_0}$ if $P_i = P_o$ then $\dfrac{4T}{r} = \dfrac{\sigma^2}{2\epsilon_0}$

Example:

Brass has a tensile strength 3.5×10^8 N/m^2. What charge density on this material will be enough to break it by electrostatic force of repulsion? How many excess electrons per square Å will there then be? What is the value of intensity just out side the surface?

Solution:

We know that electrostatic force on a charged conductor is given by $\dfrac{dF}{ds} = \dfrac{\sigma^2}{2\varepsilon_0}$

So, the conductor will break by this force if, $\dfrac{\sigma^2}{2\varepsilon_0}$ > Breaking strength i.e.,

$\sigma^2 > 2 \times 9 \times 10^{-12} \times 3.5 \times 10^8$

i.e. $\sigma_{min} = \left(3\sqrt{7}\right) \times 10^{-2} = 7.94 \times 10^{-2} \left(C/m^2\right)$

Now as the charge on an electron is 1.6×10^{-19} C, the excess electrons per m^2

Further as in case of a conductor near its surface $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{7.94 \times 10^{-2}}{9 \times 10^{-12}} = 8.8 \times 10^9$ V/m

Conductor and It's Properties [For Electrostatic Condition]

(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics, conductors are always equipotential surfaces.

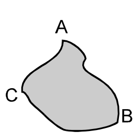
(iii) Charge always resides on outer surface of conductor.

(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

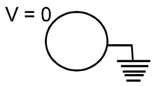
(vi) Electric lines of force never enter into conductors.

(vii) Electric field intensity near the conducting surface is given by formula = $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$



$\vec{E}_A = \dfrac{\sigma_A}{\varepsilon_0}\hat{n}$; $\vec{E}_B = \dfrac{\sigma_B}{\varepsilon_0}\hat{n}$ and $\vec{E}_C = \dfrac{\sigma_C}{\varepsilon_0}\hat{n}$

(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.

(x) When two conductors are connected there will be charge flow till their potential becomes equal.

(xi) Electric pressure at the surface of a conductor is given by formula $P = \frac{\sigma^2}{2\varepsilon_0}$ where σ is the local surface charge density.

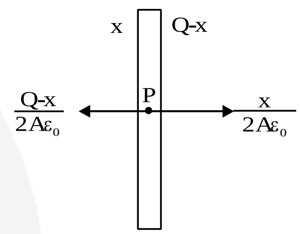
Example:

Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet s given a charge then the charge distributes equally on its two surfaces.

Solution:

Let there is x charge on left side of sheet and Q−x charge on right side of sheet.

Since point P lies inside the conductor so $E_P = 0$



$$\frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} = 0 \Rightarrow \frac{2x}{2A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

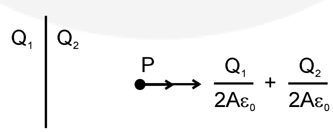
So, charge is equally distributed on both sides

Example:

If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Solution:

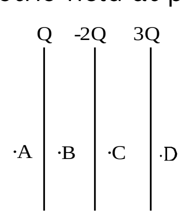
Electric field at point P : $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$



[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example:

Three large conducting sheets placed parallel to each other at finite distance contains charges Q, −2Q and 3Q respectively. Find electric field at points A, B, C, and D



Solution:

$E_A = E_Q + E_{-2Q} + E_{3Q}$.

(i) Here E_Q means electric field due to 'Q'.

$$E_A = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}, \text{ towards left}$$

(ii) $E_B = \dfrac{Q-(-2Q+3Q)}{2A\varepsilon_0}$, towards right = 0

(iii) $E_C = \dfrac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \dfrac{-4Q}{2A\varepsilon_0} = \dfrac{-2Q}{A\varepsilon_0}$, towards right $\Rightarrow \dfrac{2Q}{A\,\varepsilon_0}$ towards left

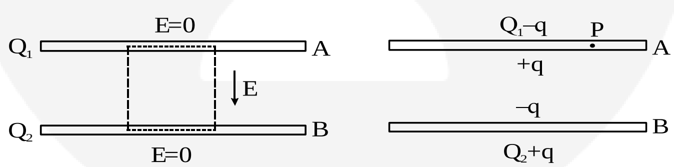
(iv) $E_D = \dfrac{(Q-2Q+3Q)}{2A\varepsilon_0} = \dfrac{2Q}{2A\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$, towards right

Example:

Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove that the charges on the inner facing surfaces are of equal magnitude and opposite sign. Also find the charges on inner & outer surfaces.

Solution:

Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.



The distribution should be like the one shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma / (2\varepsilon_0)$, the electric field at P due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward); due to the charge $+q = \dfrac{q}{2A\varepsilon_0}$ (upward), due to the charge $-q = \dfrac{q}{2A\varepsilon_0}$ (downward), and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$E_p = \frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

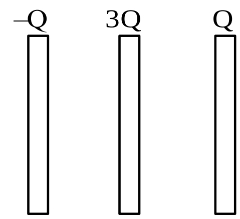
As the point P is inside the conductor, this field should be zero.

Hence, $Q_1 - q - q + q - Q_2 - q = 0 \Rightarrow q = \dfrac{Q_1 - Q_2}{2}$

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost, surfaces get equal charges and the facing surfaces get equal and opposite charges.

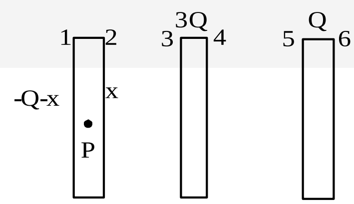
Example:

Figure shows three large metallic plates with charges – Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



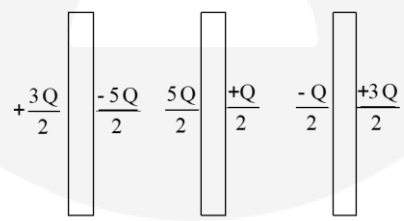
Solution:

We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge (– Q – x). The electric field inside the metal plate is zero so fields at P is zero.



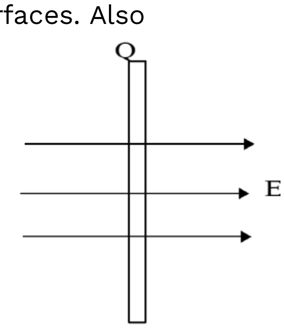
Resultant field at P : $E_P = 0 \Rightarrow \frac{-Q-x}{2A\varepsilon_0} = \frac{x+3Q+Q}{2A\varepsilon_0} \Rightarrow -Q - x = x + 4Q \Rightarrow x = \frac{-5Q}{2}$

Note: We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by gauss theorem also. Remember this it is important result. Thus, the final charge distribution on all the surfaces is:



Example:

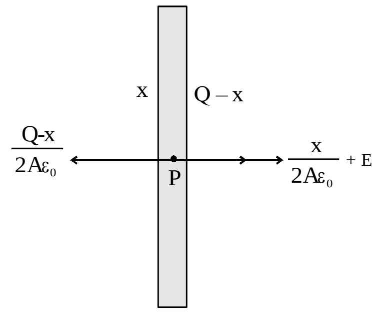
An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces. Also



Solution:

Let there is x charge on left side of plate and Q − x charge on right side of plate

$E_P = 0 \Rightarrow \frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0} \Rightarrow \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E \Rightarrow x = \frac{Q}{2} - EA\varepsilon_0$



So, charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and other side $\frac{Q}{2} + EA\varepsilon_0$

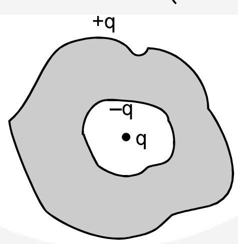
The resultant electric field on the left and right side of the plate.

On right side $E = \frac{Q}{2A\varepsilon_0} + E$ towards right and on left side

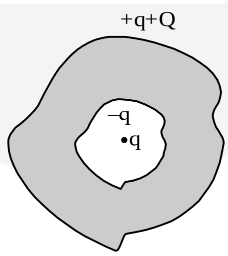
$\frac{Q}{2A\varepsilon_0} - E$ towards left.

Some Other Important Results for a Closed Conductor

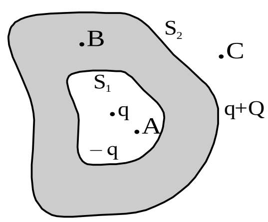
(i) If a charge q is kept in the cavity then −q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using gauss theorem)



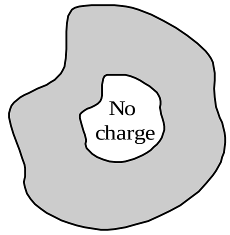
(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then −q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using gauss theorem)



(iii) Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B, C) is zero. Resultant field due to **q + Q on S_2 and any other charge outside S_2**, at any point inside of surface S_2 (like A, B) is zero

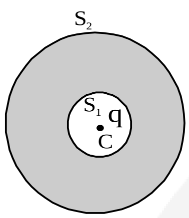


(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.



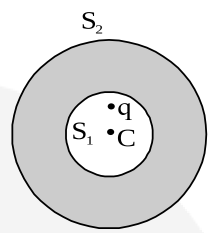
(v) Charge distribution for different types of cavities in conductors

(A)



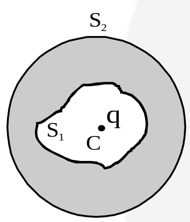
charge is at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(B)



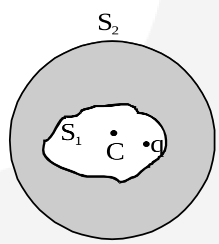
charge is not at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(C)



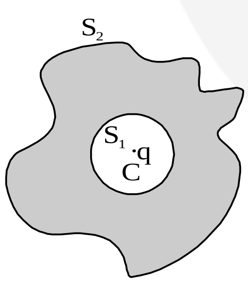
charge is at the centre of S_2
(S_2 $\rightarrow$ spherical)

(D)



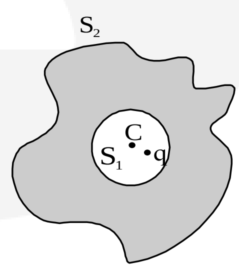
charge is not at the centre of S_2
(S_2 $\rightarrow$ spherical)

(E)



charge is at the centre of
S_1 (Spherical)

(F)



charge not at the centre
of S_1(Spherical)

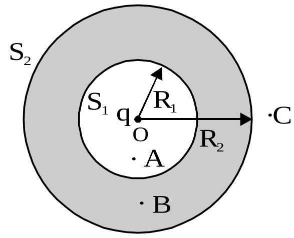
Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (iii) We can show that

Case	A	B	C	D	E	F
S_1	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform
S_2	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform

Note: In all cases charge on inner surface $S_1 = -q$ and on outer surface $S_2 = q$. The distribution of charge on S_1 will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on S_2 may change if some charges(s) is/are kept outside the conductor.

Example:

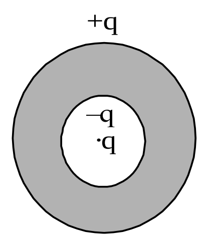
An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q at the centre as shown in figure



(i) Find $\vec{E}$ and V at points A, B and C

(ii) If a point charge Q is kept out side the sphere at a distance 'r' (>> R_2) from centre then find out resultant force on charge Q and charge q.

Solution:



At point A: $V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1}$, $\vec{E}_A = \frac{Kq}{OA^3}\overrightarrow{OA}$

At point B: $V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}$, $E_B = 0$;

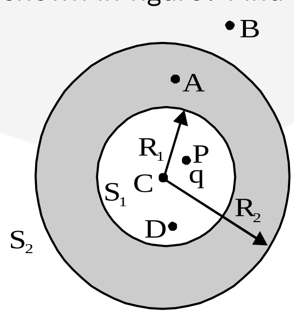
At point C: $V_C = \frac{Kq}{OC}$, $\vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$

(ii) Force on point charge Q: $\vec{F}_Q = \frac{KqQ}{r^2}\hat{r}$ (r = distance of 'Q' from centre 'O')

Force on point charge q: $\vec{F}_q = 0$ (using result (iii) & charge on S_1 uniform)

Example:

An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in figure? Find out the following:



(i) V_C (ii) V_A (iii) V_B (iv) E_A

(v) E_B (vi) force on charge Q if it is placed at B

Solution:

(i) $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$

(ii) $V_A = \frac{Kq}{R_2}$

(iii) $V_B = \frac{Kq}{CB}$

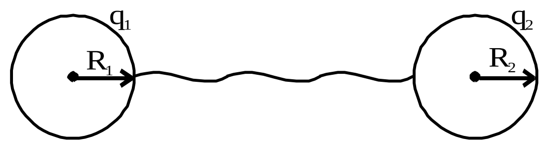
(iv) $E_A = 0$ (point is inside metallic conductor)

(v) $E_B = \frac{Kq}{CB^2}\hat{CB}$

(vi) $F_Q = \frac{KQq}{CB^2}\hat{CB}$

(vi) **Sharing of Carges**

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and separated by large distance, are joined by a conducting wire. Let final charges on spheres are q_1 and q_2 respectively.



Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2};\ \frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad \text{...(i)}$$

and $q_1 + q_2 = Q_1 + Q_2$ ...(ii)

from (i) and (ii) $q_1 = \frac{(Q_1+Q_2)R_1}{R_1+R_2}$; $q_2 = \frac{(Q_1+Q_2)R_2}{R_1+R_2}$

ratio of charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}; \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$

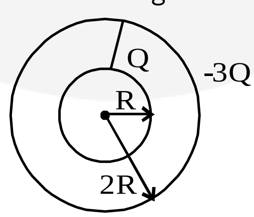
ratio of surface charge densities $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Ratio of Final Charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

Ratio of Final Surface Charge Densities $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Example:

The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing. Find out the charge on each sphere after that.



Solution:

After cutting the wire, the potential of both the shells is equal

Thus, potential of inner shell $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$

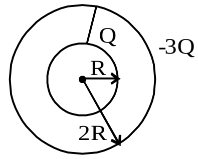
and potential of outer shell $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$

As $V_{out} = V_{in} \Rightarrow \frac{-KR}{R} = \frac{K(x-2Q)}{2R} \Rightarrow -2Q = x - 2Q \Rightarrow x = 0$

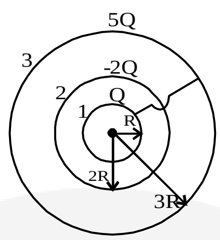
So, charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

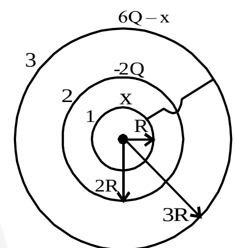
Example:

Find charge on each spherical shell after joining the inner most shell and outer most shell by a conducting wire. Also find charges on each surface.



Solution:





Let the charge on the innermost sphere be x.

Finally, potential of shell 1 = Potential of shell 3

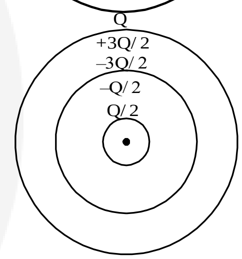
$$\frac{Kx}{R}+\frac{K(-2Q)}{2R}+\frac{K(6Q-x)}{3R}=\frac{KQ}{3R}+\frac{k(-2q)}{3R}+\frac{k(5Q)}{3R}$$

3x –3Q + 6Q – x = 4Q ; 2x = Q ; x = $\frac{Q}{2}$

Charge on innermost shell = $\frac{Q}{2}$,

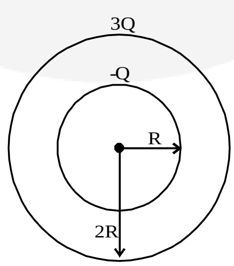
charge on outermost shell = $\frac{5Q}{2}$ middle shell = –2Q

Final charge distribution is as shown in figure.



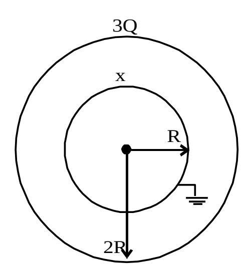
Example:

Two conducting hollow spherical shells of radii R and 2R carry charges –Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded?



Solution:

When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.



$\frac{Kx}{R}+\frac{K3Q}{2R}=0$

x = $\frac{-3Q}{2}$, the charge that has increased

= $\frac{-3Q}{2}-(-Q)=\frac{-Q}{2}$ hence charge flows into the Earth = $\frac{Q}{2}$

Example:

An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss?

Solution:

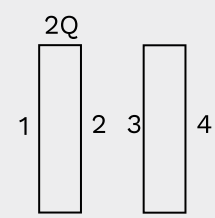
When two conducting spheres of equal radius are connected charge is equally distributed on them.

So, we can say that heat loss of system

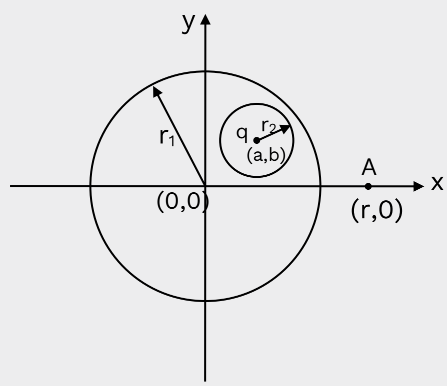
$$\Delta H = U_i - U_f = \left(\frac{Q^2}{8\pi\varepsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\varepsilon_0 R} + \frac{Q^2/4}{8\pi\varepsilon_0 R}\right) = \frac{Q^2}{16\pi\varepsilon_0 R}$$

Concept Builders-6

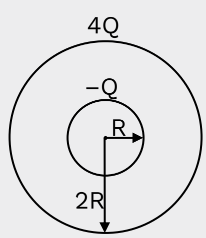
Q.1 Two metallic plates are placed at a small separation d. Left plate is given charge 2Q. Find the final charges appearing on the surfaces 1,2,3 and 4 respectively.



Q.2 A neutral spherical conductor with centre at origin has a spherical cavity with center at coordinates (a, b) as shown in figure. A charge q is placed at the centre of cavity. Find the Electric field at a point A at (r, 0).



Q.3 Two conducting hollow spherical shells of radii R and 2R carry charges –Q and 4Q respectively. How much charge will flow into the earth if outer shell is grounded?



ANSWERS KEY FOR CONCEPT BUILDER

Concept Builder 1:

1. $\frac{3KQq}{a^2}$ downwards, **2.** $\frac{d}{3}$ from Q

Concept Builder 2:

1. $\frac{4\sqrt{2}KQ}{a^2}\hat{i}$ **2.** $\frac{KQ}{a^2}(\sqrt{2}+\frac{1}{2})$

Concept Builder 3:

1. $\frac{q}{2\varepsilon_0}$ **2.** zero **3.** $\frac{2R\lambda}{\varepsilon_0}$

Concept Builder 4:

1. $\frac{Kq^2}{a}(4+\sqrt{2})$ **2.** $v = \sqrt{\frac{Kq^2}{mr}}$ **3.** $-14xy\hat{i} - 2\hat{j} + 3z^2x\hat{k}$

Concept Builder 5:

1. $qa\sqrt{3}\hat{j}$ **2.** $\frac{2kp\cos\theta}{r^3}\hat{i} - \frac{kp\sin\theta}{r^3}\hat{j}$ **3.** 2pE

Concept Builder 6:

1. Q, Q, −Q, Q **2.** $\frac{kq}{r^2}$ **3.** 3Q

Some Worked out Examples

Example 1:

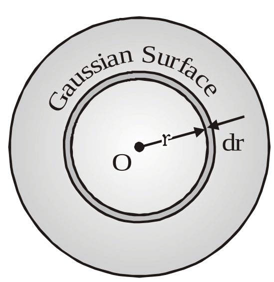
For a spherically symmetrical charge distribution, electric field at a distance r from the centre of sphere is $\vec{E} = kr^7\hat{r}$, where k is a constant. What will be the volume charge density at a distance r from the centre of sphere?

(A) $\rho = 9k\varepsilon_0 r^6$ (B) $\rho = 5k\varepsilon_0 r^3$

(C) $\rho = 3k\varepsilon_0 r^4$ (D) $\rho = 9k\varepsilon_0 r^0$

Answer: (A)

Solution:



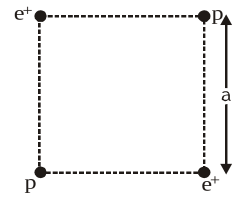
By using Gauss law $\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0} \Rightarrow (E)(4\pi r^2) = \frac{\int \rho(4\pi r^2 dr)}{\epsilon_0}$

(Note: Check dimensionally that $\rho \propto r^6$)

$$(kr^7)(4\pi r^2) = \frac{\int \rho(4\pi r^2 dr)}{\epsilon_0} \Rightarrow k\varepsilon_0 r^9 = \int \rho r^2 dr$$

Example 2:

Two positrons (e^+) and two protons (p) are kept on four corners of a square of side a as shown in figure. The mass of proton is much larger than the mass of positron. Let q denotes the charge on the proton as well as the positron then the kinetic energies of one of the positrons and one of the protons respectively after a very long time will be–



(A) $\frac{q^2}{4\pi\epsilon_0 a}\left(1+\frac{1}{2\sqrt{2}}\right), \frac{q^2}{4\pi\epsilon_0 a}\left(1+\frac{1}{2\sqrt{2}}\right)$

(B) $\frac{q^2}{2\pi\epsilon_0 a}, \frac{q^2}{4\sqrt{2}\pi\epsilon_0 a}$

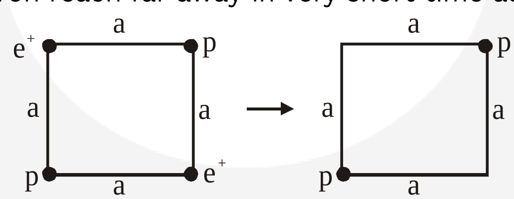
(C) $\frac{q^2}{4\pi\epsilon_0 a}, \frac{q^2}{4\pi\epsilon_0 a}$

(D) $\frac{q^2}{2\pi\epsilon_0 a}\left(1+\frac{1}{4\sqrt{2}}\right), \frac{q^2}{8\sqrt{2}\pi\epsilon_0 a}$

Answer: (D)

Solution:

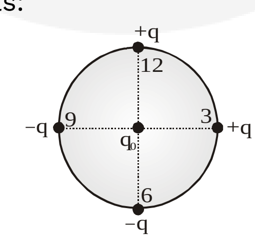
As mass of proton >>> mass of positron so initial acceleration of positron is much larger than proton. Therefore, positron reach far away in very short time as compare to proton.



$$2K_{e^+} = \left(\frac{4kq^2}{a} + \frac{2kq^2}{a\sqrt{2}}\right) - \frac{kq^2}{a\sqrt{2}} \Rightarrow K_{e^+} = \frac{q^2}{2\pi\epsilon_0 a}\left(1+\frac{1}{4\sqrt{2}}\right) \text{ and } 2K_p = \frac{kq^2}{a\sqrt{2}} - 0 \Rightarrow K_p = \frac{q^2}{8\sqrt{2}\pi\epsilon_0 a}$$

Example 3:

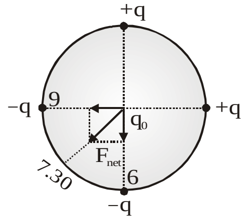
Four charges are placed at the circumference of a dial clock as shown in figure. If the clock has only hour hand, then the resultant force on a charge q_0 placed at the centre, points in the direction which shows the time as:



(A) 1:30 (B) 7:30 (C) 4:30 (D) 10:30

Answer: (B)

Solution:



Example 4:

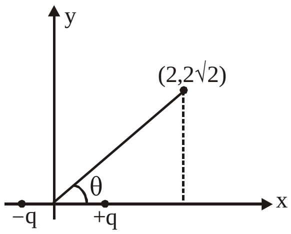
A small electric dipole is placed at origin with its dipole moment directed along positive x-axis. The direction of electric field at point $(2, 2\sqrt{2}, 0)$ is

(A) along z-axis (B) along y-axis

(C) along negative y-axis (D) along negative z-axis

Answer: (B)

Solution:



$\tan\theta = \frac{y}{x} = \sqrt{2}; \cot\theta = \frac{1}{\sqrt{2}}$ Also $\tan\alpha = \frac{\tan\theta}{2} = \frac{1}{\sqrt{2}} = \cot\theta \Rightarrow \theta + \alpha = 90°$ i.e., $\vec{E}$ is along positive y-axis.

Example 5:

Uniform electric field of magnitude 100 V/m in space is directed along the line y = 3 + x. Find the potential difference between point A (3, 1) & B (1,3).

(A) 100 V (B) $200\sqrt{2}$V (C) 200 V (D) zero

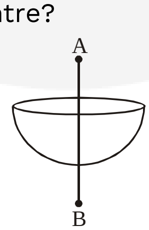
Answer: (D)

Solution:

Slope of line AB = $\frac{3-1}{1-3} = -1$ which is perpendicular to direction of electric field.

Example 6:

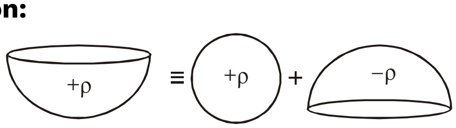
The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density ρ. If the electric field at a point 2R distance above its centre is E then what is the electric field at the point which is 2R below its centre?



(A) $\frac{\rho R}{6\varepsilon_0} + E$ (B) $\frac{\rho R}{12\varepsilon_0} - E$ (C) $\frac{-\rho R}{6\varepsilon_0} + E$ (D) $\frac{\rho R}{24\varepsilon_0} + E$

Answer: (B)

Solution:



Apply principle of superposition

Electric field due to a uniformly charged sphere = $\frac{\rho R}{12\varepsilon_0}$; $E_{resultant} = \frac{\rho R}{12 \epsilon_0} - E$

Example 7:

A metallic rod of length l rotates at angular velocity ω about an axis passing through one end and perpendicular to the rod. If mass of electron is m and its charge is –e then the magnitude of potential difference between its two ends is

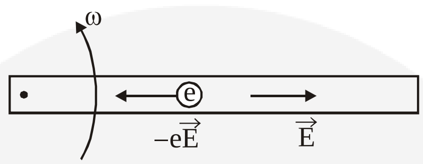
(A) $\frac{m\omega^2\ell^2}{(2e)}$ (B) $\frac{m\omega^2\ell^2}{e}$ (C) $\frac{m\omega^2\ell}{e}$ (D) None of these

Answer: (A)

Solution:

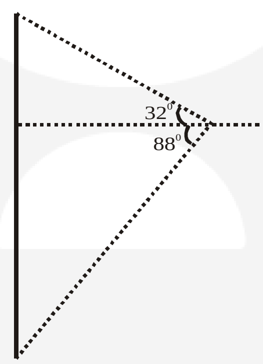
When rod rotates the centripetal acceleration of electron comes from electric field $E = \frac{mr\omega^2}{e}$

Thus, $\Delta V = -\int \vec{E}.dr = -\int_0^\ell \frac{mr\omega^2}{e}dr = \frac{m\omega^2\ell^2}{2e}$



Example 8:

Consider a finite charged rod. Electric field at Point P (shown) makes an angle θ with horizontal dotted line then angle θ is:



(A) 60° (B) 28°

(C) 44° (D) information insufficient

Answer: (B)

Solution:

Required angle = $\frac{\theta_2 - \theta_1}{2} = \frac{88° - 32°}{2} = \frac{56°}{2} = 28°$

Example 9:

The electric potential in a region is given by the relation $V(x) = 4 + 5x^2$. If a dipole is placed at position (–1, 0) with dipole moment $\vec{p}$ pointing along positive Y-direction, then

(A) Net force on the dipole is zero.

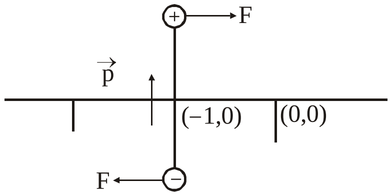
(B) Net torque on the dipole is zero

(C) Net torque on the dipole is not zero and it is in clockwise direction

(D) Net torque on the dipole is not zero and it is in anticlockwise direction

Answer: (AC)

Solution:

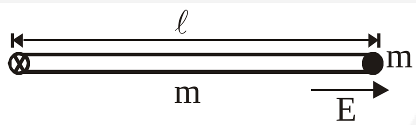


$V(x) = 4 + 5x^2 \Rightarrow \vec{E} = 10x\hat{i}$

$\therefore$ Net force will be zero and torque not zero and rotation will be along clockwise direction

Example 10 to 12:

A thin homogeneous rod of mass m and length ℓ is free to rotate in vertical plane about a horizontal axle pivoted at one end of the rod. A small ball of mass m and charge q is attached to the opposite end of this rod. The whole system is positioned in a constant horizontal electric field of magnitude $E = \frac{mg}{2q}$. The rod is released from shown position from rest.

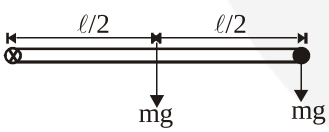


10. What is the angular acceleration of the rod at the instant of releasing the rod?

(A) $\frac{8g}{9\ell}$ (B) $\frac{3g}{2\ell}$ (C) $\frac{9g}{8\ell}$ (D) $\frac{2g}{9\ell}$

Answer: (C)

Solution:



By taking torque about hinge $I\alpha = mg\left(\frac{\ell}{2}\right) + mg(\ell)$ when $I = \frac{m\ell^2}{3} + m\ell^2 \Rightarrow \alpha = \frac{9g}{8\ell}$

11. What is the acceleration of the small ball at the instant of releasing the rod?

(A) $\frac{8g}{9}$ (B) $\frac{9g}{8}$ (C) $\frac{7g}{8}$ (D) $\frac{8g}{7}$

Answer: (B)

Solution:

Acceleration of ball = $\alpha\ell = \left(\frac{9g}{8\ell}\right)\ell = \frac{9}{8}g$

12. What is the speed of ball when rod becomes vertical?

(A) $\sqrt{\frac{2g\ell}{3}}$ (B) $\sqrt{\frac{3g\ell}{4}}$ (C) $\sqrt{\frac{3g\ell}{2}}$ (D) $\sqrt{\frac{4g\ell}{3}}$

Answer: (C)

Solution:

From work energy theorem $\frac{1}{2}I\omega^2 = mg\left(\frac{\ell}{2}\right) + mg\ell - qE\ell$

$$\frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega^2 = \frac{3}{2}mg\ell - \frac{mg\ell}{2} \Rightarrow \frac{2}{3}m\ell^2\omega^2 = mg\ell \Rightarrow \omega = \sqrt{\frac{3g}{2\ell}}$$

Speed of ball = $\omega\ell = \sqrt{\frac{3g\ell}{2}}$

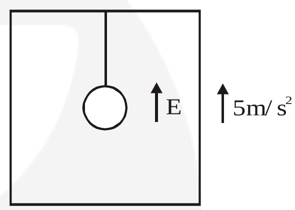
Example 13:

A simple pendulum is suspended in a lift which is going up with an acceleration of 5 m/s^2. An electric field of magnitude 5 N/C and directed vertically upward is also present in the lift. The charge of the bob is 1 μC and mass is 1 mg. Taking $g = \pi^2$ and length of the simple pendulum 1m, find the time period of the simple pendulum (in sec).

Answer: 2

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$



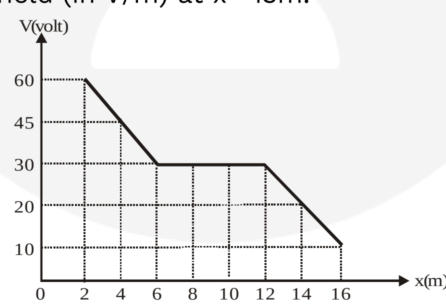
$$g_{eff} = g - \frac{qE}{M} + 5 = 15 - \frac{1\times 5\times 10^{-6}}{1\times 10^{-6}}$$

$g_{eff} = 10 = \pi^2$

T = 2 sec

Example 14:

The variation of potential with distance x from a fixed point is shown in figure. Find the magnitude of the electric field (in V/m) at x =13m.



Answer: 5

Solution:

$$E = -\frac{dV}{dx} = \frac{20}{4} = 5 \text{ volt/meter}$$

Example 15:

The energy density u is plotted against the distance r from the centre of a spherical charge distribution on a log-log scale. Find the magnitude of slope of obtianed straight line.

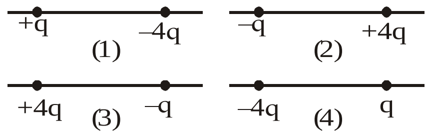
Answer: 2

Solution:

$$u = \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\left(\frac{q}{4\pi\in_0 r}\right)^2 = \frac{q^2}{32\pi^2\in_0 r^2} \Rightarrow \log u = \log\left(\frac{q^2}{32\pi^2\in_0 r^2}\right) = \log k - 2\log r$$

Example 16:

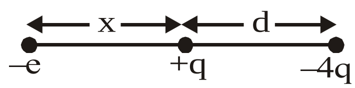
The figure shows four situations in which charges as indicated (q > 0) are fixed on an axis. How many situations is there a point to the left of the charges where an electron would be in equilibrium?



Answer: 2

Solution:

For (1)

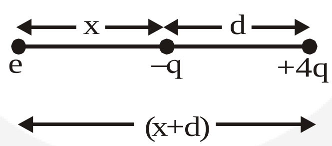


Let the electron be held at a distance x from +q charge.

For equilibrium $\frac{q(-e)}{4\pi\epsilon_0 x^2} = \frac{(-e)(-4q)}{4\pi\epsilon_0 (x+d)^2}$

We can find value of x for which $F_{net} = 0$ which means that electron will be in equilibrium.

For (2)



For equilibrium $\frac{(-e)(-q)}{4\pi\epsilon_0 x^2} = \frac{(-e)4q}{4\pi\epsilon_0 (x+d)^2}$

We can find value of x for which $F_{net} = 0$ which means that electron will be in equilibrium.
In case (3) and (4) the electron will not remain at rest, since it experiences a net non–zero force.

OR

Equilibrium is always found near the smaller charge

Example 17:

An electric field is given by $\vec{E} = \left(y\hat{i} + x\hat{j}\right)\frac{N}{C}$. Find the work done (in J) in moving a 1C charge from $\vec{r}_A = \left(2\hat{i} + 2\hat{j}\right)$ m to $\vec{r}_B = \left(4\hat{i} + \hat{j}\right)$m.

Answer: 0

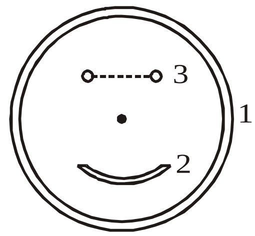
Solution:

A= (2, 2) and B = (4, 1); $W_{A\to B} = q\left(V_B - V_A\right) = q\int_A^B dV = -\int_A^B q\vec{E}.d\vec{r}$

$= q\int_A^B \left(y\hat{i} + x\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right) = q\int_A^B \left(ydx + xdy\right) = -q\int_{(2,2)}^{(4,1)} d\left(xy\right) = -q\left[xy\right]_{(2,2)}^{(4,1)} = -q\left[4-4\right] = 0$

Example 18:

The arrangement shown consists of three elements.



(i) A thin rod of charge $-3.0\ \mu C$ that forms a full circle of radius 6.0 cm.

(ii) A second thin rod of charge $2.0\ \mu C$ that forms a circular arc of radius 4.0 cm and concentric with the full circle, subtending an angle of 90° at the centre of the full circle.

(iii) An electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude 1.28×10^{-21}C-m.

Find the net electric potential in volts at the centre.

Answer: 0

Solution:

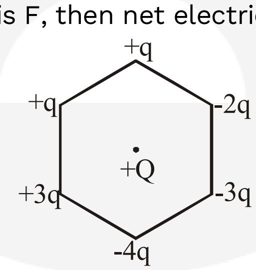
Potential due to dipole at the centre of the circle is zero.

Potentials due to charge on circle = $V_1 = \dfrac{K.(-3\times10^{-6})}{6\times10^{-2}}$

Potential due to arc $V_2 = \dfrac{K.(2\times10^{-6})}{4\times10^{-2}}$ Net potential = $V_1 + V_2 = 0$

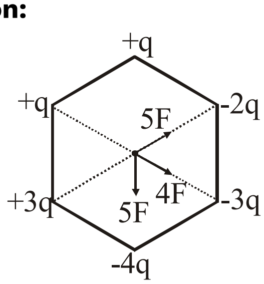
Example 19:

Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F, then net electric force on the +Q is nF. Find the value of n.



Answer: 9

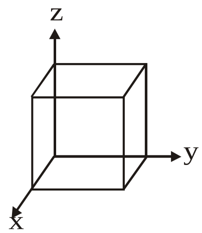
Solution:



$F_{net} = 9F$

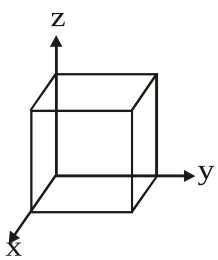
Example 20:

Electric field in a region is given by $\vec{E} = -4x\hat{i} + 6y\hat{j}$. The charge enclosed in the cube of side 1m oriented as shown in the diagram is given by $\alpha\epsilon_0$. Find the value of α.



Answer: 2

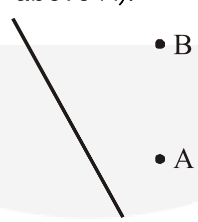
Solution:



ϕ = (6y) Area – (4x) Area = 6 × 1 × $(1)^2$ – 4 × 1 × $(1)^2$ = 2 therefore $\frac{q}{\varepsilon_0} = 2 \Rightarrow q = 2\varepsilon_0$

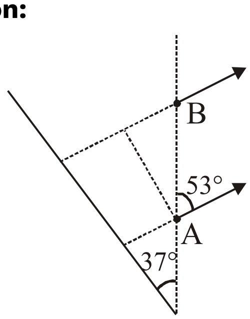
Example 21:

An infinite plane of charge with $\sigma = 2\epsilon_0 \frac{C}{m^2}$ is tilted at a 37° angle to the vertical direction as shown below. Find the potential difference, $V_A – V_B$ in volts, between points A and B at 5 m distance apart. (where B is vertically above A).



Answer: 3

Solution:



$$E = \frac{\sigma}{2\varepsilon_0} = 1 \text{ N/C}$$

$$\therefore V_B - V_A = -\int \vec{E}.d\vec{\ell} = \left(\frac{\sigma}{2\epsilon_0}\right)(5\cos 53^\circ) = 3V$$

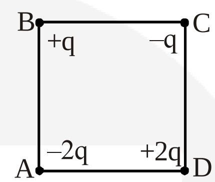
Objective Exercise - I

Single Correct Type Questions

Properties of Charges and Coulomb's Law

1. Mid way between the two equal and similar charges, we placed the third equal and similar charge. Which of the following statements is correct, concerned to the equilibrium along the line joining the charges?
(A) The third charge experienced a net force inclined to the line joining the charges
(B) The third charge is in stable equilibrium
(C) The third charge is in unstable equilibrium
(D) The third charge experiences a net force perpendicular to the line joining the charges

2. Four charges are arranged at the corners of a square ABCD, as shown. The force on a +ve charge kept at the centre of the square is
(A) zero
(B) along the diagonal AC
(C) along diagonal BD
(D) perpendicular to the side AB

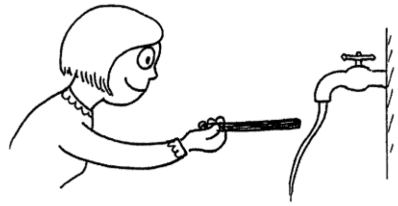


3. If an object made of substance A rubs an object made of substance B, then A becomes positively charged and B become negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?
(A) B becomes positively charged and C becomes positively charged
(B) B becomes positively charged and C becomes negatively charged
(C) B becomes negatively charged and C becomes positively charged
(D) B becomes negatively charged and C becomes negatively charged

4. Two equal negative charges are fixed at the points [0, a] and [0, –a] on the y-axis. A positive charge Q is released from rest at the points [2a, 0] on the x-axis. The charge Q will
(A) move to the origin and stop there
(B) move to the origin, cross the origin and then move to infinitely.
(C) execute simple harmonic motion about the origin.
(D) execute oscillatory but not simple harmonic motion.

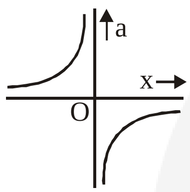
5. Two free positive charges 4q and q are a distance ℓ apart. What charge Q is needed to achieve equilibrium for the entire system and where should it be placed from charge q?
(A) $Q = -\frac{4q}{9}$ in between 4q and q at $\frac{\ell}{3}$ from 4q
(B) $Q = -\frac{4q}{9}$ in between 4q and q at $\frac{2\ell}{3}$ from 4q
(C) $Q = \frac{4q}{9}$ in between 4q and q at $\frac{\ell}{3}$ from 4q
(D) $Q = \frac{4q}{9}$ in between 4q and q at $\frac{2\ell}{3}$ from 4q

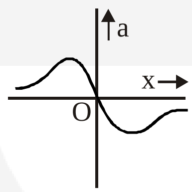
6. In normal cases thin stream of water bends toward a negatively charged rod. When a positively charged rod is placed near the stream, it will bend in the

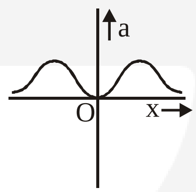


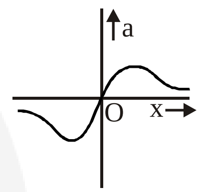
(A) Opposite direction.
(B) Same direction.
(C) It won't bend at all.
(D) Can't be predicted.

7. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the + x-axis, passes through O and moves far away from O. Its acceleration a is taken as positive in the positive x-direction. The particle's acceleration a is plotted against its x-coordinate. Which of the following best represents the plot?

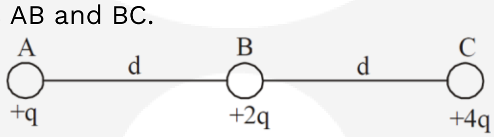
(A) 

(B) 

(C) 

(D) 

8. Three charges +q, +2q and + 4q are connected by strings as shown in the figure. What is ratio of tensions in the strings AB and BC.



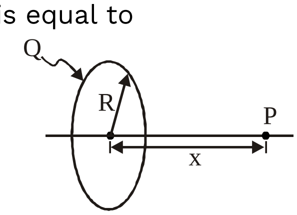
(A) 1 : 2
(B) 1 : 3
(C) 2 : 1
(D) 3 : 1

9. A charge q is placed at the centroid of an equilateral triangle. Three equal charges Q are placed at the vertices of the triangle. They system of four charges will be in equilibrium if q is equal to:

(A) $\frac{-Q}{\sqrt{3}}$
(B) $\frac{-Q}{3}$
(C) $-Q\sqrt{3}$
(D) $\frac{Q}{\sqrt{3}}$

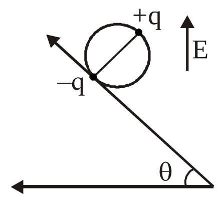
Electric Field :

10. A small particle of mass m and charge –q is placed at point P on the axis of uniformly charged ring and released. If R >> x, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to



(A) $\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$
(B) $\sqrt{\frac{qQx}{4\pi\varepsilon_0 mR^4}}$
(C) $\frac{qQ}{4\pi\varepsilon_0 mR^3}$
(D) $\frac{qQx}{4\pi\varepsilon_0 mR^4}$

11. A wheel having mass m has charges +q and −q on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E =

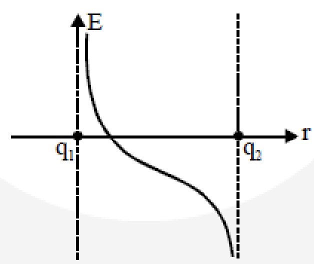


(A) $\frac{mg}{q}$ (B) $\frac{mg}{2q}$ (C) $\frac{mg\tan\theta}{2q}$ (D) None of these

12. A point charge 50 μC is located in the XY plane at the point of position vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. What is the electric field at the point of position vector $\vec{r} = 8\hat{i} - 5\hat{j}$

(A) 1200 V/m (B) 0.04 V/m (C) 900 V/m (D) 4500 V/m

13. The variation of electric field between the two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightward direction of electric field as positive) as shown in the figure. Then the correct statement is :-



(A) q_1 and q_2 are positive and $q_1 < q_2$
(B) q_1 and q_2 are positive and $q_1 > q_2$
(C) q_1 is positive and q_2 is negative and $q_1 < q_2$
(D) q_1 and q_2 are negative and $q_1 < q_2$

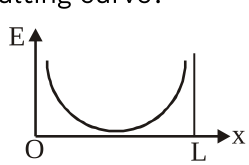
14. A point charge q is placed at origin. Let $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ be the electric field at three points A (1, 2, 3), B (1, 1, −1) and C(2, 2, 2) due to charge q. Consider the following statements and choose correct alternative.

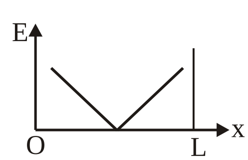
(i) $\vec{E}_A \perp \vec{E}_B$

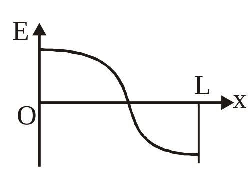
(ii) $|\vec{E}_B| = 4|\vec{E}_C|$

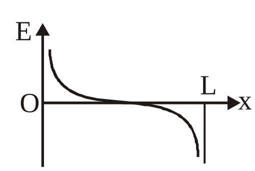
(A) only [i] is correct (B) only [ii] is correct
(C) both [i] and [ii] are correct (D) both [i] and [ii] are wrong

15. Two identical point charges are placed at a separation of L. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than L. Which of the following best represents the resulting curve?

(A)


(B)


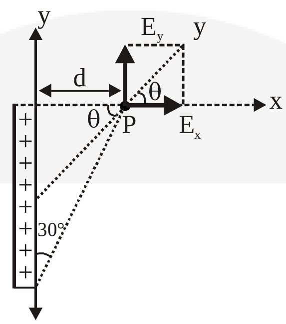
(C)


(D)


16. A nonconducting ring of radius R has uniformly distributed positive charge Q. A small part of the ring, of length d, is removed (d << R). The electric field at the centre of the ring will now be
(A) directed towards the gap, inversely proportional to R^3.
(B) directed towards the gap, inversely proportional to R^2.
(C) directed away from the gap, inversely proportional to R^3.
(D) directed away from the gap, inversely proportional to R^2.

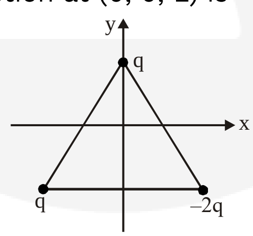
17. The maximum electric field at a point on the axis of a uniformly charged ring is E_0. At how many points on the axis will the magnitude of electric field be $E_0/2$:-
(A) 1 (B) 2 (C) 3 (D) 4

18. In the given figure the direction (θ) of $\vec{E}$ at point P due to uniformly charged finite thin rod will be



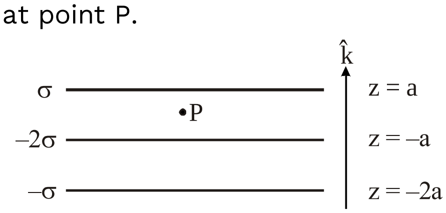
(A) at an angle of 30° from x-axis
(B) at an angle of 45° from x -axis
(C) at an angle of 60° from x -axis
(D) None of these

19. An equilateral triangle wire frame of side L having 3 point charges at its vertices is kept in x-y plane as shown. Centroid of the triangle coincides with the origin. Component of electric field due to the configuration in z direction at (0, 0, L) is



(A) $\frac{9\sqrt{3}kq}{8L^2}$ (B) $\frac{9kq}{8L^2}$ (C) zero (D) None of these

20. Three large parallel plates have uniform surface charge densities as shown in the figure. Find out electric field intensity at point P.



(A) $-\frac{4\sigma}{\epsilon_0}\hat{k}$ (B) $\frac{4\sigma}{\epsilon_0}\hat{k}$ (C) $-\frac{2\sigma}{\epsilon_0}\hat{k}$ (D) $\frac{2\sigma}{\epsilon_0}\hat{k}$

21. A charged particle having some mass is resting in equilibrium at a height H above the centre of a uniformly charged non-conducting horizontal ring of radius R. The force of gravity acts downwards. The equilibrium of the particle will be stable

(A) for all values of H

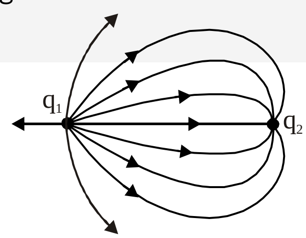
(B) only if $H > \frac{R}{\sqrt{2}}$

(C) only if $H < \frac{R}{\sqrt{2}}$

(D) only if $H = \frac{R}{\sqrt{2}}$

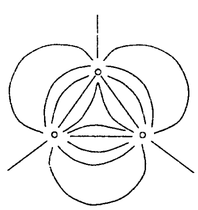
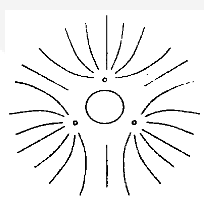
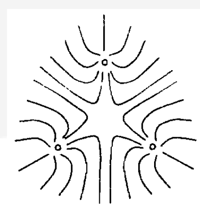
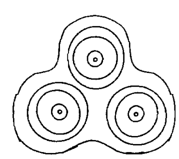
22. Select the correct statement: (Only force on a particle is due to electric field)
(A) A charged particle always moves along the electric line of force.
(B) A charged particle may move along the line of force
(C) A charge particle never moves along the line of force
(D) A charged particle moves along the line of force only if released from rest.

23. The figure shows the electric field lines in the vicinity of two point charges. Which one of the following statements concerning this situation is true?



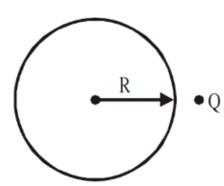
(A) q_1 is negative and q_2 is positive
(B) The magnitude of the ratio (q_2/q_1) is less than one
(C) Both q_1 and q_2 have the same sign of charge
(D) The electric field is strongest midway between the charges.

24. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(A)  (B)  (C)  (D) 

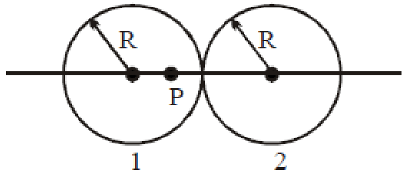
Gauss' Law :

25. **Statement 1:** A charge is outside the Gaussian sphere of radius R. Then electric field on the surface of sphere is zero.
and
Statement 2: As $\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$ for the sphere qin is zero, So $\oint \vec{E} \cdot d\vec{s} = 0$.



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

26. Figure shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centres of the spheres, at radial distance R/2 from the center of sphere 1. If the net electric field at point P is zero and Q_1 is 64 μC, what is Q_2 (in μC).

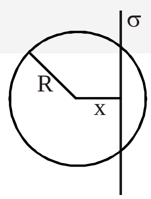


(A) 64 (B) 36 (C) 32 (D) 72

27. Electric flux through a surface of area 100 m^2 lying in the xy plane is (in V-m) if $\vec{E} = \hat{i} + \sqrt{2}\,\hat{j} + \sqrt{3}\hat{k}$

(A) 100 (B) 141.4 (C) 173.2 (D) 200

28. A large sheet of uniform charge passes through a hypothetical spherical surface. The figure shows principal section of the situation. The electric flux through the spherical surface is given by



(A) $\dfrac{\pi R^2\sigma}{\varepsilon_0}$ (B) $\dfrac{2\pi(R^2 - x^2)\sigma}{\varepsilon_0}$ (C) $\dfrac{\pi(R - x)^2\sigma}{\varepsilon_0}$ (D) $\dfrac{\pi(R^2 - x^2)\sigma}{\varepsilon_0}$

29. A uniform electric field $\vec{E} = a\hat{i} + b\hat{j}$, intersects a surface of area A. What is the flux through this area if the surface lies in the yz plane?

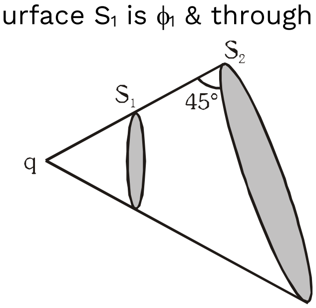
(A) a A (B) 0 (C) b A (D) $A\sqrt{a^2 + b^2}$

30. The electric field in a region is given by $\vec{E} = 200\hat{i}$ N/C for x > 0 and $-200\hat{i}$ N/C for x < 0. A closed cylinder of length 2m and cross-section area 10^2 m^2 is kept in such a way that the axis of cylinder is along X-axis and its centre coincides with origin. The total charge inside the cylinder is.

[Take : $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2m^2.N)

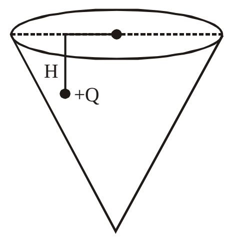
(A) zero (B) 1.86×10^{-5}C (C) 1.77×10^{-11}C (D) 35.4×10^{-8} C

31. In the given figure flux through surface S_1 is ϕ_1 & through S_2 is ϕ_2. Which is correct ?



(A) $\phi_1 = \phi_2$ (B) $\phi_1 > \phi_2$ (C) $\phi_1 < \phi_2$ (D) None of these

32. A charge +Q is located somewhere inside a vertical cone such that the depth of the charge from the free surface of the cone is H. It is found that the flux associated with the cone with the curved surface is $\frac{3Q}{5\varepsilon_0}$. If the charge is raised vertically through a height 2H, then the flux through the curved surface is



(A) $\frac{3Q}{5\in_0}$ (B) $\frac{2Q}{5\in_0}$ (C) $\frac{4Q}{5\in_0}$ (D) Zero

33. Refer to the arrangement of charges in Fig. and a Gaussian surface of radius *R* with *Q* at the centre. Then

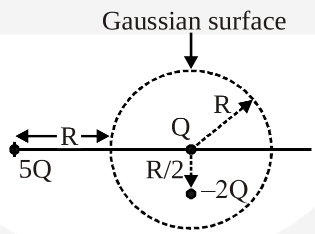


Fig.

(A) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$

(B) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$

(C) flux through the surface of sphere due to *Q* is zero.

(D) field on the surface of sphere due to –2*Q* is same everywhere.

34. A positive charge q is placed in a spherical cavity made in a positively charged sphere. Position vector of the centre of cavity relative to centre of the sphere is $\vec{\ell}$. Force on the charge q is

(A) in the direction parallel to vector $\vec{\ell}$.

(B) in the direction perpendicular to vector $\vec{\ell}$.

(C) in a direction which depends on the magnitude of charge density in sphere.

(D) direction cannot be determined from the given information.

Electric Potential Energy And Electric Potential :

35. An electric charge 10^{-8} C is placed at the point (4m, 7m, 2m). At the point (1m, 3m, 2m), the electric

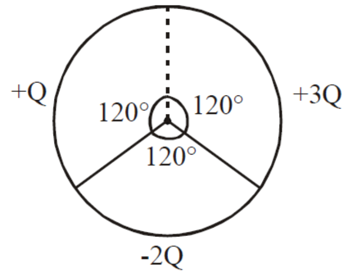
(A) potential will be 18 V

(B) field has no Y-component

(C) field will be along Z-axis

(D) potential will be 1.8 V

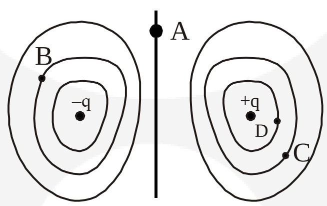
36. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the centre of curvature is:



(A) $\frac{Q}{4\pi\varepsilon_0 R}$ (B) $\frac{Q}{2\pi\varepsilon_0 R}$ (C) $\frac{2Q}{\pi\varepsilon_0 R}$ (D) $\frac{Q}{\pi\varepsilon_0 R}$

37. When a negative charge is released and moves in electric field, it moves towards a position of
(A) lower electric potential and lower potential energy
(B) lower electric potential and higher potential energy
(C) higher electric potential and lower potential energy
(D) higher electric potential and higher potential energy

38. Figure shows equi-potential surfaces for a two charge-system. At which of the labeled points will an electron have the highest potential energy?



(A) Point A (B) Point B (C) Point C (D) Point D

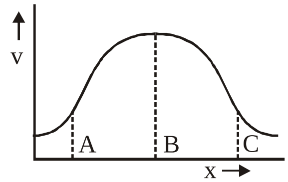
39. An infinite nonconducting sheet of charge has a surface charge density of 10^{-7} C/m^2. The separation between two equipotential surfaces near the sheet whose potential differ by 5V is
(A) 0.88 cm (B) 0.88 mm (C) 0.88 m (D) 5×10^{-7} m

40. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges are placed at (n – 1) corners. At the centre, the intensity is E and the potential is V. The ratio V/E has magnitude
(A) r n (B) r(n – 1) (C) (n – 1)/r (D) r(n – 1)/n

41. A charge of 3 coulomb experiences a force of 3000 N when placed in a uniform electric field. The potential difference between two points separated by a distance of 1 cm along the field lines is
(A) 10 V (B) 100 V (C) 1000 V (D) 9000 V

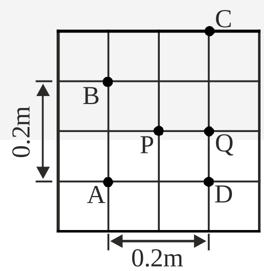
42. Uniform electric field of magnitude 100 V/m in space is directed along the line y = x + 3. Find the potential difference between point A (3, 1) & B (1, 3).
(A) 100 V (B) $200\sqrt{2}$ V (C) 200 V (D) 0 V

43. Variation of electrostatic potential along x-direction is shown in the graph. The correct statement about electric field is



(A) x component at point B is maximum
(B) x component at point A is towards positive x-axis.
(C) x component at point C is along negative x-axis
(D) x component at point C is along positive x-axis

44. Points A, B, C, D, P and Q are shown in a region of uniform electric field. The potentials at some of the points are V(A) = 2 V & V(P) = V(B) = V(D) = 5 V. The electric field in the region is

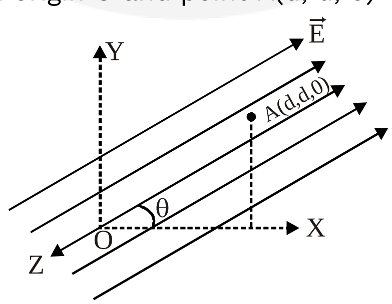


(A) 10 V/m along PQ
(B) $15\sqrt{2}$ V/m along PA
(C) 5 V/m along PC
(D) 5 V/m along PA

45. The equation of an equipotential line in an electric field is y = 2x, then the electric field strength vector at (1, 2) may be

(A) $-4\hat{i}+8\hat{j}$
(B) $4\hat{i}-8\hat{j}$
(C) $-8\hat{i}-4\hat{j}$
(D) $-8\hat{i}+4\hat{j}$

46. A uniform electric field having strength $\vec{E}$ is existing in x-y plane as shown in figure. Find the potential difference between origin O and point A(d, d, 0)



(A) Ed(cosθ + sinθ)
(B) –Ed(sinθ – cosθ)
(C) $\sqrt{2}$ Ed
(D) 2 Ed

47. In a certain region of space, the potential is given by : $V = k[2x^2 - y^2 + z^2]$. The electric field at the point(1, 1, 1) has magnitude =

(A) $k\sqrt{6}$
(B) $2k\sqrt{6}$
(C) $2k\sqrt{3}$
(D) $4k\sqrt{3}$

48. Two identical thin rings, each of radius R meter are coaxially placed at distance R meter apart. If Q_1 and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

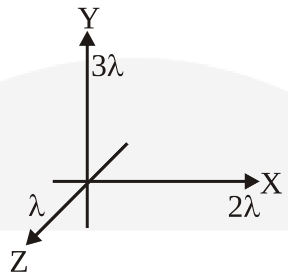
(A) zero

(B) $\frac{q(Q_1 - Q_2)\left(\sqrt{2}-1\right)}{(4\sqrt{2}\,\pi\varepsilon_0 R)}$

(C) $\frac{q\sqrt{2}(Q_1 + Q_2)}{4\,\pi\varepsilon_0 R}$

(D) $\frac{q(Q_1 - Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\,\pi\varepsilon_0 R)}$

49. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from (1, 1, 1) to (0, 1, 1) is equal to



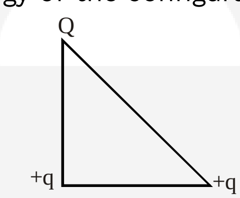
(A) $\frac{\lambda \ln 2}{2\pi\varepsilon_0}$

(B) $\frac{\lambda \ln 2}{\pi\varepsilon_0}$

(C) $\frac{3\lambda \ln 2}{2\pi\varepsilon_0}$

(D) None of these

50. Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to:



(A) $\frac{-q}{1+\sqrt{2}}$

(B) $\frac{-2q}{2+\sqrt{2}}$

(C) $-2q$

(D) $+q$

51. Consider the following conclusions regarding the components of an electric field at a certain point in space given by:

$E_x = -Ky \qquad E_y = Kx\, E_z = 0$

(I) The field is conservative.

(II) The field is non-conservative

(III) The lines of force are straight lines

(IV) The lines of force are circles

Of these conclusions

(A) II and IV are valid

(B) I and III are valid

(C) I and IV are valid

(D) II and III are valid

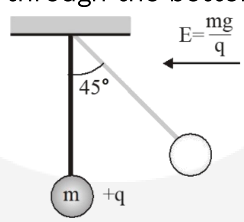
52. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.

(A) If Y is fixed, both p and E are conserved.
(B) If Y is fixed, E is conserved, but not p.
(C) If both are free to move, p is conserved but not E.
(D) If both are free, E is conserved, but not p.

53. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After along time, finally

(A) X will stop, Y will move with velocity u.
(B) X and Y will both move with velocities u/2 each.
(C) X will stop, Y will move with velocity < u.
(D) both will move with velocities < u/2.

54. In space of horizontal electric field (E = (mg)/q) exist as shown in figure and a mass m attached at the end of a light rod. If mass m is released from the position shown in figure find the angular velocity of the rod when it passes through the bottom most position



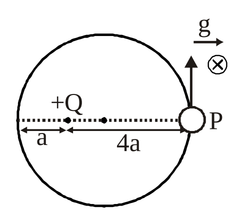
(A) $\sqrt{\frac{g}{\ell}}$ (B) $\sqrt{\frac{2g}{\ell}}$ (C) $\sqrt{\frac{3g}{\ell}}$ (D) $\sqrt{\frac{5g}{\ell}}$

55. Two identical particles of mass m carry a charge Q each. Initially one is at rest in free space and the other is projected directly towards first particle from a large distance with speed v. The closed distance of approach be

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{mv}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{4Q^2}{mv^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{mv^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{mv^2}$

56. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the frictionless ring placed in horizontal plane. In the plane of the ring a charge +Q has also been fixed as shown. The potential at the point P due to +Q is V. The velocity with which the bead should projected from the point P, so that it can complete a circle, should be greater than



(A) $\sqrt{\frac{6qV}{m}}$ (B) $\sqrt{\frac{qV}{m}}$ (C) $\sqrt{\frac{3qV}{m}}$ (D) None

57. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes 2r is

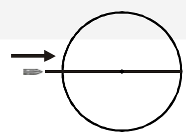
(A) $\sqrt{\frac{Qq}{4\pi\varepsilon_0 mr}}$ (B) $\sqrt{\frac{Qqm}{4\pi\varepsilon_0 r}}$

(C) $\sqrt{\frac{Qqm}{\pi\varepsilon_0 r}}$ (D) $\sqrt{\frac{Qqm}{2\pi\varepsilon_0 r}}$

58. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

(A) R (B) R/2 (C) R / 3 (D) 2 R

59. A small bullet of mass m and charge q is fired towards a solid uniformly charged large sphere of radius R and total charge +q. It enters the surface of sphere with speed u along a diameter of the sphere. Find the minimum speed u so that it can penetrate through the sphere. (Neglect all resistive forces acting on the bullet)

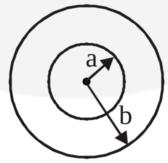


(A) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$ (B) $\frac{q}{\sqrt{4\pi\varepsilon_0 mR}}$ (C) $\frac{q}{\sqrt{8\pi\varepsilon_0 mR}}$ (D) $\frac{\sqrt{3}\,q}{\sqrt{4\pi\varepsilon_0 mR}}$

60. n small drops of same size are charged to V volts each. If they coalesce to form a single large drop, then its potential will be

(A) V/n (B) Vn (C) $Vn^{1/3}$ (D) $Vn^{2/3}$

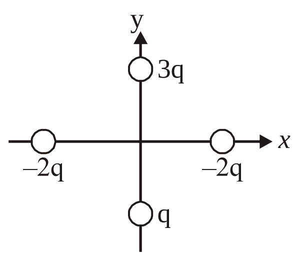
61. In the given figure if the electric potential of the inner metal sphere is 10 V & that of the outer shell is 5 V, then the potential at the centre will be:



(A) 10 V (B) 5 V (C) 15 V (D) 0 V

Electric Dipole :

62. Four charges are placed each at a distance a from origin. The dipole moment of configuration is

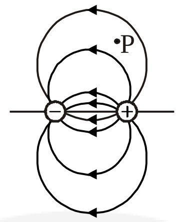


(A) $2q\,a\hat{j}$ (B) $3q\,a\hat{j}$ (C) $2aq[\hat{i}+\hat{j}]$ (D) None

63. The dipole moment of a system of charge +q distributed uniformly on an arc of radius R subtending an angle $\pi/2$ at its centre where another charge $-q$ is placed is :

(A) $\frac{2\sqrt{2}qR}{\pi}$ (B) $\frac{\sqrt{2}qR}{\pi}$ (C) $\frac{qR}{\pi}$ (D) $\frac{2qR}{\pi}$

64. Figure shows the electric field lines around an electric dipole. Which of the arrows best represents the electric field at point P ?



(A)  (B)  (C)  (D) 

65. Point P lies on the axis of a dipole. If the dipole is rotated by 90° anti clock wise, the electric field vector $\vec{E}$ at P will rotate by

(A) 90° clockwise

(B) 180° clockwise

(C) 90° anti-clockwise

(D) it depends on which side of the dipole the point P is located.

66. A and B are two points on the axis and the perpendicular bisector respectively of an electric dipole. A and B are far away from the dipole and at equal distance from it. If the field at A and B are $\vec{E}_A$ and $\vec{E}_B$ then

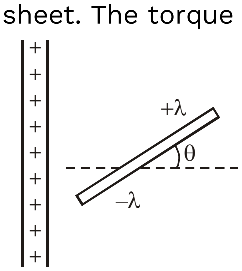
(A) $\vec{E}_A = \vec{E}_B$

(B) $\vec{E}_A = 2\vec{E}_B$

(C) $\vec{E}_A = -2\vec{E}_B$

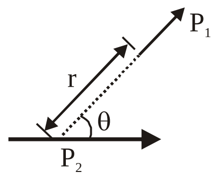
(D) $|\vec{E}_B| = \frac{1}{2}|\vec{E}_A|$ and $\vec{E}_B$ is perpendicular to $\vec{E}_A$

67. A large sheet carries uniform surface charge density σ. A rod of length 2ℓ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is



(A) 0 (B) $\frac{\sigma\lambda\ell^2}{2\varepsilon_0}\sin\theta$ (C) $\frac{\sigma\lambda\ell^2}{\varepsilon_0}\sin\theta$ (D) $\frac{\sigma\lambda\ell}{2\varepsilon_0}$

68. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be



(A) $\frac{2kP_1P_2\cos\theta}{r^3}$ (B) $\frac{-2kP_1P_2\cos\theta}{r^3}$ (C) $\frac{-2kP_1P_2\sin\theta}{r^3}$ (D) $\frac{-4kP_1P_2\cos\theta}{r^3}$

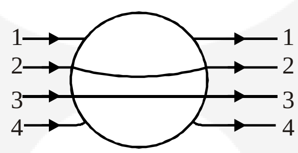
69. An electric dipole (dipole moment p) is placed at a radial distance r>> a (where a is dipole length) from a infinite line of charge having linear charge density +λ. Dipole moment vector is aligned along radial vector $\vec{r}$, force experienced by dipole is:

(A) $\frac{\lambda p}{2\pi\varepsilon_0 r^2}$, attractive (B) $\frac{\lambda p}{2\pi\varepsilon_0 r^3}$, attractive

(C) $\frac{\lambda p}{2\pi\varepsilon_0 r^2}$, repulsive (D) $\frac{\lambda p}{2\pi\varepsilon_0 r^3}$, repulsive

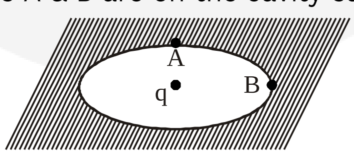
Conductors :

70. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :



(A) 1 (B) 2 (C) 3 (D) 4

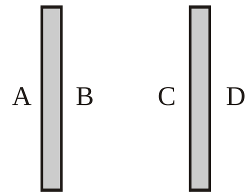
71. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the center of the cavity. The points A & B are on the cavity surface as shown in the figure. Then



(A) electric field near A in the cavity = electric field near B in the cavity
(B) charge density at A = charge density at B
(C) potential at A = potential at B
(D) total electric field flux through the surface of the cavity is q/ε_0

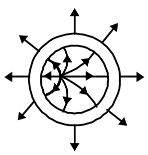
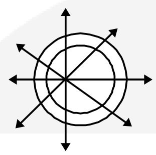
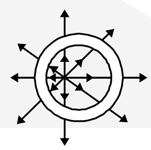
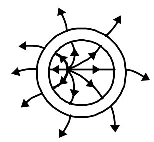
72. A conducting sphere of radius r has a charge. Then
(A) The charge is uniformly distributed over its surface, if there is an external electric field.
(B) Distribution of charge over its surface will be non uniform if no external electric field exists in space.
(C) Electric field strength inside the sphere will be equal to zero only when no external electric field exists
(D) Potential at every point of the sphere must be same

73. Two large metal plates each of area A carry charges +2q and −q and face each other as shown in the figure. The plates are separated by a small distance d. Charges appearing on surfaces A, B, C, D are

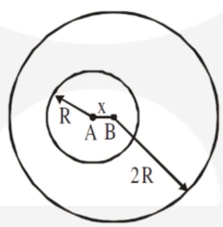


(A) 0.5q, 1.5q, −1.5q, and +0.5q
(B) q, q, −q and zero
(C) zero, 2q, −q and zero
(D) q, q, zero and −q.

74. A point charge q is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct ?

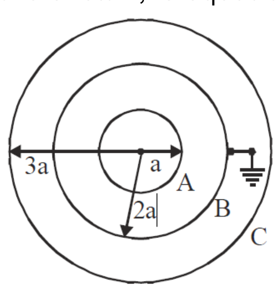
(A)  (B)  (C)  (D) 

75. Figure shows two shells of radii R and 2R. The inner shell (centre at A) is nonconducting and uniformly charged with charge Q while the outer shell (centre at B) is conducting and uncharged, the potential at the point B is:-



(A) Zero
(B) $\frac{KQ}{R}$
(C) $\frac{KQ}{x}$
(D) None of these

76. Figure shows a system of three concentric metal shell's A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then the final charge on the shell B, is equal to :

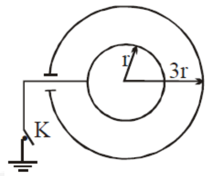


(A) $-\frac{4Q}{13}$
(B) $-\frac{8Q}{11}$
(C) $-\frac{5Q}{3}$
(D) $-\frac{3Q}{7}$

77. Three concentric metallic spherical shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities $-\sigma$, $+\sigma$, and $-\sigma$ respectively. The potential of shell A is :

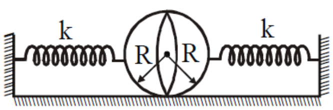
(A) $(\sigma/\varepsilon_0)[a + b - c]$
(B) $(\sigma/\varepsilon_0)[a - b + c]$
(C) $(\sigma/\varepsilon_0)[b - a - c]$
(D) None

78. Figure shows two conducting thin concentric shell of radii and 3r. The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to :-



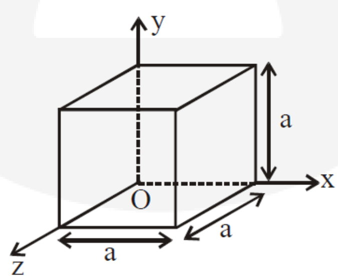
(A) $-q/3$
(B) $q/3$
(C) $3q$
(D) $-3q$

79. Two non-conducting hemispherical surfaces, which are having uniform charge density σ are placed on horizontal surface as shown in figure. Assuming springs are ideal, calculate compression in each spring if both the hemispherical surface are just touching each other.



(A) $\frac{\sigma^2 R^2}{2\varepsilon_0 k}$
(B) R
(C) $\frac{\sigma^2 \pi R^2}{2\varepsilon_0 k}$
(D) None of these

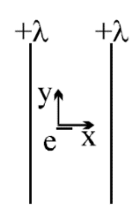
80. Electric field in a region is found to be $E = 3y\hat{j}$. The total energy stored in electric field inside the cube shown will be



(A) $9a^5\epsilon_0$
(B) $3\epsilon_0 a^5$
(C) $\frac{3}{2}\epsilon_0 a^5$
(D) 0

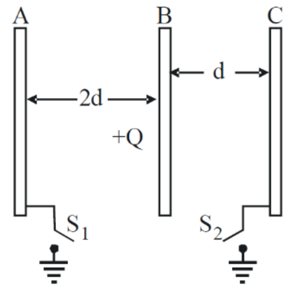
One or More Than One Correct Type Questions

81. An electron is placed just in the middle between two long fixed line charges of charge density $+\lambda$ each. The wires are in the x-y plane (Do not consider gravity)



(A) The equilibrium of the electron will be unstable along x-direction
(B) The equilibrium of the electron will be neutral along y-direction
(C) The equilibrium of the electron will be stable along z-direction
(D) The equilibrium of the electron will be stable along y-direction

82. Three identical, parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. +Q charge is given to B.



(A) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1
(B) If S_2 is closed with S_1 open, a charge of amount Q will pass through S_2
(C) If S_1 and S_2 and closed together, a charge of amount Q/3 will pass through S_1, and a charge of amount 2Q/3 will pass through S_2
(D) All the above statements are incorrect

83. If there were only one type of charge in the universe, then

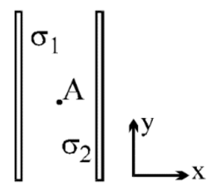
(A) $\oint_s \vec{E}.\vec{dS} \neq 0$ on any surface

(B) $\oint_s \vec{E}.\vec{dS} = 0$ if the charge is outside the surface

(C) $\oint_s \vec{E}.\vec{dS}$ could not be defined

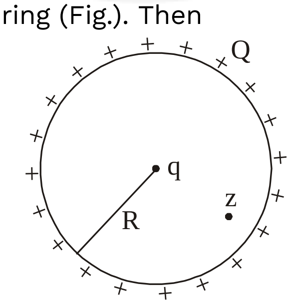
(D) $\oint_s \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface

84. Two large conducting sheets are kept parallel to each other as shown. In equilibrium, the charge density on facing surface is σ_1 and σ_2. What is the value of electric field at A.



(A) $\frac{\sigma_1}{\varepsilon_0}\hat{i}$
(B) $-\frac{\sigma_2}{\varepsilon_0}\hat{i}$
(C) $\frac{\sigma_1+\sigma_2}{2\varepsilon_0}\hat{i}$
(D) $\frac{\sigma_1-\sigma_2}{2\varepsilon_0}\hat{i}$

85. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring (Fig.). Then



(A) If q > 0 and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.
(B) If q < 0 and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.
(C) If q < 0, it will perform SHM for small displacement along the axis.
(D) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for q > 0.

86. Three point charges Q, 4 Q and 16 Q are placed on a straight line 9 cm long. Charges are placed in such a way that the system has minimum potential energy. Then

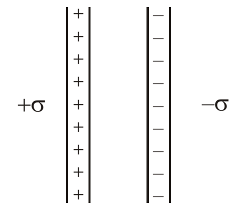
(A) 4 Q and 16 Q must be at the ends and Q at a distance of 3 cm from the 16 Q.

(B) 4 Q and 16 Q must be at the ends and Q at a distance of 6 cm from the 16 Q.

(C) Electric field at the position of Q is zero.

(D) Electric field at the position of Q is $\frac{Q}{4\pi\varepsilon_0}$

87. Two infinite sheets of uniform charge density $+\sigma$ and $-\sigma$ are parallel to each other as shown in the figure. Electric field at the



(A) points to the left or to the right of the sheets is zero.

(B) midpoint between the sheets is zero.

(C) midpoint of the sheets is σ / ε_0 and is directed towards right.

(D) midpoint of the sheet is $2\sigma / \varepsilon_0$ and is directed towards right.

88. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (–1, 0, 0) and (0, –1, 0).

(A) The potential at the origin is zero.

(B) The field at the origin is zero.

(C) The potential at all points on the z-axis, other than the origin, is zero.

(D) The field at all points on the z-axis, other than the origin acts along the z-axis.

89. An electric dipole moment $\vec{p} = (2.0\hat{i} + 3.0\hat{j})\mu Cm$ is placed in a uniform electric field

$\vec{E} = (3.0\hat{i} + 2.0\hat{k}) \times 10^5 NC^{-1}$

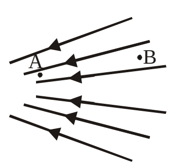
(A) The torque that $\vec{E}$ exerts on $\vec{p}$ is $(0.6\hat{i} - 0.4\hat{j} - 0.9\hat{k})$ Nm.

(B) The potential energy of the dipole is –0.6 J.

(C) The potential energy of the dipole is 0.6 J.

(D) If the dipole is rotated in the electric field, the maximum potential energy of the dipole is 1.3 J.

90. Which of the following is true for the figure showing electric lines of force? (E is electrical field, V is potential)



(A) $E_A > E_B$ (B) $E_B > E_A$ (C) $V_A > V_B$ (D) $V_B > B_A$

91. Charges Q_1 and Q_2 lies inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.

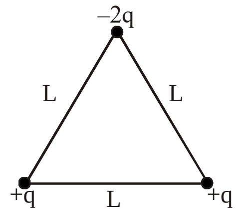
(A) If Q_1 changes, both E and ϕ will change.

(B) If Q_2 changes, E will change but ϕ will not change.

(C) If $Q_1 = 0$ and $Q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.

(D) If $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

92. Three points charges are placed at the corners of an equilateral triangle of side L as shown in the figure.



(A) The potential at the centroid of the triangle is zero.

(B) The electric field at the centroid of the triangle is zero.

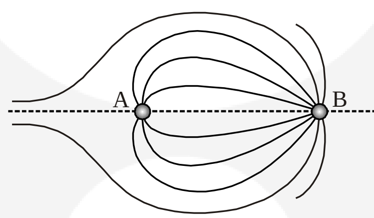
(C) The dipole moment of the system is $\sqrt{2}\,qL$

(D) The dipole moment of the system is $\sqrt{3}\,qL$

Comprehension Type Questions

Paragraph for Question No. 93 to 94

In the figure, electric field lines are shown for an isolated system of two charges A and B. Study the lines of forces shown and answer the following questions.



93. From the figure you can certainly conclude that

(A) Both of them have same magnitude and opposite sign.

(B) Both of them have opposite sign and magnitude of A is greater than that of B.

(C) Both of them have opposite sign and magnitude of B is greater than that of A.

(D) Both of them have opposite sign but information is insufficient to predict their relative magnitudes.

94. Charges A and B have

(A) magnitudes in the ratio 5: 3 respectively

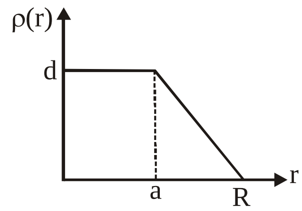
(B) magnitudes in the ratio 3:2 respectively

(C) magnitudes in the ratio 3:5 respectively

(D) magnitudes in the ratio 2:3 respectively

Paragraph for Question No. 95 to 97

The nuclear charge (Ze) is non–uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



95. The electric field at r = R is :

(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a

96. For a = 0, the value d (maximum value of ρ as shown in the figure) is :

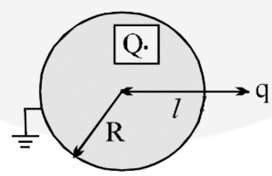
(A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

97. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies :

(A) a = 0 (B) $a = \frac{R}{2}$ (C) a = R (D) $a=\frac{2R}{3}$

Paragraph for Question No. 98 to 100

There is a cubical cavity inside a conducting sphere of radius R. A positive point charge Q is placed at the centre of the cube and another positive charge q is placed at a distance $\ell(>R)$ from the centre of the sphere. The sphere is earthed



98. Charge induced on the inner surface of cavity is

(A) –Q, uniformly distributed
(B) –Q, non-uniformly distributed
(C) –(Q+q) non-uniformly distributed
(D) None

99. Net charge on the outer surface of conducting sphere is

(A) +Q (B) $Q - qR/\ell$ (C) $-qR/\ell$ (D) None

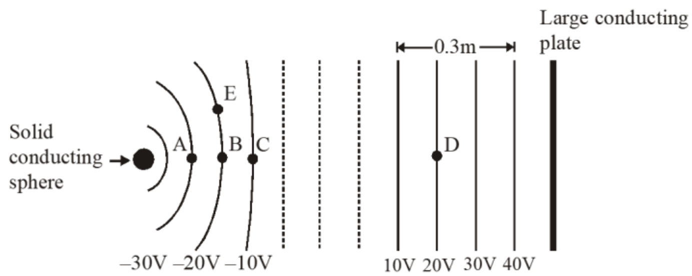
100. Potential at a point inside the cavity is :

(A) Zero (B) positive (C) negative (D) Can not be determined

Paragraph for Question No. 101 to 103

The sketch below shows cross–sections of equipotential surfaces between two charged conductors that are shown in solid black. Some points on the equipotential surfaces near the conductors are marked as A, B, C,...... The arrangement lies in air.

[Take ε_0 = 8.85 × 10^{-12} $C^2/N\ m^2$]



101. Surface charge density of the plate is equal to

(A) 8.85 × 10^{-10} C/m^2
(B) –8.85 × 10^{-10} C/m^2
(C) 17.7 × 10^{-10} C/m^2
(D) –17.7 × 10^{-10} C/m^2

102. A positive charge is placed at B. When it is released

(A) no force will be exerted on it
(B) it will move towards A
(C) it will move towards C
(D) it will move towards E

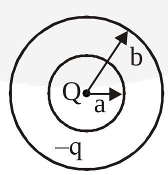
103. How much work is required to slowly move a –1 μC charge from E to D ?

(A) 2 × 10^{-5} J
(B) –2 × 10^{-5} J
(C) 4 × 10^{-5} J
(D) –4 × 10^{-5} J

Paragraph for Question No. 104 to 105

A spherical shell with an inner radius a and an outer radius b is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge –q is placed on the shell.

104. Charge –q is distributed on the surfaces as



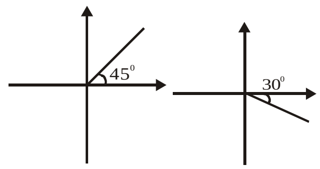
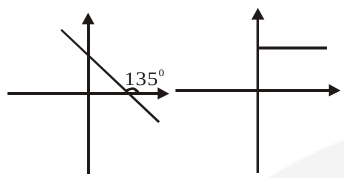
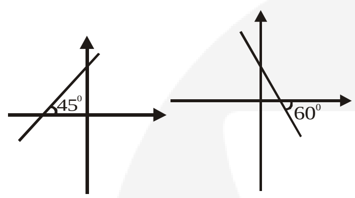
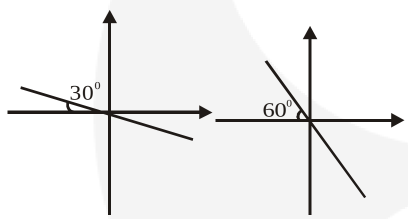
(A) –Q on the inner surface, –q on outer surface
(B) –Q on the inner surface, – q + Q on the outer surface
(C) +Q on the inner surface, –q –Q on the outer surface
(D) The charge –q is spread uniformly between the inner and outer surface.

105. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance R(a < R < b) from the centre of the shell is (where $K = \frac{1}{4\pi\varepsilon_0}$)

(A) 0
(B) $\frac{KQ}{a}$
(C) $K\frac{Q-q}{R}$
(D) $K\frac{Q-q}{b}$

Matrix Match Type Question

106. Column–I shows graphs of electric potential V versus x and y in a certain region for four situations. Column–II shows the range of angle which the electric field vector makes with positive x-direction

Column–I : V versus x, V versus y		**Column–II : Range of angle**	
(A)		(P)	$0^0 \le \theta < 45^0$
(B)		(Q)	$45^0 \le \theta < 90^0$
(C)		(R)	$90^0 \le \theta < 135^0$
(D)		(S)	$135^0 \le \theta < 180^0$

ANSWER KEY

1.	(B)	**2.**	(D)	**3.**	(C)	**4.**	(D)	**5.**	(B)	**6.**	(B)	**7.**	(B)
8.	(B)	**9.**	(A)	**10.**	(A)	**11.**	(B)	**12.**	(D)	**13.**	(A)	**14.**	(C)
15.	(D)	**16.**	(A)	**17.**	(D)	**18.**	(A)	**19.**	(C)	**20.**	(C)	**21.**	(B)
22.	(B)	**23.**	(B)	**24.**	(C)	**25.**	(D)	**26.**	(D)	**27.**	(C)	**28.**	(D)
29.	(A)	**30.**	(D)	**31.**	(A)	**32.**	(B)	**33.**	(A)	**34.**	(A)	**35.**	(A)
36.	(B)	**37.**	(C)	**38.**	(B)	**39.**	(B)	**40.**	(B)	**41.**	(A)	**42.**	(D)
43.	(D)	**44.**	(B)	**45.**	(D)	**46.**	(A)	**47.**	(B)	**48.**	(B)	**49.**	(B)
50.	(B)	**51.**	(A)	**52.**	(B)	**53.**	(A)	**54.**	(B)	**55.**	(B)	**56.**	(A)
57.	(B)	**58.**	(C)	**59.**	(B)	**60.**	(D)	**61.**	(A)	**62.**	(A)	**63.**	(A)
64.	(B)	**65.**	(A)	**66.**	(C)	**67.**	(B)	**68.**	(B)	**69.**	(A)	**70.**	(D)
71.	(C)	**72.**	(D)	**73.**	(A)	**74.**	(A)	**75.**	(B)	**76.**	(B)	**77.**	(C)
78.	(B)	**79.**	(C)	**80.**	(C)	**81.**	(A,B,C)	**82.**	(A,B,C)	**83.**	(B,D)	**84.**	(A,B,D)
85.	(A,B,C)	**86.**	(B,C)	**87.**	(A,C)	**88.**	(B,D)	**89.**	(A,B,D)	**90.**	(A,D)	**91.**	(A,B,C)
92.	(A,D)	**93.**	(C)	**94.**	(C)	**95.**	(A)	**96.**	(B)	**97.**	(C)	**98.**	(B)
99.	(C)	**100.**	(B)	**101.**	(A)	**102.**	(B)	**103.**	(D)	**104.**	(B)	**105.**	(D)

106. A→S; B→P; C→R; D→Q

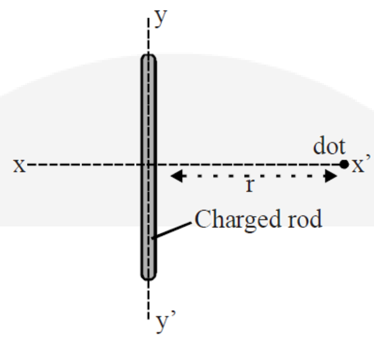
Objective Exercise - II

Single Correct Type Questions

1. There are four concentric metallic shells A, B, C and D of radii a, 2a, 3a and 4a respectively. Shells B and D are given charges +q and –q respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is

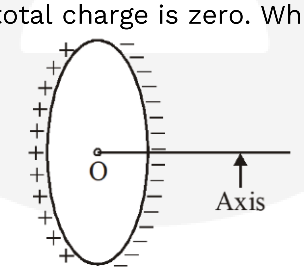
(A) $\frac{Kq}{2a}$ (B) $\frac{Kq}{3a}$ (C) $\frac{Kq}{4a}$ (D) $\frac{Kq}{6a}$

2. A uniformly charged rod is kept on y-axis with centre at origin, as shown. Which of the following actions will increase the electric field strength at the position of the dot.



(A) Make the rod longer without changing the charge
(B) Make the rod shorter without changing the charge
(C) Make the rod shorter without changing the linear charge density
(D) Rotate the rod about yy'

3. The figure shows a nonconducting ring which has positive and negative charge on uniformly distributed on it such that the total charge is zero. Which of the following statements is true?

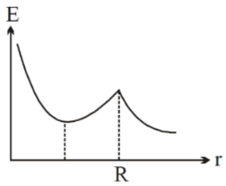


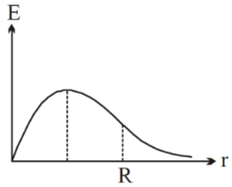
(A) The potential at all the points on the axis will be zero.
(B) The electric field at all the points on the axis will be zero.
(C) The direction of electric field at all points on the axis will be along the axis.
(D) If the ring is placed inside a uniform external electric field then net torque and force acting on the ring would be zero.

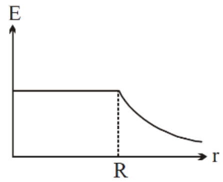
4. A system consists of uniformly charged sphere of radius R and a surrounding medium filled by a charge with the volume density $\rho = \frac{\alpha}{r}$, where α is a position constant and r is the distance from the centre of the sphere. The charge of the sphere for which electric field intensity E out side the sphere is independent of r is

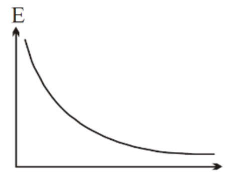
(A) $\frac{\alpha}{2\varepsilon_0}$ (B) $\frac{2}{\alpha\varepsilon_0}$ (C) $2\pi\alpha R^2$ (D) αR^2

5. A spherical insulator of radius R is charged uniformly with a charge Q throughout its volume and contains a point charge $\frac{Q}{16}$ located at its centre. Which of the following graphs best represent qualitatively, the variation of electric field intensity E with distance r from the centre.

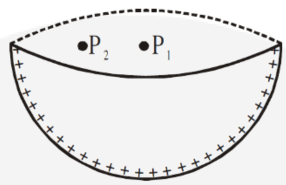
(A) 

(B) 

(C) 

(D) 

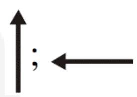
6. Consider a uniformly charged hemispherical shell shown below. Indicate the directions (not magnitude) of the electric field at the central point P_1 and an off center point P_2 on the drumhead of the shell.



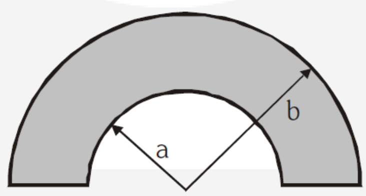
(A) 

(B) 

(C) 

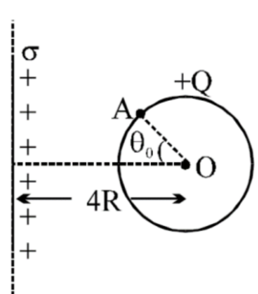
(D) 

7. A non-conducting semicircular disc (as shown in figure) has a uniform surface charge density σ. The electric potential at the centre of the disc:-



(A) $\frac{\sigma}{2\pi\varepsilon_0}\frac{\ell n(b/a)}{(b-a)}$ (B) $\frac{\sigma(b-a)}{2\varepsilon_0}$ (C) $\frac{\sigma(b-a)}{4\varepsilon_0}$ (D) $\frac{\sigma(b-a)}{4\pi\varepsilon_0}$

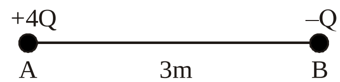
8. A conducting sphere of radius R and charge Q is placed near a uniformly charged non-conducting infinitely large thin plate having surface charge density σ. Then find the potential at point A (on the surface of sphere) due to charge on sphere (here $\frac{1}{4\pi\varepsilon_0}, \theta_0 = \frac{\pi}{3}$)



(A) $K\frac{Q}{R} - \frac{\sigma}{4\varepsilon_0}R$ (B) $K\frac{Q}{R} - \frac{\sigma R}{\varepsilon_0}$ (C) $K\frac{Q}{R}$ (D) None of these

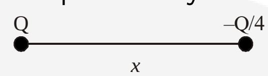
Multiple Correct Type Questions

9. Two fixed charges 4Q (positive) and Q (negative) are located at A and B, the distance AB being 3 m.



(A) The point P where the resultant field due to both is zero is on AB outside AB.
(B) The point P where the resultant field due to both is zero is on AB inside AB.
(C) If a positive charge is placed at P and displaced slightly along AB it will execute oscillations.
(D) If a negative charge is placed at P and displaced slightly along AB it will execute oscillations.

10. Two point charges Q and – Q/4 are separated by a distance x. Then



(A) potential is zero at a point on the axis which is x/3 on the right side of the charge – Q/4
(B) potential is zero at a point on the axis which is x/5 on the left side of the charge – Q/4
(C) electric field is zero at a point on the axis which is at a distance x on the right side of the charge-Q/4
(D) there exist two points on the axis where electric field is zero.

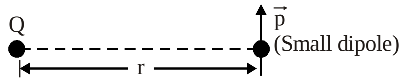
11. At distance of 5cm and 10cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then
(A) potential at its surface is 150V.
(B) the charge on the sphere is $(5/3) \times 10^{-10}$C.
(C) the electric field on the surface is 1500 V/m.
(D) the electric potential at its centre is 225V.

12. A proton and a deuteron are initially at rest and are accelerated through the same potential difference. Which of the following is true concerning the final properties of the two particles?
(A) They have different speeds (B) They have same momentum
(C) They have same kinetic energy (D) none of these

13. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct?
(A) The entire charge will appear on its outer surface.
(B) All points on the conductor will have the same potential.
(C) All points on its surface will have the same charge density.
(D) All points near its surface and outside it will have the same electric intensity.

14. An electric dipole is placed at the centre of a sphere. Mark the correct answer
(A) The flux of the electric field through the sphere is zero
(B) The electric field is zero at every point of the sphere.
(C) The electric potential is zero everywhere on the sphere.
(D) The electric potential is zero on a circle on the surface.

15. For the situation shown in the figure below (assume r >> length of dipole) mark out the correct statement(s).



(A) Force acting on the dipole is zero.

(B) Force acting on the dipole is approximately $\frac{pQ}{4\pi\varepsilon_0 r^3}$ and is acting upward

(C) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in clockwise direction.

(D) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in anti-clockwise direction

16. Four identical particles each having mass m and charge q are placed at the vertices of a square of side ℓ. All the particles are free to move without any friction and released simultaneously from rest. Then

(A) At all instant, the particles remains at vertices of square whose edge length is changing

(B) The configuration is changing (not remaining square) as the time passes

(C) The speed of the particles when one of the particles get displaced by $\frac{\ell}{\sqrt{2}}$ is $\sqrt{\frac{q^2}{8\pi\varepsilon_0 m\ell}\left(2+\frac{1}{\sqrt{2}}\right)}$

(D) Speed of the particles can not be found

17. A particle of mass m and charge q is thrown in a region where uniform gravitational field and electric field are present. The path of particle

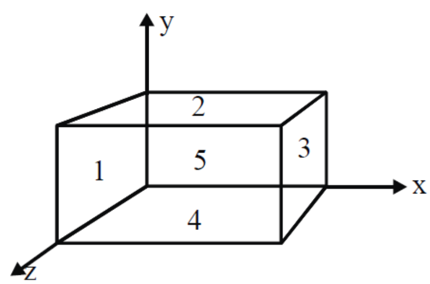
(A) may be a straight line

(B) may be a circle

(C) may be a parabola

(D) may be a hyperbola

18. In a region of space, the electric field $\vec{E} = E_0 x\hat{i} + E_0 y\hat{j}$. Consider an imaginary cubical volume of edge 'a' with its edges parallel to the axes of coordinates. Now,



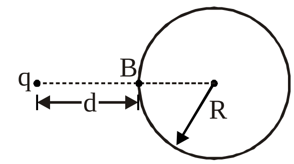
(A) The total electric flux through the faces 1 and 3 is E_0a^3

(B) The charge inside the cubical volume is $2\varepsilon_0E_0a^3$

(C) The total electric flux through the faces 2 and 4 is $2E_0a^3$

(D) The charge inside the cubical volume is $\varepsilon_0E_0a^3$

19. For the situation shown in the figure below, mark out the correct statement(s)



Hollow neutral conductor

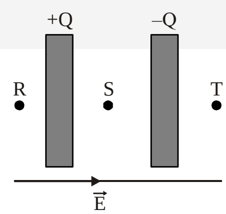
(A) Potential of the conductor is $\frac{q}{4\pi\varepsilon_0(d+R)}$

(B) Potential of the conductor is $\frac{q}{4\pi\varepsilon_0 d}$

(C) Potential of the conductor can't be determined as nature of distribution of induced charges is not known

(D) Potential at point B due to induced charges is $\frac{-qR}{4\pi\varepsilon_0(d+R)d}$

20. Two large thin conducting plates with small gap in between are placed in a uniform electric field E (perpendicular to the plates). Area of each plate is A and charges +Q and –Q are given to these plates as shown in the figure. If points R, S and T as shown in the figure are three points in space, then the



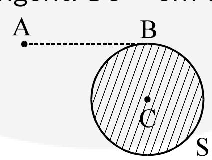
(A) field at point R is E

(B) field at point S is E

(C) field at point T is $\left(E+\frac{Q}{\varepsilon_0 A}\right)$

(D) field at point S is $\left(E+\frac{Q}{A\varepsilon_0}\right)$

21. S is a solid neutral conducting sphere. A point charge q of 1×10^{-6}C is placed at point A. C is the centre of sphere and AB is a tangent. BC = 3m and AB = 4m.



(A) The electric potential of the conductor is 1.8 kV

(B) The electric potential of the conductor is 2.25 kV

(C) The electric potential at B due to induced charges on the sphere is – 0.45 kV.

(D) The electric potential at B due to induced charges on the sphere is 0.45 kV.

22. The electric field in space is $4\hat{i}-3\hat{j}$ N/C. Comment on each of the following.

(A) The Potential at (4,3,0) is 7V if the potential at the origin is taken to be zero.

(B) The energy contained in a sphere of radius 1 m is $\frac{25}{54}$ nJ

(C) The potential at (4, 3, 0) is – 7V if potential at origin is taken to be zero

(D) The field exerts no net force on a dipole

Assertion and Reason Type Questions

23. **Statement-1 :** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)

Statement-2 : Electric lines of force represents path of charged particle which is released from rest in it.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

24. **Statement-1:** In a given arrangement of charges, an extra charge is placed outside the Gaussian surface. In the Gauss Theorem $\oint \vec{E}.d\vec{s} = \frac{Q_{in}}{\varepsilon_0}$: value of Q_{in} remains unchanged whereas electric field $\vec{E}$ at the Gaussian surface is changed.

Statement-2: Electric field $\vec{E}$ at any point on the Gaussian surface is due to inside charge only.

(A)Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement 1.

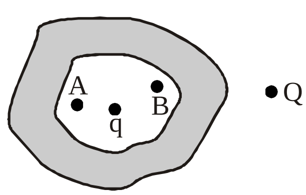
(B)Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C)Statement-1 is true, statement-2 is false.

(D)Statement-1 is false, statement-2 is true.

25. **Statement-1:** A point charge q is placed inside a cavity of conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q is pushed away from conductor, the potential difference ($V_A - V_B$) between two points A and B within the cavity of conductor remains constant.

Statement-2: The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor.



(A)Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B)Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

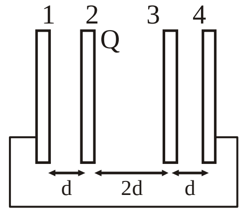
(C)Statement-1 is true, statement-2 is false.

(D)Statement-1 is false, statement-2 is true.

Comprehension Type Questions

Paragraph for Question No. 26 to 28

Four metallic plates are placed as shown in the figure. Plate 2 is given a charge Q whereas all other plates are uncharged. Plates 1 and 4 are joined together. The area of each plate is same.



26. The charge appearing on the right side of plate 3 is

(A) zero
(B) +Q/4
(C) −3Q/4
(D) Q/2

27. The charge appearing on right side of plate 4 is

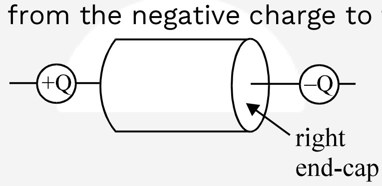
(A) zero
(B) −Q/4
(C) −3Q/4
(D) +Q/2

28. The potential difference between plates 1 and 2 is

(A) $\frac{3}{2}\frac{Qd}{\varepsilon_0 A}$
(B) $\frac{Qd}{\varepsilon_0 A}$
(C) $\frac{3}{4}\frac{Qd}{\varepsilon_0 A}$
(D) $\frac{3Qd}{\varepsilon_0 A}$

Paragraph for Question No. 29 to 30

The figure applies to the following two questions. Positive and negative charges of equal magnitude lie along the symmetry axis of a cylinder. The distance from the positive charge to the left end-cap of the cylinder is the same as the distance from the negative charge to the right end -cap.



29. What is the flux of the electric field through the closed cylinder ?

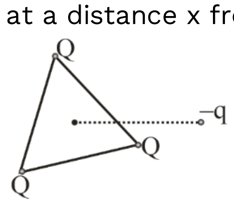
(A) 0
(B) + Q / ε_0
(C) + 2Q / ε_0
(D) − Q / ε_0

30. What is the sign of the flux through the right end-cap of the cylinder ?

(A) Positive
(B) Negative
(C) There is no flux through the right end-cap.
(D) None of these

Paragraph for Question No. 31 to 34

Three charged particles each of +Q are fixed at the corners of an equilateral triangle of side 'a'. A fourth particle of charge −q and mass m is placed at a point on the line passing through centroid of triangle and perpendicular to the plane of triangle at a distance x from the centre of triangle



31. Force on the fourth particle is

(A) $\frac{1}{4\pi \in_0} \frac{9\sqrt{3}\ Qx}{(3x^2 + a^2)^{3/2}}$

(B) $\frac{1}{4\pi \in_0} \frac{3\sqrt{3}\ Qx}{(3x^2 + a^2)^{3/2}}$

(C) $\frac{1}{4\pi \in_0} \frac{2\sqrt{2}\ Qx}{(2x^2 + a^2)^{3/2}}$

(D) $\frac{1}{4\pi \in_0} \frac{4\sqrt{2}\ Qx}{(2x^2 + a^2)^{3/2}}$

32. Value of x for which the force is maximum is

(A) $\frac{a}{\sqrt{3}}$ (B) $\frac{a}{\sqrt{2}}$ (C) $\frac{a}{\sqrt{6}}$ (D) $\frac{a}{\sqrt{5}}$

33. If particle is released from a distance 'x' its speed when it reaches the centre of triangle is

(A) $\sqrt{\frac{6\sqrt{3}Qq}{4\pi \in_0 m}\left[\frac{1}{a} - \frac{1}{(a^2 + 3x^2)^{1/2}}\right]}$

(B) $\sqrt{\frac{3\sqrt{3}Qq}{4\pi \in_0 m}\left[\frac{1}{a} - \frac{1}{(a^2 + 3x^2)^{1/2}}\right]}$

(C) $\sqrt{\frac{\sqrt{3}Qq}{4\pi \in_0 m}\left[\frac{1}{a} - \frac{1}{(a^2 + 3x^2)^{1/2}}\right]}$

(D) $\sqrt{\frac{\sqrt{3}Qq}{2\pi \in_0 m}\left[\frac{1}{a} - \frac{1}{(a^2 + 3x^2)^{1/2}}\right]}$

34. For small oscillation the period of oscillation of fourth particle is

(A) $2\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}\ Q}}$

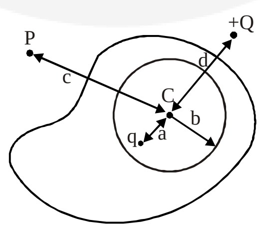
(B) $\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}\ Q}}$

(C) $2\pi\sqrt{\frac{2\pi \in_0 ma^3}{9\sqrt{3}\ Q}}$

(D) $2\pi\sqrt{\frac{\pi \in_0 ma^3}{9\sqrt{3}\ Q}}$

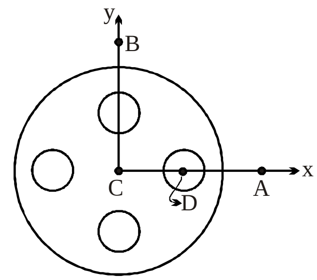
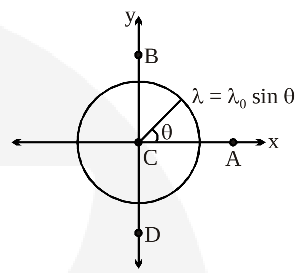
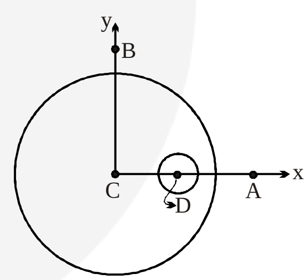
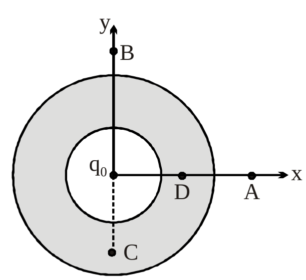
Matrix Match Type Questions

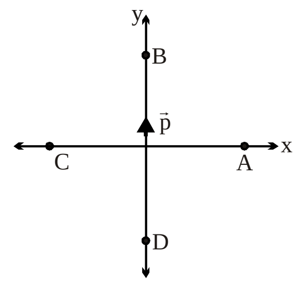
35. In the shown figure the conductor is uncharged and a charge q is placed inside a spherical cavity at a distance a from its centre (C). Point P and charge +Q are as shown. a, b, c, d are known.



	Column-I		Column-II
(A)	Electric field due to induced charges on the inner surface of cavity at point P	(P)	zero
(B)	Electric potential due to charges on the inner surface of cavity and q at P	(Q)	non-zero
(C)	Electric field due to induced charges on the outer surface of conductor and Q at C	(R)	value can be stated with the given data.
(D)	Electric potential due to induced charges on the inner surface of cavity at C	(S)	value cannot be stated from the given data

36. Column-II shows some charge distributions and column-I has some statement about electric field at four points A, B, C, D. Match column-I with column-II.

Column-I	Column-II
(A) $\vec{E}_A$ has x component only	(P)  Solid non conducting sphere of radius R of volumetric charge density ρ with four symmetrical cavities. All the five sphere's centre lie in same plane.
(B) $\vec{E}_B$ has y component only	(Q)  A very small circular filament lying in xy-plane. Points B, C and D are at large distance compared to radius of circle.
(C) $\vec{E}_C$ has y component only	(R)  A charged spherical conductor with a cavity in it.
(D) $\lvert\vec{E}_D\rvert$ is zero	(S)  A hollow thick spherical charge conductor with a concentric cavity. Charge q_0 placed inside at centre of cavity.

(T) 

A small electric dipole placed at origin. A,B,C and D are four points at large distance from origin.

ANSWER KEY

1.	(D)	**2.**	(B)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(A)	**9.**	(A,D)	**10.**	(A,B,C)	**11.**	(A,C,D)	**12.**	(A,C)	**13.**	(A,B)	**14.**	(A,D)
15.	(B,C)	**16.**	(A,C)	**17.**	(A,C)	**18.**	(A,B)	**19.**	(A,D)	**20.**	(A,D)	**21.**	(A,C)
22.	(B,C,D)	**23.**	(C)	**24.**	(C)	**25.**	(A)	**26.**	(B)	**27.**	(D)	**28.**	(C)
29.	(A)	**30.**	(A)	**31.**	(A)	**32.**	(C)	**33.**	(D)	**34.**	(A)		

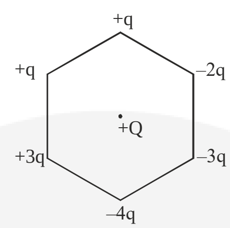
35. A→QS; B→PR; C→PR; D→QR

36. A→P,R,S; B→P,Q,R,S,T; C→Q,T; D→R,S

Properties of Charge and Coulomb's Law:

1. Two particles of charges –2q and q are fixed at points A and B, ℓ distance apart. Where should a positive test charge be placed on the line connecting the charges for it to be in equilibrium?

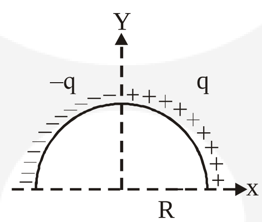
2. Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F, then net electric force on the +Q is nF. Find the value of n.



3. Three charges 4q, Q and q are in a straight line in the position of 0, $l/2$ and l respectively. The resultant force on q will be zero, if Q =

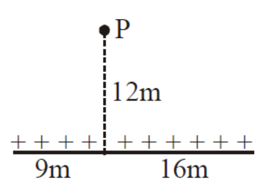
Electric Field:

4. Find the electric field at centre of semicircular ring shown in figure.



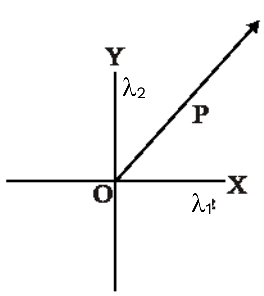
5. A clock face has negative charges –q, –2q, –3q, ..., –12q fixed at the positions of the corresponding numerals on the dial. Assume that the clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

6. Find the magnitude of electric field (in N/C) due to a line charge of $\lambda = 2\sqrt{2}$ nC/m at a point P as Shown.

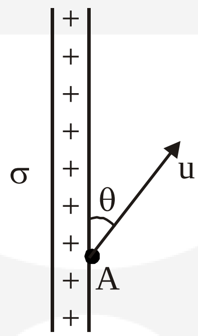


7. A charge $+10^{-9}$ C is located at the origin in free space and another charge Q at (2, 0, 0). If the X-component of the electric field at (3, 1, 1) is zero, calculate the value of Q. Is the Y-component zero at (3, 1, 1)?

8. Two mutually perpendicular infinite wires along x-axis and y-axis carry charge densities λ_1 and λ_2. The electric line of force at P is along the line $y = \frac{1}{\sqrt{3}}x$, where P is also a point lying on the same line then find λ_1/λ_2



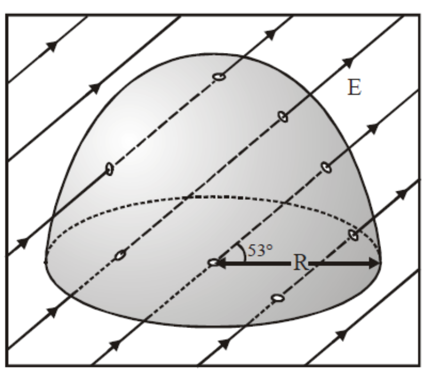
9. A particle of mass m and negative charge q is thrown in a gravity free space with speed u from the point A on the large charged sheet of uniform surface charge density σ, as shown in figure. Find the maximum distance from A on sheet where the particle can strike.



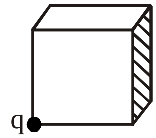
Gauss's Law:

10. Calculate the magnitude of electrostatic force on a charge placed at a vertex of a triangular pyramid (4 vertices, 4 faces), if 4 equal charges (q) are placed at all four vertices of pyramid of side 'a'.

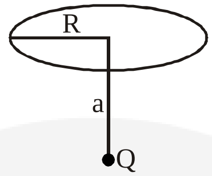
11. A uniform electric field E = 500 N/C passes through a hemispherical surface of radius R = 1.2 m as shown in figure. The net electric flux (in S.I. units) through the hemispherical surface only is $2N\pi$. Then find the value of N.



12. The length of each side of a cubical closed surface is ℓ. If charge q is situated on one of the vertices of the cube, then find the flux passing through shaded face of the cube.



13. A point charge Q is located on the axis of a disc of radius R at a distance a from the plane of the disc. If one fourth (1/4th) of the total flux from the charge passes through the disc, find the relation between a & R.

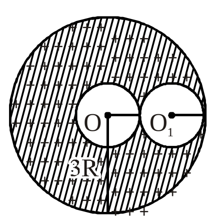


14. Consider a triangular surface whose vertices are three points having co–ordinate A(2a, 0, 0), B(0, a, 0), C(0, 0, a). If there is a uniform electric field $E_0\hat{i} + 2E_0\hat{j} + 3E_0\hat{k}$ then flux linked to triangular surface ABC is $\frac{E_0 a^2}{2} \times N$. Find the value of N.

15. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8×10^3 Nm^2/C.(here $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\text{-}m^2$)
(A) What is the net charge inside the box?
(B) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?

16. An electric field is given by $\vec{E} = 4\hat{i} + 3\left(y^2 + 2\right)\hat{j}$ pierces gaussian cube of side 1m placed at origin such that one of its corners is at origin & rest of sides are along positive side of coordinate axis. If the magnitude of net charge enclosed is $n\varepsilon_0$ then n (in SI units) will be equal to.

17. A thick shell with inner radius R and outer radius 3R has uniform charge density $\rho c/m^3$. It has a spherical cavity of radius R as shown in the figure. The electric field at the centre O_1 of the cavity is



Electric Potential Energy And Electric Potential:

18. A circular ring of radius R with uniform positive charge density λ per unit length is fixed in the Y-Z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(\sqrt{3}R, 0, 0)$ on the positive X-axis directly towards O, with initial velocity *v*. Find the smallest value of the speed *v* such that the particle does not return to P.

19. Consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) to figure (ii).

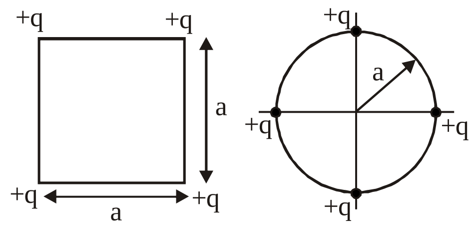


fig (i) fig (ii)

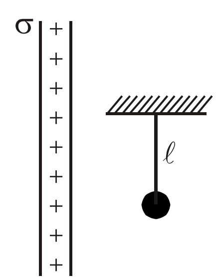
20. Three charges 0.1 coulomb each are placed on the corners of an equilateral triangle of side 1 m. If the energy is supplied to this system at the rate of 1 kW, how much time would be required to move one of the charges onto the midpoint of the line joining the other two?

21. A point charge +q and mass 100 gm experiences a force of 100 N at a distance 20 cm from a straight infinitely long uniformly charged straight wire. If the charge particle is released there, find its speed when it is at a distance 40 cm from wire.

22. A charge + Q is uniformly distributed over a thin ring of radius R and very large mass. A particle of charge – Q and mass m starts from rest at a point on the axis of the ring far away from the centre of the ring. Find the velocity of this particle at the moment it passes through the centre of the ring.

23. A small ball of mass 2×10^{-3} Kg having a charge of 1 μC is suspended by a string of length 0. 8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

24. A simple pendulum of length ℓ and bob mass m is hanging in front of a large nonconducting sheet of surface charge density σ. If suddenly a charge +q is given to the bob in the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



25. There are 27 drops of a conducting fluid. Each has radius r and they are charged to a potential V_0. They are then combined to form a bigger drop. Find its potential.

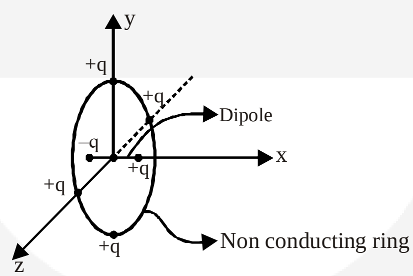
26. **An electric field** $\left(-30\hat{i} + 20\hat{j}\right)\text{Vm}^{-1}$ exists in the space. If potential at the origin is zero then find the potential at (5m, 3m) in volts.

27. The electric field strength depends only on the x, y and z coordinates according to the law $E = \frac{a(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2 + y^2 + z^2)^{3/2}}$, where a = 122.5 SI unit and is a constant. Find the potential difference (in volt) between (3, 2, 6) and (0, 3, 4).

28. A particle of mass m and charge – q moves along a diameter of a uniformly charged sphere of radius R and carrying a total charge + Q. Find the frequency of SHM of the particle if the amplitude does not exceed R.

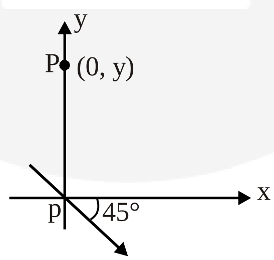
Electric Dipole:

29. An electric dipole placed along x-axis with its dipole moment vector pointing along positive x-axis, is placed at the centre of wire frame as shown in figure. A fixed circular wire frame lies in y-z plane with 20 identical charges of magnitude q, kept symmetrically fixed to wire frame. What is work done (in joule) by an external agent in slowly turning dipole through 180° from initial orientation of dipole moment vector.



Radius of ring : R = 1 cm
Length of dipole : r = 1 mm
Charged : q = 1μC

30. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point P(0, y).



31. A dipole of dipole moment $\vec{p} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ is placed at point A(2,–3,1). The electric potential due to this dipole at the point B(4,–1,0) is (ab) × 10^9 volt here 'a' represents sign (for negative answer select 0 for positive answer select 1). Write the value of $(a+b)^2$. The parameters specified here are in S.I. units.

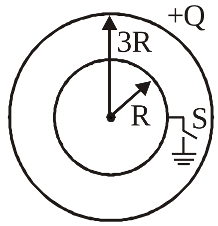
32. A charge +Q is fixed at the origin of the coordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.

(A) Calculate the K.E. of the dipole when it reaches to a point (d, 0)

(B) Calculate the force on the charge +Q due to the dipole at this moment.

Conductors:

33. Two thin conducting shells of radii R and 3R are shown in figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of switch S. Find the charge attained by the inner shell.



34. Consider two concentric conducting spheres of radii a & b (b > a). Inside sphere has a positive charge q_1. What charge should be given to the outer sphere so that potential of the inner sphere becomes zero? How does the potential vary between the two spheres & outside?

35. A conducting liquid bubble of a and thickness t (t<< a) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet.

36. Two infinite horizontal rods with linear charge density $+\lambda$ are kept apart by distance d. An electron e is kept at the mid point between the two rods. On being given slight vertical displacement (in the plane perpendicular to the plane of rods). Find the time period of this oscillatory motion. Is it SHM. (Neglect gravity)

ANSWER KEY

1. $a = \ell(1+\sqrt{2})$ outside line segment AB and near to B

2. 9

3. $-q$

4. $-\dfrac{4kq}{\pi R^2}\hat{i}$

5. 9.30

6. 3

7. $-\left[\dfrac{3}{11}\right]^{3/2} \times 3 \times 10^{-9}$C, No.

8. 1/3

9. $\dfrac{2\varepsilon_0 u^2 m}{q\sigma}$

10. $F = \dfrac{q^2\sqrt{6}}{4\pi\varepsilon_0 a^2}$

11. 288

12. $\dfrac{q}{24\varepsilon_0}$

13. $a = \dfrac{R}{\sqrt{3}}$

14. N = 11

15. (A) 7.08×10^{-8} (b) No

16. 3

17. $E_{O_1} = \dfrac{7\rho R}{12 \epsilon_0}$

18. $\sqrt{\dfrac{\lambda q}{2\varepsilon_0 m}}$

19. $-\dfrac{kq^2}{a}\left(3-\sqrt{2}\right)$

20. 1.8×10^5 sec

21. $20\sqrt{\ln 2}$

22. $\sqrt{\dfrac{2kQ^2}{mR}}$

23. 5.86 m/s

24. $2\tan^{-1}\left(\dfrac{\sigma q}{2\varepsilon_0 mg}\right)$

25. $9V_0$

26. 90

27. 7

28. $\dfrac{1}{2\pi}\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$

29. 0

30. $\dfrac{kP}{\sqrt{2}y^3}(-\hat{i}-2\hat{j})$

31. 4

32. (A) $K.E = \dfrac{P}{4\pi\varepsilon_0}\dfrac{Q}{d^2}$, (B) $\dfrac{QP}{2\pi\varepsilon_0 d^3}$ along positive x-axis

33. $-Q/3$

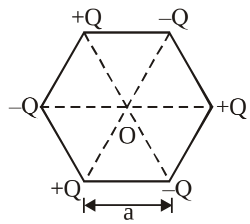
34. (i) $q_2 = -\dfrac{b}{a}q_1$ (ii) $\left[\begin{array}{l} V_r = \dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{a}\right);\ a \le r \le b \\ V_b = \dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{b}-\dfrac{1}{a}\right);\ r = b \\ V_r = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{r}+\dfrac{q_2}{r}\right);\ r \ge b \end{array}\right.$

35. $V' = \left(\dfrac{a}{3t}\right)^{1/3} V$

36. $2\pi\sqrt{\dfrac{\pi\varepsilon_0 md^2}{4\lambda e}}$

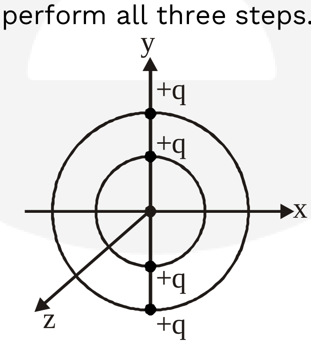
Subjective Exercise - II

1. Six charges are placed at the vertices of a regular hexagon as shown in the figure. Find the electric field on the line passing through O and perpendicular to the plane of the figure as a function of distance x from point O.



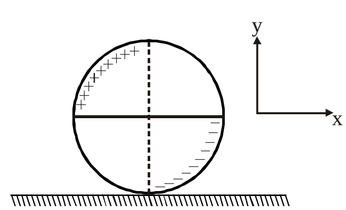
2. Three uncharged conducting large plates are placed parallel to each other in a uniform electric field. Find the induced charge density on each surfaced of each plate.

3. Two concentric rings of radii r and 2r are placed with centre at origin. Two charges +q each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle 90° about Z-axis then it is again rotated by 90° about Y-axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by 90° about X-axis, find the total work required to perform all three steps.

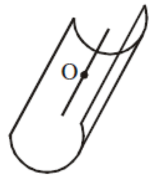


4. Small identical balls with equal charges are fixed at vertices of regular 2008 - gon with side a. At a certain instant, one of the balls is released & a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each ball.

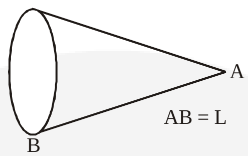
5. A nonconducting ring of mass m and radius R is charged as shown. The charged density i.e. charged per unit length λ is. It is then placed on a rough nonconducting horizontal surface plane. At time t = 0, a uniform electric field $\vec{E} = E_0\hat{i}$ is switched on and the ring start rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.



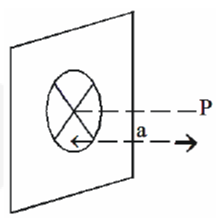
6. A thin long strip whose cross-section is a semicircle carries a uniform surface charge of density σ on its inner surface. Find the electric field at a point O located midway on its axis.



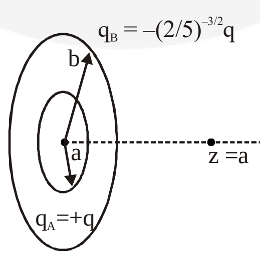
7. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to take a test charge q from infinity to apex A of cone. The slant length is L.



8. An infinitely long non-conducting plane of charge density σ has circular aperture of certain radius carved out from it. The electric field at a point which is at a distance 'a' from the centre of the aperture is $\sigma/2\sqrt{2}\varepsilon_0$. Find the radius of aperture.

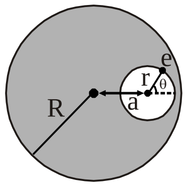


9. Two concentric rings, one of radius a and the other of radius b have the charges +q and $-(2/5)^{-3/2}q$ respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at z = a is in equilibrium.



10. A positive charge Q is uniformly distributed throughout the volume of a nonconducting sphere of radius R. A point mass having charge +q and mass m is fired towards the centre of the sphere with velocity *v* from a point at distance r(r > R) from the centre of the sphere. Find the minimum velocity *v* so that it can penetrate R/2 distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

11. A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centres of the sphere and the cavity is a. An electron e is released inside the cavity at an angle $\theta = 45°$ as shown. How long will it take to touch the sphere again?



12. The electric potential in a region is given by $V(x, y, z) = ax^2 + ay^2 + abz^2$. 'a' is a positive constant of appropriate dimension and b, a positive constant such that V is volts when x, y, z are in m. Let b = 2. The work done by the electric field when a point charge $+4\mu C$ moves from the point (0, 0, 0.1m) to the origin is $50\mu J$. The radius of the circle of the equipotential curve corresponding to V = 6250 volts and $z = \sqrt{2}$ m is α m. Fill α in OMR sheet.

13. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m & positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find itsc equilibrium position.

14. A nonuniform but spherically symmetric distribution of charge has a charge density ρ given as follow:

$\rho = \rho_0 (1 - r/R)$ for $r \le R$,

$\rho = 0$ for $r \ge R$,

where $\rho_0 = 3Q / \pi R^3$ is a constant.

(a) Show that the total charge contained in the charge distribution is Q.

(b) Show that, for the region defined by $r \ge R$, the electric field is identical to that produced by a point charge Q.

(c) Obtain an expression for the electric field in the region $r \le R$.

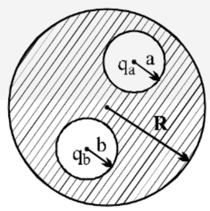
(d) Compare your results in parts (b) and (c) for r = R.

15. Two fixed charges −2Q and Q are located at the point with coordinates (−3a, 0) and (3a, 0) respectively in the x-y plane (a) Show that all the points in the x-y plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its centre. (b) Give the expression for the potential $V_{(x)}$ at a general point on the x-axis and sketch the function $V_{(x)}$ on the whole x -axis. (c) If a particle of charge +q starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when its dose so.

16. Four point charges +8μC, –1 μC, –1 μC and + 8μC are fixed at the points $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y–axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the x–direction. Its speed at x = +∞ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given: $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2).

17. A spherical shell has uniform charge density 8.8×10^{-11} C/m^2. If a pin hole is made in the surface of the shell then find the electric field in the hole in N/C. (Take $\epsilon_0 = 8.8 \times 10^{-12}$ S.I. units)

18. A conducting sphere of radius R has two spherical cavities of radius a and b. The cavities have charges q_a and q_b respectively at their centres. Find:



(a) The electric field and electric potential at a distance r

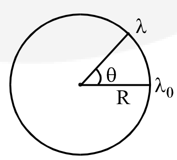
(i) r (distance from O, the centre of sphere > R)

(ii) r (distance from B, the centre of cavity b) < b

(b) Surface charge densities on the surface of radius R, radius a and radius b

(c) What is the force on q_a and q_b?

19. A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\theta$, where λ_0 is the value of λ at $\theta = 0$. Find net electric dipole moment for this charge distribution.



ANSWER KEY

1. zero **2.** $-\varepsilon E$, εE and so on **3.** $W_{\text{first step}} = \left(\frac{8}{3} - \frac{4}{\sqrt{5}}\right)\frac{Kq^2}{r}$, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$

4. $\sqrt{4\pi\varepsilon_0 Ka}$ **5.** $\lambda RE_0\hat{i}$ **6.** $\frac{\sigma}{\pi\varepsilon_0}$

7. $\frac{Qq}{2\pi\varepsilon_0 L}$ **8.** a **9.** 2

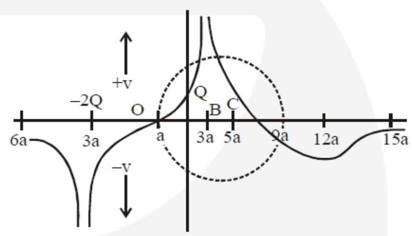
10. $\left[\frac{2KQq}{mR}\left(\frac{r-R}{r} + \frac{3}{8}\right)\right]^{1/2}$ **11.** $\sqrt{\frac{6\sqrt{2}\,mr\varepsilon_0}{e\rho a}}$ **12.** $\sqrt{6}$

13. (a) $H = \frac{4a}{3}$ (b) $U = mg\left[2\sqrt{h^2 + a^2} - h\right]$ equilibrium at $h = \frac{a}{\sqrt{3}}$

14. (c) $\left(\frac{kQr}{R^3}\right)\left(\frac{4R - 3r}{R}\right)$

15. (a) Radius = 4a & center (5a, 0)

(b) $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{|(x+3a)|} + \frac{Q}{|(x-3a)|}\right]$



(c) speed $v = \sqrt{\frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{2ma}}$

16. v_0 = 3 m/s.; K.E. at the origin $= (27 - 10\sqrt{6}) \times 10^{-4}$ J approx 2.5×10^{-4} J

17. 5

18. (a) (i) $V = \frac{k(q_a + q_b)}{r}, E = \frac{k(q_a + q_b)}{r^2}$; (ii) $\frac{k(q_a + q_b)}{R} + \frac{kq_b}{r} - \frac{kq_b}{b}; \frac{kq_b}{r^2}$

(b) $\sigma_R = \left(\frac{q_a + q_b}{4\pi R^2}\right), \sigma_a = -\frac{q_a}{4\pi a^2}; \sigma_b = -\frac{q_b}{4\pi b^2};$

(c) f = 0

19. $(\pi R^2 \lambda_0)$

JEE-Main (Previous Year Questions)

1. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by **[AIEEE-2010]**

(1) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(2) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(3) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

(4) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

2. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a,b are constants. Then the charge density inside the ball is: **[AIEEE-2011]**

(1) $-24\pi a\varepsilon_0 r$

(2) $-6\pi a\varepsilon_0 r$

(3) $-24\pi a\varepsilon_0$

(4) $-6 a\varepsilon_0$

3. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is: **[AIEEE 2011]**

(1) zero

(2) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 + \frac{1}{\sqrt{5}}\right)$

(3) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{2}{\sqrt{5}}\right)$

(4) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$

4. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE-2012]**

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero.

Statement-1: When a charge 'q' is taken from the centre to the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement-2: The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.

(2) Statement-1 is true Statement-2 is false.

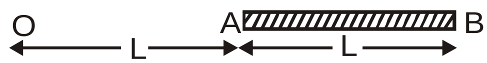
(3) Statement-1 is false Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

5. Two charges, each equal to q, are kept at x = − a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y <<a) along the y-axis, the net force acting on the particle is proportional to: **[JEE(Main)-2013]**

(1) y (2) −y (3) $\frac{1}{y}$ (4) $-\frac{1}{y}$

6. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is: **[JEE(Main)-2013]**

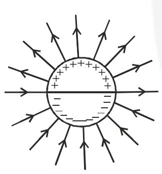
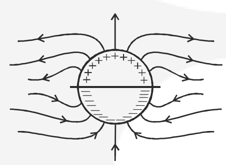
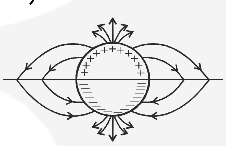
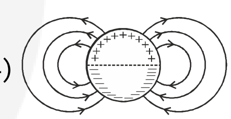


(1) $\frac{Q}{8\pi \epsilon_0 L}$ (2) $\frac{3Q}{4\pi \epsilon_0 L}$ (3) $\frac{Q}{4\pi \epsilon_0 L \ln 2}$ (4) $\frac{Q \ln 2}{4\pi \epsilon_0 L}$

7. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is: **[JEE(Main)-2014]**

(1) 120 J (2) −120 J (3) − 80 J (4) 80 J

8. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge −σ in the lower half. The electric field lines around the cylinder will look like figure given in; (figures are schematic and not drawn to scale) **[JEE(Main)-2015]**

(1)  (2)  (3)  (4) 

9. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE(Main)-2015]**

(1) $R_1 \neq 1$and $(R_2 - R_1) > (R_4 - R_3)$ (2) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(3) $2R < R_4$ (4) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

10. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[JEE(Main)-2016]**

(1) $\frac{Q}{2\pi(b^2 - a^2)}$ (2) $\frac{2Q}{\pi(a^2 - b^2)}$ (3) $\frac{2Q}{\pi a^2}$ (4) $\frac{Q}{2\pi a^2}$

11. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis, When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is: **[JEE(Main)-2017]**

(1) 30° (2) 45° (3) 60° (4) 90°

12. Three charges +Q, q, + Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x= 0, is zero, then value of q is: **[JEE(Main)-2019]**

(1) +Q/2 (2) −Q/2 (3) −Q/4 (4) +Q/4

13. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is: **[JEE(Main)-2019]**

(1) $\frac{R}{\sqrt{5}}$ (2) R (3) $\frac{R}{\sqrt{2}}$ (4) $R\sqrt{2}$

14. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$ where A and a are constants. If Q is the total charge of this charge distribution, the radius R is: **[JEE(Main)-2019]**

(1) $a\log\left(1-\frac{Q}{2\pi aA}\right)$ (2) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(3) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (4) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

15. Two point charges $q_1(\sqrt{10}\mu C)$ and $q_2(-25\mu C)$ are placed on the x-axis at x = l m and x = 4 m respectively. The electric field (in V/m) at a point y = 3m on y-axis is,

$\left[\text{take } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2C^{-2}\right]$ **[JEE(Main)-2019]**

(1) $(-63\hat{i}+27\hat{j})\times10^2$ (2) $(63\hat{i}-27\hat{j})\times10^2$

(3) $(-81\hat{i}-81\hat{j})\times10^2$ (4) $(81\hat{i}-81\hat{j})\times10^2$

16. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be: **[JEE(Main)-2019]**

(1) $\frac{Q}{4\pi\varepsilon_0}\frac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$ (2) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

(3) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$ (4) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$

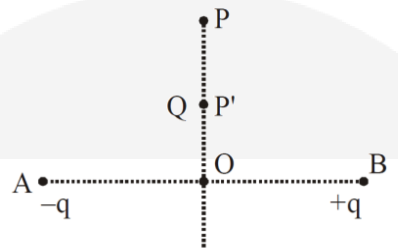
17. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, −2) and (0, −2). The work required to put a fifth charge Q at the origin of the coordinate system will be:

[JEE(Main)-2019]

(1) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (2) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (3) $\frac{Q^2}{4\pi\varepsilon_0}$ (4) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

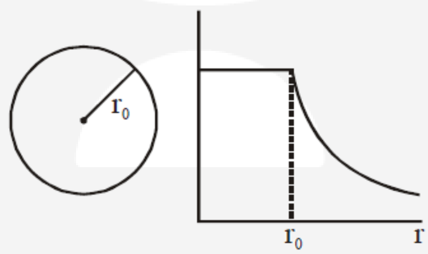
18. Charges −q and +q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where
OP = y and y >> 2a. The charge Q experiences and electrostatic force F. If Q is now moved along the equatorial line to P' such that OP' = $\left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2a\right)$:

[JEE(Main)-2019]



(1) 9F (2) 27F (3) $\frac{F}{3}$ (4) 3F

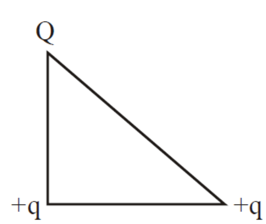
19. The given graph shows variation (with distance r from centre) of: **[JEE(Main)-2019]**



(1) Electric field of uniformly charged spherical shell
(2) Electric field of uniformly charged sphere
(3) Potential of a uniformly charged sphere
(4) Potential of a uniformly charged spherical shell

20. Three charges Q + q and +q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, it the value of Q is:

[JEE(Main)-2019]



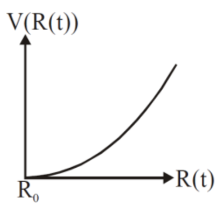
(1) −2q (2) $\frac{-q}{1+\sqrt{2}}$ (3) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (4) +q

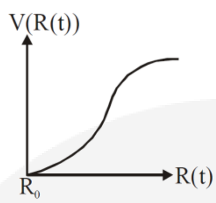
21. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole?

[JEE(Main)-2019]

(1) -7×10^{-27} J   (2) -20×10^{-18} J   (3) -9×10^{-20} J   (4) -10×10^{-29} J

22. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V(R(t)) of the distribution as a function of its instantaneous radius R (t) is: **[JEE(Main)-2019]**

(1)


(2)


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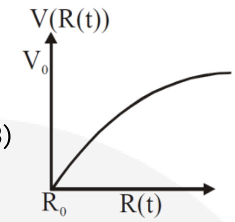

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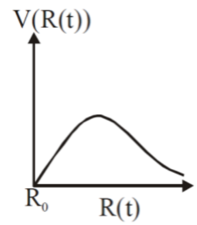
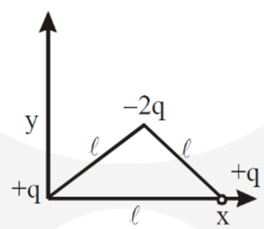


23. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure: **[JEE(Main)-2019]**



(1) $-\sqrt{3}q\ell\hat{j}$ (2) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (3) $\sqrt{3}q\ell\frac{\hat{j}-\hat{i}}{\sqrt{2}}$ (4) $2q\ell\hat{j}$

24. The bob of a simple pendulum has mass 2 g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is: (take g = 10 m/s^2) **[JEE(Main)-2019]**

(1) $\tan^{-1}$ (0.5)   (2) $\tan^{-1}$ (0.2)   (3) $\tan^{-1}$ (5.0)   (4) $\tan^{-1}$ (2.0)

25. A solid conducting sphere, having a charge Q is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollows shell be V. If the shell is now given a charge of −4Q, the new potential difference between the same two surfaces is: **[JEE(Main)-2019]**

(1) −2V   (2) V   (3) 4V   (4) 2V

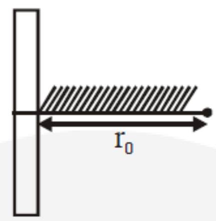
26. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is: **[JEE(Main)-2019]**

(1) 180 V   (2) −520 V   (3) 320 V   (4) −48 V

27. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientaition, then its angular frequency ω is: **[JEE(Main)-2019]**

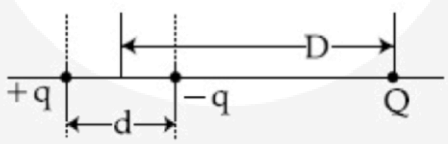
(1) $\sqrt{\frac{2qE}{md}}$ (2) $\sqrt{\frac{qE}{md}}$ (3) $\sqrt{\frac{qE}{2md}}$ (4) $2\sqrt{\frac{qE}{md}}$

28. A positive point charge is released form rest at a distance r_0 from a positive line charge with uniform density. The speed (υ) of the point charge, as a function of instantaneous distance r from line charge, is proportional to : **[JEE(Main)-2019]**



(1) $\upsilon \propto ln\left(\frac{r}{r_0}\right)$ (2) $\upsilon \propto e^{+r/r_0}$ (3) $\upsilon \propto \sqrt{ln\left(\frac{r}{r_0}\right)}$ (4) $\upsilon \propto \left(\frac{r}{r_0}\right)$

29. A system of three charges are placed as shown in the figure: **[JEE(Main)-2019]**



(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$ (2) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{2D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$ (4) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

30. Four point charges − q, + q, + q and − q are placed on y-axis at y= − 2d, y = −d, y = + d and y = + 2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as: **[JEE(Main)-2019]**

(1) $E \propto \frac{1}{D}$ (2) $E \propto \frac{1}{D^4}$ (3) $E \propto \frac{1}{D^2}$ (4) $E \propto \frac{1}{D^3}$

31. A uniformly charged ring of radius 3a and total charge q is placed in ay-plane centred at origin. A point charge q is moving towards the ring along the 2-axis and has speed υ at z = 4a. The minimum value of υ such that it crosses the origin is : **[JEE(Main)-2019]**

(1) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\epsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\epsilon_0 a}\right)^{1/2}$

(3) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\epsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\epsilon_0 a}\right)^{1/2}$

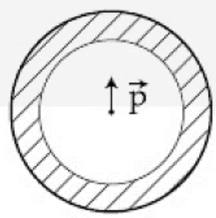
32. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is: $\left[\text{Take } \frac{1}{4\pi \in_0} = 9 \times 10^9 Nm^2C^{-2}\right]$ **[JEE(Main)-2019]**

(1) 1.0 m/s (2) 1.5×10^2 m/s (3) 3.0×10^4 m/s (4) 2.0×10^3 m/s

33. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V = 0 at infinity) **[JEE(Main)-2019]**

(1) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{-\vec{p}}{4\pi \in_0 d^3}$ (2) $0, \frac{\vec{p}}{4\pi \in_0 d^3}$ (3) $\frac{|\vec{p}|}{4\pi \in_0 d^2}, \frac{\vec{p}}{4\pi \in_0 d^3}$ (4) $0, \frac{-\vec{p}}{4\pi \in_0 d^3}$

34. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carrier charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case **[JEE(Main)-2019]**



(1) surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(2) electric field outside the shell is the same as that of a point charge at the centre of the shell

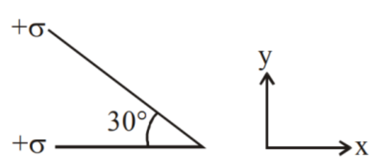
(3) surface charge density on the inner surface of the shell is zero everywhere s

(4) surface charge density on the outer surface depends on $|\vec{p}|$

35. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r)=kr$, where r is the distance from the centre. Two charges A and B, of – Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then: **[JEE(Main)-2019]**

(1) $a = 2^{-1/4}R$ (2) $a = 8^{-1/4}R$ (3) $a = \frac{R}{\sqrt{3}}$ (4) $a = \frac{3R}{2^{1/4}}$

36. Tow infinite planes each with uniform surface charge density + σ are kept in such a way that the angle between them is 30º. The electric field in the region shown between them is given by: **[JEE(Main)-2020]**

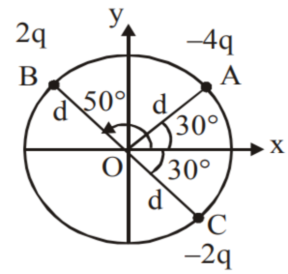


(1) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$ (2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$ (4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

37. Three charged particle A, B and C with charges $-4q$, $2q$ and $-2q$ are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is:

[JEE(Main)-2020]

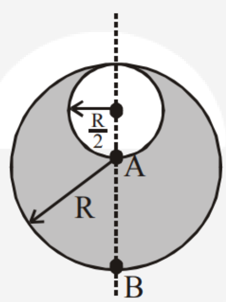


(1) $\dfrac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (2) $\dfrac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(3) $\dfrac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (4) $\dfrac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

38. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\dfrac{R}{2}$ is carved out of it, as shown, the ratio $\left|\dfrac{\vec{E}_A}{\vec{E}_B}\right|$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is :

[JEE(Main)-2020]



(1) $\dfrac{18}{54}$ (2) $\dfrac{21}{34}$

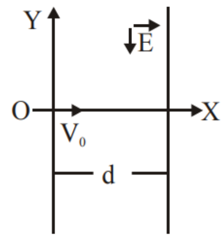
(3) $\dfrac{17}{54}$ (4) $\dfrac{18}{34}$

39. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C. m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to: **[JEE(Main)-2020]**

(1) $(-\hat{i} + 3\hat{j} - 2\hat{k})$ (2) $(+\hat{i} - 3\hat{j} - 2\hat{k})$

(3) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (4) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

40. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x=d. Equation of path of charge particle in the region x>d is: **[JEE(Main)-2020]**



(1) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$ (2) $y = \frac{qEd}{mV_0^2}(x - d)$ (3) $y = \frac{qEd}{mV_0^2}x$ (4) $y = \frac{qEd^2}{mV_0^2}x$

41. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and -3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively: **[JEE(Main)-2020]**

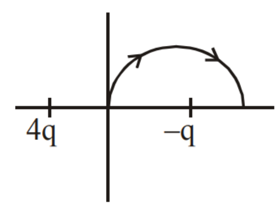
(1) 6 μC and 3 μC (2) +4.5 μC and -4.5 μC

(3) 3 μC and 6 μC (4) 4.5 μC and on both

42. A two point charges 4q and $-q$ are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a thirds point charge 'q' is taken from the origin to x =d along the semicircle as shown in the figure, the energy of the charger will : **[JEE(Main)-2020]**

(1) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$ (2) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

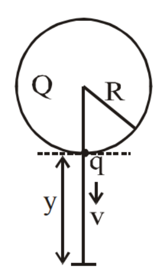
(3) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$ (4) decrease by $\frac{3q^2}{4\pi\varepsilon_0 d}$



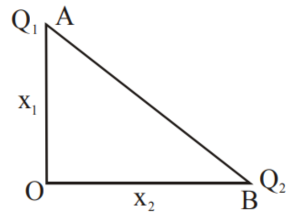
43. A solid sphere of radius R carries a charge (Q + q) distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then: (assume the remaining portion to be spherical). **[JEE(Main)-2020]**

(1) $v^2 = 2y\left[\frac{qQ}{4\pi\in_0 R(R+y)m} + g\right]$ (2) $v^2 = y\left[\frac{qQ}{4\pi\in_0 R^2ym} + g\right]$

(3) $v^2 = 2y\left[\frac{qQR}{4\pi\in_0 (R+y)^3m} + g\right]$ (4) $v^2 = y\left[\frac{qQR}{4\pi\in_0 R(R+y)m} + g\right]$

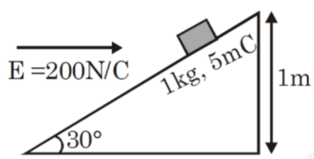


44. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to: **[JEE(Main)-2020]**



(1) $\frac{x_2^2}{x_1^2}$ (2) $\frac{x_1^3}{x_2^3}$ (3) $\frac{x_1}{x_2}$ (4) $\frac{x_2}{x_1}$

45. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1kg and charge 5 mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom. [g = 9.8 m/s², $\sin 30° = \frac{1}{2}; \cos 30° = \frac{\sqrt{3}}{2}$] **[JEE(Main)-2021]**



(1) 0.92 s (2) 0.46 s (3) 2.3 s (4) 1.3 s

46. An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55×10^5 V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess?
(consider g = 9.81 m/s²) **[JEE(Main)-2021]**

(1) 48.8×10^{11} (2) 1.73×10^{10} (3) 17.3×10^{10} (4) 1.73×10^{12}

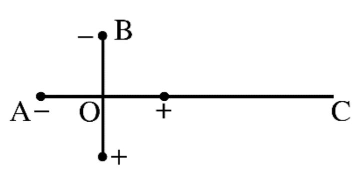
47. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole. **[JEE(Main)-2021]**

(1) 815.1 nC (2) 8.8 nC (3) 0.785 mC (4) 4.44 μC

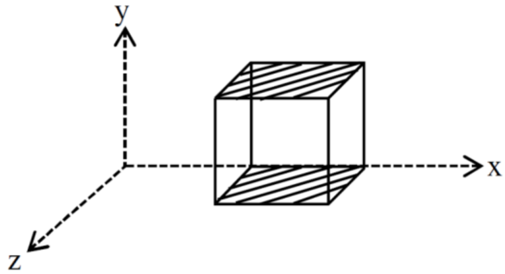
48. Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37° with the axis. The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is: (take $\sin 37° = \frac{3}{5}$) **[JEE(Main)-2021]**

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$

(3) $\frac{2}{3}$ (4) $\frac{4}{3}$



49. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is: **[JEE(Main)-2021]**



(1) 3.8×10^{-11} C (2) 8.3×10^{-11} C (3) 3.8×10^{-12} C (4) 8.3×10^{-12} C

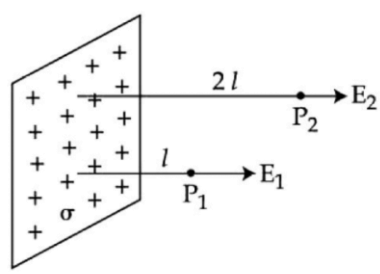
50. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be : **[JEE(Main)-2022]**
(Given g = 9.8 m/s^2)
(1) 1.6×10^{-9} C (2) 2.0×10^{-9} C (3) 3.2×10^{-9} C (4) 0.5×10^{-9} C

51. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7} C area placed on a horizontal table with a separation of L between then such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L.
[Use g = 10 ms^{-2}] **[JEE(Main)-2022]**
(1) 12 cm (2) 10 cm (3) 8 cm (4) 5 cm

52. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) resolves around the cylinder in a circular path. The kinetic energy of the particle is : **[JEE(Main)-2022]**

(1) $\frac{\rho q R^2}{4\varepsilon_0}$ (2) $\frac{\rho q R^2}{2\varepsilon_0}$ (3) $\frac{q\rho}{4\varepsilon_0 R^2}$ (4) $\frac{4\varepsilon_0 R^2}{q\rho}$

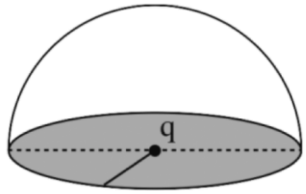
53. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are: **[JEE(Main)-2022]**



(1) $E_1 = \sigma/\varepsilon_0$, $E_2 = \sigma/2\varepsilon_0$ (2) $E_1 = 2\sigma/\varepsilon_0$, $E_2 = \sigma/\varepsilon_0$
(3) $E_1 = E_2 = \sigma/2\varepsilon_0$ (4) $E_1 = E_2 = \sigma/\varepsilon_0$

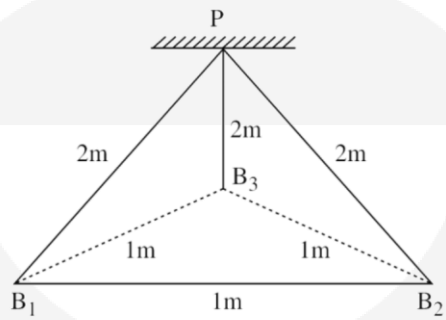
54. 27 identical drops are charged at 22V each. They combine to form a bigger drop. The potential of the bigger drop will be___ V. **[JEE(Main)-2022]**

55. If a charge q is placed at the centre a closed hemispherical non-conducting surface, the total flux passing through the flat surface would be: **[JEE(Main)-2022]**



(1) $\frac{q}{\varepsilon_0}$ (2) $\frac{q}{2\varepsilon_0}$ (3) $\frac{q}{4\pi\varepsilon_0}$ (4) $\frac{q}{2\pi\varepsilon_0}$

56. Three identical charged balls each of charge 2C are suspended from a common point P by silk threads of 2m each (as shown in figure). They form an equilateral triangle of side 1 m.
The ratio of net force on a charged ball to the force between any two charged balls will be: **[JEE(Main)-2022]**



(1) 1 : 1 (2) 1 : 4 (3) $\sqrt{3} : 2$ (4) $\sqrt{3} : 1$

57. Given below are two statements:
Statement-I: A point charge is brought in an electric field. The value of electric field at a point near to the charge may increase if the charge is positive.
Statement-I: An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.
Choose the correct answer from the option given below: **[JEE(Main)-2022]**
(1) Both statement-I and statement-II are true.
(2) Both statement-I and statement-II is false.
(3) Statement-I is true but statement-II is false.
(4) Statement-I is false but statement-II is true.

ANSWER KEY

1.	(2)	**2.**	(4)	**3.**	(4)	**4.**	(3)	**5.**	(1)	**6.**	(4)	**7.**	(3)
8.	(4)	**9.**	(2,3)	**10.**	(4)	**11.**	(3)	**12.**	(3)	**13.**	(3)	**14.**	(4)
15.	(2)	**16.**	(3)	**17.**	(4)	18.	(2)	**19.**	(4)	**20.**	(3)	**21.**	(1)
22.	(3)	**23.**	(1)	**24.**	(1)	**25.**	(2)	**26.**	(1)	**27.**	(1)	**28.**	(3)
29.	(4)	**30.**	(2)	**31.**	(4)	**32.**	(4)	**33.**	(4)	**34.**	(2)	**35.**	(2)
36.	(4)	**37.**	(4)	**38.**	(4)	**39.**	(3)	**40.**	(1)	**41.**	(1)	**42.**	(3)
43.	(1)	**44.**	(3)	**45.**	(4)	**46.**	(2)	**47.**	(4)	**48.**	(3)	**49.**	(2)
50.	(2)	**51.**	(1)	**52.**	(1)	**53.**	(3)	**54.**	198	**55.**	(2)	**56.**	(4)
57.	(1)												

JEE-Advanced (Previous Year Questions)

1. Three concentric metallic spherical shells of radii R, 2R, 3R, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is: **[IIT JEE-2009]**

(A) 1 : 2 : 3 (B) 1 : 3 : 5

(C) 1 : 4 : 9 (D) 1 : 8 : 18

2. Under the influence of the Coulomb field of charge +Q, a charge −q is moving around it in an elliptical orbit. Find out the correct statement(s). **[IIT JEE-2009]**

(A) The angular momentum of the charge −q is constant.

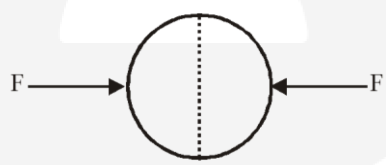
(B) The linear momentum of the charge −q is constant.

(C) The angular velocity of the charge − q is constant.

(D) The linear speed of the charge −q is constant.

3. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = Kr^a$, where K and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at r = R, find the value of a. **[IIT JEE-2009]**

4. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **[IIT JEE-2010]**



(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$

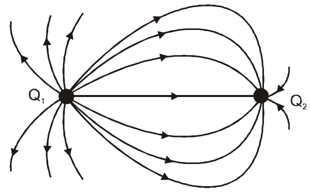
(C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

5. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m/s. Given g = 9.8 m s^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is: **[IIT JEE-2010]**

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C

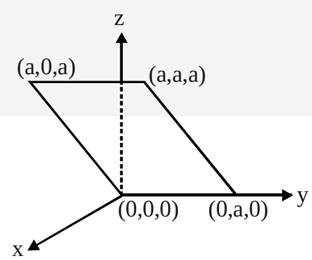
(C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

6. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that: **[IIT JEE-2010]**



(A) $|Q_1| > |Q_2|$
(B) $|Q_1| < |Q_2|$
(C) at a finite distance to the left of Q_1 the electric field is zero
(D) at a finite distance to the right of Q_2 the electric field is zero

7. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is: **[IIT JEE-2010]**



(A) $2E_0a^2$

(B) $\sqrt{2}\,E_0a^2$

(C) E_0a^2

(D) $\dfrac{E_0a^2}{\sqrt{2}}$

8. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then **[IIT JEE-2011]**

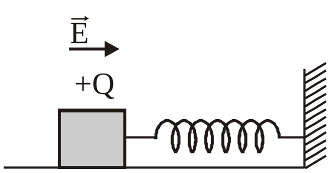
(A) $E_A^{inside} = 0$

(B) $Q_A > Q_B$

(C) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$

(D) $E_A^{on\ surface} < E_B^{on\ surface}$

9. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field ***E*** is switched-on as shown, then the SHM of the block will be **[IIT JEE-2011]**



(A) of the same frequency and with shifted mean position.
(B) of the same frequency and with the same mean position.
(C) of changed frequency and with shifted mean position.
(D) of changed frequency and with the same mean position.

10. Which of the following statement(s) is/are correct? **[IIT JEE-2011]**

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

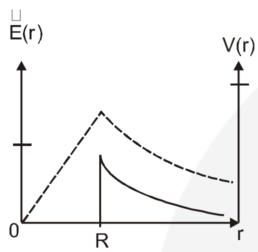
(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

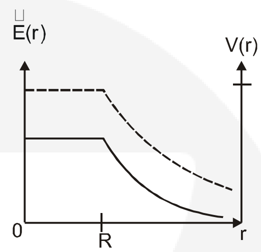
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

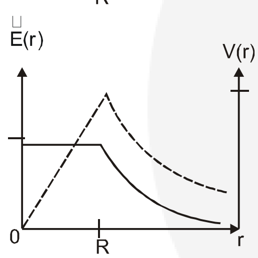
(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

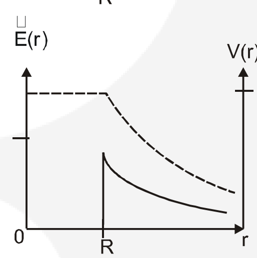
11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

[IIT JEE-2012]

(A)


(B)


(C)


(D)


Paragraph Question No. 12 and 13

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begins to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p. all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[IIT JEE-2012]**

12. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression or ω_p.

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

13. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4\times10^{27}\ m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$. Where these quantities are in proper SI units.

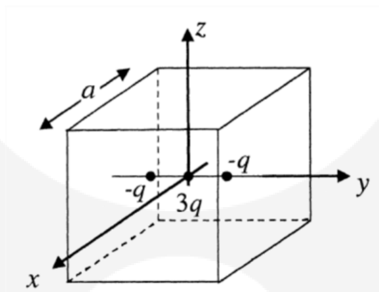
(A) 800 nm
(B) 60 nm
(C) 300 nm
(D) 200 nm

14. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

[IIT JEE-2012]

(A) 1×10^{-5} V
(B) 1×10^{-7} V
(C) 1×10^{-9} V
(D) 1×10^{-10} V

15. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, −q at $\left(0, -\frac{a}{4}, 0\right)$, +3q at (0, 0, 0) and −q at $\left(0, +\frac{a}{4}, 0\right)$, Choose the correct option(s) **[IIT JEE-2012]**



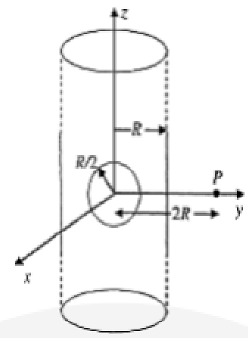
(A) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

(B) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

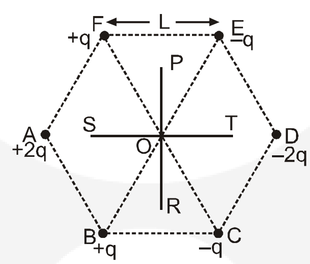
(D) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$

16. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is: **[IIT JEE-2012]**



17. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct?

[IIT JEE-2012]



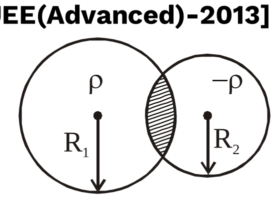
(A) The electric field at O is 6K along OD
(B) The potential at O is zero
(C) The potential at all points on the line PR is same
(D) The potential at all points on the line ST is same.

18. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be **[JEE(Advanced)-2013]**

(A) −4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

19. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region : **[JEE(Advanced)-2013]**

(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction



20. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **[JEE(Advanced)-2014]**

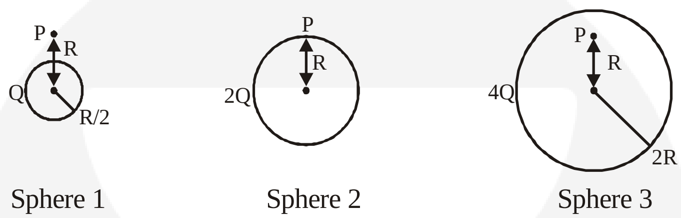
(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

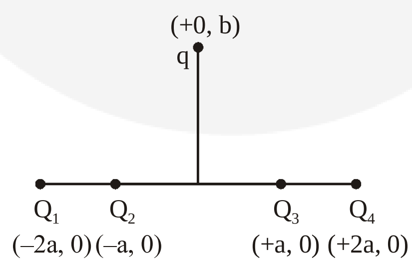
(D) $E_2(r_0/2) = 4E_3(r_0/2)$

21. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **[JEE(Advanced)-2014]**



(A) $E_1 > E_2 > E_3$
(B) $E_3 > E_1 > E_2$
(C) $E_2 > E_1 > E_3$
(D) $E_3 > E_2 > E_1$

22. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = −2a, −a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. **[JEE(Advanced)-2014]**



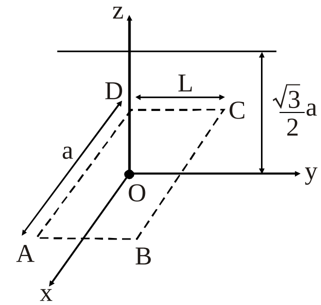
	List I		List II
P.	Q_1, Q_2, Q_3 Q_4 all positive	**1.**	+x
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	**2.**	−x
R.	Q_1, Q_4 positive ; Q_2, Q_3 negative	**3.**	+y
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	**4.**	−y

Code:

(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

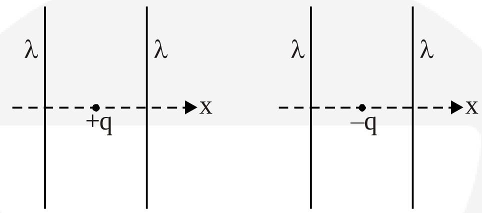
23. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its center at the origin is $\frac{\lambda L}{n\varepsilon_0}$

(ε_0 = permittivity of free space), then the value of n is



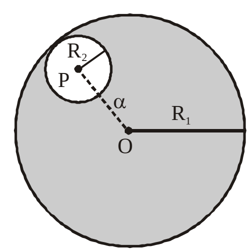
[JEE(Advanced)-2015]

24. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are) **[JEE(Advanced)-2015]**



(A) Both charges execute simple harmonic motion.
(B) Both charges will continue moving in the direction of their displacement.
(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.
(D) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

25. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = α = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



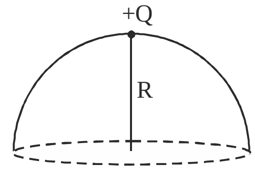
[JEE(Advanced)-2015]

(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$
(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$
(C) $\vec{E}$ is uniform, its magnitude is independent of α but its direction depends on $\vec{\alpha}$
(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{\alpha}$

26. A length scale (ℓ) depends on the permittivity (ε) of dielectric material. Boltzmann constant K_B, the absolute temperature T, the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for ℓ is (are) dimensionally correct? **[JEE(Advanced)-2016]**

(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

27. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct? **[JEE(Advanced)-2017]**



(A) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

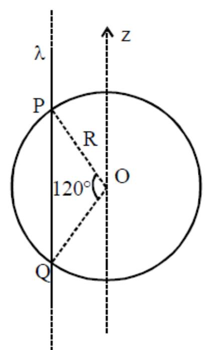
(B) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(C) The component of the electric field normal to the flat surface is constant over the surface

(D) The circumference of the flat surface is an equipotential

28. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120º at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true?

[JEE(Advanced)-2018]



(A) The electric flux through the shell is $\sqrt{3}\,R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}\,R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

29. A particle, of mass 10^{-3} kg and charge 1.0C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t\,\hat{i}$ where E_0 = 1.0 NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent time is ________

[JEE(Advanced)-2018]

30. The electric field E is measured at a point P(0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations, Match the functions in List-I with the related charge distributions in List-II. **[JEE(Advanced)-2018]**

	List-I		**List-II**
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with pint charges Q at $(0,0,\ell)$ and – Q at $(0,0,-\ell)$. Take $2\ell << d$
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying uniform linear charge density parallel to the x-axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y=0, z = -\ell)$ has a charge density $-\lambda$, Take $2\ell<<d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(A) $P \rightarrow 5$; $Q \rightarrow 3, 4$; $R \rightarrow 1$; $S \rightarrow 2$
(B) $P \rightarrow 5$; $Q \rightarrow 3$; $R \rightarrow 1,4$; $S \rightarrow 2$
(C) $P \rightarrow 5$; $Q \rightarrow 3, 4$; $R \rightarrow 1,2$; $S \rightarrow 4$
(D) $P \rightarrow 4$; $Q \rightarrow 2,3$; $R \rightarrow 1$; $S \rightarrow 5$

31. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ ($\alpha << 1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?
[JEE(Advanced)-2019]

(A) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$.

(B) The potential at the center of the shell is reduced by $2\alpha V_0$

(C) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(D) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}$R from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$.

32. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?

[JEE(Advanced)-2019]

[$\in_0$ is the permittivity of free space]

(A) If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$

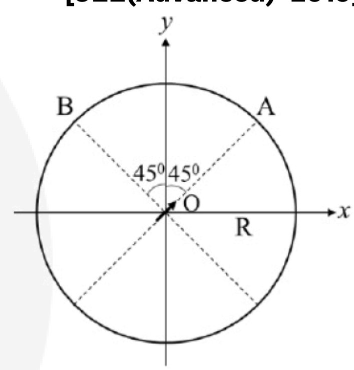
(B) If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\in_0$

(C) If $h > 2R$ and $r = 4R/5$ then $\phi = Q/5\in_0$

(D) If $h > 2R$ and $r > R$ then $\phi = Q/\in_0$

33. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

($\in_0$ is permittivity of free space. R >> dipole size) **[JEE(Advanced)-2019]**



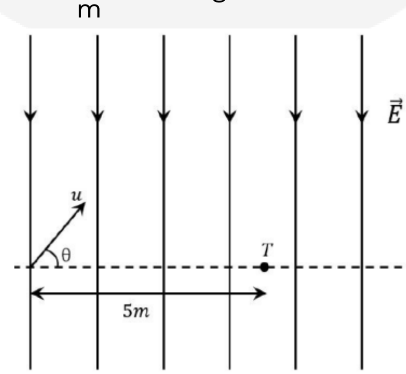
(A) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(B) $R = \left(\dfrac{p_0}{4\pi\in_0 E_0}\right)^{1/3}$

(C) The magnitude of total electric field on any two points of the circle will be same

(D) Total electric field at point B is $\vec{E}_B = 0$

34. A uniform electric field, $\vec{E} = -400\sqrt{3}\hat{y}$ N/C is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10}\times10^6\,ms^{-1}$. This particle is aimed to hit a target T, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10}\,Ckg^{-1}$. Then: **[JEE(Advanced)-2020]**



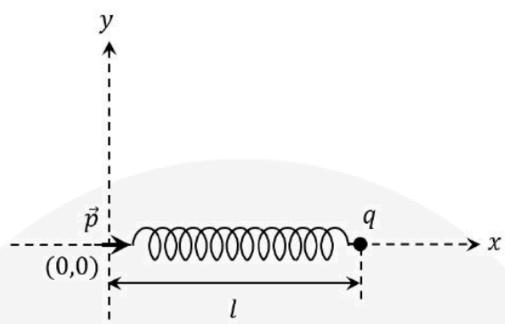
(A) The particle will hit T if projected at an angle 45º from the horizontal

(B) The particle will hit T if projected either at an angle 30º or 60º from the horizontal

(C) Time taken by the particle to hit T could be $\sqrt{\frac{5}{6}}$ μs as well as $\sqrt{\frac{5}{2}}$ μs

(D) Time taken by the particle to hit T is $\sqrt{\frac{5}{3}}$ μs

35. One end of a spring of negligible unstretched length and spring constant k is fixed at the origin (0,0). A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge q is fixed at the origin, the spring gets stretched to a lenght ℓ and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta\ell << \ell$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of δ is. **[JEE(Advanced)-2020]**

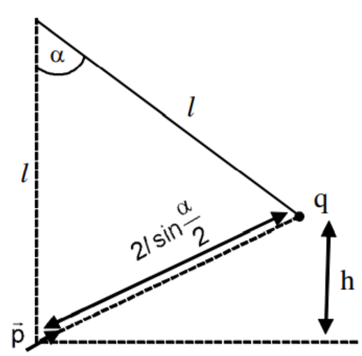


36. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0.\left(1-\frac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux though another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\frac{\phi_0}{\phi}$ is __________.

[JEE(Advanced)-2020]

37. A point charge q of mass m is suspended vertically by a string of length ℓ. A point dipole of dipole moment $\vec{p}$ is now brought towards q from infinity so that the charge moves sway. The final equilibrium position of the system including the direction of the dipole, the angles and distances is shown int the figure below. If the work done in bringing the dipole to this position is N × (mgh), where g is the acceleration due to gravity, then the value of N is ____. (Note that for three coplanar forces keeping a point mass in equilibrium, $\frac{F}{\sin\theta}$ is the same for all forces, where F is any one of the forces and θ is the angle between the other two forces)

[JEE(Advanced)-2020]



38. Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is α. The spheres are now immersed in a dielectric liquid of density 800 kg m^{-3} and dielectric constant 21. If the angle between the strings remains the same after the immersion, then: **[JEE(Advanced)-2020]**

(A) Electric force between the spheres remains unchanged

(B) Electric force between the spheres reduces

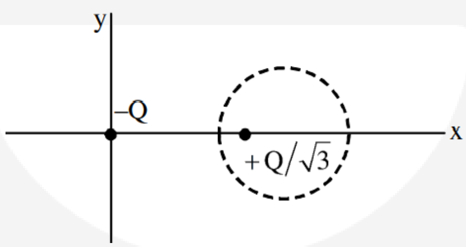
(C) Mass density of the spheres is 840 kg m^{-3}

(D) The tension in the strings holding the spheres remains unchanged

Question stem for Question Nos. 39 and 40

Two point charges $-Q$ and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0,0) and a point (2,0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b,0). All lengths are measured in meters.

[JEE(Advanced)-2020]



39. The value of R is _____ meter.

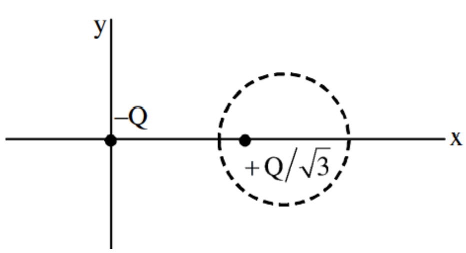
40. The value of b is _____ meter.

Question stem for Question Nos. 27 and 28

Two point charges $-Q$ and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0,0) and a point (2,0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential

V = 0 in the xy-plane with its center at (b,0). All lengths are measured in meters.

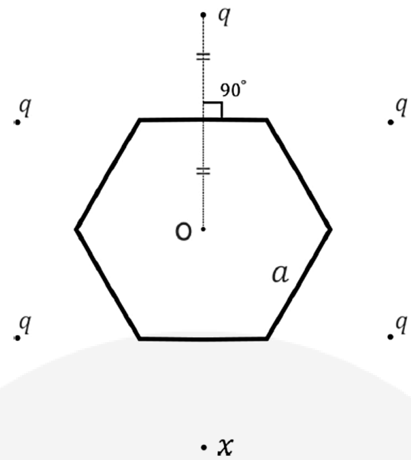
[JEE(Advanced)-2021]



27. The value of R is _____ meter.

28. The value of b is _____ meter.

29. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is(are) correct in SI units? **[JEE(Advanced)-2022]**

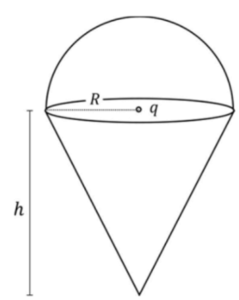
(A) When x = q, the magnitude of the electric field at O is zero.

(B) When x = −q, the magnitude of the electric field at O is $\frac{q}{6\pi \epsilon_0 a^2}$.

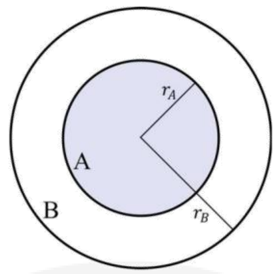
(C) When x = 2q, the potential at 0 is $\frac{7q}{4\sqrt{3}\pi \epsilon_0 a}$.

(D) When x = −3q, the potential at 0 is $-\frac{3q}{4\sqrt{3}\pi \epsilon_0 a}$.

30. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6 \epsilon_0}$ (in SI units). The value of n is ________. **[JEE(Advanced)-2022]**



31. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B, the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units. **[JEE(Advanced)-2022]**



Which of the following statement(s) is(are) correct?

(A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

(B) if $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$.

(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$.

ANSWER KEY

1.	(B)	**2.**	(A)	**3.**	(2)	**4.**	(A)	**5.**	(D)	**6.**	(AD)	**7.**	(C)
8.	(ABCD)	**9.**	(A)	**10.**	(C or CD)			**11.**	(D)	**12.**	(C)	**13.**	(B)
14.	(C)	**15.**	(ACD)	**16.**	6	**17.**	(ABC)	**18.**	(BD)	**19.**	(CD)	**20.**	(C)
21.	(C)	**22.**	(A)	**23.**	6	**24.**	(C)	**25.**	(D)	**26.**	(BD)	**27.**	(AD)
28.	(BD)	**29.**	2[1.99, 2.01]			**30.**	(B)	**31.**	(D)	**32.**	(ABD)	**33.**	(BD)
34.	(BC)	**35.**	3.14	**36.**	6.40	**37.**	2	**38.**	(BC)	**39.**	1.73	**40.**	3.00
27.	1.73	**28.**	3.00	**29.**	(ABC)	**30.**	3	**31.**	(B)				