PERFORMANCE

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PRACTICAL - 01

Topic :- Limits and Continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$

4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5) Examine the continuity of the following function

i] $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, $0 < x \le \dfrac{\pi}{2}$

$= \dfrac{\cos x}{\pi - 2x}$, $\dfrac{\pi}{2} < x < \pi$

at $x = \dfrac{\pi}{2}$

ii) $f(x) = \dfrac{x^2 - 9}{x - 3}$ $0 < x < 3$

$= \dfrac{x^2 - 9}{x + 3}$ $6 \le x < 9$

$= x + 3$. $3 \le x < 6$

at $x = 3$ & $x = 6$

6) Find value of k so that the function $f(x)$ is continuous at the indicated point

i) $f(x) = \dfrac{1 - \cos 4x}{x^2}$, $x < 0$

$= k$, $x = 0$

at $x = 0$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$

$= k$, $x = 0$

at $x = 0$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$

$= k$, $x = -\pi/3$

at $x = \pi/3$

7) Discuss the continuity of the following function which of these function have removable discontinuity? Redefine function so as to remove the discontinuity

1) i) $f(x) = \frac{1-\cos 3x}{x \tan x}$, $x \neq 0$; $= 9$, $x = 0$ } at $x = 0$

2) ii) $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}$, $x \neq 0$

3) $= \frac{\pi}{60}$, $x = 0$ } at $x = 0$

4) 8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$

for $x \neq 0$ is continuous at $x = 0$

5) Find $f(0)$

9) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$

is continuous at $x = \pi/2$

Find $f(\frac{\pi}{2})$

Solutions :-

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})} \right]$

$\lim_{x \to a} \left[\frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})} \right]$

$\frac{1}{3} \lim_{x \to a} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \lim_{x \to a} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}} = \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$

$\frac{1}{3} \times \frac{\cancel{4}^{2} \sqrt{a}}{\cancel{2}_{1} \sqrt{3a}}$

$= \frac{2}{3\sqrt{3}}$

ii) $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0}\left[\frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right]$

$\lim_{y\to 0}\left(\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right)$

$\lim_{y\to 0}\left(\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}\right)$

$\lim_{y\to 0}\left(\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}\right)$

$\lim_{y\to 0}\left(\frac{1}{\sqrt{a}(2\sqrt{a})}\right)$

$= \frac{1}{2a}$

iii) $\lim_{x\to \pi/6}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$

$x \to \pi/6 = h \quad x = h+\pi/6 \quad h\to 0$

$\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$ By using,

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$.

$\lim_{h\to 0}\frac{(\cos h\cdot\cos\pi/6)-\sinh\sin\pi/6)-\sqrt{3}(\sin h\cos\pi/6 + \cos h\sin\pi/6}{\pi-6\left(\frac{6h+\pi}{6}\right)}$

$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}\cdot\sin h\frac{1}{\sqrt{2}}-\sqrt{3}\left(\sin\sqrt{3}/_2 + \cos h\frac{1}{\sqrt{2}}\right)}{\cancel{\pi}-6h+\cancel{\pi}}$

$\lim_{h\to 0}\frac{\cos h\cancel{\frac{\sqrt{3}}{2}}h - \sin\frac{h}{2}-\sin\frac{3h}{2}-\cos\cancel{\frac{\sqrt{3}}{2}}h}{-6h}$

$\lim_{h\to 0}\frac{\cancel{-}\sin\frac{4h}{2}}{\cancel{+}6h}$

$\lim_{h\to 0}\frac{\sin 4h}{\frac{12}{3}h} = \frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h}$

$= \frac{1}{3}\times 1$

$\boxed{= 1/3}$

iv) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

$\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+x^2+1} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2+5}}\right]$

$\lim_{x\to\infty} \dfrac{(\cancel{x^2}+5-\cancel{x^2}+3)}{(\cancel{x^2}+3-\cancel{x^2}-1)} \times \dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{\sqrt{x^2+5}+\sqrt{x^2-3})}$

2 $\dfrac{8}{2}\lim_{x\to\infty}\left(\dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right)$

3 $\dfrac{8}{2}\lim_{x\to\infty}\left(\dfrac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}\right)$

4 $\dfrac{\cancel{8}^{4}}{\cancel{2}_{1}}\lim_{x\to\infty}\dfrac{\sqrt{x^2\cancel{(1+3/x^2)}}+\sqrt{\cancel{x^2}\cancel{(1+1/x^2)}}}{\sqrt{x^2\cancel{\left(1+\frac{5}{x^2}\right)}}+\sqrt{x^2\cancel{\left(1-\frac{3}{x^2}\right)}}}$

5

$= 4$

5) i) $f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 < x \le \frac{\pi}{2} \\ \dfrac{\cos x}{\pi - 2x} & \frac{\pi}{2} < x < \pi \end{cases}$ at $x = \frac{\pi}{2}$

a) $\therefore f\left(\frac{\pi}{2}\right) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$

$F\left(\frac{\pi}{2}\right) = 0$

$\therefore f$ at $x = \pi/2$ define

b) Right Hand Limit:-

$\lim_{x\to\frac{\pi}{2}^+} f(x)$

$\lim_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

Put $x - \dfrac{\pi}{2} = h$, $x = \dfrac{\pi}{2} + h$

as $x \to \dfrac{\pi}{2}$ $h \to 0$

$\lim_{x\to 0^+} \dfrac{\cos(\pi/2 + h)}{\pi - 2(\pi/2 + h)}$

$\lim_{x\to 0^+} \dfrac{-\sin h}{\pi - \pi - 2h}$

$\lim_{x\to 0^+} \dfrac{-\sin h}{-2h} = \dfrac{1}{2}$

c) Left Hand Limit

$\lim_{x \to \pi/2^-} f(x)$

$\lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cos x}{\sqrt{2}\ \sin x}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos \frac{\pi}{2}$

$= 0$

$\therefore$ LHL $\neq$ RHL

$\therefore$ F is not continuous at $x = \frac{\pi}{2}$

iii) $f(x) = \frac{x^2-9}{x-3}$, $0 < x < 3$
$= x+3$, $3 \le x \le 6$
$= \frac{x^2-9}{x^2+3}$, $6 \le x < 9$
at $x = 3$, $x = 6$

Sol^n:

a) For $x=3$

$f(3) = x+3$
$= 3+3$
$f(3) = 6$ $\therefore$ f is defined $x=3$

b) R.H.L :

$\lim_{x \to 3^+} f(x)$

$\lim_{x \to 3^+} x+3$

$\lim_{x \to 3^+} 3+3$

$= 6$

c) L.H.S :

$\lim_{x \to 3^-} f(x)$

$\lim_{x \to 3^-} \frac{x^2-9}{x-3}$

$= \lim_{x \to 3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$= \lim_{x \to 3^-} (x+3)$

$= 6$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\therefore$ From a, b, & c

F is a continous fuction

* For $x = 6$

$f(6) = \frac{x^2+9}{x+3}$

$= \frac{36-9}{6+3}$

$= \frac{27}{9}$

$f(6) = 3$

* R.H.L

$\lim_{x \to 6^+} = \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = 6-3 = 3$

* L.H.L

$\lim_{x \to 6^-} x+3$

$= \cancel{3-6}\ 6+3$

$= 9$

$L.H.L \neq R.H.L$

$\therefore$ The Function is not a continous funtion

6)
i) $f(x) = \frac{1-\cos 4x}{x^2}$, $x<0$
$= K$, $x=0$ } at $x=0$

Solution:-

F is continous at $x=0$

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = K$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$

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$2(2)^2 = K$

$2 \times 4 = K$

$\boxed{K = 8}$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= K$ $x = 0$ } at $x = 0$

Solution:-

$\lim_{x \to 0} (\sec^2 x)^{\cos^2 x} = K$

$\lim_{x \to 0} (1 + \tan^2 x)^{\cot^2 x} = K$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}} = K$

$e = K$

$\boxed{K = e}$

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iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \frac{\pi}{3}$
$= K$, $x = \frac{\pi}{3}$ } at $x = \pi/3$

Solution:-

F is continous at $x = \pi/3$

$\lim_{x \to \frac{\pi}{3}} f(x) = f(\pi/3)$

$= \lim_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x} = K$

Put $x - \pi/3 = h$

$x = \pi/3 + h$ as $x \to \frac{\pi}{3} \to h \to 0$

$= \lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$= \lim_{h \to 0} \frac{\sqrt{3} - \left(\frac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \tan h}\right)}{\pi - \pi - 3h}$

$= \lim_{h \to 0} \frac{\sqrt{3}(1 - \tan \pi/3 \tan h) - \tan \pi/3 - \tan h}{(-3h)(1 - \tan \pi/3 \tan h)}$

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$$= \lim_{h\to 0} \frac{\sqrt{3}(1-\sqrt{3}\tan h) - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \lim_{h\to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \lim_{h\to 0} \left(\frac{\tan h}{h}\right)\left(\frac{1}{1-\sqrt{3}\tan h}\right)$$

$$= \frac{4}{3}\left(\frac{1}{1-\sqrt{3}(0)}\right)$$

$$\boxed{K = \frac{4}{3}}$$

7) i) $f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$; $= 9$ $x = 0$ } at $x = 0$.

solution:-

$$\lim_{x\to 0} f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 3/2}{x\tan x}$$

$$= \frac{2\left(\frac{\sin^2 3/2 x}{\cancel{3/2} x^2}\right) \cdot x^2}{x \cdot \frac{\tan x}{x^2} \cdot x^2}$$

$$= 2\lim_{x\to 0} \frac{(3/2)^2}{1} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \neq 9 = f(0)$$

$\therefore$ f is not continuous at $x = 0$

* Redefine Function

$f(x) = \frac{1-\cos 3x}{x\tan x}$ $x \neq 0$; $\frac{9}{2}$ $x = 0$.

Now, $\lim_{x\to 0} f(x) = f(0)$

$\therefore$ F has removable discontinuity at $x = 0$

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Q 7) consider

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin x^\circ}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$3\lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\right)\lim_{x\to 0}\frac{\sin(\pi x/180)}{x}$$

$$= 3\log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

$\therefore$ F is continuous at $x = 0$

8) F is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0}\frac{(e^{x^2}-1)+(1-\cos x)}{x^2}$$

$$\lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{2\sin^2 x/2}{x^2}$$

Multiply 2 on Numerator & Denominator

Applying Limit,

$$= \log e + 2\cdot\frac{1}{4}$$

$$= 2(1) + \cancel{2}\times\frac{1}{\cancel{4}\,2}$$

$$= \frac{3}{2} = f(0)$$

$$f(0) = \frac{3}{2}$$

9) Solution:-

$\because$ F is continuous at $x = \pi/2$

$$\lim_{x\to\pi/2} f(x) = f(\pi/2)$$

$$\lim_{x\to\pi/2}\frac{\sqrt{2}-\sqrt{1+\sin x}}{(1-\sin^2 x)}\times\frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}} = f(\pi/2)$$

$$\lim_{x\to\pi/2}\frac{2-(1+\sin x)}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})} = f(\pi/2)$$

$$\therefore \lim_{x\to\pi/2}\frac{(\cancel{1-\sin x})}{(\cancel{1-\sin x})(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})} = f(\pi/2)$$

$$\therefore \lim_{x\to\pi/2}\frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})} \quad \therefore \boxed{f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}}$$

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PRACTICAL - NO 2

TOPIC :- Derivation

Q.1 Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$ ii) $\operatorname{cosec} x$ iii) $\sec x$

Q.2 If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x>0$, at $x=2$
then find 'f' is differenhable or not?

Q.3. If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \ge 3$ at $x=3$
then find 'f' is differentiable or not?

Q.4 If $f(x) = 8x-5$, $x<2$
$= 3x^2-4x+7$, $x \ge 2$ at $x=2$
then find 'f' is differentiable or not?

Solutions :-

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

$$= \lim_{x\to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan}$$

Put $x-a=h$
$x = a+h$
as $x\to a$, $h\to a$

$$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{(\tan a - a - h) - \cancel{(1+\tan a \tan(a+h))}}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = \qquad -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall\, a \in R$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \dfrac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$= \lim_{x \to a} \dfrac{1/\sin x - 1/\sin a}{x - a}$

$= \lim_{x \to a} \dfrac{\sin a - \sin x}{(x-a)\sin a \sin x}$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \dfrac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

$= \lim_{h \to 0} \dfrac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim_{h \to 0} -\dfrac{\sin h/2}{h/2} \times 1/2 \times \dfrac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

$= -1/2 \times \dfrac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$= \dfrac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$

iii) sec x

$f(x) = \sec x$

$Df(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \dfrac{\sec x - \sec a}{x - a}$

$= \lim_{x \to a} \dfrac{1/\cos x - 1/\cos a}{x - a}$

$= \lim_{x \to a} \dfrac{\cos a - \cos x}{(x-a)\cos a \cos x}$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \dfrac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

$= \lim_{h \to 0} \dfrac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

$= \lim_{h \to 0} \dfrac{-2\sin\left(\frac{2a+h}{2}\right)\sin -h/2}{\cos a \cos(a+h) \times \left(\frac{-h}{2}\right)} \times \dfrac{-1}{2}$

$= \dfrac{-1}{2} \times \dfrac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$

$= -1/2 \times -2\, \dfrac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$

Q.2) Solution:-

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-8}{x+2}$

$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$

RHD:-

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 2+2 = 4$

$Df(2^+) = 4$

RHD = LHD

$\therefore$ F is differentiable at $x = 2$

Q.3) Solution:-

L.HD:-

$Df(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$

$Df(3^-) = 4$

R.H.D

$Df(3^+) = \lim_{x \to 3^+} \frac{F(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} x(x+6)-3(x+6)$

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$

$= 3+6 = 9$

$Df(3^+) = 9$

RHD $\neq$ L.H.D

F is not differentiable at $x = 3$

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Q4) Solution :-

LHD :

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{x-2}$

$= 8$

$Df(2^-) = 8$

RHD :

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - \cancel{6x} + 2x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2 = 8$

L.H.D = R.H.D

f is differentiable at x

09/12/19

9/12/19

Practical No. 03

Topic :- Application of Derivative

Q.1 Find the intervals in which function is increasing decreasing.

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = 69 - 24x - 9x^2 + 2x^3$

v) $f(x) = x^3 - 27x + 5$

Q.2) Find the interval in which function is concave upwards or concave downwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Solution :-

Q1)

i) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x - 5$

∵ f is increasing iff $f(x) > 0$

∴ $3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

∵ ~~f(x)~~ f is increasing iff $f(x) > 0$

∴ $2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in \cancel{(-\infty, 2)}\,(2, \infty)$

and f is decreasing iff $f'(x) < 0$

∴ $2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

∴ $f'(x) = 6x^2 + 2x - 20$

∵ f is increasing if $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and,

f is decreasing if $f'(x) < 0$

∴ $6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(3x - 5)(x + 2) < 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-2, 5/3)$

iv) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + \cancel{4x} \; x - 4 > 0$

$x(x-4) + 2(x-4) > 0$

$(x-4)(x+2) > 0$

$-1 \quad 4$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and,

if f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$

$-1 \quad 4$

$\therefore x \in (-1, 4)$

v) $f(x) = x^3 - 27x + 5$

$f'(x)\ 3x^2 - 27$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$-$

$-3 \quad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and if

f is decreasing iff $f'(x) < 0$

$\therefore 3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-3, 3)$

Q.2)

i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = \cancel{6 - 12x}$

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$\therefore x > 1/2$

$\therefore x \in (1/2, \infty)$

ii) $y = (x^4 - 6x^3 + 12x^2 + 5x + 7)$

$f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

~~Ans~~

$x \in (-\infty, 1) \cup (2, \infty)$

iii) $y = x^3 - 27x + 5$

$f(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$x \in (0, \infty)$

iv) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) \ngtr 0$

$\therefore$ There exist ~~no~~ interval

$x \in (-1/6, \infty)$

v) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

16/12/17

23/12/19

Practical No: 4

Topic - Application of Derivative and Newton's Method

Q1 Find maximum & minimum value of following functions.

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ in $[-\frac{1}{2}, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q2] Find the root of following equation by Newton's Method (Take 4 iteration only) correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q1 i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$\boxed{= 8}$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum value at $x = 1$

$f(1) = 3 - 5(+1)^3 + 3(1)^5$

$= 6 - 5$

$\boxed{= 1}$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$\boxed{= 5}$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore$ f has maximum value at $x = -1$

$\therefore$ f has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$ $\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= \cancel{8 - 13} - 4 + 1$

$\boxed{= -3}$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$\boxed{= 18 > 0}$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$\boxed{= -19}$

$\therefore f''(-1) = 12(-1) - 6$

$= 12 - 6$

$\boxed{= -18 < 0}$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1) - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$\boxed{= 8}$

$\therefore$ f has maximum value 8 at $x = -1$ and minimum value -19 at $x = 2$

Q.2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ ~~x_0~~

$f'(x) = 3x^2 - 6x - 55$

$\therefore$ By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$\boxed{x_1 = 0.1727}$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$\boxed{= -0.0829}$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$\boxed{= -55.9467}$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$\boxed{= 0.1712}$

$f(x_2) = (0.1712)^3 - 3(0.1712)^3 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$\boxed{= 0.0011}$

$f'(x_2) = 3(0.1712)^3 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$\boxed{= -55.9393}$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$\boxed{= 0.1712}$

$\therefore$ The root of the equation is $\boxed{0.1712}$

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ii) $f(x) = x^3 - 4x - 9$ [2, 3]

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$\boxed{= -9}$

$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$\boxed{= 6}$

Let $x_0 = 3$ be the initial approximation.
So By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$
$= 3 - 6/23$
$\boxed{= 2.7392}$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$\boxed{= 0.596}$

$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$\boxed{= 18.5096}$

$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 2.7392 - 0.596/18.5096$
$\boxed{= 2.7071}$

$f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$\boxed{= 17.9851}$

$= 2.7071 - \cancel{0.0102}\ \dfrac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = \boxed{2.7015}$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.1758 - 10.806 - 9$
$\boxed{= -0.0901}$

$f'(3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$\boxed{= 17.8943}$

$x_4 = 2.7015 + 0.0901/17.8943$
$= 2.7015 + 0.0050$
$\boxed{= 2.7065}$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1, 2]

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 1.8(4) - 20 + 17$
$= 8 - 7.2 - 3$
$= -2.2$

Let $x_0 = 2$ be initial approximation.
By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230 = \boxed{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$\boxed{= 0.6755}$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$\boxed{= -8.2164}$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164$

$= 1.577 + 0.0822$

$\boxed{= 1.6592}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$\boxed{= 0.0204}$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$\boxed{= -7.7143}$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.6592 + 0.0204/7.7143$

$\boxed{= 1.6618}$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$\boxed{= 0.0004}$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$\boxed{= -7.6977}$

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$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.6618 + 0.0004/7.6977$

$\boxed{= 1.6618}$

PRACTICAL-05

TOPIC: Integration:

Q.1) Solve the following Integration.

1) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x}+1)\,dx$

3) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

4) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

5) $\int t^7 \sin(2t^4)\,dt$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

8) $\int \dfrac{\cos x}{3\sqrt{5\sin^2 x}}\,dx$

9) $\int e^{\cos^2 x} \sin 2x\,dx$

10) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right) dx$

Solutions:

1) $I = \int \frac{du}{\sqrt{u^2+2u-3}}$

$= \int \frac{du}{\sqrt{(u+1)^2-(2)^2}}$

comparing with,

$\int \frac{du}{\sqrt{u^2-a^2}} = u^2 = (u+1)^2$

$I = \log|u+\sqrt{u^2+a^2}|+C$

$= \log|u+1+\sqrt{(u+1)^2-(2)^2}|+C$

2) $I = \int (4e^{3u}+1)\,du$

$= \int 4e^{3u}du + \int 1\,du.$

$= \frac{4e^{3u}}{3} + u + C$

3) $I = \int (2u^2 - 3\sin u + 5\sqrt{u})\,du$

$= 2\int u^2 - 3\int \sin u + 5\int \sqrt{u}\,du$

$= \frac{2}{3}u^3 du + 3\cos u + 5 \times \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}u^3 + 3\cos u + \frac{10}{3}u^{3/2} + C$

4) $I = \int \frac{u^3+3u+4\,du}{\sqrt{u}}$

$= \int \frac{u^3+3u+4\,du}{\sqrt{u}}$

$= \int \left(\frac{u^3}{u^{1/2}} + \frac{3u}{u^{1/2}} + \frac{4}{u^{1/2}} \right) du$

$= \int u^{5/2}du + 3\int u^{1/2}du + 4\int u^{-1/2}du.$

$= \frac{2}{7}u^{7/2} + 2u^{3/2} + 8\sqrt{u} + C$

5) $I = \int t^7 \sin(2t^4)\,du.$

Let $t^4 = u$

$4t^3 dt = du$

$= \frac{1}{4}\int 4t^3 t^4 \sin(2t^4)\,dt$

$= \frac{1}{4}(u \cdot \sin(2u))\,du$

$= \frac{1}{4}\left[u\int \sin 2u - \int\left\{\int \sin 2u \cdot \frac{d}{du}u\right\}\right]$

$= \frac{1}{4}\left[-\frac{u\cos 2u}{2} + \frac{1}{2}\int \cos 2u \cdot 1\right]$

$= \frac{1}{4}\left[-\frac{\cos 2u}{2} + \frac{1}{4}\sin 2u\right] + C$

$= \frac{-1}{8} u \cos 2u + \frac{1}{16} \sin 2u + c$

$= \frac{-1}{8} t^4 \cos(2t^4) + \frac{1}{16} \sin(2t^4) + C$

6) $I = \int \sqrt{u}\,(u^2 - 1)\,du$

$= \int \sqrt{u}\,(u^2-1)\,du$

$\int (\sqrt{u} \cdot u^2 - \sqrt{u})\,du$

$\int (u^{5/2} - \sqrt{u})\,du$

$= \frac{2}{7} u^{7/2} - \frac{2}{3} u^{3/2} + C$

7) $I = \int \frac{1}{u^3} \sin\left(\frac{1}{u^2}\right) du$

Let $\frac{1}{u^3} = t$

$u^{-2} = t$

$-\frac{2}{u^3}\,du = dt$

$I = -\frac{1}{2} \int -\frac{2}{u^3} \sin\left(\frac{1}{u^2}\right) du$

$= -\frac{1}{2} \int \sin t$

$= -\frac{1}{2}(-\cos t) + c$

$= \frac{1}{2} \cos t + c$

Resubstitution $t = \frac{1}{u^2}$

$I = \frac{1}{2} \cos\left(\frac{1}{u^2}\right) + C$

8) $I = \int \frac{\cos u}{3\sqrt{\sin^2 u}}$

Let ~~cos~~ $\sin u = t$

$\cos u\,du = dt$

$I = \int \frac{dt}{3\sqrt{t^2}}$

$= \int \frac{dt}{t^2/3}$

$= \int t^{-2/3}\,dt$

$= -3t^{1/3} + c$

$= 3(\sin u)^{1/3} + c$

$= 3 \cdot \sqrt[3]{\sin u} + C$

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9) $I = \int \cos^2 x \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-2\sin 2x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos 2x} \, dx$

$= -\int e^t \, dt$

$= e^t + c$

Resubstituting $t = \cos^2 x$

$I = \int e$

$I = -e^{\cos^2 x} + c$

10) $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

Let $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x) \, dx = dt$

$(x^2 - 2x) \, dx = dt/3$

$I = \int 1/t \, dt$

$= \frac{1}{3} \int dt/t$

$= \frac{1}{3} \log t + c$

Resubstituting $t = x^3 - 3x^2 + 1$

$I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

AV
27/01/2022

PRACTICAL - 06

Topic :- Application of Integration and Numirical Integration

Q.1 Find the length of the following curve

1) $x = t - \sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$

2) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

3) $y = x^{3/2}$ in $[0, 4]$

4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2 Using Simpson's Rule solve the following

1) $\int_0^2 e^{x^2} \, dx$ with $n = 4$

2) $\int_0^4 x^2 \, dx$ with $n = 4$

3) $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ with $n = 6$

1] $x = t - \sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$

$\frac{dx}{dt} = 1 - \cos t$, $\frac{dy}{dt} = \sin t$.

$I = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \int \sqrt{(1-\cos t)^2 + \sin^2 t}$

$= \int \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}$

$= \int \sqrt{1 - 2\cos t + 1}$

$= \int \sqrt{2(1-\cos t)}$

$= \int \sqrt{2 \times 2 \sin^2 t/2}$

$= \int_0^{2\pi} \sqrt{4 \sin^2 t/2}$

$= \int_0 2 \sin t/2$

$= -2 \left[\frac{\cos t/2}{1/2} \right]$

$= -2 \left[2 \cos t/2 \right]_0^{2\pi}$

$= -4 \left[\cos t/2 \right]_0^{2\pi}$

$= -4 [\cos \pi - \cos 0]$

$= -4 [-1 - 1]$

$= -4 [-2]$

$= 8$

2) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

Solution:- ~~$I = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$~~ $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$\frac{dy}{dt} = 2 \int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2} \, dx$

$= 2 \int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx$

$= 4 \int_0^2 \frac{1}{\sqrt{4 - x^2}} \, dx$

$= 4 \left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

3] $y = x^{3/2}$ in $[0, 4]$

Sol^n:- $f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4} x$

$I = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$

$= \int_0^2 \sqrt{1 + \frac{9}{4} x} \, dx$

Put $u = 1 + \frac{9}{4} x$, ~~$du = \frac{9}{4}$~~ dx.

$L = \int_1^{1 + 9/4 x} \frac{4}{9} \sqrt{u} \, du$

$= \left[\frac{4}{9} \cdot \frac{2}{3} (u^{3/2}) \right]_1^{1 + \frac{9}{4} x}$

$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$

4) $x = 3\sin t$, $y = 3\cos t$

Sol^n $\frac{dx}{dt} = 3\cos t$, $\frac{dy}{dt} = -3\sin t$.

$L = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cot)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$\cancel{= \int_0^{2\pi} \sqrt{9\sin^2 t}}$

$= \int_0^{2\pi} 3\sqrt{x}\, dt$

$= 3\int_0^{2\pi} x\, dt$

$= 3[x]_0^{2\pi}$

$= 3[2\pi - 0]$

$L = 6\pi$

5] $x = \frac{1}{6}y^3 + \frac{1}{2y}$, $y = [1, 2]$

sol^n:- $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$,

$= \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \cancel{\frac{1}{2}} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$ units

Q.2]
i] $\int_0^2 e^{u^2} du$ with $n=4$

Sol^n:- $a=0$, $b=2$, $n=4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

u	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule:-

$\int_a^b f(u)du = \frac{h}{3}[(y_0+y_n)+4(y_1+y_3+y_5 \cdots y_{n-1})+2(y_2+y_4+y_6+\cdots y_{n-2})]$

$\int_0^2 e^{u^2} du = \frac{0.5}{3}[(1+54.5981)+4(1.2840+9.4877)+2(2.7182+54.5981)]$

$= \frac{0.5}{3}[55.5981 + 43.0868 + 114.6326]$

$= 1.1779$

ii) $\int_0^4 u^2 du$ $n=4$

$\Delta u = \frac{4-0}{4} = 0$

$\int_a^b f(u)\,du = \frac{\Delta u}{3}(y_0+4y_1+4y_3+y_4)$

$= \frac{1}{3}[y(0)+4(1)^2+4(2)^2+4(3)^2+4^2)$

$= \frac{1}{3}(y(0)+4(1)^2+4(2)^2+4(3)^2+4^2)$

$= \frac{1}{3}(0^2+4(1)^2+2(2)^2+4(3)^2+4^2)$

$= \frac{63}{3}$

3) $\int_0^{\pi/3} \sqrt{\sin u}\, du$ with $n=6$

sol^n:- $\Delta u = \frac{b-a}{n}$

$\Delta u = \frac{b-a}{n} = \frac{\pi/3-0}{6} = \frac{\pi}{18}$

u	0	$\pi/8$	$2\pi/8$	$3\pi/8$	$4\pi/18$	$5\pi/18$
y	0	0.4169	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin u}\, du = \frac{\Delta u}{3}(y_0+4(y_1+y_3+y_5)+2(y_2+y_4))$

$= \frac{\pi/18}{3}(0+4(0.4167+0.707+0.875)+0.930$

$= 2(0.584+0.801)+0.930$

$= 0.683$

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PRACTICAL - 07

TOPIC :- Differential Equation :-

Q.1] Solve the following differential equation :-

1) $x\frac{dy}{dx} + y = e^x$

Sol^n :- $\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x \, dx}$

$= x$

$Y(IF) = \int Q(x)(IF)\,dx + C$

$= \int \frac{e^x}{x} \cdot x \cdot dx + C$

$= \int e^x \, dx + C$

$xy = e^x + C$

2) $e^x\frac{dy}{dx} + 2e^x y = 1$

Sol^n :- $\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\quad Q(x) = e^{-x}$

~~$Y(IF) = \int Q(x)(IF)$~~ $dx + C$

$=$

$IF = e^{\int P(x)\,dx}$
$= e^{\int 2\,dx}$

$= e^{2x}$

$Y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int e^{-x} \cdot e^{2x}\,dx + C$

$= \int e^x\,dx + C$

$y \cdot e^{2x} = e^x + C$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

Sol^n :- $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x)$ $\quad Q(x) = \frac{\cos x}{x^2}$

$IF = e^{\int P(x)\,dx}$

$=$ ~~$e^{\int 2(x)\,dx}$~~ $e^{\int 2(x)\,dx}$

$= x^2$

$Y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \frac{\cos x}{x^2} - x^2\,dx + C$

$= \int \cos x + C$

$x^2 y = \sin x + C$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Sol^n:- $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$p(x) = 3/x \quad Q(x) = \sin x/x^3$

$p(x) = \int p(x)\,dx$

$= \int 3/x \, dx$

$= x^3$

$IF = e^{\int p(x)dx}$

$= x^3$

$Y = (IF) = \int Q(x)(IF)\,dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + C$

$= \int \sin x + C$

$x^3 y = -\cos x + C$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2$

Sol^n:- $\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x}$

$= 2x e^{-2x}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$Y(IF) = \int Q(x)(IF)\,dx + C$

$= \int 2x e^{-2x} e^{2x} + C$

$= \int 2x + C$

$y e^{2x} = x^2 + C$

6) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

Sol^n:- $\sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy = 0$

$\frac{\sec^2 x dx}{\tan x} = -\frac{\sec^2 y}{\tan y} dy$

$\int \frac{\sec^2 x dx}{\tan x} dx = \int -\frac{\sec^2 y}{\tan y} dy$

$\log|\tan x| = \log|\tan y| + C$

$\log|\tan x - \tan y| + C$

$\tan x = \tan y$

7) $\frac{dy}{dx} = \sin^x (x-y+1)$

Sol^n:- Put $x-y+1 = v$

$x - y + 1 = v$

$1 - \frac{dy}{dx} = dv$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$$1-\frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1-\sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x-y+1) = x+c$$

8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Sol^n:- Put $2x+3y = v$

$$2+\frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$\frac{1}{3}\left(\frac{dv}{du}-2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{(v+2)}$$

$$\int \frac{(v+2)}{(v+1)}\,dv = 3dx$$

$$\int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3dx$$

$$v + \log|v| = 3x + c$$

$$2x+3y+\log|2x+3y+1| = 3x+C$$

$$3y = x - \log|2x+3y+1| + c$$

PRACTICAL - 08

TOPIC :- Euler's Method

Q1) $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$. To Find $y(2)$ =

Sol^n :- $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore$ $Y(2) = 9.8215$

Q.2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$, To Find $y(1)$ = ?

Sol^n :- $x_0 = 0$ $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	~~1.04~~	0.2
1	0.2	0.2	1.04~~4~~	0.408
2	0.4	0.408	1.~~1664~~ 1.1664	0.6412
3	0.6	0.6412	1.4111 ~~1.85256~~	0.9234
4	0.8	0.9234	1.85256	1.2939
5	1	1.2939		

$Y(1) = 1.2939$

Q.3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $h = 0.2$ Find $y(1)$ = ?

Sol^n :- $x_0 = 0$ $y(0) = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$Y(1) = 1.5051$

Q4) $\frac{dy}{dx} = 3x^2 + 1$ $Y(1) = 2$ Find $y(2)$ $h = 0.5$ and $h = 0.25$

Sol^n :- $y_0 = 2$ $x_0 = 1$ $h = 0.5$

(i)

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$Y(2) = 7.875$

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② $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.996		

$y(2) = 299.9966$

Q5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

Solⁿ :- $x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$Y(1.2) = 3.6$

PRACTICAL-09

Topic :- Limits & Partial order derivatives.

Q1 Evaluate the limits:

1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

Solⁿ :- $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

Applying limits

$= \frac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1) + 5}$

$= \frac{-64 + 12 + 1 - 1}{4 + 5}$

$= \frac{-52}{9}$

2) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

Solⁿ :- $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

Applying limits

$= \frac{(0+1)((2)^2 + (0)^2 - 4(2))}{2 + 3(0)}$

$= \frac{1(4 + 0 - 8)}{2}$

$= \frac{4-8}{2} = \frac{-4}{2} = -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2-z^2}{x^3-x^2yz}$

Solⁿ:- $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2-z^2}{x^3-x^2yz}$

Applying limit

$= \dfrac{(1)^2-(1)^2-(1)^2}{(1)^3-(1)^2(1\times1)}$

$= \dfrac{1-1}{1-1} = \dfrac{0}{0}$

∴ Limit does not exist

Q.2 Find f_x, f_y for each of the following f

i) $f(x,y) = xye^{x^2+y^2}$

Solⁿ:-

$f_x = y(1\cdot e^{x^2+y^2}) + xy(e^{x^2+y^2}\cdot 2x)$

$= ye^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$f_y = x(1\cdot e^{x^2+y^2}) + xy(e^{x^2+y^2}\cdot 2y)$

$= x\cdot e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

∴ $f_x = ye^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$f_y = x\cdot e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \cos y\, e^x$

$f_y = e^x - \sin y$

∴ $f_y = -\sin y e^x$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2 3x^2 - 3y\,2x + 0 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q.3) Using definition find values of $f_x\, f_y$ at $(0,0)$ for $f(x,y) = \dfrac{2x}{1+y^2}$

Solⁿ:- $f_x(a,b) = \lim_{h\to0} \dfrac{f(a+h,b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h\to0} \dfrac{f(a,b+h) - f(a,b)}{h}$

$f_x(0,0) = \lim_{h\to0} \dfrac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to0} \dfrac{2h-0}{h}$

$= 2$

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$$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{0-0}{h}$$

$$= 0$$

$\therefore f_x = 2$, $f_y = 0$

Q.4. Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

Sol^n:- $f_{xx} = \dfrac{d^2}{dx^2}$ $\qquad f_{yy} = \dfrac{d^2 f}{dy}$

Applying u/v rule

$$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$$

$$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$$

$$f_x = \frac{x(-xy - 2y^2 + 2xy)}{x^4}$$

$\cancel{f_x = \frac{2xy - 2y^2}{x^3}} = xy$ $\quad \therefore f_x = \dfrac{x^2 y - 2xy^2}{x^4}$

$$f_{xx} = \frac{x^3(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$$

$$\cancel{f_{xx}} = \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$$

$$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8} \quad \text{—— (1)}$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} \quad \text{—— (2)}$$

$$f_{xy} = \frac{d}{dy}\left(\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{—— (3)}$$

$$f_{yx} = \frac{d}{dx}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \, d/dx\,(2y - x) - (2y - x)\,\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{—— (4)}$$

From equation (3) & (4)

$f_{xy} = f_{yx}$

2) $f(x,y) = x^3 + 3x^2 y^2 - \log(x^2+1)$

Sol^n:- $f_x = \dfrac{d}{dx}(x^3 + 3x^2 y^2 - \log(x^2+1))$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$f_y = \frac{d}{dy}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 0 + 6x^2y - 0$

$= 6x^2y$

* $f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1 \frac{d(2x)}{dx} - 2x\frac{d(x^2+1)}{dx}}{(x^2+1)^2}\right)$ — (1)

* $f_{yy} = \frac{d}{dy}(6x^2y)$ — (2)

* $f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$ — (3)

$= 12xy$

* $f_{yx} = \frac{d}{dx}(6x^2y)$ — (4)

$= 12xy$

From (3) & (4)

$f_{xy} = f_{yx}$

3) $f(x,y) = \sin(xy) + e^{x+y}$

→ $f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

∴ $f_{xx} = \frac{d}{dx}\left(y\cos(xy) + e^{x+y}\right)$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — (1)

* $f_{yy} = \frac{d}{dy}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — (2)

* $f_{xy} = \frac{d}{dy}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — (3)

* $f_{yx} = \frac{d}{dx}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (4)

∴ From (3) & (4)

$f_{xy} \neq f_{yx}$

Q.5. Find the linearization of $f(x,y)$ at given point

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

→ Sol^n :- $f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

Soln:- $f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$ $\quad f_y = 0 - 0 + \sin x$

f_x at $(\pi/2, 0) = -1 + 0 = -1$ $\quad f_y$ at $(\pi/2, 0) = \sin \pi/2 = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

PRACTICAL - 10

TOPIC :- Direction derivative, Gradien vector & Maxima ~~and~~, Minima, Tangent and Normal vector.

Q.1 Find the directional derivative of the following function at given points and in the direction of given vector

i) $f(x,y) = x+2y-3 \quad a=(1,-1) \quad u=3i-j$

Sol^n :- $u = 3i-j$, which is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + 3h/\sqrt{10}\right), \left(-1 - h/\sqrt{10}\right)$

$f(a+hu) = \left(1 + 3h/\sqrt{10}\right) + 2\left(-1 - h/\sqrt{10}\right) - 3$

$= 1 + 3h/\sqrt{10} - 2 - 2h/\sqrt{10} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\cancel{-4} + h/\sqrt{10} + \cancel{4}}{\cancel{h}}$

$$\boxed{D_u f(a) = \frac{1}{\sqrt{10}}}$$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i+5j$

Sol^n : Here $u = i+5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2+(5)^2} = \sqrt{1+25} = \sqrt{26}$

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = 4^2 - 4(3) + 1$

$= 16 - 12 + 1$

$= 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + h/\sqrt{26}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} \cancel{+5-5}}{h}$$

$$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$\boxed{\therefore D_u f(a) \; \frac{25h}{26} + \frac{36}{\sqrt{26}}}$$

iii) $2x + 3y$ $\quad a = (1,2)$, $u = (3i+4j)$

Solution:-

Here $u = 3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Unit vector along u is $u/|u| = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(2,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h(3/5, 4/5)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} \cancel{+8-8}}{\cancel{h}}$$

$$\boxed{= \frac{18}{5}}$$

Q.2 Find gradient vector for the following funⁿ at given points.

i) $f(x,y) = x^y + y^x$ $\quad a = (1,1)$

Solⁿ:- $f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, \; x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, \; 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $\quad a = (1,-1)$

Solⁿ:

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, \; 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \; \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{\pi}{\cancel{4}_2}(\cancel{-2})\right)$

$= \left(\frac{1}{2}, \; \frac{-\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\quad a = (1,-1,0)$

Solution:-

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1))0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3) Find the equation of tangent and normal to each of the following using curves at given points

i) $x^2\cos y + e^{xy} = 2$ $\quad$ at $(1,0)$

Sol^n:- $f_x = \cos y\, 2x + e^{xy}\, y$

$f_y = x^2(-\sin y) + e^{xy}\cdot x$

$(x_0, y_0) = (1, 0)$ $\therefore x_0 = 1 \quad y_0 = 0$

Eq^n tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0\, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 1$

$= 0 + 1\cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

This is the required of equation of tangent.

Eq^n of Normal:

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$\therefore 1 + 2y + d = 0$ $\quad$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\boxed{d = -1}$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ $\quad$ at $(2,-2)$

Sol^n:-

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

Scanned by CamScanner

Eqn of tangent

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(-1(y+2))=0$

$2x-2-y-2=0$

$2x-y-4=0$ - Required eqn of tangent

Eqn of Normal

$ax+by+c=0$

$bx+ay+d=0$

$=-1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\boxed{\therefore d=6}$

Q4) Find the eqn of tangent and normal line to each of the following surface.

i) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

Soln:- $f_x=2x-0+0+z$

$f_x=2x+z$

$f_y=0-2z+3+0$

$=2z+3$

$f_z=0-2y+0+x$

$=-2y+x$

$(x_0,y_0,z_0)=(2,1,0)$

$\therefore x_0=2$, $y_0=1$, $z_0=0$

$f_x(x_0,y_0,z_0)=2(2)+0=4$

$f_y(x_0,y_0,z_0)=2(0)+3=3$

$f_z(x_0,y_0,z_0)=-2(1)+2=0$

Eqn of tangent

$f_x(x-x_0)+f_y(y-y_0)\ \ f_z(z-z_0)=0$

$=4(x-2)+3(y-1)+0(z-0)=0$

$=4x-8+3y-3=0$

$4x+3y-11=0$ → This required eqn of tangent

Eqn of normal at $(4,3,-11)$

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$=\frac{x-2}{4}=\frac{y-1}{3}=\frac{z+11}{0}$$

ii) $3xyz-x-y+z=-4$ at $(1,-1,2)$

$3xyz-x-y+z=0$ at $(1,-1,2)$

Soln:-

$f_x=3yz-1-0+0+0$

$=3yz-1$

$f_y=3xz-1$

$f_z=3xy+1$

Scanned by CamScanner

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

* Eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

This is required eq^n of the tangent

* Eq^n of the normal at $(-7, 5, -2)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5) Find the local maxima and minima for the following function.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

Sol^n

$f_x = 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$ — (1)

$f_y = 0$

$2y - 3x - 4$

$2y - 3x - 4 = 0$ — (2)

Multiply equation 1 with 2

$4x - \cancel{2y} = -4$

$\cancel{2y} - 3x = 4$

$x = 0$

Substitute value of x in eq^n (1)

$2x - y + 2 = 0$

$2(0) - y + 2 = 0$

$-y + 2 = 0$

$-y = -2$

$\boxed{y = 2}$

∴ critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$ ∴ f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$\boxed{f = -4}$

iii) $f(x, y) = 2x^4 + 3x^2 y - y^2$

Sol^n

$f(x) = 8x^3 + 6xy$

$f(y) = 3x^2 - 2y$

$fx = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — (1)

$fy = 0$

$3x^2 - 2y = 0$ — (2)

Multiply eq^n (1) with 3

(2) with 4

$12x^2 + 9y = 0$

$- \quad 12x^2 - 8y = 0$

$17y = 0$

$\boxed{y = 0}$

Substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$\boxed{x = 0}$

Critical points is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0)$

$f_{xy} = 0$

r at $(0, 0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ and $rt - s^2 = 0$

$f(x, y)$ at $(0, 0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$f = 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

Solution:

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \therefore 2x + 2 = 0$

$x = \frac{-2}{2}$

$\therefore \boxed{x = -1}$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2}$

$\boxed{y = 4}$

$\therefore$ Critical Points are $(-1, 4)$

$f_{xx} = r = 2$

$f_{yy} = t = -2$

$f_{xy} = s = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$\therefore f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$17 + 30 - 70$

$37 - 70$

$\boxed{= +33}$

29/01/2021