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* Matrices *

Matrices are rectangular arrays of no., symbols or characters where all of this elements are arranged in each row and column. An array is a collection of items arranged at diffⁿ locations. Matrices are identified by their order. The order of matrices is given in the form of no. of rows x no. of columns. Matrices in maths are useful in solving numerous prblm of linear equation and many more.

$$\begin{array}{c} \xrightarrow{\text{Rows}} \\ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \end{array} \quad \Big\downarrow \text{column}$$

Order of matrix = no. of row x no. of column.

* Properties of matrices

1) $A + B = B + A$ (Commutative)
2) $(A+B)+C = A + (B+C)$ .... (Associative)
3) $AB \neq BA$ (Not commutative)
4) $(AB).C = A.(BC)$ ... (Associative)
5) $A(B+C) = AB + AC$ .... (Distributive)
6) $(A^T)^T = A$.
7) $(A+B)^T = A^T + B^T$
8) $(AB)^T = B^T . A^T$

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* Types of Matrices IMP

1) Row matrix
2) Column matrix
3) Square matrix
4) Rectangular Matrix.
5) Diagonal Matrix
6) Zero or Null Matrix.
7) Unit or identity matrix
8) Symmetric matrix
9) Skew - Symmetric matrix.
10) orthogonal matrix.
11) Idempotent matrix
12) upper triangular matrix
13) lower triangular Matrix
14) Singular matrix.
15) Non- Singular matrix.

* Determinant of a matrix

Determinant of a matrix is a no. associated with that Square matrix, the determinant of a matrix can only be calculated for Square matrix it is represented by |A|. The determinant of a matrix is calculated by adding the product of the Elements of a matrix with their co-factors

Q.1] Prove that $AB \neq BA$, where

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ -1 & 0 & 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 1 \\ 5 & 7 \\ 6 & -1 \\ 2 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 3 & 1 & 4 \\ -1 & 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 5 & 7 \\ 6 & -1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+15+6+8 & 2+21-1+8 \\ 3+0+18+10 & -1+0-3+10 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 30 \\ 31 & 6 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 \\ 5 & 7 \\ 6 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 & 4 \\ -1 & 0 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{-6+15+6+8} & \cancel{3+0+18+10} \\ \cancel{2+21-1+8} & \cancel{-1+0-3+10} \end{bmatrix}$$

$$= \begin{bmatrix} -6-1 & -9+0 & -3+3 & -12+5 \\ 10-7 & 15+0 & 5+21 & 20+35 \\ 12+1 & 18 & 6-3 & 24-5 \\ 4-2 & 6+0 & 2+6 & 8+10 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -9 & 0 & -7 \\ 3 & 15 & 26 & 55 \\ 13 & 18 & 3 & 19 \\ 2 & 6 & 8 & 18 \end{bmatrix}$$

$\therefore AB \neq BA$

(Q.2) Prove that given matrix is orthogonal.

$$A = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix}$$

$$\therefore A \cdot A^T = I$$

$$\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3}+\frac{1}{6}+\frac{1}{2} & \frac{1}{3}-\frac{2}{6}+0 & \frac{1}{3}+\frac{1}{6}-\frac{1}{2} \\ \frac{1}{3}-\frac{2}{6}+0 & \frac{1}{3}+\frac{4}{6}+0 & \frac{1}{3}-\frac{2}{6}+0 \\ \frac{1}{3}+\frac{1}{6}-\frac{1}{2} & \frac{1}{3}-\frac{2}{6}+0 & \frac{1}{3}+\frac{1}{6}+\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.3] Show that A is orthogonal matrix

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$A \cdot A^T = I$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta + 0 + \sin^2\theta & 0+0+0 & -\sin\theta \cdot \cos\theta + 0 + \sin\theta\cos\theta \\ 0 + 0 + 0 & 0+1+0 & 0+1+0 \\ -\sin\theta\cos\theta + 0 + \sin\theta\cos\theta & 0+0+0 & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Trace of a matrix

Trace of a matrix is a sum of the principal diagonal element of a square matrix. Trace of a matrix is only found in the case of a square matrix bcoz diagonal element exist only in square matrices

* Find the trace of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

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Rank of a matrix

is given by the maximum no. of linearly independant rows or columns of a matrix. The rank of a matrix is always less than equal to the totw no. of rows or column present in a matrix. A square matrix has linearly independant rows or columns if the matrix is non-singular i.e determinants is not equal to zero. Since a zero matrix has no linearly independant rows or columns, its rank is zero

rank of a matrix =
$\rho(A)$ = no. of rows - no. zero rows.

Q Find the rank of matrix of: by row echleleon form.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix}$$

$R_3 - 2R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3 - 1$

Q $A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$

$R_2 - 2R_1$, $R_3 - 4R_1$

$A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$

$R_3 \rightarrow R_3 \times \left(\frac{1}{3}\right)$

$A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -1 & 1 & 3 \end{bmatrix}$

$R_3 \rightarrow 2R_3 - R_2$

$A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 3 - 1$
$= 2$

Q. find the rank of a matrix

$A = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 3 & 5 & 1 & 2 \\ 5 & -1 & 2 & 2 \\ 2 & 6 & 5 & 5 \\ 1 & 3 & -3 & -1 \end{bmatrix}$

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* Inverse of a matrix

The inverse of a matrix is another matrix that, when multiplied by the given matrix yields the multiplicative identity. For a matrix A and its inverse of the identy property holds

$$\boxed{A \cdot A^{-1} = A^{-1} A = I}$$

For any matrix A, it's inverse is denoted by

$$\boxed{A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)}$$

* Principal component analysis (PCA).

1] PCA is a dimensionality reduction and ML method used to simplify a large dataset into a smaller set while still maintaining significante pattern & trends

2] PCA are new variables that are constructed as linear combinations or mixtures of the initial variables

3] PCA tries to put maximum possible info^n in the first component, then maximum remaining info^n and so on.

* How do you do a PCA

1] Standardized the range & continous initial variables.

2] Compute the co-variance matrix to identify correlations.

3] Compute the Eigen vectors & Eigen values of the co-variance matrix to identify the Principal components.

4] Create a feature vector to decide which principal components to keep.

5] Re-cast the data along the principal component axis.

* Ex. of PCA

1] Image compression - It reduce image dimensionality for efficient storage without loosing critical information.

2] Customer segmentation - It clusters customers based on behaviour for targeted marketing.

3] Financial data analysis - It analyse co-variance in ~~asset~~ asset, returns for portfolio optimization

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* **Eigen values & Eigen vectors**

Eigen Values

$A = [a_{ij}]$ be a square matrix.

- The ~~characteristi~~ characteristic Eq^n of A is $|A - \lambda I| = 0$
- The roots of the characteristic Eq^n are called Eigen values of A.

Eigen Vectors

$A = [a_{ij}]$ be a square matrix of order 'n'

if there Exist a non-zero vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

Such that $AX = \lambda X$, then

the vector X is called an Eigen vector of A corresponding to the Eigen value λ.

Q. Find all the Eigen values, Eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$$

Sol^n → Characteristic Eq^n of A is

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(2-\lambda)(-1-\lambda)+1] + 1[3(-1-\lambda)+2]$$
$$+ 4[3-2(2-\lambda)] = 0$$

$$\Rightarrow (1-\lambda)[-2-2\lambda+\lambda+\lambda^2+1] + 1[-3-3\lambda+2]$$
$$+ 4[3-4+2\lambda] = 0$$

$$\Rightarrow (1-\lambda)[\lambda^2-\lambda-1] + 1[-3\lambda-1] + 4[2\lambda-1]$$

$$\Rightarrow \lambda^2-\lambda-1-\lambda^3+\lambda^2+\lambda-3\lambda-1+8\lambda-4 =$$

$$\Rightarrow -\lambda^3+2\lambda^2+5\lambda-6 = 0$$

$$\Rightarrow \lambda^3-2\lambda^2-5\lambda+6 = 0$$