EE5471: Interpolation

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1 Introduction

In this lab we will study interpolation. The codes are in C but we will call them from Python and graph the results.

2 Polynomial Interpolation

We will use the C code in Numerical Recipes for interpolation. It has been linked to Python for your convenience. This python script has been uploaded in the Moodle site as `polint.py`.

1. In python sample $\sin\left(x+x^2\right)$ from 0 to 1 at 5 points. Use these points as your table and do fourth order interpolation on

   ```
   xx=linspace(-0.5,1.5,200)
   ```

   Since all the points in the table are used for 4^{th} order interpolation, this allows you to see what the effect of choosing a window that is not centred about the desired `xx` value.

2. Sample the same function at 30 points from 0 to 1. Repeat the same interpolation. How does the accuracy change? What is the change due to?

3. With the same table of values, vary the order of interpolation. How does the error vary. Plot the error vs x for different orders in a semi log plot. Explain the curves you get.

4. In python, create a table containing the values of the following truncated fourier series

   $$f(x) = \sum_{k=1}^{5} \frac{\sin(2k+1)x}{2k+1}$$

   for x going from $-\pi$ to π in 101 steps. Use Can you conclude from looking at the series whether *the full series* is continuous at all x? How many continuous derivitives will it have? Is this confirmed by the numerical plot? Remember back to the properties of a fourier series

5. Perform 5^{th} order interpolation on a grid of 999 points covering the same range of 0 to 2π. Plot the error vs x. Where is the error concentrated? What happens at $\pm\pi$? Given that the function is periodic, can we improve the error?

6. Vary the interpolation order n from 3 to 20 and determine the way the maximum error varies with order. Is the dependence on interpolation order the same or different from the case in question 3? Why?

7. We require a 6 digit accurate method to compute the function

   $$f(x) = \frac{\sin(\pi x)}{\sqrt{1-x^2}} \tag{1}$$

   between 0.1 and 0.9. The function is known *exactly* (to 15 digits) at certain locations, $x_k = x_0 + kdx$, $k = 0, \ldots, n$ where n is the order of interpolation required.

(a) Convert the function to a table, spaced 0.05 apart, sampling it from 0.1 to 0.9.

(b) In Python, plot this function and determine its general behaviour. Is it analytic in that region? What is the radius of convergence? What is the nature of the function's behaviour near ± 1?

(c) Use the `polint` routine to interpolate the function at a thousand points between 0.1 and 0.9 for different orders. What order gives 6 digit accuracy? Explain the convergence in terms of the table spacing and the ROC.

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$\langle$*code* (never defined)$\rangle$
$\langle$*code1* (never defined)$\rangle$