DEPARTMENT OF EE - IIT MADRAS

Assignment 6
Spatial Coupling and Threshold Saturation

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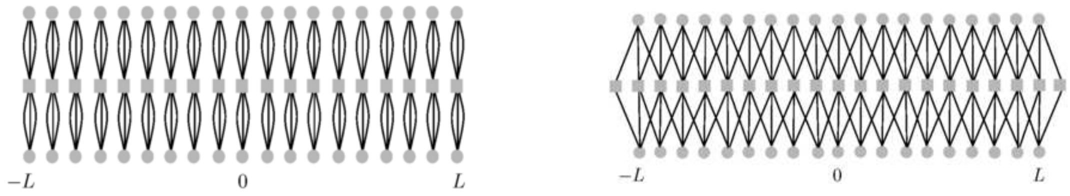
1 PROBLEM STATEMENT

Compute the uncoupled threshold for a protograpgh code. Compute the Area threshold of the spatially coupled extension of the code, and compare the two.

2 SPATIAL COUPLING

Here, we use the Protograph code [3,3], that is 2 variable nodes of degree 3 connected to a single check node of degree 6. This is a rate-$\frac{1}{2}$ code. A horizontally repeated version of the protograph code is shown in Figure (2.1a). The spatially coupled version of this code involves spreading the edges of the protograph among its nearest neighbours. At the terminal nodes, we add extra check nodes to complete the chain. Figure (2.1b) shows the coupled version of the same [3,3] code.

Spatially coupled protographs have a higher threshold than their base protograph (under BP decoding). We first motivate this claim, and then give the method to find the new threshold. Say we are operating just above the BP threshold. We look only at the terminal nodes, and ignore the information from the rest of the graph.



(a) Uncoupled Code

(b) Coupled Code

Figure 2.1: Uncoupled and Coupled versions of the Base Protograph Code

This forms a small protograph by itself, and can be decoded (if below its threshold). Since every node is connected only to its neighbours, we can propagate this decoded information like a wave throughout the rest of the graph, allowing stage-wise decoding.

To compute the threshold, we can plot the EXIT-chart under BP decoding. The BP-threshold ϵ_{BP} is computed using methods from the earlier assignment. The threshold is the ϵ which causes the 2-EXIT curves to touch. For the spatially coupled code, we allow the 2 curves to intersect. We compute the **Area Threshold** ϵ_{Area} as the ϵ which causes the areas to be equal.

An alternative formulation is to define a potential function for spatially coupled codes as

$$U(x,\epsilon) = \int_0^x \left(z - \epsilon\lambda\left(1-\rho\left(1-z\right)\right)\right)\rho'(1-z)dz \tag{2.1}$$

It has been shown in [1], that ϵ_{Area} causes the potential function to touch $U(x) = 0$ at exactly one non-zero point, and stays positive. This provides an easy way to compute the area threshold for any protograph. The authors also show ϵ_{Area} corresponds to the MAP threshold for the protograph.

3 Threshold Calculation

To find the BP threshold, we use **proto_thresh_bec_brute.m** from Assignment 3. The code we use has $\lambda(x) = x^2; \rho(x) = x^5$. For this protograph, $\epsilon_{BP} = 0.4294$. The Potential function based threshold computation is implemented in the script **spatcoup.m**.

Figure (3.1) shows a plot of the potential function for various values of ϵ. For this spatially coupled protograph, $\epsilon_{Area} = 0.48815$

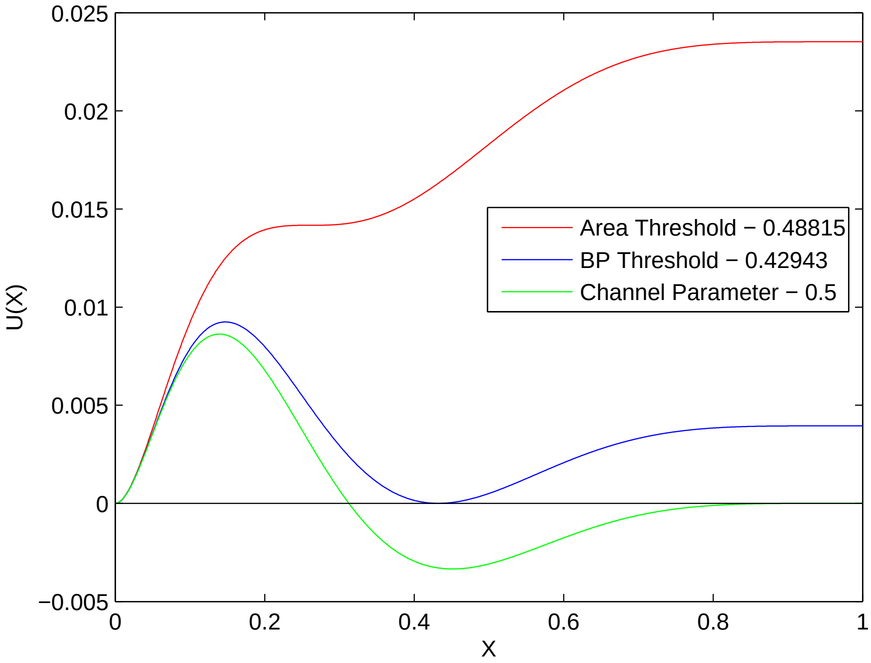


Figure 3.1: Potential Function plot for various values of ϵ

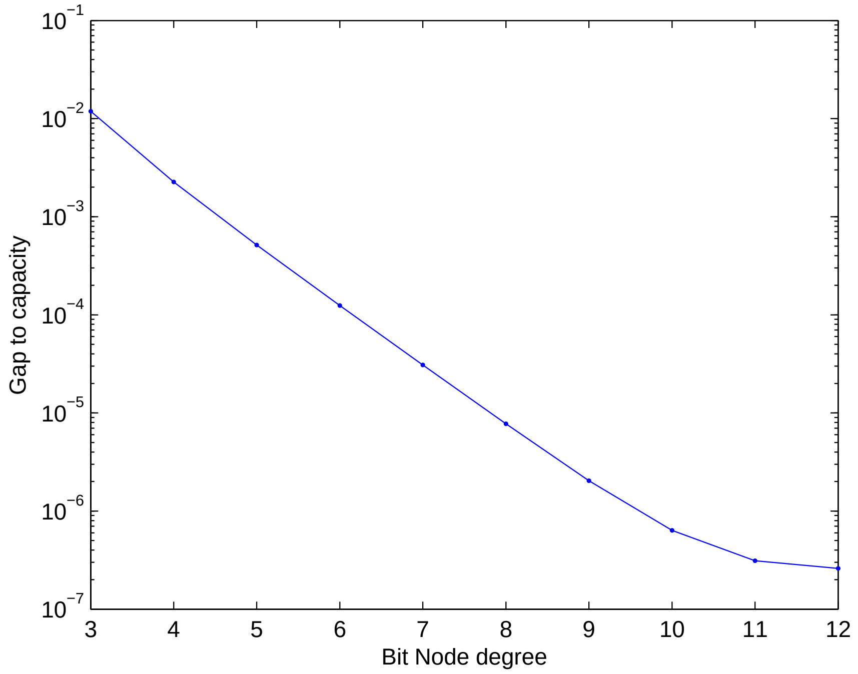


Figure 4.1: Gap to capacity as a function of bit-node degree

4 Capacity Achieving

Kudekar et.al [2] have shown that Spatially Coupled codes can achieve capacity. We construct a sequence of spatially coupled codes of increasing bit-node degree. The check node degree is $2x$ the bit-node degree. We compute the Area threshold for every such protograph. The script to implement this is **spatcoup_capacity.m**

Figure (4.1) shows how the Gap to Capacity varies with the bit-node degree of the spatially coupled code. The Gap to capacity here is $0.5-\epsilon_{Area}$. We observe an exponential fall in the Gap to capacity with increasing bit-node degree. The saturation-like behaviour at the bottom is from MATLAB precision effects, and can be remedied by taking more sample points. As the node degrees tend to infinity in a spatially coupled code, the threshold appears to approaches capacity.

References

[1] EPFL tutorial on Spatially Coupled codes, *ISIT 2013* - http://ipg.epfl.ch/doku.php?id=en:publications:scc_tutorial

[2] S. Kudekar, T. Richardson, R. Urbanke; *Spatially Coupled Ensembles Universally Achieve Capacity under Belief Propagation* - http://arxiv.org/abs/1201.2999

[3] Images used in Figure 2.1 - http://ita.ucsd.edu/wiki/index.php?title=Spatially_coupled_codes