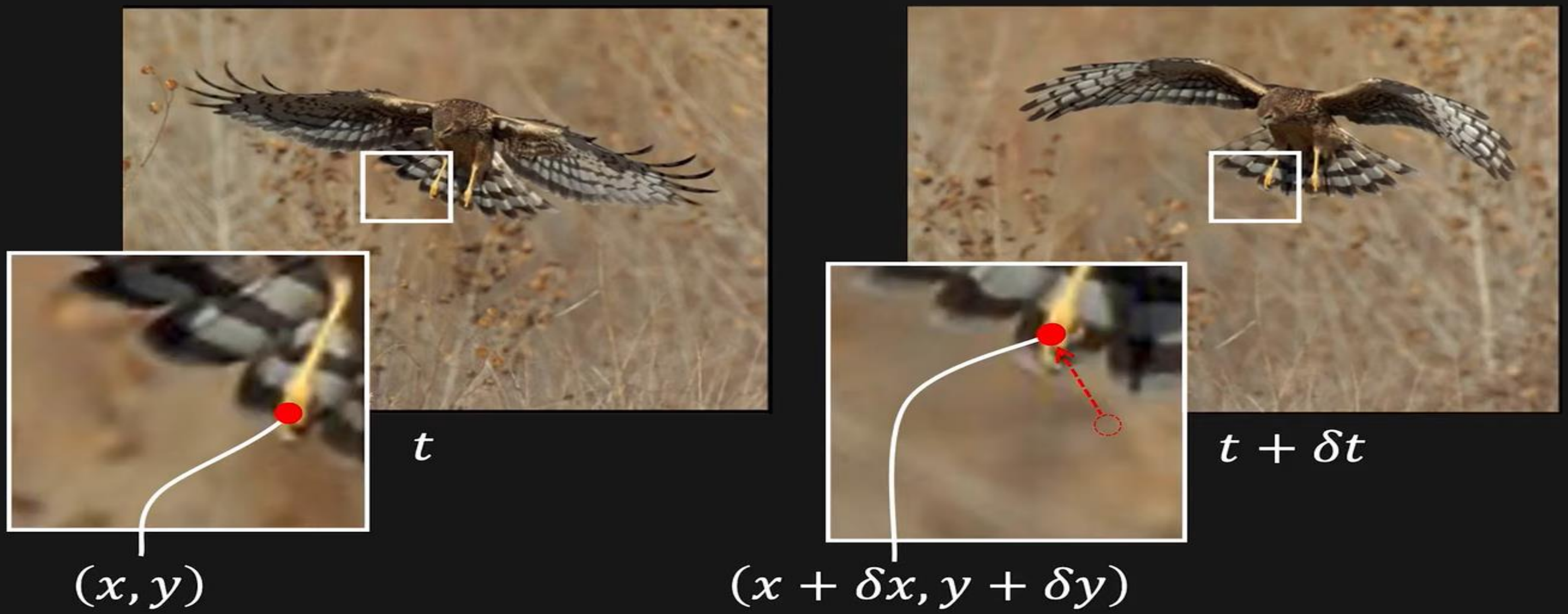
Motion estimation: Optic flow

Optic flow is the **apparent** motion of objects or surfaces

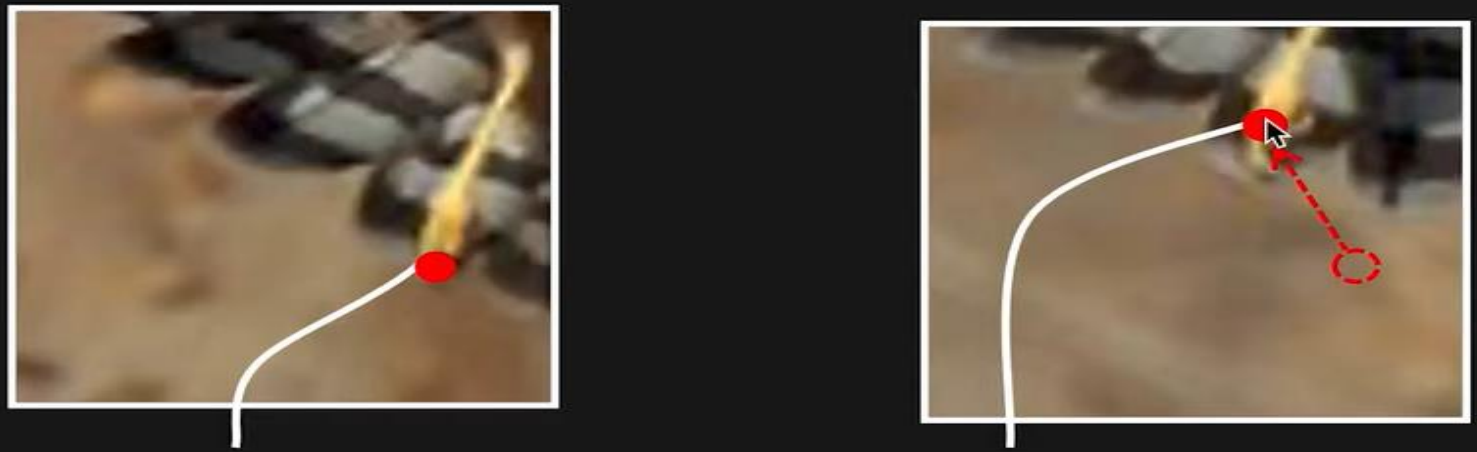
Optical Flow



Displacement: $(\delta x, \delta y)$

Optical Flow: $(u, v) = \left(\frac{\delta x}{\delta t}, \frac{\delta y}{\delta t}\right)$

Optical Flow Constraint Equation



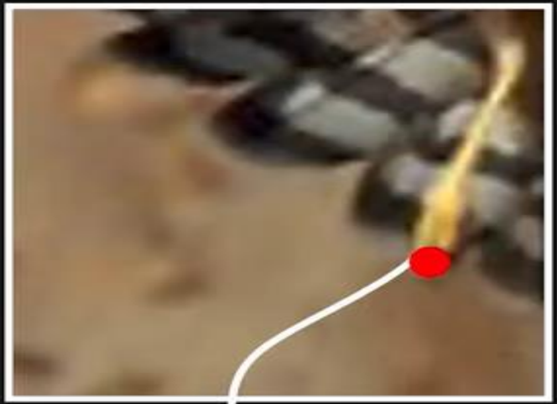
$$I(x, y, t) \qquad\qquad I(x + \delta x, y + \delta y, t + \delta t)$$

Assumption #1:

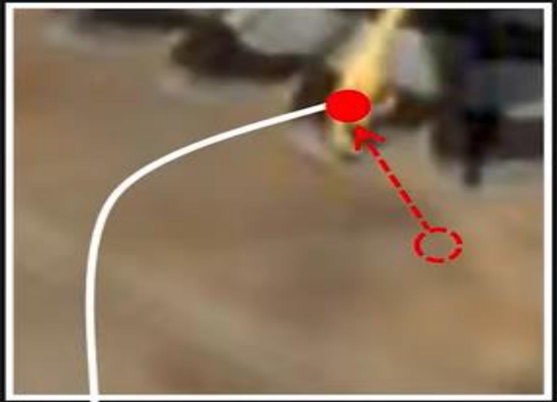
Brightness of image point remains constant over time

$$\boxed{I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t)}$$

Optical Flow Constraint Equation



$I(x, y, t)$



$I(x + \delta x, y + \delta y, t + \delta t)$

Assumption #2:

Displacement $(\delta x, \delta y)$ and time step δt are small

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

$$\boxed{I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x\delta x + I_y\delta y + I_t\delta t}$$

Taylor Series Expansion

Expand a function as an infinite sum of its derivatives

$$f(x+\delta x) = f(x) + \frac{\partial f}{\partial x}\delta x + \frac{\partial^2 f}{\partial x^2}\frac{\delta x^2}{2!} + \cdots + \frac{\partial^n f}{\partial x^n}\frac{\delta x^n}{n!}$$

If δx is small:

$$f(x+\delta x) = f(x) + \frac{\partial f}{\partial x}\delta x + \boxed{O(\delta x^2)} \rightarrow \text{Almost Zero}$$

For a function of three variables with small $\delta x, \delta y, \delta t$:

$$f(x+\delta x, y+\delta y, t+\delta t) \approx f(x,y,t) + \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial t}\delta t$$

Optical Flow Constraint Equation

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) \quad \text{-----------} \; (1)$$

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x \delta x + I_y \delta y + I_t \delta t \quad \text{-----------} \; (2)$$

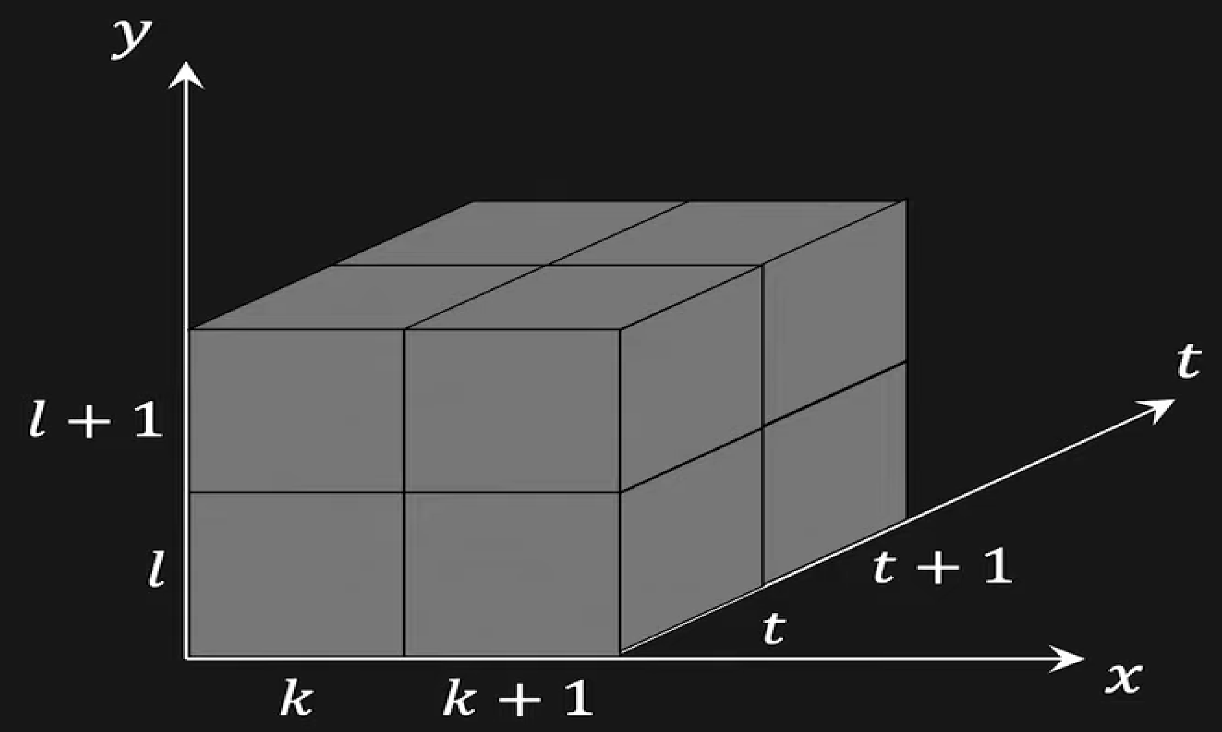
Subtract (1) from (2): $I_x \delta x + I_y \delta y + I_t \delta t = 0$

Divide by δt and take limit as $\delta t \to 0$: $I_x \dfrac{\partial x}{\partial t} + I_y \dfrac{\partial y}{\partial t} + I_t = 0$

Constraint Equation: $\boxed{I_x u + I_y v + I_t = 0}$ (u, v): Optical Flow

(I_x, I_y, I_t) can be easily computed from two frames

Computing Partial Derivatives I_x, I_y, I_t



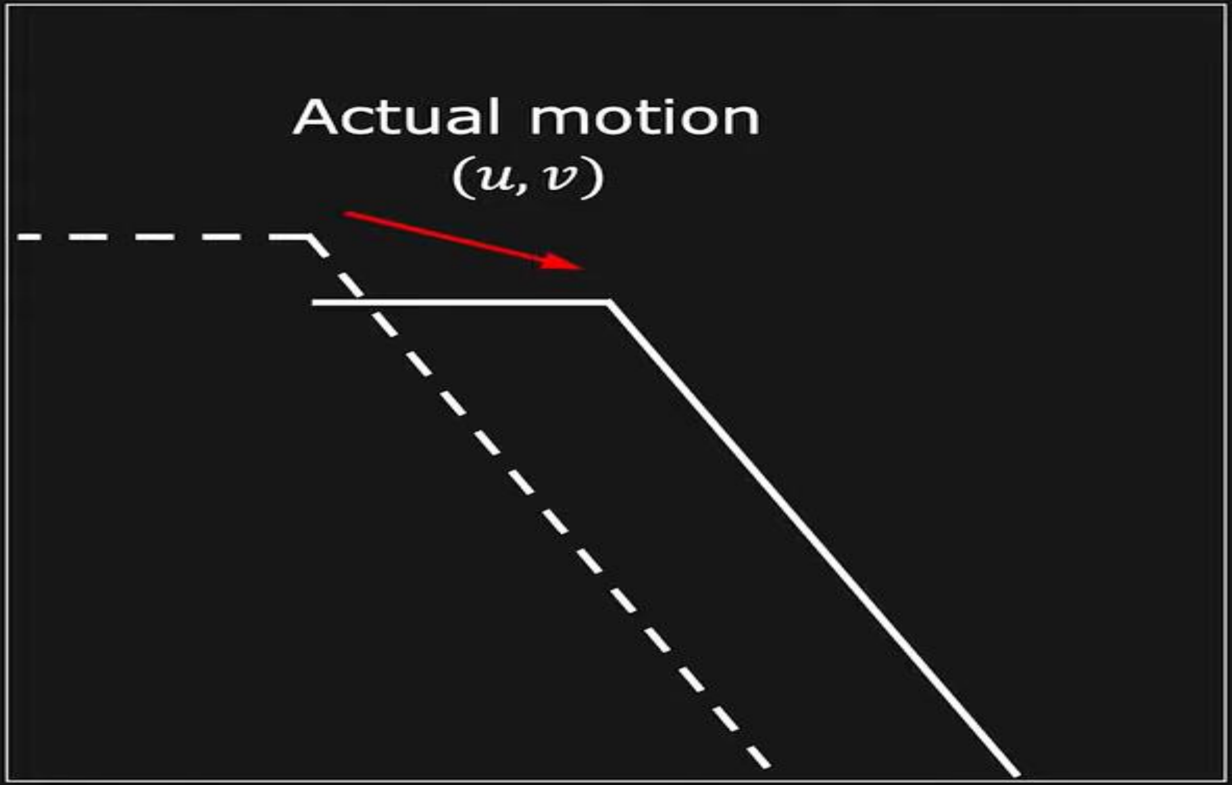
$$I_x(k,l,t) =$$

$$\frac{1}{4}[I(k+1,l,t) + I(k+1,l,t+1) + I(k+1,l+1,t) + I(k+1,l+1,t+1)]$$

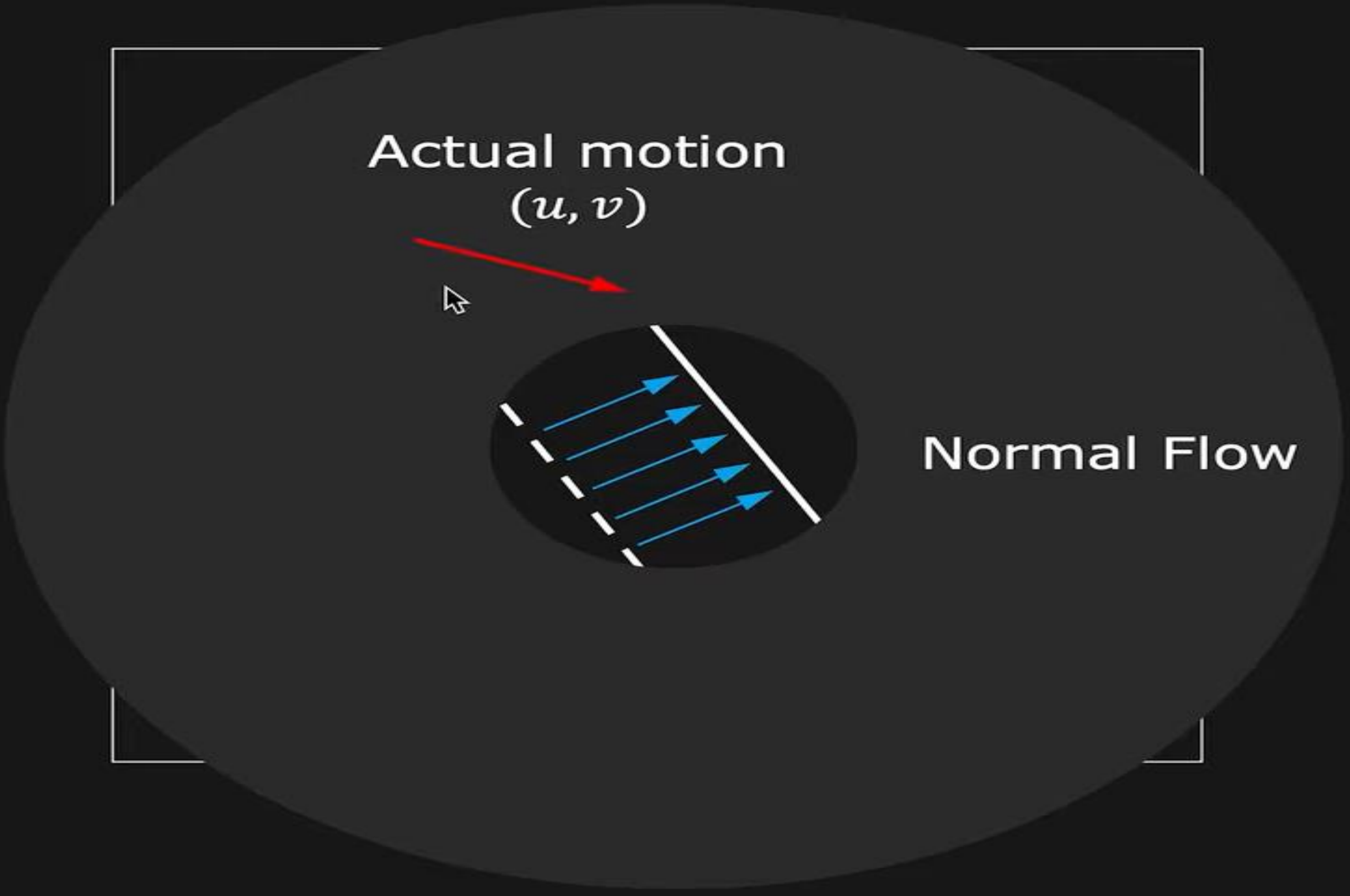
$$-\frac{1}{4}[I(k,l,t) + I(k,l,t+1) + I(k,l+1,t) + I(k,l+1,t+1)]$$

Similarly find $I_y(k,l,t)$ and $I_t(k,l,t)$

Aperture Problem



Aperture Problem



Locally, we can only determine normal flow!

Optical Flow is Under Constrained

Constraint Equation: $\boxed{I_x u + I_y v + I_t = 0}$

2 unknowns, 1 equation.

Lucas-Kanade Solution

Assumption: For each pixel, assume Motion Field, and hence Optical Flow (u, v), is constant within a small neighborhood W.

W

That is for all points $(k, l) \in W$:

$$I_x(k,l)u + I_y(k,l)v + I_t(k,l) = 0$$

Lucas-Kanade Solution

For all points $(k,l) \in W$: $I_x(k,l)u + I_y(k,l)v + I_t(k,l) = 0$

Let the size of window W be $n \times n$

In matrix form:

$$\boxed{\begin{bmatrix} I_x(1,1) & I_y(1,1) \\ I_x(k,l) & I_y(k,l) \\ \vdots & \vdots \\ I_x(n,n) & I_y(n,n) \end{bmatrix}} \boxed{\begin{bmatrix} u \\ v \end{bmatrix}} = \boxed{\begin{bmatrix} I_t(1,1) \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{bmatrix}}$$

A	$\mathbf{u}$	B
(Known)	(Unknown)	(Known)
$n^2 \times 2$	2×1	$n^2 \times 1$

n^2 Equations, 2 Unknowns: Find Least Squares Solution

Least Squares Solution

Solve linear system: $A\mathbf{u} = B$

$$A^T A\mathbf{u} = A^T B$$

(Least-Squares using Pseudo-Inverse)

In matrix form:

$$\underbrace{\boxed{\begin{bmatrix} \sum_W I_x I_x & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y I_y \end{bmatrix}}}_{A^T A} \underbrace{\boxed{\begin{bmatrix} u \\ v \end{bmatrix}}}_{\mathbf{u}} = \underbrace{\boxed{\begin{bmatrix} -\sum_W I_x I_t \\ -\sum_W I_y I_t \end{bmatrix}}}_{A^T B}$$

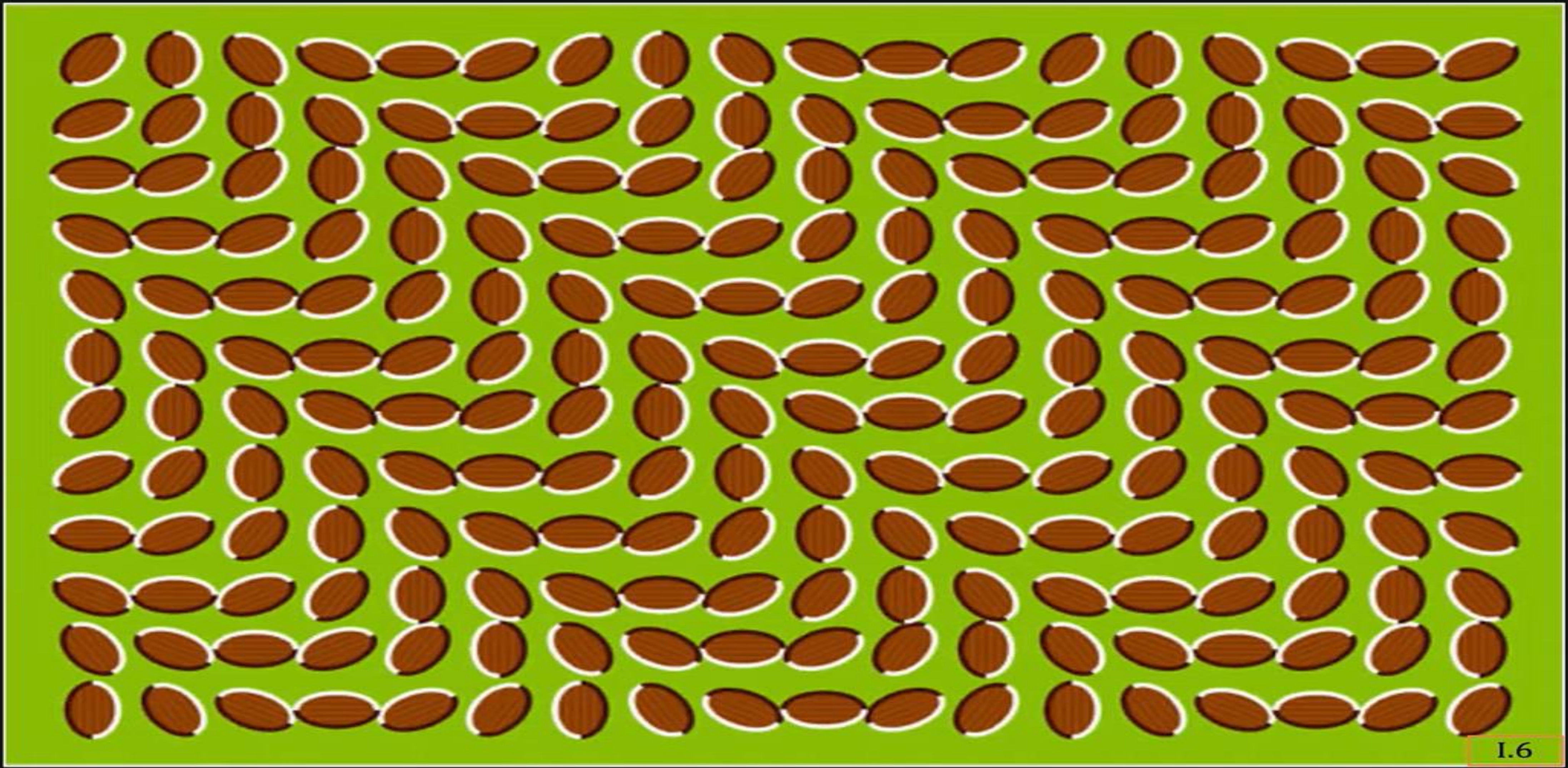
Indices (k, l) not written for simplicity

$A^T A$	$\mathbf{u}$	$A^T B$
(Known)	(Unknown)	(Known)
2×2	2×1	2×1

$$\boxed{\mathbf{u} = (A^T A)^{-1} A^T B}$$

Fast and Easy to Solve

Motion Illusions



Donguri Wave Illusion

Explanation !!

The illusion is created due to the way our eyes and brain process visual information. The circular shapes in the image create an optical illusion called the "Kanizsa triangle," which is a visual phenomenon where our brain perceives a triangle shape even when it is not actually there. In the Donguri wave illusion, the Kanizsa triangles are arranged in a wave pattern, which creates the illusion of movement.