Transformation of Random Variable :

(1) $\underline{x} \sim p(\,)$

(2) $\underline{y} = f(\underline{x})$

what is the distribution of $\underline{y}$?

$$\underline{y} = f(\underline{x}) = \underline{A}\underline{x} + \underline{b}$$

$$\mathbb{E}[\underline{y}] = \mathbb{E}[\underline{A}\underline{x} + \underline{b}] = \underline{A}\underline{\mu} + \underline{b}$$

$$f(\underline{x}) = \underline{a}^T \underline{x} + b \Rightarrow \mathbb{E}[\underline{a}^T \underline{x} + b] = \underline{a}^T \underline{\mu} + b$$

$$\text{cov}[\underline{y}] = \text{cov}[\underline{A}\underline{x} + \underline{b}] = \underline{A} \underline{\Sigma} \underline{A}^T$$

$$\underline{\Sigma} = \text{cov}[\underline{x}]$$

$$\text{var}[\underline{a}^T \underline{x} + b] = \underline{a}^T \underline{\Sigma}\, \underline{a}$$

=

Probability mass fn^n.

$$f(x) = y :$$

$$p_y(y) = \sum_{x : f(x) = y} p_x(x)$$

$f(X) = 1 \quad X = \text{even}$

$f(X) = 0 \quad X = \text{odd}$

$p_x(X)$ is uniform on $\{1, \ldots, 10\}$

$$p_y(1) = \sum_{x \in \{2, 4, \ldots 10\}} p_x(x) = 0.5$$

$$p_y(0) = 0.5$$

$x \longrightarrow y$

$\{1, 2, 3, 4, \ldots, 10\} \xrightarrow{f(\,)} \{0, 1\}$

Prob. density fn^n {use CDF}

$$P_y(y) \triangleq P(Y \le y)$$

$$= P(f(x) \le y)$$

$$= P(x \in \{x \mid f(x) \le y\})$$

=

$$p_y(y) \triangleq \frac{d}{dy} P_y(y)$$

$$p_y(y) = \frac{dx}{dy} p_x(x) \; ; \; x = f^{-1}(y)$$

$$p_y(y) = p_x(x) \left| \frac{dx}{dy} \right|$$

Multivariate change of variable.

$R^n \to R^n$

Jacobian matrix $J = J_{x\to y} \triangleq \dfrac{\partial(y_1,\ldots,y_n)}{\partial(x_1,\ldots,x_n)}$

$$\triangleq \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \\ \frac{\partial y_n}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_n} \end{pmatrix}$$

$|\det J|$

$\underline{y} \to \underline{x}$:

$$p_y(\underline{y}) = p_x(\underline{x}) \left| \det\left(\frac{\partial x}{\partial y}\right) \right| =$$

$$= p_x(\underline{x}) \left| \det J_{y\to x} \right|.$$

=

Central Limit Theorem :

N random variable with pdf $p(x_i)$ mean $= \mu$ & variance $= \sigma$.

all are iid

$$S_N = \sum_{i=1}^{N} X_i$$

$$p(S_N = s) = \frac{1}{\sqrt{2\pi N\sigma^2}} \exp\left(-\frac{(s-N\mu)^2}{2N\sigma^2}\right)$$

$$Z_N \triangleq \frac{S_N - N\mu}{\sigma\sqrt{N}} = \frac{\bar{X}-\mu}{\sigma/\sqrt{N}}$$