① Student t-distribution :

$$T(x \mid \underbrace{\mu}_{\text{mean}}, \underbrace{\sigma^2}_{\text{Scale parameter}}, \underbrace{\nu}_{\text{degree of freedom}}) \propto \left[1 + \frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right]^{-\frac{(\nu+1)}{2}}$$

mean = μ

mode = μ

var = $\dfrac{\nu\sigma^2}{(\nu-2)}$

Heavy tails.

Laplace Distribution

② $$Lap(x \mid \mu, b) \triangleq \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$$

mean = μ

mode = μ

var = $2b^2$

③ Gamma distribution

$$Ga(T \mid shape = a, rate = b) \triangleq \frac{b^a}{\Gamma(a)} T^{a-1} e^{-Tb}$$

$$\Gamma(x) \triangleq \int_0^\infty u^{x-1} e^{-u} \, du$$

mean = a/b ; mode = $\dfrac{a-1}{b}$; var = $\dfrac{a}{b^2}$

④ Beta distribution :

$$Beta(x \mid a, b) = \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}$$

$$B(a,b) \triangleq \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

⑤ Pareto Distribution : long tail / heavy tail

$$Pareto(x \mid k, m) = k m^k x^{-(k+1)} \mathbb{I}(x \geq m)$$

Joint Probability Distribution :

① Covariance b/w X & Y

$$\mathrm{cov}[X, Y] \triangleq \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$$
$$= \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$\mathrm{cov}[\underline{X}]$

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix} = \begin{pmatrix} \mathrm{var}[X_1] & \mathrm{cov}[X_1, X_2] & \cdots & \mathrm{cov}[X_1, X_d] \\ \mathrm{cov}[X_2, X_1] & \mathrm{var}[X_2] & \cdots & \mathrm{cov}[X_2, X_d] \\ \vdots & & & \\ \mathrm{cov}[X_d, X_1] & \mathrm{cov}[X_d, X_2] & \cdots & \mathrm{var}[X_d] \end{pmatrix}$$

Correlation coeff.

$$\mathrm{corr}[X, Y] \triangleq \frac{\mathrm{cov}[X, Y]}{\sqrt{\mathrm{var}[X]\,\mathrm{var}[Y]}}$$

correlation matrix

$$\underline{R} = \begin{pmatrix} \mathrm{corr}[X_1, X_2] & \mathrm{corr}[X_1, X_2] & \cdots & \mathrm{corr}[X_1, X_d] \\ \vdots & \vdots & & \vdots \\ \mathrm{corr}[X_d, X_1] & \mathrm{corr}[X_d, X_2] & \cdots & \mathrm{corr}[X_d, X_d] \end{pmatrix}$$

Multivariate Gaussian :

multivariate normal (MVN)

$$\mathcal{N}(\underline{x} \mid \underline{\mu}, \underline{\Sigma}) \triangleq \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\underline{x} - \underline{\mu})^T \underline{\Sigma}^{-1} (\underline{x} - \underline{\mu})\right]$$

$\underline{\mu} = \mathbb{E}[\underline{x}] \in \mathbb{R}^D$

$\underline{\Sigma} = \mathrm{cov}[\underline{x}]$ is $D \times D$

$\underline{\Lambda} = \underline{\Sigma}^{-1}$